A Marxian Model of the Breakdown of Capitalism

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Abstract: This paper sets out a Marxian model that is based on the one by Stephen Marglin with one sector and continuous substitution. It is extended by adding technical progress and land as a factor of production. It is then shown that capital accumulation causes the preconditions for the breakdown of capitalism to emerge; that is, it causes the organic composition of capital to rise, the rate of profit to fall and the rate of exploitation to rise. A compressed history of the idea of the breakdown of capitalism is then set out and an explanation is given as to how the model relates to this and how it may serve as the basis for further research.

JEL Classification: B24, E11, O41.

I. Introduction.

This paper sets out a Marxian model in which the preconditions for the breakdown of capitalism emerge. That is, it shows how the accumulation of capital will cause the organic composition of capital to rise, the rate of profit to fall and the rate of exploitation to rise. For convenience this causal chain will be referred to as the central relations.

Duncan Foley in his introduction to Marx's economics (1986 p. 10) makes the distinction between a theory, which is a possibly internally contradictory set of ideas about the world, and a model, which is a logically correct but necessarily more limited description. The construct presented here is a Marxian model in the sense that it is based solely on elements found in Marx's writing, contains no internal contradictions and is not necessarily consistent with every statement that Marx made. Specifically it is a

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version of Stephen Marglin's (1984) fixed wage, one sector neo-Marxian model with continuous substitution that is expanded by adding technical progress and land as a factor of production.[2] In this context, under a simple assumption on the production function, it is shown that the central relations are valid.

The paper is composed of four sections and two appendixes. The first appendix contains detailed discussions of a number of issues that are important but tangential to the main argument of the paper. The second appendix is principally devoted to the proof of the theorem.

With regard to the sections, Section II sets out the model and gives an intuitive explanation of the result. Section III traces the theme of the central relations and the breakdown of capitalism in the writing of Marx and Marxist writers. Marx first thought that the movements of the rates of profit and exploitation would bring on the crisis that would end capitalism, but later he gave more weight to the rise in the organic composition because this centralised and concentrated the ownership of capital. Marxian writers moved though a variety of phases but since the 1970s they have focused on movements in the rate of profit and been divided into two camps by their reaction to the Okishio theorem with the profit squeeze group accepting that the rate can fall only if the wage rises and the capital logic group attempting to avoid this conclusion. As a result, the theme of the breakdown of capitalism has, for the most part, disappeared from the Marxian discourse.

Section IV which concludes makes two points, one concerning models and the other methodology. With respect to modelling, the establishment of the central relations in the context of a fixed wage model may allow the formalisation of many of the breakdown arguments as, for example, the one connected with concentration and centralisation. This would serve to bring the end of capitalism back into the Marxian discourse. With respect to methodology one must ask what the value is of a model that assumes a counter factual constant wage and has as a major conclusion the empirically improbable breakdown of capitalism? The answer is that it allows us to see where Marx went wrong and, at the same time, judge how fragile the current success of capitalism is.

II. The Model and the Result

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[2] Of the total references to land or labour in the indexes Capital and Theories of Surplus Value, 34% refer to land.

The model and the results are set out in this section. The model is an augmented and specialised version of Marglin's (1984, chap. 9) one sector neo-Marxian model with continuous substitution. It is augmented by adding technical progress and land and it is specialised by using a particular production function and the simplest version of the savings function.

The specific production function that produces output Y is CES in a Cobb-Douglas capital/labour aggregate and land, where K, L, and M are capital, labour and land.

(1) $$Y=[(K L^{1-} e^{- t})^- +(1-)(Me^t)^-]^{-1/}$$

The aggregate experiences factor augmenting technical progress at rate > 0 while land experiences it at rate 0. The elasticity of substitution between the aggregate and land is $= \dfrac{1}{1+}$, -1 , where 0 , 1 are parameters and t is time. This may be interpreted either as a net production function or a gross production function with zero depreciation (see appendix I.3). The constant real wage w and the rental on capital r are equal to their respective marginal products,

(2) $$w = Y/ L$$

(3) $$r = Y/ K.$$

The rate of return on investment in land, which is its marginal product plus the capital gain $\dot{P}$ divided by the price, is equal to the return on capital,[3]

(4) $$r = \frac{Y}{M} +\dot{P} / P,$$

where P is the price in terms of the good. Capitalists are assumed to own the land as well as capital. Their rate of profit R is defined as

(5) $$R = \frac{Y +\dot{P}M - wL}{K + PM} .$$

It is easily shown that r = R (see lemma 1, appendix II), that is that the rate of profit is equal to the marginal product of capital. Savings are provided only by capitalists who save all their income. Their savings are equal to the accumulation of wealth,

(6) $$\dot{P} M + \dot{K} = Y +\dot{P} M - wL.$$

The assumption that all profits are saved removes $\dot{P} M$ from the accumulation equation and immensely simplifies the model. This seems justified since the effect of changing

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[3] $\dot{x}$ is the derivative of x with respect to time.

land prices on capitalist saving seems to play no role in Marx's writing. It is assumed that factor augmenting technical progress is slower for land than for the aggregate,

(7) $\qquad\qquad\qquad\qquad < \quad .$

Marx (TSV vol. III, 1972, pp. 300-1) specifically assumed that this was the case and, moreover, one must have this if the fixed stock of land is to lead, in some cases, to a falling rate of profit. Finally the definitions of the two other Marxian concepts are as follows. The organic composition of capital is $K/L = k$. There are two definitions of the rate of exploitation, e_i :

$$(8) \qquad e_1 = \frac{rK + \dfrac{Y}{M} M}{wL} = \frac{Y}{wL} - 1$$

$$(9) \qquad e_2 = \frac{L - L_m}{L_m} = \frac{L}{L_m} - 1$$

where necessary labour L_m is given by

$$(10) \qquad wL = \left[\left(K\, L_m^{1-}\, e^{\,t} \right)^{-} + (1 -)\left(Me^{\,t} \right)^{-} \right]^{-1/} .$$

e_1 corresponds to surplus value divided by variable capital. e_2 corresponds to surplus labour divided by necessary labour, that is the labour needed to produce the wage bill. (See appendix I.4 for a general discussion of these definitions.) This concludes the presentation of the model.

The model yields a single differential equation in K in the following manner. (2) determines L as a function of K and t, $L(K,t)$. Thus output also depends on these two variables, $Y(K,L,t) = \underline{Y}(K,L(K,t),t)$. Substituting these into (6) gives the non-autonomous differential equation

$$(11) \qquad \dot{K} = f(K,t) \qquad K(0) = K_0 > 0$$

where K_0, the initial capital stock, is assumed to be positive. The initial-value problem (11) has a solution $K(t)$. Taking account of the dependence of L on K and t, this solution implies the time paths of the key variables, $k(t)$, $R(t)$, $e_1(t)$ and $e_2(t)$.

The characteristics of these time paths are given by the following theorem.

Theorem: For the model of equation (1)-(10), there exists a t^*, such that, for $t > t^*$:

a) If $\quad > 0 (\quad < 1)$ then

$\qquad k \qquad$ and $\quad \dot{k} > 0$,

$$R \quad 0 \quad \text{and} \quad \dot{R} < 0,$$

$$e_i \quad \text{and} \quad \dot{e}_i > 0 \quad i = 1,2.$$

b) If $= 0$, ($= 1$) then

$$k \quad O \quad \text{and} \quad \dot{k} < 0,$$

$$R \quad \text{and} \quad \dot{R} > 0,$$

$$e_i = \text{a constant,} \quad i = 1,2.$$

c) If < 0 (> 1) then

$$k \quad O \quad \text{and} \quad \dot{k} < 0,$$

$$R \quad \text{and} \quad \dot{R} > 0,$$

$$e_i \quad e_i^* \quad \text{and} \quad \dot{e}_i < 0, \quad e_i^* \text{ a constant, } i = 1,2.[4]$$

The proof is quite complicated and is left to appendix II but the outline can be sketched verbally. For all values of   we have two properties. First K approaches infinity since profits are always positive. Second the "ratio" of efficiency units of land to the aggregate $\dfrac{M e^{\;t}}{K\; L^{1-}\; e^{\;t}}$ approaches zero. Suppose it did not. Then it must be that L approaches zero and thus K/L also approaches infinity.[5] The wage increases with a rise in K/L and decreases with a rise in the ratio of the aggregate to land. Thus the approach of K/L to infinity and the bound on the rise of the ratio imply that the wage approaches infinity which is impossible.

First consider how the value of the elasticity of substitution   effects the movements of the organic composition of capital. On the one hand let < 1. In this case the slowest growing factor dominates and eventually K will grow a rate . Now suppose that K/L is constant. Then L grows at   and the aggregate at   +   so that the ratio falls at rate . Since < 1, prices move faster than quantities so that the wage falls faster than   because of this and only rises by   because of technical progress. Thus, to keep the wage constant K/L , the organic composition of capital, must rise. On the other hand, if > 1, the fastest growing factor dominates. If K/L were constant the marginal product of labour would grow at . Thus to keep the wage constant K/L, this time, must fall.

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[4] x   means that the limit of x as t approaches infinity is infinity etc.
[5] Since $>$, if L did not approach zero, then the ratio would approach zero.

The movements of the rate of profit follow directly from the movements of the organic composition. The Cobb-Douglas form of the aggregate means that the ratio of the marginal products of labour and capital depends only on K/L. Thus from the previous paragraph the rate of profit falls if < 1 and rises if > 1.

Finally consider the movements of the first definition of exploitation. On the one hand let < 1. We saw that K/L must rise to keep the wage from falling. But the effect of this is blunted by the relative growth of the aggregate that lowers the marginal products of the factors it contains. To compensate for this, the efficiency units of land per capita must grow as well. Output per man depends on the aggregate per man and efficiency units of land per man. Since both of these increase, output per man and the rate of exploitation in terms of the first definition increases as well. On the other hand let > 1. We saw that K/L must fall to keep the wage from rising. Since > 1 and since the aggregate is the faster rising factor this effect dominates so that output per man and the first version of the rate of exploitation fall. The logic for the movement of the second definition is more involved but basically the same.

While I hope that the reader will agree that this is a Marxian model in the sense defined in the introduction, he or she may strongly feel that this is not the way Marx saw things; in particular the neo-classical production function and the importance of land may rankle. In a paper that is forthcoming in this journal I have provided an extensive justification for the model as a representation of Marx's thought. Specifically I use Marglin's (1984) arguments to justify the production function, the arguments of a considerable number of writers to substantiate Marx's linking of the falling rate of profit to resource scarcity and finally extensive quotations from Marx to show that he thought pressure on the land would be an important cause of the revolution that would destroy capitalism. (See appendices I.5 and I.1 for discussions of one of the writers (Meek) and the relation of the model to Ricardo.) I understand that this position may conflict with the reader's long held opinions and that in the end it may not be found convincing but I hope that he or she will look at my forthcoming paper before dismissing the position out of hand.

III. The Central Relations and the Breakdown of Capitalism in the Works of Marx and Marxist Writers.

This section gives a compressed description of the way in which the theme of the central relations and the breakdown of capitalism varied in Marx's writing over his life time and has since undergone continuous change in the works of Marxist writers. The object is to provide the reader with a convenient summary rather than make any original points. The section on Marx is taken from Simon Clark's excellent book (1994) while the section on Marxist writers is taken from the relevant chapters of Michael Howard and John King's monumental "History of Marxian Economics". Only in the description of the 1990s does the author's opinion intrude.

1. Marx.

The central relations played an important role in Marx's writing about the breakdown of Capitalism. Two themes appear: that of crises, periods in which the rates of profit and growth and the level of employment all fall rapidly, and that of secular trends. The weight that Marx placed on these two themes as causes of the end of capitalism varied over his lifetime. According to Clarke, crises were more important during the middle period that started with the failure of the 1848 revolutions and ended with the abandonment of the Grundrisse in March of 1858. The emphasis was stronger on secular trends in the earlier period which included the Manifesto and the later period in which Marx wrote the notebooks from which volumes II and III of Capital and Theories of Surplus Value were taken and volume I itself.

Clarke sums up the Grundrisse as follows: He notes that underconsumption, overproduction and disproportionality are all intertwined.[6] He says that one may find a number of scenarios. First (p. 166), that the falling rate of profit may lead to overproduction and thus a crisis, but he notes that neither the mechanism which causes the fall nor the way this produces overproduction is demonstrated. Second (p. 171-2), he describes a scenario that operates through disproportionality. He stresses that Marx did not think that a mere fall in the rate of profit could cause a fall in investment and thus a crisis. Rather the route is that a rising organic composition of capital will lead to disproportionality and, in addition, "the falling rate of profit will introduce disproportionalities as capital is diverted into speculative channels." Thus, concludes Clarke, the falling rate of profit and the rising organic composition of capital are linked to crises but not in a clear way.

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[6]These are different causes of crises: underconsumption is when there is a lack of demand for consumer goods usually due to low worker income; overproduction is when there is an excess supply of all goods;

In addition Clark (p. 172) states that there are three other points that are of the same level of importance as the preceding scenarios and which are linked to the rising organic composition of capital and rate of surplus value and the falling rate of profit. They are: first, that the falling rate of profit intensifies class conflict as capitalists put pressure on workers in an attempt to maintain profits; second, that the rising rate of surplus value increases the vulnerability of the economy to crises as a greater proportion of aggregate demand is made up of the more volatile investment demand;[7] and third, the rising organic composition of capital and mass of surplus value is associated with the centralisation of ownership and the pauperisation of an ever wider strata of the population. These last three points are relevant, not for the generation of specific crises, but as explanations of the secular trends.

In the period between 1850 and 1857 Marx and Engels expected that capitalism would end with the next crisis. When the crisis of 1857 passed without event, their perspective changed. In this sense the Grundrisse is a work of transition, it is both an attempt to understand the reason that crises occur because they had seemed so important and the beginning of the attempt to understand the secular forces which would lead to the end of capitalism (p. 175).

Turning to the notebooks of 1861-5, there is a change in content and emphasis. According to Clarke's summary (p. 240-5), the driving force is the increase in the organic composition of capital that has two effects. First, the centralisation of ownership is given more stress. Capitalists, to avoid having their profit rate fall, attempt to increase the rate of surplus value. This leads to greater centralisation as the smaller capitalists are at a disadvantage, both technically and with regard to access to credit, and are forced into risky ventures. Second, the rising organic composition of capital leads to the creation of the reserve army of the unemployed but the description of this is mixed, in a confusing way, with the falling rate of profit. Thus the rising rate of surplus value has disappeared as a cause of instability, the effect of the rising organic composition of capital on centralisation has received more stress and the reserve army, linked with the falling rate of profit, has made its appearance.

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and disproportionality is when there is disequilibrium so that some goods are in excess supply while others are in excess demand.

[7] Clarke says mass of surplus value here but on p. 168 where he is describing the phenomenon in detail he says rate. Moreover the logic requires the proportion of capitalist spending to worker spending to rise. For these two reasons it seems permissible to insert rate in the paraphrase.

Finally, turning to Volume I, the falling rate of profit has completely disappeared and the increase in the organic composition of capital has assumed centre stage. This has two effects: the first is, again, the centralisation of ownership where the link between the rising organic composition of capital and the technical efficiency of large units of capital is stressed. The second is that the appearance of the reserve army is now concisely explained without reference to the falling rate of profit (p. 252). Clarke gives a long exposition of the interaction between wages and accumulation (pp. 249-259) the gist of which, I think, is that, although it might require a crisis to motivate them, capitalists react to a potential shortage of labour by increasing the organic composition of capital in a way that re-establishes the reserve army and ensures that capitalism can continue to expand.

The importance of these two effects is expressed in the "absolute general law of capitalist accumulation which defines the necessary polarisation of capitalist society between the growing power and wealth of capital on the one hand, and the growing misery of the mass of the population, expressed in the growth of the relative surplus population which constitutes the reserve army of labour and the pauperised layer of the working class on the other." (p. 251).

To summarise: the falling rate of profit, the rising rate of exploitation and the rising organic composition of capital all play their roles in breakdown of capitalism. The falling rate of profit and the rising rate of exploitation have heavier weight in the middle period in which Marx saw the end of capitalism as coming suddenly as the result of a crisis, while the rising organic composition of capital comes to the fore in the later period when Marx gave more emphasis to secular trends.

2. Marxist Writers.

Marxist writers have used both the central relations and pure underconsumptionism as the basis for their explanations of the breakdown of capitalism. In addition these explanations have been influenced by both economic and intellectual events. A brief review is now presented.

The orthodox Marxian position was set out by Karl Kautsky in the Erfurt Programme of 1891 and his book "The Class Struggle " of 1892. Kautsky sees the breakdown as being brought on by the increasing misery of the proletariate, the growing inequality of wealth and income and the evermore-frequent crises. These are caused in part by a concentration of capital, which is brought on by the falling rate of profit, but mainly by a lack of aggregate demand. Under attack from Edward Bernstein, Kautsky shifted

ground considerably: he admitted that the immiseration was only relative and, forgetting his previous writings (according to H&K), denied that there was a breakdown theory. H&K summarise this as "Kautsky's discussion of crises reveals the dominance of underconsumptionist ideas in Marxist thought before 1914 together with the marginality of the falling rate of profit and wide spread suspicion of disproportionality models of crisis." (vol. 1, p. 84). [vol. 1, pp. 67-84][8]

Rudolf Hilferding reacted to both the centralisation of economic power in Germany and the growing international tension with "Finance Capital" of 1910. His theory emphasises the centralisation of control of industry. Crises are worsened because of disproportionality caused by the slowness of cartels to react and also by the falling rate of profit brought on by the rising organic composition of capital. But the centralised control allows the crises to be muted so that capitalism's problem lies elsewhere. The falling rate of profit, this time caused by rising wages on a tightening home labour market, will bring on imperialistic expansion and increasingly violent conflicts out of which will emerge the dictatorship of the proletariat. (Kautsky concurred but thought the spur to imperialism would be, rather than labour, a scarcity of agricultural resources.) H&K note that Hilferding's book has had an influence second only to that of Capital. [vol. 1, pp. 94-103]

The First World War and the Russian revolution did not spark quality innovations in the breakdown idea. Rosa Luxemburg saw imperialism as a reaction to insufficient demand. Otto Bauer used Marx's reproduction schemes to show the flaws in her argument and attributed the breakdown either to shortages of labour and resources if expansion abroad was blocked or imperialistic wars if it was not. [vol. 1, pp. 106-26] Finally H&K find the Russian work on these issues "actually rather unimpressive" both because of their interest in the development of the Solviet Union and because of the heavy weight of Leninism (vol. 1, p. 297).

The next major work on the breakdown of capitalism was that of Henryk Grossman, published in 1929. He extended the number of periods in Bauer's model, which has an increasing organic composition of capital and a falling rate of profit, and found that the model eventually broke down because the surplus value was insufficient for the required investment and because capitalists were unable to consume. (Thus Bauer had a breakdown model without realising it.) Grossman's argument was deeply flawed but

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[8] The reference in square brackets shows from where the information in the preceding paragraph was

striking for three reasons: it was the first time that Marx's volume III arguments had been used as the basis of a breakdown theory, it was the first time that the cause of the breakdown had been located in production and accumulation rather than realisation and circulation and, finally, this second notion has become a fundamental tenant of a major stream of thought in Marxian political economy. [vol. 1, pp. 316-32]

The attempts by Marxists to understand the depression of the 1930s demonstrated a change in the concept of the end of capitalism. H&K note (vol. 2, p. 4) that "In general the breakdown thesis in its strict sense-the assertion that profitable growth would soon be impossible, for narrowly economic reasons-was more widely supported before 1914 than thereafter,". Rather it was thought that the depression was symptomatic of a further decline in the ability of capitalism to function well. The reasons the writers gave varied between individuals but generally were a mixture of two arguments, each with two variants: the falling rate of profit which was caused either by a rising organic composition of capital or rising wages; and realisation problems which were described either in pure underconsumptionist terms or by disporportionality linked at times to a rising rate of surplus value. The Marxist analysts used phrases like death agony, impending downfall and pre-requisites for the revolutionary crisis, but they hung back from using the actual word breakdown. [vol . 2, pp. 3-23]

The Keynesian revolution was the first intellectual event to influence Marxism and, according to H&K, it was a "watershed" (vol. 2, p. 105). Previously explanations of the end of capitalism had been based on both underconsumptionist ideas and the central relations. But the implication of Keynesianism was that the underconsumptionist problems of capitalism could be remedied by government policy. This caused a divide, with those that followed the Kalecki-Sraffa version of Marxism being considered as nothing more that left Keynesians, while the "wide" majority of Marxist economists concentrated on the falling rate of profit as the principle source of capitalist problems. Paul Baran and Paul Sweezy fought a long rear guard action for underconsumption as an explanation of the end of capitalism that culminated in "Monopoly Capitalism" of 1966, but by the early 1970s most Marxists turned away from this type of explanation (vol. 2, pp. 123-4). With the advent of Keynesianism, the Marxists had lost half of their intellectual weaponry. [vol. 2, pp. 101-5]

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taken.

The 1970s saw combination of an intellectual event and an economic one: The importance of the Okishio theorem began to be appreciated and, at the same time, the long boom ended and it became clear that the rate of profit was falling. That is, it became clear that, at least in terms of the most general model available, Marx's explanation of the falling rate of profit was incorrect and, at the same time, the actual behaviour of the rate of profit demanded analysis. Two distinct reactions emerged: First a group of writers defended the idea that the fall in the rate of profit was rooted in the nature of capital. This line of thought started with Grossman, continued with Paul Mattick and Roman Rosdolsky and was represented in the 1970s by a widely read article by David Yaffe (vol. 2, p. 144) and an attack on the Okishio theorem by Anwar Shaikh (vol. 2, p. 317). This literature which is know as the capital logic school, tends to hark back to Marx, take the end of capitalism seriously and, except in the case of Shaikh, ignore the Okishio theorem.

The second and larger profit squeeze group accepted the Okishio theorem and interpreted the fall in the rate of profit as a consequence of rising wages. Writing in the early 1970s, Andrew Glyn and Bob Sutcliff, among others, attributed these to increasing union militancy. The emphasis shifted in the 1980s with Thomas Wiesskoff blaming the fall on things like capitalist's inability to defend themselves against rising social security payments, Glyn and two colleagues citing labour shortages (overaccumulation) and both noting the effects of rising resource prices. Generally this work is descriptive and there is no mention of the end of capitalism. [vol. 2, pp. 318-22]

In the 1990s, after the publication of H&K's "History", both groups continued to develop. Taking the direction of the capital logic school, Allan Freeman (1995, 1998) and Andrew Kliman (1996, 1999) have attacked the Okishio theorem in terms of the conventional model by trying to show that, in spite of a constant real wage, the rate of profit may fall if what Hahn and Mathews (1964) called equilibrium dynamics is used to describe the path of the economy instead of the traditional steady state analysis. On a somewhat different front, Fred Moseley (1992, 1997) has argued, without a formal model, that capitalist hiring of unproductive labour, particularly sales personnel, can cause the rate of profit to fall without wage increases. Generally these works maintain the end of capitalism tone and Moseley's (1997) piece contains an assessment that could have been written in the 1930s if not before 1914.

In the profit squeeze camp there have been improvements in empirical work and developments on the theoretical side. A problem with the empirical work has been the

tendency to see rises in the organic composition of capital as causing a fall in the rate of profit without simultaneously taking account of the compensating rise in labour productivity. In a paper of great originality, Gerard Duménil and Domenic Lévy (1995) set out a model of endogenous technical change that internalises this relation and uses it to reproduce the actual movements of the wage and the rate of profit. On the theoretical side there have been a number of papers that show how capital accumulation may cause both a rise in the wage and a fall in the rate of profit[9], but the wage movements had not been convincingly integrated until Gilbert Skillman (1997), in a break through paper, used the Rubinstein-Wolinsky matching and bargaining framework to make the connection. Finally Duménil and Lévy (1999) have used their model of technical change to try to reproduce the central relations, albeit in the context of rising wages. The end of capitalism is hardly mentioned in this literature, primarily because the rising real wages go directly against the traditional Marxian descriptions of this event.[10]

To summarise, the central relations are still an important Marxist theme but the end of capitalism is not. Initially the explanations of the breakdown of capitalism had been based primarily on underconsumptionist ideas, but the advent of Keynesianism made this untenable and capitalism's long survival softened the notion of a breakdown itself. In the 1970s the fall in the rate of profit focused Marxist attention on this variable so that interest in the central relations was maintained. But on a theoretical level the necessity of having rising wages and perhaps, on a practical level, the evident good health of capitalism itself, caused the theme of the end of capitalism to virtually disappear from the main current of Marxist thought.

IV Conclusion.

The model of this paper is important for two reasons: first it provides a hitherto missing account of the central relations in the context of a constant wage and second it opens the door to an ambitious Marxian research program.

The intrinsic problem in modelling the central relations in the context of a Marxian model is the generation of the dynamics. The neo-classical growth model does this by

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[9] See Duménil and Lévy (1993, 1995), Skott (1992) and Michl (1994, 1999).

[10] Two extreme reactions are indicative of the tension between the need to assume rising wages and the idea of the end of capitalism: Foley (1986, p. 156) sees socialism being brought on, not by the increasing misery of the workers, but because their situation has so improved that they are prepared to take over. On the other extreme, Laibman (1997) somewhat as Marx did, repeatedly stresses the worsening conditions of the workers while, at the same time using a model which, when looked at carefully, exhibits rising wages. (See Laibman (1998, p. 100) for an admission.)

having the exogenously determined rate of growth of labour be different from the endogenously determined rate of growth of capital when the model is out of steady state. The natural Marxian assumptions of a fixed real wage and an infinitely elastic supply of labour that characterise the formal Marxian models of the 1970s and 1980s[11] meant that this mechanism could not operate. The only equilibrium of these models was a steady state in which the labour supply increased in tandem with the capital stock and the organic composition of capital, the rate of exploitation and the rate of profit all remained constant. Duménil and Lévy (1999) broke this impasse in an eminently Marxian way by inserting their model of endogenous technical change. Specifically, in the face of symmetric technical possibilities, the relatively higher labour share generates technical change that raises the organic composition of capital and, in steady state, the wage in a way that the share remains constant so that the rate of profit falls. They thus reproduce two out of the three central relations but at the cost of a rising wage. The model of this paper returns to a version of the neo-classical mechanism: the infinitely elastic labour supply still adjusts to keep the real wage constant but now it is the difference between an exogenously determined rate of growth of land in efficiency units and the endogenously determined rate of growth of capital that supplies the dynamics. This mechanism generates the complete central relations in the context of a constant wage.

There are three ways in which this model can be used: to serve as a basis for the theme of the end of capitalism, to model past revolutions and current revolutionary situations and, finally, to gain insights into the presently successful capitalism of the industrialised world.

First the model could be extended to produce formalisations of the various accounts of the breakdown of capitalism. The following three examples each emphasise a different element of the central relations. For the first example one could insert a model of cycles that is based on the relative instability of investment spending and then use the rising rate of surplus value to show how the cycles would worsen over time. This would formalise one of the arguments that Marx put forward in the Grundrisse. For the second one could insert a mechanism that caused the income distribution to narrow as the organic composition of capital rose. This would parallel the arguments made by Marx when he emphasised trends and also those of the Erfurt programme. Finally, for the

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[11] See Morishima (1973), Steadman (1977), Roemer (1981) and Marglin(1984).

third, one could insert a minimum level of capitalist consumption and show how the falling rate of profit would cause a crisis within the capitalist class. This would detail the argument made by Grossman and more loosely fit the ideas of the capital logic school. In these ways and others the model could serve as a foundation for structures that would bring the theme of the end of capitalism back into the Marxian discourse.

The second way in which the model can be used is for the analysis of revolutions. The model is suitable for this both because of the subsistence wage assumption and because land plays an important role, but one extension must be made. Population pressure on the land played a role in Marx's writing on revolutions, Theda Skocpol´s seminal work (1979) emphasised the importance of this for the French (p.56), Russian (p.132) and Chinese (p.47) revolutions, and this theme has continued in the descriptions of recent revolutionary situations in Vietnam (Popkin, 1988), Latin America (Wickham-Crowley, 1992), southern Mexico and the former Zaire (Renner,1997). However a common theme of all these later writers (not to mention Lenin and Mao) is the difference between the roles played by the peasants and the urban workers. All these events had deep economic causes and much is known about what happened but they have not been formally analysed. If the model of the present paper were extended so as to treat peasants as a separate group, it could provide the basis for a formal description of these events.

The third way in which the model can be used is to analyse the capitalism of the industrialised world. Two problems immediately arise: First the actual situation of rising real wages fits badly with the assumption of constant subsistence wages of the model, and second the generally successful operation of the capitalist system calls into question the relevance of any attempt to explain how the system will end.

With regard to the first problem it is clear that the model will have to be extended to permit rising wages. One way to do this is to make the value of labour power or the level of the subsistence wage an endogenous variable in line with Francis Green's (1991) interpretation of Marx's labour market. In my forthcoming paper I have shown that this leads to a model with either rising or falling wages depending on the size of decreasing returns to land relative to technical progress. A revolution may occur only if the parameters are such that technical progress is weaker.

A much more radical change, but one which is very much in the spirit of Marx, would be to add a second type of capitalist who accumulated human rather than physical capital. The force of this would be, since human capital cannot be centralised, that this

would largely mitigate the wealth centralising force of the rising organic composition of capital. This in turn could be used to show why the events described in the three examples have not happened. With regard to the first, the thwarting of the centralisation of wealth would stop investment from becoming a larger portion of demand and thus break the mechanism that was responsible for the growing instability. With regard to the second, the distribution of wealth would not narrow so much because of the nature of human capital. This, incidentally, gives a deep insight into why revolutionary tendencies persist in the third world where human capital accumulation has been thwarted. Finally, with regard to the third, the empirical correlation between the ownership of physical and human capital would suggest a reason why the capitalist class has not experienced noticeable distress during periods of falling profits.

Careful perusal of these solutions to the first problem leads also to a resolution of the second problem, that of the relevance of models of the end of capitalism in the context of its current actual success. One may think, at first glance, that finding out what went wrong with the Marxian predictions can hardly be seen as a salubrious Marxian research programme. But this is the wrong reaction. Neo-classical research generally analyses how capitalism works without considering why it exists. The Marxist perspective is to understand why capitalism exists and how it might disappear. The examples above show that this research programme leads exactly to answers to this type of question. For example it leads to the conclusion that perhaps capitalism owes its continuing existence to the accumulation of human capital, a conclusion that never could have come from neo-classical analysis.

Appendix I.

1. Marx and Ricardo.

A natural criticism of the model is that it is Ricardian rather than Marxian since fall in the rate of profit is connected with a growing scarcity of land. Because this criticism was not addressed in the forthcoming paper it is convenient to deal with it now. It is argued, on the basis of his commentaries in Theories of Surplus Value, that Marx distinguished himself from Ricardo by his assumption that capital was important in agriculture, that the model reflects this distinction and finally, that it supports exactly the difference in results that Marx emphasised.

Marx's main comments on the falling rate of profit are contained in chapter XVI. A careful look at these allows them to be transformed into the language of modern growth

theory. The greater part of the chapter is devoted to various consequences of Ricardo's failure to consistently take account of constant capital. Of particular importance is the point that prices of individual goods are not, as Ricardo thinks, equal to their values but rather are determined so that the rate of profit will be the same in all sectors. Then on pages 466-7 Marx sets out his basic criticisms. Ricardo identifies the determination of the rate of profit with the return on land that pays no rent. This is because he identifies price with value and disregards constant capital in agriculture so the rate of profit is merely the ratio of what remains after the worker has received his "equivalent" to that equivalent. But this is wrong because agricultural produce generally sells at a cost-price which is different from its value and is determined so that the rate profit in this sector will be the same as that on non-agricultural capital. Marx's general conclusion is that the rate of profit is determined generally in the economy and not specifically in the agricultural sector. In modern terms Ricardo has a two sector model in which agricultural products are produced by land and labour and manufactures are produced by these two factors plus capital while Marx prefers a three factor one sector description. In this sense the model of this paper is Marxian rather than Ricardian.

At the beginning of his discussion of Ricardo's theory of the falling rate of profit, in a much quoted statement, Marx announces the difference between his and Ricardo's theories.

"The rate of profit falls, although the rate of surplus-value remains the same or rises, because the proportion of variable capital to constant capital decreases with the development of the productive power of labour. The rate of profit falls, not because labour becomes less productive, but because it becomes more productive." (p. 439)

What is not often pointed out is that Marx offers virtually no justification of this ringing pronouncement. Finally on p. 468 he mentions unproductive labour, flirts with the idea of cyclic overproduction and then tells the reader he must wait for the section on the competition of capitals. However the theorem of this paper provides a precise justification for the difference that Marx claimed existed between himself and Ricardo: The rate of profit falls but labour productivity (defined as Y/L) and the rate of surplus value rise, both of these movements because the ratio of variable to constant capital (defined as wL/K) decreases.

2. Technical Progress and the Classical Model.

The model of this paper is closely related to the classical model. The results make a substantial contribution to a slender but pithy debate that has stretched over one hundred and fifty years.

The starting point is the work of John Stuart Mill. He considered at great length, in the context of the classical model, whether technical progress could counteract the effects of population growth and allow the economy to grow at a constant rate with a wage above subsistence level and a constant positive rate of profit. He thought that this was a theoretical possibility but that in practice the force of population growth would overwhelm that of technical progress (Mill (1965), vol. 2, Book I, chap. 13 and vol. 3, Book IV, chap. 3).

Johansen (1967), referring to Ricardo and Smith rather than Mill, built a classical model that provided a theoretical basis for Mill's thoughts. He showed that it was possible to have a steady state in which technical progress just balanced population growth. He further supported Mill's intuition by showing that higher potential capacity for population growth would imply a lower equilibrium salary. However Johansen's case was special in that he used a Cobb-Douglas production function. This meant that one did not know if the results would hold for other production functions or what type of technical progress was being supposed.

Samuelson (1978) again touched on this theme. In the context of the same model, he stated that if the technical progress were land augmenting then there would be a steady state with the wage above subsistence and a positive constant rate of profit.

These results leave open the question of what would happen if the technical progress was other than land augmenting. The model of the present paper is not exactly the classical model: the momentary labour supply and its rate of growth do not depend on the wage, and capital and labour do not enter the production in fixed proportions. This difference means nothing can be said about the behaviour of the wage. But the model is close enough to the classical model to allow a provisional extension of the results.

The results of the present model show a surprising break in the tendency to find theoretical support for Mill's position. It seems to me that Samuelson's statement would continue to hold as long as the rate of capital/labour augmenting progress remained below that of land. But the moment it rises above things change radically. Steady state is lost and, depending on whether the elasticity of substitution is greater or less than 1, the system either explodes or collapses. In fact one has the somewhat paradoxical result

that a rise in the rate of capital/labour augmenting technical progress may lead to a rapid descent of the rate of profit.

3. The Importance of the Absence of Depreciation.

One interpretation of the model is that there is zero depreciation. A natural question is whether the addition of depreciation would cause problems for the result? Since I have not worked this through my answer must be speculative. There are two ways of looking at this.

The first way is to ask what would happen if one merely added depreciation to the current model? A number of things might happen. In the case of < 1, if the marginal product of capital fell to the rate of depreciation, growth might stop in finite time. Also, since depreciation lowers profitability, in the case of > 1 the profit might not always grow. Finally consider the rate of exploitation. For the case of > 1, K/L decreases and so does the rate of exploitation. The negative effect of depreciation on capitalists falls as K/L does. Thus adding depreciation would work against the tendency of the rate of exploitation to fall. In the same way it would work against the rise in the rate of exploitation in the < 1 case. Summarising, with depreciation added, in the case of < 1 the model might stop growing, in the case of > 1 the rate of profit might stop rising, and in both cases the rate of exploitation might move differently. But it must be emphasised that all of this might not be true since I have not worked through the model for this case.

The second way to look at this is to suppose that Y is a net production function so that depreciation is already taken into account. In this case one can calculate the gross production function G(.),

$$G(.) = Y(K,L,M,t) + \ K.$$

One sees that this is a slightly unusual production function, but that it satisfies all the normal assumptions. With this interpretation, the answer is that the results are not affected by depreciation.

The real point is the following. The model is really just an example and there are no strict rules for choosing between examples. I started with a one parameter model, judiciously chosen, and showed that the value of the parameter had important and simple effects on the behaviour of the model. The exercise carried out in the second paragraph seems pointless to me. The alternative is to start from a general production function and find general conditions that imply the results. This would be hard but not

impossible. I have not done it because the purpose here is to convince the readers of the fruitfulness of the approach and not the generality of the results.

4. The Rate of Exploitation and the Organic Composition of Capital.

Marx gave two definitions of the rate of exploitation: first, surplus value / value of labour power and second surplus labour / necessary labour. These definitions are equivalent in models with fixed coefficients and no joint production. Morishima (1974) and Roemer (1981, chap. 2) used the second definition to define the rate of exploitation in a Von Neuman model where values do not exist. Marglin (1984) based his definition of the rate of exploitation in models with continuous substitution on the first definition. Finally Petith (1997) noticed that the two definitions were not equivalent in models with continuous substitution.

With regard to the first definition, one has to decide what surplus value is in terms of this model. Marx made it clear that surplus value included rent paid to landlords (Theories of Surplus Value, part II p. 453) so that it would seem that this should be added. The problem arises with respect to capital gains from land. I am not aware of any comments by Marx and furthermore the relation between value and capital gains would seem to verge on the metaphysical. Besides, to include capital gains one would have to solve for the path of prices that would be an added technical problem. For all these reasons I have decided on the definition (8).

With regard to the second definition, I have decided to include it, in spite of its being less tractable, because it seems to be more relevant when talking about revolution. Roemer (1982, chap. 7) makes the point that a worker is exploited if there is another situation in which he would be better off. The first definition makes no reference to another situation. The second does. It shows the proportion by which the workers could reduce the working day if they got rid of the capitalists.

The definition of the organic composition of capital is the ratio of the value of constant capital to the value of labour power. There are a number of points: First, in a one good model the value cancels so it is just the ratio of constant capital to the wage bill both in physical units. This means that one does not have to worry about the distinctions between the organic, the value, and the technical compositions of capital. Second, constant capital is defined as the depreciation of fixed capital plus the amount of raw materials that are used. Since there is neither depreciation nor raw materials, constant capital would seem to be zero. Instead I take constant capital as the stock of physical capital. This is frequently done (see Moseley (1991, chap. 1)), and is in the

spirit of the way Marx used the concept, that is the organic composition of capital was connected with employment and centralisation.

A much more delicate point is whether land should be included in constant capital, i.e. K + PM. With regard to employment it would seem clear that it should not be. The combination of accumulation of physical capital and the capital to labour ratio determine employment. But with regard to centralisation the issue is less clear. If accumulation causes the price of land to rise, then more must be spent on plots of land and this could increase centralisation. In the end constant capital does not include land mainly because of the technical problems.

5. The Relation to Meek's Work.

Meek (1967) set out a numerical example in which the increasing organic composition of capital made the rate of profit first rise and then fall. Meek made reference to capital accumulation, technical change, and the presence of land. It is possible to see the model of this paper as being behind his numerical example. Thus it is of interest to see if, in the case of $\ < 1$, there is an initial rise in the rate of profit before the final fall. Meek's proposition in appendix II shows that this is, in fact, the case. Thus this model provides the basis for an interpretation of Meek (see Petith (1997b) for details).

Appendix II. (A detailed version is available from the author.)

1. The Proof of the Theorem.

<u>Lemma 1.</u> r = R.

Proof: (4) implies $rPM = \dfrac{Y}{M} M + \dot{P} M$. Thus R $= \dfrac{\dfrac{Y}{K}K + \dfrac{Y}{M}\dot{M} + \dot{P}M}{K + PM}$ = r, this follows from (5); linear homogeneity and (2); and, finally, (3) and the implication of (4).

<u>The Case of $\ \ 0$.</u>

There are three basic relations in the model that relate to w, $\dot{K}$ and L_m. However these can be expressed in a number of ways. These will be set out at the beginning to avoid cluttering the arguments with tedious calculations:

First consider the way the relation w = Y/ L, from (2), can be stated. Define

$$ m = \frac{Me^{\ t}}{K\ L^{1-}\ e^{\ t}} \ , $$

m is the land-aggregate ratio in efficiency units. Using (1) and (2)

$$(A1) \quad w = \left[\left(K\ L^{1-}\ e^{t}\right)^{-} + (1-\ )\left(Me^{t}\right)^{-} \right]^{-\frac{1+}{}} \left(K\ L^{1-}\ e^{t}\right)^{-\ -1}(1-\ )K\ L^{-}\ e^{t}$$

$$(A2)\ w =\ (1-\ )\ +(1-\ )\ \frac{Me^{t}}{K\ L^{1-}\ e^{t}}\ ^{-\ -\frac{1+}{}}\ \frac{K}{L}\ e^{t}.$$

Using the new variable

$$(A3)\quad w =\ (1-\ )\left[\ +(1-\ )m^{-}\ \right]^{-\frac{1+}{}} k\ e^{t}.$$

Or rearranging this

$$(A4)\quad w =\ (1-\ )\left[\ m\ +(1-\ )\right]^{-\frac{1+}{}} m^{1+}\ k\ e^{t}.$$

Calculating Y/K and using (3) and (A1) gives

$$(A5)\qquad r =\ \frac{}{1-\ }\ wk^{-1}.$$

Second, consider the expression for $\dot{K}$. From (6) $\dot{K} = Y - wL$ so that expressing the RHS in terms of m

$$(A6)\qquad\qquad \dot{K} = \left[\ m\ +(1-\ )\right]^{-\frac{1+}{}}\left[\ m\ +(1-\ )\right]Me^{t}.$$

From (A2) with $C_1 = \dfrac{w}{(1-\ )}$

$$(A7)\qquad\qquad k\ e^{t} = C_1\left[\ +(1-\ )m^{-}\ \right]^{\frac{1+}{}}$$

$$(A8)\qquad\qquad k = C_1^{\frac{1}{}}\left[\ +(1-\ )m^{-}\ \right]^{\frac{1+}{}\frac{1}{}} e^{-\ -t}$$

and since $m^{-1}Me^{t} = k^{-1}Ke^{t}$, (A6) implies

$$(A9)\qquad \dot{K} = C_1^{-\frac{1}{}}\left[\ +(1-\ )m^{-}\ \right]^{-\frac{1+}{}\frac{1}{}}\left[\ +(1-\ )m^{-}\ \right]Ke^{-t}.$$

Finally a different approach is useful when dealing with e_1 and e_2. Define

$$y =\ \frac{L_m}{L}^{(\ -1)},\qquad x =\ \frac{K}{L}\ e^{t},\qquad z = \frac{M}{L}e^{t}\ .$$

Then dividing (10) by L and raising both sides to the power $-$ gives

$$(A10)\qquad w^{-} =\ x^{-}\ y +(1-\ )z^{-}\ .$$

Manipulating (A1) and substituting the definitions of x and z gives

$$(A11)\qquad\qquad (1-\ )z^{-} = C_1^{-\frac{}{1+}} x^{-\frac{2}{1+}} -\ x^{-}\ .$$

Substituting this into (A10) gives

(A12)
$$y = \frac{w^-}{} x^- - \frac{C_1^{-\overline{1+}}}{} x^{\overline{1+}} + 1.$$

(A12) gives essentially e_2 in terms of x alone. Calculation Y/L in terms of x and z and using (A11) gives

(A13)
$$\frac{Y}{L} = \left[x^- + (1-)z^- \right]^{-\frac{1}{}} = C_1^{\frac{1}{1+}} x^{\overline{1+}},$$

that is e_1 in terms of x. This completes the preliminaries.

Although it seems reasonable that a solution should exist, given the sometimes-unexpected nature of non-linear differential equations, it is safer to check.

<u>Lemma 2</u>, For the initial-value problem (11) there exists a unique solution K(t) defined on $0 \quad t < $. (The argument is valid for $=0$ as well as 0.)

Proof: The proof uses the lemma of p. 8 and Theorem 1.7 of Grimshaw (1990). To save space these results are not set out and the argument only sketched. Choose $D = \{\underline{K} < K < \overline{K}\}$ and $I = \{-1 < t < \overline{T}\}$ where $\overline{T}$ is arbitrary, $0 < \underline{K} < K_0$ and $\overline{K}$ is defined so that K cannot reach $\overline{K}$ for $t < \overline{T}$. Then m is defined as a function of K and t, m(K, t) which has the m following properties:

(A14) m(K,t) is defined on the closure of (D,I), is positive, continuous, has continuous partials and is bounded.

Finally define f(K,t) from (A9). (A14) and the form of (A9) show that f(K,t) satisfies the conditions of the lemma of p.8 so that it satisfies a Lipschitz condition on (D,I). An application of Theorem 1.7 then shows that a solution to (11) exists and is unique on

$0 \quad t < \overline{T}$.

<u>Lemma 3.</u> $\dot{K} > 0$ for all t .

Proof: The argument that $\dot{K} > 0$ is simple. Define L(K,t) by (A6). Note that

(A15) L(K,t) > 0 for all t when K > 0:

Suppose that L(K,t*) = 0 for some t*. Then, since K > 0, $\lim_{t \to t^*} m = \lim_{t \to t^*} k = $ so that

RHS (A3)   which is impossible.

Then, since K(t) > 0 for all t (because $K_0 > 0$ and $\dot{K} \quad 0$) , (A15) implies that m(t) > 0 for all t and the result follows from (A6).

<u>Lemma 4.</u> m   0.

Proof: Suppose m   A > 0. By (7) and lemma 2, L   0 and k   so the RHS (A3)

which is impossible.

<u>Lemma 5.</u> Let   > 0. For any , ' > 0 there is a t* such that for t > t*

(A16) $\dot{K}$ > $(C_2 -)e^{t}$, $\dot{K}$ < $(C_2 +)e^{t}$

(A17) $K(t)$> $\dfrac{C_2 - }{ }$ $- '$ e^{t} $K(t)$< $\dfrac{C_2 + }{ }$ $+ '$ e^{t}

where $C_2 = (1-)^{-1/ } M$.

Proof: By lemma 4 and   > 0 the coefficient of e^{t} in (A6) approaches C_2 as t

approaches infinity. This gives (A16).

Integrating the first of the inequalities in (16) from t^0 to t, and , for any ' > 0,

choosing t* so that, for t > t*,

$$ - K(t^0)e^{-t} + \dfrac{C_2 - }{ } e^{- (t - t^0)} > - '$$

gives the first inequality of (A17). The other follows similarly.

<u>Lemma 6.</u> k   or 0 and R   0 or   as   > 0 or < 0.

Proof: We show first that

(A18)   k   or 0 as   > 0 or < 0,

the rest of the lemma then follows immediately from (A5) and lemma 1.

First let   > 0. Writing $m^{1+ }$ k e^{t} in terms of M, K and k, substituting for K with the

inequality (A17) and putting this into (A4) shows that k   .

Second when   < 0 it is immediate from (A3) that k   O.

<u>Lemma 7.</u> Let   < 0, then e_1 and e_2 approach constants.

Proof: (A3) and lemma 4 imply that x   a constant. (A13) and (8) then imply the result

for e_1 while (A12) and (9) imply it for e_2.

<u>Lemma 8.</u> Let   > 0, then e_1 and e_2 .

Proof: From (A3) and lemma 4, x   . (A13) and (8) imply the result for e_1. x   and

(A11) imply that z^- 0, (A10) implies y   , and, finally (9) implies the result for e_2.

<u>Lemma 9.</u> Let   < 0 and choose an   > 0. Then there exists a t* and constants C_4, $C_5( )$

and $C_6(t*,)$ such that for t > t*

(A19) $\dfrac{\dot{K}}{K}$ > $(C_4 -)e^{-t}$

(A20) $K(t)$< $C_6(t*,)e^{C_5( )-e^{-t}}$

where $C_4 = C_1^{-1}\ {}^{-\frac{1+}{}\frac{1}{}}$, $C_5(\ ) = C_4 +\ $, and $C_6(t^*,\ ) = K(t^*)e^{-C_5(\ )-e^{-t^*}}$.

Proof: Writing (A9) as $\dot{K} = g(m)Ke^{-t}$ and noting that $g(m)\quad C_4$ by lemma 4 gives (A19). A(20) follows from integrating (A19).

<u>Lemma 10.</u> Let $\ < 0$, then there is a t^* such that $t > t^*$ implies $\dot{k} < 0$ and $\dot{R} > 0$.

Proof: Write (A2) as

$$C_1 =\ + (1-\ )\ \frac{M}{Kk^{-1}e^{(-\ )t}}{}^{-\frac{1+}{}}\ k\ e^{\ t} = G(K, k , t)$$

Differentiating totally with respect to t

(A21) $$\dot{k} = -\frac{1}{G_k}\ G_K \dot{K} + G_t\ .$$

Since clearly $G_k > 0$, the lemma will follow from lemma 1 and (A5) once it is shown that

(A22) $$G_K \dot{K}\quad 0$$

(A23) $$G_t\quad C_1\ > 0.$$

Using lemma 9 and calculating G_K we have

(A24)
$$-\left[\ + (1-\ )m^-\ \right]^{-\frac{1+}{}-1}(1+\ )(1-\ )M^- k^{-(1-\ )+}\ e^{[\ (\ -\ )+\ ]t}$$

$$C_6(t^*,\ )\ e^{C_5(\ )-e^{-t}}\ (C_4 -\ )e^{-t}\quad H(t) < G_K \dot{K} < 0$$

where H(t) is the term that precedes the for an arbitrary > 0, a t^* which is determined by as in lemma 9 and $t > t^*$, and where the dependence of m and k on t is taken into account in the definition of H(t). Using (A8) and lemma 4 one has

$$\lim_{t} H(t) = C_8 \lim_{t} e^{at} e^{-be^{-t}}$$

where $a = -\ -\left[-\ (1-\ )+\ \right]+\ (\ -\ )+\ +-\ > 0$, $b = -\ C_5(\ )-\ > 0$,

$C_8 = -\ {}^{-\frac{1+}{}-1}(1+\ )(1-\ )M^- C_7^{-(1-\ )+}\ C_6(t^*,\ )\ (C_4 -\ )$ and $C_7 = C_1^{\frac{1}{}\ -\frac{1+}{}\frac{1}{}}$. Taking logs and using L'Hospital's rule, one has

$$\lim_{t} H(t) = 0,$$

which, from (A24), implies (A22). Finally calculation of G_t, (A7) and lemma 4 imply (A23) immediately.

__Lemma 11.__ Let $\;>0$. Then $\dot{e}_1 > 0, \dot{e}_2 > 0$.

Proof: (A2) can be written as

(A25) $\qquad C_1 = \; + (1 - \;)\dfrac{M^{-}}{x^{-1}\, v^{-}}^{-\frac{1+}{}} \quad x \quad J(x, v.)$

where $v = Ke^{--t}$. Since $\dot{v} > 0$ from lemma 3 and (7) and since a calculation shows that $J_v < 0$ and $J_x > 0$, the differentiation of (A25) gives $\dot{x} > 0$ and the result for e_1 follows from (A13) and (8).

   Now consider the result for e_2. Differentiating (A12) and dropping the second term in the square brackets in (A25) to create an inequality gives

(A26)

$$\dfrac{dy}{dx} = \dfrac{x^{-1}}{}\, w^{-} \; - \dfrac{1}{+1} C_1^{-\frac{}{1+}} x^{-\frac{2}{+1}}$$

$$> \dfrac{x^{-1}}{}\, w^{-} \; - \dfrac{1}{+1} C_1^{-\frac{}{1+} - \frac{2}{1+}} \; - \; = \dfrac{x^{-1}}{}\, w^{-} \; - \dfrac{1}{1+}\dfrac{w^{-}}{(1-\;)} \; > 0.$$

The differentiation with respect to time of (A12) and $\dot{x} > 0$ then imply $\dot{y} > 0$ and the result follows from (9) and the definition of y.

__Lemma 12.__ Let $\;>0$. Then there is a t* such that $t > t*$ implies $\dot{k} > 0$ and $\dot{R} < 0$.

Proof: Once it is shown that $\dot{k} > 0$ the lemma will follow from lemma 1 and (A5). Thus from (A21), since $G_k > 0$ the lemma will follow once it is shown that there is a t* such that $G_K \dot{K} + G_t < 0$ for $t > t*$. First consider $G_K \dot{K}$. A calculation and (A7) show that

(A27) $\qquad G_K \dot{K} = -C_1 \big[\; m \; + (1 - \;)\big]^{-1} (1 + \;)(1 - \;)\dfrac{\dot{K}}{K}.$

Choose $\;, \;'$ such that

(A28) $\qquad\qquad 1 - \dfrac{C_2 - }{C_2 + \; + \;'}\,(1 + \;) - \quad \leq 0.$

Then from lemma 5 there is a t* such that for $t > t*$

$$\frac{\dot{K}}{K} > \frac{C_2 -}{\underline{C_2 +} +} , = \frac{C_2 -}{C_2 + +'} .$$

Then, putting this inequality into (A27), taking the limit and using lemma 4

$$\lim_t G_K \dot{K} \quad -C_1(1+ \ ) \quad \frac{C_2 -}{C_2 + +'} .$$

Next consider G_t. A calculation and (A7), taking the limit and using lemma 4 gives

$$\lim_t G_t = C_1 \left[\ -(1+ \ )(\ - \ ) \right].$$

Putting the last two inequalities together and using (A28) gives, as was to be shown,

$$\lim_t G_K \dot{K} + G_t \quad C_1 \quad - \quad +(1+ \ ) \quad 1 - \frac{C_2 -}{C_2 + +'} \quad < 0.$$

Lemma 13. Let > 0. Then there exists a t* such that $t > t^*$ implies that $\dot{e}_1 > 0$ and $\dot{e}_2 > 0$.

Proof: As in lemma 11, $\dot{x} > 0$ for $t > t^*$. The result for e_1 then follows from (A13) and (8). Next write (A2) as a function of m and x. Lemma 4 implies that x . (A26) then implies that $dy/dx > 0$. This and $\dot{x} > 0$ imply that $\dot{y} > 0$ so that the result for e_2 follows from (9) and the definition of y.

The case of $= 0.$

Lemma 14. Let $= 0$. Then k O and R .

Proof: In this case the production function has the form

(A29) $\qquad Y = K \quad L^{(1- \ )} M^{1-} e^{\ t}$

where $= +(1- \ )$. A calculation shows

(A30) $\qquad r = \frac{}{1-} \frac{L}{K} w.$

Using the wage equals the marginal product of labour condition

(A31) $\qquad \frac{L}{K}^{1-(1- \ )} = \frac{1}{C_1} K^{\ -1} M^{1-} e^{\ t},$

so that the specific path of K must be found. Substitute (A31) into the basic equation for $\dot{K}$ to get

(A32) $\qquad \dot{K} = C_9 K^{\overline{1-(1- \ )}} e^{\frac{1}{1-(1- \ )} t}.$

where $C_9 = \left(\dfrac{1}{C_1}\right)^{\frac{(1-\ )}{1-(1-\ )}} M^{\frac{1}{1-(1-\ )}}$. Integrating this from t to t^0 gives

$$(A33) \qquad K(t) = \left[K^{1-a}(t^0)e^{-bt} - C_9 \frac{1-a}{b}\ e^{-b(t-t^0)} - 1 \right]^{\frac{1}{1-a}} e^{\frac{b}{1-a}t}$$

where $a = \dfrac{\ }{1-(1-\ )}$ and $b = \dfrac{\ }{1-(1-\ )}$ and $a < 1$. Taking the limit, for any small $\ $ there is a t^* such that $t > t^*$ implies that

$$K(t) > \left[\frac{1-a}{b} C_9 - \ \right]^{\frac{1}{1-a}} e^{\frac{b}{1-a}t}$$

Substituting this into (A31),

$$\left(\frac{L}{K}\right)^{1-(1-\ )} > C_{10} e^{\left(\ -1\right)\frac{b}{1-a}+\ t}$$

where $C_{10} = \dfrac{1}{C_1} \left[\dfrac{1-a}{b} C_9 - \ \right]^{\frac{-1}{1-a}} M^{1-\ }$. Thus $\dot{k}\ \ O\ $ and, from (A30) and lemma 1, $\dot{R}\ \ \ $.

<u>Lemma 15.</u> Let $\ = 0$. There exists a t^* such that $t > t^*$ implies $\dot{k} < 0$ and $\dot{R} > 0$.

Proof: From (A33)

$$(A34) \qquad K^{\ -1}(t) e^{\ t} = A(t)^{\frac{\ -1}{\ -a}},$$

where $A(t) = K^{1-a}(t^0) e^{\frac{1-a}{\ -1}t} - C_9 \frac{1-a}{b}\left[e^{bt^0 + \frac{1-a}{\ -1}t} - e^{b+\frac{1-a}{\ -1}t} \right]$. A calculation shows that there is a t^* such that, for $t > t^*$, $\dot{A} < 0$. Thus, from (A34), $K^{\ -1}e^{\ t}\ > 0$ for $t > t^*$. This and (A31) imply the result for $\dot{k}$ and thus (A30) and lemma 1 imply it for $\dot{R}$.

<u>Lemma 16.</u> Let $\ = 0$. Then e_1 and e_2 are constants.

Proof: Replacing (1) with (A29) in (10) and dividing by L gives

$$(A35) \qquad w = \ \left(\frac{K}{L}\right)^{\ } \left(\frac{L_m}{L}\right)^{(1-\ )} \left(\frac{M}{L}\right)^{1-\ } e^{\ t}.$$

On the other hand the wage equals the marginal product of labour implies

$$(A36) \qquad w = (1-\ )\ \left(\frac{K}{L}\right)^{\ } \left(\frac{M}{L}\right)^{1-\ } e^{\ t}.$$

(A35), (A36) and (9) imply the result for e_2. From (A29) $\dfrac{Y}{L} = \left(\dfrac{K}{L}\right)\left(\dfrac{M}{L}\right)^{1-} e^{t}$ so that the result for e_1 follows from (A36) and (8).

Proof of the Theorem: The theorem follows from lemmas 6-8 and 10-16.

2. Proof of Meek's Proposition.

From the solution to (11) one can write the path of R, $R(K_0,t)$, as a function of t and the initial condition K_0.

Meek's Proposition. For the model (1)-(10) let > 0. Then $\lim_{K_0 \to 0} \dot{R}(K_0, 0) > 0$ and there exists a t* such that $t > t^*$ implies $\dot{R}(K_0, t) < 0$.

That is, when the elasticity of substitution between the aggregate and land is less than one and the initial capital stock is small enough, capital accumulation first causes the rate of profit to rise and then, afterwards, fall.

Proof: From (A2)

$$(A37) \quad \left(\frac{w}{(1-)}\right)^{-\frac{1+}{}} = k^{-\frac{}{1+}} e^{-\frac{}{1+} t} + (1-)M^{-} e^{-t} k^{-(1-)-\frac{}{1+}} e^{-\frac{}{1+} t} K .$$

Differentiating this with respect to time, letting $t = 0$ and taking the limit as K approaches zero gives

$$(A38) \quad 0 = -\frac{}{1+} k^{-\frac{}{1+}-1}\dot{k} + (1-)M^{-} k^{-(1-)-\frac{}{1+}}\left[\lim_{K \to 0} K^{-1}\dot{K} + \left(k^{-\frac{}{1+}} - \frac{}{1+}\right)\right] .$$

Since $\dot{K} = Y - wL$, from (1) and (A1) we have

$$(A39) \quad K^{-1}\dot{K} = \left[k^{(1-)} + (1-)M^{-} K \right]^{-\frac{1+}{}} K \left[k^{(1-)} + (1-)M^{-} K \right] .$$

Let $t = 0$. From (A37) $\lim_{K \to 0} k > 0$ and finite. Thus (A39) implies $\lim_{K \to 0} K^{-1}\dot{K} = 0$. Thus (A38) and the positive finite limit of k imply $\lim_{K \to 0} \dot{k} < 0$. The first part of the result now follows from (A5) and lemma 1, and the second part from the previous theorem.

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