

Advertising, Brand Loyalty and Pricing*

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Abstract

I construct a model in which an oligopoly first invests in persuasive advertising in order to induce brand loyalty to consumers who would otherwise buy the cheapest alternative on the market, and then competes in prices. Despite ex-ante symmetry, at equilibrium, there is one firm which chooses a lower advertising level, while the remaining ones choose the same higher advertising. For the endogenous profile of advertising expenditure, there are a family of pricing equilibria with at least two firms randomizing on prices. The setting offers a way of modelling homogenous product markets where persuasive advertising creates subjective product differentiation and changes the nature of subsequent price competition. The pricing stage of the model can be regarded as a variant of the Model of Sales by Varian (1980) and the two stage game as a way to endogenize consumers heterogeneity raising a robustness question to Varian's symmetric setting.

JEL: D21, D43, L11, L13, M37

Keywords: oligopoly, advertising, price dispersion, brand loyalty

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1 Introduction

The interest in the economic analysis of advertising is continuously resuscitated by the amazing diversity of media¹ and by the large amounts invested in advertising. The US 100 largest advertisers spent a total of USD 90.31 billion on advertising in 2003, with amounts ranging between USD 0.317-3.43 billions.² The most advertised segments include many consumer goods: Beer, cigarettes, cleaners, food products, personal care, and soft drinks. In many of these markets the goods are nearly homogenous, and the high amounts mentioned above seem to suggest that advertising, rather than increasing the demand, redistributes the buyers among sellers.

In the present paper, I study the strategic effect of persuasive advertising in homogenous product markets. For this purpose, I construct a model of two-dimensional competition in non-price advertising and prices. Firms first invest in advertising in order to induce brand loyalty to consumers who would otherwise purchase the cheapest alternative on the market, and then compete in prices for the remaining brand indifferent consumers. At equilibrium, prices exhibit dispersion being random draws from asymmetric distributions. The variation in the price distributions is reflected by the expected profits and, in consequence, the advertising levels chosen by the firms are asymmetric. There is one firm choosing a lower advertising level, while the remaining firms choose the same higher advertising. For this profile of advertising expenditure, there are a family of pricing equilibria with at least two firms randomizing on prices. One limiting equilibrium has all firms randomizing; the other one has only two firms randomizing and the others choosing monopoly pricing with probability 1. As the number of rivals increases, more firms prefer to price less aggressively, counting on their loyal bases rather than undercutting in order to capture the indifferent market. Firms compensate a larger first stage advertising investment by means of less aggressive pricing, and a lower advertising by more aggressive second stage prices. Asymmetry follows from the fact that firms are counterbalancing their advertising and pricing decisions.

The setting proposes a way of modelling homogenous product markets where persuasive advertising creates subjective product differentiation and changes the nature of subsequent price competition. It also offers a new perspective on the coexistence of advertising and price dispersion. The market outcome turns out to be asymmetric, despite a priori symmetry of the firms.

¹Magazines, newspapers, television, radio, internet or outdoor ads.

²Advertising Age, June 28, 2004, 100 Leading National Advertisers.

The model predicts an asymmetric advertising expenditure profile more adequate for small oligopolies. The results relate to the carbonated cola drinks market in the US, where Coca Cola and Pepsico invest similar large amounts in advertising, while Cadbury-Schweppes has lower advertising expenditure. Similarly, in the US sport drinks market, Pepsico highly advertises its product Gatorade, while Coca Cola promotes less its product Powerade.

The basic model in this paper draws from the persuasive view on advertising that goes back to Kaldor (1950).³ More recently, Friedman (1983) and Schmalensee (1972,1976) dealt with oligopoly competition in models where advertising increases selective demand.⁴ Schmalensee (1972) explores the role played by promotional competition in differentiated oligopoly markets where price changes are infrequent. This article departs from his work assuming that competition takes place in both advertising and prices. It turns out that advertising can significantly relax price competition.

Much empirical work explored whether homogenous goods advertising is informative and affects the primary (industry) demand, or is persuasive and affects only the selective (brand) demand. The results are often contradictory, and they seem to vary across industries. The persuasive view was supported by Baltagi and Levin (1986) in the US cigarette industry, and by Kelton and Kelton (1982) for US brewery industry. Using inter-industry data, they report a strong effect of advertising on selective demand.⁵ More recent studies, using disaggregated data (at industry or brand level), show that advertising is meant to decrease consumers' price sensitivity. For instance, Krishnamurthi and Raj (1985) find that brand demand becomes more inelastic once advertising increases.⁶

Using a model where advertising creates vertical differentiation, Sutton (1991) points out to the existence of two-tier markets as a result of differences in consumer tastes.⁷ In his

³Robinson (1933), Braithwaite (1928), Galbraith (1958, 1967) and Packard (1957,1969) also contributed to this view.

⁴Von der Fehr and Stevik (1998) and Bloch and Manceau (1999) analyze the role of persuasive advertising in differentiated duopoly markets, and Tremblay and Polasky (2002) show how persuasive advertising may affect price competition in a duopoly with no real product differentiation.

⁵Lee and Tremblay (1992) find no evidence that advertising promotes beer consumption in the US. Nelson and Moran (1995), in an inter-industry study of alcoholic beverages, conclude that advertising serves to reallocate brand sales.

⁶Pedrick and Zufryden (1991) report a strong direct effect of advertising exposure over brand choice in the yogurt industry. However, some recent empirics point out towards the dominance of experience over advertising. See Bagwell (2003) for a review.

⁷He focuses on the concentration-market size relationship in industries where firms incur endogeneous sunk

study, frozen-food industry illustrates the emergence of dual structures (where high-advertisers coexist with nonadvertisers) in advertising-intensive markets. As in Sutton's model, here post advertising consumers tastes are heterogenous.⁸ However in my model not all consumers rank the advertised products the same (advertising induces subjective "horizontal" differentiation), and the number of advertising-responsive consumers is endogenous. I show that a dual structure emerges albeit the indifferent consumer and the loyal markets are not independent.

Although it takes a different view on advertising, my article shares a number of technical features with part of the informative advertising literature dealing with price dispersion phenomena. A seminal article by Stigler (1961) revealed the role of informative advertising in homogenous product markets, and related it to price dispersion.^{9,10}

The pricing stage of the game can be viewed as a modified version of the model of sales by Varian (1980), in the sense that the total base of captured consumers is shared asymmetrically instead of being evenly split among the firms. The two-stage game offers a way of endogeneizing consumers' heterogeneity: It turns out that the symmetric outcome does not obtain, raising a robustness question to Varian's (1980) symmetric model.

Baye, Kovenock and de Vrier (1992) present an alternative way of checking the robustness of Varian's symmetric setting. They construct a metagame where both firms and consumers are players, and asymmetric price distributions cannot form part of a subgame perfect equilibrium. In my model, only firms are making decisions and the subgame perfect equilibria of the game are asymmetric.

Narasimhan (1988) derived the mixed pricing equilibrium in a price-promotions model where two brands act as monopolists on loyal consumer markets and compete in a common market of brand switchers. The pricing stage of my model extends his setting to oligopoly, when the switchers are extremely price sensitive.

Section 2 describes the model, while section 3 derives the equilibrium in the pricing stage costs, and contrasts these results with those in markets with exogeneous setup costs.

⁸In his setting, nonretail buyers do not respond to advertising, unlike retail customers.

⁹Butters (1977), Grossman and Shapiro (1984), McAfee (1994), Robert and Stahl (1993), Roy (2000) construct price dispersion models where oligopolists use targeted advertising to offer information about their products to consumers who are completely uninformed or incur costly search to collect information.

¹⁰Recently, clearinghouse models have been used to explain persistent price dispersion in internet markets, see Baye and Morgan (2001). The present setting can be linked to virtual markets: in the presence of price-comparison sites, firms may have incentives to engage (prior to price competition) in costly search frustration activities (obfuscation). See Ellison and Ellison (2001).

and comments on the chosen strategies. Section 4 presents the equilibrium emerging in the advertising stage, and defines the outcome of the sequential game. In section 5, I propose an alternative interpretation of the model. Sections 6 and 7 discuss the setting and present some concluding remarks. All the proofs missing from the text are relegated to an Appendix.

2 The Model

There are n **firms** selling a homogenous product. All firms have the same constant marginal cost. In the first period, firms choose simultaneously and independently an advertising expenditure and, in the second period, they compete in prices. Let α_i be the advertising expenditure chosen by firm i .

This model deals with non-price advertising. Each firm promotes its product to induce subjective differentiation and generate brand loyalty.¹¹ The fraction of consumers that are loyal to firm i depends on the advertising expenditure profile. At the end of the first stage, the advertising choices become common knowledge.

In the second stage, firm i chooses the set of prices that are assigned positive density in equilibrium and the corresponding density function. Let $F_i(p)$ be the cumulative distribution function of firm i 's offered prices. The price charged by a firm is a draw from its price distribution.

I assume that there is a continuum of **consumers**, with total measure 1, who desire to purchase one unit of the good whenever its price does not exceed a common reservation value r . After advertising takes place, part of the consumers remain indifferent (possibly because no advertising reached them or, alternatively, no advertising convinced them) and the remaining ones become loyal to one brand or another. The indifferent consumers view the alternatives on the market as perfect substitutes and all purchase from the lowest price firm. The loyal consumers are split amongst the advertised brands. The size of each group is determined by the total advertising investment on the market. The total number of loyal consumers, U , is assumed to be an increasing and concave function of the aggregate advertising expenditure of the firms, $U = U(\sum_i \alpha_i)$, with $\lim_{\sum_i \alpha_i \rightarrow \infty} U(\sum_i \alpha_i) = 1$.¹² Thus, with higher advertising more

¹¹Advertising may carry some emotional content that potentially touches people and makes them develop loyalty feelings for the product. Although there is no real differentiation among the products they elicit different associations due to advertising. See Section 6 for a more detailed discussion on this view.

¹²The empirical studies often present evidence that advertising is subject to diminishing returns to scale. See the reviews by Scherer and Ross (1990) and Bagwell (2003).

consumers join the captured (loyal) group. One may think that more advertising would be more convincing. The captured consumers are split amongst the firms according to a market sharing function depending on the advertising expenditure of the firms. Let it be $S(\alpha_i, \alpha_{-i}) = S_i \in [0, 1]$, satisfying the following properties:¹³

1. $\sum_{i=1, \dots, n} S_i = 1$;
2. $\frac{\partial S_i}{\partial \alpha_i} \geq 0$ with strict inequality if $\exists j \neq i$ s.t. $\alpha_j > 0$ or if $\alpha_j = 0$, for all j ;
3. $\frac{\partial S_i}{\partial \alpha_j} \leq 0$ with strict inequality if $\alpha_i > 0$;
4. $S(\alpha_i, 0) = 1$ if $\alpha_i > 0$;
5. $S(0, \alpha_{-i}) = 0$ if $\exists j \neq i$ s.t. $\alpha_j > 0$ and $S(0, 0) = 0$;
6. $S(\alpha_i, \alpha_{-i})$ is homogeneous of degree 0;
7. $S(\alpha_i, \alpha_{-i}) = S(\alpha_i, \tilde{\alpha}_{-i})$ whenever $\tilde{\alpha}_{-i}$ is obtained from α_{-i} by permutation.

The symmetry of the loyal market sharing function follows from the symmetry of the firms. Conditions 1, 4 and 5 require that all loyal consumers be split amongst the firms with positive advertising expenditure. Conditions 2 and 3 require that a higher own advertising increase the market share of a firm, whenever it is below 1, and a higher rival advertising decrease the share of a firm whenever it is above 0. Condition 6 requires that the share profile remain unchanged to multiplications of the advertising by the same factor.¹⁴ Condition 7 states that the market share of a firm is determined by the rivals' advertising levels and not by their identity.

Finally, the remaining consumers form the base of indifferent consumers, denominated by I . Hence, each firm faces an indifferent base $I = 1 - U(\sum_i \alpha_i)$ and a particular captured or locked-in base $U_i = S(\alpha_i, \alpha_{-i}) U(\sum_i \alpha_i)$. Firms cannot price discriminate between these two types of consumers.¹⁵

¹³This function is used by Schmalensee (1976). An example of loyal market share function satisfying these conditions is $S(\alpha_i, \alpha_{-i}) = \frac{\alpha_i}{\sum_j \alpha_j}$.

¹⁴However, for instance, doubling the advertising expenditure leads to an increase in the *size* of the loyal market, although the sharing rule does not change. Hence, escalating advertising would increase the captured base of each firm, all else equal.

¹⁵Post advertising, some consumers face infinite costs of switching from their favorite brand to another, while the others have no switching costs. Persuasive advertising induces a psychological switching cost.

The timing of the game can be justified by the fact that I deal with non-price advertising. Firms are building-up a brand identity through advertising expenditure. A change in the brand advertising takes time, whereas firms can modify their prices almost continuously.

The concept of equilibrium used in the model is subgame perfect Nash equilibrium. The two stage advertising-pricing game is solved backwards.

3 The Pricing Stage

In the first stage, firms simultaneously choose the advertising investments. In the second stage, knowing the whole profile of advertising expenditure, firms choose prices. In this section I solve the pricing game for an arbitrary weakly ordered profile of advertising expenditure. Let U_i be the loyal base of consumers captured by firm i , and let $I = 1 - U = 1 - \sum_i U_i$ be the group of brand switchers who buy the lowest priced brand. Without loss of generality, assume $U_i \geq U_j$ whenever $i \leq j$ with $i, j \in N = \{1, 2, \dots, n\}$.¹⁶

For any firm $i \in N$ only the prices in the interval $A_i = [c, r]$ are relevant, with c being the common constant marginal cost of production. Pricing at $p_i < c$ firms would make negative profits and pricing at $p_i > r$ firms would make zero profits.

The profit function of firm i , $i = 1, 2, \dots, n$ is given by:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)(U_i + I) & \text{if } p_i \leq r \text{ and } p_i < p_j, \forall j \neq i; \\ (p_i - c)(U_i + I\varphi) & \text{if } p_i \leq r, p_i = p_j, j \in M \subseteq N \\ & \text{and } p_i < p_k, \forall k \in N \setminus M \text{ with } \varphi = \frac{1}{|M|}; \\ (p_i - c)U_i & \text{if } p_i \leq r \text{ and } \exists j \neq i \text{ s.t. } p_j < p_i. \end{cases} \quad (1)$$

The firms choose the prices that maximize their payoffs taking as given the pricing strategies of the rivals.

At this stage, firms count with a loyal group and at the same time they compete for the remaining brand indifferent consumers. However, if all firms price above marginal costs, there are incentives to undercut in order to win the switchers market. Pricing at marginal cost cannot form part of an equilibrium as firms can always obtain monopoly profits on their loyal base. The tension between the incentives to undercut in order to win the indifferent market and those to extract surplus from the loyal leads to the following result.

¹⁶Loyal consumers buy their favourite brand whenever its price does not exceed their reservation value. To characterize the off the equilibrium path, assume that, in case their favourite is priced above the reservation value, loyal purchase a random product whenever is priced below reservation value.

Proposition 1 *The game $(A_i, \pi_i; i \in N)$ has no pure strategy Bertrand-Nash equilibrium.*

Existence of a mixed strategy equilibrium can be proven by construction.¹⁷ This approach gives also the functional forms of the equilibrium pricing strategies of the firms.

A mixed strategy for firm i is defined by a function, $f_i : A_i \rightarrow [0, 1]$, which assigns a probability density $f_i(p) \geq 0$ to each pure strategy $p \in A_i$ so that $\int_{A_i} f_i(p) dp = 1$.

Let $\widehat{S}_i = [L_i, H_i]$ be the support of the equilibrium price distribution of firm i . This means that $f_i(p) > 0$ for all $p \in \widehat{S}_i$. Denominate by $F_i(p)$ the cumulative distribution function related to $f_i(p)$.

At a price p , a firm sells to its loyal market and is the winner of the indifferent market provided that p is the lowest price. Consequently, its expected demand is given by:

$$D_i(p) = (1 - \sum_i U_i) \prod_{j \neq i} (1 - F_j(p)) + U_i.$$

The following lemmas present a number of features of the price distributions. The lowest bound of the price supports has to be shared by at least two firms. If only one firm puts positive density on the minimal lower bound, this firm has incentives to move the density to the second lowest bound. In-between the two prices the firm faces no competition and its profits are increasing. When selling to its maximal market (loyal group plus the switchers) at a specific (low) price a firm makes profits equal to monopoly profits on its loyal base. A firm does not put positive density at prices below this specific level. At any price in its support a firm faces a positive probability of losing the switchers. Hence all firms put positive density on the monopoly price. If all firms have an atom at the upper bound, there is a positive probability of a tie and, therefore, incentives to undercut. An analogous reasoning indicates that the distribution function should be continuous below the upper bound.

Lemma 1 *Let $L = L_k = \min_i \{L_i\}$. Then $\exists j \neq k$, such that $L_j = L_k = L$. Moreover for each firm*

$$L_i - c \geq (r - c) \frac{U_i}{U_i + I}.$$

Proof. If $L_k = \min_i \{L_i\}$ and $\nexists j$, s.t. $L_j = L_k$, choosing a lower bound in the interval (L_k, L') , with $L' = \min_{i \neq k} \{L_i\}$, does not decrease firm k 's probability of being the winner of

¹⁷It is guaranteed as the pricing game satisfies the conditions found by Dasgupta and Maskin (1986, p. 14, Thm.5). The emergence of mixed-strategy equilibria when there is a mass of consumers with zero switching costs and there are consumers with positive switching costs was also pointed out by Klemperer (1987).

the indifferent consumers, and strictly increases its expected profits. Then, $\exists j \neq k$, such that $L_j = L_k$. Suppose firm i chooses a price p such that $p - c < (r - c) \frac{U_i}{U_i + I}$. Then, the maximal profits of firm i when pricing at p are $(p - c)(I + U_i) < (r - c)U_i$. The RHS represents the certain profit of firm i when it sells only to its loyal base at the monopoly price (its minmax value). Then, $L_i - c \geq (r - c) \frac{U_i}{U_i + I}$ for all i . ■

Lemma 2 $H_i = r, \forall i = 1, \dots, n$.

Proof. At any price $p_i > r$, $\pi_i(p_i, p_{-i}) = 0$. This implies $H_i \leq r$. Let $H_m = \min_{j \in N} \{H_j\}$. Suppose by contradiction that $H_m < r$.

I show first that (a) $\nexists j \in N$ such that $f_j(p) > 0$ for $p \in (H_m, r)$. Suppose such j existed. Notice that $(1 - F_m(p)) = 0$ for $p \in (H_m, r)$. Then, $\pi_j(p) = (p - c)U_j < (r - c)U_j$, and firm j would deviate.

To complete the proof I show that (b) $A = \{i \mid i \in N \text{ and } H_i = H_m\} = \emptyset$.

That is, $H_m < r \Leftrightarrow A \neq \emptyset$. By (a), $H_j = r$ for $j \in N \setminus A$. (b1) Let $|A| = 1 \Leftrightarrow A = \{m\}$. Then, because $\prod_{j \neq m} (1 - F_j(H_m)) = \prod_{j \neq m} (1 - F_j(r - \varepsilon))$, follows that

$$(H_m - c)(U_m + I \prod_{j \neq m} (1 - F_j(H_m))) < (r - c - \varepsilon)(U_m + I \prod_{j \neq m} (1 - F_j(r - \varepsilon))),$$

and firm m would deviate. (b2) Let $|A| > 1$. If $\exists j \in A$ s.t. $pr(p_j = H_m) = 0$, then the previous argument applies to $i \in A, i \neq j$ and there are incentives to deviate. If $\forall j \in A, pr(p_j = H_m) \neq 0$, there is positive probability of a tie at H_m and any firm j has incentives to deviate to $H_m - \varepsilon$ for $\varepsilon > 0$ small. ■

Lemma 3 *There is at least one firm that does not have an atom at r .*

Proof. Assume to the contrary that all firms assign positive probability to pricing at r , equal to $\phi_i \in (0, 1]$. Then by pricing at r , firm k makes profits of $(r - c)(U_k + I\phi)$, where $\phi = \frac{1}{n} \prod_{i \neq k} \phi_i$. But, pricing at $r - \varepsilon$, firm k makes profits of $(r - c - \varepsilon)(U_k + I) > (r - c)(U_k + I\phi)$ for $0 < \varepsilon < \frac{I(1-\phi)}{I+U_k}(r - c)$. This proves that $\exists k \in N$, s.t. $F_k(r) = 1$. ■

Lemma 4 $F_i(p)$ is continuous on $[L_i, r)$.

Proof. A heuristic proof follows, and a formal one is presented in the Appendix. If firm i has a jump at $p \in [L_i, r)$, then there is $\varepsilon > 0$ such that for $j \neq i$, $F_j(p) = F_j(p + \varepsilon)$. Otherwise, firm j would increase its expected profit by choosing $p - \delta$, instead of $p_j \in (p, p + \varepsilon)$. This contradicts the optimality of p given that firm i would only increase its profits by moving the

mass to $p + \varepsilon$. Notice that this argument fails at $p = r$ because profits at $r + \varepsilon$ are equal to zero.

■

Remark 1 *Lemmas 1-4 do not rule out the existence of degenerate distributions for some firms.*

I propose the following asymmetric pricing equilibrium, valid for any weakly ordered profile of loyal bases.

Conjecture 1 *For $U_1 \geq \dots \geq U_{n-1} \geq U_n$, the supports are $\widehat{S}_n = [L, r]$, $\widehat{S}_{n-1} = [L, r]$ and $\widehat{S}_k = \{r\}$ for $k = 1, 2, \dots, n-2$. In addition, $F_k(p) = 0$ for $p < r$ and $F_k(p) = 1$ for $p \geq r$. The cdf's $F_n(p)$ and $F_{n-1}(p)$ are continuous on $[L_i, r)$ and $F_{n-1}(r) < 1$.*

Note that by Lemma 3, $L_i - c \geq (r - c) \frac{U_i}{I + U_i}$ for all i . Also $L_k - c \geq (r - c) \frac{U_{n-1}}{I + U_{n-1}}$ for any $k \leq n - 2$. Then firm n has no incentives to put positive densities to prices $p \in [(r - c) \frac{U_n}{I + U_n} + c, (r - c) \frac{U_{n-1}}{I + U_{n-1}} + c)$ because all these prices are strictly dominated by $p = (r - c) \frac{U_{n-1}}{I + U_{n-1}} + c$, which does not decrease the probability of winning the indifferent consumers (I), but strictly increases firm's expected profits. Therefore, $L = (r - c) \frac{U_{n-1}}{I + U_{n-1}} + c$.

At equilibrium a firm should be indifferent among all the strategies (prices) that form the support of its distribution function. Firm n should be indifferent between any price $p \in [L, r)$ and pricing at the lower bound of its support, L . It follows that:

$$(p - c) [(1 - F_{n-1}(p)) I + U_n] = (L - c) (I + U_n) \Rightarrow$$

$$F_{n-1}(p) = \frac{(p - L) (U_n + I)}{I (p - c)}.$$

Similarly, firm $n - 1$ should be indifferent between any price $p \in [L, r)$ and pricing at the upper bound of its support, r . This gives:

$$(p - c) [(1 - F_n(p)) I + U_{n-1}] = (r - c) U_{n-1} \Rightarrow$$

$$F_n(p) = \frac{(p - c) (U_{n-1} + I) - (r - c) U_{n-1}}{I (p - c)}.$$

For the conjectured strategies to be an equilibrium: i) expected profit of any firm should be constant at all prices in the support of its distribution, ii) distribution functions should be well defined and, iii) no firm should have incentives to price outside its support.

i) *Constant profit conditions.*

The expected profits are:

$$\begin{aligned}\pi_k(r) &= (r - c) U_k \quad \forall k \in N \setminus \{n - 1, n\}, \\ E\pi_{n-1}(p) &= (r - c) U_{n-1} \quad \forall p \in \widehat{S}_{n-1}, \\ E\pi_n(p) &= (L - c) (U_n + I) \quad \forall p \in \widehat{S}_n.\end{aligned}$$

Consider pricing in the interval $[L, r)$. Only firms n and $n - 1$ choose these prices. Using the distribution functions it can be easily shown that this requirement is fulfilled.

ii) *Properties of the distribution functions.*

The distribution functions are increasing ($F'_{n-1}(p) > 0$, $F'_n(p) > 0$) with $F_{n-1}(L) = F_n(L) = 0$. Firm $n - 1$ puts positive probability on pricing at r equal to $\frac{U_{n-1} - U_n}{U_{n-1} + I}$, while the cdf of firm n is continuous on $[L_{n-1}, r]$. For any firm $k = 1, 2, \dots, n - 2$, the degenerate distribution functions are well defined.

iii) *Deviation outside the support.*

For firms n and $n - 1$, deviating outside the support means pricing above r or pricing below L . But, all such prices are strictly dominated by pricing in the interval $[L, r]$.

Consider firm $k = 1, 2, \dots, n - 2$. Deviating to price $p < r$, it makes profits:

$$\begin{aligned}(p - c) (U_k + I (1 - F_n(p)) (1 - F_{n-1}(p))) &= \\ (p - c) U_k + (r - p) U_k \frac{[(L - c) I - (p - L) U_n] U_{n-1}}{I (p - c) U_k}.\end{aligned}$$

Its expected profit at r is $(r - c) U_k = (r - p) U_k + (p - c) U_k$. Let $g(p) = \frac{[(L - c) I - (p - L) U_n] U_{n-1}}{I (p - c) U_k}$ and notice that $g'(p) = -\frac{(L - c) U_{n-1} (I + U_n)}{p - c} < 0$ and $g(L) = \frac{U_{n-1}}{U_k} \leq 1$. It follows that deviation to prices in the interval $[L, r)$ is not profitable. Deviation to prices below L is trivially unprofitable. Hence, no firm has incentives to price outside its support. This completes the proof of next result.

Proposition 2 *The following distribution functions represent a mixed strategy Nash equilibrium of the pricing subgame $(A_i, \pi_i; i \in N)$.*

$$\begin{aligned}F_n(p) &= \begin{cases} 0 & \text{for } p < L = (r - c) \frac{U_{n-1}}{I + U_{n-1}} + c \\ \frac{(I + U_{n-1})}{I} - \frac{(r - c) U_{n-1}}{I (p - c)} & \text{for } L \leq p \leq r \\ 1 & \text{for } p \geq r \end{cases}, \\ F_{n-1}(p) &= \begin{cases} 0 & \text{for } p < L = (r - c) \frac{U_{n-1}}{I + U_{n-1}} + c \\ \frac{(I + U_n)}{I} - \frac{(r - c) U_{n-1} (I + U_n)}{I (p - c) (I + U_{n-1})} & \text{for } L \leq p \leq r \\ 1 & \text{for } p \geq r \end{cases},\end{aligned}$$

$$F_k(p) = \begin{cases} 0 & \text{for } p < r \\ 1 & \text{for } p \geq r \end{cases} \quad \text{for } \forall k = 1, 2, \dots, n-2.$$

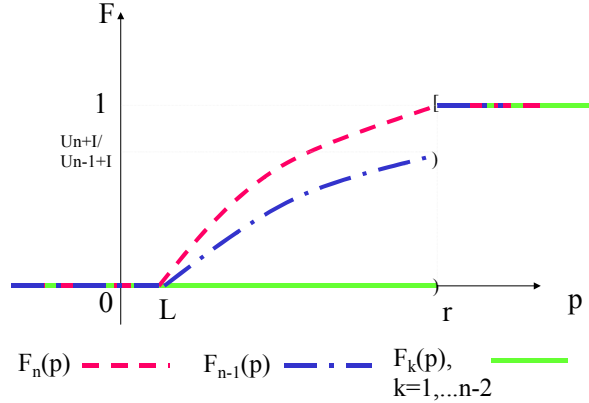


Figure 1: Price distributions in Proposition 2

The price of firm k ($k = 1, 2, \dots, n-2$) first order stochastically dominates the price of firm $n-1$, while the latter stochastically dominates the price of firm n .

When charging a price $p < r$, a firm i with $U_i > U_j$, $i, j \in N$, loses $\Delta^- = (r-p)U_i > (r-p)U_j$, while facing the same potential gain as firm j in the indifferent market $\Delta^+ = (p-c)I$. Thus, firm i is less aggressive than firm j because it tends to lose more. Given that with $n \geq 3$ the price rivalry augments, it turns out that the firms with higher loyal base $U_k \geq U_{n-1}$, $k = 1, \dots, n-2$, choose to maintain monopoly pricing, r , with probability 1. The mass point at r in the distribution of firm $n-1$ increases in the difference between the two lowest loyal bases, $F_{n-1}(r) = 1 - \frac{U_{n-1}-U_n}{U_{n-1}+I}$.

The firms that choose degenerate distributions make deterministic profits $\pi_k(r) = (r-c)U_k$ for $k = 1, \dots, n-2$. The expected profits of the remaining firms are $E\pi_{n-1}(p) = (r-c)U_{n-1}$ and $E\pi_n(p) = (L-c)(U_n+I) = (r-c)U_{n-1}\frac{U_n+I}{U_{n-1}+I}$.

Notice that $(r-c)U_{n-1} \geq E\pi_n(p) \geq (r-c)U_n$. Then, $E\pi_i(p) \geq E\pi_j(p)$ whenever $U_i \geq U_j$ (or $i \leq j$). Firm $n-1$, despite of choosing a mixed pricing strategy, makes in equilibrium expected profit equal to the profit it would make on its loyal base if it chose monopoly pricing. By contrast, at equilibrium, firm n has expected profit higher than its monopoly profit on its loyal base.

The equilibrium in Proposition 2 predicts price dispersion in relatively small markets. When the number of firms increases, so does the number of firms that permanently choose monopoly pricing, and the dispersion in prices tends to become insignificant. The higher the number of

competitors the lower the chances of an individual firm to win the indifferent market. When the number of competitors is higher more firms prefer to rely on their locked-in markets and act as monopolists rather than engage in aggressive pricing.

Narasimhan (1988) offers an explanation for price dispersion in competitive markets based on consumer loyalty. He restricts attention to a duopoly. The present paper offers an extension of his setting to oligopoly. With arbitrary weakly ordered profiles of loyal groups, only the two lowest firms engage in price promotions¹⁸ and, for this reason, the potential of this model to explain market wide price promotions is limited when the number of firms increases.

The next Proposition presents a uniqueness result.

Proposition 3 *If firms employ convex supports, the equilibrium stated in Proposition 2 is the unique equilibrium that applies to **any** weakly ordered profile of loyal bases.*

However, when $n > 2$, for particular profiles of loyal groups, there are other pricing equilibria, as well. *In all equilibria the firms make the same expected payoffs.* This allows to solve the reduced form game in the first stage. I present another equilibrium for the profile $U_n < U_{n-1} = \dots = U_1$, which turns out to be relevant for the present analysis.

Proposition 4 *If the loyal bases, U_i , $i = 1, 2, \dots, n$, satisfy $U_i = \bar{U}$ for $i = 1, 2, \dots, n-1$ and $U_n < \bar{U}$, there exists a pricing equilibrium with all firms randomizing over the same convex support. The firms choose prices according to the following distributions:*

$$F_i(p) = \begin{cases} 0 & \text{for } p < L = (r-c) \frac{\bar{U}}{I+\bar{U}} + c \\ 1 - \left[\frac{(L-c)(I+U_n)}{I(p-c)} - \frac{U_n}{I} \right]^{\frac{1}{n-1}} & \text{for } L \leq p < r \\ 1 & \text{for } p \geq r \end{cases}$$

for $i = 1, 2, \dots, n-1$,

$$F_n(p) = \begin{cases} 0 & \text{for } p < L = (r-c) \frac{\bar{U}}{I+\bar{U}} + c \\ 1 - \left[\frac{(r-p)\bar{U}}{I(p-c)} \right] \left[\frac{L(I+U_n)}{I(p-c)} - \frac{U_n}{I} \right]^{\frac{2-n}{n-1}} & \text{for } L \leq p \leq r \\ 1 & \text{for } p \geq r \end{cases} .$$

Proof. Directly follows from the constant profit condition. ■

¹⁸Pricing below the monopoly level with positive probability is interpreted as a price promotion.

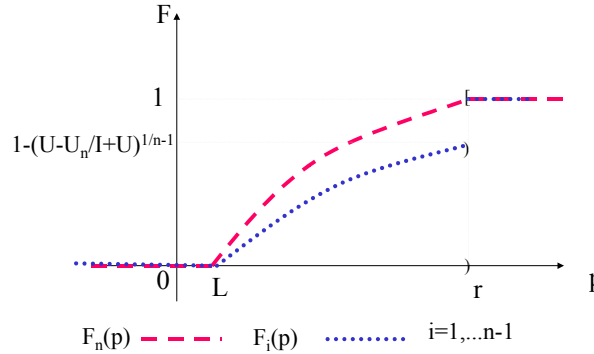


Figure 2: Price distributions in Proposition 4

Remark 2 For this specific profile of loyal bases, there are other equilibria in-between the ones in Propositions 2 and 4. They have a number $k < n - 2$ of firms choosing r with probability 1 and the rest randomizing over the same convex support. Firms $n - 1$ and n necessarily randomize.

It can be seen from Proposition 4 and Figure 2 that price of firm n is stochastically dominated by price of firm i ($i = 1, \dots, n - 1$). The same intuition as before holds, but given that $U_i = \bar{U}$ for $i = 1, 2, \dots, n - 1$ it is possible that all firms randomize at equilibrium.

The pricing subgame maybe understood as a variant of Varian’s “Model of Sales”, with asymmetric captured consumer bases. “A Model of Sales” is meant to describe markets which exhibit price dispersion, despite the existence of at least some rational consumers.¹⁹ The model interprets sales as a way to discriminate between consumers who are assumed to come in two types, informed and uninformed. All consumers have a common reservation value and they purchase a unit of the good whenever the price does not exceed the valuation, the uninformed ones choose randomly a shop and the informed ones buy from the cheapest seller. Varian restricts attention to the symmetric equilibrium of the symmetric game (with uninformed consumers evenly split amongst the firms).²⁰ However, there exists a family of asymmetric equilibria of the symmetric game.²¹

¹⁹Varian is concerned with understanding “temporal price dispersion” rather than “spatial price dispersion”. That is, intertemporal changes in the pricing of a given firm rather than cross-sectional price volatility.

²⁰If the optimal pricing distributions are continuous and the supports are convex, then the equilibrium is symmetric (Proposition 9 of Varian (1980), p.658). See the Appendix for a transcription of this Proposition.

²¹A comprehensive analysis of all the asymmetric equilibria of the symmetric game is provided by Baye, Kovenock and deVries (1992).

The present setting offers a way of endogenizing the creation of locked-in consumers, and it raises a robustness question to Varian's symmetric setting because it turns out that at equilibrium the captured bases are asymmetric.

Extending Varian's model, Baye, Kovenock and de Vries (1992) construct a metagame in which consumers are also players. In the first stage, uninformed consumers and firms move simultaneously. Firms choose a price distribution and the uninformed consumers decide from which firms to purchase. In the second stage, the informed consumers choose the seller they will buy from. Given that the asymmetric price distributions can be ranked by first-order stochastic dominance, they show that the unique subgame perfect equilibrium of the extensive game is the symmetric one. However, this follows from the equilibrium consistency requirement that a firm with higher expected price cannot have a larger uninformed consumer base.

4 Advertising Expenditure Choices

In this section, I derive the equilibrium of the reduced form game in the first stage where oligopolists simultaneously choose an advertising expenditure. Their payoffs are the profits emerging in the pricing stage minus the chosen advertising expenditure. The *gross* of advertising cost profits are:

$$\begin{aligned} E\pi_j(p) &= (r - c)U_j, & \forall p \in \widehat{S}_j \text{ and } \forall j \in N \setminus \{n\}; \\ E\pi_n(p) &= (L - c)(U_n + I) = (r - c)U_{n-1} \frac{(U_n + I)}{(U_{n-1} + I)}, & \forall p \in \widehat{S}_n. \end{aligned}$$

The loyal consumer group of firm i (U_i) is defined as a share $S(\alpha_i, \alpha_{-i})$ of the total number of loyal consumers given by $U(\sum_i \alpha_i)$. That is,

$$U_i(\alpha_i, \alpha_{-i}) = S(\alpha_i, \alpha_{-i})U(\sum_j \alpha_j).$$

Each firm may invest in generating loyal consumers and the total number of brand loyalists on the market is determined by the aggregate expenditure. The advertising technology is imperfect, so that there is always a fraction of the consumers who are not persuaded (or reached) by advertising.²² This fraction forms the brand indifferent group (I) and buys the cheapest product,

$$I = 1 - \sum_i U_i(\alpha_i, \alpha_{-i}) = 1 - U(\sum_i \alpha_i).$$

²²Alternatively, one can think that these consumers enter the market after advertising took place.

Under the assumptions made so far, the loyal group of a firm is increasing in own advertising. The incremental consumers may proceed from the brand indifferent group ($U'(\sum_i \alpha_i) > 0$) or from the rival loyal groups ($\frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \geq 0$). The last source leaves open the possibility of reciprocal cancellation across brands.²³ An increase in rival advertising has two conflictive effects on the loyal base of a firm. There is a positive effect due to the increase in the total number of loyals ($U'(\sum_i \alpha_i) > 0$) and a negative one due to a decrease in the share of the firm ($\frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_j} \leq 0$). I assume that the overall effect of rival advertising is negative.²⁴

Assumption 1:

$$\frac{\partial^2 U_i}{\partial \alpha_i \partial \alpha_j} < 0; \quad \frac{\partial^2}{\partial \alpha_n \partial \alpha_j} \left(\frac{U_{n-1}(U_n + I)}{(U_{n-1} + I)} \right) < 0 \quad \text{and} \quad \frac{\partial}{\partial \alpha_i} U_i(0, \alpha_{-i}) > \frac{1}{r - c}.$$

The first and second conditions are sufficient for an equilibrium to exist.²⁵ The last condition is necessary for a non-trivial result. In the remainder of this paper, the first subscript refers to the identity of the firm, and the second to the partial derivative. That is, $S_{i,j} = \frac{\partial S_i}{\partial \alpha_j} = \frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_j}$ and $U_{j,j} = \frac{\partial U_j}{\partial \alpha_j} = \frac{\partial (S_j(\alpha_j, \alpha_{-j})U(\sum_i \alpha_i))}{\partial \alpha_j}$.

Firms $j = 1, 2, \dots, n-1$ maximize the expected profit net of advertising expenditure:

$$\pi_j^{net} = (r - c) S_j U(\sum_i \alpha_i) - \alpha_j.$$

Firm n maximizes its expected profit net of advertising costs:

$$\pi_n^{net} = (r - c) S_{n-1} U(\sum_i \alpha_i) \frac{(S_n U(\sum_i \alpha_i) + I)}{(S_{n-1} U(\sum_i \alpha_i) + I)} - \alpha_n.$$

The FOC of the maximization problems above implicitly define $\alpha_n^*(\alpha_{-n})$ and $\alpha_j^*(\alpha_{-j})$ for $j = 1, 2, \dots, n-1$. In addition, $\alpha_l^* = \alpha_j^* = \alpha^*, \forall j, l \in N \setminus \{n\}$.

A symmetric equilibrium ($\alpha_n^* = \alpha_j^* = \alpha^*$) exists if, when choosing α^* , firm n does not have incentives to decrease, and the other firms do not have incentives to increase.

$$\frac{\partial}{\partial \alpha_n} \pi_n^{net}(\alpha^*) \geq 0 \quad \text{and} \quad \frac{\partial}{\partial \alpha_j} \pi_j^{net}(\alpha^*) \leq 0 \quad \text{for all } j = 1, 2, \dots, n-1.$$

In Appendix B it is shown that this requirement leads to a contradiction. Together with the optimization problem above, this proves the following result.

²³Metwally (1975,1976) and Lambin (1976) find empirical evidence in this sense.

²⁴The market size is normalized to 1. It is possible that advertising raises all firms' market shares, but seems more reasonable to be so if it generates a market size expansion.

²⁵Concavity of the profit functions guarantees well defined best responses, while strategic substitutability of advertising investments guarantees that at the asymmetric equilibrium candidate firms do not leapfrog each other.

Proposition 5 *Under Assumption 1, in any pure strategy Nash equilibrium of the reduced form advertising game, $\alpha_n < \alpha_i = \alpha_j$ for $\forall i, j \in \{1, 2 \dots n-1\}$. The values of α_n and α_i ($\forall i \in \{1, 2 \dots n-1\}$) are implicitly defined by the FOCs.*

Proposition 6 *The advertising expenditure in Proposition 5, together with any of the pricing strategy profiles in Proposition 2, Proposition 4, or Remark 2 represent the subgame perfect equilibria of the two stage game.*

The asymmetric equilibrium may seem counterintuitive and arbitrary, however it follows from the fact that firms weight their first and second stage decisions. A larger first stage investment triggers less aggressive second stage pricing, while a lower investment imposes a more aggressive pricing strategy.

To illustrate Proposition 5, let $S(\alpha_i, \alpha_{-i}) = \frac{\alpha_i}{\sum_j \alpha_j}$ for $\forall i \in N$, $U(\sum_i \alpha_i) = \frac{\sum_i \alpha_i}{1 + \sum_i \alpha_i}$, and $c = 0$. Table 1 presents the equilibrium advertising expenditure, the size of the indifferent group, and the net profits of the firms, for different values of n and r .²⁶

The sharing rule in the example, $S(\alpha_i, \alpha_{-i}) = \frac{\alpha_i}{\sum_j \alpha_j}$ maybe interpreted as a probability that an arbitrary loyal consumer chooses firm i .²⁷

Although they are identical, at equilibrium, firms choose asymmetric advertising expenditure. This asymmetry follows from the choice of mixed pricing strategies. There is one firm with a strictly lower advertising level (firm n in the example) and with the lowest loyal group. All other firms choose the same higher level of advertising and have equal larger loyal consumer bases. It is difficult to rank the *net* profits of the firms; however, in the examples above, the low advertiser makes lower profits than its rivals. This need not be always the case: The levels of net profits depend on the number of firms, on the advertising technology, and on the monopoly margins.

The lowest loyal group firm prices more aggressively, has higher probability of being the winner of the indifferent market, and makes lower *gross* expected profits. All other firms price less aggressive and make equal gross of advertising expenditure profits equal to monopoly profit on their loyal market.

²⁶For computational convenience, the chosen advertising technology is a very effective one, resulting in a large loyal market (U).

²⁷This probability is nonincreasing in i and thus a firm with higher advertising is more likely to be chosen by a loyal consumer.

Table 1:

n	$r - c$	$\alpha_j, j \neq n$	α_n	$U_j, j \neq n$	U_n	I	$\pi_j^{net}, j \neq n$	π_n^{net}
5	100	16.56	11.79	.209	.149	.012	4.39	3.46
5	25	4.48	.21	.234	.011	.052	1.37	1.08
4	100	19.22	15.37	.259	.207	.013	6.74	5.64
4	25	5.09	1.58	.285	.088	.055	2.03	1.43
3	100	22.53	19.62	.343	.298	.015	11.76	10.43
3	25	5.82	3.11	.369	.197	.063	3.41	2.45

5 A Random Utility Interpretation

In this section I offer a possible microfoundation to the demand system considered in the case of a duopoly. In particular, I argue that a random utility model can lead to the proportional market sharing function $S_i = \frac{\alpha_i}{\sum_j \alpha_j}$ used in the examples, when consumers respond to advertising and prices, but are short-sighted. With a duopoly there is a unique mixed pricing equilibrium where both firms randomize, and $E(p_i | p_i < r) = E(p_j | p_j < r)$. In the marketing literature, price dispersion models are used to explain price promotions. The expected price conditional on a brand being priced lower than r is thought of as the average discounted price, while $1 - F(r)$ measures the frequency of discounts. Considering that loyal consumers are myopic and care only about the average discount and advertising, the proportional market sharing function, $S_i = \frac{\alpha_i}{\sum_j \alpha_j}$, can be derived from a random utility model.²⁸

6 Discussion

While there is no doubt that advertising plays an important informative role in the economy, not less numerous are the occasions in which it does not provide any relevant information on price or product characteristics. In many homogenous product markets, advertising only operates a redistribution of the consumers among the sellers. Many advertising campaigns and a great deal of the TV spot advertising have rather an emotional content and try to attract consumers associating the product with attitudes or feelings that have no relevant relation to the product

²⁸Consumers have idiosyncratic preferences over the brands and each consumer chooses the one that has the greatest brand advertising-average discount differential (see Anderson, De Palma and Thisse (1992)).

or its consumption.²⁹

Loewenstein (2001) remarks “[w]hile conventional models of decision making can make sense of advertisements that provide information about products (whether informative or misleading), much advertising- for example, depicting happy, attractive friends drinking Coca-Cola seem to have little informational content. Instead such advertising seems to be intended to create mental associations that operate in both directions, causing one to think that one should be drinking Coca-Cola if one is with friends (by evoking a choice heuristic) and to infer one that one must be having fun if one is drinking Coca-Cola (playing on the difficulty of evaluating one’s hedonic state).”³⁰ He points out to the importance of situational construals, people “often seem to behave according to a two-stage process in which they first attempt to figure out what kind of situation they are in and then adopt choice rules that seem appropriate for that situation”. Persuasive advertising could be a way to influence such categorization through mental associations. As an example of association, Pepsi’s 1997 GenerationNext campaign defines itself as being “about everything that is young and fresh, a celebration of the creative spirit. It is about the kind of attitude that challenges the norm with new ideas, at every step of the way”.³¹ Similarly, Perrier significantly strengthened its position in the French mineral water market with its advertising campaign directed to the younger generation, under the slogan “Perrier c’est fou” (“Perrier is crazy”) making the product be perceived as very fashionable.³² These considerations support the persuasive view on advertising and offer a justification for the stylized setting here.

²⁹Explaining the content of Chevrolet TV ads for automobiles “Heartbeat of America”, broadcasted in 1988, General Motors advertising executive Sean Fitzpatrick observed that they “may look disorganized, but every detail is cold-heartedly calculated. People see the scenes they want to identify with.[...] It’s not verbal. It’s not rational. It’s emotional, just the way people buy cars.” (see Scherer and Ross 1990 p.573, originally from “On the Road again, with a Passion”, New York Times October 10th, 1988) Also, commenting on the new trends in television advertising, James Twitchell Professor of advertising at the University of Florida noted that “Advertising is becoming art. You don’t need it, but it’s fun to look at” (see Herald Tribune, January 10th, 2003 p.7).

³⁰In the same spirit Camerer, Loewenstein and Prelec (forthcoming) notice “Prevailing models of advertising assume that ads convey information or signal a product’s quality or, for “network” or “status” goods, a product’s likely popularity. Many of these models seem like strained attempts to explain effects of advertising without incorporating the obvious intuition that advertising taps the neural circuitry of reward and desire”.

³¹See www.pepsi.com. Coke relies on more traditional values in its “Coca Cola...Real” campaign. See also Tremblay and Polasky (2002).

³²See J. Sutton (1991), p.253. Even more relevant for the present discussion is the subsequent campaign of Perrier under the (only literary) meaningless slogan “Ferrier c’est pou”, a partial toggle of its initial slogan.

The present analysis suggests that high advertisers tend to have higher prices. Often, blind tests show that consumers perceive highly advertised brands as different.³³ The model also predicts the existence of a group of heavy advertisers and of a low advertiser. This is compliant with the empirical evidence that markets with significant advertising have a two-tier structure. The endogenous asymmetric advertising profile is more suitable for small oligopolies. In the US sport drinks market, Coca Cola and Pepsico are the two major suppliers. In 2002, Pepsico invested 125 millions to promote its product Gatorade, while Coca Cola invested 11 millions to advertise its product Powerade.

Several extensions to this work are worth mentioning. A major limitation of the present model is the extreme post-advertising heterogeneity of consumers: Loyal consumers are extremely advertising responsive, while the indifferent ones continue to be extremely price sensitive. One may consider a smoother distribution of advertising-induced switching costs. Also, here, indifferent consumers are aware about existence of all products. Possibly it is more realistic to assume that the indifferent consumers know only the prices of some sellers.

7 Conclusions

The present article proposes a way to model the effects of persuasive advertising on price competition in a homogenous product market. I solve a two stage game in which an oligopoly competes, first, in persuasive advertising and, then, in prices. Advertising results in the creation of a loyal group attached to the firm. The equilibrium outcome exhibits price dispersion, although it is possible to have in equilibrium up to $n - 2$ firms choosing monopoly pricing with probability 1. The endogenous advertising choices of the firms reflect the asymmetry in the mixed pricing strategies and, at equilibrium, there is one firm that chooses a lower level of advertising and the remaining ones choose the same higher advertising. In expected terms, the low advertiser prices more aggressively, while the intensive advertisers choose higher prices. Firms balance their first and second stage choices and, as a result, the market outcome is asymmetric despite initial symmetry of firms. The setting shows how persuasive advertising may create a two-tier market and be successfully used to relax price competition.

³³“Double-blind experiments have repeatedly demonstrated that consumers cannot consistently distinguish premium from popular-priced beer brands, but exhibit definite preferences for the premium brands when labels are affixed-correctly or not.” (see Scherer and Ross 1990 p.582)

8 Appendix

8.1 Appendix A

Proof of Proposition 1. Assume (p_i^*, p_{-i}^*) is a pure strategy Nash equilibrium profile. Then, by the definition of such equilibrium, $\nexists p_i$ such that $\pi_i(p_i, p_{-i}^*) > \pi_i(p_i^*, p_{-i}^*)$. Let i be such that $p_i^* \leq p_k^*, \forall k \neq i$. Consider first the case where $p_i^* = p_j^* \neq c$, where $p_j^* = \{\min p_k^* \mid k \in N \setminus \{i\}\}$. Then $\pi_i(p_i^*, p_{-i}^*) = (p_i^* - c)(U_i + I\varphi) < \pi_i(p_i^* - \varepsilon, p_{-i}^*) = (p_i^* - c - \varepsilon)(U_i + I)$. Hence, $\exists p_i = p_i^* - \varepsilon$, for $0 < \varepsilon < \frac{(p_i^* - c)(1 - \varphi)I}{U_i + I}$, such that $\pi_i(p_i, p_{-i}^*) > \pi_i(p_i^*, p_{-i}^*)$. This argument fails at $p_i^* = p_j^* = c$, but $\pi_i(c, p_{-i}^*) = 0 < \pi_i(r, p_{-i}^*)$. Consider, finally, the case $p_i^* < p_j^*$, where $p_j^* = \{\min p_k^* \mid k \in N \setminus \{i\}\}$. Then $\pi_i(p_i^*, p_{-i}^*) = (p_i^* - c)(U_i + I) < \pi_i(p_i^* + \varepsilon, p_{-i}^*) = (p_i^* - c + \varepsilon)(U_i + I)$ whenever $\varepsilon < p_j^* - p_i^*$. Then, $\exists p_i = p_i^* + \varepsilon$, for $0 < \varepsilon < p_j^* - p_i^*$, such that $\pi_i(p_i, p_{-i}^*) > \pi_i(p_i^*, p_{-i}^*)$. The cases presented above complete the proof of nonexistence of a pure strategy Bertrand-Nash equilibrium in the game $(A_i, \pi_i; i \in N)$. ■

Proof of Lemma 4. I prove that $F_i(p)$ is continuous on $[L_i, r)$ by contradiction. Assume there is a mass point in the equilibrium distribution of firm i at price $p < r$ and let the mass be equal to ξ . For simplicity, let $c = 0$.

Case 1. Suppose $\exists j$ pricing in the interval $(p, p + \varepsilon]$. Then, the variation in the profit of firm j when pricing at $p - \varepsilon$ instead of $p + \varepsilon$ is:

$$\begin{aligned} & (p - \varepsilon)[U_j + I\Pi_{k \neq j}(1 - F_k(p - \varepsilon))] - (p + \varepsilon)[U_j + I\Pi_{k \neq j}(1 - F_k(p + \varepsilon))] \geq \\ & -2\varepsilon U_j + \Pi_{k \neq i, j}(1 - F_k(p - \varepsilon)) \begin{bmatrix} I(p - \varepsilon)(1 - F_i(p - \varepsilon)) \\ -I(p + \varepsilon)(1 - F_i(p + \varepsilon)) \end{bmatrix} = \\ & -2\varepsilon U_j + \Pi_{k \neq i, j}(1 - F_k(p - \varepsilon)) \begin{bmatrix} I(p + \varepsilon)(F_i(p + \varepsilon) - F_i(p - \varepsilon)) \\ -2\varepsilon I(1 - F_i(p - \varepsilon)) \end{bmatrix}. \end{aligned}$$

Then, $E\pi_j(p - \varepsilon) - E\pi_j(p + \varepsilon) \rightarrow \Pi_{k \neq i, j}(1 - F_k(p))Ip\xi > 0$ when $\varepsilon \rightarrow 0$. This contradicts the optimality of the equilibrium strategy of firm j .

Case 2. Suppose $\nexists j$ pricing in the interval $(p, p + \varepsilon]$, then

$$E\pi_i(p + \varepsilon) = (p + \varepsilon)U_i + I(p + \varepsilon)\Pi_{k \neq i}(1 - F_k(p)) > pU_i + Ip\Pi_{k \neq i}(1 - F_k(p)).$$

This contradicts the optimality of the equilibrium strategy of firm i and completes the proof. ■

Proof of Proposition 3. Suppose there exists a Nash equilibrium in prices different than the one in Proposition 2. Let \widehat{S}_i^* be the associated supports of the price distributions. For simplicity, let $c = 0$.

By Lemma 2, $\{r\} \subseteq \widehat{S}_i^*$ for all i . Let $K = \{i \mid \{r\} \neq \widehat{S}_i^*\}$.

By Proposition 1 and Lemma 1, $|K| \geq 2$.

a) Assume, first, $|K| > 2$.

By Lemma 3, $\exists l \in N$, such that $f_l^*(r) = 0$. Notice that for all $i \in N \setminus K$, $f_i^*(r) = 1$. Hence, $l \in K$.

By Lemma 1, $\exists i, k \in N$, s.t. $L_i = \min \widehat{S}_i^* = L_k = \min \widehat{S}_k^*$

a1) Suppose that $l = i$. Let $k, h \in K$, $k \neq l \neq h$ and $L_l = L_k$.

1.1. Consider, first, the case $L_h > L_l = L_k$. Then,

$$p(U_h + I \prod_{j \neq h} (1 - F_j(p))) = rU_h \text{ and } p(U_k + I \prod_{j \neq k} (1 - F_j(p))) = rU_k.$$

It follows that $\frac{1 - F_k(p)}{1 - F_h(p)} = \frac{U_h}{U_k}$ or $F_k(p) = \frac{U_k - U_h}{U_k} + \frac{U_h}{U_k} F_h(p)$.

By Lemma 4, $F_h(L_h) = 0 \Rightarrow F_k(L_h) = \frac{U_k - U_h}{U_k}$. Then, because $L_h > L_k$ $F_k(L_h) > 0 \Leftrightarrow U_k > U_h$.

If firm h deviates to L_k , then for sure wins and makes profits of $L_k(U_h + I)$. But, $L_k \geq \frac{U_k r}{U_k + I} > \frac{U_h r}{U_h + I} \Rightarrow L_k(U_h + I) > \frac{U_h r}{U_h + I}(U_h + I) = U_h r$, where the RHS of the equality is the profit of firm h when it prices on its support. This proves that this cannot be an equilibrium.

1.2. Consider that $L_h = L_l = L_k$. Following the equilibrium conditions above $F_h(L_h) = 0 \Rightarrow F_k(L_h) = \frac{U_k - U_h}{U_k} \Rightarrow U_k = U_h$. This proves that this cannot be an equilibrium for **any** profile of weakly ordered loyal bases.

a2) Suppose that $i \neq l \neq k$. Then, $L_l > L_i = L_k$. Then, using the above equilibrium condition, follows that $U_k = U_i$. So this cannot be an equilibrium for **any** profile of weakly ordered loyal bases.

b) Finally, by construction, the equilibrium presented in Proposition 2 is unique for $|K| = 2$. ■

Proposition (Varian (1980), p.658): If each store's optimal strategy involves zero probability of a tie, and $f(p) > 0$ for all $p^* \leq p < r$, then each store must choose the same strategy.

Notation: $f(p)$ represents the pricing density, p^* represents the average cost associated with serving the whole informed market plus the proportional part of the uninformed one and r is the common reservation value of the consumers.

8.2 Appendix B

Proof of Proposition 5. The advertising choices, in the reduced form game, follow from the FOC's of the maximization problems (see Proposition 5):

$$\frac{\partial \pi_i^{net}}{\partial \alpha_i} = 0 \Leftrightarrow U_{i,i} = \frac{1}{(r-c)}$$

$$\frac{\partial \pi_n^{net}}{\partial \alpha_n} = 0 \Leftrightarrow$$

$$\frac{(U_n + I)(U_{n-1} + I)U_{n-1,n} + U_{n-1} \left[(U_{n,n} - U') (U_{n-1} + I) - (U_{n-1,n} - U') (U_n + I) \right]}{(U_{n-1} + I)^2} = \frac{1}{(r-c)}$$

Suppose to the contrary that $\alpha_n^* = \alpha_j^* = \alpha^*$ for all $j \neq n$. Then, should hold that:

$$\frac{\partial}{\partial \alpha_n} \pi_n^{net}(\alpha^*) \geq 0 \text{ and } \frac{\partial}{\partial \alpha_j} \pi_j^{net}(\alpha^*) \leq 0 \text{ for all } j = 1, 2, \dots, n-1.$$

The above inequalities imply the following one:

$$U_{i,i} \leq \frac{U_{n-1,n}(U_n+I)}{U_{n-1}+I} + \frac{U_{n-1}[(U_{n,n}-U')(U_{n-1}+I)-(U_{n-1,n}-U')(U_n+I)]}{[U_{n-1}+I]^2}$$

Notice that $S_i(\alpha^*) = S_j(\alpha^*)$ and $S_{i,i}(\alpha^*) = S_{j,j}(\alpha^*)$ for $\forall i, j$. In effect, $U_i(\alpha^*) = U_j(\alpha^*)$ and

$U_{i,i}(\alpha^*) = U_{j,j}(\alpha^*)$ for $\alpha_n^* = \alpha_j^* = \alpha^*$, given that $U_i = S_i U$ and $U_{i,i} = S_{i,i} U + S_i U'$. Then, the

last inequality becomes: $U_{i,i} \leq U_{n-1,n} + \frac{U_{n-1}[U_{n,n}-U_{n-1,n}]}{(U_{n-1}+I)} \Leftrightarrow$

$$S_{i,i} U + S_i U' \leq S_{n-1,n} U + S_{n-1} U' + S_{n-1} U \frac{S_{n,n} U + S_n U' - S_{n-1,n} U - S_{n-1} U'}{S_{n-1} U + 1 - U} \Leftrightarrow$$

$$S_{i,i} U \leq S_{n-1,n} U + S_{n-1} U \frac{S_{n,n} U - S_{n-1,n} U}{S_{n-1} U + 1 - U} \Leftrightarrow$$

$$(S_{i,i} - S_{n-1,n}) U (1 - U) \leq 0 \Leftrightarrow$$

As $S_{i,i} > 0$ and $S_{i,j} < 0$ then the inequality holds only if $U = 0$ or $U = 1$. But,

$\lim_{\Sigma_i \alpha_i \rightarrow \infty} U(\Sigma_i \alpha_i) = 1$ and $\frac{\partial U_i(0, \alpha_{-i})}{\partial \alpha_i} > 0$. Then, $\alpha_n < \alpha_j = \alpha, \forall j \neq n$. ■

Proof of Proposition 6. To make sure that the candidate maximum defines the first stage strategies in a subgame perfect equilibrium, firm $j = 1, \dots, n-1$ should not have incentives to leapfrog firm n , and firm n should not have incentives to leapfrog its rivals.

Consider firm n . Its profits are:

$$\pi_n(\alpha) = \begin{cases} (r-c) U_{n-1}(\alpha, \alpha_i^*) \frac{U_n(\alpha, \alpha_i^*) + I}{U_{n-1}(\alpha, \alpha_i^*) + I} - \alpha & \text{if } \alpha \leq \alpha_i^* \\ (r-c) U_n(\alpha, \alpha_i^*) - \alpha & \text{if } \alpha > \alpha_i^* \end{cases},$$

where α is the choice of firm n and α_i^* is the equilibrium choices of firms $j = 1, \dots, n-1$.

Consider the case $\alpha > \alpha_i^*$, then the following is true:

$$(r-c) \frac{\partial U_n}{\partial \alpha_n}(\alpha, \alpha_i^*) - 1 \leq (r-c) \frac{\partial U_n}{\partial \alpha_n}(\alpha, \alpha_i^*, \alpha_n^*) - 1 = 0.$$

The LHS is the first derivative of the profit with respect to own choice and the RHS equality is the FOC of firm j in equilibrium. The inequality follows from strict concavity of U_i

$\left(\frac{\partial U_j}{\partial \alpha_j}(\alpha_i^*, \cdot) > \frac{\partial U_j}{\partial \alpha_j}(\alpha, \cdot) \right)$ and from strategic substitutability of advertising choices $\left(\frac{\partial U_i}{\partial \alpha_i}(\cdot, \alpha_i^*) > \frac{\partial U_i}{\partial \alpha_i}(\cdot, \alpha_n^*) \right)$. Then, firm n has incentives to decrease.

When $\alpha \leq \alpha_i^*$, the profit function is strictly concave and, hence, maximized at $\alpha = \alpha_n^*$.

Consider firm $j = 1, \dots, n-1$. Its profits are:

$$\pi_j(\alpha) = \begin{cases} (r-c) U_n(\alpha, \alpha_i^*) - \alpha & \text{if } \alpha \geq \alpha_n^* \\ (r-c) U_n(\alpha, \alpha_i^*, \alpha_n^*) \frac{U_j(\alpha, \alpha_i^*, \alpha_n^*) + I}{U_n(\alpha, \alpha_i^*, \alpha_n^*) + I} - \alpha & \text{if } \alpha < \alpha_n^* \end{cases}$$

where α is the choice of firm n , and α_i^* and α_n^* are the equilibrium choice of firms $j = 1, \dots, n-1$ and, respectively, n .

Consider $\alpha < \alpha_n^*$, then the following is true:

$$\frac{\partial \pi_j(\alpha)}{\partial \alpha_j}(\alpha, \alpha_i^*, \alpha_n^*) > \frac{\partial \pi_n(\alpha)}{\partial \alpha_n}(\alpha_n^*, \alpha_i^*, \alpha_i^*) = 0.$$

The LHS is the first derivative of the profit with respect to own choice and the RHS equality is the FOC of firm n in equilibrium. The inequality follows from strict concavity of π_j $\left(\frac{\partial \pi_i}{\partial \alpha_i}(\alpha_n^*, \cdot) < \frac{\partial \pi_i}{\partial \alpha_i}(\alpha, \cdot)\right)$ and from the strategic substitutability of the advertising choices $\left(\frac{\partial \pi_j}{\partial \alpha_j}(\cdot, \alpha_i^*) < \frac{\partial \pi_j}{\partial \alpha_j}(\cdot, \alpha_n^*)\right)$. Then, firm j has incentives to increase.

When $\alpha \geq \alpha_n^*$, the profit function is strictly concave and, hence, maximized at $\alpha = \alpha_j^*$. ■

9 References

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