Aggregate R&D Expenditure

and Endogenous Economic Growth*

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Resumen

El objetivo de este trabajo es analizar teórica y empíricamente el papel que juega el gasto agregado en actividades de I+D en el crecimiento económico. Para ello, se propone una tecnología de innovación definida en unidades de gasto. De este modo se obtiene crecimiento endógeno sostenido aunque no exista crecimiento de la población. Esto nos permite analizar el efecto de ciertas políticas fiscales. Para la estimación econométrica utilizaremos una especificación obtenida directamente del modelo teórico. Más en concreto, dicha especificación se obtiene a partir de la condición de libre entrada del sector de innovacion y de la solución de la ecuacion dinámica que describe la evolución del precio de las patentes.

Abstract

The aim of this paper is to theoretically and empirically analyze the role that aggregate R&D-expenditures play in economic growth. We introduce a technology of innovation based on R&D-expenditures instead of labor to see how this consideration generates sustainable growth determined endogenously, even if population growth does not exist. Therefore, it seems relevant to analyze the effects of different fiscal policies. For the empirical analysis we make use of an econometric model obtained from the decentralized equilibrium. More precisely, the specification is obtained using the free-entry condition that the competitive equilibrium states for the R&D-activity and the policy function defining the dynamic evolution of patentees' price.

1. Introduction.

This paper presents an R&D-based model of economic growth in the line of those proposed by Romer (1990), Grossman-Helpman (1991) and Aghion-Howitt (1992). In this type of models the research and development efforts made by profit-maximizing agents are the basis for technological advance, which underlies sustained growth. Moreover, these models adopt some kind of monopolistic power to generate a surplus that can be assigned to the innovation activity. In particular, this activity creates non-rival ideas that are used in the form of patents to produce new capital goods by monopolistic firms. Finally, these goods will be complementary inputs in the production of consumption goods. In this way, the introduction of new capital goods does not reduce the marginal productivity of the pre-existing ones, and so the innovation activity generates perpetual growth through the increase in the number of capital goods.

The aim of the paper is to theoretically and empirically analyze the role that aggregate R&D-expenditures play in the growth of per capita income. Generally, the R&D-based growth models consider an R&D technology that uses (skilled) labor as the unique input, so that the quantity and the quality of labor determines the production of R&D, and therefore the growth rate of the economy. We instead are interested in analyzing the effects of expenditures in this activity, i.e., we investigate whether the income that individuals devoted to finance the innovation activity translates into a growth of per capita income. Actually, the innovation process is intensive in human capital. However, we are interested in analyzing the private choice of investing income in this process. The economy must channel the individuals' savings to finance the R&D-activity. The question is what are the determinants of this decision. Therefore, by R&D-expenditure we understand the income devoted to finance the total cost of the R&D-activity, i.e., wages, infrastructure, capital and so on used in the innovative process.

In the present model, the endogenous accumulation of wealth is the source of perpetual growth of per capita income. The fraction of savings that individuals allocate to finance the innovation process generates some number of intermediate capital goods. These capital goods finally increase the production of consumption goods and the profits obtained by the monopolistic firms producing the capital goods. This increase of profits is absorbed by the innovative firms through the patents, so that the return by unit of R&D-expenditure also increases. Therefore, the increase in monopoly profits encourages investment in the R&D-activity, and so in the R&D-expenditures as well. In other words, our model exhibits endogenous and sustained growth, and R&D-expenditure together with monopoly power are the key to this growth.

Moreover, following Jones (1995), we adopt a specification for the R&D-technology that incorporates two kinds of externalities. First, the productivity of the R&D-expenditure depends on the level of ideas discovered in the economy. The second externality considered in the paper comes from the aggregate R&D-expenditure. The latter externality makes the assumption that the marginal productivity of the aggregate R&D-expenditure decreases when this expenditure grows. This means that the higher the aggregate R&D-expenditure is, the larger the number of firms developing the innovation activity will be, and so the smaller the probability that each firm has of discovering new ideas will be as well.

This R&D-technological specification also permits us to avoid the problem of the scale effects common in the R&D-based models of economic growth. If we assume a growth rate of ideas that is linear in the R&D-expenditures, the growth rate of per capita income increases in the same proportion as the level of resources devoted to the R&D-activity. The latter assumption, which is used in most of the literature to generate sustained growth, is hard to reconcile with the empirical evidence. The R&D-expenditures of the developed countries have grown drastically in the last three decades, but their growth rates have been roughly constant. Therefore, we assume that the growth rate of ideas in the total economy is strictly concave in the aggregate R&D-expenditure and inversely related to the aggregate stock of ideas. However, the growth rate of ideas in a particular firm is linear in its own R&D-expenditure.

In this paper, we analytically characterize the steady-state equilibrium and the dynamic behavior of the economy. Thus, we state the structural or parameter conditions that guarantee the existence, uniqueness and stability of equilibrium, together with the conditions under which the endogenous saving decisions generate sustained growth of income per capita. In this sense, it seems relevant to analyze the effects of different fiscal policies on the long-run growth rate of per capita income. In particular, we must investigate if the fiscal policies directly affecting R&D-expenditure decisions are not neutral for growth. We must theoretically contrast if a tax on wealth reduces the long-run growth rate or if a tax-credit to the physical capital investment and a tax-credit to R&D-expenditure increase this rate.

Another goal of the paper is to analyze how R&D-expenditure affects productivity growth using cross-country data. To that purpose, we make use of an econometric model derived from the theoretical one. In particular, we estimate an equation obtained from the solution of the dynamic system that describes the behavior of the economy. This equation expresses the growth of income as a function of the growth rates of human capital, physical capital and R&D-expenditure, and some temporal and fixed components. Moreover, this empirical work permits us to estimate the parameter of the production technologies and R&D-activity.

As Busom (1994) points out, there have been many studies that have estimated the impact of R&D-investment on productivity at the firm and industry level. However, there are not enough papers that contrast the evidence of those endogenous models, analyzing the effect of R&D using international data. These previous cross-country empirical studies are based either on "ad-hoc" equations about the Total Factor Productivity [see Coe and Helpman (1995), Coe, Helpman and Hoffmaister (1995), Engelbrecht (1997)] or on convergence equations approximately similar to the one developed by Mankiw, Romer and Weil (1992), [see, e.g., Lichtenberg (1992), de la Fuente (1995)]. In the present paper we estimate an econometric specification that is not a convergence equation. Furthermore, the econometric model is obtained from the decentralized equilibrium derived from the optimizing behavior of consumers and the profit maximization of firms. Thus, one contribution of our analysis is that we directly derive a structural econometric model from an R&D endogenous growth model.

The paper is organized as follows. In Section 2 we present the model. The market equilibrium is defined in Section 3. Section 4 characterizes the balanced growth equilibria, whereas Section 4 analyzes the local stability of these equilibria. The effects of fiscal policy on the long-run growth rate are analyzed in Section 6. In Section 7 we build an econometric model which relates the growth rate of per capita income with the level and the growth rate of R&D-expenditures. In Section 8 we present and discuss the empirical results obtained from the estimation of the previous model. We present the summary of the main findings of the paper in Section 9.

2. The Model.

We consider an R&D-based growth model with an infinite horizon and continuous time. The economy consists of five types of economic agents: the producers of the final good, the discoverers of new ideas, the intermediate firms, the consumers, and finally, the Government. We now present the behavior of each of these five types of agents in more detail.

2.1. Final good sector.

Consider a Cobb-Douglas production function that in each moment of time is represented by:

$$Y_t = H_t^{\beta} L_t^{1-\alpha-\beta} \int_0^{N_t} X_{it}^{\alpha} di, \qquad (2.1)$$

where Y_t is the final output in period t, and the production factors are H_t, L_t, X_{it} which represent the human capital level, the labor force and the total variety of intermediate goods.

Assume that the stock of human capital and the labor force grow at exogenous and constant rates, η and n respectively. That is,

$$\frac{\dot{H}}{H} = \eta \quad \text{and} \quad \frac{\dot{L}}{L} = n. \tag{2.2}$$

A competitive firm solves the following maximization problem:

$$\max_{\{H_t, L_t, X_t\}} \quad \bar{P}_{Yt} Y_t - \bar{w}_{Ht} H_t - \bar{w}_{Lt} L_t - \int_0^{N_t} \bar{P}_{it} X_{it} di,$$

where $\bar{P}_Y$ is the price of final goods, $\bar{w}$ represents the wage rate and $\bar{P}_i$ is the rental price of producer durable i. The first order conditions imply the following conditional demand functions:

$$\beta \bar{P}_{Yt} Y_t H_t^{-1} = \bar{w}_{Ht} \tag{2.3a}$$

$$(1 - \alpha - \beta) \bar{P}_{Yt} Y_t L_t^{-1} = \bar{w}_{Lt} \tag{2.3b}$$

$$\alpha \bar{P}_{Yt} H_t^{\beta} L_t^{1-\alpha-\beta} X_{it}^{\alpha-1} = \bar{P}_{it}, \qquad \text{for all } i = 0 \ldots . \mathrm{N}_t \tag{2.3c}$$

Let's take $\bar{P}_Y$ as the numeraire. Therefore, we can normalize all prices by $\bar{P}_Y$, thus the relative price of the intermediate goods, p_{it}, is expressed by

$$\alpha H_t^{\beta} L_t^{1-\alpha-\beta} X_{it}^{\alpha-1} = p_{it} \quad \text{for all } i = 0 \ldots . \mathrm{N}_t, \tag{2.4}$$

where the demand elasticity of the capital goods is $1/(1-\alpha)$, and the demand function is

$$X_{it} = \left(\frac{\alpha H_t^{\beta} L_t^{1-\alpha-\beta}}{p_{it}} \right)^{1/1-\alpha}. \tag{2.4´}$$

2.2. R&D sector.

This sector consists of a large number of equal and competitive firms. They take the savings of consumers to finance their research projects. With these projects, each of these firms attempts to discover new ideas, which will adopt the form of designs for new capital goods. The variation in the number of designs achieved by each firm is then given by the following expression:

$$\dot{N}_{it} = \xi R_{it}, \tag{2.5}$$

where N_{it} denotes the stock of ideas or designs achieved by firm i, R_{it} represents the investment of firm i, i.e., the individual research expenditure, and ξ is the productivity parameter of the R&D-activity. This productivity parameter follows a Poisson distribution. It measures how many designs the firm can obtain for one unit of the R&D-expenditure. This parameter depends on the aggregate R&D-expenditure R_t and the aggregate stock of designs N_t. On the one hand, the aggregate R&D-expenditure encourages the R&D effort, and so too the entry of firms in the R&D sector. In this way, the aggregate R&D-expenditure increases the competition for discovering new ideas, so that the number of designs obtained for each unit of expenditure decreases. On the other hand, the stock of ideas increases the productivity of the R&D activity. Hence, the productivity parameter ξ can be parameterized as

$$\xi = \gamma R_t^{\lambda - 1} N_t^{\varphi}, \tag{2.6}$$

where λ and φ belong to $(0,1)$.

In the aggregate, the variation on the total number of designs is then given by the following expression:

$$\dot{N} = \gamma R_t^{\lambda} N_t^{\varphi}. \tag{2.7}$$

From now on, we will assume that the R&D process exhibits constant returns to scale at the aggregate level, i.e., $\lambda + \varphi = 1$.

Before closing this subsection, we must note that the stock of ideas or designs is now determined by intentional R&D-expenditure made by the households. Consumers are owners of the R&D firms. After having invested R_t units of income, they are the owners of the new design that this investment produces. Thus, the return of this investment will be the flows that the designs will yield. Each design is sold at a price P_{Dt} to an intermediate or capital-goods producer. Therefore the free-entry condition can be stated as follows:

$$R_t(1 - s_R) = P_{Dt}\dot{N}, \tag{2.8}$$

where s_R is the rate at which the government subsidizes investment in research.

2.3. Intermediate goods sector.

The intermediate sector is composed of an infinite number of firms on the interval $[0, N_t]$ that have purchased a design from the R&D sector. These firms transform a part of the consumer's savings into physical capital. This sector produces the durable goods that are available to be used in final goods production at any time. Consider the simplest production function:

$$X_{it} = S_{it}, \tag{2.9}$$

where S_{it} represents savings, which are rented at a rate r_t, driven to the production of the intermediate good i at time t. We are assuming that the capital is putty-putty, so that the firm transforms units of durable goods back into general capital.

These goods X_i are everlasting and do not depreciate. Firm i is the only seller of capital good i, that is, each capital good producer is a local monopoly. Therefore, the monopoly price is a simple markup over marginal cost, determined by the positive elasticity of demand $1/(1-\alpha)$, according to (2.4). This means:

$$p_{it} = r_t/\alpha \,. \tag{2.10}$$

Government subsidizes the physical capital accumulation reducing the production cost of these intermediate goods by s_k. Note that a subsidy to physical capital accumulation might be imposed directly on consumers as subsidy on investment or on producers as subsidy on investment cost. Since the price p_{it} is the same for all firms, and the output in equilibrium is also equal for each different good, the net profits obtained for each monopolist are identical and given by:

$$\Pi_{it} = p_{it}X_{it} - r_{it}S_{it}(1-s_k) = X_t\left(r_t/\alpha - r_t(1-s_k)\right) = r_t X_t\left(\frac{1-\alpha(1-s_k)}{\alpha}\right) = \Pi_t \,. \tag{2.11}$$

The decision to produce a new capital good also depends on a comparison between the price that the firms must pay for the use of the design, P_{Dt}, and the discounted stream of net revenue. Because the market for designs is competitive, their price will be bid down until it is equal to the present value of the monopolist's net profit. Therefore, at every moment in time the following must hold:

$$P_{Dt} = \int_t^{\infty} e^{-\int_t^{\tau} r_s ds}\,\Pi_\tau \, d\tau \,. \tag{2.12}$$

Because of symmetry, each firm sets the same price and sells the same quantity of its producer durable goods. Therefore we can express the total physical capital stock of economy K_t by:

$$K_t = N_t X_t \,. \tag{2.13}$$

2.4. Households.

We consider a representative household which has time-separable preferences with a constant subjective rate of time preferences, ρ, and an instantaneous utility function

$$U(C) = \left(C^{1-\sigma} - 1\right)\big/(1-\sigma)\,, \tag{2.14}$$

where $\sigma > 0$ denotes the inverse of the constant inter-temporal elasticity of substitution.

The consumer at each time t is burdened not only with a lump-sum tax T_t but also with an income tax τ. Therefore, the household is subject to the following temporal sequence of instantaneous budget constraints:

$$\dot{B} = (r_t B_t + w_t)(1-\tau) - C_t + T_t, \tag{2.15}$$

where B_t represents its stock of wealth

$$B_t = K_t + P_{Dt} N_t. \tag{2.16}$$

The households distribute their assets between both types of investment. That is, they invest in the production of intermediate goods which means they accumulate new physical capital, and invest in research activities. The share decision is made as a function of the arbitrage conditions which the equilibrium market states.

Thus, given $K(0) = K_0$, and $N(0) = N_0$, and imposing the non-negative constraints $C(t) \geq 0$, $R(t) \geq 0$, $K(t) \geq 0$ and $N(t) \geq 0$, the standard dynamic optimization problem with control variable C, and state variable B, is expressed by the corresponding Hamiltonian function:

$$H(C_t, B_t, \mu_t) = e^{-\rho t}\left\{\frac{C_t^{1-\sigma}-1}{1-\sigma} + \mu_t\left((r_t B_t + w_t)(1-\tau) - C_t + T_t\right)\right\}. \tag{2.17}$$

Therefore, the optimal plan for a household is a set of paths $\{C_t, S_t, R_t, K_t, N_t\}$ that satisfy the following necessary conditions:

$$C_t^{-\sigma} = \mu_t, \tag{2.18}$$

$$\dot{\mu}_t - \rho\mu_t = -\mu_t r_t (1-\tau), \tag{2.19}$$

and the transversality condition which ensures that the discounted value of the utility function is bounded:

$$\lim_{t \to \infty} e^{-\rho t} U_t(C) = 0. \tag{2.20}$$

2.5. Government.

The behavior of the government is very simple. It neither consumes nor issues interest-bearing bonds. Its balanced budget constraint at each moment in time is given by:

$$T_t = \tau(r_t B_t + w_t) - s_k N_t X_t - s_R R_t \tag{2.21}$$

It is assumed that the income tax parameter and both subsidy rates are kept constant over time, and only marginal variations in discrete moments in time can be introduced. Hence, the lump-sum tax T_t is the adjusting parameter of the government's budget constraint.

3. The Market Equilibrium.

From now on, and to analyze the dynamics of the system, we will suppose that human capital and the labor force are fixed and equal to one. We adopt this assumption for simplicity.

Substituting $p_{it} = r_t / \alpha$ in the expression (2.4), we compute the value of r_t which the intermediate goods firms must to pay to the consumers:

$$r_t = \alpha^2 K_t^{\alpha-1} N_t^{1-\alpha} . \tag{3.1}$$

Moreover, remember that equilibrium condition (2.8) implies that

$$R_t = N_t \left(\gamma P_{Dt} / (1 - s_R)\right)^{1/1-\lambda} . \tag{3.2}$$

Thus, differentiating equation (2.12) with respect to time we obtain:

$$\dot{P}_D = P_{Dt} r_t - \Pi_t . \tag{3.3}$$

Using (2.11) the monopoly profits can be substituted by their expression. Thus, we can find the dynamic evolution of the patent's price, which also represents an arbitrage condition that relates the returns of the two types of investments:

$$\dot{P}_D = P_{Dt} r_t - r_t X_t \left(\frac{1 - \alpha(1 - s_k)}{\alpha} \right). \tag{3.4}$$

From (2.13), the aggregate production function (2.1) can be transformed into

$$Y_t = H_t^{\beta} L_t^{1-\alpha-\beta} K_t^{\alpha} N_t^{1-\alpha} . \tag{3.5}$$

The savings assigned to the intermediate sector is totally transformed into an increase of the capital stock, that is $\dot{K} = S$, because that depreciation does not exist here. Thus the law of motion for capital stock is given by

$$\dot{K} = Y_t - C_t - R_t , \tag{3.6}$$

where C_t is the total consumption of the households.[1]

[1] If we consider a positive rate of depreciation, savings should be used not only to produce new types of

Therefore according to equilibrium conditions (3.1) and (3.2), and the dynamical equations (3.4) and (3.6), jointly with the first order condition (2.16), the dynamic system that defines the equilibrium paths of the economy is composed of four differential equations that describe the optimal behavior of the physical capital stock, the number of designs, the consumption and the patent's price, respectively. In other words, given the initial endowments of both designs and physical capital, the market equilibrium of the economy is defined by the following system of differential equations:

$$\frac{\dot{K}}{K} = K_t^{\alpha-1} N_t^{1-\alpha} - C_t / K_t - (N_t / K_t)\left(\gamma P_{Dt} / (1 - s_R)\right)^{1/1-\lambda}, \tag{3.7a}$$

$$\frac{\dot{N}}{N} = \gamma\left(\gamma P_{Dt} / (1 - s_R)\right)^{1/1-\lambda}, \tag{3.7b}$$

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left((1-\tau)\alpha^2 K_t^{\alpha-1} N_t^{1-\alpha} - \rho\right), \tag{3.7c}$$

$$\frac{\dot{P}_D}{P_D} = \alpha^2 K_t^{\alpha-1} N_t^{1-\alpha} - \alpha\left(1 - \alpha(1 - s_k)\right) K_t^{\alpha} N_t^{-\alpha} P_{Dt}^{-1}. \tag{3.7d}$$

The dynamic system (3.17) reaches the steady state when N, K and C grow at a constant rate, and P_D is constant over time. Thus, to simplify the study of the transitional dynamics, we transform the previous system (3.7) into one that presents a stationary equilibrium. Thus, we consider the following two ratios: K/N and C/K, that will not grow in the steady state. In order to check that, we first rewrite (3.7a) as follows:

$$\rho + \sigma\frac{\dot{C}}{C} = \alpha^2 K_t^{\alpha-1} N_t^{1-\alpha}. \tag{3.8}$$

In the balanced growth path the consumption grows at a constant rate. Therefore, the expression (3.8) only holds in the balanced growth path when the stock of capital and the number of designs grow at the same rate.

On the other hand, from (3.2), (3.5), and using the previous conclusion, we can show that R, Y and K also grow at the same rate in the balanced growth path. Thus, rewriting the household's budget constraint as $Y/K = \dot{K}/K + C/K + R/K$, we can conclude that the ratio C/K is constant in the steady state.

Therefore, defining $X = K/N$, $Z = C/K$ the dynamic behavior of the economy can also be represented by the following reduced dynamic system in X, Z and P_D

$$\dot{X} = X^{\alpha} - ZX - \left(\gamma P_D / (1 - s_R)\right)^{(1/1-\lambda)} - \gamma^{1/1-\lambda}\left(P_D / (1 - s_R)\right)^{(\lambda/1-\lambda)} X, \tag{3.9a}$$

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capital goods, but also to replace the depreciation. However, the results of this paper would still remain valid.

$$\dot{Z} = \frac{Z}{\sigma}\left(\alpha^2 X^{(\alpha-1)}(1-\tau)-\rho\right) - ZX^{(\alpha-1)} + Z^2 + \left(\gamma\, P_D/(1-s_R)\right)^{1/1-\lambda} ZX^{-1}, \tag{3.9b}$$

$$\dot{P}_D = \alpha^2 X^{(\alpha-1)} P_D - \alpha\left(1-\alpha(1-s_k)\right) X^{\alpha}. \tag{3.9c}$$

Since the initial value of X is totally defined by the initial stocks of physical capital and the number of designs, given these initial stocks the dynamic system (3.9) and the transversality condition (2.20) define the market equilibrium paths. They completely characterize both the transitional dynamics and the steady-state equilibrium of the economy.

4. The Balanced Growth Path.

The long run equilibrium of this economy is given by Balance Growth Paths along which, as we saw above, N, K, C, R, and Y grow at a constant and equal rate, and P_D is constant over time. In terms of the reduced system, this means that X, Z, P_D stay constant over time. Therefore, the steady state ($\bar{X}, \bar{Z}, \bar{P}_D$) can be computed by equating the growth rates of these three variables to zero, from which the following system of equations is obtained:

$$\bar{X}^{\alpha-1} - \bar{Z} - \left(\gamma\bar{P}_D/(1-s_R)\right)^{(1/1-\lambda)} \bar{X}^{-1} - \gamma\left(\gamma\bar{P}_D/(1-s_R)\right)^{(\lambda/1-\lambda)} = 0, \tag{4.1}$$

$$\frac{1}{\sigma}\left(\alpha^2\bar{X}^{(\alpha-1)}(1-\tau)-\rho\right) - \bar{X}^{(\alpha-1)} + \bar{Z} + \left(\gamma\bar{P}_D/(1-s_R)\right)^{(1/1-\lambda)}\bar{X}^{-1} = 0, \tag{4.2}$$

$$\alpha^2\bar{X}^{(\alpha-1)} - \alpha\left(1-\alpha(1-s_k)\right)\bar{X}^{\alpha}\bar{P}_D^{-1} = 0. \tag{4.3}$$

It is difficult to calculate the analytical solution of this system of equations. However, we can still analyze the properties of it. The following result proves the existence and the uniqueness of the steady-state equilibrium.

PROPOSITION 4.1. *(i) If $\sigma \geq 1$, there always exists a unique balanced growth path. (ii) When $\sigma \in (0,1)$, a balanced growth path exists if the following condition holds:*

$$\frac{\rho}{\sigma(1-\sigma)}\left(\frac{1-\alpha(1-s_k)}{(1-\alpha)}\right)^{(1-\alpha)} - \gamma^{1/1-\lambda}\left(\frac{(1-\alpha)}{\alpha(1-s_R)}\left(\frac{\alpha^2(1-\sigma)(1-\tau)}{\rho}\right)^{1/(1-\alpha)}\right)^{(\lambda/1-\lambda)} - \frac{\rho}{\sigma} > 0.$$

Otherwise, no balanced growth path exists.

Proof: See Appendix A

Note that in this model there is endogenously determined sustainable growth, even if there is no population growth.[2] From (3.7b) we can affirm that the growth rate in the steady-state equilibrium is given by the following expression:

$$g^* = \gamma\left(\frac{\gamma\,\bar{P}_D}{(1-s_R)}\right)^{\frac{1}{1-\lambda}}, \tag{4.4}$$

which is directly obtained from equation (3.7b).

5. Stability of the Balanced Growth Path.

To examine the stability of the balanced equilibrium path, we set the fiscal parameters equal to zero. Let us define $x=\ln(K/N)$, $z=\ln(C/K)$ and $P=\ln(P_D)$, then the dynamic behavior of the economy can also be represented by the transformed dynamic system in x, z and P:

$$\dot{x} = e^{(\alpha-1)x} - e^{z} - \gamma^{\frac{1}{1-\lambda}} e^{-x+P\left(\frac{1}{1-\lambda}\right)} - \gamma^{\frac{1}{1-\lambda}} e^{P\left(\lambda/1-\lambda\right)} \equiv \Psi(x,z,P), \tag{5.1a}$$

$$\dot{z} = \frac{1}{\sigma}\left(\alpha^{2} e^{(\alpha-1)x} - \rho\right) - e^{(\alpha-1)x} + e^{z} + \gamma^{\frac{1}{1-\lambda}} e^{-x+P\left(\frac{1}{1-\lambda}\right)} \equiv \Omega(x,z,P), \tag{5.1b}$$

$$\dot{P} = \alpha^{2} e^{(\alpha-1)x} - \alpha(1-\alpha)e^{\alpha x-P} \equiv \Phi(x,z,P). \tag{5.1c}$$

Therefore, to study the local stability of the Balanced Growth Path, we will use the log-linearization of the three dimensional dynamic system around the steady state equilibrium:

$$\begin{pmatrix}\dot{x}\\ \dot{z}\\ \dot{P}\end{pmatrix} = \begin{pmatrix}\Psi_{\bar{x}} & \Psi_{\bar{z}} & \Psi_{\bar{P}}\\ \Omega_{\bar{x}} & \Omega_{\bar{z}} & \Omega_{\bar{P}}\\ \Phi_{\bar{x}} & \Phi_{\bar{z}} & \Phi_{\bar{P}}\end{pmatrix}\begin{pmatrix}x-\bar{x}\\ z-\bar{z}\\ P-\bar{P}\end{pmatrix}. \tag{5.2}$$

The elements of the coefficient matrix are the partial derivatives of $(\dot{x},\dot{z},\dot{P})$ with respect to x, z and P evaluated in the steady state. The sign of the eigenvalues of this matrix determines the dynamics around the steady state. The following result characterizes the convergence of the economy to its steady-state equilibrium.

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[2] In Jones' model sustainable growth only exists if the rate of population growth is positive. This model has been dubbed the "semi-endogenous" growth model for this reason because economic policies do not affect the long run growth rate.

PROPOSITION 5.1 *The coefficient matrix of (5.2) evaluated at the unique balanced growth path has two eigenvalues with positive real parts and one eigenvalue with a negative real part. Hence, the balanced growth equilibrium is locally saddle-path stable.*

Proof: See appendix B.

6. The Fiscal Policy.

The introduction of a technology of innovation based on R&D-expenditure instead of labor generates endogenous growth. For this reason this model permits us to analyze the effect of the introduction of some fiscal policy. Specifically we will study the long run impacts caused by a marginal variation of subsidies to R&D-expenditure, subsidies to investment and the modification of the income tax. To that purpose, we suppose that the economy is initially on the balanced growth path, and suddenly the government decides to implement an unanticipated, permanent, marginal increase in one of the fiscal parameters. We will analyze the impacts on the steady state equilibrium using comparative static arguments.

PROPOSITION 6.1. *Consider the system (4.1) to (4,3) defining the steady-state equilibrium (A3), and the associated growth rate (4.4). Then*

(i) The long run effects of a marginal change in the subsidy to R&D-activity are:

$$\frac{d\overline{X}}{ds_R} < 0, \qquad \frac{d\overline{P}_D}{ds_R} < 0, \qquad \frac{d\overline{Z}}{ds_R} \gtrless 0, \qquad \frac{dg^*}{ds_R} > 0.$$

(ii) The long run effects of a marginal change in the subsidy to physical capital investment are:

$$\frac{d\overline{X}}{ds_K} < 0, \qquad \frac{d\overline{P}_D}{ds_K} > 0, \qquad \frac{d\overline{Z}}{ds_K} \gtrless 0, \qquad \frac{dg^*}{ds_K} > 0.$$

(iii) The long run effects of a marginal change in the income tax are:

$$\frac{d\overline{X}}{d\tau} < 0, \qquad \frac{d\overline{P}_D}{d\tau} < 0, \qquad \frac{d\overline{Z}}{d\tau} \gtrless 0, \qquad \frac{dg^*}{d\tau} < 0.$$

Proof: See Appendix C.

Property 6.1 confirms the growth effects of fiscal policy in our model. In particular we observe that τ has a negative effect on g^*, whereas s_k and s_R have a positive effect. Note that the government can stimulate long run growth either directly through a subsidy to R&D-expenditure, or indirectly by means of a subsidy to physical capital accumulation.

The growth effects of τ and s_R are quite trivial, however it seems necessary to add some comments about the effects of s_k. The subsidy to physical capital accumulation reduces the production cost of the intermediate capital goods. Hence, the net profits obtained by monopolistic firms producing each good increase. This can induce two opposite, in some sense, type of behaviors from intermediate firms. They can increase either the quantity produced of the existing capital good or the demand of new designs. Since in this kind of models the intermediate capital goods are complementary in the final production, increase the variety of these goods is better for growth than increase the quantity produced of the existing ones. Therefore, the subsidy to physical capital has positive effect on the demand of designs, which translates into an increase in the patentee's price, and then, in a rise in the R&D effort. Observe that this subsidy may also increase the production of the actual capital goods, however the result states that, even in this case, the growth in the variety of capital goods is larger than the increase in quantity produced of each capital good. Furthermore, we can show that there exists a relationship between the growth effects of both subsidies, and these effects depend on the size of those subsidies.

PROPOSITION 6.2. *Let* $s_R > 0$ *and* $s_k > 0$. *Consider a marginal change in both* s_R *and* s_k. *Then,*

$$\frac{dg^*}{ds_R} \gtreqless \frac{dg^*}{ds_k} \text{ if } s_R \gtreqless \frac{2\alpha - 1}{\alpha} - s_k .$$

Proof: See Appendix C.

Notice that for the traditional estimated value $\alpha \approx 0.36$, the inequality $s_R > \left((2\alpha - 1)/\alpha\right) - s_k$ holds. This fact can easily be shown with the following corollary.

COROLLARY 6.3. *There exists an* $\bar{\alpha} = 1/\left(2 - s_k\right)$, *such that for all values of* α *smaller than* $\bar{\alpha}$, $dg^*/ds_R > dg^*/ds_k$.

Finally, from the above results, the following states a negative impact that the taxes on both types of investment have on economic growth.

COROLLARY 6.4. *Let* τ_k *and* τ_R *be the rates of taxes on physical capital income and on the return of R&D activity, respectively. Then, the following holds:*

$$\frac{dg^*}{d\tau_k} < 0, \qquad \frac{dg^*}{d\tau_R} < 0.$$

Proof. The introduction of taxes on the return of R&D activity and physical capital income would imply that the free condition (2.8) and the monopolistic profits would be substituted by $R_t(1-s_R) = P_{Dt}\dot{N}(1-\tau_R)$ and $\Pi_{it} = p_{it}X_{it} - r_{it}S_{it}(1+\tau_k - s_k)$, respectively. Thus, the taxes would offset the subsidies to R&D-expenditures and physical capital accumulation if the tax rates were equal to the subsidy rates. QED

7. The Econometric Model.

In this section we use this theoretical benchmark to analyze the empirical evidence about the sources of growth, specially to find an econometric specification that allows us to empirically test the role that the aggregate R&D-expenditure plays in these endogenous growth models. To do that we will not consider the existence of fiscal policies.

Running a Cobb-Douglas production function expressed in differences is a traditional way to do this. In our model the production function can be expressed by:

$$Y_t = F\left(H_t, L_t, K_t, N_t\right) = H_t^{\beta} L_t^{1-\alpha-\beta} K_t^{\alpha} N_t^{\eta}, \tag{7.1}$$

then, expressed in growth rates:

$$\frac{\dot{Y}}{Y} = \beta\frac{\dot{H}}{H} + (1-\alpha-\beta)\frac{\dot{L}}{L} + \alpha\frac{\dot{K}}{K} + \eta\frac{\dot{N}}{N}. \tag{7.2}$$

However, since the state of technology N is not an observable variable, we must find an expression which permits us to approximate it. To that purpose, we will use two equations obtained from the theoretical model, which relates the technology growth rate with some observable variables such as, for instance, the R&D-expenditure. First, we will use the free-entry condition for the R&D sector (2.8). Differentiating this equation with respect to time, we have an expression relating the growth rate of technology with the growth rates of aggregate R&D-expenditure and the price of designs; i.e.,

$$\frac{\dot{N}}{N} = \frac{\dot{R}}{R} - \frac{1}{1-\lambda}\frac{\dot{P}_D}{P_D}. \tag{7.3}$$

Hence, we will use this relationship to approximate the evolution of technology. From the market equilibrium we observe that the growth rate of technology depends on how much firms increase their expenditure on research and how the price of these new designs rises. Note that we also introduce the role of the R&D-expenditure, which does not participate directly in the production function. On the other hand, the growth rate of the patentee's price is one of the equations of the dynamic system that describes the behavior of the economy. Thus, the second step to obtain the approximation of the growth rate of technology (that we will estimate) is to solve this system. In particular, we will use the policy function of the prices with respect to the transformed variable $x = \ln(K/N)$.[3] Then, after some calculations we can write the technology growth rate as a linearly approximated function of the R&D-expenditure growth rate and a dynamic factor. This last component depends on the distance between the initial level and the steady state of the ratio from capital stock to the number of designs:

$$\frac{\dot{N}}{N} \approx \frac{\dot{R}}{R} - \frac{1}{1-\lambda}(\Phi_{\bar{x}} \quad - \Phi_{\bar{x}})\left(\frac{1}{e_P}\right)(x_0 - \bar{x})\exp(-\theta t). \tag{7.4}$$

In this expression θ denotes the absolute value of the stable eigenvalue of the dynamic system (5.1), $\left(1 \quad e_p\right)'$ is the associated eigenvector, and $\Phi_{\bar{x}}$ is an element of the Jacobian matrix (5.2).[4]

Note that the second part of the right hand side is a number which depends on the time t. Therefore we can approximate the growth rate of the technology, using a directly observable component, which is controlled by economic agents, that is the R&D-expenditure growth rate $\dot{R}/R$, and a temporal component, $a_i \exp(-\theta t)$. Note that this temporal component includes a constant factor, a_i, which depends on the initial level and the steady state value of the ratio capital stock-technology. Thus, if

$$-\eta \frac{1}{1-\lambda}(\Phi_{\bar{x}} \quad - \Phi_{\bar{x}})\left(\frac{1}{e_P}\right)(x_0 - \bar{x})\exp(-\theta t) = a_i \exp(-\theta t),$$

we can then transform (7.2) into

$$\frac{\dot{Y}}{Y} = \beta \frac{\dot{H}}{H} + (1-\alpha-\beta)\frac{\dot{L}}{L} + \alpha \frac{\dot{K}}{K} + \eta \frac{\dot{R}}{R} + a_i \exp(-\theta t). \tag{7.5}$$

The fixed effect a_i is different for each country, for this reason to estimate it we will divide the total sample of countries into seven homogeneous subsamples, and then we will use one dummy for each of them. These groups are the following, North America (USA and Canada),

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[3] This policy function tells us that the patentee's price level is determined by the distance between the initial level and the steady state of the ratio from capital stock to the number of designs.

[4] See Appendix D for a detailed explanation.

Oceania (New Zealand and Australia), Japan, the poorest EC countries (Greece, Portugal, Ireland and Spain), the central EC countries (Belgium, Italy, Netherlands, France, Germany, UK, Denmark and Austria) the Scandinavian EC countries (Finland and Sweden) and finally the European non-EC countries (Norway and Switzerland). We include these seven dummies to estimate a_i because its value depends on the difference between the initial value and the steady state of the ratio x on each country.

The following step is to try to analyze which elements determine this component $a_i \exp(-\theta t)$. To that purpose, we use the same equations of the model used to derive equation (7.5), but without considering the policy functions, i.e., we directly substitute in the dynamic equation of prices as Appendix D shows. In this way, we approximate the growth rate of technology by

$$\frac{\dot{N}}{N} = \frac{\dot{R}}{R} - \frac{\alpha^2 e^{(\alpha-1)\bar{x}}}{(1-\lambda)}\left[(1-\lambda)\ln(R) + \lambda \ln(N) - \ln(K) - \ln\gamma + \ln(\alpha/(1-\alpha))\right]. \tag{7.6}$$

Finally, substituting (7.6) into (7.2), and using (3.5) to replace $\ln(N)$, we obtain

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \beta\frac{\dot{H}}{H} + (1-\alpha-\beta)\frac{\dot{L}}{L} + \alpha\frac{\dot{K}}{K} + \eta\frac{\dot{R}}{R} - \\ &\quad - \frac{\alpha^2 e^{(\alpha-1)\bar{x}}}{(1-\lambda)}\big(\lambda\ln(Y) + \lambda\beta\ln(H) + \lambda(1-\alpha-\beta)\ln(L) \\ &\quad + (\eta+\alpha\lambda)\ln(K) - \eta(1-\lambda)\ln(R) + (1-\alpha)(\ln\gamma - \ln(\alpha/(1-\alpha)))\big). \end{aligned} \tag{7.7}$$

Note that this equation is similar to (7.5) where the temporal component of the technology, $a_i \exp(-\theta t)$, is determined by the initial levels of income, physical capital stock, total human capital stock, the total labor, the R&D-expenditure, and the technology parameters. We also include dummies for each group of countries as before because we must estimate $e^{(\alpha-1)\bar{x}}$ which depends on the steady state of the ratio x of each country.

Next, we will present the results of the different estimations. We use the Non Linear Squares Method.

8. The Empirical Results.

The model is estimated using pooled cross-section data for a sample of 21 OECD countries with five observations for each country, which correspond to the differences for the years 1965, 1970, 1975, 1980, 1985 and 1990.

Income, population, labor force and investment rates data are taken from the Summers-Heston Penn World Table 5.6. These data are expressed in real terms corrected for differences in purchasing power. The information covers the time period from 1965 to 1990 at five year intervals.

Data for human capital stocks is obtained from the revised Barro and Lee data set (1996) . The proxy used for the human capital is the estimated years of schooling. Concerning the physical capital stock, we use the information given by Summers and Heston for one sample. Data about R&D-expenditure comes from de la Fuente (1997).

Column I of Table 1 shows the results obtained from the estimation of equation (I), which is the specification corresponding to equation (7.5). We observe that the coefficients of human and physical capital are positive and significant, (in 0.19 and 0.27 respectively). Note that t-statistics are in parenthesis below the corresponding coefficients. Concerning R&D-activity, Table 1 shows a strong positive correlation between the growth of the total R&D-expenditure and the growth of the GDP. The estimated coefficient is significant, and its value is 0.08. On the other hand, we also see that the sign of the different a_i's, which are not in the table, is positive for all groups of countries except for New Zealand and Australia.[5] Therefore, we can conclude that, in general, during the transition to the steady state the ratio K / N is larger than its stationary value.[6] It means that there is an over-accumulation of physical capital with respect the state of the technology. In other words, this conclusion reveals that the technology level of these countries is lower with respect to their physical capital stock.

To end with, note that the estimated value of θ gives us an approximated measure of how the distance between the initial values of the variable $x = \ln(K/N)$ and its steady state is reduced. Unfortunately, in our estimation this coefficient seems to be not significant.

Column (II) shows the outcomes of the estimation of equation (II), which is the specification corresponding to equation (7.7).[7]

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[5] We obtain seven coefficients, each of them represents the value of this component for each group of countries. The magnitude of these coefficients is very similar, except for Japan and the poorest group of EC countries which are larger. This result means that these groups of countries are in a similar position with respect to their long run equilibrium. However, only the coefficients of Japan and the poorest group of EC countries have significant coefficients.

[6] See Appendix E for a proof of this statement.

[7] This specification also includes the estimation of a parameter which depends on the steady state of the ratio capital stock-number of designs. Although the table does not show the values of these seven coefficients, we want to remark here that the value of these coefficients is equal for all groups of countries. This result can be interpreted as follows: all countries of the sample have the same steady state for this ratio, although as footnote 5 indicates, the initial values are different for some of them.

Table 1. NLS Estimation Results.

	(I)		(II)
β	0.19 (2.5)	β	0.05 (1.3)
α	0.27 (2.6)	α	0.22 (3.1)
η	0.08 (2.2)	η	0.08 (2.6)
θ	0.36 (1.1)		
		λ	0.94 (1.1)

(I) $$\Delta y_{it} = \beta \Delta h_{it} + (1-\alpha-\beta)\Delta l_{it} + \alpha \Delta k_{it} + \eta \Delta r_{it} + \exp(-\theta t)\sum a_i dum + \varepsilon_t$$

(II) $$\Delta y_{it} = \beta \Delta h_{it} + (1-\alpha-\beta)\Delta l_{it} + \alpha \Delta k_{it} + \eta \Delta r_{it} - \frac{\alpha^2 \sum e^{(\alpha-1)\dot{x}_i} dum}{(1-\lambda)} \Big(\lambda y_{t-5} + \lambda\beta h_{t-5} + \lambda(1-\alpha-\beta) l_{t-5} + (\eta + \alpha\lambda) k_{t-5} - \eta(1-\lambda) r_{t-5} + (1-\alpha)\big(\ln\gamma - \ln(\alpha/(1-\alpha))\big)\Big) + \varepsilon_t .$$

With respect to this second estimation, we can see that the coefficient of human capital is smaller than before and not significant. However, this result is not quite surprising. There are recent empirical articles where the evidence on the relation between human capital and economic growth is puzzling [see de la Fuente (1996), Freire-Serén (1999)]. Regarding the growth rate of the aggregate R&D-expenditure, it has a significant and positive coefficient like in the first estimation. Hence, we can conclude that the results about the coefficient of the aggregate R&D-expenditure is robust to the specification. From this equation (7.7) we also estimate the value of λ (and the outcome is 0.94). Note that λ represents the elasticity of the R&D-expenditure with respect to the state of technology. Thus, a high value of λ means there is not decreasing return on the R&D-expenditure. Therefore, the value of λ tells us that the R&D-expenditure is very important for the generation of new technologies, and the evidence would confirm that R&D-expenditure affects economic growth through its participation in the production of new designs. However, this estimated coefficient is not significant.

9. Conclusion.

In this paper we have shown that the introduction of R&D-expenditure can generate sustainable growth in the traditional R&D-based models of economic growth. Thus, the continuous growth of the per capita income of countries can partially depend on the individuals' choice of the amount of income devoted to finance the total cost of the R&D-activity. The introduction of a technology of innovation based on R&D-expenditure instead of labor, generates endogenous growth. For this reason this model has permitted us to analyze the growth effects of some fiscal policies. More specifically, we have found that not only the introduction of a tax-deduction to the R&D investment will encourage the innovation activity, but also the introduction of a tax-deduction to the physical capital production. Thus, in the environment defined for our model, the physical capital subsidy positively affects the long run growth rate because it provides incentives to increase the variety of capital goods.

The paper also has empirically analyzed how R&D-expenditure affects productivity growth. To that purpose, we have made use of an econometric model derived from the theoretical one. More precisely, we have found an expression that can approximate the growth rate of technology through the R&D-expenditure and other dynamical components. This expression has been obtained using the free-entry condition that the competitive equilibrium states for the R&D-activity and the policy function defining the dynamic evolution of patentees' price. At this point, using cross-country data, we have found a positive and significant effect, and this evidence seems to be quite robust. Then, the estimated coefficient corresponding to the R&D regressor reveals a strong positive relationship between the growth of total R&D-expenditure and the growth of the GDP. The estimated value of this coefficient tells us that a 1% rise in the aggregate R&D-expenditure will increase the real GDP by 0.08%. Finally, we have tried to estimate the value of the parameters defining the innovation technology. The results suggest that the elasticity of the R&D-expenditure is close to one. Nevertheless, this estimated coefficient does not appear significant.

APPENDICES

A. Proof of Proposition 4.1.

From (4.3) we derive the steady-state value $\overline{X}$ as a function of $\overline{P}_D$, i.e.,

$$\overline{X} = \frac{\alpha}{\left(1-\alpha(1-s_k)\right)}\overline{P}_D. \qquad \text{(A1)}$$

Second, introducing (A1) into equation (4.1), we also obtain the stationary value of $\overline{Z}$ as a function of $\overline{P}_D$:

$$\overline{Z} = \left(\frac{\alpha \overline{P}_D}{1-\alpha(1-s_k)}\right)^{(\alpha-1)} - \frac{1-\alpha(1-s_k)}{\alpha}\left(\frac{\gamma \overline{P}_D^{\,\lambda}}{(1-s_R)}\right)^{(1/1-\lambda)} - \left(\frac{\gamma \overline{P}_D}{(1-s_R)}\right)^{(\lambda/1-\lambda)}. \tag{A2}$$

Finally, we substitute the previous stationary values of $\overline{Z}$ and $\overline{X}$ in (4.2) to obtain the following equation:

$$\frac{(1-\tau)\alpha^2}{\sigma}\left(\frac{\alpha \overline{P}_D}{1-\alpha(1-s_k)}\right)^{(\alpha-1)} - \gamma^{1/1-\lambda}\left(\frac{\overline{P}_D}{(1-s_R)}\right)^{(\lambda/1-\lambda)} - \frac{\rho}{\sigma} = 0, \tag{A3}$$

which implicitly expresses the stationary value of $\overline{P}_D$ only as a function of the parameters. The number of roots of (A3) gives us the number of balanced growth paths of the economy. In order to obtain this number of roots, we define the following valued function of $\overline{P}_D$ from (A3):

$$G(P_D) = \frac{(1-\tau)\alpha^2}{\sigma}\left(\frac{\alpha P_D}{1-\alpha(1-s_k)}\right)^{(\alpha-1)} - \gamma^{1/1-\lambda}\left(\frac{P_D}{(1-s_R)}\right)^{(\lambda/1-\lambda)} - \frac{\rho}{\sigma}, \tag{A4}$$

whose domain is $(0,\infty)$. We can easily check that the previous function satisfies the following properties:

(i) It is a decreasing function in P, i.e., $G'(P) < 0$ for all P,

(ii) $\lim_{x\to 0^+} G(P) = +\infty$,

(iii) $\lim_{x\to +\infty} G(P) = -\infty$.

The ordinary equation (A3) has the unique root, belong to $(0,\infty)$. Hence, the uniqueness of BGP is proved.

Now, we will prove the existence of a BGP. To that purpose, we must check whether this balanced growth path satisfies the transversality condition (2.20). Using the F.O.C. (2.18), this transversality condition can be rewritten as $\lim_{t\to\infty} e^{-\rho t}\mu_t C_t = 0$. Note that this condition holds when the following inequality is satisfied:

$$\lim_{t\to\infty}\left(-\rho + \frac{\dot{\mu}}{\mu} + \frac{\dot{C}}{C}\right) < 0. \tag{A5}$$

Substituting for the growth rate of both state and co-state variables, we obtain that inequality (A5) is equivalent to

$$(1-\sigma)(1-\tau)\alpha^2\overline{X}^{(\alpha-1)} - \rho < 0. \tag{A6}$$

When $\sigma \geq 1$ inequality (A6) always holds, so that the transversality condition (2.18) is satisfied. However, when $\sigma \in (0,1)$, inequality (A6) holds if and only if

$$\overline{X} > \left(\alpha^2(1-\sigma)(1-\tau)/\rho\right)^{1/(1-\alpha)}. \tag{A7}$$

Using (A1) inequality (A7) can be expressed in terms of $\overline{P}_D$ as follows:

$$\overline{P}_D > \frac{(1-\alpha)}{\alpha}\left(\alpha^2(1-\sigma)(1-\tau)/\rho\right)^{1/(1-\alpha)}. \tag{A8}$$

Therefore, when $\sigma \in (0,1)$ the tranversality condition holds if the unique root of equation (A3) satisfies inequality (A8). In other words, denote by A:

$$A = \frac{(1-\alpha)}{\alpha}\left(\alpha^2(1-\sigma)(1-\tau)/\rho\right)^{1/(1-\alpha)},$$

since $G^{'}(\,) < 0$, and $G(\overline{P}_D) = 0$, we know that $G(A) > 0$. By substituting A for $\overline{P}_D$ in (A4), we obtain:

$$\frac{\rho}{\sigma(1-\sigma)}\left(\frac{1-\alpha(1-s_k)}{(1-\alpha)}\right)^{(1-\alpha)} - \gamma^{1/1-\lambda}\left(\frac{(1-\alpha)}{\alpha(1-s_R)}\left(\frac{\alpha^2(1-\sigma)(1-\tau)}{\rho}\right)^{1/(1-\alpha)}\right)^{(\lambda/1-\lambda)} - \frac{\rho}{\sigma} > 0. \tag{A9}$$

Hence when $\sigma \in (0,1)$ the root of equation (A3) defines a BGP if condition (A9) holds. Otherwise, no BGP exists. The proposition is then proved. QED

B. Proof of Proposition 5.1.

We first calculate the coefficients of Jacobian matrix (5.2). Differentiating the dynamic equations in system (5.1) with respect to x, z and P, and evaluating these derivatives at $\left(\bar{x}, \bar{z}, \overline{P}\right)$, we obtain

$$\Psi_{\bar{x}} = \left.\frac{\partial \dot{x}}{\partial x}\right|_{(\bar{x},\bar{z},\overline{P})} = -(1-\alpha)e^{(\alpha-1)\bar{x}} + (\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)},$$

$$\Psi_{\bar{z}} = \left.\frac{\partial \dot{x}}{\partial z}\right|_{(\bar{x},\bar{z},\overline{P})} = -e^{\bar{z}},$$

$$\Psi_{\overline{P}} = \left.\frac{\partial \dot{x}}{\partial P}\right|_{(\bar{x},\bar{z},\overline{P})} = -\frac{(1+\lambda)}{(1-\lambda)}(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)},$$

$$\Omega_{\bar{x}} = \left.\frac{\partial \dot{z}}{\partial x}\right|_{(\bar{x},\bar{z},\overline{P})} = -\frac{(1-\alpha)}{\sigma}\alpha^2 e^{(\alpha-1)\bar{x}} + (1-\alpha)e^{(\alpha-1)\bar{x}} - (\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)},$$

$$\Omega_{\bar{z}} = \left.\frac{\partial \dot{z}}{\partial z}\right|_{(\bar{x},\bar{z},\bar{P})} = e^{\bar{z}},$$

$$\Omega_{\bar{P}} = \left.\frac{\partial \dot{z}}{\partial P}\right|_{(\bar{x},\bar{z},\bar{P})} = \frac{1}{(1-\lambda)}(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\ e^{\bar{x}(\lambda/1-\lambda)},$$

$$\Phi_{\bar{x}} = \left.\frac{\partial \dot{P}}{\partial x}\right|_{(\bar{x},\bar{z},\bar{P})} = -\alpha^2 e^{(\alpha-1)\bar{x}},$$

$$\Phi_{\bar{z}} = \left.\frac{\partial \dot{P}}{\partial z}\right|_{(\bar{x},\bar{z},\bar{P})} = 0,$$

$$\Phi_{\bar{P}} = \left.\frac{\partial \dot{P}}{\partial P}\right|_{(\bar{x},\bar{z},\bar{P})} = \alpha^2 e^{(\alpha-1)\bar{x}}.$$

We now compute the three eigenvalues of Jacobian matrix (5.2), which are the solutions θ_i of the associated characteristic polynomial

$$-\theta^3 + A\theta^2 - B\theta + C = 0, \tag{B1}$$

where A and C are respectively the trace and the determinant of Jacobian matrix (5.2), and

$$B = \Psi_{\bar{x}}\Omega_{\bar{z}} + \Psi_{\bar{x}}\Phi_{\bar{P}} + \Omega_{\bar{z}}\Phi_{\bar{P}} - \Phi_{\bar{x}}\Psi_{\bar{P}} - \Psi_{\bar{z}}\Omega_{\bar{x}}. \tag{B2}$$

In order to characterize the local stability of system (5.1), we must determine the sign of the real parts of each θ_i. To that purpose, we will use Routh's theorem.[8] In this particular case, the theorem says that the number of roots with positive real parts is equal to the number of variations of sign in the following sequence: -1, A, $-B + C/A$, C. Hence, to apply Routh's theorem, we first consider the following results:

RESULT 1. *The determinant of Jacobian matrix (5.2) is always negative.*

Proof: From the Jacobian matrix we know that the determinant is given by

$$C = \Psi_{\bar{x}}\Omega_{\bar{z}}\Phi_{\bar{P}} + \Psi_{\bar{z}}\Omega_{\bar{P}}\Phi_{\bar{x}} - \Psi_{\bar{P}}\Omega_{\bar{z}}\Phi_{\bar{x}} - \Psi_{\bar{z}}\Omega_{\bar{x}}\Phi_{\bar{P}}. \tag{B3}$$

Substituting for the derivatives computed at the beginning of this Appendix in (B3), we can obtain

$$C = -\frac{(1-\alpha)}{\sigma}\alpha^2\alpha^2 e^{2(\alpha-1)\bar{x}} e^{\bar{z}} - \frac{\lambda}{(1-\lambda)}(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\ \alpha^2 e^{(\alpha-1)\bar{x}} e^{\bar{z}} e^{\bar{x}(\lambda/1-\lambda)},$$

[8] See Grantmacher (1960) for more details.

which is negative. QED

RESULT 2. *The trace of Jacobian matrix (5.2) is positive if* $\sigma > (1-\alpha)$.

Proof: From the Jacobian matrix we calculate the trace as follows:

$$A = \Psi_{\bar{x}} + \Omega_{\bar{z}} + \Phi_{\bar{p}} . \tag{B4}$$

Substituting for the derivatives computed at the beginning of this Appendix in (B4), we can obtain

$$A = -(1-\alpha)e^{(\alpha-1)\bar{x}} + (\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)} + e^{\bar{z}} + \alpha^2 e^{(\alpha-1)\bar{x}} . \tag{B5}$$

We know from (4.2) that the following relation holds:

$$\begin{aligned} &-(1-\alpha)e^{(\alpha-1)\bar{x}} \\ &\qquad = -\frac{(1-\alpha)}{\sigma}\alpha^2 e^{(\alpha-1)\bar{x}} + \frac{\rho(1-\alpha)}{\sigma} - (1-\alpha)e^{\bar{z}} - (1-\alpha)(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)}. \end{aligned}$$

Therefore, introducing the previous equality in (B5), we can rewrite the trace as

$$A = \frac{\rho(1-\alpha)}{\sigma} + \alpha e^{\bar{z}} + \alpha(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, e^{\bar{x}(\lambda/1-\lambda)} + \alpha^2 e^{(\alpha-1)\bar{x}} \left[\frac{\sigma-(1-\alpha)}{\sigma}\right],$$

which is positive when $\sigma > (1-\alpha)$. QED

RESULT 3. B *is negative if* $\sigma < (1-\alpha)$.

Proof: Substituting for the derivatives computed at the beginning of this Appendix in (B2), we can rewrite B as follows:

$$\begin{aligned} B = &-(1-\alpha)\alpha^2 e^{2(\alpha-1)\bar{x}} \\ &\qquad -\alpha^2(\gamma\alpha/(1-\alpha))^{1/1-\lambda}\, \frac{2\lambda}{(1-\lambda)} e^{\bar{x}(\lambda/1-\lambda)} e^{(\alpha-1)\bar{x}} - \alpha^2 e^{(\alpha-1)\bar{x}} e^{\bar{z}} \left[\frac{(1-\alpha)-\sigma}{\sigma}\right], \end{aligned}$$

which is negative when $\sigma < (1-\alpha)$. QED

Therefore, using the previous three results, we can prove Proposition 5.1. To this purpose, we analyze the following two cases:

(i) $\sigma > (1-\alpha)$. Since the trace and the determinant of Jacobian matrix (5.2) are respectively positive and negative, two changes of sign in Routh's sequence always occur. Hence, we have only one eigenvalue with a negative real part, so that the steady state is a saddle point.

(ii) $\sigma < (1-\alpha)$. In this case, the determinant of Jacobian matrix (5.2) and B are both negative, whereas the trace can be either positive or negative. If the trace were positive, we know from case (i) that the steady state is a saddle point. On the other hand, if the trace were negative, the third element of Routh's sequence is positive, so that the same result holds.

C. Proof of Propositions 6.1 and 6.2.

First, applying the Implicit Function Theorem to equation (A3), we can calculate the impact of the subsidies to investments in R&D and physical capital accumulation, and the income tax on $\bar{P}_D$ as follows

$$\frac{d\bar{P}_D}{ds_R} = -\frac{\delta G(\bar{P}_D)/\delta s_R}{\delta G(\bar{P}_D)/\delta \bar{P}_D} = \frac{\lambda \left(\gamma / (1-s_R) \right)^{1/1-\lambda} \bar{P}_D^{(\lambda/1-\lambda)}}{(1-\lambda) D_P} < 0, \tag{C1}$$

$$\frac{d\bar{P}_D}{ds_k} = -\frac{\delta G(\bar{P}_D)/\delta s_k}{\delta G(\bar{P}_D)/\delta \bar{P}} = \frac{(\alpha-1)\alpha^3 (1-\tau)}{\sigma\left(1-\alpha(1-s_k)\right) D_P} \left(\frac{\alpha \bar{P}_D}{\left(1-\alpha(1-s_k)\right)} \right)^{(\alpha-1)} > 0, \tag{C2}$$

$$\frac{d\bar{P}_D}{d\tau} = -\frac{\delta G(\bar{P}_D)/\delta \tau}{\delta G(\bar{P}_D)/\delta \bar{P}_D} = \frac{\alpha^2 (1-\tau)(1-s_k)^{1-\alpha}}{\sigma D_P} \left(\frac{\alpha}{(1-\alpha)} \bar{P}_D \right)^{(\alpha-1)} < 0, \tag{C3}$$

where

$$D_P = \frac{\delta G(\bar{P}_D)}{\delta \bar{P}_D} = \frac{(\alpha-1)\alpha^2 (1-\tau)}{\sigma} \left(\frac{\alpha}{\left(1-\alpha(1-s_k)\right)} \right)^{(\alpha-1)} \bar{P}_D^{(\alpha-2)}$$

$$- \frac{\lambda}{1-\lambda} \gamma^{1/1-\lambda} (1-s_R)^{-\lambda/1-\lambda} \left(\bar{P}_D\right)^{(2\lambda-1/_{1-\lambda})} < 0.$$

Using (A1) we can also compute the impact of the three fiscal policies on $\bar{X}$ as follows:

$$\frac{d\bar{X}}{ds_R} = \frac{\alpha}{(1-\alpha)(1-s_k)} \frac{d\bar{P}_D}{ds_R} < 0, \tag{C4}$$

$$\frac{d\bar{X}}{ds_k} = \frac{\alpha}{\left(1-\alpha(1-s_k)\right)} \frac{d\bar{P}_D}{ds_k} - \frac{\alpha^2}{\left(1-\alpha(1-s_k)\right)^2} \bar{P}_D$$

$$= \frac{\alpha^2 \lambda \gamma^{1/(1-\lambda)} (1-s_R)^{-\lambda/(1-\lambda)} \bar{P}_D^{(2\lambda-1)/(1-\lambda)}}{\left(1-\alpha(1-s_k)\right)^2 (1-\lambda) D_P} < 0, \tag{C5}$$

$$\frac{d\bar{X}}{d\tau} = \frac{\alpha}{\left(1-\alpha(1-s_k)\right)} \frac{d\bar{P}_D}{d\tau} < 0. \tag{C6}$$

With respect to the effect of the fiscal policies on $\bar{Z}$, we differentiate (A2) with respect to s_R, s_k and τ. However, one can easily check that the signs of the derivatives are ambiguous.

Finally, to analize the impact of the fiscal policies on the steady-state growth rate, we differentiate (4.4) with respect to s_R, s_k and τ. Hence, using the previous results, we can prove that

$$\frac{dg^*}{ds_R} = \frac{\gamma}{1-\lambda}\left(\frac{\gamma \bar{P}_D}{(1-s_R)^{(\lambda-2)}}\right)^{1/(1-\lambda)} + \frac{\gamma}{1-\lambda}\left(\frac{\gamma \bar{P}_D^{(2-\lambda)}}{(1-s_R)}\right)^{1/(1-\lambda)} \frac{d\bar{P}_D}{ds_R}$$

$$= \frac{\alpha^2(\alpha-1)(1-\tau)}{\sigma(1-\lambda)D_P}\left(\frac{\gamma}{(1-s_R)}\right)^{(2-\lambda)/(1-\lambda)} \left(\frac{\alpha}{\left(1-\alpha(1-s_k)\right)}\right)^{\alpha-1} \bar{P}_D^{(\alpha-2)+(1/(1-\lambda))} > 0, \tag{C7}$$

$$\frac{dg^*}{ds_k} = \frac{\gamma}{1-\lambda}\left(\frac{\gamma \bar{P}_D^{\lambda}}{(1-s_R)}\right)^{(1/(1-\lambda))} \frac{d\bar{P}_D}{ds_k} > 0, \tag{C8}$$

$$\frac{dg^*}{d\tau} = \frac{\gamma}{1-\lambda}\left(\frac{\gamma \bar{P}_D^{\lambda}}{(1-s_R)}\right)^{(1/(1-\lambda))} \frac{d\bar{P}_D}{d\tau} < 0. \tag{C9}$$

Before closing this section, we establish the relationship between the growth effects of s_R and s_k. After some trivial algebra, we can write:

$$\frac{dg^*}{ds_R} - \frac{dg^*}{ds_k} =$$

$$\frac{\gamma\alpha^2(\alpha-1)(1-\tau)}{\sigma(1-\lambda)D_P}\left(\frac{\gamma}{(1-s_R)}\right)^{1/(1-\lambda)} \left(\frac{\alpha}{1-\alpha(1-s_k)}\right)^{\alpha-1} \bar{P}_D^{\alpha-2+(1/(1-\lambda))}\left(\frac{1}{(1-s_R)} - \frac{\alpha}{\left(1-\alpha(1-s_k)\right)}\right),$$

hence, the following can be checked:

$$Sing\left[\frac{dg^*}{ds_R} - \frac{dg^*}{ds_k}\right] = Sing\left[(1-2\alpha) + \alpha s_k + \alpha s_R\right]. \tag{C10}$$

D. The Econometric Model.

In this Appendix, we derive expressions (7.5) and (7.7) used in the econometric model. We start with expression (7.4). From the main text we know that the production function is given by:

$$Y_t = F\left(H_t, L_t, K_t, N_t\right) = H_t^{\beta} L_t^{1-\alpha-\beta} K_t^{\alpha} N_t^{\eta}, \tag{D1}$$

where N is an empirically unobservable variable. We will use relations (2.7) and (3.2) to obtain an approximation for $\dot{N}/N$. In particular, to facilitate the explanation we consider the following notation $m_t = R_t / N_t$. Thus, differentiating with respect to time (3.2) we obtain

$$\frac{\dot{m}}{m} = \frac{1}{1-\lambda}\frac{\dot{P}_D}{P_D} = \frac{1}{1-\lambda}\dot{P}. \tag{D2}$$

Introducing (5.1c) into (D2), we can rewrite:

$$\frac{\dot{m}}{m} = \frac{1}{1-\lambda}\left(\alpha^2 e^{(\alpha-1)x} - \alpha(1-\alpha)e^{\alpha x - P}\right) \equiv G(x, P). \tag{D3}$$

At this point, we make the first order Taylor approximation of (D3) around the steady state, i.e.,

$$G(x, P) \approx G(\bar{x}, \bar{P}) + \begin{pmatrix} G_{\bar{x}} & G_{\bar{P}} \end{pmatrix} \begin{pmatrix} x - \bar{x} \\ P - \bar{P} \end{pmatrix}, \tag{D4}$$

where $\begin{pmatrix} G_{\bar{x}} & G_{\bar{P}} \end{pmatrix}$ is the Jacobian of $G(x, P)$ evaluated in the steady state. We also note that $G(\bar{x}, \bar{P})$ is zero. Moreover, from (5.1c) we can write

$$G(x, P) = \frac{1}{1-\lambda}\Phi(x, z, P), \tag{D5}$$

where from Appendix B we know that $\Phi_{\bar{z}}$ is zero and $\Phi_{\bar{P}} = -\Phi_{\bar{x}}$. Hence, we can rewrite (D4) as follows

$$G(x, P) \approx \frac{1}{1-\lambda}\begin{pmatrix} \Phi_{\bar{x}} & -\Phi_{\bar{x}} \end{pmatrix} \begin{pmatrix} x - \bar{x} \\ P - \bar{P} \end{pmatrix}. \tag{D6}$$

We also know from the stability properties of Section 5 that the linear approximation of the policy function of P is given by:

$$(P - \bar{P}) = (x_0 - \bar{x}) e_P \exp(-\theta t), \tag{D7}$$

where θ denotes the absolute value of the stable eigenvalue of Jacobian matrix (5.2), and $e_P = \Phi_{\bar{x}} / (-\theta + \Phi_{\bar{x}})$ is the third component of the associated eigenvector. Thus, equation (D5) transforms into

$$G(x,P) \approx \frac{1}{1-\lambda}\left(\Phi_{\bar{x}} \quad -\Phi_{\bar{x}}\right)\begin{pmatrix}(x_0-\bar{x})\exp(-\theta t)\\ (x_0-\bar{x})e_P\exp(-\theta t)\end{pmatrix}. \tag{D8}$$

Using (D3) and (D8), $\dot{m}/m$ can be approximated by

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda}\left(\Phi_{\bar{x}} \quad -\Phi_{\bar{x}}\right)\begin{pmatrix}1\\ e_P\end{pmatrix}(x_0-\bar{x})\exp(-\theta t). \tag{D9}$$

Moreover, since $\dot{m}/m = \dot{R}/R - \dot{N}/N$, we can state that

$$\frac{\dot{N}}{N} \approx \frac{\dot{R}}{R} - \frac{1}{1-\lambda}\left(\Phi_{\bar{x}} \quad -\Phi_{\bar{x}}\right)\begin{pmatrix}1\\ e_P\end{pmatrix}(x_0-\bar{x})\exp(-\theta t). \tag{D10}$$

Therefore, expression (7.5) in the main text was obtained.

We now derive equation (7.7) used in the econometric model. From (5.2), and using Appendix B, we can see that

$$\dot{P} = \alpha^2 e^{(\alpha-1)\bar{x}}\left((P-x)-(\bar{P}-\bar{x})\right). \tag{D11}$$

Furthermore, differentiating with respect to time (3.2), we also obtain

$$\dot{P} = (1-\lambda)\left(\frac{\dot{R}}{R} - \frac{\dot{N}}{N}\right). \tag{D12}$$

Hence, introducing (D12), (3.2) and (A1) into (D11), and noting that $x = \ln(K) - \ln(N)$, we obtain after simple algebra

$$\frac{\dot{N}}{N} = \frac{\dot{R}}{R} - \frac{\alpha^2 e^{(\alpha-1)\bar{x}}}{(1-\lambda)}\left[(1-\lambda)\ln(R) + \lambda\ln(N) - \ln(K) - \ln\gamma + \ln(\alpha/(1-\alpha))\right]. \tag{D13}$$

Therefore, expression (7.6) in the main text was obtained.

E. The over-accumulation of physical capital during the transition.

In this Appendix, we show that the empirical result of a positive fixed effect implies that there is an over-accumulation of physical capital with respect to the level of technology during the transition to the steady-state equilibrium. In other words, we want to prove the if $a_i > 0$ then $(x-\bar{x}) > 0$. First, we note from Appendix B that

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda}\left(\Phi_{\bar{x}} \quad -\Phi_{\bar{x}}\right)\begin{pmatrix}x-\bar{x}\\ P-\bar{P}\end{pmatrix} = \frac{1}{1-\lambda}\Phi_{\bar{x}}\left((x-\bar{x})-(P-\bar{P})\right). \tag{E1}$$

Furthermore, since the linear approximation around the steady state of the policy function of the P can be written as $(P-\bar{P}) = e_P(x-\bar{x})$, then we can transform (E1) into

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda}\Phi_{\bar{x}}(x-\bar{x})(1-e_p) = -\frac{1}{1-\lambda}\frac{\theta\,\Phi_{\bar{x}}}{(-\theta+\Phi_{\bar{x}})}(x-\bar{x}), \tag{E2}$$

where we have used the equality $e_P = \Phi_{\bar{x}} / (-\theta + \Phi_{\bar{x}})$. Thus, since $\Phi_{\bar{x}} < 0$ and $\theta > 0$, then $\dot{m}/m < 0$ if and only if $(x-\bar{x}) > 0$. Therefore, noting from Appendix B that $a_i \exp(-t) = -\eta\, \dot{m}/m$, we prove that $a_i > 0$ implies $(x-\bar{x}) > 0$.

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