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In whose backyard? A generalized bidding approach*

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Abstract

We analyze situations in which a group of agents (and possibly a designer) have to reach a decision that will affect all the agents. Examples of such scenarios are the location of a nuclear reactor or the siting of a major sport event. To address the problem of reaching a decision, we propose a one-stage multi-bidding mechanism where agents compete for the project by submitting bids. All Nash equilibria of this mechanism are efficient. Moreover, the payoffs attained in equilibrium by the agents satisfy intuitively appealing lower bounds.

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1.- Introduction

The location of major attractions or events has frequently been the subject of heated debate. Even more problematic are decisions regarding noxious facilities, such as dump sites, environmentally hazardous plants, nuclear power generators and the like. The bodies entrusted with carrying out these projects face a non-trivial problem in deciding upon the socially efficient location.

Every two years, the International Olympic Committee must deal with the problem of selecting hosts for the summer and winter games. Choosing a host is a procedure which is constantly under review and which has to be updated in order to bring about the "best possible" outcome. Up until about a year ago, the decision was made by a vote based on bids and plans submitted by candidate cities. However, the recent controversy surrounding the choice of Salt lake City to host the 2002 winter games led to the design of a new procedure for choosing the host of the 2006 winter games. This procedure called for the selection of two finalist cities (in this case, Turin and Sion), immediately after which the host was selected by secret ballot (in this case, Turin).

The FIFA executive committee faced a similar issue when it had to choose the site of the 2006 World Cup games. In July 2000 it was decided that Germany would host the games. This decision was reached by a vote after considering the offers of several potential hosts. However, the decision triggered great opposition by the South African Soccer Association and heated discussions about the whole choice process ensued. Soon after, the president of FIFA suggested that the system for awarding the FIFA world cup should be reviewed and possibly altered in order to achieve a better outcome.

When we turn to the problems of locating noxious facilities, similar issues arise. For example, about a year ago, a strong controversy on the location of a nuclear waste repository erupted in the US. In February 2000 the Senate had decided that nationwide nuclear waste would be shipped to the Yucca mountain site in Nevada (conditional to it being approved as a high-level nuclear waste repository). Despite the attractive compensation package, the State of Nevada voiced vehement opposition. President Clinton vetoed the bill, and in May 2000 the Senate failed to overturn it. As such, the major problem facing the US nuclear industry on where to site a high-level radioactive waste repository remains virtually unresolved.

These and many other similar scenarios fall within a large class of situations in which a group of agents and a designer have to reach a decision that will affect all of the agents directly or indirectly. More concretely, when a single project has to be awarded to one out of n possible agents, this creates an array of external effects in addition to the immediate effect on the chosen agent. Each agent's payoff depends not only on whether he is awarded the project, but also on the actual identity of the agent who is granted the project.

A decision is efficient if it maximizes the sum of benefits when all the externalities are taken into account. Reaching an efficient decision is trivial when the designer knows all the externalities. However, it is often the case that the parties concerned possess much more information than the designer. Hence, the designer faces a non-trivial problem if she wishes to optimally choose one agent over the rest. Moreover, from a normative point of view, even if the designer has all the information, it is easier to justify the use of a fixed procedure to reach a decision, than to modify the procedure for various cases in light of information revealed to (or already in the hands of) the designer.

To address the problem of reaching an efficient decision, we construct a *multi-bidding mechanism* where agents compete for the project by submitting bids. The equilibrium outcomes of the mechanism not only determine the agent to whom the project is awarded, but also generate a system of transfer payments that will internalize the externalities associated with the decision reached.

The one-stage mechanism proceeds as follows. Each agent submits a vector of bids, the components of which can be interpreted as the amount of money the agent is willing to pay to the others if he is awarded the project. The agent also indicates whether he "really" wants the project. We define the "net bid" of an agent as the difference between the sum of the bids he submitted minus the sum of the bids other agents submitted for him. The agent with the highest net bid is chosen as the winner. If several agents have the highest net bid, the winner is randomly chosen among those agents with the highest net bid who have announced that they "really" want the project. If there is no such agent, the winner is randomly chosen among the agents with the highest net bid. The winner pays the bids to the other agents and proceeds with the project.

This mechanism is simple and straightforward. We show that all Nash equilibrium outcomes of this mechanism are efficient. That is, the project is always awarded to an efficient agent. Furthermore, we provide a detailed analysis of the structure of the equilibria outcomes and their ranking by the participating agents. In particular, we show that, at equilibrium, each agent's payoff is at least the expected payoff he would obtain in a situation where all the agents have the same probability of developing the project.

Similar problems have been addressed in the literature. In an early paper Samuelson (1980) studied a class of procedures suggested by Knaster and Steinhaus by which indivisible objects get divided. He presented, as an example, the problem of locating a nuclear power plant in one of several communities. However, the analysis concentrated only on normative issues and left open the problems caused by strategic behavior.

Later contributions have emphasized strategic considerations, usually by analyzing environments with either complete or incomplete information. Kunreuther and Kliendorfer (1986) suggested a sealed-bid mechanism for siting noxious facilities, for an incomplete information setting where each agent knows his own preferences but does not know the preferences of the other agents. Each agent announces a single number indicating the compensation he would like to receive were he "awarded" the project. The agent requiring the lowest compensation is awarded the project, after which the required compensation is collected from all the other agents and paid to the "winner." The outcomes realized by max-min strategies are then shown to be efficient in those environments where each agent is indifferent between all the outcomes, as long as he is not the host.

O'Sullivan (1993) considered the problem of siting a noxious facility in one of two candidate cities. He analyzed the results of a sealed-bid auction where the city with the low bid hosts the facility and receives the highest bid as compensation. O'Sullivan demonstrates that symmetric Bayes-Nash equilibria of the bidding game yield an efficient outcome when the cost parameters of the two cities are independently drawn and each city is informed only of its own cost of hosting the site.

Ingberman (1995) added a further dimension to the problem of siting a noxious facility by relating the cost to the agents to their distance from the noxious facility. He analyzed the process of majority agreement by using an auction approach and concluded that decisions reached in this manner would not be efficient.

Rob (1989) approached the siting problem from a mechanism design point of view. The designer's role was assumed by a firm that has to decide where to build its plant. Each location is privately informed of the cost imposed on it, were it to host the plant. Rob (1989) characterizes the optimal mechanism among all the mechanisms satisfying incentive compatibility and individual rationality. The firm has to design a mechanism that, to each cost vector reported by the locations, associates a randomized decision rule and an expected compensation for each location. The author points out that, as is customary in such environments, the resulting mechanism could lead to inefficient outcomes. Furthermore he notes, that the mechanism has more than one equilibrium and the properties of equilibria other than the one considered are not analyzed.

For the complete information setting Jackson and Moulin (1992) considered an environment where a set of agents decides whether to carry out a particular public project that yields a benefit to each agent. At every subgame perfect equilibrium of the mechanism they design, the public project is undertaken exactly when its total benefit outweighs its cost. Moreover, the mechanism allows the designer to realize a variety of cost distribution rules.

Jehiel, Moldovanu, and Stacchetti (1996) analyzed a more complex environment where the benefit (or cost) to an agent depends on the identity of the agent carrying out the project. In their setting, one seller has an object he wants to sell to one of n agents. First, the authors assume the seller is aware of all the benefits and show how he could construct a revenue-maximizing selling mechanism. Then in the incomplete information setting they characterize the incentive compatible and individually rational mechanisms that maximize the seller's revenue.

In the environment we analyze, the physical characteristics are similar to those of Jehiel, Moldovanu, and Stacchetti (1996), but the informational structure resembles that of Jackson and Moulin (1992). The multi-bidding mechanism we propose can be applied without any knowledge of the particular environment in which it is being used. In contrast, the outcome function of the mechanism proposed by Jehiel, Moldovanu, and Stacchetti (1996) is very sensitive to the particulars of the environment to which it is applied.

The paper proceeds as follows: In section 2 we present the basic environment and the multi-bidding mechanism. In section 3 we study the outcomes generated by the mechanism and provide a full characterization of all equilibrium payoffs. Section 4 concludes and offers further directions of research.

2.- The environment and the multi-bidding mechanism

We consider a set of agents $N = \{1,\ldots, n\}$ one of which has to develop a project. It could be a set of cities, one of which will host the Olympics, or a set of states one of which will end up hosting a nuclear dumpsite. The project carried out by the agent affects the agent's utility and also imposes (positive or negative) externalities on the other agents. The utility (payoff) of agent j if project i is chosen is given by v^i_j.

Project i is *efficient* (or alternately agent i is efficient) if:

$$\sum_{j=1}^{n} v_j^i \geq \sum_{j=1}^{n} v_j^k \quad \text{for all } k \in N.$$

We denote by E the set of efficient projects, and by I the set of inefficient projects, $E \cup I = N$. Also, we denote by V^e the maximum value that can be attained by developing the project, that is,

$$V^e = \sum_{j=1}^{n} v_j^i \quad \text{for some } i \in E.$$

Although each agent knows all the v_j^i's, the planner who has to choose the agent that will carry out the project does not know them. Alternately, we can assume that the planner, while possibly having some partial information, wants to design a system that will be applicable to any such situation.

We construct a *multi-bidding mechanism* in which the agents compete for the project. The project is awarded to the suitably defined highest bidder. Informally, the mechanism is described as follows: Each agent i announces $(n-1)$ bids, one for each of the other agents. Furthermore, he announces whether he "really" wants to have the project. The agent with the highest net bid is chosen as the winner. We define the "net bid" of an agent as the difference between the sum of the bids he makes to the others minus the sum of the bids the others make to him. In case of a tie, the winner is chosen randomly among those agents with the highest net bid that have announced that they "really" want the project. If there is a tie but none of the agents with the highest net bid announced that he really wanted the project, the winner is randomly chosen among all the agents with the highest net bid. Once the winner is identified, he pays the bids to the other agents and proceeds with the project.

A key feature of our mechanism is that we allow for more than one bid by each agent is. This gives the agents the freedom to construct their strategies in a way that reflects the actual externalities prevailing in the environment. Forcing the agents to submit a single bid would make it impossible for them to "fine-tune" their strategy and would lead to inefficient equilibria.

The bids can be naturally interpreted as transfer payments (positive or negative) offered for the right to carry out the project. Let us consider the case where one of several cities is to host either a “good” or a “bad” project. In this case, we can think of v^j_i as the utility of city i if city j ends up having the facility. Typically, v^j_i is a negative (positive) number if $j \neq i$, and a positive (negative) number if $j = i$ for a “good” (“bad”) facility. The bid b^j_i can be interpreted as the amount of money city j is ready to give (pay) to city i for city j to get the facility.

We now describe the multi-bidding mechanism more formally:[1]

Each agent $i \in N$ makes bids b^i_j in R for every $j \neq i$ and announces $m_i \in \{1, 0\}$. Hence, a strategy for agent i is a vector $((b^i_j)_{j\neq i}, m_i)$ in R^{n-1}x$\{1, 0\}$ ($m_i = 1$ is “interpreted” to mean that the agent would really like to carry out the project).

For each $i \in N$, we let $B^i = \sum_{j\neq i} b^i_j - \sum_{j\neq i} b^j_i$ denote the net bid of agent i. The set of agents with the highest net bid is denoted by $\Omega = \{ i \in N \mid B^i \geq B^k \text{ for all } k \in N \}$. Finally, $\Omega_1 = \{ i \in \Omega \mid m_i = 1\}$ denotes the set of agents with the highest net bid and who really want to carry out the project. The winner α is randomly chosen among the agents in Ω_1 if Ω_1 is non-empty. Otherwise, he is randomly chosen in Ω.

The final payment to any agent i other than the winner α is given by $v^\alpha_i + b^\alpha_i$, whereas agent α receives $v^\alpha_\alpha - \sum_{i\neq\alpha} b^\alpha_i$ as his final payment.

[1] A similar bidding procedure was used by Pérez-Castrillo and Wettstein (1999) as part of a non-cooperative implementation of the Shapley value of cooperative games with transferable utility.

3.- Equilibrium outcomes of the mechanism

We are mainly interested in the equilibrium outcomes generated by the mechanism; that is, which agent will end up hosting the project? As this is a game of complete information, we analyze the resulting Nash equilibria. We start by proving several important properties satisfied by all equilibrium strategies. The first Lemma shows that in equilibrium the net bid of any agent must be zero. This is used in the second Lemma to derive a set of inequalities satisfied by all equilibrium strategies.

Lemma 1.- In any NE of the multi-bidding mechanism, the net bid of any agent is zero.

Proof.- If $\Omega = N$ the claim is satisfied since $\sum_{i \in N} B^i = 0$. Otherwise, we can show that any agent i in Ω_1 (or any agent in Ω, if Ω_1 is empty) can change his bids so as to decrease the sum of payments in case he wins. Furthermore, these changes can be made without altering the sets Ω and Ω_1. Hence, agent i maintains the same (positive) probability of winning, and obtains a higher expected payoff.

Since $\Omega \neq N$, there is an agent $j \notin \Omega$. Let agent $i \in \Omega$ change his strategy by announcing: $b^{i'}{}_k = b^i{}_k + \delta$ for all $k \in \Omega$ and $k \neq i$; $b^{i'}{}_j = b^i{}_j - |\Omega|\delta$; and $b^{i'}{}_l = b^i{}_l$ for all $l \notin \Omega$ and $l \neq j$. The new net bids are: $B^{i'} = B^i - \delta$; $B^{k'} = B^k - \delta$ for all $k \in \Omega$ and $k \neq i$; $B^{j'} = B^j + |\Omega|\delta$ and $B^{l'} = B^l$ for all $l \notin \Omega$ and $l \neq j$. If δ is small enough, so that $B^j + |\Omega|\delta < B^i - \delta$ (bear in mind that $B^j < B^i$), then $B^{l'} < B^{i'} = B^{k'}$ for all $l \notin \Omega$ (including j) and for all $k \in \Omega$. Therefore, Ω does not change. Since there has not been any change in the announcement m_i, the set Ω_1 does not change either. However,

$\sum_{h \neq i} b_h^i - \delta < \sum_{h \neq i} b_h^i$. *Q.E.D.*

The multi-bidding mechanism not only enables each agent to influence whether he himself will develop the project, but also to affect the identity of the agent to be in charge when it is not himself. Lemma 1 reinforces this characteristic of the mechanism. It stresses that, at equilibrium, the bids are such that each agent can decide the identity of the winner by slightly decreasing or increasing his bid. Indeed, since all net bids are equal, agent i can make agent j the certain winner by slightly increasing b^i_j. Similarly, agent i can make sure that agent j does not win by slightly decreasing b^i_j.

Lemma 2.- If project k is chosen as a NE outcome of the multi-bidding mechanism, the following conditions must be satisfied:

$$v_k^k - \sum_{j \neq k} b_j^k \geq v_k^i + b_k^i \quad \text{for all } i \neq k, \tag{1}$$

$$v_i^k + b_i^k \geq v_i^i - \sum_{j \neq i} b_j^i \quad \text{for all } i \neq k, \tag{2}$$

$$v_h^k + b_h^k \geq v_h^i + b_h^i \quad \text{for all } h \neq i, h \neq k \neq i. \tag{3}$$

Proof.- Since the net bid of any agent is zero, it cannot be the case that agent k would strictly prefer any agent i different from k to win. If this were the case, agent k could make agent i win by slightly decreasing his bid for agent i, and obtain a larger payoff in contradiction to him being at a NE. Hence, the inequalities in (1) must be satisfied at a NE. A similar argument makes it clear that it cannot be the case that agent i would strictly prefer himself to win rather than k, leading to inequalities (2). Finally, it cannot be the case that agent h would strictly prefer any agent i different from k and from h to win, leading to the inequalities in (3). *Q.E.D.*

The inequalities derived in Lemma 2 are now used to show that all Nash equilibrium outcomes are efficient.

Theorem 1.- In every NE of the multi-bidding mechanism, the project chosen is efficient.

Proof.- Assume that project k was chosen. Take any project i different from k. Consider the equations (1)-(3) corresponding to this particular project i. Summing up these equations we get:

$$\sum_{j\in N} v_j^k - \sum_{j\neq k} b_j^k + \sum_{j\neq k} b_j^k \geq \sum_{j\in N} v_j^i + \sum_{j\neq i} b_j^i - \sum_{j\neq i} b_j^i \ ,$$

i.e., $\sum_{j\in N} v_j^k \geq \sum_{j\in N} v_j^i$.

Therefore project k is efficient. *Q.E.D.*

Hence the multi-bidding mechanism leads to an efficient outcome and optimally awards the project even though the authorities in charge lack precise information on the externalities associated with each location.

The efficiency of any equilibrium outcome is now used to derive further properties of the equilibrium strategies as outlined in the following lemmas.

Lemma 3.- At the NE of the multi-bidding mechanism, only efficient agents send in $m = 1$. Moreover, if there are inefficient agents (i.e., $E \neq N$), at each such NE at least one efficient agent sends in $m = 1$.

Proof.- Since in equilibrium all net bids are equal, if there were an inefficient agent i that sent in $m_i = 1$, there would be a positive probability that that inefficient project is chosen at equilibrium. This is in contradiction to the finding (Theorem 1) that only efficient projects are chosen in equilibrium. Similarly, if there are inefficient agents and in equilibrium no efficient agent sends in $m = 1$, once more there would be a positive probability that an inefficient project is chosen in equilibrium. *Q.E.D.*

Lemma 4.- In any NE of the multi-bidding mechanism, all agents are indifferent as to the identity of the project chosen as long as the project is efficient.

Proof.- Assume we are at a NE where an efficient project k was chosen. Furthermore, there is another efficient project i. Summing up as in Theorem 2, we get that all the inequalities we sum up must be satisfied as equalities. Hence, at this NE, agents i and k get the same payoff regardless of who carries out the project, as does any other agent. It is also the case that any agent j different from k or i gets the same payoff whether k or i were assigned the project. *Q.E.D.*

Now we can use the above properties to provide a full characterization of all equilibrium strategies.

Lemma 5.- A set of strategies constitutes a NE of the multi-bidding mechanism if and only if the following three properties hold:

(a) The net bid of every agent is equal to zero.

(b) Given the equilibrium bids, an agent's payoff is maximal if an efficient project is chosen, and it is the same regardless of which among the efficient projects is chosen.

(c) Only efficient agents send the message $m = 1$. Moreover, if the set of efficient projects does not coincide with N, then at least one efficient agent sends the message $m = 1$.

Proof.- The "only if" part has been shown in the previous lemmas. We now prove that the conditions are also sufficient. Suppose that the strategies of the agents satisfy conditions (a) – (c). We prove that there is no agent i in N who profits by deviating from these strategies.

Notice first that conditions (b) and (c) imply that every agent is indifferent to the identity of the project chosen among the projects in Ω_1 (or maybe in Ω, if $E = N$). Therefore, given the bids, no agent can gain by a change in the set of agents who sent the message $m = 1$. If an agent decides to increase his total bid, his project will be chosen with certainty (because of (a)). However, since the previous proposer yielded a maximal payoff, his payoff will decrease. Similarly, an agent cannot gain by manipulating the bids so that a particular project (or group of projects) is chosen with certainty. Manipulating the bids will not change his final payoff, and the original composition of Ω_1 was optimal for him. Similarly no agent can gain by changing his m announcement, or by simultaneously modifying his bids and choice of m.

Q.E.D.

The characterization of the equilibrium strategies enables us to calculate explicit lower bounds on the payoffs accruing to the players in any equilibrium outcome. Denote by u_i the final level of utility that agent i obtains by playing the multi-bidding mechanism. We know that u_i is the final payoff of agent i when an efficient agent wins. Also, denote by $\underline{v}_i$ the expected utility agent i obtains if there is a probability of $1/n$ that each agent carries out the project. Formally,

$$\underline{v}_i = \frac{1}{n}\sum_{j=1}^{n} v_i^j \quad .$$

The level $\underline{v}_i$ has a natural interpretation. Given that the environment is one in which every agent has the same *a priori* rights, a natural and fair (although not efficient) procedure for allocating the project is to choose randomly each agent with the same probability. The value $\underline{v}_i$ is the expected payoff of agent i when this random procedure is implemented. The following theorem shows that these utility levels provide a lower bound on the payoffs received by the agents at the equilibria of the multi-bidding mechanism.

Theorem 2.- In every NE of the multi-bidding mechanism, the final utility of any agent i is greater or equal to his average valuation, that is, $u_i \geq \underline{v}_i$ for all i in N.

Proof.- Consider an equilibrium outcome in which project k is chosen. We have shown that equations (1) – (3) must hold. Let us change the notation in equation (3) to write:

$$v_i^k + b_i^k \geq v_i^h + b_i^h \qquad \text{for all } h \neq i,\ h \neq k \neq i. \qquad (3')$$

To examine the payoff received by an agent i different than k we sum up equation (2) and equation (3') for all h different from i to get:

$$v_i^k + b_i^k + (n-1)(v_i^k + b_i^k) \geq \sum_{j \in N} v_i^j - \sum_{j \neq i} b_j^i + \sum_{j \neq i} b_i^j .$$

By Lemma 1, $\sum_{j \neq i} b_j^i = \sum_{j \neq i} b_i^j$, and hence $v_i^k + b_i^k \geq \frac{1}{n} \sum_{j \in N} v_i^j = \underline{v}_i$.

We now consider the payoff received by agent k, $v_k^k - \sum_{j \neq k} b_j^k$. We sum up equation (1) for all i different from k to get:

$$(n-1)\left(v_k^k - \sum_{j \neq k} b_j^k \right) \geq \sum_{j \neq k} v_k^j + \sum_{j \neq k} b_k^j .$$

Hence, $(n-1)\left(v_k^k - \sum_{j \neq k} b_j^k \right) + \left(v_k^k - \sum_{j \neq k} b_j^k \right) \geq \sum_{j \neq k} v_k^j + \sum_{j \neq k} b_k^j + \left(v_k^k - \sum_{j \neq k} b_j^k \right).$

Using Lemma 1 this implies that:

$$v_k^k - \sum_{j \neq k} b_j^k \geq \frac{1}{n} \sum_{j \in N} v_k^j = \underline{v}_k \ . \qquad \textit{Q.E.D.}$$

Theorems 1 and 2 highlight the main properties of the equilibrium outcomes. All equilibrium outcomes are efficient and they guarantee for each agent at least his expected utility when the probability of carrying out the project is uniformly distributed among all the agents. Hence all equilibrium outcomes belong to the set P, where $P = \left\{ u \in R \,\middle|\, \sum_{i \in N} u_i = V^e \text{ and } u_i \geq \underline{v}_i \right\}$.

Theorem 3 shows that any vector in P can be realized as an equilibrium outcome of our mechanism. Thus we provide a full characterization of the set of NE outcomes.

Theorem 3.- Any $u \in P$ can be realized as a Nash equilibrium outcome of the multi-bidding mechanism.

Proof.- The proof proceeds as follows: First we show that both the set of equilibrium bidding strategies and of equilibrium payoff vectors are convex. Second, we show by construction that for any agent $i \in N$ there exists an equilibrium where the payoff for any agent j other than i is $\underline{v}_j$. We conclude by noting that these payoff vectors are the extreme points of the set P and as such any point in P can be expressed as a convex combination of these vectors.

Let $S^1 = (\beta^1, m^1)$ and $S^2 = (\beta^2, m^2)$ be two strategy tuples that are NE for the bidding mechanism where, $\beta^1 = ((b_j^{1i})_{j \neq i})_{i=1,\ldots,N}$ and $\beta^2 = ((b_j^{2i})_{j \neq i})_{i=1,\ldots,N}$ are the bidding strategies used and (m^i) the n-tuple of 0-1 messages sent in these equilibria. We now prove that the bidding strategies β_λ given by: $b_j^i = \lambda b_j^{1i} + (1-\lambda) b_j^{2i})$, together with m^1 (or m^2) are a NE for any $0 \leq \lambda \leq 1$.

The net bid of any agent in β_λ is zero:

$$B_\lambda^i = \sum_{j\neq i}\left(\lambda b_j^{1i} + (1-\lambda)b_j^{2i}\right) - \sum_{j\neq i}\left(\lambda b_i^{1j} + (1-\lambda)b_i^{2j}\right) =$$

$$\lambda\left(\sum_{j\neq i} b_j^{1i} - \sum_{j\neq i} b_i^{1j}\right) + (1-\lambda)\left(\sum_{j\neq i} b_j^{2i} - \sum_{j\neq i} b_i^{2j}\right) = 0 + 0 = 0.$$

Since m^1 was part of a Nash equilibrium only efficient agents send $m = 1$.

Now we take any efficient agent, say $1 \in E$. Agent i's ($i \neq 1$) payoff if 1 wins is higher than his payoff if any other agent $j \neq i$ wins:

$$v_i^1 + \lambda b_i^{11} + (1-\lambda)b_i^{21} = \lambda\left(v_i^1 + b_i^{11}\right) + (1-\lambda)\left(v_i^1 + b_i^{21}\right) \geq$$

$$\lambda\left(v_i^j + b_i^{1j}\right) + (1-\lambda)\left(v_i^j + b_i^{2j}\right) = v_i^j + \lambda b_i^{1j} + (1-\lambda)b_i^{2j} \quad \text{for all } j \neq i.$$

Furthermore,

$$v_i^1 + \lambda b_i^{11} + (1-\lambda)b_i^{21} = \lambda\left(v_i^1 + b_i^{11}\right) + (1-\lambda)\left(v_i^1 + b_i^{21}\right) \geq$$

$$\lambda\left(v_i^i - \sum_{j\neq i} b_j^{1i}\right) + (1-\lambda)\left(v_i^i - \sum_{j\neq i} b_j^{2i}\right) = v_i^i - \sum_{j\neq i}\left(\lambda b_j^{1i} + (1-\lambda)b_j^{2i}\right)$$

Hence agent i's payoff is maximized if project 1 is chosen. Also notice that in the two previous expressions the inequalities become equalities if project j or project i, respectively, are efficient.

To see that agent 1's payoff is maximized as well by the choice of project 1 note that

$$v_1^1 - \lambda\sum_{j\neq i} b_j^{11} - (1-\lambda)\sum_{j\neq i} b_j^{21} = \lambda\left(v_1^1 - \sum_{j\neq i} b_j^{11}\right) + (1-\lambda)\left(v_1^1 - \sum_{j\neq i} b_j^{21}\right) \geq$$

$$\lambda\left(v_1^i + b_1^{1i}\right) + (1-\lambda)\left(v_1^i + b_1^{2i}\right) \geq v_1^i + \lambda b_1^{1i} + (1-\lambda)b_1^{2i} \quad \text{for any } i \neq 1.$$

By Lemma 5, β_λ and m^1 are a NE. From the above calculations we also see that the payoff vector associated with the β_λ equilibrium is the convex combination with weights λ and $(1-\lambda)$ of the payoffs corresponding to the S^1 and S^2 equilibria.

We now proceed to construct the "extreme" equilibrium points. Take any agent $j \in N$. The bidding strategies in the best equilibrium for agent j are given by:

$$b_i^k = \underline{v}_i - v_i^k \quad \text{for } i \neq j, k.$$

$$b_j^k = \sum_{s \neq j} v_s^k - \sum_{s \neq j} \underline{v}_s \quad \text{for } k \neq j.$$

Also, the players' strategy include a vector of messages such that at least one efficient agent sends $m = 1$, while all inefficient agents send $m = 0$.

Given the bids, the net bid of any agent $i \neq j$ is zero:

$$B^i = \sum_{k \neq i,j} \left(\underline{v}_k - v_k^i\right) + \sum_{s \neq j} v_s^i - \sum_{s \neq j} \underline{v}_s - \sum_{k \neq i} \left(\underline{v}_i - v_i^k\right) =$$

$$\sum_{k \neq i,j} \underline{v}_k - \sum_{s \neq j} \underline{v}_s - \sum_{k \neq i} \underline{v}_i - \sum_{k \neq i,j} v_k^i + \sum_{s \neq j} v_s^i + \sum_{k \neq i} v_i^k = -n\underline{v}_i + \sum_{k \in N} v_i^k = 0.$$

Since $B^i = 0$ for all $i \neq j$, B^j is also zero.

To show this is a NE we now prove that all the conditions of Lemma 5 are satisfied. We need to show that given these strategies each agent's payoff is maximized if any efficient agent carries out the project. The payoff for any agent $i \neq j$ is:

$$b_i^k + v_i^k = \underline{v}_i \quad \text{if } k \neq i \text{ wins,}$$

$$v_i^i - \sum_{k \neq i,j} \left(\underline{v}_k - v_k^i\right) - \sum_{s \neq j} v_s^i + \sum_{s \neq j} \underline{v}_s = \underline{v}_i \quad \text{if } i \text{ wins.}$$

Therefore, independent of the identity of the winner, the payoff to every player different from j is the same. Therefore, their payoffs are maximized if an efficient agent carries out the project. As for *j*, he receives the difference between the total surplus minus the previous payoffs. Therefore, *j* prefers that any efficient project be carried out. Therefore, all the conditions of Lemma 5 are satisfied, so the previous strategies are part of a NE that yields the payoff vector that agent *j* prefers the most.

The payoff vectors corresponding to the equilibria we have constructed are the extreme points of the set *P* and as such any point *p* in *P* can be expressed as a convex combination of these vectors. Applying the same convex combination to the bidding strategies yields a NE of the game with the associated payoff vector coinciding with *p*. *Q.E.D.*

The following corollary sets our results in the framework of implementation theory.

Corollary 1.- The multi-bidding mechanism implements in Nash Equilibrium the set of utility vectors given by:

$$\left\{ (u_1,\ldots,u_n) \in R^n \mid \sum_{i \in N} u_i = V^e \text{ and } u_i \geq \underline{v}_i \text{ for all } i \text{ in } N \right\}.$$

Proof.- Follows directly from theorems 1, 2, and 3.

3.- Conclusion

We have addressed the problem of achieving an efficient outcome in the presence of externalities. The underlying problem might be the siting of a major sport event, the election of a department chairman, or the location of a nuclear reactor. We assumed that the agents are fully informed and that an uniformed designer has to reach a decision that will affect all of them. We then constructed a simple and straightforward mechanism whose Nash equilibria realize any efficient outcome, that satisfies intuitively appealing lower bounds on the payoffs attained in equilibrium by the participating agents.

It should be emphasized that all Nash equilibrium outcomes are efficient. The mechanism proposed here improves previous constructions in that it can handle a large set of environments. It involves a single stage of play with each agent being able to express his preferences by sending a multi-dimensional message.

The complete information assumption, while restrictive, might approximate realistic environments where the agents taking part in the mechanism are much better informed than the designer. For example, the designer could be a university president and the agents members of a department, one of whom should be designated as a chairman. The complete information framework allows the derivation of a powerful efficiency result and can serve as a benchmark for the study of environments with incomplete information.

The mechanism, as specified, could operate in environments with asymmetric information as well. Determining the properties of the resulting equilibria in such environments is a topic for further research.

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