Long Run Unemployment, Growth and Inflation*

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Abstract

This paper presents an endogenous growth model of a monetary economy with unemployment due to the existence of unions in the labor market. We assume that total factor productivity depends on a public input and that the services derived from this public input depend on the level of employment, i.e., there is an externality accruing from the average amount of employment. We show that, if this externality is positive then an expansive monetary policy decreases unemployment. On the contrary, if the externality is negative then a contractive monetary policy or a decrease in the tax rates decrease unemployment. If there is no such externality, the model predicts a vertical long run Phillips curve as the one obtained with a competitive labor market but this relationship vanishes if the externality exists.

Key Words: Inflation, Endogenous Growth, Unemployment, Unions, Productive Government Spending, Seignorage, Income Taxes.

JEL Classification: E24, E31, E63, J51, O42

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Abstract

Este artículo presenta un modelo de crecimiento endógeno de una economía monetaria con paro debido a la existencia de sindicatos en el mercado de trabajo. Suponemos que la productividad total de los factores depende de un input público y que los servicios derivados de este input dependen del nivel de empleo, es decir, existe una externalidad dependiente del nivel de empleo medio. Demostramos que si la externalidad es positiva una política monetaria expansiva disminuye el desempleo. Si la externalidad es negativa una política monetaria contractiva o una disminución de los impuestos disminuyen el paro. Si la externalidad no existe el modelo predice una curva de Phillips a largo plazo vertical como la obtenida con un mercado de trabajo competitivo pero esta relación se rompe si existe la externalidad.

1. Introduction

This paper develops an endogenous growth model of a monetary economy with a non-competitive labor market in order to analyze how fiscal and monetary policy and the bargaining power of workers affect the growth, inflation and unemployment rates in the long run. The effect of monetary and fiscal shocks in models with nominal price rigidities and imperfect competition in the output market is now an active field of research. This literature is associated with "the return of the Phillips curve" (Galí (1999), Clarida, Galí and Gertler (1999)). In general, in this literature, it is assumed that the labor market is competitive and without frictions but Cooley and Quadrini (1999) present a model with matching frictions in the labor market. In contrast with this literature, our concern is not to analyze the short run fluctuations of these variables and, hence, there are no nominal rigidities in the model.

There is not an extensive theoretical literature on long run growth models with unemployment. Classical references are Pissarides (1990), Bean and Pissarides (1993) and Aghion and Howitt (1994), all of them with matching frictions in the labor market. In the political science literature, the seminal paper by Przeworski and Wallerstein (1988) is also a model of growth and unemployment. Usually, these papers analyze the effects of different exogenous variables on both the economic growth rate and on unemployment. More recently, Eriksson (1997), using an endogenous growth model with matching frictions in the labor market, analyzes the effects of the bargaining power of the worker and taxes on capital income, among other exogenous variables, on both the economic growth rate and on unemployment. Ramos and Sanchez-Losada (1999) analyze, using an endogenous growth model with human capital and unions in the labor market, if the economic growth rate in a unionized economy is higher or lower than the economic growth rate of the same economy with a competitive labor market. Finally, Daveri and Tabellini (1997) study the effects of taxes on growth and unemployment in a model with unions in the labor market.

Because of the nature of the questions studied, none of the models mentioned above is an economy with money. In fact, growth models with money and unemployment are the exception in the literature. Pissarides (1990, chapter 2) studies the effect of monetary policy in an exogenous growth model with a labor market with matching frictions. In this model, the demand side is based on the dynamic IS-LM system. Compared with the paper by Pissarides, the model presented in this paper has a more micro-founded money demand based on a cash-in-advance constraint. Moreover, growth is endogenous which means that simultaneously the growth rate and the unemployment rate are endogenously determined. Finally, unemployment in the model is not frictional but it is due to the presence of unions in the labor market that negotiate the wage with firms. More precisely, we consider a right-to-manage model.

In our model, the government finances, by means of both seignorage and income taxes, the unemployment benefit and a public input that increases the labor marginal product. We assume that average employment in the economy affects the services derived from the public input. With this assumption, we show that long run unemployment depends on the monetary policy. Then, we explore under which conditions an expansionary government policy may reduce the long run unemployment rate and which government policies may implement full employment. We also study how government policies affect the growth rate and the inflation rate and the shape of the long run Phillips curve. Finally, we compare the results derived in this paper with the ones obtained in models with a competitive labor market and we find out substantial differences.

The rest of the paper is organized as follows. In Section 2, we study how firms and unions set the wage. In Section 3, we introduce the consumer and the government. In Section 4, we obtain the equilibrium amount of employment and the government expenditures to private capital ratio and we study the effects of government policies on employment. In Section 5, we derive the equilibrium growth and inflation rates and we study the effects of government policies in these two rates. Finally, Section 6 summarizes the main results.

2. Wage Setting

In this section, we study the wage setting process which is based on a wage bargaining between firms and unions. In order to describe this bargaining, we first present the production technology and then firms' and unions' behavior. The technology is characterized by the following production function:

$$F(K(t),G(t),L(t)) = \psi\left(t\right)K(t)^{\alpha}\overline{K}(t)^{1-\alpha}L(t)^{\beta},\ \alpha\in\left(0,1\right),\beta\in\left(0,1\right),\ \alpha+\beta<1, \tag{2.1}$$

where $K(t)$ is the capital stock, $\overline{K}(t)$ is an externality accruing from the average capital stock, $L(t)$ is the number of workers and, finally, $\psi\left(t\right)$ measures total factor productivity and it is given by

$$\psi\left(t\right)=A+B\left(\frac{G\left(t\right)}{\overline{K}\left(t\right)}\right)^{\gamma}\overline{L}\left(t\right)^{\delta},\ \gamma\in\left(0,1\right),\delta+\beta\geqslant 0,\ \beta>\delta,\ A>0,\ B>0.$$

where $G(t)$ is productive government spending and $\overline{L}(t)$ is the rate of employment. Note that total factor productivity depends positively on the public input. However, it decreases with the average stock of capital in the economy. This is justified by means of a congestion effect of the average stock of capital on the services derived from the public input. Moreover, there is an externality accruing from the fraction of employed workers in the economy, if $\delta > 0$, total factor productivity increases with the fraction of employed workers. Thus, we introduce a positive externality on production due to a reduction on unemployment. On the contrary, if $\delta < 0$, total factor productivity decreases with the fraction of employed workers which is justified by means of a congestion effect.[1] Finally, $\psi(t)$ is such that makes production depend on the quantity of private production factors even if $G(t) = 0$.

We denote the firms´profits by

$$\Pi(t) = F\left(K(t), G(t), L(t)\right) - w(t)L(t) - r(t)K(t),$$

where $w(t)$ is the real wage at time t and $r(t)$ is the real interest rate at time t. We assume that there is a large number of firms in the economy and they are price takers, thus, profit maximization implies

$$w(t) = F_L(t) = \beta\psi(t)\, K(t)^{\alpha}\overline{K}(t)^{1-\alpha}L(t)^{\beta-1}. \tag{2.2}$$

The assumption of a large number of firms implies that they do not take the labor externality into account when maximizing profits. We also assume that both the capital stock and the public input are considered constant by unions when setting the wage and, then, equation (2.2) characterizes the labor demand $\widetilde{L}^d(w(t))$. We assume that there is a central union that bargains the same wage for all firms, i.e., we have a centralized wage setting system. If the wage were set unilaterally by this union, it would set it in order to maximize the expected income of a worker

$$\widetilde{L}^d(w(t))(1-\tau)w(t) + \left(1 - \widetilde{L}^d(w(t))\right) d(t),$$

where τ is the income tax rate which is constant with time, and $d(t)$ is the unemployment benefit that an unemployed worker gets. We assume that the labor supply is inelastic with respect to the wage and equal to one. Thus, the amount of unemployment is $1-\widetilde{L}^d(w(t))$. The first order condition of the union maximization problem is given by

$$w(t) = \frac{d(t)}{(1-\tau)\left(1 + F_{LL}(t)\frac{L(t)}{F_L(t)}\right)},$$

[1] Barro and Xavier Sala-i-Martín (1992), Fisher and Turnovsky (1998) and Glomm and Ravikumar (1994) have introduced a congestion effect accruing from both the average stock of capital and the average number of workers in the economy. In contrast, we introduce an externality accruing from the fraction of employed workers in the economy.

where

$$F_{LL}(t)=(\beta-1)\,\beta\psi\,(t)\,K(t)^{\alpha}\overline{K}(t)^{1-\alpha}L(t)^{\beta-2}+\beta\frac{\partial\psi\,(t)}{\partial L}K(t)^{\alpha}\overline{K}(t)^{1-\alpha}L(t)^{\beta-1}. \tag{2.3}$$

Note that the central union takes the externality accruing from the fraction of employed workers into account when it sets the wage because by setting the same wage for all firms it determines employment and, then, the fraction of employed workers[2]. If the owners of the firms would set the wage in order to maximize profits, they would set the wage equal to zero. For this reason, we assume the following short cut rule for the wage setting:

$$w(t)=\left(\frac{\varphi d(t)}{1-\tau}\right)\left(\frac{1}{1+F_{LL}(t)\frac{L(t)}{F_L(t)}}\right), \tag{2.4}$$

where $0<\varphi<1$ is the unions bargaining power.[3] We also assume that the unemployment benefit is a constant fraction, $\upsilon\in(0,1)$, of production

$$d(t)=\upsilon F(K(t),G(t),L(t)). \tag{2.5}$$

Using equations (2.2), (2.4) and (2.5), we get the following equation

$$1+F_{LL}(t)\frac{L(t)}{F_L(t)}=\left(\frac{\varphi\upsilon}{1-\tau}\right)\left(\frac{F(t)}{F_L(t)}\right). \tag{2.6}$$

Assuming a symmetric equilibrium, that is $\overline{K}(t)=K(t)$ and $\overline{L}(t)=L(t)$, and substituting (2.1), (2.2) and (2.3) in (2.6) we obtain

$$\frac{G\,(t)}{K\,(t)}=\left(\frac{AL\,(t)^{-\delta}}{B}\right)^{\frac{1}{\gamma}}\left(\frac{\widehat{\beta}-L\,(t)}{L\,(t)-\widehat{\delta}}\right)^{\frac{1}{\gamma}}, \tag{2.7}$$

where $\widehat{\beta}=\frac{(1-\tau)\beta^2}{\varphi\upsilon}$ and $\widehat{\delta}=\frac{(1-\tau)\beta(\delta+\beta)}{\varphi\upsilon}$. Note that the ratio $\frac{G}{K}$ is positive if and only if $L\in\left(\min\left\{\widehat{\beta},\widehat{\delta}\right\},\max\left\{\widehat{\beta},\widehat{\delta}\right\}\right)$. From now on, we impose this constraint on the domain of L.

[2] Because total labor supply is equal to one then the fraction of employed workers is equal to the number of workers and, hence, taking into account the externality implies $\overline{L}(t)=L(t)$.

[3] Of course, in order to be rigorous for the derivation of the wage, it should be obtained from the maximization of the weighted Nash product of the expected income of a worker and profits but, for simplicity, we follow this short cut approach also followed by Aghion and Howitt(1994) and Eriksson (1997), among others. Actually, the maximization of the weighted Nash product would not modify the results.

Equation (2.7) determines implicitly the function $L = \widetilde{L}_E(\frac{G}{K}, \tau, \varphi)$, we call to this function the employment equation which says the demand of labor set by the wage setting rule given the ratio of government spending to capital, the tax rate and the bargaining power of workers.

What is important for the solution of the model are the partial derivatives of this function with respect to the ratio $\frac{G}{K}$, φ and τ. The sign of these partial derivatives is given in the following proposition:

Proposition 2.1. *Along the employment equation,* $\frac{\partial \widetilde{L}_E}{\partial \frac{G}{K}} \gtreqless 0$ *if* $\delta \gtreqless 0$, $\frac{\partial \widetilde{L}_E}{\partial \tau} < 0$ *and* $\frac{\partial \widetilde{L}_E}{\partial \varphi} < 0$.

Proof. The proof follows by means of using the implicit function theorem in equation (2.7).

Note that both the wage and the marginal productivity of labor increase with the ratio of government spending to capital. When $\delta > 0$, the increase in the ratio $\frac{G}{K}$ makes the wage increase less than the marginal productivity of labor and, hence, the amount of labor decreases along the employment equation. The opposite occurs when δ is negative. Note also that the amount of employment does no depend on the ratio $\frac{G}{K}$ when δ is equal to zero. Finally, employment is decreasing with both the income tax and with the bargaining power of workers. The reason is that while the wage increases with these two variables, the marginal productivity of labor is not affected and then employment decreases. In the following section, we complete the description of the economy by means of characterizing the behavior of both the consumers and the government.

3. Consumers and Government

There are a continuum of identical infinitely lived consumers distributed in the interval $[0, 1]$. Each consumer, i, is endowed with one unit of time that is inelastically supplied, $k_i(t)$ units of capital and $M_i(t)$ units of nominal money balances in each period. Each consumer solves the following problem: given $I_i(t)$, $\Pi_i(t)$, $P(t)$ and $r(t)$, choose $c_i(t)$, $k_i(t)$ and $M_i(t)$ in order to maximize

$$Max \int_0^\infty e^{-\varrho t} \ln(c_i(t))dt$$

$$st$$

$$P(t)(c_i(t) + \dot{k}_i(t)) + \dot{M}_i(t) = P(t)[(1-\tau)r(t)k_i(t) + I_i(t) + (1-\tau)\Pi_i(t)],$$
$$P(t)\phi(c_i(t) + \dot{k}_i(t)) = M_i(t),$$

where $c_i(t)$ is consumption of the individual i in period t, $k_i(t)$ is the stock of private capital of the individual i in period t, $M_i(t)$ is the amount of nominal money balances, $P(t)$ is the price level in period t, $\Pi_i(t)$ are profits of the individual i in period t, ρ is the subjective discount rate, ϕ are the money requirements to purchase one unit of goods and I_i is the labor income at period t. We assume that I_i is equal to $(1-\tau)w(t)$ when the consumer works in period t and it is equal to $d(t)$ when the consumer is unemployed in period t. The first constraint is the budget constraint and the second one is the cash-in-advance (CIA) constraint.

Let $M_i(t) = P(t)m_i(t)$ where $m_i(t)$ is the demand of real money balances. It follows that $\frac{\dot{M}_i(t)}{P(t)} = \dot{m}_i(t) + \pi(t) m_i(t)$ where $\pi(t)$ is the inflation rate. Using this equation, we can rewrite the budget constraint as

$$c_i(t) + \dot{k}_i(t) + \dot{m}_i(t) + \pi(t) m_i = (1-\tau)r(t)k_i(t) + I_i(t) + (1-\tau)\Pi_i(t). \quad (3.1)$$

Solving the problem of consumer i, we obtain the following differential equation for the evolution of the consumption growth rate of individual i, $\zeta_i(t)$:[4]

$$\dot{\zeta}_i(t) = (\zeta_i(t) + \rho)(\zeta_i(t) + \pi(t) + \rho + \frac{1}{\phi} + \frac{\dot{r}(t)}{r(t)}) - \frac{(1-\tau)r(t)}{\phi}. \quad (3.2)$$

In a symmetric equilibrium, the consumption growth rate of each individual is the same, that is $\zeta_i(t) = \zeta(t)$ for all i. It follows that the rate of growth of aggregate consumption is also equal to $\zeta(t)$.[5] This means that the evolution of the rate of growth of aggregate consumption in a symmetric equilibrium is given by

$$\dot{\zeta}(t) = (\zeta(t) + \rho)(\zeta(t) + \pi(t) + \rho + \frac{1}{\phi} + \frac{\dot{r}(t)}{r(t)}) - \frac{(1-\tau)r(t)}{\phi}. \quad (3.3)$$

The government obtains revenues by means of both seignorage and collecting taxes on the production. With these revenues, the government finances productive government spending and the unemployment benefits. Thus, the government budget constraint is

$$P(t)[d(t)(1-L(t)) + G(t)] = \dot{M}(t) + P(t)\tau F(t),$$

[4]It can be shown that the transversality conditions hold along any equilibrium path that converges in the balanced growth path (BGP). Moreover, both the CIA constraint and the budget constraint are binding along these equilibrium paths. A complete derivation of equation (3.2) can be obtained upon request.

[5]Let $C(t)$ be aggregate consumption. Thus, $C(t) = \int_0^1 c_i(t)\, di$. Differentiating this equation with respect to time we get $\dot{C}(t) = \int_0^1 \dot{c}_i(t)\, di$. Dividing by $C(t)$, we obtain $\frac{\dot{C}(t)}{C(t)} = \frac{1}{C(t)} \int_0^1 c_i(t) \frac{\dot{c}_i(t)}{c_i(t)} d_i$. If $\frac{\dot{c}_i(t)}{c_i(t)} = \zeta(t)$ for all i then $\frac{\dot{C}(t)}{C(t)} = \zeta(t)$.

where $M(t)$ is the money supply at time t.We assume that the money supply grows at a constant and positive rate, μ. This means that $\dot{M}(t) = \mu M(t)$. Using the previous equation and equation (2.5), the government budget constraint can be rewritten as

$$G(t) = \mu m(t) + (\tau - \upsilon(1 - L(t)))\, F(t)\,. \tag{3.4}$$

Finally, the constant growth of the money supply implies that

$$\pi(t) = \mu - \frac{\dot{m}(t)}{m(t)}. \tag{3.5}$$

4. Equilibrium Employment

We assume that markets are in equilibrium. Thus, the aggregate capital demand must be equal to the aggregate capital supply, $\sum k_i(t) = K(t)$, and the aggregate money demand must be equal to the aggregate money supply, $\sum m_i(t) = m(t)$. Combining these two equilibrium conditions with equations (3.1) and (3.5), we obtain the aggregate consumer's budget constraint

$$C(t) + \dot{K}(t) = (1 - L(t))d(t) + (1 - \tau)F(t) - \mu m(t)\,. \tag{4.1}$$

Next, from the individual CIA constraint, we obtain the aggregate CIA constraint

$$(C(t) + \dot{K}(t))\phi = m(t). \tag{4.2}$$

Combining equations (4.2) and (4.1), we get

$$m(t) = \frac{\phi\left((1 - L(t))d + (1 - \tau)F\right)}{1 + \phi\mu}. \tag{4.3}$$

Next, substituting the previous equation into the government budget constraint, equation (3.4), and using equation (2.1), we obtain

$$\frac{G(t)}{K(t)} = \left(\frac{\tau + \mu\phi - \upsilon(1 - L(t)}{1 + \mu\phi)}\right) L(t)^{\beta}\psi(t)\,. \tag{4.4}$$

Equation (4.4) is the equilibrium government budget constraint equation $\frac{G(t)}{K(t)} = \widetilde{\frac{G}{K}}(L(t), \tau, \mu)$. It says the ratio of government spending to capital given the amount of employment, the tax rate and the rate of growth of money in nominal terms . From now on, we will assume that $\tau + \mu\phi > \upsilon$. This assumption makes the ratio $\frac{G}{K}$ positive for any value of L. The following proposition computes the partial derivatives of the function $\widetilde{\frac{G}{K}}$:

Proposition 4.1. *Along the equilibrium government budget constraint equation,* $\frac{\partial \widetilde{\frac{G}{K}}}{\partial L} > 0$, $\frac{\partial \widetilde{\frac{G}{K}}}{\partial \tau} > 0$ *and* $\frac{\partial \widetilde{\frac{G}{K}}}{\partial \mu} > 0$.

Proof. These results are obtained by means of using the implicit function theorem on equation (4.4)

Proposition 4.1 says that, along the government budget constraint, the amount of government spending per unit of capital increases with the amount of employment, the value of the tax rate and the value of the money growth rate. Next, we define the equilibrium amount of employment, L^*, and the equilibrium value of the ratio of government spending to private capital, $\frac{G}{K}^*$, as follows.

Definition 4.2. *Let* L^* *and* $\frac{G}{K}^*$ *be such that* $\frac{G}{K}^* = \widetilde{\frac{G}{K}}(L^*, \tau, \mu)$ *and* $L^* = \widetilde{L}_E\left(\frac{G}{K}^*, \tau, \varphi\right)$.

Note that both L^* and $\frac{G}{K}^*$ are constant along the equilibrium path. Unicity and existence of this equilibrium values are discussed in the following proposition:

Theorem 4.3. *If* $\delta < 0$ *at most one equilibrium exists. Moreover, if* $\delta < 0$ *and* $\widehat{\beta} < 1$ *then a unique equilibrium exists. If* $\delta > 0$ *and* $\widehat{\delta} < 1$ *then at least one equilibrium exists.*

Proof. See the appendix.

This theorem provides conditions that guarantee the existence of an equilibrium. However, the equilibrium might not be unique when $\delta > 0$. The reason is as follows. In this economy, unions set the wage and firms choose the amount of labor given the wage and the expected marginal productivity of labor. If firms expect that the marginal productivity of labor will be small, they will demand a small quantity of labor. This makes unemployment be large and, hence, productive government spending will be reduced. Because of the reduction in productive government spending, the marginal productivity of labor will be small in equilibrium. Note that the firms expectations on a small marginal productivity of labor hold in equilibrium. From this equilibrium, we can construct another one when $\delta > 0$. Assume that firms expect that the marginal productivity of labor will increase more than the wage income. As a consequence, they increase the amount of labor and, because of the government budget constraint, the ratio of government spending to private capital will increase. From the employment equation, we know that an increase in this ratio makes the marginal productivity of labor increase more than the wage income when $\delta > 0$. Therefore, it is possible to construct multiple equilibrium paths driven by different expectations when $\delta > 0$.

However, multiple equilibria requires strong assumptions on the parameter values. Thus, from now on, we assume that the parameters take values that make the equilibrium unique and we study the effects on employment of both government policies and of the workers bargaining power. These effects are displayed in the proposition below.

Proposition 4.4. *If* $\delta < 0$ *then* $\frac{\partial L^*}{\partial \mu} < 0$, $\frac{\partial \frac{G}{K}^*}{\partial \mu} > 0$ *and* $\frac{\partial L^*}{\partial \tau} < 0$. *If* $\delta > 0$ *then* $\frac{\partial L^*}{\partial \mu} > 0$ *and* $\frac{\partial \frac{G}{K}^*}{\partial \mu} > 0$. *Finally,* $\frac{\partial L^*}{\partial \varphi} < 0$ *and* $\frac{\partial \frac{G}{K}^*}{\partial \varphi} < 0$.

Proof. The proof follows from Proposition 2.1 and Proposition 4.1.

Proposition 4.3 says that an expansionary monetary policy increases employment if and only if $\delta > 0$. The reason is as follows. An increase in the money growth rate does not affect the firm and the union decisions. Therefore, the employment equation is not modified. Instead, the increase in μ makes the amount of government spending financed using the same quantity of employment larger. Thus, the ratio of government spending to capital increases with μ. The increase in this ratio implies that the marginal product of labor increases more than the wage if δ is positive and, hence, L^* increases when δ is positive. Instead, the increase in this ratio has the opposite effect on labor when δ is negative. An increase in the workers bargaining power affects negatively the equilibrium amount of employment because it increases the wage without modifying the marginal productivity of labor. Concerning the effects of an increase in the income tax rate, note that an increase in τ affects negatively the amount of employment along the employment equation. Moreover, it also increases the ratio of government spending to capital financed given the quantity of labor. When δ is negative, both the decrease of labor along the employment equation and the increase in the ratio $\frac{G}{K}$ make employment to decrease. This explains why the equilibrium amount of employment decreases as τ increases when δ is negative. Instead, the relation between employment and the income tax is ambiguous when δ is positive. If δ is positive, labor decreases along the employment equation but the increase in the ratio $\frac{G}{K}$ makes employment to increase. This yields that the effects on employment of an increase in the income tax are ambiguous when δ is positive. Next, we present a particular case, imposing constraints on the value of the parameters, in order to show how the tax rate affects the equilibrium amount of employment when $\delta > 0$.

Proposition 4.5. *Assume that* $A = B = \gamma = 1$ *and* $\delta = 1 - \beta$ *then* $\frac{\partial L^*}{\partial \tau} \lesseqgtr 0$ *if and only if* $\tau \gtreqless \overline{\tau}$ *where*

$$\overline{\tau} = 1 - \sqrt{\frac{(1 + \phi\mu)\, v\varphi}{\beta\,(1 - \beta)}}.$$

Proof. Combining equations (2.7) and (4.4), we obtain the equilibrium amount of employment, L^*, as a function of the tax rate and we can obtain $\frac{\partial L^*}{\partial \tau}$.

An inverse U-shaped curve relates the amount of employment with the income tax rate when δ is positive. This means that employment increases with the income tax when the ratio of government spending to capital is small and, hence, an increase in this ratio has a strong positive effect on employment. Thus, we have found a case where increasing the amount of productive government spending may reduce unemployment even though the increase in government spending is financed by means of an income tax.

The previous results imply that the effects on employment of increasing productive government spending depend on the source of government revenues. It follows that full employment may also depend on the combination of financing instruments used by the government. In order to illustrate how full employment depends on the combinations of fiscal and monetary policies, we impose constraints on the value of the parameters and we derive the following result:

Proposition 4.6. *Assume that $A = B = \gamma = 1$. If $\delta = -\beta$ then full employment requires $\varphi v < \beta^2$ and it can be obtained when government revenues accrue from either tax revenues or seignorage. If $\delta = 1 - \beta$ then full employment requires $\varphi v < \beta$. More precisely, if $\varphi v \in (\beta^2, \beta)$ then full employment requires that at least a fraction of government revenues is obtained from seignorage and if $\varphi v < \beta^2$ then full employment requires that at least a fraction of government revenues is obtained from tax revenues.*

Proof. Combining equations (2.7) and (4.4), we can obtain the equilibrium amount of employment, L^*, as a function of the government policy parameters. Next, remember that the domain of L^* belongs to the following interval: $\left(\min\left\{\widehat{\delta}, \widehat{\beta}\right\}, \max\left\{\widehat{\delta}, \widehat{\beta}\right\}\right)$. Finally, we obtain the combination of the government policy parameters that make $L^* = 1$ and that do not restrict the domain of L^* so that they make $\max\left\{\widehat{\delta}, \widehat{\beta}\right\} > 1$.

It follows that full employment can only be obtained when the unions bargaining power and the unemployment benefit are sufficiently small. Interestingly, either positive tax rates or a positive money growth rate are a requirement to obtain full employment when δ is positive. Moreover, if the unions bargaining power is large then full employment might only be obtained when the government obtains revenues by means of printing money. This result suggests that long run unemployment may be due to restrictive monetary policies.

5. The Rate of Inflation and the Rate of Growth

In the previous section, we have shown how the equilibrium amount of employment depends both on the government policies and on the bargaining power of workers. In this section, we will use these results in order to study how the economic growth rate and the inflation rate are affected by these variables. We start analyzing the effect on the interest rate. The interest rate is equal to the marginal product of capital, hence

$$r^* = \alpha (L^*)^\beta \left(A + B \left(\frac{G^*}{K} \right)^\gamma (L^*)^\delta \right). \tag{5.1}$$

Note that r^* is constant along the equilibrium because L^* and $\frac{G}{K}^*$ are constant. The sign of the partial derivatives of the interest rate with respect to both the money growth rate and to the unions bargaining power are displayed in the following proposition:

Proposition 5.1. $\frac{\partial r^*}{\partial \mu} > 0$ *and* $\frac{\partial r^*}{\partial \varphi} < 0$.

Proof. Combine equations (5.1) and (2.7) in order to obtain the equilibrium interest rate as a function of L^*. The sign of the partial derivatives follows from the results in Proposition 4.3.

An increase in the money growth rate or a decrease in the unions bargaining power increase the equilibrium interest rate. Instead, it can be shown that the effects on the interest rate of an increase in the tax rate are ambiguous. This is in contrast with the existing literature. In particular, Barro (1990) has shown that the interest rate increases with the income tax rate when government spending is a productive input, the labor market is competitive and the labor supply is exogenous. Next, we derive the equilibrium of the economy. Combining equations (2.1), (2.5), (4.1) and (4.2) we obtain

$$\frac{\dot{K}(t)}{K(t)} = \xi - X(t), \tag{5.2}$$

where $X(t) = \frac{C(t)}{K(t)}$ and

$$\xi = \left(\frac{(1 - L^*)v + (1 - \tau)}{1 + \phi\mu} \right) \frac{r^*}{\alpha}.$$

We can rewrite equation (4.3) as $m(t) = \phi\xi K(t)$ which implies that $\frac{\dot{m}(t)}{m(t)} = \frac{\dot{K}(t)}{K(t)}$. Combining equations (3.5) and (5.2), we get $\pi(t) = \mu - \xi + X(t)$. Next, combining the previous equation and (3.3), we obtain

$$\dot{\zeta}(t) = (\rho + \zeta(t))(\zeta(t) + \mu - \xi + X(t) + \rho + \frac{1}{\phi}) - \frac{(1-\tau)r^*}{\phi}. \quad (5.3)$$

Finally, from the definition of $X(t)$, we derive $\dot{X}(t) = \zeta(t) - \xi + X(t)$. This differential equation and equation (5.3) characterize the dynamic equilibrium. Using these differential equations, it can be shown that the dynamic equilibrium does not exhibit transition. Thus, the equilibrium is always the balanced growth path (BGP).[6] Along the BGP, consumption, government spending, capital and production grow at the same constant growth rate. This economic growth rate is

$$\zeta^* = \frac{(1-\tau)r^*}{\phi(\mu + \rho) + 1} - \rho,$$

and the inflation rate is $\pi^* = \mu - \zeta^*$.

We proceed to study the growth effects of government policies and of the unions bargaining power. First, an increase in the bargaining power of workers, as we said, reduces the interest rate and, hence, the economic growth rate decreases. The growth effects of any government policy can be divided into a direct effect and an indirect effect. Concerning the direct effect, note that the tax rate reduces the net of taxes capital gains and, hence, agents reduce the accumulation of capital which deters growth. Seignorage also reduces the accumulation of capital since it makes investment purchases more expensive. Again, this deters growth. Thus, the direct effect of any government policy negatively affects the economic growth rate. Indirectly, government policies also affect the economic growth rate by means of changing the equilibrium interest rate. In Proposition 5.1, we saw that an expansive monetary policy implies an increase in the interest rate. This means that there is a trade off between the direct and indirect effects which may result into an inverse U-shaped curve that relates the growth rate with the money growth rate. Interestingly, if $\delta > 0$, we may have situations that an expansive monetary policy implies more employment and a higher rate of growth. Next, we proceed to analyze the growth effects of fiscal policy. In those models with a competitive labor market and productive government spending, an increase in the tax rate implies a trade-off between the direct and the indirect effects that typically results into an inverse U-shaped curves that relate the economic growth rate with the tax rate.[7] These curves do no longer need to exist in our model with a non competitive labor

[6] Remember that we have imposed a symmetric equilibrium. Because there is no transition, it follows that the growth rate must always be at the BGP which is common for all individuals. This means that the assumption of a symmetric equilibrium is not contradicted.

[7] See Barro (1990).

market. The following example is a case where $\delta < 0$ that yields an inverse U-shaped curve relating the growth rate with the money growth rate and a negative relation between the rate of growth and the tax rate.

Proposition 5.2. *If $A = B = \gamma = 1$ and $\delta = -\beta$ then $\frac{\partial \varsigma^*}{\partial \tau} < 0$ and $\frac{\partial \varsigma^*}{\partial \mu} \gtreqless 0$ if and only if $\mu \lesseqgtr \overline{\mu}$ where*

$$\overline{\mu} = \frac{(1-\beta)\,\phi\rho - v\widehat{\beta} - \beta}{\beta\phi}.$$

Proof. Combine equations (5.1), (2.7) and (4.4) in order to obtain the growth rate as a function of the government policy parameters. Then, the sign of the partial derivatives can be obtained.

An increase in the income tax rate reduces the growth rate. Moreover, $\overline{\mu}$ is actually negative when we consider plausible parameter values.[8] This means that, for these plausible cases, an increase in the money growth rate always reduces the economic growth rate.

When $\delta > 0$, it is easy to find examples of inverse U-shaped curves relating the economic growth rate with the money growth rate and with the tax rate. In these examples, the values of the government policy parameters that maximize the economic growth rate are different than the ones obtained when the labor market is competitive. These differences are illustrated in Figures 1 and 2.

[Insert Figures 1 and 2]

Figure 1 displays the growth rate as a function of the money growth rate in the following two cases: a competitive labor market (full employment) and the non competitive labor market (unemployment) of our model when $\delta > 0$. In both cases, the growth rate displays an inverse U-shaped curve. However, the growth maximizing money growth rate is larger when the labor market is not competitive than when it is. This difference is due to the positive effects on employment of an increase in the money growth rate which accelerate economic growth. Figure 2 displays the growth rate as a function of the tax rate in the same two cases. Again, the growth rate displays an inverse U-shaped curve. Note that the growth maximizing tax rate is larger when the labor market is competitive than when it is not. This difference arises since an increase in the tax rate may reduce employment and, hence, decrease the economic growth rate. We conclude that the growth effects of government policies obtained in models with full employment

[8] $1-\beta$ and ϕ are smaller than one. Moreover, plausible values of ρ are smaller than 0.1. Finally, plausible values of β are larger than 0.5. This means that $\overline{\mu}$ is negative.

cannot be extrapolated into actual economies without taking into account the effects of government policies into unemployment.

The effects of a fiscal policy on the inflation rate are derived from the following equation: $\pi^* = \mu - \zeta^*$. Note that an increase in the tax rate will increase inflation if and only if it decreases the growth rate. Thus, in the examples presented above, when $\delta < 0$, the inflation rate increases with the tax rate and when $\delta > 0$, there exists a U-shaped curve relating the inflation rate with τ. Concerning the effects of monetary policy, note that an increase in the money growth rate makes inflation larger when it reduces the economic growth rate. Thus, in the examples presented above, if $\delta < 0$, the inflation rate increases with the money growth rate and if $\delta > 0$ it exhibits an ambiguous relationship.

Summarizing, the previous examples suggest that there is no conflict among maximizing employment and growth and minimizing inflation when $\delta < 0$. In this case, a contractive fiscal or monetary policy will decrease unemployment and inflation and will increase growth. This is not true when $\delta > 0$. In this case, an expansive monetary policy implies more employment but the effect on growth and inflation is ambiguous which means that it is possible to find situations where an expansive monetary policy implies less unemployment, more growth and less inflation.

Finally, we focus on the long-run Phillips curve obtained when there are changes in the bargaining power of workers and monetary policy, that is, on the relationship between unemployment and inflation in the long run due to changes in these two exogenous variables. Note that if the bargaining power of workers changes, we get a Phillips curve with a positive slope, i.e., more inflation and more unemployment. With respect to monetary policy, note that if δ is equal to zero then the inflation rate increases with the money growth rate and unemployment does not depend on the money growth rate. Thus, we find the classical vertical long run Phillips curve, that is, the same relationship that we find in an endogenous growth model with money and a competitive labor market. However, when δ is different from zero, monetary policy simultaneously affects both the inflation rate and unemployment. When $\delta < 0$, the examples presented above imply a positive relationship between inflation and unemployment and, hence, the Phillips curve has a positive slope. When $\delta > 0$, an expansive monetary policy implies less unemployment but the effect on inflation is ambiguous and, hence, the Phillips curve may have a positive or a negative slope. Concluding, in our model with a non competitive labor market, the long run Phillips curve obtained by means of changing the monetary policy is always vertical when there is no externality of labor over productive government spending. However, this verticality disappears when the externality is introduced.

6. Main Results

We have studied the equilibrium of an endogenous growth model of a monetary economy with unemployment due to the existence of unions. In this model, we have assumed that total factor productivity depends on a public input and that the services derived from this public input are affected by the total amount of labor, i.e., there is an externality of labor over productive government spending.

We have shown that if the externality is positive then an expansive monetary policy decreases unemployment. On the contrary, if the externality is negative then a contractive monetary policy or a decrease in the tax rates reduce unemployment. We also find cases where full employment requires an active monetary or fiscal policy and where, contrary to the result obtained in Barro (1990), an increase in the income tax does not increase the interest rate. An increase in the bargaining power of workers always increases unemployment and inflation and decreases growth.

When the externality is negative, the study of a simple case indicates that a contractive monetary policy or a decrease in taxes implies more growth and less inflation. When the externality is positive, we obtain cases with inverse U-shaped curves relating the economic growth rate with the money growth rate and with the tax rate. In this situation, it is possible to have situations where an expansive monetary policy implies more growth and less inflation. Also, in this case, we find examples where the monetary and fiscal policies that maximize growth are different from the ones obtained when the labor market is competitive. If there is no such externality, the model predicts a vertical long run Phillips curve due to monetary policy as the one obtained with a competitive labor market but this relationship disappears if the externality exists. Changes in the bargaining power of workers imply a Phillips curve with positive slope.

To conclude, the results obtained suggest that the observed cross-country differences among the unemployment, inflation and growth rates may not only be due to differences in the technology or in the unions bargaining power but they may also be due to differences in government policies.

Appendix

Proof of Theorem 4.3

In order to prove existence of an equilibrium, note that an equilibrium is a value of L for which the budget constraint crosses the employment equation. Next, realize that the budget constraint is a continuous function that takes positive and finite values for any value of L. The employment equation is a continuous function that takes values between zero and infinite when the domain of L belongs to the following interval: $\left(\min\left\{\widehat{\delta},\widehat{\beta}\right\},\max\left\{\widehat{\delta},\widehat{\beta}\right\}\right)$. The domain of L belongs to this interval when $\widehat{\delta}<1$ if $\delta>0$ and when $\widehat{\beta}<1$ if $\delta<0$. It follows that if these conditions hold then the budget constraint crosses the employment equation at least once. This proves existence. In order to prove unicity when $\delta<0$, note that the budget constraint is an increasing function and the employment equation is a decreasing function. This guarantees unicity when $\delta<0$. ■

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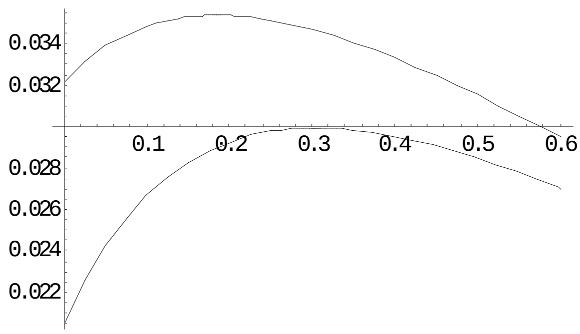
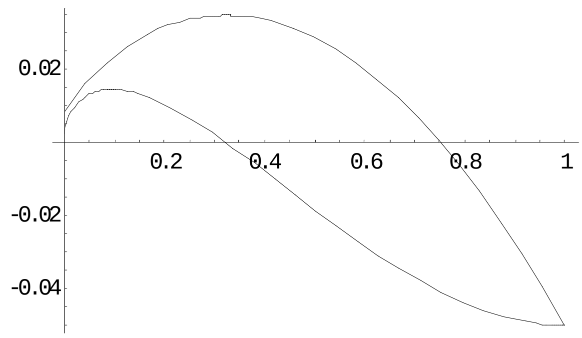
Figure 1



The upper line displays the growth rate as a function of the money growth rate when there is a competitive labor market. The lower line displays the growth rate as a function of the money growth rate when there is unemployment. The economy is characterized by the following parameter values: $\alpha = 0.4$, $\beta = 0.5$, $\delta = 0.25$, $\gamma = 0.5$, $\rho = 0.05$, $\phi = 1$, $A = 0.1$, $B = 0.7$, $v = 0.5$, $\varphi = 0.53$, $\tau = 0.25$.

Figure 2



The upper line displays the growth rate as a function of the tax rate when there is a competitive labor market. The lower line displays the growth rate as a function of the tax rate when there is unemployment. The economy is characterized by the following parameter values: $\alpha = 0.4$, $\beta = 0.5$, $\delta = 0.25$, $\gamma = 0.5$, $\rho = 0.05$, $\phi = 1$, $A = 0.1$, $B = 0.7$, $v = 0.5$, $\varphi = 0.6$, $\mu = 0.05$.