Georgescu-Roegen versus Solow/Stiglitz and the Convergence to the Cobb-Douglas

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Abstract: The value of the elasticity of substitution of capital for resources is a crucial element in the debate over whether continual growth is possible. It is generally held that the elasticity has to be at least one to permit continual growth and that there is no way of estimating this outside the range of the data. This paper presents a model in which the elasticity is determined endogenously and may converge to one. It is concluded that the general opinion is wrong: that the possibility of continual growth does not depend on the exogenously given value of the elasticity and that the value of the elasticity outside the range of the data can be studied by econometric methods.

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I. Introduction.

This paper is a contribution to the debate over whether it is possible to have continually growing consumption with a finite amount of resources by substituting capital for resources in production. In the 1974 Review of Economic Studies symposium, Stiglitz wrote the essential neoclassical treatment. He showed that, with population growth, technical progress, exhaustible resources and a Cobb-Douglas production function, the competitive/optimal path would, depending on the parameter values, be characterized by growing per capita consumption. This paper together with one by Solow, which also used a Cobb-Douglas production function, have been the object of numerous attacks, the latest of which is the Georgescu-Roegen versus Solow/Stiglitz forum organized by Daly (1997a,b). Generally two claims have been made: first, that econometric methods are inadequate for the determination of the value of the elasticity at the crucial high capital-resource ratios since these lie outside the range of current data and second, that materials balance and thermodynamic methods, which overcome this problem, show that the elasticity will eventually be less than one and that technical progress will be insufficient to permit continual growth of per capita income. In the present paper a model is formulated in which research can be directed at either maintaining a high elasticity of substitution or increasing the efficiency of the factors of production. It is shown that the production function will converge to either a Cobb-Douglas or a Leontief. The conclusions are first,

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that econometric methods can, by confirming this tendency, predict the value of the elasticity outside the range of the data; second, that the conclusion of the materials balance and thermodynamic arguments that the elasticity must eventually be less than one is incorrect; and finally, that, while resource scarcity will not slow growth by lowering the elasticity, it may do so indirectly because the cost of maintaining the elasticity lowers the rate of growth of factor productivity.

The paper is organized as follows: The model and the results are set out in section II and the debate is reviewed in section III. The implications of the model for the debate are given in the conclusion, section IV.

II. The Model and Results.

The two components of the model are a CES production function in land and labour and a generalized innovation possibility frontier (IPF) which shows how an increase in effective quantities of factors can be had only at the cost of lower substitutability.[2] The time paths of the factor growth rates and the elasticity of substitution are generated by choosing the values of these variables in each period to maximize the growth of income.

The IPF may be interpreted as a compressed form of an endogenous growth model. Labour is a human capital-physical capital aggregate and land is renewable resources. An overlapping generations structure generates savings which are devoted either to research or factor accumulation. This interpretation is approximate since, according to the usual assumptions, the development of these models depends on the stocks of the specific factors which are absent from the IPF. I think that these stocks could be added, with difficulty, to the model without changing the basic results. The IPF cannot be interpreted as a compressed form of an exhaustible resource model since the stock of the resource plays a crucial role and it too is absent from the IPF. This is commented on at length in the conclusion.

The nature of substitution and technical progress impose restrictions on the way in which a change in the elasticity affects the production function. This is illustrated in figure 1. The lines in the figure that go through the point A are isoquants of the production function. The point A is the factor input bundle of the previous period. An increase in the elasticity of substitution shifts the isoquant in the manner indicated in 1a.

Figure 1 about here.

There are two conditions that this shift must satisfy : a) The point A must not move. This is because a change in the elasticity of substitution should not affect production if no substitution takes place between periods. b) The slope of the isoquant at the point A should not change. If it did, it would imply that research aimed at increasing the elasticity

[2]It is more general than that of Kennedy (1964) since it includes substitution as well as factor augmentation.

actually made production less efficient for some input ratios, which is impossible. This is illustrated in figure 1b.

The CES production function is given by

$$Y = \alpha[\delta L^{-\rho} + (1-\delta)M^{-\rho}]^{-1/\rho}$$

where Y, L, and M are output, labour and land; $\alpha > 0$, $1 > \delta > 0$, and $-1 \leq \rho \leq \infty$ are the efficiency, distribution and elasticity parameters; and $\sigma = 1/(1+\rho)$ is the elasticity. The condition that the slope of the isoquant does not change at the previous endowment point is

$$\frac{1-\delta}{\delta}\left(\frac{M}{L}\right)^{-\rho} = \frac{1-\delta_+}{\delta_+}\left(\frac{M}{L}\right)^{-\rho_+} \tag{1}$$

where the variables of the previous period bear no subscript and those of the current period bear a $_+$. The condition that the output remains unchanged at the previous inputs is

$$\alpha[\delta L^{-\rho}+(1-\delta)M^{-\rho}]^{-1/\rho} = \alpha_+[\delta_+ L^{-\rho_+}+(1-\delta_+)M^{-\rho_+}]^{-1/\rho_+}. \tag{2}$$

Let $\gamma = \frac{1-\delta}{\delta}\left(\frac{M}{L}\right)^{-\rho}$ and $\hat{L} = \frac{L_+}{L}$ etc. γ is the relative share of land and $\hat{L}$ is one plus the rate of growth of labour etc., called the rate of growth for short. Using (1) and (2)

$$\hat{Y} = \left[\frac{1}{1+\gamma}\hat{L}^{-\rho_+} + \frac{\gamma}{1+\gamma}\hat{M}^{-\rho_+}\right]^{-1/\rho_+} \equiv \underline{H}(\hat{L}, \hat{M}, \rho_+, \gamma).$$

In addition, (1) implies

$$\gamma_+ = \gamma\left(\frac{\hat{L}}{\hat{M}}\right)^{\rho_+}.$$

The IPF has the form $G(\hat{L}, \hat{M}) = F(\rho)$ where, from this point on, ρ without a subscript will refer to the current period and where G and F satisfy conditions (3),

$$F > 0;\ F' > 0;\ F'' < 0;\ \lim_{\rho\to\infty} F = c.$$

(3) $G_i > 0$, $i = \hat{L}, \hat{M}$; G is smoothly convex and linear homogeneous; and

$$\lim_{\hat{l}\to\infty} G_{\hat{L}} / G_{\hat{M}} = \infty,\ \lim_{\hat{l}\to 0} G_{\hat{L}} / G_{\hat{M}} = 0.$$

where c is a positive constant, G_i is the i^{th} partial of G, $\hat{l} \equiv \hat{L} / \hat{M}$ and smoothly convex means that the derivitive of the slope level curve exists for $0 < \hat{l} < \infty$. Letting $\tilde{\rho} \equiv -\rho$ represent the elasticity and measuring cost positively, $-dF/d\tilde{\rho} = F' > 0$ is the marginal cost of elasticity in terms of productivity which, because $d(-dF/d\tilde{\rho})/d\tilde{\rho} = -F'' > 0$, is increasing. Even when the elasticity is zero only limited factor growth is possible. Finally the marginal cost of $\hat{L}$ in terms of $\hat{M}$, $G_{\hat{L}} / G_{\hat{M}}$, is increasing in $\hat{L}/\hat{M}$ and reaches infinity when the ratio does.

The model is now given by

$$(4) \qquad \begin{cases} \underset{\hat{L},\hat{M},\rho}{\text{Max}}\, \underline{H}\left(\hat{L},\hat{M},\rho,\gamma\right) \text{ suchthat } G\left(\hat{L},\hat{M}\right) = F(\rho). \\ \gamma_+ = \gamma\left(\hat{L}/\hat{M}\right)^{\rho}. \end{cases}$$

where G and F satisfy (3). It is a difference equation: given γ, the solution to the problem yields $\hat{L}$, $\hat{M}$ and ρ, from which γ_+ is calculated.[3]

A steady state occurs when γ is constant over time[4]; this implies that $\hat{L}$, $\hat{M}$ and ρ are constant over time as well. There are three types of steady states: the ones that occur when $\gamma = 0$ or ∞; the one with $\rho = \infty$ and $\hat{l} = 1$; and the ones with $\rho = 0$. This is explained in the following three propositions.[5]

<u>Proposition 1.</u> For the model (4) let $0 < \gamma < \infty$. There is a unique steady state value of γ, γ^* such that $\rho = \infty$ and $\hat{l} = 1$. All other steady state values of γ must have $\rho = 0$.

<u>Proposition 2.</u> For the model (4), $\gamma = 0$ and $\gamma = \infty$ are steady states with $\hat{l} = \infty$ and 0 respectively.

<u>Proposition 3.</u> For the model (4) suppose there exists γ^{**} such that $\rho = 0$ is the solution to the maximum problem of (4). Then $\hat{l}$ is bounded away from 0, 1 and ∞ and γ^{**} is a steady state.

Each of these types of steady states deserves a comment. The ones associated with $\gamma = 0$ and ∞ will be shown to be unstable and thus lack importance. Next, the steady state with both factors growing at the same rate and a Leontief production function is intuitively satisfying since, in this case, there is no substitution of one factor for the other and thus no need to expend resources maintaining a positive elasticity. Finally, the result that the only other possible type of steady state must have a Cobb-Douglas production

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[3]One might favor an alternative model in which the change in elasticity was costly and discounted total income was maximized. This does not appear clearly better to me since a new technique might easily imply that methods of substitution had to be found from scratch and, furthermore, the uncertainty over the long run character of technical advance makes it unlikely that the effect of current research on factor availability in the far future will weigh heavily in present decisions.

[4]From this point on γ may also refer to the relative share of land in general.

[5]All proofs are given in the appendix.

function is both unexpected and central to the debate over the long run size of the elasticity. The intuition of this important result is straight forward. Since the slope of the isoquant at the previous endowment point cannot change, neither can the relative share. But, because the production function is Cobb-Douglas, the relative share cannot change during the movement to the new endowment point either. Thus it remains constant and the system is in steady state.

The analysis of the existence of a steady state with $\rho = 0$ and of the stability properties generally is difficult and is set out in the context of the specialized IPF with

$$G(\hat{L},\hat{M}) = (\hat{L}^2+\hat{M}^2)^{1/2}. \tag{5}$$

Note that the conditions imposed by (3) on G are still satisfied. This IPF permits the maximum problem of (4) to partially solved for $\hat{L}$ and $\hat{M}$ as functions of γ and ρ and the corresponding specialized model to be written as

$$\max_{\rho} H(\rho, \gamma) \tag{6}$$

$$\gamma_{+} = \gamma^{2/(2+\rho)} \tag{7}$$

where $h(\rho,\gamma) \equiv \left[\left(\frac{1}{1+\gamma}\right)^{\frac{2}{2+\rho}} + \left(\frac{\gamma}{1+\gamma}\right)^{\frac{2}{2+\rho}}\right]^{\frac{\rho+2}{2\rho}}$ and $H(\rho,\gamma) \equiv h(\rho,\gamma)F(\rho)$.

The FOC of (6) gives ρ as a function of γ, $\rho(\gamma)$. The conditions that can be imposed on F to ensure the existence of steady states with $\rho = 0$ are described by the following proposition.

<u>Proposition 4.</u>

a) If F satisfies $F'(0)/F(0) < 0.0549$ then $\rho(\gamma) = 0$ is satisfied by four values of γ: $1/\gamma_2$, $1/\gamma_1$, γ_1, and γ_2 which obey $1/\gamma_2 < 1/\gamma_1 < 1 < \gamma_1 < \gamma_2$, and conversely.

b) If, in addition, F satisfies $-F''(0)/F'(0) > 1$ then $\rho = 0$ is a local maximum at these values of γ.

c) If, in addition to the condition of a), F satisfies $h(0,\gamma)/h(\rho,\gamma) > F(\rho)/F(0)$ for $-1 \leq \rho \leq \infty$, $\rho \neq 0$, then $\rho = 0$ is a global maximum at these values and they are steady states of the model of (4) with G given by (5).

The conditions of a) and b) are the important ones. It is clear that one needs a global condition like that of c), but it gives very little information about what is demanded of $F(\rho)$ and, in addition, for local stability only the local condition of b) is used. With regard to the condition of a), F'/F can be interpreted as the marginal cost of elasticity in terms of a percentage change in factor augmentation. If this is too high at $\rho = 0$, it will always pay to reduce the elasticity at this point. With regard to the condition of b), writing $H = h(\rho,\gamma)F(\rho)$ shows that a large value of $-F''(0)/F'(0)$ will ensure that $H_{\rho\rho}(0,\gamma) < 0$.

The stability characteristics of the seven steady states are given in the following proposition.

Proposition 5. For the model (4) with G given by (5), suppose that all seven steady states exist. In this case:

a) if $-F''(0)/F'(0) > 1$ then 0, $1/\gamma_1$, γ_1 and ∞ are unstable, and $1/\gamma_2$ and γ_2 are locally stable; and

b) if $\lim_{\gamma \to 1, \rho \to \infty} \frac{d\left[\rho^2 F'(\rho)/\ F(\rho)\right]}{d\rho} \frac{\rho^2}{\ln \gamma} \neq 0$ then 1 is locally stable.

A schematic phase diagram with these characteristics is given in figure 2. A similar phase diagram, which arises from a specific example of the function F which satisfies all the required conditions, is given in Petith (1998). The equilibria at B, D, and F are stable,

Figure 2 about here.

those at A, C, E and G are not. The equilibria at B and D correspond to a Cobb-Douglas while that at D corresponds to a Leontief. The type of production function that eventually emerges depends on the initial conditions: if $\gamma < 1/\gamma_1$ or $\gamma > \gamma_1$ then eventually a Cobb-Douglas will emerge with land and labour growing at different rates; while if $1/\gamma_1 < \gamma < \gamma_1$ then a Leontief will emerge with land and labour growing at the same rate.

Finally a parameter C, which indicates the level of resource scarcity, is inserted into the IPF and its effect on the model analysed. The conventional form for the IPF would be

$$\tilde{G}(\hat{L}, \hat{M}, C_p) + \tilde{F}(\tilde{\rho}, C_e) = S$$

where $\tilde{G}$ and $\tilde{F}$ are the cost functions, S denotes the resources available, $\tilde{\rho} = -\rho$ is the measure of the elasticity produced and C_p and C_e are the cost parameters for productivity and elasticity. On the other hand the IPF analysed will be

$$G(\hat{L}, \hat{M}) = F(\rho, C)/C \,. \tag{8}$$

With this form, the analysis of the effects of C is straight forward. On the other hand a considerable effort must be made to justify this in terms of the conventional form.

First suppose that C is interpreted as C_p. Subtracting $\tilde{F}$ from both sides of the first form and multiplying both sides of the second by C shows that $G(\hat{L}, \hat{M})C$ corresponds to $\tilde{G}(\hat{L}, \hat{M}, C)$ and $F(\rho,C)$ to $S - \tilde{F}(\tilde{\rho})$. The interpretation of $F(\rho,C)$ is the resources that are left over after $\tilde{\rho}$ has been produced. In this case C should be dropped from F.

<u>Remark 1.</u> If C is interpreted as the cost of productivity, then the implied cost functions $\tilde{G}$ and $\tilde{F}$ have the following properties: $\tilde{G}_i > 0$, $\tilde{G}_{ii} > 0$, $\tilde{G}_C > 0$, $\tilde{F}' > 0$ and $\tilde{F}'' > 0$. In addition F(ρ,C) does not depend on C.

That is, marginal costs are positive and increasing, the cost of productivity increases in C and the resources that are left over after $\tilde{\rho}$ of elasticity is produced are not affected by the cost of producing productivity. The remark follows directly from the properties of G and F.

Next suppose that C is interpreted as C_e. Now subtracting $\tilde{F}$ from both sides of the first form shows that G corresponds to $\tilde{G}$ and F(ρ,C)/C has the interpretation of the resources left over after $\tilde{\rho}$ of elasticity has been produced. This time considerable care must be taken with the form of F to ensure that the implied cost function $\tilde{F}$ has the normal properties. In addition it will be convenient to allow for the possibility that $\tilde{F} = 0$ for $\tilde{\rho}$ less than a given quantity, that is that a certain amount of elasticity is given free by nature.

Define:

$$\rho^*(C) = \rho \text{ such that } F(\rho,C)/C = S \text{ and } F(\rho',C)/C < S \text{ for } \rho' < \rho.$$

That is - ρ*(C) is the maximum elasticity that is available without cost when the cost parameter has a value C. Now consider following conditions that may be imposed of F.

$$F(\rho,C) > 0 \text{ defined on } -1 \le \rho \le \infty,\ 0 < C.$$

$$\text{For } \rho < \rho^*(C)\text{: } F_\rho > 0,\ F_{\rho\rho} < 0,\ F_C = F_{C\rho} = 0.$$

(9) $$\text{For } \rho = \rho^*(C)\text{: } F^-_\rho > 0,\ F^-_C = F^-_{C\rho} = 0;\ F^+_\rho = 0,\ F^+_C = F/C.$$

$$\text{For } \rho > \rho^*(C)\text{: } F_\rho = 0,\ F_C = F/C.$$

$$\text{There exists a } \bar{C} \text{ such that } F(\rho, \bar{C})/\bar{C} < S \text{ for all } \rho \text{ and } \lim_{\rho\to\infty} F(\rho, \bar{C})/\bar{C} = S.$$

Here F^-_ρ is the limit of $F_\rho(\rho', C)$ when ρ' approaches ρ*(C) from the left etc.; note that all the limits refer to ρ and not C.

<u>Remark 2.</u> Let F(ρ, C)/C be interpreted as S - $\tilde{F}(\tilde{\rho},C)$ and satisfy the conditions (9). If $0 < C \le \bar{C}$, then:

a) For $\tilde{\rho} \le -\rho^*(C)$: $\tilde{F} = 0$.

b) For $-\rho^*(C) < \tilde{\rho}$: $\tilde{F} > 0$, $\tilde{F}_{\tilde{\rho}} > 0$, $\tilde{F}_{\tilde{\rho}\tilde{\rho}} > 0$, $\tilde{F}_C > 0$.

c) $d(-\rho^*(C))/dC < 0$, and $\lim_{C\to\bar{C}} \rho^*(C) = \infty$.

That is a) for the elasticity less than or equal to the greatest free elasticity, the cost is zero; b) for larger elasticity the cost is positive and increasing in C and the marginal cost is positive and increasing; the maximum free elasticity falls as the cost parameter increases; and when $C = \bar{C}$, there is no free elasticity.

Now with the IPF in this form the only effect of an increase in the cost parameter is to slow productivity growth. This is shown in proposition 6.

Proposition 6. If the IPF (8) is used in the model (4), then the values of ρ, $\hat{1}$ and γ_+ are independent of the value of C. In addition, if C is interpreted either as C_p or as C_e and $\rho \leq \rho^*(C)$, then $\hat{L}$ and $\hat{M}$ are negatively related to C; otherwise they are independent of it.

Proposition 6. allows the effects of growing resource scarcity to be calculated. First note that when $C = \bar{C}$, F has the form assumed in propositions 1-5. But, according to proposition 6, γ_+ is independent of the value of C. Thus the results of propositions 1-5, which concern the steady states and their stability properties, continue to hold when $C < \bar{C}$ as well.

Let growing resource scarcity be represented by an increase in the value of C.

Corrollary to Proposition 6. Increased resource scarcity has no effect either on the elasticity or on the relative growth rates of factors either in of out of steady state. Let elasticity be costly, if increased resource scarcity makes it more costly to produce either productivity growth or elasticity, then it slows the growth of productivity of both factors proportionally, otherwise it has no effect.

In brief, the overall picture that emerges from the model is the following. In the long run, depending on the initial conditions, we expect an elasticity of either zero or one to emerge. Increasing resource scarcity may slow the rate of growth of output, but it will have no effect on the elasticity.

III. A Brief Review of the Debate.

Stiglitz (1979 p36) notes that those who make prognostications of the future can be divided into optimists and pessimists. In the current round of the debate over whether economic activity is threatened by resource scarcity, the focus of the attack of the pessimists has been the papers written by Solow and Stiglitz for the 1974 Review of Economic Studies symposium on exhaustible resources. Both used a Cobb-Douglas and, in addition, Stiglitz assumed positive population growth and technical progress and showed that continual growth of per capita consumption was a distinct possibility. The trick is that, with a Cobb-Douglas, output can grow although the input of resources becomes vanishingly small as long as the quantity of capital grows sufficiently.

This property of the Cobb-Douglas has been attacked by the pessimists as violating the laws of physics. The essential point is that output consists of physical goods which can not be made out of a vanishingly small amount of resources regardless of how much capital is available. This point was initially made by Geogescu-Roegen in a number of publications e.g. (1975, 1979). It has intermittently been revived over the years with the most notable reoccurrence being the Georgescu-Roegen versus Solow/Stiglitz forum organized by Daly (1997a,b). In it Daly attacked, Solow and Stiglitz defended and then a large number of interested parties made comments.

While the essential idea of the pessimists is easy to grasp and has often been alluded to, the fashioning of a convincing argument is a considerable challenge which has not often been attempted: It must be shown that regardless of what happens in the future, the laws of physics imply that continuous growth is not possible. Basically there are two approaches, materials balance and thermodynamics, both of which were mentioned by Georgescu-Roegen (1975). The materials balance approach concentrates on the finite quantity of material available on earth, while the thermodynamic approach is based on the fact that production lowers the amount of energy available in a closed system.

The only formal model that uses materials balance to show that continual growth is impossible is that of Gross and Veendorp (1990). They adopt Stiglitz's framework, define output in terms of the exhaustible resource, assume that output can not be greater than the resource input and then show that continual growth of output and consumption is not possible. This in itself is not unexpected, given the finite quantity of resources. What is surprising, however, is that they then add technical progress, the factor that, in Stiglitz's model, allows growing per capita income, and show that in most cases continual consumption growth still is impossible. I think that the result is flawed, but it is impressive and deserves more interest than it seems to have received.

There are two formal models which attempt to use thermodynamics. The second, by Islam (1984), adds resource scarcity to Houthakker's (1955) demonstration from first principles that the production function must be Cobb-Douglas. He shows that, with this addition, the elasticity of substitution between resources and capital must be less than one for large outputs. The idea is deep and engaging but the presentation is obscure in the extreme and has not been followed up.

The first by Ayres and Miller (1980) is, to my mind, the only completely convincing formal argument that perpetual growth of consumption and output is impossible. They accept the equivalency of energy and matter and measure all output and all inputs in terms of negentropy (a measure of energy available). Technical progress increases the efficiency of production but, since output and inputs are measured in the same units, there is an upper limit: the output cannot contain more negentropy than the inputs. Their conclusion is that once the minerals of the earth's crust have been exhausted, production cannot exceed the amount of energy received from extraterrestial sources. Thus, in spite of technical progress, continual growth of production and consumption is impossible.

All three of these contributions derive production functions in which the elasticity of substitution is less than one. The papers by Gross and Veendorp and by Ayres and Miller, which are the ones well connected to the debate, then show that this makes continual growth of consumption and production impossible and that technical progress can not reverse this conclusion. Finally they both see their contributions as rebuttals of the Solow/Stiglitz position. These papers are far from being the most polemic, but I think they must be seen as the heart of the pessimist position.

The pessimists have centered their attacks on a caricature of the optimists and have shown no awareness of the considerable change that has taken place within this camp. For this reason and since no summary of the positions of the optimists exists,[6] it is worth while reviewing the positions of Solow, Stiglitz and a third contributor to the symposium, Partha Dasgupta.

The basis of Solow's optimism, an optimism which has always appeared as a justification for the framework in which he analyses intergenerational equity, has changed considerably over the years. In (1974a) and (1974b) he thought that continual growth of consumption could be sustained by capital-resource substitution since the elasticity appeared to be greater than one. Then, in a number of papers, (1986), (1991) and (1992), his position gradually shifted. In his (1997) note, he still mentions the possibility of capital-resource substitution, but his emphasis is elsewhere. He thinks that growth can be supported "for a long time" by two types of technical progress: that which increases the output of renewable resources like fisheries, and that which allows the use of plentiful exhaustible resources like the development of nuclear fission.

Stiglitz's (1974a) and (1974b) models generated continual growth but contained no justification for the assumption of unitary elasticity. However in his (1979) paper he completely clarified his position which has, despite some later ambiguity, remained unchanged to the present. He divided the question of continual growth into two: under what conditions was it possible? and are these conditions empirically satisfied? He thought that his (1974) papers had answered the first: the elasticity of substitution had to be at least one and the rate of technical progress had to be sufficient. He thought that only econometricians could answer the second and this only within a horizon of between twenty five to one hundred years. Within this time range he was optimistic about the size of the elasticity since there was no evidence that the share of resources was increasing.

After this he changed his tack and wrote three papers, (1981), (1982) and (1983) with Daspupta and other authors which utilized the backstop technology. With this one invests in research and, after a time, a substitute of infinite supply is found for the exhaustible resource. The example given was the technical-break through which allowed the substitution of plentiful coal for increasingly scarce timber in nineteenth century British

[6]For example, two recent surveys, Toman, Pezzey and Krautkraemer (1995) and Stern (1997) contain no detailed description of the optimists' positions.

iron manufacture. Since research can be thought of as capital accumulation, this is a complex combination of resource-resource and capital-resource substitution. Unlike pure capital-resource substitution it depends on there being a plentiful supply of alternative resources and, for this reason, it does not justify perpetual growth. On the other hand, given the availability of resources in general, it is an argument in favor of a very long growth horizon.

After this Stiglitz left the issue and only has returned briefly in his (1997) note. In it he mentions resource-resource substitution which is a clear reference to the long run backstop argument. But then he insists that the horizon of fifty to sixty years is the important one and, within this, he emphasizes the importance of technical progress that increases the efficiency of resource use and his optimism over the possibility of capital-resource substitution. Thus, with some ambiguity, he seems largely to have returned to the position he took in his (1979) paper.

The views of the third of the optimists, Partha Dasgupta, are characterized, first, by a consistently long run perspective, second by a move from optimisim to pessimism over the elasticity and third, by the reverse move with regard to technical progress. Dasgupta, along with Geoffery Heal wrote one of the 1974 symposium papers and a book in 1979. The (1974) paper noted that the elasticity of substitution was important for planning, that on current evidence its value was greater than one, but that there was no way of knowing its value for resource-capital ratios outside the historic range (p206). In the book the position is strengthened. Their justification of the assumption of a constant population, the finite size of the world, shows that they are focusing on long run problems. They identify the elasticity of substitution as the key variable: if it is greater than one there is no problem while if it is less there is no way to avoid doom (pp 199-200). They think that the backstop model is the correct way to model technical progress in relation to resources but are generally pessimistic: " Perpetual technical progress, while unlikely, is not an absurd notion" (p199 and also p206). With regard to the size of the elasticity, they are equivocal but generally optimistic. They note (p206) that currently the value seems to be greater that one but they caution against extrapolation. Then, in their discussion of thermodynamics, they say that these considerations imply an elasticity less than one (p211). But immediately after this they justify their continued use of the Cobb-Douglas by arguing in an unclear way that energy is included in capital so that the shapes of the isoquants can not be determined by thermodynamic considerations. Thus, while the argument is muddy, the tone is decidely optimistic with regard to the size of the elasticity of substitution.

Dasgupta then moved away from optimism about the elasticity of subtitution between capital and resources and toward optimism about the possibility of sustained growth supported by technical progress which permitted the substitution of exhaustible resources which are scarce by those which are plentiful. First between 1977 and 1983, he coauthored at least four papers on the backstop technology, (1977), (1981), (1982) and (1983). Then in 1993 he published what might be called a position paper on the subject. He started by clearly identifying himself as an elasticity pessimist, claiming that, for high resource-capital ratios, the elasticity of substitution was less than one, citing in support (with a short memory lapse) the section on thermodynamics in his joint book. Then he made three points: First, the substitution that is important is not capital-resource but that of plentiful for scarce resources, a process which requires increases in technical knowledge. Second the working of markets will provide incentives for the prerequisite technical progress to take place. And third, that the totality of minerals in the earth's crust, with the exception of phosphate rock (1300 years), fossil fuels (2500 years) and manganese (13,000 years) are sufficient, at the current rate of use, for at least the next million years. Thus Dasgupta's position is, with the recycling of phosphate and manganese and the development of cheap energy sources, that the world should not experience problems due to resource shortages in this time scale.

To summarize the development of the three optimists: Solow and Dasgupta moved from the position that perpetual growth was possible because of capital-resource substitution to the position that growth is possible for a long period, a million years according to Dasgupta, because of a combination of capital investment in research and resource-resource substitution. Stiglitz flirted with this view, but has largely returned to the position that the hundred year horizon is the only one of interest for predictions and that, within this horizon, technical progress and the elasticity of substitution are such that there will be no resource scarcity induced crisis.

What is the actual state of the debate? Pezzey (1992 p339) has characterized it as being between two models, one which permits sustained growth (eg. Dasgupta and Heal (1979)) and one that does not (eg. Ayres and Miller (1980)) and then calls for empirical research to decide which assumptions are more justified. I disagree. Rather I think that it is the level of the time scale that distinguishes the participants and that it is disinterest and, at times, incomprehension rather than disagreement which characterizes the debate. At the lowest time scale come Stiglitz, a large group of resource econometricians and the advocates of sustainability, whose works are respectively reviewed by Slade, Kolstad and Weiner (1993) and by Toman, Pezzey and Krautkraemer (1995) and Stern (1997). Each of these participants has shown only minimal interest in the works of the others. In particular it is noteworthy that the econometricians, althought they have the techniques (if perhaps not the data) have completely eschewed any attempt to answer the elasticity question that Stiglitz posed. At the next level come Solow and Dasgupta and at the last level, where material balance and thermodyanmic considerations become important, come the pessimists for whom, some what unjustly, Georgescu-Roegen and Daly may be taken as spokesmen.[7] Georgescu-Roegen (1979) agrees that econometric techniques cannot be used for long run elasticity estimation but criticizes Stiglitz's hundred year horizon as being too short to be of interest and claims that material balance and thermodynamic methods will allow the longer horizon to be studied. Solow (1997), replying to Daly's (1997a) call for the use of these tools, states that the required time horizon is too long for the results to be of "practical interest". One would expect that Daly, in his reply (1997b), would argue that the horizon is, indeed, short enough. But instead he reacts as if Solow were at the last level and was merely unaware of the difficulties that this causes for his Cobb-Douglas production function. The same inability to engage the opponent characterizes Ayres and Miller's (1980) criticism of Stiglitz's hundred year horizon and Gross and Veendorp's (1990) criticism of the optimist position generally. They use the tools of the last level to attack the optimists without seeming to realize that it is the relevance of the level that must be defended. At the level where material balance and thermodynamic considerations become important, everyone who has expressed an opinion is already an elasticity pessimist.

IV. Conclusion.

In this section two issues are treated: first the relevance of the model of section II to the debate described in secton III; and second, the contribution that the model makes to that debate.

With regard to the relevance, if we think in terms of generic models, the one of this paper and the one on which the debate is based belong to different groups. According to the first, resources are a constant flow of services while according to the second, they are a finite quantity of services. When there is no technical progress and no population growth the two groups differ in an important way. For the first, it is obvious that a constant per capita consumption may be maintained, while for the second it may be maintained only if the elasticity of substitution is at least one (Stiglitz 1974a). Thus, while one might claim that the present model has implications for the size of the elasticity in exhaustible resource models as well, in the context of no technical progress or population growth one can not use the present model to discuss directly whether constant consumption is possible in the context of exhaustible resources.

However the situation changes radically in the context of technical progress and population growth: the exhaustible resource model can be made to approximate the fixed flow of services model. Although the following statements are slighty speculative, it is difficult to see how they could be very wrong. The idea is that in the exhaustible resource model one chooses the initial quantity and the rate of fall of a resource so that the resource

[7] One need only read the contributions of the interested parties in the forum to get a flavor of the disagreement in the pessimist camp.

will be just used up at infinity. By choosing both of these close to zero one can approximate a constant resource flow as closely as desired. When one then adds technical progress one has a resource flow that approximately grows at the rate of technical progress. Thus the flow of services from resources has approximately the same character in both types of model.

In this context one can see how the relation between the elasticity of substitution and the possibility of constant per capita consumption is the same in both types of models: First, in the Cobb-Douglas case, constant per capita consumption is possible only in the case in which the rate of resource augmenting technical progress is greater than the rate of population growth. Stiglitz (1974a prop.4) showed this for the exhaustible resource model and it is easy to show for the constant land model. Second, if the elasticity of substitution is less than one, then each factor has a distinct rate of technical progress. Once again in both cases the rate of growth is that of the slowest growing factor. This would imply, in both cases that the rate of growth of output and consumption would be limited by the rate of technical progress in resources. Third, if the elasticity of substitution is greater than one , then resources are not important in either model. It follows that once one has introduced technical progress and one wants to talk of long run behavior, little is gained by treating the more complicated case of exhaustible resources. In both cases the issues are the elasticity of substitution and the rates of technical progress.

With regard to the contribution to the debate, it is that the elasticity of substitution should converge to either zero or one. Thus, for all time scales, we should expect that the production function will approximately be either Leontief or Cobb-Douglas. This goes against the positions of all the participants in the debate with the exception of Stiglitz who considered only a hundred year horizon. How can all of these participants have been wrong?

First, there are the group who view the elasticity of substitution as a parameter to be established by econometric techniques. It is a common opinion of this group that these techniques can not establish the value of the parameter for the high capital-resource ratios that will be relevant for the future exactly because we do not, now, have data in that range, e.g. Georgescu-Roegen (1979), Dasgupta and Heal (1979 p206) and Slade, Kolstad and Weiner (1993 p959). This lack of long run relevance might be thought to excuse the absence of econometric attempts to answer the elasticity question. But this general position is incorrect. The argument of the paper provides a testable hypothesis: if there is a difference between the rates of growth of resources and the capital-labour aggregate then one ought to observe a convergence of the elasticity.[8] If this was confirmed, one

[8] Stevenson (1980) is the only attempt that I know of to test the consequences of having a IPF. However the test was performed on micro data, only concerned productivity growth rates and, although the results were in the right direction, they were statistically insignificant.

would have econometric evidence that the long run value of the elasticity is one, even outside the range of the data.

Second, there is a large group of pessimists that hold that for high values of the capital-resource ratio the elasticity of substitution must be less than one for materials balance or thermodynamic reasons. These include Dasgupta (1993 p1115), Ayers (1978 chap.3), Ayers and Miller (1980), Gross and Veendrop (1990) and clearly Daly (1997a,b). The model implies that these authors are incorrect as well in the sense that, depending on the initial conditions, the elasticity may well be one. Since this conclusion seem to fly in the face of physical science it requires some justification.

How can a growing quantity of output be made with a vanishingly small quantity of resources? The answer lies in what we mean by output. Both Ayres and Miller (1980) and Gross and Veendrop (1990) defined output in terms of a physical characteristic. They were forced to do this in order to demonstrate that continuous growth was not possible. But output is, rather, an index of a bundle of goods which change with a movement along an isoquant and which are only related by the fact that they all give the same utility to the consumer.[9] It is not hard to imagine that, as one moves along an isoquant, the bundle changes in such a way that a constant utility is produced by a vanishingly small amount of resources so long as the amount of capital is sufficiently large. This point was first made by Ayres (1978, chap.3) and reiterated by Stiglitz (1997 p267). What the model of this paper has shown is that technical progress will be just such as to reveal those techniques which ensure that the elasticity of substitution will be exactly one.

Apart from indicating where the positions of the participants may be incorrect, what is the positive contribution of the paper to the debate? It is to show that the size of the elasticity is completely irrelevant to the question of whether constant or growing per capita consumption is possible. To put the point as strongly as possible consider the following. Suppose the production function is Cobb-Douglas, the capital-labour aggregate is growing much faster than land and, due to an increase in the cost of producing elasticity connected with increasing resource scarcity, the rate of growth of output has dropped below that of population growth. From the perspective of the participants in the debate one would be tempted to say," Its lucky that the elasticity is one because, if it fell even slightly, the rate of growth would drop precipitously to that of land". But this statement completely misunderstands the situation since it is exactly the increased cost of keeping the elasticity equal to one that has caused the problem in the first place.

Appendix.

[9]This can be formalized using the notion of an exact quantity index which is explained by Diewert (1993) for example.

<u>Proof of Proposition 1.</u> Let $f(\hat{l})$ be the slope of the level curve of G. Define $\gamma^* \equiv -f(1)$. Let $\gamma < \gamma^*$. The tangency condition between G and $\underline{H}$ is

(A1) $\qquad f(\hat{l}) = -\gamma\, \hat{l}^{\rho+1}.$

This defines the function $g(\gamma,\rho) = \hat{l}$ with the following properties: $\lim\limits_{\gamma\to\gamma^*} g = 1$, $g_\gamma < 0$, $\lim\limits_{\gamma\to\gamma^*} g_\rho = 0$ and $\lim\limits_{\rho\to\infty} g = 1$. Write the maximand as

$$\overline{H}(\gamma,\rho) \equiv \left[\frac{1}{1+\gamma}\left(g(\gamma,\rho)\right)^{-\rho} + \frac{\gamma}{1+\gamma}\right]^{-\frac{1}{\rho}} \frac{F(\rho)}{G\left(g(\gamma,\rho),1\right)}.$$

Then as $\gamma \to \gamma$ $\lim \overline{H}_\rho > 0$ so that $\rho \to \infty$ and $\hat{l} \to 1$; and similarly if $\gamma > \gamma^*$.

Next suppose that $\gamma^{**} \neq \gamma^*$ is a steady state such that, as $\gamma \to \gamma^{**}$, $\rho \to \infty$. Then $\hat{l} \to 1$ and, from (A1), $\gamma\hat{l}^{\rho+1} \to \gamma^*$. Since $\gamma \to \gamma^{**}$, $\hat{l}^\rho \not\to 1$ and the difference equation of (4) implies that γ^{**} is not a steady state, contradiction.

Finally suppose that there is a steady state $\gamma^{**} \neq \gamma^*$ such that $\rho \neq 0$. If $\hat{l} = 1$ then (A1) implies that $\gamma^{**} = \gamma^*$, while if $\hat{l} \neq 1$ then $\hat{l}^\rho \neq 1$ so that γ^{**} is not a steady state by the difference equation of (4).

<u>Proof of Proposition 2.</u> Let $\gamma \to 0$. That $\gamma = 0$ is a steady state will follow from the fact that $\gamma\hat{l}^\rho \to 0$ and the difference equation of (4).

$\rho \to \infty$: suppose $\rho \to \rho^* < \infty$, then $\underline{H} \to \hat{L}$ and the solution is clearly $\rho = \infty$, contradiction. $\hat{l} \to \hat{l}^* \geq 1$ since if not (A1) would be violated. Thus $\gamma\hat{l}^\rho \not\to \infty$ and if $\gamma\hat{l}^\rho \not\to 0$ then $\hat{l} \to \hat{l}^*$, $1 < \hat{l}^* < \infty$: if $\gamma\hat{l}^\rho \to \infty$ then (A1) implies $\hat{l} \to 0$, contradiction; if $\gamma\hat{l}^\rho \not\to 0$ then (A1) implies $\hat{l} \to \hat{l}^* < \infty$; and writing the maximand as $\left(\frac{1}{1+\gamma} + \frac{1}{1+\gamma}\gamma g^\rho\right)^{-\frac{1}{\rho}} \hat{L} \to \hat{L}$ shows that $g \to 1$ can not be the limiting solution..

To prove the proposition multiply the expression for $\overline{H}_\rho$ by $G[\,]^{1/\rho}/F$ and set it to zero, where [] is the expression in square brackets in the definition of $\overline{H}$. This gives

$$0 = \ln[\,]\frac{1}{\rho} + [\,]^{-1}\frac{1}{1+\gamma} g^{-\rho}\ln g + [\,]^{-1}\frac{1}{1+\gamma} g^{-\rho-1}\rho g_\rho + \rho\frac{F'}{F} - \frac{G_{\hat{L}}}{G}\rho g_\rho$$

where a calculation shows $g_\rho = -\dfrac{\gamma\hat{l}^{\rho+1}\ln\hat{l}}{f' + (\rho+1)\gamma\hat{l}^{\rho}}$. Suppose that $\gamma g^\rho \not\to 0$. Then $g^{-\rho} \to 0$; and also $g \to g^*$, $1 < g^* < \infty$ and the smoothness of G imply that ρg_ρ approaches a finite non-zero limit. Thus the first three terms on the RHS of the previous equation approach zero while the last two approach a positive limit, contradiction.

<u>Proof of Proposition 3.</u> (A1) with $\rho = 0$ gives the bounds on $\hat{l}$, the steady state follows from the difference equation of (4).

<u>Proof of Proposition 4.</u>

Part a). The FOC can be written as*[10]

$$\Phi(\rho,\gamma) \equiv \theta(\rho,\gamma) + \frac{1}{\rho^2}\varphi(\rho,\gamma) = 0$$

where

$$\theta(\rho,\gamma) = \frac{F'(\rho)}{F(\rho)}(\gamma^{-1/(2+\rho)} + \gamma^{1/(2+\rho)})$$

$$\varphi(\rho,\gamma) = \hat{\varphi}(\rho,\gamma) + \hat{\varphi}(\rho,\frac{1}{\gamma})$$

$$\hat{\varphi}(\rho,\gamma) = \gamma^{1/(\rho+2)} \ln\left(\frac{1}{1+\gamma}\gamma^{\rho/(2+\rho)} + \frac{\gamma}{1+\gamma}\right).$$

When $\rho = 0$, the FOC takes the form*

(A2) $$8\tilde{F}\frac{(1+\gamma)^2}{\gamma} = (\ln\gamma)^2$$

where $\tilde{F} = F'(0)/F(0)$.

First let $\gamma > 1$. To find the condition on $\tilde{F}$ such that (A2) will have solutions, plot the RHS and the LHS of (A2) for various values of $\tilde{F}$. This is done in figure A1. The point

Figure A1 about here.

of tangency is at $\gamma = \gamma^o$ where the slopes are equal and (A2) is satisfied. These two conditions imply that

(A3) $$\ln\gamma = 2\frac{\gamma+1}{\gamma-1}.$$

This can be solved for γ^o. From (A2) the corresponding value of $\tilde{F}$ is $\tilde{F}^* = .0549$.* From the graph and the specific forms of the LHS and the RHS it follows that there are two solutions, γ_1 and γ_2 when $\tilde{F} < \tilde{F}^*$.

Next let $\gamma < 1$. If (A2) is satisfied by γ, it is also satisfied by $1/\gamma$. Thus if $\tilde{F} < \tilde{F}^*$, $1/\gamma_2$ and $1/\gamma_1$ are also solutions.

Finally, it is clear from the graph that the existence of the four solutions imply that $\tilde{F}^* < .0549$.

Part b). Calculations and the condition of b) show that*

(A4) $$\Phi_\rho(0,\gamma) < -(\ln\gamma)^2\gamma^{\frac{1}{2}}\frac{1}{1+\gamma}\frac{1}{24}(\ln\gamma)\frac{\gamma-1}{\gamma+1} < 0.$$

On the other hand, from the manipulations* that lead to the construction of Φ from $\partial H/\partial\rho = 0$, we see immediately that

$$H_\rho = \left[(\frac{1}{1+\gamma})^{2/(\rho+2)} + (\frac{\gamma}{1+\gamma})^{2/(\rho+2)}\right]^{-(3\rho+2)/2\rho} F(\rho)(\frac{1}{1+\gamma})^{2/(2+\rho)}\gamma^{1/(2+\rho)}\Phi(\rho,\gamma).$$

Since $\Phi(0,\gamma) = 0$ at $\gamma = \gamma_1$ etc., for these values of γ we have that $H_{\rho\rho}(0,\gamma)$ is given by

$$\lim_{\rho\to 0}\left\{\left[(\frac{1}{1+\gamma})^{2/(\rho+2)} + (\frac{\gamma}{1+\gamma})^{2/(\rho+2)}\right]^{-(3\rho+2)/2\rho} F(\rho)(\frac{1}{1+\gamma})^{2/(2+\rho)}\gamma^{1/(2+\rho)}\right\}\Phi_\rho(0,\gamma)$$

where an application of l'Hospital's rule shows that the limit term is positive. Thus (A4) implies that $H_{\rho\rho} < 0$ for $\gamma = \gamma_1$ etc..

[10] An asterisk indicates points at which routine calculations have been suppressed. They are set out in Petith (1998).

Part c. Obvious.

It is convenient to start the proof of proposition 5 with the following lemma.

Lemma. Let the condition of a) of proposition 4 be satisfied, if - F''(0)/F'(0) > 1 then $\rho'(\gamma_2) > 0$, $\rho'(\gamma_1) < 0$, $\rho'(1/\gamma_1) > 0$, $\rho'(1/\gamma_2) < 0$.

Proof: From the FOC, when $\rho = 0$

(A5) $$\rho'(\gamma) = -\frac{\Phi_\gamma(0,\gamma)}{\Phi_\rho(0,\gamma)} .$$

Tedious calculations* show

(A6) $$\Phi_\gamma(0,\gamma) = \frac{1}{8}\frac{1}{1+\gamma}(\ln\gamma)\,\gamma^{-1/2}\left[(\ln\gamma)\frac{\gamma-1}{\gamma+1} - 2\right].$$

Thus, from (A4)

(A7) $$\text{sign } \rho'(\gamma) = \text{sign } \Phi_\gamma = \text{sign } \frac{1}{8}\frac{1}{1+\gamma}(\ln\gamma)\,\gamma^{-1/2}\left[(\ln\gamma)\frac{(\gamma-1)}{\gamma+1} - 2\right] .$$

From (A3)

(A8) $$2 = (\ln\gamma^o)\frac{(\gamma^o-1)}{\gamma^o+1} = (\ln 1/\gamma^o)\frac{1/\gamma^o-1}{1/\gamma^o+1}$$

and furthermore, letting $f(\gamma) = \ln(\gamma)\frac{(\gamma-1)}{\gamma+1}$,we have

(A9) $$f'(\gamma) = \ln(\gamma)\frac{2}{(\gamma+1)^2} + \frac{1}{\gamma}\frac{\gamma-1}{\gamma+1} \gtrless 0 \text{ as } \gamma \gtrless 1 .$$

(A7), (A8), (A9) and the location of γ^o relative to γ_1 and γ_2 from the proof of a) of the previous proposition now demonstrate the lemma.

Proof of Proposition 5.

Part a).The local stability condition is

(A10) $$1 > \frac{d\gamma_+}{d\gamma} > -1 .$$

Differentiating (7) and setting $\rho = 0$ gives

(A11) $$\frac{d\gamma_+}{d\gamma} = 1 - \frac{1}{2}\gamma(\ln\gamma)\rho'(\gamma).$$

Existence, part a) of proposition 4 and the condition of this part allow the use of the lemma which, together with (A10) and (A11), shows that $1/\gamma_1$ and γ_1 are unstable.

Next consider the local stability of $\gamma = 0$. At $1/\gamma_2$

$$\frac{d(\gamma_+ - \gamma)}{d\gamma} = \frac{d\gamma_+}{d\gamma} - 1 < 0$$

by (A11) and the lemma. Since $\gamma_+ = \gamma$ at $\gamma = 1/\gamma_2$, $\gamma_+ > \gamma$ immediately to the left of $1/\gamma_2$. Thus $\gamma_+ > \gamma$ on $0 < \gamma < 1/\gamma_2$ since, on this interval, there are no steady states and γ_+ is a continuous function of γ. Thus $\gamma = 0$ is unstable and similarly for $\gamma = \infty$.

Now consider $1/\gamma_2$ and γ_2. From the lemma for these values of γ, the stability condition is $1-\frac{1}{2}(\ln\gamma)\rho'(\gamma) > -1$, or $\gamma(\ln\gamma)\,\rho'(\gamma) < 4$. Now

$$\gamma(\ln\gamma)\rho'(\gamma) = \frac{\gamma(\ln\gamma)\Phi_\gamma}{-\Phi_\rho}$$

$$< \gamma(\ln\gamma)\frac{1}{8}\frac{1}{1+\gamma}(\ln\gamma)\,\gamma^{-1/2}\Big[(\ln\gamma)\frac{\gamma-1}{\gamma+1} - 2\Big]\Big[(\ln\gamma)^2\gamma^{1/2}\frac{1}{1+\gamma}\frac{1}{24}(\ln\gamma)\frac{\gamma-1}{\gamma+1}\Big]^{-1}$$

$$= 3\Big[(\ln\gamma)\frac{\gamma-1}{\gamma+1} -2\Big]\Big[(\ln\gamma)\frac{\gamma-1}{\gamma+1}\Big]^{-1} < 4$$

from (A4), (A5), and (A6). Thus $1/\gamma_2$ and γ_2 are locally stable.

Part b). Differentiating (7) and using proposition 1

(A12) $$\underset{\gamma\to 1}{\text{Lím}}\frac{d\gamma_+}{d\gamma} = \underset{\gamma\to 1,\rho\to}{\text{Lím}}\left\{-\frac{2}{(2+\rho)^2}\gamma^{2/(2+\rho)}(\ln\gamma)\rho'(\gamma)\right\}.$$

Multiplying Φ by ρ^2 and dividing it by $\gamma^{-\frac{1}{2+\rho}}+\gamma^{\frac{1}{2+\rho}}$ gives the FOC as $0 = \Gamma^*(\rho)+\Gamma^{**}(\rho,\gamma)$ where $\Gamma^*(\rho) = \rho^2F'(\rho)/F(\rho)$ etc. A calculation shows* $\Gamma^{**}_\gamma(\ \ ,1) = 0$ and

$$\Gamma^{**}_\rho = -\frac{2\rho}{(2+\rho)^3}\frac{\gamma^{2/(2+\rho)}}{(1+\gamma^{2/(2+\rho)})^2}(\ln\gamma)^2,$$

so that $(\rho^2/\ln\gamma)\Gamma^{**}_\rho \to 0$ as $\gamma \to 1$ and $\rho \to \infty$. Thus as $\gamma \to 1$ and $\rho \to \infty$,

$$-\frac{2}{(2+\rho)^2}\gamma^{2/(2+\rho)}(\ln\gamma)\rho'(\gamma) = \frac{2\rho^2}{(2+\rho)^2}\gamma^{2/(2+\rho)}\frac{\Gamma^{**}_\gamma}{[\rho^2/\ln(\gamma)](\Gamma^*_\rho+\Gamma^{**}_\rho)} \to 0$$

by the condition of b). This shows that $\gamma = 1$ is stable by (A10) and (A12).

Proof of Remark 2.

a) and b) are straight forward.

c) For C', C" < $\bar{C}$, consider $\rho' = \rho^*(C')$, $\rho'' = \rho^*(C'')$, $\rho' < \rho''$. The idea is to go from ρ', C' to ρ'', C"; the limit of $\Delta\rho/\Delta C$ will give $d\rho^*/dC$. Break the move into ρ',C' to ρ',C" and then ρ',C" to ρ'',C". The first and second moves give approximately $[-F(\rho',C''),/C''^2](C''-C')$ and $[F_\rho(\rho',C'')/C''][\rho^*(C'')-\rho^*(C')]$. Since the sum of the changes in F/C is zero we have

$$\frac{d\rho}{dC} = \lim_{C'\to C''}\frac{\rho(C'')-\rho(C')}{C''-C'} = \lim_{C'\to C''}\frac{F(\rho^*(C'),C'')/C''}{F_\rho(\rho^*(C'),C'')} = \frac{F(\rho^*(C''),C'')/C''}{F_{\bar{\rho}}(\rho(C''),C')} > 0$$

which proves the first part.

To prove the second part suppose that there is a $\bar{\rho}$ such that $\rho^*(C) \leq \bar{\rho}$ for all C < $\bar{C}$. Note that if $\rho_1 > \bar{\rho}$ then

$$F(\bar{\rho},C)/C = F(\rho_1,C)/C = S > F(\rho_1,\bar{C})/\bar{C} = \big[F(\bar{\rho},\bar{C})/\bar{C}\big] + \int_{\bar{\rho}}^{\rho_1}\big[F_\rho(\rho.\bar{C})/\bar{C}\big]d\rho,$$

where the integral is positive and bounded away from zero since $F_\rho(\rho,\bar{C}) > 0$ for all ρ. On the other hand, the existence of $\partial(F(\rho,C)/C)/\partial C$ at $\rho = \bar{\rho}$, $C = \bar{C}$ implies that $F(\bar{\rho},C)/C$ approaches $F(\bar{\rho},\bar{C})/\bar{C}$ as C approaches $\bar{C}$ which contradicts the previous line. This proves the second part.

Proof of Proposition 6. Just as in proposition 1, the first order conditions can be solved for $\hat{l} = g(\gamma,\rho)$ which does not depend on C. In the same way as before the maximand can then be written as

$$\bar{H}(\gamma,\rho) \equiv \left[\frac{1}{1+\gamma}\big(g(\gamma,\rho)\big)^{-\rho} + \frac{\gamma}{1+\gamma}\right]^{-\frac{1}{\rho}} \frac{F(\rho,C)/C}{G\big(g(\gamma,\rho),1\big)} \equiv \tilde{H}(\rho,\gamma)F(\rho,C)/\,C.$$

The first order condition is then

$$\tilde{H}_\rho F/\,C + \tilde{H}F_\rho\,/\,C = 0 \Leftrightarrow \tilde{H}_\rho F + \tilde{H}F_\rho = 0.$$

First let C be interpreted as C_p, then the first order condition is independent of C by Remark 1. Next let C be interpreted as C_e so that the conditions (9) are imposed on F. Then if $\rho > \rho^*(C)$ or $\rho = \rho^*(C)$ and we take the limit of the derivitive of the maximand as ρ approaches $\rho^*(C)$ from the right, we have that $F_\rho = F_\rho^+ = 0$ and that F/C is independent of C. If $\rho < \rho^*(C)$ or if $\rho = \rho^*(C)$ and ρ approaches $\rho^*(C)$ from the left $F_C = F_{C\rho} = F_C^- = F_{C\rho}^- = 0$. Thus in both cases the first order condition is, again, independent of C. This proves the first statement.

To prove the second, note that

$$\hat{M} = \frac{1}{G(g(\gamma,\rho),1)}F(\rho,C)/\,C, \qquad \frac{\partial\hat{M}}{\partial C} = \frac{1}{G}\left(F_C\,/\,C - F\,/\,C^2\right).$$

Let C be interpreted as C_p, then $\partial\hat{M}/\partial C < 0$ by Remark 1. Next let C be interpreted as C_e so that conditions (9) hold. $\partial\hat{M}/\partial C < 0$ if $\rho < \rho^*(C)$ or if $\rho = \rho^*(C)$ and the partial is taken from the right since then $F_C = F_C^- = 0$.[11] Otherwise $\partial\hat{M}/\partial C = 0$ since $F_C = F_C^+ = F/C$. Since $\hat{l}$ is independent of C, the same statements hold for $\partial\hat{L}/\partial C$.

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[11] Above F_C^- was interpreted as the limit as ρ'' approached ρ*(C') from the left. But it can also be interpreted as a limit as C'' approaches $C' = \rho^{*-1}(\rho')$ from the right. Draw a graph with ρ and C on the horizontal and vertical axes. With the definition of the text it is shown in the proof of remark 2 that $d\rho^*(C)/dC > 0$. Any sequence of points that approaches ρ*(C'),C' from above the line ρ*(C) has as a limit of F_C, F_C^-. But this approach can be carried out by having C approach C' from above, that is to say, from the right.

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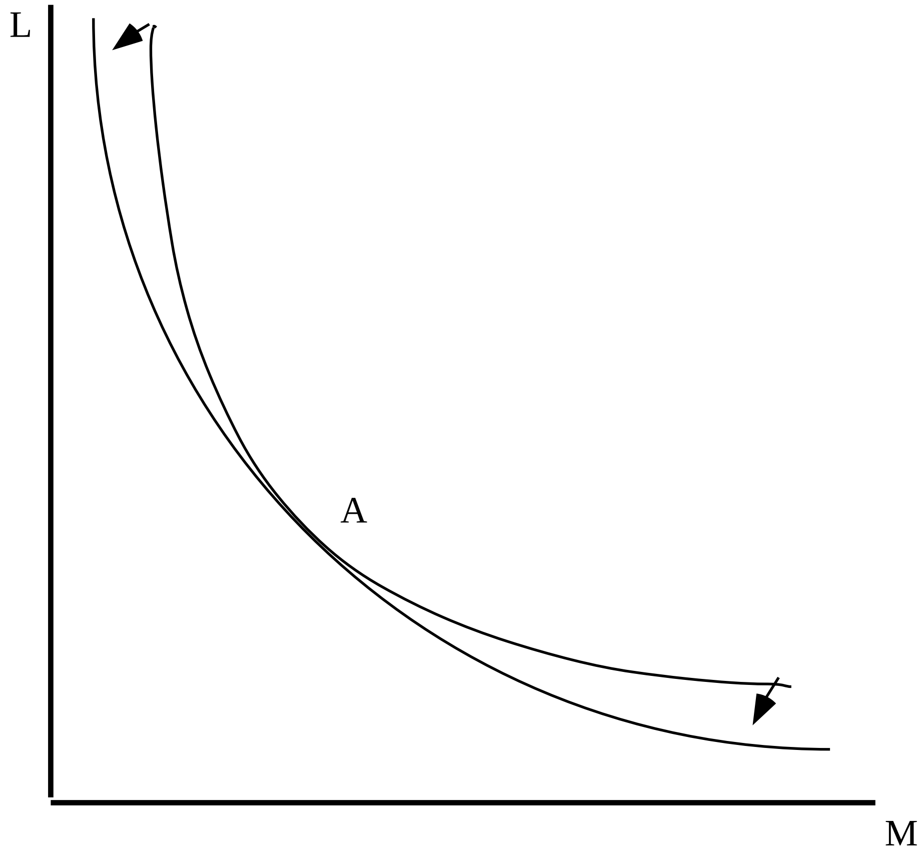
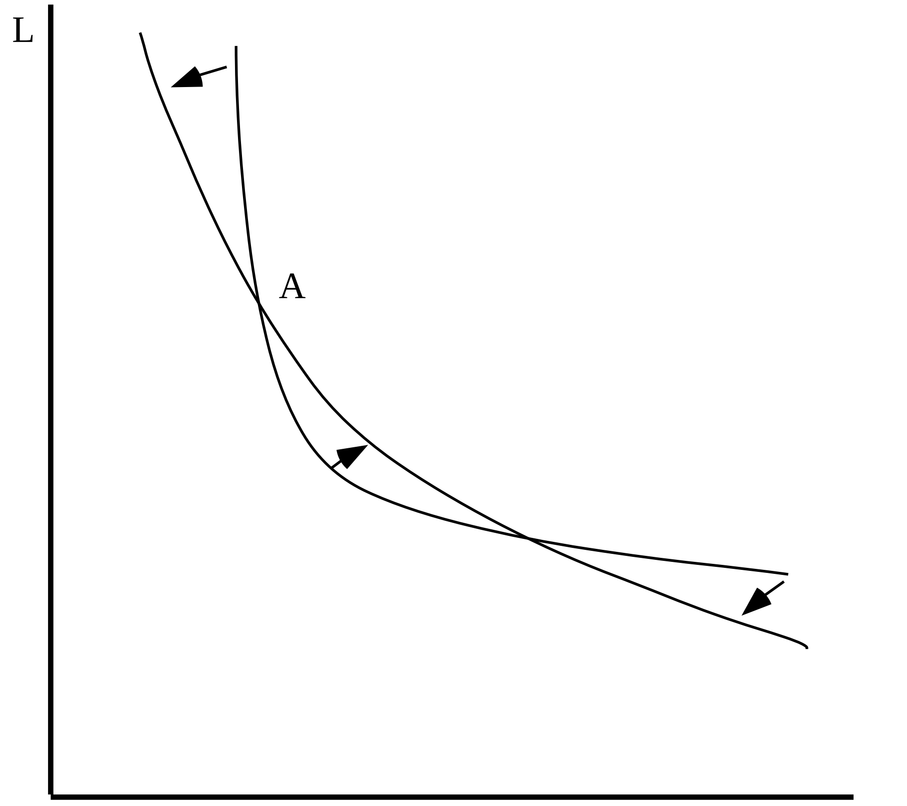


a) Possible Isoquant Shift



b) Impossible Isoquant Shift

Figure 1.

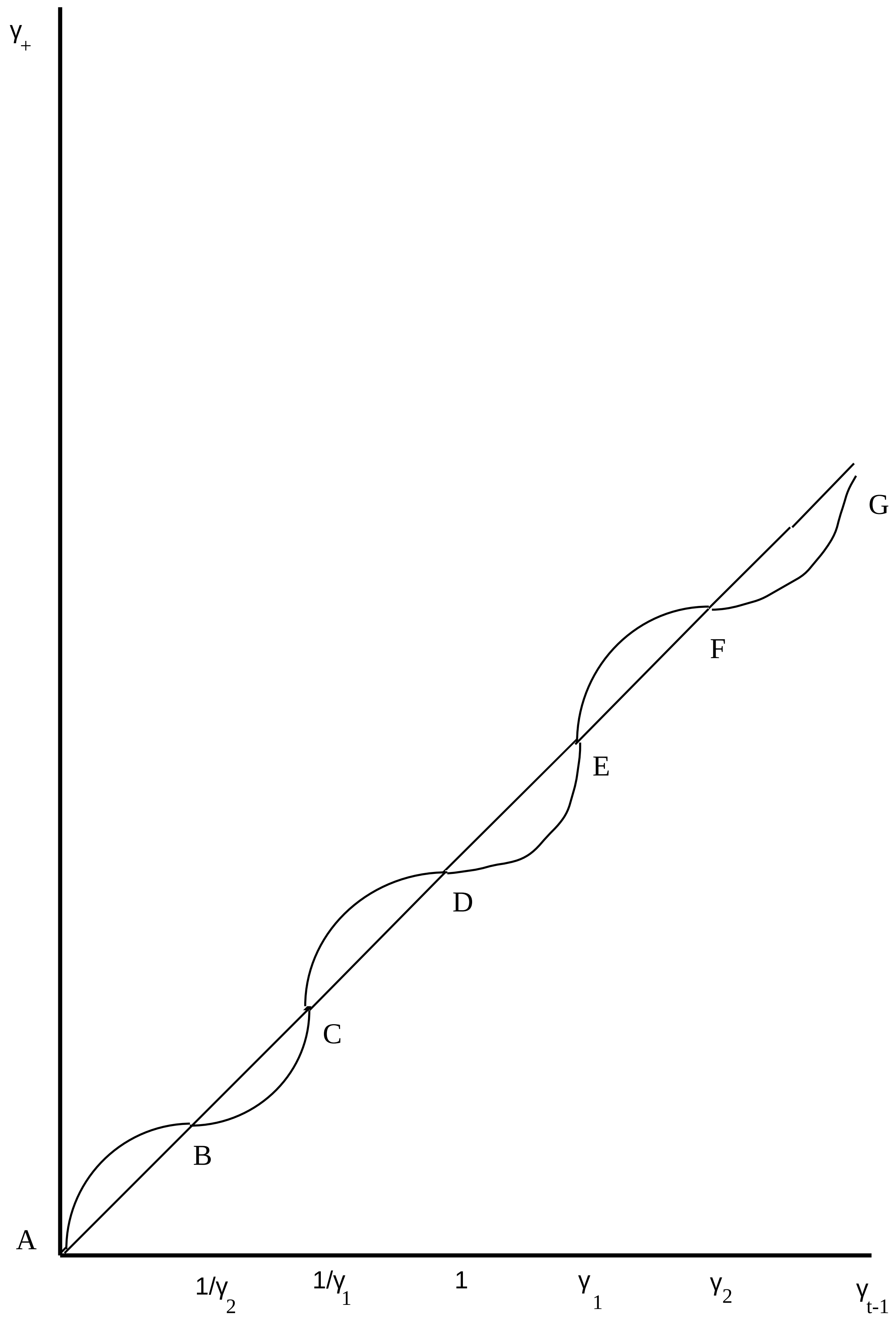


Figure 2. The Phase Diagram for Model of Equations (6) and (7).

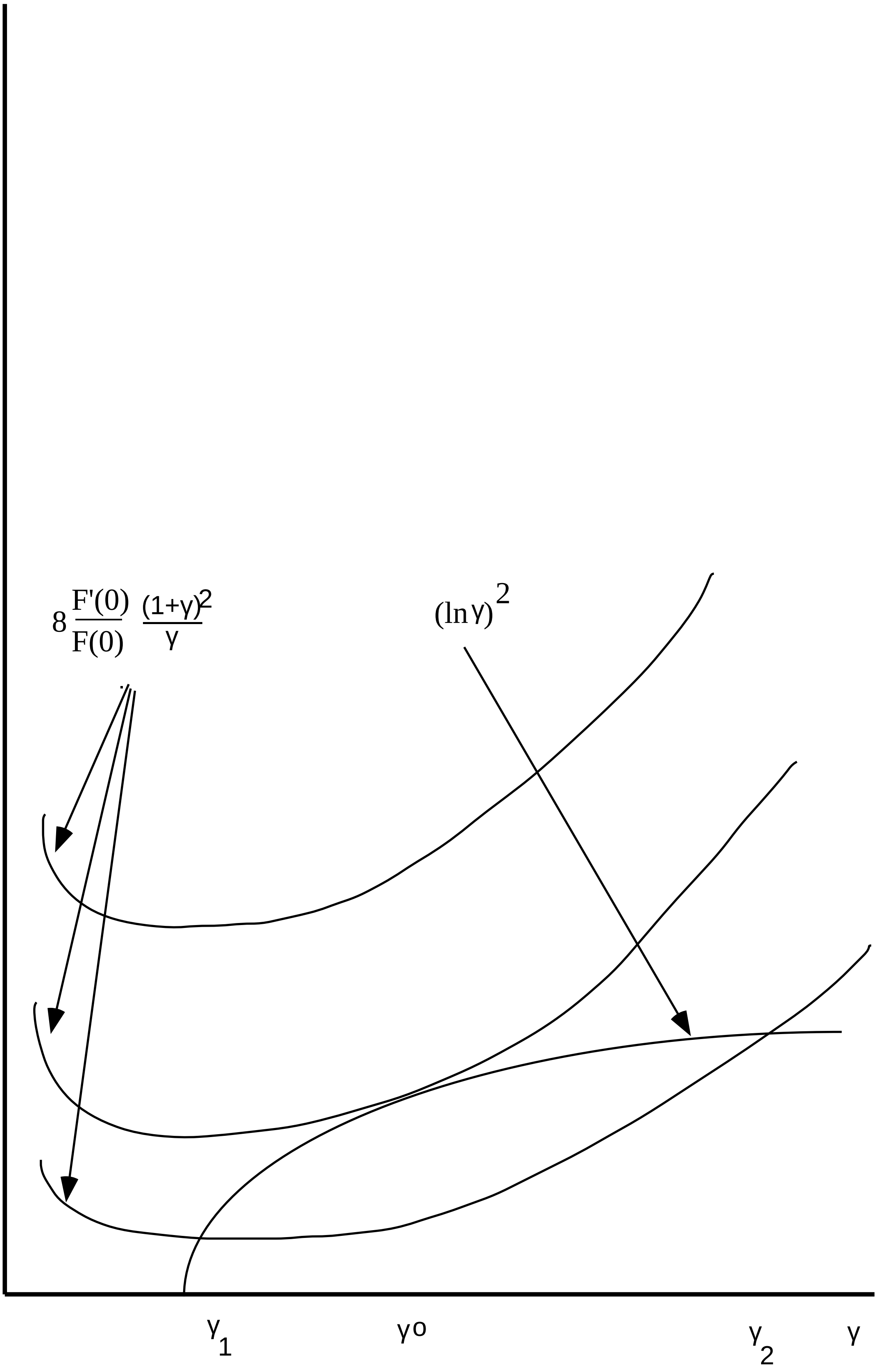


Figure A1. The Graph of $\Phi(0,y) = 0$.