Growth, Unemployment and the Wage Setting Process

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Abstract

We analyze, using an standard OLG model with a non competitive labour market, under which conditions a lower unemployment rate is associated with a higher unemployment benefit. We also study the dynamics of growth and unemployment and we check if there is no relationship between growth and unemployment in the long run. The main results are: 1) If the government cares sufficiently about unemployed workers, then a lower unemployment rate is associated with a higher unemployment benefit. 2) The relationship between growth and unemployment in the long run is weak in the sense that only the rate of growth of productivity from all the parameters of the model affects both in the long run when in wage setting process past wages and the present unemployment benefit are taken into account.

Abstract

Analizamos, con un modelo de generaciones solapadas standard y mercado de trabajo no competitivo, bajo que condiciones una tasa de paro mas baja esta asociada con un subsidio de paro mas alto. Tambien estudiamos la dinamica del crecimiento y el paro y comprobamos si en el largo plazo estan relacionados. Los resultados principales son: 1) Si el gobierno se preocupa lo suficientemente de los trabajadores desempleados una tasa de paro mas baja esta asociada con un subsidio de paro mas alto. 2) La relacion entre crecimiento y paro en el largo plazo es debil en el sentido de que solo la tasa de crecimiento de la productividad, entre todos los parametros del modelo, afecta a las dos variables en el largo plazo, cuando en el proceso de determinacion de salarios se tienen en cuenta los salarios pasados y presentes.

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1 Introduccion

The prevailing paradigm about the long run relationship between growth an unemployment in the seventies comes from two independent models. On the one hand, the Friedman-Phelps model which says that the economy in the long run converges to a natural rate of unemployment which depends on the institutional characteristics of the labour market. On the other hand, any of the growth models of the decade (Solow, Infinite Horizon or OLG) with perfect competition in the labor market, which say that, with an exogenous constant rate of labour aumenting technological progress, the long run rate of growth depends on the the exogenous rate of population growth and the exogenous rate of technological progress. According to these results one may think that there is no relationship between growth and unemployment in the long run, but this is an unwarranted conclusion because these are two different models and the definition of long run is also different.

In the eighties and the nineties this relationship has been analyzed by diferent authors. Pissarides [8] in chapter 2 presents a model with matching frictions in the labour market where growth, via the creation of more vacancies, has a long run positive effect on employment (the capitalization effect), that is, a negative relationship between growth and unemployment in the long run. Aghion and Howitt [1] add to this positive effect of growth on employment a negative one arguing that growth, via the increase in productivity, will increase the inflow rate into unemployment (the reallocation effect). Considering together these two effects they obtain a hump-shaped relationship between growth and unemployment in the long run.

Bean and Pissarides [2], using an OLG model with also a labour market with matching frictions and with a technology capable of yielding endogenous unbounded growth (Romer [10]), analyze how the rate of growth and the rate of unemployment in the short run change when there is a change in hiring costs, in taxes, in the propensity to consume and in the relative bargaining strengh of workers. This model ignores these causality effects of growth on unemployment and growth and unemployment are jointly determined endogenous variables that depend on exogenous variables. The results are that a reduction in hiring cost and a decreases in taxes implies more growth and less unemployment, an increase in the propensity to consume lowers the rate of growth and leaves unemployment unaffected and an increase in the bargaining strenght of workers has ambiguos effect on growth and unemployment. Using the same technology but in a union-monopoly model (McDonald and Solow [6]) Daveri and Tabellini [3] get that an increase in labour taxes implies less growth and more unemployment.

In the model presented in this paper we also follow the union-monopoly model but, different from the Daveri and Tabellini's paper, we endogenize the behaviour of the government which allows to study a comparative static question not analyzed in the literature: how a government that cares more about employed workers affects the equilibrium of the economy and, consequently, under which conditions there is lower unemployment with a higher unemployment benefit. On the other hand, we do not take into account the aggregate capital externality of the Romer's model (Romer [10]) and we allow for a constant exogenous rate of productivity in order to clarify its effect on the long run rate of growth and unemployment. Finally, we introduce the possibility of taking into account past wages in the wage setting process and, as we will see, this has a significative effect on the long run rate of growth and unemployment. This is one of the main contributions of this paper. This purpouse has determined our option for the union-monopoly model instead of the matching model, more suitable for analizing the effect of the destruction of jobs due to the technological progress on growth and unemployment.

That a higher exogenous unemployment benefit is associated with more unemployment is one of the standard results of partial equilibrium static models with unions (see Oswald [7] and Layard, Nickell and Jackman [5]). Sorolla-i-Amat [11] presents a general equilibrium static model where a higher exogenous unemployment benefit implies more unemployment. In Sorolla-i-Amat [12] the same result is presented using an infinite horizon model. In the present paper the unemployment benefit is endogenous. On the one hand, it is true that, if the government increases the unemployment benefit, the union asks for a higher wage and there is more unemployment, but, on the other hand, if there is less unemployment, then the government can pay a higher unemployment benefit and the final relationship depends on both effects.

There are other papers that study similar questions using dynamic models with wage setting. Przeworski and Wallerstein [9] and Sorolla-i-Amat [12] analyze with an infinite horizon model what is the effect of a more proworker government on the unemployment level. The main difference between both papers is that in Sorolla-i-Amat's paper the economy is a market economy.

The paper is organized as follows: in section 2 we present the individual agents: firms and consumers and we define the market equilibrium for a given wage. Section 3 presents the collective agents: the union and the government. Section 4 defines the Nash equilibrium of the economy and shows under which conditions a higher unemployment benefit is associated with a lower unemployment rate. Section 5 analyzes the behaviour of wages, growth and unemployment in two special cases: In the first case, the union takes into account only previous wages in the wage setting process. In the second case only present variables are considered by the union. Section 6 studies the long run rate of growth and unemployment when the union weights the previous wage and the present unemployment benefit when setting the wage. Last section is a summary of the results. All proofs are contained in the appendix.

2 The Individual Agents

2.1 The Firm

There is only one firm with Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha};$$

where $0 < \alpha < 1$, Y_t is output in period t, K_t is the capital stock in period t, L_t is employment in period t, A_t is "knowlegde" or the "productivity of labour" in period t and $t = 0, 1, 2, ...$ We assume that A_t changes according to the equation $A_{t+1} = (1 + g)A_t$ where g is the rate of growth of productivity. Output and capital are the same produced good, and capital in period t is the output saved in period t-1. For simplicity we assume complete depreciation of the stock of capital. The real wage in period t is denoted by w_t and the rate of interest in period t by R_t. The firm maximizes profits every period. The formal program is the following:

Program F_t: Given w_t, R_t, choose K_t and L_t in order to maximize:

$$K_t^{\alpha}(A_t L_t)^{1-\alpha} - (1 + R_t)K_t - w_t L_t.$$

As it is well known, the zero profits condition implies a relationship between the rate of interest and the real wage given by:

$$1 + R_t = \tilde{R}(w_t) = \alpha[\frac{w_t}{(1-\alpha)A_t}]^{\frac{\alpha-1}{\alpha}}. \tag{1}$$

If the zero profits condition holds then the relationship between the demand of capital, K_t^d, and the demand of labor, L_t^d, is given by the optimal capital/effective labor ratio function $k_t^d = \frac{K_t^d}{A_t L_t^d}$ and, if the production function is Cobb-Douglas, k_t^d is given by:

$$k_t^d = \tilde{k}(w_t) = [\frac{w_t}{(1-\alpha)A_t}]^{\frac{1}{\alpha}}; \tag{2}$$

The function $\tilde{R}(w_t)$ is called the factor-price frontier (see Diamond [4]). The function $\tilde{k}(w_t)$ gives the optimal capital/effective labor ratio for a given wage. It is easy to check that $\tilde{R}' < 0$ and $\tilde{k}' > 0$, which means that if the wage increases the rate of interest decreases and the capital effective labor ratio increases, i.e, the technology becomes mores capital intensive. We call to the term $\frac{w_t}{A_t}$ the wage per unit of effective labour and we denote it by ω_t. The output supply in period t is denoted by Y_t^s.

2.2 The Individuals

We follow an standard overlappping generation model (Diamond [4]). There are N_t individuals born in period t. Population growths at rate n, thus $N_t = (1+n)N_{t-1}$, individuals live for two periods and, then, at time t there are N_t individuals in the first period of their lives and N_{t-1} individuals in the second period of their lives.

When young, each individual supplies inellastically one unit of labor and divides his income, $I_{1,t}$, between consumption and saving. When old, the individual consumes the saving and any interest he or she earns. In period zero a generation of old people provides, as saving, the quantity of K_0 units of capital. We denote the consumption in period t of young and old individuals as $C_{1,t}$ and $C_{2,t}$ respectively, thus, the utility of an individual born in period t, U_t depends on $C_{1,t}$ and $C_{2,t+1}$. The saving in period t of a young individual is denoted by $S_{1,t}$. The program of an individual born in period t is the following:

Program C_t: Given w_t and R_{t+1}, choose $C_{1,t}$, $S_{1,t}$ and $C_{2,t+1}$ in order to maximize:

$$U_t(C_{1,t}, C_{2,t+1}) = \log C_{1,t} + \frac{1}{1+\rho} \log C_{2,t+1};$$

subject to:

$$C_{1,t} + S_{1,t} = I_{1,t};$$
$$C_{2,t+1} = (1 + R_{t+1})S_{1,t}.$$

The solution to program Program C_t gives the optimal demand function of good in period t and period $t+1$ and the optimal supply of saving. With a logarithmic utility function the optimal saving does not depen on R_{t+1} and the saving function, $\tilde{S}^s_{1,t}(I_t)$, is:

$$\tilde{S}^s_{1,t}(I_t) = \frac{I_{1,t}}{2+\rho} = cI_{1,t}; \tag{3}$$

where $c = \frac{1}{2+\rho}$ is the saving's propensity.

We assume that an individual born in period t, who supplies one unit of labor, can be either employed or unemployed. If employed, his income I_t is equal to the net real wage after taxes $(1-\tau_t)w_t$, where τ_t is the tax rate on an employed worker in period t. If unemployed, his income is equal to the unemployment benefit in period t s_t. The saving of an employed in period t, $\tilde{S}^s_{w,t}((1-\tau_t)w_t)$, and the saving of an unemployed, $\tilde{S}^s_{u,t}(s_t)$, are given by:

$$\tilde{S}^s_{w,t}((1-\tau_t)w_t) = c(1-\tau_t)w_t \; ; \tilde{S}^s_{u,t}(s_t) = cs_t. \tag{4}$$

Old people are not taxed, thus the consumption of an old who was employed in period t is $(1+R_{t+1})c(1-\tau_t)w_t$ and the consumption of an old unemployed in period t is $(1+R_{t+1})cs_t$.

2.3 Equilibrium in the Input Markets

Now we define the effective quantity of labor employed in period t as $L_t^e = min(L_t^d, N_t)$. Then unemployment in period t is given by $N_t - L_t^e$ and the rate of employment in period t, u_t, by $\frac{N_t - L_t^e}{N_t}$.

We define the aggregate saving supply in period t-1, S_{t-1}^s, as:

$$S_{t-1}^s = \tilde{S}_{w,t-1}^s((1-\tau_{t-1})w_{t-1})L_{t-1}^e + \tilde{S}_{u,t-1}^s(s_{t-1})(N_{t-1} - L_{t-1}^e). \tag{5}$$

Recall that aggregate saving in period t-1 is equal to the supply of capital in period t, K_t^s. Thus, we have $K_0^s = K_0$ and $K_t^s = S_{t-1}^s$ for all $t \geq 1$.

Using (4) and (5) we have that for all $t \geq 1$:

$$\begin{array}{c} K_t^s = \tilde{K}_t^s(w_{t-1}, \tau_{t-1}, s_{t-1}, L_{t-1}^e, N_{t-1}) = \\ c((1-\tau_{t-1})w_{t-1}L_{t-1}^e + s_{t-1}(N_{t-1} - L_{t-1}^e)) \quad . \end{array} \tag{6}$$

Equation (6) says that the supply of capital in period t, that is, the supply of saving in period t-1, is equal to aggregate labour income times the saving's propensity.

We assume that the interest rate in period t, R_t^*, balances demand and supply in at least one of the inputs markets, that is:

Definition 2.3.1 *Given w_{t-1}, L_{t-1}^e and w_t; R_t^* is such that:*

$$K_t^d = \tilde{K}_t^s(w_{t-1}, \tau_{t-1}, s_{t-1}, L_{t-1}^e, N_{t-1}) \tag{7}$$

and

$$L_t^d \leq N_t; \tag{8}$$

or

$$L_t^d = N_t \tag{9}$$

and

$$K_t^d \leq \tilde{K}_t^s(w_{t-1}, \tau_{t-1}, s_{t-1}, L_{t-1}^e, N_{t-1}). \tag{10}$$

Equations (7) and (8) say that the capital market is in equilibrium and that there is an excess supply of labor. Equations (9) and (10) that the labor market is in equilibrium and that there is an excess supply of capital.

The equilibrium interest rate, R_t^*, the competitive wage, w_t^c, the inputs' demand functions and the effective quantity of labor employed are given by the following proposition:

Proposition 2.3.1 *Given* w_{t-1}, τ_{t-1}, s_{t-1}, L^e_{t-1}, N_{t-1} *and* w_t; $1 + R^*_t = \tilde{R}(w_t)$. *Moreover there exists* $w^c_t = (1-\alpha)(A_t)^{1-\alpha}[\frac{S^s_{t-1}}{N_t}]^\alpha$ *such that: If* $w_t > w^c_t$ *then:*

$$K^d_t = \tilde{K}^s_t(.) = S^s_{t-1} \tag{11}$$

and

$$L^e_t = L^d_t = \frac{K^d_t}{A_t\tilde{k}(w_t)} = \frac{A_t^{\frac{1-\alpha}{\alpha}} S^s_{t-1}}{[\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}}} < N_t. \tag{12}$$

If $w_t = w^c_t$ *then:*

$$K^d_t = \tilde{K}^s_t(w_{t-1}, L^e_{t-1}) = S^s_{t-1} \tag{13}$$

and

$$L^e_t = L^d_t = N_t. \tag{14}$$

If $w_t < w^c_t$ *then:*

$$L^e_t = L^d_t = N_t \tag{15}$$

and

$$K^d_t = A_t L^d_t \tilde{k}(w_t) = \frac{N_t}{A_t^{\frac{1-\alpha}{\alpha}}}[\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}} < \tilde{K}^s_t(.). \tag{16}$$

Proposition 2.3.1 says that the equilibrium interest rate is given by the zero profits condition $1 + R^*_t = \tilde{R}(w_t)$. Moreover, if the wage is greater than the competitive wage, the capital market is in equilibrium, the effective quantity of labor employed is given by the labor demand (12) and there is always unemployment. In this case, if the wage in period t increases then the labor demand in period t decreases. This may seem strange because we are in the particular case in which saving do not depend on the interest rate. The explanation is that an increase in the wage switches to a more capital intensive technique and labor is reduced. This fact can be seen in equation (12) where K^d_t does not depend on R_t but if w_t increases then $\tilde{k}(w_t)$ decreases. Note also that the elasticity labor demand is $-\frac{1}{\alpha} < -1$, and, thus, the wage bill before taxes $w_t L^d_t(.)$ decreases when w_t increases.

If the capital market is in equilibrium one can show that the output market is in equilibrium and then aggregate output in period t, Y_t, is given by:

$$Y_t = Y^s_t = \frac{S^s_{t-1}}{[\frac{w_t}{A_t(1-\alpha)}]^{\frac{1-\alpha}{\alpha}}} = \frac{A_t^{\frac{1-\alpha}{\alpha}} S^s_{t-1}}{[\frac{w_t}{1-\alpha}]^{\frac{1-\alpha}{\alpha}}}. \tag{17}$$

In this case the elasticity of output with respect to the wage is $-\frac{1-\alpha}{\alpha}$, that is, in order to increase aggregate employment by the percentage of $\frac{1}{\alpha}$ aggregate output must increase by the percentage of $\frac{1-\alpha}{\alpha}$ (Okun's Law).

Finally, if the wage is less than the competitive wage, the labor market is in equilibrium and there is an excess supply of capital. In this last case, we have an excess supply of agregate savings, which means that young individuals must save less and consume more. Thus, in order to be rigurous, we should specify how to raction saving in this situation. Nevertheless, we omit this point because, as we will see in the next section, this situation never holds.

3 The Collective Agents

3.1 The Union

In each period t there is one union that sets the wage taking into account that it will affect the effective quantity of labor employed in the way described by proposition 2.3.1. We assume that the union cares about employment and compares the wage in period t with the wage obtained in period t-1 and with the unemployment benefit obtained in period t. These assumptions are formalized by the following program:

Program U_t: Given w_{t-1}, τ_{t-1}, s_{t-1}, L^e_{t-1}, N_{t-1}, τ_t and s_t choose, w_t in order to maximize:

$$S_t = ((1-\tau_t)w_t - \gamma_t(1-\tau_{t-1})w_{t-1} - (1-\gamma_t)s_t)L^e_t.$$

Note from the objective function of the union, S_t, that the higher the wage after taxes in period t, the higher the welfare of the union. The parameter γ_t weights the way in which the wage after taxes of the previous period and the unemployment benefit are compared with the wage after taxes of the period. We assume that $0 \leq \gamma_t \leq 1$. If $\gamma_t = 1$ the union compares the wage after taxes of the period only with the wage after taxes of the previous period. If $\gamma_t = 0$ the union compares the wage after taxes of the period only with the unemployment benefit. When γ_t increases the unemployment benefit weights less in the comparison. The effective quantity of labor employed appears as a factor in S_t meaning that if employment increases, the welfare of the union also increases.

The motivation for this apparently ad hoc utility function is the following: In the two extreme cases the union compares the wage after taxes of the period either with the wage after taxes of the previous period or with the unemployment benefit of the period. The assumption that $0 \leq \gamma_t \leq 1$ is because we think that both parameters play an important role in the wage setting process. In other words, we think the when unions (workers) set wages they take into account past wages and the current unemployment benefit. We use the simplest utility funtion that reflects this comparison where γ_t gives the weight given to past wages and the unemployment benefit. Note also that when $\gamma_t = 1$ the union maximizes the increase in wages after taxes of all employed workers and when $\gamma_t = 0$ the union maximizes the wedge times employment. The solution obtained in this last case is the same than the solution obtained if the union maximizes either the expected income of the worker or the sum of all workers income, which are the most common utility functions used for unions. The solution to Program U_t is given by the following proposition:

Proposition 3.1.1 *There exists $\bar{w}_t \geq 0$ such that if $w_t^c < \bar{w}_t$ then*

$$w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1} + (1-\gamma_t)s_t}{(1-\alpha)(1-\tau_t)} > w_t^c. \tag{18}$$

If $w_t^c \geq \bar{w}_t$ and $\gamma_t = 1$ then $w_t = w_t^c$. If $w_t^c \geq \bar{w}_t$ and $0 \leq \gamma_t < 1$ then there exists $\bar{s}_t \geq 0$ such that if $s_t > \bar{s}_t$ then

$$w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1} + (1-\gamma_t)s_t}{(1-\alpha)(1-\tau_t)} > w_t^c; \tag{19}$$

and if $s_t \leq \bar{s}_t$ then $w_t = w_t^c$.

Proposition 3.1.1 gives the best reply function of the union which is denoted by: $\tilde{w}_t(w_{t-1}, \tau_{t-1}, \tau_t, s_t, \gamma_t)$. Proposition 3.1.1 says that if the competitive wage is low enough then the union always sets a wage that implies unemployment. If the competitive wage is high and $\gamma_t = 1$ the union always sets the competitive wage. If the competitive wage is high and $0 \leq \gamma_t < 1$, then there are low values of the unemployment benefit where the union sets the competitive wage. It is easy to check from equations (18) and (19) that, if there is unemployment, $\tilde{w}_t$ is increasing in τ_t and when $0 \leq \gamma_t < 1$ it is increasing in s_t. Note also that if $\gamma_t = 1$ and $\tau_{t-1} = \tau_t$ then $w_t = \frac{w_{t-1}}{(1-\alpha)}$ that is, the growth rate of the real wage is given by: $\frac{1}{(1-\alpha)} - 1$. If $\gamma_t = 0$ then $w_t = \frac{s_t}{(1-\alpha)(1-\tau_t)}$. In this last case if $s_t = (1-\alpha)(1-\tau_t)w_{t-1}$ then we have $w_t = w_{t-1}$, that is, real wage rigidity along time. We have also real wage rigidity in this case if s_t and τ_t are constant over time.

3.2 The Government

We assume that if there is unemployment in period t the government sets the tax rate and the unemployment benefit in order to maximize a utility function that depends on the income of employed and unemployed workers. We assume also that the government balances its budget at period t.

The formal program of the government is the following:

Program G_t: Given $w_t > w_t^c$ and L_t^e, choose τ_t and s_t in order to maximize:

$$G_t = ((1-\tau_t)w_t L_t^e)^{\beta_t}(s_t(N_t - L_t^e))^{1-\beta_t},$$

subject to:

$$s_t(N_t - L_t^e) = \tau_t w_t L_t^e.$$

We assume, then, that the government maximizes a social welfare function that depends on the income of people affected by its actions. This utility function is similar to the used in the standard social planner program where the social planner maximizes also the utility of a representative agent. Because in this model we have heterogeneity of workers (employed and unemployed) the income of both appears in the utility function. We use as argument income instead of utility (log of income) because it simplies the solution. The parameter β_t weights the welfare of employed and unemployed workers in the utility function of the government. If β_t increases we say that the government becomes more proemployed workers. We assume that $0 \leq \beta_t \leq 1$. Note that the income of the old people in period t, $(1 + R_t)cw_{t-1}L^e_{t-1}$, do not appear in the utility function of the government. This is because we assume that the government only cares for the welfare of the people affected by its actions when there is unemployment. However, one can check, that the solution of the government's program will not change if we introduce this term as a factor.

The approach of the social welfare function for the government with different weights has been used in other models with heterogeinity of agents (Przeworski and Wallerstein [9]). The other alternative standard approach, not followed in this paper, is to assume that the government represents the median voter.

It is easy to check that the solution to Program G_t is given by:

$$\tau_t = 1 - \beta_t, \tag{20}$$

and

$$s_t = \frac{(1 - \beta_t)w_t L^e_t}{N_t - L^e_t}. \tag{21}$$

If there is full employment the government does nothing, which means $\tau_t = 0$ and $s_t = 0$.

Note that the government decides both the tax rate and the unemployment benefit and its optimal choice implies a constant tax rate and the amount of the unemployment benefit changes depending on the amount of employment. How the unemployment benefit is set is really important and it will affect the dynamics of the model. Other assumptions used in the literature is to set the unemployment benefit proportional to wages ($z = \lambda w$, Pissarides [8]) or proportional to aggregate output ($s = \sigma y$, Daveri and Tabellini [3]).

The behaviour of the government is expressed by the best reply functions of the government, $\tilde{\tau}_t(w_t, \beta_t)$ and $\tilde{s}_t(w_t, \beta_t)$. Note that, if there is unemployment, the taxe rate does not depend on the wage and that it is decreasing in β. The variable decided by the government that depend on the wage is the unemployment benefit. It is easy to check that $\tilde{s}_t$ is decreasing in w_t and β_t. Note also, from (21), that when there is few unemployment, the unemployment benefit is really big, thus, it can be the case, that the real wage after taxes is less than the unemployment benefit. This unpleasant characteristic can be eliminated introducing in the program of the government the constraint $(1-\tau_t)w_t \geq s_t$. For simplicity when proving the existence of equilibrium in the next section, we do not introduce this contraint in program G_t.

If the government has a balanced budget constraint and the wage in period t is greater than the competitive wage then, using (12) the effective quantity of labor employed is given by:

$$L_t^e = \frac{cw_{t-1}L_{t-1}^e}{A_t[\frac{w_t}{A_t(1-\alpha)}]^{\frac{1}{\alpha}}} = \frac{c(1-\alpha)^{\frac{1}{\alpha}}(\frac{w_{t-1}}{A_{t-1}})L_{t-1}^e}{(1+g)(\frac{w_t}{A_t})^{\frac{1}{\alpha}}} = \frac{A_t^{\frac{1-\alpha}{\alpha}}cw_{t-1}L_{t-1}^e}{[\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}}} < N_t. \quad (22)$$

If there is unemployment in periods $t-1$ and t it is easy to compute that the rate of growth output in period t, $y_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$, the rate of growth of employment in period t, $l_t = \frac{L_t^e - L_{t-1}^e}{L_{t-1}^e}$ and the rate of unemployment, u_t, are given by:

$$y_t = \frac{c(1-\alpha)^{\frac{1}{\alpha}}}{(\frac{w_t}{A_t})^{\frac{1-\alpha}{\alpha}}} - 1 = \frac{cA_t^{\frac{1-\alpha}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}}{w_t^{\frac{1-\alpha}{\alpha}}} - 1. \quad (23)$$

$$l_t = \frac{c(1-\alpha)^{\frac{1}{\alpha}}(\frac{w_{t-1}}{A_{t-1}})}{(1+g)(\frac{w_t}{A_t})^{\frac{1}{\alpha}}} - 1 = \frac{cA_t^{\frac{1-\alpha}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}w_{t-1}}{w_t^{\frac{1}{\alpha}}} - 1. \quad (24)$$

$$u_t = 1 - \frac{A_t^{\frac{1-\alpha}{\alpha}}cw_{t-1}L_{t-1}^e}{N_t[\frac{w_t}{(1-\alpha)}]^{\frac{1}{\alpha}}}. \quad (25)$$

Note that, from equations (23),(24) and (25), if w_t increases then y_t and l_t decrease and u_t increases. Note in fact that the evolution of the output growth rate is given by the evolution of the wage per unit of effective labor. If the wage per unit of effective labor increases along time the rate of growth of output decreases. Note also that if the saving propensity increases, both the output growth rate and the unemployment rate decreases. Note, finally, that an increase in the supply of labor, N_t, increases the unemployment rate but, if there is unemployment, do not affect the rate of growth of output.

4 Nash Equilibrium

Definition 4.1 *Given w_{t-1}, τ_{t-1}, γ_t, and β_t; w_t^*, τ_t^* and s_t^* is an Nash Equilibrium if*

$$w_t^* = \tilde{w}_t(w_{t-1}, \tau_{t-1}, \tau_t^*, s_t^*, \gamma_t); \quad (26)$$

$$\tau_t^* = \tilde{\tau}_t(w_t^*, \beta_t); \quad (27)$$

$$s_t^* = \tilde{s}_t(w_t^*, \beta_t). \quad (28)$$

Theorem 4.1 *There exists w'_t and $w"_t$ such that if $w^c_t < w'_t$ then an equilibrium with unemployment always exists and it is unique.*

If $\gamma_t = 1$ and $w'_t \leq w^c_t < w"_t$ there exists an equilibrium with unemployment and an equilibrium with full employment, if $w"_t \leq w^c_t$ there exists a unique equilibrium with full employment.

If $0 \leq \gamma_t < 1$ and $w'_t \leq w^c_t$ there exists an equilibrium with unemployment and an equilibrium with full employment.

The representation of these situations are given by Figures 1.a, 1.b and 2.a, 2.b and 2.c. We denote the equilibrium with unemployment as $w^*_{t,u} = \hat{w}_t(w_{t-1}, \tau_{t-1}, \gamma_t, \beta_t)$, $\tau^*_{t,u} = \tau_t(\beta_t)$, and $s^*_{t,u} = \hat{s}_t(w_{t-1}, \tau_{t-1}, \gamma_t, \beta_t)$. The special case $\gamma_t = 1$ is analyzed in the next section. When $0 \leq \gamma_t < 1$ we can prove the following theorem.

Theorem 4.2 *Comparative Statics. The sign of the equilibrium partial derivatives is given by the following table:*

If $u_t < \frac{1-\beta_t}{\alpha}$ then:

$\frac{\partial \hat{w}_t}{\partial \beta_t} < 0 \quad \frac{\partial \tilde{\tau}_t}{\partial \beta_t} < 0 \quad \frac{\partial \hat{s}_t}{\partial \beta_t} > 0$.

If $u_t > \frac{1-\beta_t}{\alpha}$ then:

$\frac{\partial \hat{w}_t}{\partial \beta_t} < 0 \quad \frac{\partial \tilde{\tau}_t}{\partial \beta_t} < 0 \quad \frac{\partial \hat{s}_t}{\partial \beta_t} < 0$.

If $(1-\tau_t)w_{t-1} > s_t$ then:

$\frac{\partial \hat{w}_t}{\partial \gamma_t} > 0 \quad \frac{\partial \tilde{\tau}_t}{\partial \gamma_t} = 0 \quad \frac{\partial \hat{s}_t}{\partial \gamma_t} < 0$.

If $(1-\tau_t)w_{t-1} < s_t$ then:

$\frac{\partial \hat{w}_t}{\partial \gamma_t} < 0 \quad \frac{\partial \tilde{\tau}_t}{\partial \gamma_t} = 0 \quad \frac{\partial \hat{s}_t}{\partial \gamma_t} > 0$.

The interpretation of Theorem 4.2 is the following. First, the more proemployed workers the government the lower the equilibrium wage and, by (23) and (25), the higher the output growth rate and the lower the unemployment rate. Second, if the unemployment rate is low enough, $u_t < \frac{1-\beta_t}{\alpha}$, then the more proemployed workers the government the higher the unemployment benefit, that is, we will have more employment and a higher unemployment benefit with a more proemployed workers government. Note that the unemployment rate depends on β_t, this means that, in principle, we can not say when this situation happens. A sufficient condition is to have $\frac{1-\beta_t}{\alpha} > 1$, that is, $\beta_t < 1 - \alpha$. We can interpret this result as follows: if the government is really prounemployed workers and it becomes more proemployed workers the economy will have less unemployment and a higher unemployment benefit, that is, if the government cares sufficiently about unemployed workers a lower unemployment rate is associated with a higher unemployment benefit. Finally, if the real wage after taxes of the previous period is greater that the unemployment benefit of the period the higher the union cares about the wage of the previous period the higher the equilibrium wage rate and the unemployment rate and the lower the growth rate and the unemployment benefit.

5 Special Cases

In general it is not possible to compute the equilibrium $w^*_{t,u}$, $\tau^*_{t,u}$ and $s^*_{t,u}$ except in the special cases: $\gamma_t = 1$ and $\beta_t = \beta$ for all t, and $\gamma_t = 0, \beta_t = \beta$ for all t. These two extreme cases have an easy interpretation. In the first case, $\gamma_t = 1$ and $\beta_t = \beta$ for all t, the union only takes into account the wage after taxes of the previous period, when setting the wage. In the second case the union only takes into account the unemployment benefit of the period. In both cases the government gives always the same weight to the welfare of employed workers.

When $\gamma_t = 1$ and $\beta_t = \beta$ for all t we can prove the following proposition:

Proposition 5.1 *There exists $\bar{N}_t$ such that if $N_t > (\leq)\bar{N}_t$ then there exists a unique equilibrium with unemployment (full employment) in period t and $\frac{w^*_t - w^*_{t-1}}{w^*_{t-1}} = \frac{\alpha}{1-\alpha}$. If there is unemployment in each period t ($N_t > \bar{N}_t$ for all t) and $g < \frac{\alpha}{1-\alpha}$, the rate of growth of output, y_t, and the rate of growth of employment, l_t, decrease with time and there is some period t' such that for all $t \geq t'$ the rate of unemployment, u_t, increases. If $g = \frac{\alpha}{1-\alpha}$, y_t and l_t are constant. If $l_t < n$ then u_t increases with time. If $l_t = n$ then u_t is constant. If $l_t > n$ then u_t decreases. If $g > \frac{\alpha}{1-\alpha}$, y_t and l_t increase with time and there is some period t" such that for all $t \geq t'$ the rate of unemployment, u_t, decreases.*

Proposition 5.1 says that if the rate of growth of wages is less than the rate of productivity growth then there is an increase in the rates of growth of output and employment and the rate of unemployment decreases after some time. This condition implies that the rate of growth of the wage per unit of effective labour is negative, that is, the wage per unit of effective labor decreases with time. The bad situation is when the rate of growth of wages is greater than the rate of productivity growth, i. e. when the wage per unit of effective labour increases, in this case growth decreases and unemployment increases. From the proof of proposition 5.1 it is easy to see that we can have situations with a positive rate of growth of output and a negative rate of growth of employment.

When $\gamma_t = 0$ and $\beta_t = \beta$ for all t, we can prove the following proposition:

Proposition 5.2 *If $\gamma_t = 0$ and $\beta_t = \beta$ for all t there always exists an equilibrium with unemployment and an equilibrium with full employment. The unemployment rate at the equilibrium with unemployment is the same for all t and is given by $u_t = \frac{1-\beta}{1-\alpha\beta}$. Moreover if there is always unemployment, the wage per unit of effective labour, $\omega^*_t = \frac{w_t}{A_t}$, and the unemployment benefit per unit of effective labour, $\sigma^*_t = \frac{s_t}{A_t}$, converge to a long run wage and unemployment benefit per unit of effective labour, $\omega^*_{L-R} = (1-\alpha)^{\frac{1}{1-\alpha}}(\frac{c}{(1+n)(1+g)})^{\frac{\alpha}{1-\alpha}}$ and $\sigma^*_{L-R} = \beta(1-\alpha)^{\frac{2-\alpha}{1-\alpha}}(\frac{c}{(1+n)(1+g)})^{\frac{\alpha}{1-\alpha}}$, the long run rate of growth of output is equal to $(1+n)(1+g)-1$ and the long run rate of growth of employment is equal to n.*

The interpretation of proposition 5.2 is, if the union only cares about the unemployment benefit of the period, β remains constant along time and there is always unemployment, the unemployment rate remains constant along time and depends only on α and β. Note that the higher the β the lower the unemployment rate. The intuition for this result comes from the proof of 5.2. In this case the wage is proportional to the unemployment benefit and the unemployment benefit is proportional to the wage, which means the same equation in terms of the rate of unemployment for all t.

Moreover, if the wage and the unemployment benefit per unit of effective labour at period zero are lower (higher) than the long run wage and the unemployment benefit per unit of effective labour they increase (decrease) along time. The rate of growth of wages and the unemployment benefit in the long run is equal to g. The long-run rate of growth is equal to $(1+n)(1+g)-1$ and the long run rate of unemployment is equal to $\frac{1-\beta}{1-\alpha\beta}$, that is, there is no relationship between growth and unemployment in the long run. Note that in this specific OLG model when the labour market is competitive the long-run rate of growth is also equal to $(1+n)(1+g)-1$. This is not surprising because the constant rate of unemployment implies that the effective quantity of labour employed growths at the constant rate n and, then, the analysis of the model is identically to the case of perfect competition.

Thus, the behaviour of the economic variables of this OLG model along time when the labour market is not competitive really depends on the specific wage setting. We want also to emphasize that the specific behaviour of the government is also important for the results obtained. Let's now, alternativelly, assume that the government is commited to pay a given unemployment benefit s' constant along time and this quantity is provided taxing both employed and unemployed workers at the same tax rate τ_t. In this case the government has no choice, given the wage and the unemployment benefit, and, it only determines τ_t in order to balance its budget. On the other hand, the utility function of the union, if the unemployed workers are also taxed, changes to

$$S'_t = ((1-\tau_t)w_t - \gamma_t(1-\tau_{t-1})w_{t-1} - (1-\gamma_t)(1-\tau_t)s')L^e_t.$$

If $\gamma_t = 0$ for all t, it is easy to check, using the proof of proposition 3.1.1, that, if there is unemployment, $w_t = \frac{s'}{(1-\alpha)}$, for all t, that is, there is real wage rigidity along time. It is also easy to check, using equations (23) and (24), that the rates of growth of output and employment increase.

Note then that, depending on the assumptions about the specific wage setting process and the behaviour of the government, we have obtained the following special cases: 1) Constant rate of growth of the real wage with rates of growth of output and employment increasing or decreasing depending on the rate of growth of productivity. 2) Constant rate of unemployment with real wage per unit of effective labour increasing or decreasing to a long run wage and with the rate of growth of output increasing or decreasing to $(1+n)(1+g)$ (a case similar to the same OLG model with perfect competition in the labor market). 3) Real wage rigidity with increasing rates of growth of output and employment with time. These special cases show that the dynamics of this OLG model with a non competitive labour market may be very different depending on the assumptions about the specific wage setting process and the behaviour of the government.

6 The Case $0 < \gamma < 1$.

If $\gamma_t = \gamma$, $\beta_t = \beta$ for all t, $0 < \gamma < 1$ and we assume that there is always unemployment substituting (21) in (18) on the one hand and taking equation (22) on the other we have the equilibrium wage and the effective quantity of labour employed are given by the following system of non linear first order difference equations:

$$w_t^* = \frac{\gamma}{(1-\alpha)} w_{t-1}^* + \frac{1-\gamma}{(1-\alpha)\beta} \frac{(1-\beta) w_t^* L_t^e}{N_t - L_t^e} \tag{29}$$

$$L_t^e = \frac{A_t^{\frac{1-\alpha}{\alpha}} c w_{t-1}^* L_{t-1}^e}{[\frac{w_t^*}{1-\alpha}]^{\frac{1}{\alpha}}}. \tag{30}$$

Where the endogenous variables are L_t^e and w_t^*.

We can transform this system into the following non-linear first order system of two difference equations in terms of the wage per unit of effective labour and the rate of unemployment.

$$\omega_t^* = \frac{\gamma}{(1-\alpha)(1+g)} \omega_{t-1}^* + \frac{(1-\gamma)(1-\beta)}{(1-\alpha)\beta} \omega_t^* [\frac{1}{u_t} - 1] \tag{31}$$

$$\frac{(1-u_t)(1+n)}{(1-u_{t-1})} = \frac{c(1-\alpha)^{1-\alpha} \omega_t^*}{(1+g)(\omega_t^*)^{\frac{1}{\alpha}}}. \tag{32}$$

It is easy to compute that the steady state wage per unit of effective labor and rate of unemployment are: $\omega^* = (1-\alpha)^{\frac{1}{1-\alpha}} (\frac{c}{(1+n)(1+g)})^{\frac{\alpha}{1-\alpha}}$ and $u = \frac{(1+g)(1-\gamma)(1-\beta)}{(1+g)(1-\gamma)(1-\beta)+[(1+g)(1-\alpha)-\gamma]\beta}$. Analyzing the linearized system around the steady-state we obtain the following result:

Proposition 6.1 *There exists γ' such that if $\gamma < \gamma'$ then the linearized system converges to ω^* and u.*

In this case it is easy to see that the long run unemployment rate is less than one and that decreases with g and β. It increases with γ if and only if $g < \frac{\alpha}{1-\alpha}$.

Note that the long run wage per unit of effective labour is the same that the one obtained in the case $\gamma = 0$ which means that the long run rate of output growth is equal to $(1+n)(1+g)-1$. This means that, when both the previous wage and the unemployment benefit are taken into account when setting the wage, a change in either γ or β affects the long run rate of unemployment but keeps the long run rate of growth of output unaffected, an increase in n increases the rate of growth of output but keeps the rate of unemployment unaffected and an increase in g increases the rate of growth of output and decreases the rate of unemployment.

We can not conclude that there is no relationship between growth and unemployment in the long run in this case because there is a common variable g which affects both rates in the long run. The higher the rate of productivity growth, the higher the growth rate and the lower the unemployment rate in the long run, which implies an negative relationship between growth and unemployment in the long run. Note that this happens only when previous wages are taken into account when setting the wage. The intuition is that, in this case, the same real wage is set independently of the rate of productivity and, then, a change in g will affect employment. This result means that any government policy that increases productivity will imply more growth and less unemployment. Note also that, contrary to Daveri and Tabellini's result ([3]), an increase on workers's tax rate (decrease in β) increases unemployment but keeps output growth unaffected.

Note also that if $g > \frac{\alpha}{1-\alpha}$ and the economy converges to the steady state the higher the γ the lower the unemployment rate which means that taking into account only the unemployment benefit when setting the wage does not imply the lowest unemployment rate.

7 Final Comments

Using an standard OLG model with a non competitive labour market, where the wage is set by unions and the government pays an unemployment benefit taxing workers, we have analyzed under which conditions a lower unemployment rate is associated with a higher unemployment benefit. We have found that this situation happens if the government cares sufficiently about unemployed workers.

We have also analyzed the long run behaviour of real wages, the rates of growth of ouput and the rate of unemployment in three cases. We have found that, if the government sets both the unemployment benefit and the tax rate to employed workers and workers take into account only present variables of the period when setting the wage, the long run rate of growth depends on the rate of population growth and the rate of productivity growth. The rate of unemployment is constant along time. This constant rate of unemployment depends on how the government cares about the welfare of employed (or unemployed) workers and on the parameter of the production function. We can conclude in this case that there is no relationship between growth and unemployment in the long run.

If the government sets both the unemployment benefit and the tax rate to employed workers and workers take into account only previous wages when setting the wage, the rate of growth of real wages is constant and depends on the parameter of the production function. If it is smaller than the rate of productivity growth, the rates of growth of output and employment increase with time and, after some time, the rate of unemployment decreases.

Finally, when both previous wages and the present unemployment benefit are taken into account when setting the wage, and the weight of previous wages is small the economy converges to a long run rate of unemployment. The long run rate of growth depends on the rate of population growth and the rate of productivity growth. The long run rate of unemployment depends on the weight of previous wages when setting the wage, on how the government cares about the welfare of unemployed workers and on the rate of productivity growth. In this case the higher the rate of productivity growth, the higher the growth rate and the lower the unemployment rate in the long run. Last result means that any government policy that increases productivity will imply more growth and less unemployment.

Summarizing, we can not conclude that there is no relationship between growth and unemployment in the long-run because this only happens when the government sets both the unemployment benefit and the tax rate to employed workers and workers take into account only the unemployment benefit of the period when setting the wage. If it turns out that past wages are a relevant variable in the wage setting process the higher the rate of growth of productivity the higher the rate of growth and the lower the rate of unemployment in the long run, which implies a negative relationship between growth and unemployment. Nevertheless this relationship is weak because growth and unemployment in the long run are also affected independently by many other exogenous variables.

A Appendix.

Proof of Proposition 2.3. 1 *We know that if* $1 + R_t = \tilde{R}(w_t)$ *then* $Y_t^s = F(K_t^d, A_t L_t^d)$ *and* $\frac{K_t^d}{A_t L_t^d} = \tilde{k}(w_t)$. *The competitive wage,* w_t^c, *implies:*

$$K_t^d = K_t^s \text{ and } L_t^d = N_t, \tag{33}$$

and, then, w_t^c *satisfies:*

$$\tilde{k}(w_t^c) = \frac{K_t^s}{A_t N_t}. \tag{34}$$

Substituting (2) and (6) in (34), and solving the equation for w_t^c *we obtain:* $w_t^c = (1-\alpha)(A_t)^{1-\alpha}[\frac{S_{t-1}^s}{N_t}]^\alpha$.

If $w_t > w_t^c$ *we obtain:*

$$\tilde{k}(w_t) = \frac{K_t^d}{A_t L_t^d} > \frac{K_t^s}{A_t N_t} = \frac{S_{t-1}^s}{A_t N_t}, \tag{35}$$

because $\tilde{k}(w_t)$ *is increasing in* w_t. *If* $K_t^d = S_{t-1}^s$, *then, from (35), we obtain:* $L_t^d = \frac{K_t^d}{A_t \tilde{k}(w_t)} = \frac{A_t^{\frac{1-\alpha}{\alpha}} S_{t-1}^s}{[\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}}} < N_t$. *If, on the contrary,* $L_t^d = N_t$, *then, from (35), we have:* $K_t^d = \tilde{k}(w_t) A_t N_t > K_t^s$.

If $w_t < w_t^c$ *we have:*

$$\tilde{k}(w_t) = \frac{K_t^d}{A_t L_t^d} < \frac{K_t^s}{A_t N_t} = \frac{S_{t-1}^s}{A_t N_t}. \tag{36}$$

If $L_t^d = N_t$, *then, from (36), we have:* $K_t^d = \tilde{k}(w_t) A_t N_t = [\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}} \frac{N_t}{A_t^{\frac{1-\alpha}{\alpha}}} < K_t^s$. *If, on the contrary,* $K_t^d = S_{t-1}^s$, *then, from (36), we obtain:* $L_t^d = \frac{K_t^d}{A_t \tilde{k}(w_t)} > N_t$.

Proof of Proposition 3.1. 1 *First note that the wage set by the union will never be lower than the competitive wage. From proposition 2.3.1, if* $w_t < w_t^c$ *then* $L_t^e = N_t$. *Thus setting the wage equal to the competitive wage we still have* $L_t^e = N_t$ *and a greater wage. Now we solve the program of the union assuming that* $L_t^e = \frac{A_t^{\frac{1-\alpha}{\alpha}} S_{t-1}^s}{[\frac{w_t}{1-\alpha}]^{\frac{1}{\alpha}}}$. *It is easy to show that the* w_t *that maximizes the program of the union is the wage that maximizes the function:*

$$S_t' = ((1-\tau_t)w_t - h_t) w_t^{-\frac{1}{\alpha}}; \tag{37}$$

where $h_t = \gamma_t(1-\tau_{t-1})w_{t-1} + (1-\gamma_t)s_t$. *The first order condition for a maximum of* S_t' *is:*

$$(1-\tau_t) - \frac{1}{\alpha}\frac{(1-\tau_t)w_t - h_t}{w_t} = 0. \tag{38}$$

Solving (38) for w_t *we obtain:*

$$w_t = \frac{h_t}{(1-\alpha)(1-\tau_t)} = \frac{\gamma_t(1-\tau_{t-1})w_{t-1} + (1-\gamma_t)s_t}{(1-\alpha)(1-\tau_t)}. \tag{39}$$

Now, $w_t > w_t^c$ implies, using (39):

$$\frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)(1-\tau_t)} > (1-\alpha)(A_t)^{1-\alpha}[\frac{S_{t-1}^s}{N_t}]^\alpha. \tag{40}$$

If $\gamma_t = 1$ inequality 40 becomes:

$$w_t^c < \frac{(1-\tau_{t-1})w_{t-1}}{(1-\alpha)(1-\tau_t)} = \bar{w}_t. \tag{41}$$

If $0 \leq \gamma_t < 1$ solving 40 for s_t we obtain:

$$s_t > \frac{(1-\alpha)^2(1-\tau_t)(A_t)^{1-\alpha}[\frac{S_{t-1}^s}{N_t}]^\alpha - \gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\gamma_t)} = \bar{s}_t. \tag{42}$$

Thus, if $s_t > \bar{s}_t$ we have: $w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)(1-\tau_t)} > w_t^c$. If $s_t \leq \bar{s}_t$ then we have $L_t^e = N_t$ in which case $w_t = w_t^c$. Finally we compute when $\bar{s}_t < 0$. From (42) this inequality holds if and only if:

$$w_t^c < \frac{\gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\alpha)(1-\tau_t)} = \bar{w}_t. \tag{43}$$

Note from (41) and (43) that $\bar{w}_t \geq 0$.

Proof of Theorem 4. 1 *First we compute the best reply function of the union when $\tau_t = 1-\beta_t$. Using the proof of proposition 3.1.1 we obtain: If $w_t^c < w"_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\alpha)\beta_t}$ then $w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)\beta_t} > w_t^c$. If $w_t^c \geq w"_t$ and $\gamma_t = 1$ then $w_t = w_t^c$. If $w_t^c \geq w"_t$ and $0 \leq \gamma_t < 1$ then there exists $s"_t = \frac{(1-\alpha)\beta_t w_t^c - \gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\gamma_t)} \geq 0$ such that if $s_t > s"_t$ then $w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)\beta_t} > w_t^c$ and if $s_t \leq s"_t$ then $w_t = w_t^c$. We denote this best reply function of the union as $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 1-\beta_t, s_t, \gamma_t)$.*

Now we compute the best reply function of the union when $\tau_t = 0$. Using the proof of proposition 3.1.1 we obtain: If $w_t^c < w_t' = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\alpha)}$ then $w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)} > w_t^c$. If $w_t^c \geq w_t'$ and $\gamma_t = 1$ then $w_t = w_t^c$. If $w_t^c \geq w_t'$ and $0 \leq \gamma_t < 1$ then there exists $s_t' = \frac{(1-\alpha)w_t^c - \gamma_t(1-\tau_{t-1})w_{t-1}}{(1-\gamma_t)} \geq 0$ such that if $s_t > s_t'$ then $w_t = \frac{\gamma_t(1-\tau_{t-1})w_{t-1}+(1-\gamma_t)s_t}{(1-\alpha)} > w_t^c$ and if $s_t \leq s_t'$ then $w_t = w_t^c$. We denote this best reply function of the union as $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 0, s_t, \gamma_t)$.

Functions $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 1-\beta_t, s_t, \gamma_t)$ and $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 0, s_t, \gamma_t)$ when $\gamma_t = 1$ and $w_t^c < w_t'$, $w_t' \leq w_t^c < w"_t$ and $w"_t \leq w_t^c$ are represented in figures 1.a, 1.b and 1.c respectively. Functions $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 1-\beta_t, s_t, \gamma_t)$ and $\tilde{w}_t(w_{t-1}, \tau_{t-1}, 0, s_t, \gamma_t)$ when $0 \leq \gamma_t < 1$ and $w_t^c < w_t'$, $w_t' \leq w_t^c < w"_t$ and $w"_t \leq w_t^c$ are represented in figures 2.a, 2.b and 2.c respectively. On the other hand, we can also represent function $\tilde{s}_t$ in all figures. Looking at figure 1.a it is obvious that there is a unique equlibrium with unemployment. Looking at figure 1.b there is an equilibrium with unemployment and an equilibrium with full employment. Looking at *figure 1.c there is an equilibrium with full employment. Looking at figure 2.a it is obvious that there is a unique equlibrium with unemployment. Looking at figures 2.b and 2.c there is an equilibrium with unemployment and an equilibrium with full employment.*

Proof of Theorem 4. 2 *If there is unemployment, by 20, we have* $\tilde{\tau}_t(w_t, \beta_t) = 1 - \beta_t$ *and then* $\frac{\partial \tilde{\tau}_t}{\partial \beta_t} < 0$ *and* $\frac{\partial \tilde{\tau}_t}{\partial \gamma_t} = 0$. *We can then write the best reply function of the union as:* $\tilde{w}_t(w_{t-1}, \tau_{t-1}, \beta_t, s_t, \gamma_t)$. *By the implicit function theorem we have:*

$$\begin{bmatrix} \frac{\partial \hat{w}_t}{\partial \gamma_t} & \frac{\partial \hat{w}_t}{\partial \beta_t} \\ \frac{\partial \hat{s}_t}{\partial \gamma_t} & \frac{\partial \hat{s}_t}{\partial \beta_t} \end{bmatrix} = \frac{1}{1 - \frac{\partial \tilde{w}_t}{\partial \beta_t}\frac{\partial \tilde{s}_t}{\partial w_t}} \begin{bmatrix} \frac{\partial \tilde{w}_t}{\partial \gamma_t} & \frac{\partial \tilde{w}_t}{\partial \beta_t} + \frac{\partial \tilde{w}_t}{\partial s_t}\frac{\partial \tilde{s}_t}{\partial \beta_t} \\ \frac{\partial \tilde{s}_t}{\partial w_t}\frac{\partial \tilde{w}_t}{\partial \gamma_t} & \frac{\partial \tilde{s}_t}{\partial w_t}\frac{\partial \tilde{w}_t}{\partial \beta_t} + \frac{\partial \tilde{s}_t}{\partial \beta_t} \end{bmatrix}. \tag{44}$$

It is easy to prove that if $(1 - \tau_t)w_{t-1} > (<) s_t$ *then* $\frac{\partial \tilde{w}_t}{\partial \gamma_t} > (<) 0$. *We know that* $\frac{\partial \tilde{w}_t}{\partial \beta_t} < 0$ *and* $\frac{\partial \tilde{w}_t}{\partial s_t} > 0$. *We also know that* $\frac{\partial \tilde{s}_t}{\partial w_t} < 0$ *and* $\frac{\partial \tilde{s}_t}{\partial \beta_t} < 0$. *Substituting the sign of this partial derivatives in (44) in we obtain, when* $(1 - \tau_t)w_{t-1} > (<) s_t$ *the following matrix:* $\begin{bmatrix} \frac{\partial \hat{w}_t}{\partial \gamma_t} & \frac{\partial \hat{w}_t}{\partial \beta_t} \\ \frac{\partial \hat{s}_t}{\partial \gamma_t} & \frac{\partial \hat{s}_t}{\partial \beta_t} \end{bmatrix} = \begin{bmatrix} +(-) & - \\ - & ? \end{bmatrix}$. *This means that, by the sign of the partial derivatives, we do not know the sign of* $\frac{\partial \hat{s}_t}{\partial \beta_t}$. *In order to have these partial derivative positive, by (44), we need:*

$$\frac{\partial \tilde{s}_t}{\partial w_t}\frac{\partial \tilde{w}_t}{\partial \beta_t} > -\frac{\partial \tilde{s}_t}{\partial \beta_t}. \tag{45}$$

Computing the partial derivatives of (45) and substituting the labor demand elasticity by $-\frac{1}{\alpha}$ *we obtain:*

$$\frac{(1-\beta_t)L_t^d \frac{\alpha-1}{\alpha}(N_t - L_t^d) - (1-\beta_t)\frac{1}{\alpha}(L_t^d)^2}{(N_t - L_t^d)^2}(-\frac{w_t}{\beta_t}) > \frac{w_t L_t^d}{(N_t - L_t^d)}. \tag{46}$$

Simplifing (46) we obtain:

$$\frac{\frac{1-\beta_t}{\beta_t}\frac{1-\alpha}{\alpha} w_t L_t^d (N_t - L_t^d) + \frac{1-\beta_t}{\alpha\beta_t} w_t (L_t^d)^2}{(N_t - L_t^d)^2} > \frac{w_t L_t^d}{(N_t - L_t^d)}. \tag{47}$$

Inequality (47) is true if and only if $(1-\beta_t)L_t^d > (\alpha + \beta_t - 1)(N_t - L_t^d)$, *that is, if and only if:* $u_t < \frac{1-\beta_t}{\alpha}$.

Proof of Proposition 5. 1 *If* $\beta_t = \beta$ *for all t from the proof of theorem 4.1 we have* $w'_t = w"_t = \frac{w_{t-1}}{(1-\alpha)}$. *Then theorem 4.1 becomes: If* $w_t^c < \frac{w_{t-1}^*}{(1-\alpha)}$ *there exists a unique equilibrium with unemployment. If* $w_t^c \geq \frac{w_{t-1}^*}{(1-\alpha)}$ *there exists a unique equilibrium with full employment. Finally, it is easy to check that* $w_t^c < \frac{w_{t-1}^*}{(1-\alpha)}$ *holds if and only if* $N_t > \frac{cA_t^{\frac{1-\alpha}{\alpha}}(1-\alpha)^{\frac{2}{\alpha}} L_{t-1}^e}{(w_{t-1}^*)^{\frac{1-\alpha}{\alpha}}} = \bar{N}_t$. *Note also that in period zero we have* $w_0^* = w_0^c$.

If $w_t^* = \frac{w_{t-1}^*}{(1-\alpha)}$ *for all* t *solving this difference equation backwards we get:*

$$w_t^* = \frac{w_0^*}{(1-\alpha)^t}. \tag{48}$$

Subtituting $A_t = A_0(1+g)^t$ *and (48) in (23) we obtain:*

$$y_t = \frac{c(1-\alpha)^{\frac{1}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}}[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}t} - 1. \tag{49}$$

If $w_{t-1}^* = (1-\alpha)w_t^*$ *then, using (24),* l_t *becomes:*

$$l_t = \frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{(\frac{w_t^*}{A_t})^{\frac{1-\alpha}{\alpha}}} - 1 = \frac{cA_t^{\frac{1-\alpha}{\alpha}}(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{w_t^{*\frac{1-\alpha}{\alpha}}} - 1. \tag{50}$$

Subtituting $A_t = A_0(1+g)^t$ *and (48) in (50) we obtain:*

$$l_t = \frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}}[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}t} - 1. \tag{51}$$

We have that $u_t < (=)(>)u_{t-1}$ *if and only if* $l_t > (=)(<)n$ *which becomes, using (51):*

$$\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}}[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}t} - 1 > (=)(<)n. \tag{52}$$

From (49) and (51) we have that y_t *and* l_t *decrease if and only if* $[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}} < 1$ *that is, if and only if* $g < \frac{\alpha}{1-\alpha}$*. From (52) if* $\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}} - 1 \leq n$ *we have that* u_t *increases for all* $t \geq 0$*. If not there will exist some* $t > 0$ *such that* $l_t < n$*.*

From (49) and (51) we have that y_t *and* l_t *are constant if and only if* $[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}} = 1$ *that is, if and only if* $g = \frac{\alpha}{1-\alpha}$*. From (52) if* $\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}} - 1 < n$ *then* u_t *increases. If* $\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}} - 1 = n$ *then* u_t *is constant. If* $\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}} - 1 > n$ *then* u_t *decreases.*

From (49) and (51) we have that y_t *and* l_t *increase if and only if* $[(1+g)(1-\alpha)]^{\frac{1-\alpha}{\alpha}} > 1$ *that is, if and only if* $g > \frac{\alpha}{1-\alpha}$*. From (52) if* $\frac{c(1-\alpha)^{\frac{1+\alpha}{\alpha}}}{[\frac{w_0^*}{A_0}]^{\frac{1-\alpha}{\alpha}}} - 1 \geq n$ *we have that* u_t *decreases for all* $t \geq 0$*. If not there will exist some* $t > 0$ *such that* $l_t > n$*.*

Proof of Proposition 5. 2 *If* $\gamma_t = 0$ *it is easy to compute, using the proof of theorem 4.1 that* $w_t' = 0$ *and then* $w_t^c \geq w_t'$*. Now, if there is unemployment from* *proposition 3.1.1 and theorem 4.1 we have:*

$$w_t^* = \frac{s_t^*}{(1-\alpha)\beta}; \tag{53}$$

substituting s_t^* using proposition 5.1 we obtain:

$$w_t^* = \frac{1}{(1-\alpha)\beta}\frac{(1-\beta)w_t^* L_t^e}{N_t - L_t^e}; \tag{54}$$

and simplifying w_t^* equation 54 becomes:

$$L_t^e = \frac{(1-\alpha)\beta}{(1-\alpha)\beta + (1-\beta)} N_t; \tag{55}$$

that is,

$$u_t = \frac{1-\beta}{1-\alpha\beta}. \tag{56}$$

From (22) we have:

$$L_t^e = \frac{cA_t^{\frac{1-\alpha}{\alpha}} w_{t-1} L_{t-1}^e}{[\frac{w_t^*}{1-\alpha}]^{\frac{1}{\alpha}}}. \tag{57}$$

Substituting (55) in (57), if there is unemployment in period $t-1$ solving for w_t^* we obtain:

$$w_t^* = cA_t^{1-\alpha}(1-\alpha)[\frac{cw_{t-1}^* N_{t-1}}{N_t}]^{\alpha} = A_t^{1-\alpha}(1-\alpha)[\frac{cw_{t-1}^*}{(1+n)}]^{\alpha}. \tag{58}$$

Diving (58) by A_t we obtain:

$$\frac{w_t^*}{A_t} = (1-\alpha)[\frac{c\frac{w_{t-1}^*}{A_{t-1}}}{(1+n)(1+g)}]^{\alpha}. \tag{59}$$

If we call at the equilibrium wage per unit of efficient labor ω_t^*, that is, $\omega_t^* = \frac{w_t^*}{A_t}$ equation (59) becomes:

$$\omega_t^* = (1-\alpha)[\frac{c\omega_{t-1}^*}{(1+n)(1+g)}]^{\alpha}. \tag{60}$$

Solving (60) for ω_{L-R}^* when $\omega_t^* = \omega_{t-1}^* = \omega_{L-R}^*$ we obtain :

$$\omega_{L-R}^* = (1-\alpha)^{\frac{1}{1-\alpha}}(\frac{c}{(1+n)(1+g)})^{\frac{\alpha}{1-\alpha}}. \tag{61}$$

It is easy to check, drawing the phase diagram that the wage per unit of efficient labour converges to ω_{L-R}^*. From (22) we have in period zero

$$L_0^e = \frac{A_0^{\frac{1-\alpha}{\alpha}} K_0}{[\frac{w_0^*}{1-\alpha}]^{\frac{1}{\alpha}}}; \tag{62}$$

using (55) we have:

$$L_0^e = \frac{(1-\alpha)\beta}{(1-\alpha)\beta + (1-\beta)} N_0. \tag{63}$$

Substituting (62) in (63) and solving the equation for w_0^* we obtain:

$$w_0^* = (1-\alpha)[\frac{(1-\alpha)\beta + (1-\beta)}{(1-\alpha)\beta} \frac{A_0^{\frac{1-\alpha}{\alpha}} K_0}{N_0}]^\alpha, \tag{64}$$

that is:

$$\omega_0^* = (1-\alpha)[\frac{(1-\alpha)\beta + (1-\beta)}{(1-\alpha)\beta} \frac{K_0}{A_0 N_0}]^\alpha. \tag{65}$$

From (53) we obtain : $s_t^* = \beta(1-\alpha)w_t^*$ and defining the equilibrium unemployment benefit per unit of efficient labor, σ_t^*, as $\sigma_t^* = \frac{s_t}{A_t}$ we also have

$$\sigma_t^* = \beta(1-\alpha)\omega_t^*. \tag{66}$$

Substituting (58) and (61) in (66) we obtain:

$$\sigma_t^* = \beta^{(1-\alpha)}(1-\alpha)^{(2-\alpha)}[\frac{c\sigma_{t-1}^*}{(1+n)}]^\alpha; \tag{67}$$

and

$$\sigma_{L-R}^* = \beta(1-\alpha)^{\frac{2-\alpha}{1-\alpha}}(\frac{c}{(1+n)})^{\frac{\alpha}{1-\alpha}}. \tag{68}$$

Finally, substituting (61) in (23) we obtain that $y_{L-R}^* = (1+g)(1+n) - 1$ and the rate of growth of employment is always equal to the rate of population growth because the rate of unemployment is constant with time.

Proof of Proposition 6. 1 *We define equation 31 as $F(\omega_t, \omega_{t-1}, u_t) = 0$ and equation 32 as $G(\omega_t, \omega_{t-1}, u_{t-1}, u_t) = 0$ which implicitly define the system: $\omega_t = f_1(\omega_{t-1}, u_{t-1}), u_t = f_2(\omega_{t-1}, u_{t-1})$. Linearizing this system around the steady state and simplifying we obtain the following jacobian matrix:*

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \omega_{t-1}} & \frac{\partial f_2}{\partial u_{t-1}} \\ \frac{\partial f_2}{\partial \omega_{t-1}} & \frac{\partial f_1}{\partial u_{t-1}} \end{bmatrix} = \begin{bmatrix} \frac{\gamma + [(1-\alpha)(1+g)-\gamma]\frac{1}{u}}{\gamma + \frac{1}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}} & \frac{-[(1-\alpha)(1+g)-\gamma]\frac{\omega}{u(1-u)}}{\gamma + \frac{1}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}} \\ \frac{\frac{1+\alpha}{\alpha}\gamma\frac{(1-u)}{\omega}}{\gamma + \frac{1}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}} & \frac{\gamma}{\gamma + \frac{1}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}} \end{bmatrix}. \tag{69}$$

The eigen values, λ, are obtained computing the equation $det(\lambda I - J) = 0$. If $\gamma = (1-\alpha)(1+g)$ then $u = 1$ and $J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which means a unique eigen value $\lambda = 1$ and, then, the system does not converge to the steady state. If $\gamma < (1-\alpha)(1+g)$ then $0 < u < 1$. In this case one can compute that the eigen values will be real numbers if and only if

$$\gamma < \frac{\alpha}{4(1+\alpha)}[(1-\alpha)(1+g) - \gamma]\frac{1}{u}. \tag{70}$$

If $0 < u < 1$ *and* $\gamma < \frac{\alpha}{4(1+\alpha)}[(1-\alpha)(1+g)-\gamma]$ *then 70 holds. Last inequality holds if and only if* $\gamma < \frac{\alpha(1-\alpha)(1+g)}{4+5\alpha}$. *If* $0 < u < 1$ *then* $J = \begin{bmatrix} 0<a<1 & b<0 \\ c>0 & 0<d<1 \end{bmatrix}$. *In this case the two eigen values* λ_1, λ_2 *are such that* $\lambda_1 + \lambda_2 = a + d > 0$ *and* $\lambda_1 \lambda_2 = ad - bc > 0$. *These two inequalities imply* $\lambda_1 > 0$ *and* $\lambda_2 > 0$. *Now if* $\lambda_1 + \lambda_2 = a + d < 1$ *then we have that* $0 < \lambda_1 < 1$ *and* $0 < \lambda_2 < 1$ *which implies convergence to the steady state if both are real. Using 69 we obtain:*

$$\lambda_1 + \lambda_2 = \frac{2\gamma + [(1-\alpha)(1+g)-\gamma]\frac{1}{u}}{\gamma + \frac{1}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}}. \tag{71}$$

Using 71 we have that $\lambda_1 + \lambda_2 < 1$ *if and only if:*

$$\gamma < \frac{(1-\alpha)}{\alpha}[(1-\alpha)(1+g)-\gamma]\frac{1}{u}. \tag{72}$$

If $0 < u < 1$ *and* $\gamma < \frac{(1-\alpha)}{\alpha}[(1-\alpha)(1+g)-\gamma]$ *then 72 holds. Last inequality holds if and only if* $\gamma < (1-\alpha)^2(1+g)$. *Finally we define*
$\gamma' = min(\frac{\alpha(1-\alpha)(1+g)}{4+5\alpha}, (1-\alpha)^2(1+g))$.

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