Sincerity in Simple and Complex Voting Mechanisms

Abstract

We discuss sincere voting when voters have cardinal preferences over alternatives. We interpret sincerity as opposed to strategic voting, and thus define sincerity as the optimal behavior when conditions to vote strategically diminish. When voting mechanisms allow for only one *message type* (*simple voting mechanisms*) we show that eliminating some conditions for strategic voting, individuals' optimal behavior coincides with an intuitive and common definition of sincerity. In order to obtain a precise definition of sincerity in voting mechanisms allowing for multiple *message types* (*complex voting mechanisms*) further restrictions on strategic voting are required. We illustrate our methodological approach using approval voting (AV) as a prime example of complex voting mechanisms for which no conclusive definition of sincerity exists in the literature.

1 Introduction

In this paper we discuss we discuss a new approach to defining sincerity in voting mechanisms. A definition of sincerity is important since it allows to compare the properties of different voting rules with respect to voters' strategic behavior. Under different voting mechanisms, and given voters' preferences over alternatives, voters may be better able to favour the election of preferred outcomes by behaving strategically instead of sincerely and thus, manipulate the voting mechanism.[1] In order to provide a general definition of sincerity, our approach is to consider this strategic component of voting and eliminate it.

There exists ample literature on the definition of sincerity for different voting mechanisms and on which voting rules may achieve it.[2] Brams and Fishburn (1978) define sincere voting as non-strategic behavior in which individuals vote "directly in accordance with their preferences". The problem arises because translating preferences over alternatives to sincere votes may not be direct under some voting rules, since they may demand to structure votes in a different format than preferences may be specified.

Since the majority of the voting literature limits the analysis to ordinal preferences over alternatives, votes are normally structured in the same format as preferences and thus, this problem has not been highlighted.[3] However, it seems plausible to assume that voters may be able to quantify differences between alternatives and thus, they may have cardinal preferences over them. Under cardinal preferences, if a voting mechanism exactly required all cardinal information, the definition of "sincere voting" would be straightforward. A sincere "vote" would just be the declaration of the cardinal utility that each alternative gives to a voter.

Consider the following example. There are three alternatives x, y and z that yield the following utilities to a voter: $U(x) = 0.8$, $U(y) = 0.5$ and $U(z) = 0.1$. A voting rule that required all cardinal information would have associated as "sincere voting" the revelation of utilities $0.8, 0.5$ and 0.1 respectively.

However, the majority of voting mechanisms only require (partial) ordinal information

[1] Any voting rule is subject to strategic voting behaviour when its range has at least three alternatives and there are no dictators (Gibbard (1973), Satterwhaite (1975).

[2] Starting with Farquharson (1969).

[3] See, for example, Arrow (1951), Fishburn (1973) and Nurmi (1987).

from voters and thus a definition of sincerity may be more complicated. Votes may be understood as messages since they transmit information on the desirability of the alternatives for the voters. The translation of cardinal utilities to non-cardinal votes may then depend on the number (and type) of messages each voting mechanism allows.

If the voting mechanism only allows for one possible message type (*simple voting mechanism*) then identifying sincere behavior is not so problematic. A sincere vote would be the one that intuitively "best represents" the order of the cardinal preferences, given the restrictions of the voting mechanism. For example, the plurality rule is a clear case of a voting rule that allows for only one message type, since voters can only choose between singletons (with the meaning of a superior alternative, since the aggregation process will consider positively such singletons). Thus, sincere voting under Plurality Rule (PR) would intuitively fit with voting (in the top set) for the alternative that yields highest utility to the voter. In our example, a sincere voter under PR would then declare her real preferences by voting for alternative $\{x\}$. Intuitively, any other possible message, for example, $\{z\}$ would be a worse representation of the voter's real cardinal preferences and thus, would not be sincere.

There are however several voting rules that allow for more than one message type (*complex voting mechanisms*). In such cases, there exists ambiguity about what the best representation of cardinal preferences would be. Consider Approval Voting (AV) as an example of complex voting rules. Under AV the decision of whether to include an alternative among the "approved" ones or not may naturally depend on the difference in cardinal utility between alternatives: if the voter was only allowed to approve her "best alternative" (to choose from the set of singletons of $2^{\{x,y,z\}}$) then voting $\{x\}$ would intuitively be sincere as previously mentioned. On the other hand, if the voter was only allowed to vote for pairs of alternatives (which is what Negative Voting would do), then voting $\{x, y\}$ would be sincere as it best fits with her cardinal preferences given the restrictions. However, as AV allows voters to specify any subset of alternatives as the set of approved options, it may not be clear whether voting $\{x\}$ or $\{x, y\}$ is the sincere message, if at all.

We use a new approach to obtain precise definitions of sincere voting behavior for simple and complex voting mechanisms. We consider a voter under a hypothetical situation in which conditions to behave strategically are diminished and define sincerity as her optimal voting strategy under such conditions.

Strategic voting implies balancing the relative preference for the different alternatives against the relative likelihood of influencing the outcome of the election.[4] Notice that whether a voter assesses that her vote may affect the outcome depends on how she thinks other voters will vote. Strategic behavior may thus be enhanced the more information voters have on the strategies of other voters. Weber (1978) and Merrill and Nagel (1987) go as far as claiming that in settings where voters have little access to information concerning either the preferences of other voters or their intended behavior, voters can be presumed to vote sincerely, since the lack of information means there is no basis for voting "in some clever strategic way". Our first two results formally study this claim. Theorem 1 shows that Weber's (1978) intuition is correct for the class of voting mechanism which we define as *simple*. We show that in simple voting mechanisms the optimal strategy of a voter with no information on other voters' strategies is unique and independent of the size of the electorate. We thus define sincere voting behavior as this optimal strategy for voting rules that allow for only one message type.

However, our theorem 2 shows that the previous result cannot be directly extended to complex voting mechanisms. We show that in complex voting mechanisms the optimal strategy for any voter when information on others' strategies is eliminated may not be unique. For example, it may depend on other conditions that facilitate strategic behavior, such as the size of the electorate. Thus, it cannot be the case that we consider this optimal behavior as a precise definition of sincerity, since how sincere a vote is should not vary with the number of voters.

We thus consider new conditions that may diminish strategic behavior. A natural intuition emerging from the example used to prove Theorem 2 is that the larger the electorate the lower the manipulative effect of a strategic vote on the outcome of the election may be. Therefore, following our approach, we define sincere voting as the optimal strategy when i) there is no information on other voters' preferences over alternatives and, ii) the size of the electorate tends to infinity.

Finally, we check this definition of sincerity for a particular example of complex voting mechanisms: Approval Voting (AV). Theorem 3 shows that the optimal strategy under these conditions coincides with an *ad-hoc* definition of sincerity in AV previously discussed in the

[4]See Fisher and Myatt (2002).

literature. This definition considers "sincere" to approve all alternatives that yield (cardinal) utility above the average of the utilities. Our result thus provides new support to this intuitive definition as a consequence of eliminating those features of the problem that generate strategic behavior.

We have focused on the case of three alternatives x, y and z. Although this case is of course special, it is the simplest one allowing to differentiate between voting rules while maintaining conditions for strategic voting to appear.[5]

The rest of the paper is organized as follows. Section 2 shows the notation and the basic assumptions made. Section 3 discusses a first definition of sincerity when information on voters' preferences is eliminated and its validity for simple and complex voting mechanisms (Theorems 1 and 2). In Section 4 we present a second definition of sincerity by imposing additional requirements and we use it to describe sincerity in Approval Voting (Theorem 3). Section 5 concludes.

2 Notation and Definitions

Consider a set of n agents $\{1, 2, ..., n\}$ and a set of three alternatives $X = \{x, y, z\}$. Individuals are endowed with cardinal utilities over alternatives $U = (U_j(k))$ with $j \in \{1, 2, ..., n\}$, $k \in X$ and $U_j(k) \in [0, 1]$. For the elegance of the exposition, assume that there are not two alternatives providing the same utility to each agent.[6]

In general, every voting system does not allow voters to make explicit their utility over alternatives. Each voting mechanism has an associated codified method of communicating such utilities, which restricts and standardizes the information that voters can transmit. Assume there exists a set of messages M from which each agent has to choose one. Such message is the agent's vote and transmits information on her preferences. In this paper we consider sets of messages M containing either linear orders over X or subsets of X.[7]

Consider any bijective mapping $\sigma : X \to X$. Given a message $m \in M$, with m being a

[5] See Myerson and Weber (1993), Myerson (2002) and Dhillon and Lockwood (2004).

[6] Parallel results are obtained without such assumption, although proofs become tedious without adding further insights.

[7] Messages on linear orders or subsets of alternatives are the most common approach to voting.

linear order, then $\sigma(m)$ is a linear order such that: $x\ \sigma(m) y \Leftrightarrow \sigma(x)\ m\ \sigma(y)$. Given a message $m \in M$, with m being a subset of alternatives $m = \{x_1, ..., x_t\}$, then $\sigma(m)$ denotes a subset of alternatives such that: $\sigma(m) = \{\sigma(x_1), ..., \sigma(x_t)\}$. We say that two messages m and m' belong to the same *message type* if there exists a bijective mapping $\sigma : X \to X$ such that $m' = \sigma(m)$. We now impose an additional condition on the valid sets of messages in order to avoid voting mechanisms to be biased towards alternatives: if the set of possible messages M contains a message m then it also contains any other message of the form $\sigma(m)$, i.e., $m \in M, \sigma : X \to X$ a bijective mapping $\Longrightarrow \sigma(m) \in M$. The class of messages which belong to the same message type as m is denoted by $[m]$.

A voting mechanism $V : M^n \to 2^{\{x,y,z\}}$ can be defined as the composition of a set of messages (among which the voters can choose one) and an aggregation process of the collected messages such that some alternatives are chosen.[8] We refer to elements of M^n as $\mathbf{m} = (m_1, ..., m_n)$ with $m_j \in M$ for $j = 1, ..., n$. We naturally denote $\sigma(\mathbf{m}) = (\sigma(m_1), ..., \sigma(m_n))$. Finally, we denote, as usual, $\mathbf{m}_{-j} = (m_1, ..., m_{j-1}, m_{j+1}, ..., m_n)$.

A voting mechanism may allow a set of possible messages with several message types. We first classify voting mechanisms according to the number of message types associated to them. The crucial property to study sincerity will be whether voting mechanisms have a single or several message types associated to them.

Definition 1 *A voting mechanism is said to be simple if it only allows for one message type. Otherwise, it is said to be complex.*

Two examples of *simple* voting mechanisms are the Borda Rule and the Plurality Rule. In the former, the set of possible messages contains all linear orders over alternatives while in the latter, the set of possible messages contains all singletons, i.e., $M = \{\{x\}, \{y\}, \{z\}\}$.

A prime example of a *complex* voting mechanism is Approval Voting. We will formally define it below as we will discuss it thoroughly in the following.

Once we have discussed messages, we now briefly refer to the aggregation process. In particular, we now define some properties on how voting mechanisms may aggregate messages to select alternatives.

[8] Merril and Nagel (1987) also differentiate between balloting methods and the decision rules that produce an outcome.

Definition 2 *A voting mechanism V is Neutral in alternatives if for any permutation σ of the set of alternatives and any $\mathbf{m}$ in M^n, then $V(\sigma(\mathbf{m})) = \sigma(V(\mathbf{m}))$.*

Neutrality in alternatives implies that the names of the alternatives do not affect their election.

Our second definition refers to the monotonicity of the aggregation process. We distinguish between voting mechanisms composed by linear orders or subsets as messages.

Definition 3 *A voting mechanism V with M containing linear orders (respectively subsets of alternatives) is weakly monotonic if for any alternative x, for all $y, z \in X \backslash \{x\}$, for all $j \in \{1, ..., n\}$ and for any pair of messages' collections $\mathbf{m}$ and m' with $y\ m_j\ z \Longleftrightarrow y\ m'_j\ z$ and $x\ m_j\ y \Longrightarrow x\ m'_j\ y$, (respectively $y \in m \Longleftrightarrow y \in m'$ and $x \in m_j \Longrightarrow x \in m'_j$), then:*

$x \in V(\mathbf{m}) \Longrightarrow x \in V(\mathbf{m}')$,

$\{x\} = V(\mathbf{m}) \Longrightarrow \{x\} = V(\mathbf{m}')$.

Our monotonicity condition is mild. It just implies that if an agent's message is modified such that it *favours* an alternative x, the voting mechanism responds accordingly. Thus, if x was in the elected set before modifying agent's message in a particular way, then it is also elected under the new message. Similarly, if x is the only elected alternative then it must also be the only elected alternative under the new message.

Finally, we define Approval Voting, which is an example of a voting rule that satisfies Neutrality in alternatives and Monotonicity. We will use it in sections 3 and 4.

Definition 4 *A voting mechanism V is Approval Voting if $M = 2^{\{x,y,z\}}$ and the selected alternatives are those that maximize the number of messages in which they appear.* [9]

Using the above definitions, our goal is to define sincere voting behavior for voting mechanisms. We understand sincere voting behavior as opposed to strategic behavior. The latter

[9] In the spirit of Merril and Nagel (1987), they would claim that AV is our balloting method, while, given our definition, the outcome of the election is decided under Plurality Rule. Our definitions consider both characteristics of voting rules, i.e., the set of available messages and the way to aggregate them.

comprises the possibility of favouring the election of preferred outcomes by misrepresenting *sincere* messages. There exist some conditions that may facilitate the appearance of strategic behavior. For instance, the influence of an individual agent's message on the outcome of the election or the amount of information agents have on others' preferences over alternatives. Our approach is to define sincere voting as the optimal voting strategy when the conditions that ease strategic behavior are diminished. Since such approach requires to study how agents react to uncertainty, we impose the following two assumptions.

Assumption 1 *In the absence of information on other agents' preferences over alternatives, agents believe that any possible combination of others' messages is equally probable. Formally, for all* j *and for all* $m_{-j} \in M^{n-1}$, $p_j(m_{-j}) = (\frac{1}{\sharp M})^{n-1}$ *where* $p_j(m_{-j})$ *is the probability with which agent* j *beliefs other agents will transmit messages* m_{-j}.

Notice that the probability each agent assigns to any combination of messages by other agents clearly depends on the cardinality of the set of messages. In particular, for the case of AV, $\forall j$ and for all $\mathbf{m}_{-j} \in M^{n-1}$, $p_j(\mathbf{m}_{-j}) = \left(\frac{1}{2^{\sharp X}}\right)^{n-1}$.

Assumption 2 *Given agents' beliefs, they maximize their expected utility over alternatives.*

Assumptions 1 and 2 are a simple way for voters to resolve the uncertainty about others' preferences. Notice that we aim to strengthen conditions that eliminate strategic voting and thus, our assumptions refer to cases in which agents can not form clear expectations about how others will vote. Moreover, these assumptions may have a behavioral support. Both assumptions are also the common starting point to define *k-levels of rationality* in the literature on degrees of cognitive complexity which has found certain experimental validity.[10]

3 Sincerity and Informational Conditions

We aim to define sincere voting as the best response strategy when the possibility of strategic behavior is diminished. In particular, in this section we study whether we can define sincere voting behavior as the optimal behavior when voters do not have information on other

[10] See Stahl (1993), Stahl and Wilson (1994, 1995), McKelvey and Palfrey (1995), Costa-Gomes, Crawford and Broseta (2001) and Goeree and Holt (2004).

voters' preferences. Theorem 1 shows that when voting mechanisms are *simple,* eliminating such information uniquely identifies the optimal voting strategy, which we define as sincere. Notice that this result confirms the intuition that under simple voting mechanisms sincerity implies transmitting pieces of ordinal information contained in agents' cardinal preferences over alternatives.

Theorem 1: *Let V be a simple voting mechanism satisfying Neutrality in alternatives and Weak Monotonicity. Assume, there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions* 1 *and* 2 hold*. Then, for any number of agents n, agent i's best response (sincere behavior) is:*

- *For $M = [\overline{m}]$ with $\overline{m}$ being a linear order, the linear order such that $x\ m\ y\ m\ z \Leftrightarrow U_i(x) > U_i(y) > U_i(z)$.*

- *For $M = [\overline{m}]$ with $\overline{m}$ being a subset of alternatives, the subset of the $\sharp m$ alternatives which provide highest utility to agent i.*

Proof: We proceed to prove separately the cases in which the set of messages is the set of linear orders and the cases in which the set of messages is a collection of subsets of X.

- We first consider the case in which $M = \{\text{linear orders over } X\}$. Consider wlog. $U_i(x) > U_i(y) > U_i(z)$. Consider the linear order m such that $x\ m\ y\ m\ z$. We have to prove that m is agent i's best response independently of the number of agents in society.

We show that m is a better response than m', where $y\ m'\ x\ m'\ z$. To see this, let us analyze all the possible situations in which transmitting m' could be beneficial for agent i. Consider any combination of messages by the other agents in society, $\mathbf{m}_{-i}$. Then, given that the voting mechanism is Weakly Monotonic, we know that $x \in V(\mathbf{m}_{-i}, m') \Rightarrow x \in V(\mathbf{m}_{-i}, m)$ and $y \in V(\mathbf{m}_{-i}, m) \Rightarrow y \in V(\mathbf{m}_{-i}, m')$. We also know that $\{x\} = V(\mathbf{m}_{-i}, m') \Rightarrow \{x\} = V(\mathbf{m}_{-i}, m)$ and $\{y\} = V(\mathbf{m}_{-i}, m) \Rightarrow \{y\} = V(\mathbf{m}_{-i}, m')$. The following table specifies all possible outcomes of the election in which declaring m' instead of m may be beneficial for agent i. Any other combination of others' messages always yields a worse outcome when declaring m'. For instance, outcome $\{x, y\}$ whenever i states m yields lower utility than outcome $\{x, z\}$ whenever i states m', since $\frac{U_i(z)+U_i(x)}{2} < \frac{U_i(y)+U_i(x)}{2}$ and thus declaring m'

would not be beneficial. Notice also that not every pair of outcomes can be associated to messages m and m'. For example, the outcome $\{y,z\}$ whenever i states m and outcome $\{x,y\}$ whenever i states m' is not possible since $x \in V(\mathbf{m}_{-i}, m')$ but $x \notin V(\mathbf{m}_{-i}, m)$.

Messages	Outcome							
m	$\{x,z\}$	$\{x,y,z\}$	$\{x,z\}$	$\{x,z\}$	$\{x,y,z\}$	$\{y,z\}$	$\{z\}$	$\{z\}$
m'	$\{x,y\}$	$\{x,y\}$	$\{x,y,z\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y,z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

Notice that under cases 3), 4) and 5), m' yields higher expected utility than m only when $U_i(y) > \frac{U_i(x)+U_i(z)}{2}$.

In order to prove that message m is a better response than m', we show that, for any of the previous cases (associated to a combination of messages by the others), there exists another combination of messages by the others such that:

1. Its probability of occurrence is larger.
2. The benefit from transmitting m instead of m' is larger than the benefit from transmitting m' instead of m in the initial case.

Consider the bijection $\sigma : X \Rightarrow X$, where $\sigma(x) = y, \sigma(y) = x$ and $\sigma(z) = z$. For k, $k \in \{1, ..., 8\}$, consider the combination of others' messages $\mathbf{m}^k_{-i}$ which makes transmitting m' beneficial with respect to m. Consider also the combination of others' messages $\sigma(\mathbf{m}^k_{-i})$. By Assumption 1, individual i assigns the same probability to messages $\sigma(\mathbf{m}^k_{-i})$ and m^k_{-i}. Since $\sigma(m) = m'$ and $\sigma(m') = m$, by Neutrality in alternatives, it must be that $V\left(m, \sigma(\mathbf{m}^k_{-i})\right) = \sigma\left(V\left(m', \mathbf{m}^k_{-i}\right)\right)$ and $V\left(m', \sigma(\mathbf{m}^k_{-i})\right) = \sigma\left(V\left(m, \mathbf{m}^k_{-i}\right)\right)$. Thus, we can compute parallel cases (with equal probability) to those of the previous table. The outcomes of the voting mechanism now are:

Messages	Outcome							
m	$\{x,y\}$	$\{x,y\}$	$\{x,y,z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x,z\}$
m'	$\{y,z\}$	$\{x,y,z\}$	$\{y,z\}$	$\{y,z\}$	$\{x,y,z\}$	$\{x,z\}$	$\{z\}$	$\{z\}$
Cases	1')	2')	3')	4')	5')	6')	7')	8')

For cases k', $k \in \{1, ..., 8\}$, the benefit obtained from declaring m instead of m' is, in all the cases, at least as large as the loss for the corresponding case k. Given that any of these cases has the same probability as its counterpart, m guarantees a expected utility at least as large as m'.

Showing that any other message m'' yields lower expected utility than m follows exactly the same reasoning.[11] Thus, m is agent i's best response.

- We now consider situations in which M is a family of subsets of X. In order to have a simple voting mechanism, there only exist four possibilities:

$$M_1 = \{\phi\}, M_2 = \{X\}, M_3 = \{\{x\}, \{y\}, \{z\}\} \text{ and } M_4 = \{\{x, y\}, \{x, z\}, \{y, z\}\}.$$

M_1 and M_2 are trivial cases given that agents can not decide which message to transmit. Plurality Rule is a prime example of a voting mechanism using M_3. Negative Voting (or Antiplurality) is an example of a voting mechanism using M_4.[12] We here prove the result for M_3 and leave the analogous proof for M_4 for the reader.

Consider $M_3 = \{\{x\}, \{y\}, \{z\}\}$ and *wlog.* $U_i(x) > U_i(y) > U_i(z)$. We first show that transmitting $\{x\}$ is better than transmitting $\{y\}$. Consider any combination of messages in society, $\mathbf{m}_{-i}$. Then, given that the voting mechanism is Weakly Monotonic, we now that $x \in V(\mathbf{m}_{-i}, \{y\}) \Rightarrow x \in V(\mathbf{m}_{-i}, \{x\})$ and $y \in V(\mathbf{m}_{-i}, \{x\}) \Rightarrow y \in V(\mathbf{m}_{-i}, \{y\})$. Additionally, $\{x\} = V(\mathbf{m}_{-i}, \{y\}) \Rightarrow \{x\} = V(\mathbf{m}_{-i}, \{x\})$ and $\{y\} = V(\mathbf{m}_{-i}, \{x\}) \Rightarrow \{y\} = V(\mathbf{m}_{-i}, \{y\})$. The following table, which is in fact equivalent to the case of linear orders, specifies all possible outcomes in which transmitting $\{y\}$ may be beneficial for agent i.

[11] Since all the proofs rely in the same construction, for simplicity we explicitly exclude them. They are, however, available upon request.

[12] One is tempted to think that Negative Voting also uses M_3, given that agents transmit their least preferred alternative. However, for Negative Voting to satisfy weak monotonicity, its messages must be interpreted as transmitting all the alternatives but the least preferred one.

Messages	Outcome							
$\{x\}$	$\{x,y\}$	$\{x,y\}$	$\{x,y,z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x,z\}$
$\{y\}$	$\{y,z\}$	$\{x,y,z\}$	$\{y,z\}$	$\{y,z\}$	$\{x,y,z\}$	$\{x,z\}$	$\{z\}$	$\{z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

The analysis is parallel to the case of linear orders, but proving that $\{x\}$ strictly yields a larger expected utility than $\{y\}$. Reproducing the analysis with strategies $\{y\}$ and $\{z\}$ it can be shown that $\{y\}$ strictly yields a larger expected payoff than $\{z\}$. Thus, transmitting $\{x\}$ strictly yields a larger expected utility than $\{y\}$ and $\{z\}$, concluding the proof for M_3. □

We have therefore shown that our first definition of sincerity is appropriate for simple voting mechanisms. Voters' optimal strategy under no information conditions is to assign votes in a manner that maintains some ordinal information of their true preferences. Notice that in simple mechanisms this behavior does not depend on the weight of an individual agent's vote on the outcome of the election. However, Theorem 2 shows that the absence of information is not enough to guarantee a precise definition of sincerity for complex voting mechanisms. The reason is that best responses may depend, for instance, on the number of agents participating in the election.

Theorem 2: *There exists complex voting mechanisms for which eliminating all information on other voters' preferences does not uniquely identify an optimal voting strategy. In particular, optimal voting may depend on the size of the electorate.*

Proof: We prove it by showing that the optimal voting behavior in a particular complex voting mechanism V varies with the size of the electorate. Let V be Approval Voting. Under AV agents can transmit a large variety of messages. For example, in the case of three alternatives, AV allows for the set of messages $M = 2^{\{x,y,z\}}$. This set is composed by the following four different message types $M_1 = \{\phi\}$, $M_2 = \{X\}$, $M_3 = \{\{x\},\{y\},\{z\}\}$ and $M_4 = \{\{x,y\},\{x,z\},\{y,z\}\}$.

Assume $U_i(x) > U_i(y) > U_i(z)$. Assume, there is no information on agents' preferences over alternatives $X = \{x,y,z\}$ and assumptions 1 and 2 hold. We claim that agent i optimally transmits message $\{x, y\}$ if and only if $U_i(y) \geq \lambda(n)U_i(x)+(1-\lambda(n))U_i(z)$ with $\lambda(n) \in (0,1)$. Otherwise, agent i optimally transmits message $\{x\}$.

Let $U_i(x) > U_i(y) > U_i(z)$. In Theorem 1, we have proved that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\},\{y\},\{z\}\}$ whenever the domain of the voting rule V is M_3. Notice that using the same procedure as the proof of Theorem 1, we can indeed show that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\},\{y\},\{z\}\}$ whenever the domain of the voting rule V is $2^{\{x,y,z\}}$. Given that AV satisfies $Neutrality$ in alternatives and $Weak\ Monotonicity$, we can ensure that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\},\{y\},\{z\}\}$. A similar argument applies for M_4.

Therefore, the only messages worth considering are $\{\phi,\ X, \{x\}, \{x,y\}\}$. We first show that voting $\{x\}$ is always better than voting X (respectively ϕ). Suppose that voting X (respectively ϕ) leads to have $S \neq \{x\}$ as the set of elected alternatives.[13] If $x \in S \neq \{x\}$, then transmitting $\{x\}$ leads to have x as the unique elected outcome, which obviously dominates S for agent i. If $x \notin S$, transmitting $\{x\}$ leads either to have S as the set of elected outcomes or to have $S \cup \{x\}$ as the set of elected outcomes. This is clearly preferable to the outcome obtained when transmitting X (respectively ϕ). Thus we focus on $\{x,y\}$ and $\{x\}$. We present here the situations in which $\{x,y\}$ and $\{x\}$ could yield different outcomes:

$Messages$	Outcome					
$\{x\}$	$\{x\}$	$\{z\}$	$\{x,y\}$	$\{x,z\}$	$\{y,z\}$	$\{x,y,z\}$
$\{x,y\}$	$\{x,y\}$	$\{y,z\}$	$\{y\}$	$\{x,y,z\}$	$\{y\}$	$\{y\}$
Cases	1)	2)	3)	4)	5)	6)

The previous table shows all possible combinations of others agents' messages in which messages $\{x,y\}$ and $\{x\}$ yield different outcomes. In order for these situations to occur, the distribution of other agents' messages must satisfy the following conditions:

[13]Obviously if $S = \{x\}$ then transmitting $\{x\}$ has the same effect on the election of outcomes.

$$\begin{array}{ll} 1)\ a_x = a_y > a_z - 1 & 4)\ a_x + 1 = a_y + 1 = a_z \\ 2)\ a_x + 1 < a_y + 1 = a_z & 5)\ a_x + 1 < a_y = a_z \\ 3)\ a_x = a_y - 1 > a_z - 1 & 6)\ a_x + 1 = a_y = a_z, \end{array}$$

where a_k represents the number of times alternative k appears in other agents' messages, excluding agent i.

- Under Assumption 1, the probabilities P_q of each of these six conditions are:

$$P_1 = \frac{\sum_{t=0}^{n-1}\sum_{s=0}^{t}\binom{n-1}{t}\binom{n-1}{t}\binom{n-1}{s}}{(2^3)^{n-1}} \qquad P_2 = \frac{\sum_{t=2}^{n-1}\sum_{s=0}^{t-2}\binom{n-1}{s}\binom{n-1}{t-1}\binom{n-1}{t}}{(2^3)^{n-1}}$$

$$P_3 = \frac{\sum_{t=1}^{n-1}\sum_{s=0}^{t-1}\binom{n-1}{t-1}\binom{n-1}{t}\binom{n-1}{s}}{(2^3)^{n-1}} \qquad P_4 = \frac{\sum_{t=1}^{n-1}\binom{n-1}{t-1}\binom{n-1}{t-1}\binom{n-1}{t}}{(2^3)^{n-1}}$$

$$P_5 = \frac{\sum_{t=2}^{n-1}\sum_{s=0}^{t-2}\binom{n-1}{s}\binom{n-1}{t}\binom{n-1}{t}}{(2^3)^{n-1}} \qquad P_6 = \frac{\sum_{t=1}^{n-1}\binom{n-1}{t-1}\binom{n-1}{t}\binom{n-1}{t}}{(2^3)^{n-1}}$$

Hence, the expected utility of messages $\{x\}$ and $\{x, y\}$ under assumption 1 can be expressed in terms of these probabilities. For message $\{x\}$, the expected utility equals:

$$P_1U_i(x) + P_2U_i(z) + P_3\left(\frac{U_i(x)+U_i(y)}{2}\right) + P_4\left(\frac{U_i(x)+U_i(z)}{2}\right)$$
$$+P_5\left(\frac{U_i(y)+U_i(z)}{2}\right) + P_6\left(\frac{U_i(x)+U_i(y)+U_i(z)}{3}\right)$$

whereas for message $\{x, y\}$ the expected utility equals:

$$P_1\left(\frac{U_i(x)+U_i(y)}{2}\right) + P_2\left(\frac{U_i(y)+U_i(z)}{2}\right) + P_3U_i(y) +$$
$$P_4\left(\frac{U_i(x)+U_i(y)+U_i(z)}{3}\right) + P_5U_i(y) + P_6U_i(y)$$

Therefore, using Assumption 2, the condition for preferring to transmit $\{x, y\}$ has to be that it yields a higher expected value than transmitting $\{x\}$, i.e.,

$$\left(\frac{P_1}{2} + \frac{P_3}{2} + \frac{P_4}{6} + \frac{P_6}{3}\right) U_i(x) + \left(\frac{P_2}{2} + \frac{P_4}{6} + \frac{P_5}{2} + \frac{P_6}{3}\right) U_i(z) \leq \left(\frac{P_1}{2} + \frac{P_2}{2} + \frac{P_3}{2} + \frac{P_4}{3} + \frac{P_5}{2} + \frac{2P_6}{3}\right) U_i(y)$$

Denoting:

$$f(n) = \left(\frac{P_1}{2} + \frac{P_3}{2} + \frac{P_4}{6} + \frac{P_6}{3}\right)$$
$$g(n) = \left(\frac{P_2}{2} + \frac{P_4}{6} + \frac{P_5}{2} + \frac{P_6}{3}\right)$$
$$h(n) = \left(\frac{P_1}{2} + \frac{P_2}{2} + \frac{P_3}{2} + \frac{P_4}{3} + \frac{P_5}{2} + \frac{2P_6}{3}\right),$$

we can express the previous inequality as $\frac{f(n)}{h(n)} U_i(x) + \frac{g(n)}{h(n)} U_i(z) \leq U_i(y)$. Since $f(n) + g(n) = h(n)$, we only need to consider the function $\lambda(n) = \frac{f(n)}{h(n)}$ to conclude the proof. Since any $P_q : q = 1, ..., 6$ is different from zero, it follows that $\lambda(n) \neq 1$ and $\lambda(n) \neq 0$. $\square$

Theorem 2 shows that when voting mechanisms are not simple, the result in Theorem 1 does not hold, as optimal behavior does not only depend on the available information on candidates' preferences. In the case of ÁV, whether the alternative yielding second highest utility to an agent is included in her transmitted message depends on its *relative* cardinal utility with respect to the utilities yielded by the most and least preferred alternatives. Such dependence rests on the weights measured by the function $\lambda(n)$, which varies with the size of the electorate n. For example, if $U(x) = 0.9, U(y) = 0.7$ and $U(z) = 0.1$, basic calculus shows that an agent optimally transmits message $\{x\}$ when the size of the electorate is 2. On the other hand, the same agent transmits message $\{x, y\}$ when the size of the electorate is 3. It seems unreasonable that how sincere a voting strategy is depends on the size of the electorate. Thus, in the following subsection we further impose conditions to diminish the possibility of strategic voting in order to obtain sincere behavior.

4 Sincerity and the Size of the Electorate

The previous section shows that our first definition of sincerity, although valid for a large class of frequently used voting mechanisms, is not valid for complex voting mechanisms, as optimal behavior does not only depend on the available information on candidates' preferences.

The influence of an individual agent's vote on the outcome of an election diminishes the bigger the size of an electorate. Notice that in the proof for Theorem 2, we already identified the size of the electorate as a source of heterogeneity in optimal behavior for approval voting, which is a particular example of a complex voting mechanism.

Following our methodological approach we consider a second definition of sincerity by eliminating other sources of manipulability. According to our definition, sincere voting is identified as the optimal strategy when voters do not have information on other voters' preferences and the size of the electorate tends to infinity.

We finally check the applicability of this definition of sincere voting to a particular case: Approval Voting. Previous literature has discussed at least two *ad-hoc* definitions of sincerity for AV. The first one specifies that if one alternative is voted in the top set (approved), all alternatives that yield higher cardinal utility to the individual should also be included in the top set to be considered sincere.[14] Notice that this definition is somewhat weak as several messages would then be considered sincere. For the case of three alternatives and using the numerical example shown in the introduction, $\{x, y, z\}$ (meaning "all alternatives are approved"), ϕ (meaning "all alternatives are disapproved"), $\{x\}$ and $\{x, y\}$ would all be considered sincere under this weak definition.

Translating this argument to cardinal utilities and using our notation, we establish a definition of *weak sincerity*:

Definition 5 *Agent i's message m is Weak Sincere under AV if for all x, y such that $U_i(x) > U_i(y)$, $y \in m$ implies $x \in m$.*

We refer to such definition as *weak* because it does not determine a unique message as sincere, which may be an appealing property.

Corollary 6 *Let V be Approval Voting. Assume, there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions 1 and 2 hold. Then agent i's optimal behavior satisfies the weak definition of sincerity.*

Proof. Notice that following Theorem 2, the only possible optimal messages are $\{x\}$ and $\{x, y\}$, and thus, it easily follows that the weak definition of sincerity always holds. ■

[14]See Brams and Fishburn (1981) and Niemi (1984) .

A second and more restrictive definition identifies sincerity with voting for those alternatives that yield the individual more cardinal utility than the average of all alternatives.[15] In the numerical example, the only sincere voting representation would then be to vote for both x and y (i.e. $\{x, y\}$) since both provide more cardinal utility than the average ($0.8 > 0.47$ and $0.5 > 0.47$).

Using our notation and for the case of three alternatives, such definition can be expressed as follows:

Definition 7 *Agent i's message m is Strong Sincere under AV if for all $x \in X$:*

$$x \in m \Leftrightarrow U_i(x) \geq \frac{1}{3} \sum_{y \in X} U_i(y).$$

The strong definition of sincere voting under AV implies voting for those alternatives that yield more utility than the average of utilities. This definition, although intuitively appealing, has not been given a complete formal justification. In particular, it has been defined under a restrictive set of assumptions, such as imposing specific probabilities on the number of votes each alternative receives. As in the previous subsections, we obtain our results by precisely calculating these probabilities using a cognitive process based only on initial beliefs over individual votes. In the remainder of the paper, we show that the best response of an agent under conditions that diminish the possibility of behaving strategically is precisely voting for those alternatives that yield more than the average of utilities. Therefore, we provide stronger support for this second definition of sincerity, which uniquely determines which message is sincere in AV.

In the proof for Theorem 2, we identified agents' best response in AV in the absence of information. Under such conditions, to include in the transmitted message the alternative yielding the second highest utility partially depends on the size of the electorate through the weighting function $\lambda(n)$. Theorem 3 determines the limit of $\lambda(n)$ when n goes to infinity.

Theorem 3: *Let V be Approval Voting. Assume there is no information on agents'*

[15] See, for instance, Weber (1978), Hoffman (1982), Merrill (1983), Merrill and Nagel (1987) who present some results characterizing this behavior under a very restrictive set of assumptions.

preferences over alternatives $X = \{x, y, z\}$ *and assumptions* 1 *and* 2 hold, *then:*

$$\lim_{n \longrightarrow \infty} \lambda(n) = \frac{1}{2}.$$

Proof: Following notation introduced in the proof of Theorem 2, we want to prove that $\lim_{n \longrightarrow \infty} \lambda(n) = \lim_{n \longrightarrow \infty} \frac{f(n)}{h(n)} = \frac{1}{2}$. Given that

$$f(n) + g(n) = h(n),$$

this is equivalent to proving,

$$\lim_{n \longrightarrow \infty} \frac{f(n) - g(n)}{h(n)} = 0.$$

Notice that $f(n)$, $g(n)$ and $h(n)$ are functions of the probabilities of each of the six possible races between alternatives. Substituting their values and using basic calculus, we obtain that,

$$\frac{f(n) - g(n)}{h(n)} = \frac{\frac{1}{2}(P_1 - P_5)}{h(n)} + \frac{\frac{1}{2}(P_3 - P_2)}{h(n)} \leq \frac{\frac{1}{2}(P_1 - P_5)}{\frac{1}{2}P_5} + \frac{\frac{1}{2}(P_3 - P_2)}{\frac{1}{2}P_2} = \frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2}.$$

Hence, given the positivity of $(P_1 - P_5)$ and $(P_3 - P_2)$ (see below the combinatorial decomposition), $\frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2}$ is an upper bound for $\frac{f(n)-g(n)}{h(n)}$. Thus, proving

$$\lim_{n \longrightarrow \infty} \left(\frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2} \right) = 0,$$

implies $\lim_{n \longrightarrow \infty} \frac{f(n)-g(n)}{h(n)} = 0$.

Actually, we here prove that $\lim_{n \longrightarrow \infty} \frac{P_1 - P_5}{P_5} = 0$ and $\lim_{n \longrightarrow \infty} \frac{P_3 - P_2}{P_2} = 0$, which is stronger than what is needed. We start by proving that $\lim_{n \longrightarrow \infty} \frac{P_1 - P_5}{P_5} = 0$. Notice that the combinatorial expressions for P_1 and P_5 appear in the proof of Theorem 2.

Consider the following two standard properties of combinatorial numbers which apply to any non-negative integers k, i for $k \geq i$:

Property 1: $\begin{pmatrix} k \\ i \end{pmatrix} + \begin{pmatrix} k \\ i-1 \end{pmatrix} = \begin{pmatrix} k+1 \\ i \end{pmatrix}$.

Property 2 (symmetry): $\begin{pmatrix} k \\ i \end{pmatrix} = \begin{pmatrix} k \\ k-i \end{pmatrix}$.

In order to prove $\lim\limits_{n\longrightarrow\infty}\frac{P_1-P_5}{P_5}=0$, we first use Property 1.

$$\lim_{n\longrightarrow\infty}\frac{P_1-P_5}{P_5}=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n-1}{t}+\binom{n-1}{t-1}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}=$$

$$=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n}{t}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}.$$

We only consider the cases in which n is even (a similar reasoning would follow for the case in which n is odd). From the last expression and using Property 2 we can derive,

$$\lim_{n\longrightarrow\infty}\frac{P_1-P_5}{P_5}=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n}{t}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}=\lim_{n\longrightarrow\infty}\frac{B_1+B_2+B_3}{C_1+C_2+C_3}$$

where,

$$B_1=\sum_{t=2}^{\frac{n-2}{2}}\binom{n-1}{t}^2\left[\binom{n}{t}+\binom{n}{n-t-1}\right],$$

$$B_2=\binom{n-1}{n-2}^2\binom{n}{n-2},$$

$$B_3=\binom{n-1}{n-1}^2\binom{n}{n-1}.$$

$$C_1 = \sum_{t=2}^{\frac{n-2}{2}} \binom{n-1}{t}^2 \left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s}\right],$$

$$C_2 = \binom{n-1}{n-2}^2 \sum_{s=0}^{n-4} \binom{n-1}{s},$$

$$C_3 = \binom{n-1}{n-2}^2 \sum_{s=0}^{n-3} \binom{n-1}{s}.$$

Notice that $\lim_{n\longrightarrow\infty} \frac{B_1+B_2+B_3}{C_1+C_2+C_3}$ can be expressed as $\lim_{n\to\infty} \frac{\sum_{t=1}^{T} b_t(n)}{\sum_{t=1}^{T} c_t(n)}$, with $b_t(n)$ and $c_t(n)$ being products of combinatorial numbers.

Notice also that $\lim_{n\to\infty} \frac{\sum_{t=1}^{T} b_t(n)}{\sum_{t=1}^{T} c_t(n)} \leq \lim_{n\to\infty} \frac{b_{k_n}}{c_{k_n}}$ where k_n is the value that maximizes $\frac{b_t(n)}{c_t(n)}$ for dimension n. Therefore, it is sufficient to prove that $\lim_{n\to\infty} \frac{b_{k_n}}{c_{k_n}} = 0$.

We now identify the value k_n. It is not difficult to see that this value has to belong to B_1 and C_1. With respect to B_1, notice that by Property 2, $\binom{n}{n-t-1} = \binom{n}{t+1}$. Applying Properties 1 and 2 to B_1, we obtain:

$$B_1 = \sum_{t=2}^{\frac{n-2}{2}} \binom{n-1}{t}^2 \binom{n+1}{t+1}.$$

Hence, one of the values can be expressed:

$$\frac{b_t}{c_t} = \frac{\binom{n-1}{t}^2 \binom{n+1}{t+1}}{\binom{n-1}{t}^2 \left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s}\right]} = \frac{\binom{n+1}{t+1}}{\left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s}\right]}$$

and therefore,

$$\frac{b_{t+1}}{c_{t+1}} = \frac{\binom{n+1}{t+2}}{\left[\sum_{s=0}^{t-1}\binom{n-1}{s} + \sum_{s=0}^{n-t-4}\binom{n-1}{s}\right]}.$$

Notice also that $t+1 \leq \frac{n-2}{2}$ implies $t+2 \leq \frac{n}{2} \leq \frac{n+1}{2}$. Thus, $\binom{n+1}{t+2} \geq \binom{n+1}{t+1}$. With respect to the denominators, it is clear that

$$\left[\sum_{s=0}^{t-2}\binom{n-1}{s} + \sum_{s=0}^{n-t-3}\binom{n-1}{s}\right] - \left[\sum_{s=0}^{t-1}\binom{n-1}{s} + \sum_{s=0}^{n-t-4}\binom{n-1}{s}\right] = \binom{n-1}{n-t+3} - \binom{n-1}{t-1}.$$

By property 2 this is equivalent to $\binom{n-1}{t+2} - \binom{n-1}{t-1} > 0$, and therefore $\frac{b_t}{c_t} \leq \frac{b_{t+1}}{c_{t+1}}$. Therefore, the maximal value is obtained when $t = \frac{n-2}{2}$.

$$\lim_{n\to\infty}\frac{b_t}{c_t} \leq \lim_{n\to\infty}\frac{\binom{n+1}{\frac{n}{2}}}{\left[\sum_{s=0}^{\frac{n-6}{2}}\binom{n-1}{s} + \sum_{s=0}^{\frac{n-4}{2}}\binom{n-1}{s}\right]}.$$

Applying property 2 we obtain

$$\lim_{n\to\infty}\frac{b_t}{c_t} \leq \lim_{n\to\infty}\frac{\binom{n-1}{\frac{n}{2}}}{\sum_{s=0}^{n-1}\binom{n-1}{s} - \binom{n-1}{\frac{n-2}{2}} - \binom{n-1}{\frac{n}{2}} - \binom{n-1}{\frac{n+2}{2}}} =$$

$$\lim_{n\to\infty} \frac{\binom{n+1}{\frac{n}{2}}}{2^{n-1} - \binom{n-1}{\frac{n-2}{2}} - \binom{n-1}{\frac{n}{2}} - \binom{n-1}{\frac{n+2}{2}}} = 0.$$

The proof for $\lim_{n\to\infty} \frac{P_3 - P_2}{P_2} = 0$ is similar, and thus we omit it. This concludes the proof. □

Theorem 3 says that as the size of the electorate increases, agents' best response in AV consists in voting for those alternatives that yield more than the average of utilities. Given that we have eliminated the possibly most important components of strategic behavior, namely information on others' preferences and the weight of an individual vote in determining the outcome, we interpret such best response as sincere voting behavior under AV.

Notice that previous attempts to define sincere behavior in AV did not differentiate between the implications of Theorems 2 and 3.[16] The reason is that they assumed that the probability of a tie between the number of votes that two alternatives received was equal to the probability of one of the alternatives surpassing the other by just one vote. As a by-product of our Theorem 3, we have shown that such assumption only holds true in the limit.

Thus, in this section we have suggested a possible way of complementing the restrictions imposed in section 2 to obtain a unique definition of sincerity for complex voting mechanisms. Finally, we have also tested its validity for the particular case of AV.

5 Discussion

Identifying sincere voting behavior under a variety of voting rules is an important starting point in the discussion of adopting new voting mechanisms. A definition of sincerity is almost straightforward when simple voting mechanisms are considered. However, we have seen that under complex voting mechanisms such as Approval Voting defining sincerity is cumbersome. We conjecture that the difficulty in defining sincerity arises as a consequence of the presence of several message types in complex mechanisms.

[16] See for instance, Merrill (1979) and Hoffman (1982).

Our approach to define sincere voting behavior consists in opposing sincere behavior to strategic behavior. We methodologically contribute to obtain a formal definition of sincerity by omitting the elements that facilitate strategic behavior; namely, by increasing the size of the electorate and by eliminating information on other agents' preferences. The optimal behavior obtained under such conditions is thus what we define as sincere voting behavior.

Our aim in this paper has been to provide a definition of sincere voting behavior. Nevertheless, this is not equivalent to identifying sincere voting from the results of an election. Knowledge on the cardinal value that the alternatives yield to the voters is required in empirical tests of our results. An experiment controlling for such utility values may be a worthwhile avenue to explore how individuals vote when informational conditions or the weight of their votes are changed.

Finally, we have also contributed to the approval voting literature. Notice that following our approach, our definition of sincerity for AV coincides with the previously provided strong definition of sincerity. Our technical contribution consists in calculating optimal voting behavior by assessing explicitly the probability of each of the possible races between alternatives that can occur instead of assuming they all have the same probability. Therefore, we have provided stronger support to an intuitive definition of sincerity when agents have cardinal utilities over three alternatives.

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