Leadership in Collective Action

Joan Esteban * Esther Hauk †

October 13, 2008

Abstract

We extend the model of collective action in which groups compete for a budged by endogenizing the group platform, namely the specific mixture of public/private good and the distribution of the private good to group members which can be uniform or performance-based. While the group-optimal platform contains a degree of publicness that increases in group size and divides the private benefits uniformly, a success-maximizing leader uses incentives and distorts the platform towards more private benefits - a distortion that increases with group size. In both settings we obtain the anti-Olson type result that win probability increases with group size.

JEL-Classification: D70, D72, D74

Key-words: collective contests, leadership, group platform, incentives, sharing rules.

Acknowledgements: We are grateful to Antonio Cabrales, Sjaak Hurkens and Debraj Ray for useful comments.

*Institut d'Anàlisi Económica, CSIC; email: joan.esteban@iae.csic.es. This paper is part of the Polarization and Conflict Project CIT-2-CT-2004-506084 funded by the European Commission-DG Research Sixth Framework Programme. Member of the Barcelona GSE Research Network funded by the Government of Catalonia. Financial support from the CICYT project n. SEJ2006-00369.

†Institut d'Anàlisi Económica, CSIC; email: esther.hauk@iae.csic.es. The author acknowledges financial support from the CICYT project n. SEJ2006-01717 and the support of the Barcelona GSE Research Network and the government of Catalonia.

1 Introduction

The collective action failure —the "tragedy of the commons" in Hardin's (1968) apt expression— is a core problem in economics and in political science. It affects all goods and services that need to be produced collectively like general or local public goods or common lobbying for group specific benefits. Because production is collective, an individual's benefit from the good depends on the group she is a part of. But her costs of participating in group activities are privately incurred. This lack of coordination makes individuals contribute less than what would be optimal for the group.

Collective action poses two types of coordination problems. The first one is on the individual contribution. The second concerns the very definition of the goals of the group.[1]

The first problem is the most studied (lack of) coordination at the stage of the individual decisions on how much to contribute.[2] The difficulty — often impossibility — to coordinate and to monitor the execution of any potential agreement by the group leads to the well-known "free-rider" problem.[3]

The second coordination problem, previous to the individual contribution stage, has caught far less attention. It deals with the very definition of the specific cause the group will be asked to contribute to. There are instances in which the relevant community and the good to be produced are obvious and the regular institutions (e.g. the municipality) offer the natural channel for reaching an agreement on how to distribute the benefits. Typical examples are irrigation, sewerage or water supply.[4] However, most often there are no

[1] A third type of coordination problem which we do not study here is the endogenous formation of competing groups. Several papers have studied this problem: see Anesi (2007), Bardhan and Singh (2004) , Esteban and Ray (2008), and references therein. Anesi (2007) adds a lobbying-formation game a la Mitra (1999) (where forming a lobby involves a fixed cost F_i for group i) to the model of Esteban and Ray (2001). Anesi shows that the typical free-riding problem at the effort choice stage (which he calls moral hazard in teams) may raise large groups' equilibrium lobby size and also the total contribution to lobbying of large groups with low organizational costs. The possibility to free-ride in the second stage (choice of effort) lowers the cost of being a member of the lobby and thereby increases participation in lobbying activities. In other words, moral hazard in teams decreases individual contributions to lobbying but raises the number of contributions.

[2] The literature on collective action failure is surveyed by Sandler (1992).

[3] Monitoring and sanctions may alleviate this problem. Experimental evidence is examined in Andreoni et al. (2003), Fehr and Gächter (2000) and Masclet et al. (2003) and field evidence in irrigation systems in India in Bardhan (2000).

[4] Development economics has been very concerned with the collective production of

natural channels to facilitate coordination in the definition of the common ends. The coordination problem then is usually solved by means of external political entrepreneurs who take the lead by proposing a coalition aiming at the production of some good (or bundle of goods) and a particular distribution of the benefits.[5] Obvious examples are NGOs and most social and political movements.

The role of entrepreneurial leaders however does not come free of charge. Leaders primarily seek personal success and reappointment and this may partially clash with the goals of the individual members of the group. While helping individuals to coordinate to define a common platform they may go beyond what would be group optimal because of their thirst for success. In this paper we shall study the role of leaders in defining the group platforms.

The literature on collective action (and rent-seeking) has assumed that the goal of each group is to maximize the probability of success in obtaining a given monetary prize or budget, leaving little role for an entrepreneurial leader. That social movements —and even lobbies— just seek for monetary benefits clearly defies the most elementary casual evidence. Cornes and Sandler (1984, 1994) and Esteban and Ray (2001), instead have considered group goals that are a mixture of public and private goods. In their models, though, the public/private composition of the group platform was supposed to be exogenously given and common to all groups. Further, the benefits of the private component of the platform are supposed to be uniformly distributed within the group.

How are group platforms chosen? If the essential problem in collective action is free-riding, it is obvious that the size of the private benefits will be key in determining individual contributions. This is the focus of our paper. We study the role of leaders in setting the group platform which specifies the mixture of goods to be produced and the distribution of the divisible benefits among the members.[6] Leaders of groups with different characteristics (in our model, group size) relevant for the determination of free-riding will choose

public goods and the role of institutions. See the recent panoramic paper by Banerjee et al (2008).

[5] On leadership in collective action, see Frolich et al (1971), Calvert (1992) and Colomer (1995). Cai (2002) empirically examines the characteristics of the community that are conducive to group decision or to external leadership.

[6] Some papers have endogenized sharing rules, e.g. Lee (1995) and Ueda (2002). To our knowledge ours is the first paper to endogenize the degree of public / privateness of the goal.

their platforms in order to alleviate the collective action failure. We will assess the cost of leadership by comparing the leader's choice with the (utility-maximizing) group optimal platform.

We extend the model of Esteban and Ray (2001) and of Banerjee et al. (2008) in which groups compete for the access to a budged that finances a mixture of public/private good. In our case, the share of privateness and how this private part is distributed within the group is chosen by the leader to maximize her probability of success. In our model, the produced private goods can be distributed either uniformly as in Esteban and Ray (2001) or linked to individual performance in order to create incentives.[7] Following Nitzan (1991) we call the uniform split the *egalitarian rule* and the split linked to incentives the *relative effort rule*.[8] The success probability of each group depends on the resources contributed relative to the total. These contributions are private and hence the most studied coordination failure of collective action arises: individuals free-ride on the effort by their fellow group members. Our game has two stages. First the platform is chosen by the group leader and at the second stage individuals privately decide how much to contribute.

Our results are the following. We first solve for the group optimal platform defined as the platform that maximizes the utility of the representative group member assuming no coordination at the effort decision stage, i.e. effort contributions are private decisions and not group decisions. We show that the degree of publicness chosen is increasing in the group size and that the preferred distribution of the private benefits is the egalitarian rule over the relative effort rule in spite of the potentially beneficial effects of incentives on free-riding in the effort decision stage. Against this (group decision) benchmark we contrast the choices made by opportunistic leaders. We find that the platform chosen by the leader is biased towards more privateness

[7] Olson (1965) already mentioned the introduction of incentives as the way to solve the free-riding problem. Bandiera et al. (2005) obtain experimental evidence on the effects of using piece-rate in collective action.

[8] Lee (1995) and Ueda (2002) endogenize the choice of the sharing rule in a two-stage game. In the first stage each group chooses the sharing rule from all possible linear combinations of the egalitarian and the relative effort rule that maximizes the group's joint welfare. In the second stage individuals make their effort choices. In both papers (which assume linear benefit functions for the prize and linear cost functions for effort) all groups with a size smaller than half of the total population choose to give all the weight to the relative effort rule. Notice that the relative effort rule gives the highest weight to incentives and therefore maximizes win probabilities.

and distributes the private benefits on the basis of relative performance. This bias for privateness increases with group size. The existence of a bias in the decisions of opportunistic leaders is not a surprise since their interests are not aligned with those of the community. However, it is surprising that the divergence of goals of the leader vis-a-vis the community produces a social loss to the group only if the leader is allowed to depart from equality and establish incentives. Concerning the old discussion whether large groups are at disadvantage in collective action we obtain the anti-Olson's result that they are not: under both systems —group decision and opportunistic leaders— larger groups have a higher success probability.

The remainder of the paper is organized as follows. In Section 2 the model is presented. Section 3 solves for the equilibrium of the private contributions in the second stage of the game. Section 4 is devoted to the choice of platform in the first stage of the game. We start by solving the benchmark case of group decision and then derive the platform and distribution rule that would be chosen by an opportunistic leader. The efficiency properties of the platform are discussed and it is shown how the choice of the platform varies with group size and its effect on win probabilities is derived. Section 5 puts together the different results obtained and identifies the different losses generated by leadership. Section 6 concludes. Proofs are relegated to a technical appendix.

2 The Model

Suppose that there are G different types of public goods and the same number of types of preferences, accordingly with the type of public good individuals prefer. Individuals are assumed to derive utility from their own type of public good only. Let n be the total population and $n_1, n_2, ..., n_G$ be the population of the G types of preferences. Without loss of generality, we assume that $n_i \leq n_{i+1}$.

We assume that people with the same preferences form a group. Each group is organized as a lobby (or political party), competing with the opposing groups in view of controlling the allocation of the public budget b. Group competition is modelled as a two-stage game. In the first stage the group platform is fixed and in the second stage individuals privately decide how much to contribute.

The group platform has two ingredients: (i) the share λ of the budget

b to be allocated to the production of the group-specific public good, and (ii) the specification of how the private good produced with the remaining budget will be allocated among the group members.

We will study two different ways in which the private good is transferred to the group members: an egalitarian split and transfers intended to provide incentives for collective action. In the latter case the platform specifies that each individual receives a proportion of the group money equal to their share of total group effort.

Platforms are set by group leaders who only care about the probability of success and therefore choose the platform in view of maximizing the win probability of the group they lead irrespective of the private cost of the contributions to the group members.

In the second stage, in view of the group platform and of the contributions by the others, individuals decide how much to contribute to the collective cause. Individual contributions determine the win probabilities of each group. Finally the winning group is chosen by nature. We assume that the contribution always is a private decision and hence individuals free-ride on the effort chosen by their fellow group members.

Individuals contribute effort r_i in support of the platform of their group. We choose these units of effort so that effort is *added* across group members to yield group effort R_i. As in Esteban and Ray (2001) we model the utility cost of effort $c(r)$ as an increasing smooth, convex function with $c'(0) = 0$ and $c'(\infty) = \infty$, and with an elasticity $\eta(r)$

$$\eta(r) = \frac{rc''(r)}{c'(r)} \geq 1.^{9} \tag{1}$$

We make the standard assumption that the probability that alternative i will be chosen, p_i, equals the effort level of group i, R_i, relative to the aggregate amount of effort R exerted by *all* groups. Therefore, letting r_{ik} be the effort contributed by individual k of group i, we have that

$$p_i = \frac{R_i}{R} = \frac{\Sigma_k r_{ik}}{R}.^{10} \tag{2}$$

[9] This elasticity plays a crucial role for the main results of Esteban and Ray (2001). In particular in their model $\eta(r) \geq 1$ is a necessary and sufficient condition for the unconditional reversal of Olson's results, that is, that the win probabilities be strictly increasing in group size irrespective of the group platform.

[10] Notice that the win probability is not defined when $R_j = 0, \forall j$. We shall simply assume that in this case $p_i = \frac{n_i}{n}$.

Individual preferences are additively separable in the concave valuation of the public good, $v(.)$, and the linear valuation of the private good. On $v(.)$ we assume it be strictly concave with the Inada limit conditions: $\lim_{z\to 0} v'(z) = \infty$ and $\lim_{z\to\infty} v'(z) = 0$. Furthermore, we assume that $v'(b) \leq \frac{1}{n_G}$. This assumption simply establishes the size of prize b relative to the (largest) group size.

As already mentioned, we will study two different rules how to distribute the private good. The first rule, the egalitarian rule, consists of an egalitarian division among all group members. In this case, individual decisions cannot modify the size of the transfer. The second rule, the relative effort rule, establishes incentives to reward the effort contributed; each group member receives a proportion of the group money equal to their share of total group effort. Hence, the transfer received $\frac{(1-\lambda_i)br_{ik}}{R_i}$ does depend on individual's decisions r_{ik}.

In the case of egalitarian transfers the expected utility u_{ik} of member k of group i is given by

$$u_{ik} = p_i \left(v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) b}{n_i} \right) - c(r_{ik}) \tag{3}$$

and with incentives it is

$$u_{ik} = p_i \left(v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) br_{ik}}{R_i} \right) - c(r_{ik}). \tag{4}$$

For convenience we denote by $\omega_{ik}(\lambda_i, n_i, r_{ik})$ the payoff of member k of group i in case of victory. Therefore, either under egalitarian transfers

$$\omega_{ik}(\lambda_i, n_i, r_{ik}) = \omega_i(\lambda_i, n_i) = v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) b}{n_i}, \tag{5}$$

or under incentives

$$\omega_{ik}(\lambda_i, n_i, r_{ik}) = v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) br_{ik}}{R_i}. \tag{6}$$

In general, we will write

$$u_{ik} = p_i \omega_{ik}(\lambda_i, n_i, r_{ik}) - c(r_{ik}). \tag{7}$$

This game has two stages. First, groups or leaders choose the platform and then individuals privately decide how much to contribute. We solve the game backwards. In the next section we characterize the Nash equilibrium of the contributions game for given platforms that is played in the second stage of the game. In section 4 we shall deal with the first stage of the game.

3 Equilibrium Contributions in the Second Stage of the Game

In the second stage, given the group platforms, individuals choose their effort. We assume that each individual takes its best course of action, given the behavior of the rest of the population —both, fellow group members and the rest— and hence free-rides on the effort contributed by the other group members. Therefore, in view of (2) the effect of an increase in effort r_{ik} on the win probability will be strictly increasing and strictly concave. Indeed,

$$\frac{\partial p_i}{\partial r_{ik}} = \frac{R - \sum_k r_{ik}}{R^2} = \frac{1}{R}(1 - p_i) > 0.$$

and

$$\frac{\partial^2 p_i}{\partial r_{ik}^2} = -\frac{1}{R^2}(1 - p_i) - \frac{1}{R}\frac{\partial p_i}{\partial r_{ik}} = -\frac{2}{R^2}(1 - p_i) < 0.$$

Differentiating u_{ik} with respect to effort r_{ik} we obtain

$$\frac{\partial u_{ik}}{\partial r_{ik}} = \frac{\partial p_i}{\partial r_{ik}}\omega_{ik}(\lambda_i, n_i, r_{ik}) + p_i\frac{\partial \omega_{ik}(\lambda_i, n_i, r_{ik})}{\partial r_{ik}} - c'(r_{ik}). \tag{8}$$

The effect of r_{ik} on ω_{ik} depends on whether the sharing rule is egalitarian or the relative effort rule. In the first case the change in effort has obviously no effect on ω_{ik}. For the relative effort case ω_{ik} is strictly increasing and strictly concave in r_{ik}. Differentiating we have

$$\frac{\partial \omega_{ik}}{\partial r_{ik}} = (1 - \lambda_i)b\frac{R_i - r_{ik}}{R_i^2} > 0,$$

and

$$\frac{\partial^2 \omega_{ik}}{\partial r_{ik}^2} = -\frac{2}{R_i}\frac{\partial \omega_{ik}}{\partial r_{ik}} < 0.$$

We first show that the first order condition

$$\frac{\partial u_{ik}}{\partial r_{ik}} = 0 \tag{9}$$

implies a within-group symmetric equilibrium.

Lemma 1 *For each group i there is a unique r_i that satisfies (9) for each individual k.*

Proof See appendix.

Lemma 2 states that (9) indeed characterizes a maximum.

Lemma 2 *In each case —egalitarian and incentives— the unique r_i maximizing u_i is implicitly given by*

$$\frac{(1-p_i)}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)\,b}{n_i}\right) - c'(r_i) = 0 \text{ (egalitarian)} \tag{10}$$

$$\frac{(1-p_i)}{R}(v(\lambda_i g) + \frac{(1-\lambda_i)\,br_i}{R_i}) + p_i\frac{(1-\lambda_i)\,b(R_i - r_i)}{R_i^2} - c'(r_i) = 0 \text{ (incentives)} \tag{11}$$

Proof See appendix.

Notice, that for given R determining r_i is equivalent to determining p_i. For future use, we rewrite the FOC that characterize the best response r_i under both sharing rules as:

$$\frac{1}{R}(1-p_i)\omega_i(\lambda_i, n_i) - c'(\frac{p_i R}{n_i}) = 0 \ (egalitarian) \tag{12}$$

$$\frac{1}{R}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{(1-\lambda_i)b(n_i-1)}{n_i R} - c'(\frac{p_i R}{n_i}) = 0 \ (incentives), \tag{13}$$

where we have used the fact that

$$r_i = \frac{p_i R}{n_i}. \tag{14}$$

Expressions (12) and (13) implicitly define the win probability p_i as a function of the exogenous parameters (λ_i, n_i, b) and of the *endogenous* value of R. We shall write

$$p_i = \psi(R, \lambda_i, n_i, b) \tag{15}$$

for the two cases with egalitarianism and incentives. The following Lemma will be instrumental in proving existence and uniqueness of a Nash equilibrium of the contribution game.

Lemma 3 *$p_i = \psi(R, \lambda_i, n_i, b)$ is a continuous strictly decreasing function of R for all i.*

Proof See appendix.

Definition 1 *A Nash equilibrium of the second stage of the game is a vector $\mathbf{p}^*$ and a value R^* such that*

$$\sum_j \psi(R^*, \lambda_j, n_j, b) = 1 \tag{16}$$

and for all j

$$p_j^* = \psi(R^*, \lambda_j, n_j, b). \tag{17}$$

Notice that from the equilibrium p^*, R^* we can immediately obtain r_i^* for all $i = 1, ..., G$ using equation (14).

We can now establish the existence and uniqueness of equilibrium.

Proposition 1 *For every set of parameters (λ, n, b) there exists an equilibrium of the second stage of the game and it is unique.*

Proof See appendix.

It follows that we can write

$$R^* = \rho(\lambda, n, b). \tag{18}$$

Therefore, the equilibrium win probabilities are

$$p_i^* = \psi(\rho(\lambda, n, b), \lambda_i, n_i, b). \tag{19}$$

We are now set for the analysis of the choice of platform.

4 Choice of Platform in the First Stage

Leaders choose the platform that maximizes their probability of success, i.e. the win probability of their group. This objective does not coincide with the maximization of the expected payoff to the representative group member. In order to evaluate the potential bias introduced by political leaders we shall take the utility maximizing platform as a benchmark case. This platform would result from group decision or if leaders were altruistic. We shall therefore talk of *opportunistic* (probability maximizing) and *altruistic* (utility maximizing) leaders. We shall thus contrast the choice of platform made by an opportunistic leader, λ_i^{oh}, with the one that would have been chosen by an altruistic leader, λ_i^{ah}, where $h = e, s$ stands for the egalitarian and the relative effort sharing rules.

Leaders take into account that *all* group members will change their behavior in response to changes in λ_i. Leaders know the best reply of each individual group member and use this information when deciding on the desired platform. Notice that this best reply depends on the effort contributed by the members of the other groups —through the win probability— but not directly on the specific platform that they might have adopted. Therefore, an equilibrium will require that there exists a vector of probabilities such that all the associated platforms —and the individual contributions— be best responses to each other.

For the sake of completeness let us introduce the formal definition of equilibrium of the full two stage game.

Definition 2 *The vectors (r, λ^{kh}), $k = a, o$ and $h = e, s$, are an equilibrium if, given the vector λ^{kh}, each r_{ik} is best response and, given the vector r, each λ_i^{kh} is best response.*

4.1 Preliminaries

An opportunistic leader will choose λ_i^{oh} so as to maximize p_i. The corresponding first order condition for a maximum implies that λ_i^{oh} solves

$$\frac{dp_i}{d\lambda_i} = 0.$$

An altruistic leader instead seeks to maximize the expected utility of group members. The first order condition implies that λ_i^{ah} solves

$$\frac{du_i}{d\lambda_i} = \frac{dp_i}{d\lambda_i}\omega_i + p_i\frac{d\omega_i}{d\lambda_i} - \frac{dc}{d\lambda_i}. \qquad (20)$$

Developing the differentiation and reorganizing terms equation (20) can be rewritten as

$$\frac{du_i}{d\lambda_i} = \left(\omega_i - \frac{R}{n_i}c'(p_i\frac{R}{n_i})\right)\frac{dp_i}{d\lambda_i} + p_i\frac{d\omega_i}{d\lambda_i} - \frac{p_i}{n_i}c'\left(p_i\frac{R}{n_i}\right)\frac{dR}{d\lambda_i} = 0. \qquad (21)$$

Observe that for leadership and our benchmark case we will need to compute $\frac{dp_i}{d\lambda_i}$. Differentiating (19) we obtain,

$$\frac{dp_i}{d\lambda_i} = \frac{\partial\psi(\lambda_i)}{\partial R}\frac{dR}{d\lambda_i} + \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}.$$

We start by computing $\frac{dR}{d\lambda_i}$. Differentiating (16) with respect to R and λ_i we get

$$\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}dR + \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}d\lambda_i = 0.$$

That is

$$\frac{dR}{d\lambda_i} = -\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}\frac{1}{\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}} \qquad (22)$$

Reorganizing and substituting above, we obtain

$$\frac{dp_i}{d\lambda_i} = \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}\left(1 - \frac{\frac{\partial\psi(\lambda_i)}{\partial R}}{\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}}\right). \qquad (23)$$

By Lemma 1 we know that p_i is a strictly decreasing function of R. It follows that the fraction within the braces is positive and strictly less than 1. So, the braces are always positive and less than 1. Hence,

Remark 1 *The sign of* $\frac{dp_i}{d\lambda_i}$ *is equal to the sign of* $\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}$.

Furthermore, in view of (22) we also have the following useful result:

Remark 2 *The sign of* $\frac{\partial R}{\partial\lambda_i}$ *is equal to the sign of* $\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}$.

Note that in the derivation of (22) and (23) we have not made any assumption on whether the distribution of the private benefits was egalitarian or based on incentives. Hence, Remarks 1 and 2 are valid for both cases. These two remarks will be essential in characterizing the optimal platforms.

4.2 The Group Optimal Benchmark

We start with the benchmark case of the choice made by altruistic leaders maximizing the well-being of the representative group member. We will first derive the group optimal platform under an egalitarian sharing rule λ_i^{ae}. Then we will show that the egalitarian sharing rule is part of the overall preferred group optimal platform. In other words we will prove that altruistic leaders will not introduce the relative effort rule. Finally, we will derive some properties of the group optimal sharing rule relative to group size.

We first examine the chosen platform under an egalitarian sharing of the private good. Recalling the first order condition for a maximum, we have that λ_i^{ae} solves

$$\frac{du_i}{d\lambda_i} = \frac{dp_i}{d\lambda_i}\omega_i + p_i\frac{d\omega_i}{d\lambda_i} - \frac{dc}{d\lambda_i} = 0.$$

We can easily compute that

$$\frac{d\omega_i}{d\lambda_i} = b\left(v'(\lambda_i b) - \frac{1}{n_i}\right).$$

Since $v(.)$ is a strictly concave function and because of our assumption that $v'(b) \leq \frac{1}{n_G}$ it is immediate that there is a unique $\lambda_i^o \in (0,1]$ that maximizes $\omega(b, \lambda_i, n_i)$.

Clearly, when the win payoff ω_i is maximal so are the incentives to supply effort and hence the win probability. However, an altruist leader should also take into account the individual cost of supplying such effort. The following result shows that an altruist leader still would choose to maximize the equilibrium win payoff.

Proposition 2 *Under an egalitarian division the optimal platform chosen by an altruistic leader λ_i^{ae} maximizes the equilibrium win payoff. This platform is implicitly defined by*

$$n_i v^{'}(\lambda_i^{ae} b) = 1. \tag{24}$$

Proof See appendix.

Notice that λ_i^{ae} is independent of all the endogenous variables and hence independent of the platforms chosen by the other groups under any type of leadership.

Let r_i^{ae} be the effort contributed by each individual under this platform. The expected equilibrium payoff to each group member will be

$$u_i^{ae} = \frac{n_i r_i^{ae}}{R_{i-} + n_i r_i^{ae}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{ae}). \tag{25}$$

We shall now compare this utility with the utility of a representative group member under the relative effort rule u_i^{as} which will allow us to determine the overall group optimal platform. The expected equilibrium payoff to each group member under the relative effort rule is

$$u_i^{as} = \frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{as}, n_i) - c(r_i^{as}) \tag{26}$$

where λ_i^{as} is the group optimal degree of publicness under the relative effort rule and r_i^{as} is the corresponding contributed effort.

Notice that in a second stage equilibrium all members of a group i contribute the same effort and hence each group member receives an equal share $\frac{1}{n_i}$ of the private good. Therefore for a given λ_i the equilibrium win payoff $\omega_i = v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}$ is the same under the relative effort rule and under the egalitarian rule. Of course, the resources contributed under the two sharing rules will in general not be the same. This will allow us to compare u_i^{as} and u_i^{ae} without characterizing λ_i^{as}.

We have shown that λ_i^{ae} maximizes $\omega(.)$. Therefore,

$$\frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{as}) \geq u_i^{as}.$$

But since for λ_i^{ae} individuals choose $r_i^{ae} \neq r_i^{as}$, by revealed preference we have that

$$u_i^{ae} = \frac{n_i r_i^{ae}}{R_{i-} + n_i r_i^{ae}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{ae}) > \frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{as}) \geq u_i^{as}.$$

The preference of equality over incentives is not conditioned by the platforms of the other groups. We thus have proven the following Proposition.

Proposition 3 *Let groups choose their own platform. Irrespective of the platforms adopted by the other groups and of their type of leadership, groups will choose the platform consisting of* λ_i^{ae} *and the egalitarian distribution of the private good.*

Proposition 3 has the remarkable implication that if groups were able to self-organize they would choose an egalitarian distribution of the private good even if efficient incentives to individual performance were available. It follows that incentives are a sign that groups were not able to self-organize. In the following section we will show that incentives are in the personal interest of opportunist leaders. But before doing so we want to examine the overall group optimal platform in more detail. In particular, we are interested how it changes with group size and how it affects win probabilities depending on group size. From expression (24) the following result is immediate.

Proposition 4 *The group optimal platform chosen by an altruistic leader* λ_i^{ae} *is increasing in the group size,* n_i*.*

This proposition tells us that the larger the group the more "socially minded" will be their platform. Therefore, small groups will appear as greedier than large group. This platform choice also affects the win probabilities of the groups. The following proposition generalizes Esteban and Ray (2001)'s result.

Proposition 5 *Under altruistic leadership the win probability* p_i *is increasing in the group size,* n_i*.*

Proof See appendix.

Observe that in Esteban and Ray (2001) some restrictions are needed for win probabilities to always increase with group size. In particular for nearly linear cost functions the social component of the platform - which is identical for all groups - has to be sufficiently large. We do not need this restriction. In our model larger groups choose a larger share of publicness in their platform and thereby mitigate the free-rider problem by reducing the effect of group size on the payoff of group members. This is why larger groups succeed in having higher win probabilities.

4.3 Opportunistic Leaders

Opportunistic leaders seek to maximize their win probability. We start by characterizing the chosen λ_i^{oh} under the two different sharing rules and then show that opportunistic leaders would opt for the introduction of incentives.

We first characterize the optimal platform by an external leader under an egalitarian division.

Proposition 6 *Under an egalitarian division the optimal platform chosen by an external, opportunistic leader coincides with the group-optimal platform. This platform λ_i^{oe} is implicitly defined by*

$$n_i v'(\lambda_i^{oe} b) = 1. \tag{27}$$

Proof See appendix.

The intuition for this result is quite straightforward. The effort contributed by individuals is an increasing function of the payoff in case of victory. Hence, opportunistic leaders will choose the composition public/private that maximizes the value of ω_i. Notice that in the absence of individualized incentives leaders act as if they were concerned with the well-being of their constituency. Without incentives there would be no cost to external, opportunistic leadership. Our previous results for group decision apply here: the public good share and the win probability increase with group size.

Opportunistic leaders constrained to be egalitarian choose platforms such that larger groups have higher win probabilities. This is achieved by assigning a larger share of the budget to public goods the larger is the group. But this is an efficient choice by the leader only in as much as the distribution of the private good is restricted to be egalitarian.

Opportunistic leaders might do better by introducing incentives that link the amount of the private good received by each individual to her supply of effort. But now, the larger the share of the public good, the smaller the budget that can be used as incentives for collective action. We shall now examine this trade-off and check whether large groups continue to be more "socially minded" and have higher win probabilities.

As a quick reminder, let us recall the implicit characterization of the optimal individual choice with incentives to effort. Transcribing (13) we have

$$\frac{1}{R}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{(1-\lambda_i)b(n_i-1)}{n_i R} - c'(\frac{p_i R}{n_i}) = 0 \ .$$

This implicitly defines

$$p_i = \xi(R, \lambda_i, n_i).$$

The equilibrium R^* is obtained from the condition

$$\sum_j \xi(R^*, \lambda_j, n_j, b) = 1.$$

which implicitly defines

$$R^* = \phi(\lambda, n).$$

Hence, the equilibrium win probabilities will be

$$p_i = \xi\left(\phi(\lambda, n), \lambda_i, n_i\right).$$

External leaders will choose λ_i in order to maximize this equilibrium p_i. Accordingly with Remark 1

$$sign\frac{dp_i}{d\lambda_i} = sign\frac{\partial\xi(\lambda_i)}{\partial\lambda_i}.$$

Differentiating (13) we obtain

$$\begin{aligned}\frac{\partial\xi(\lambda_i)}{\partial\lambda_i} &= \frac{\frac{1}{R}(1-p_i)\left[bv'(\lambda_i b) - \frac{b}{n_i}\right] - \frac{b(n_i-1)}{Rn_i}}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''\left(\frac{p_i R}{n_i}\right)} \\ &= \frac{\frac{(1-p_i)b}{Rn_i}\left[n_i v'(\lambda_i b) - \frac{n_i - p_i}{1-p_i}\right]}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''\left(\frac{p_i R}{n_i}\right)}.\end{aligned} \tag{28}$$

The sign of (28) depends on the sign of its numerator only. This is strictly decreasing in λ_i, strictly positive for $\lambda_i = 0$ and strictly negative for $\lambda_i = 1$. Hence, we have the following Proposition.

Proposition 7 *Let there be incentives to effort, then an external leader will choose the unique* λ_i^{os} *satisfying*

$$n_i v'(\lambda_i^{os} b) = \frac{n_i - p_i}{1 - p_i} = 1 + \frac{n_i - 1}{1 - p_i}. \tag{29}$$

Observe that the need for incentives does not make leaders precipitate complete privateness. However, as shown by the following proposition, the use of incentives by opportunistic leaders distorts platforms towards less publicness than under an egalitarian split.

Proposition 8 $\lambda_i^{os} < \lambda_i^{oe}$. *Furthermore,* λ_i^{os} *maximizes waste* R.

Proof See appendix.

Under the relative effort rule opportunistic leaders give too much weight to the private good and therefore overuse incentives relative to what is optimal for the group.

Let us now examine how this distortion affects the relationship between group size and the publicness of the platform and the win probability in a given equilibrium. To do this, consider the implicit characterization of the optimal platform chosen by a leader is given by (29). In any given equilibrium —and hence for R fixed— this equation has to hold for all groups, i.e. for all n_i. Therefore, we can obtain the relationship between publicness and group size by differentiating with respect to λ and n in (29) while holding R constant. We find that with individual incentives to effort leaders choose platforms that still give larger groups a higher win probability.

Proposition 9 *Let there be incentives to the supply of effort, then the win probability increases with group size, but the degree of publicness of the platform decreases with size.*

Proof See appendix.

The efficiency of the larger groups is at the cost of reducing publicness. Notice that now it is thanks to an increased share of private goods that larger groups succeed to have higher win probabilities. With incentives smaller groups appear to be more socially minded.

But would we observe incentives under external leadership? We shall now show that, unlike the case of group decision, opportunistic leaders prefer the use of incentives, irrespective of the platforms and leadership of the other groups.

Proposition 10 *Taking the behavior of the other leaders as fixed, all leaders can increase their win probability by introducing incentives.*

Proof See appendix.

We wish to remark two points here. First, leaders prefer to introduce incentives *irrespective of whether the other leaders are using incentives or not*. Secondly, note that moving from egalitarianism to incentives increases the win probability because it also increases the amount of effort contributed by the individual group members.[11] From Proposition 10 we directly have the following proposition.

Proposition 11 *Let opportunistic leaders be able to choose the platform, including the option between an egalitarian distribution of the private good or incentives. Then, there is a unique equilibrium in which all opportunistic leaders use the platform λ_i^{os} (hence with incentives).*

5 Individual Incentives and the Costs of Leadership

We now summarize the previous results to assess the cost of external leadership.

The first and basic point to stress is that opportunistic political entrepreneurs can have a negative bias only through the use of incentives. If political entrepreneurs were restricted to use the egalitarian sharing rule they would choose the group optimal platform. It is due to the use of incentives that the diverging goals of the leader and the community materialize and produce inefficient biases. In contrast, in group decision incentives are not used even if available in spite of the potentially beneficial effects of incentives on free-riding in the effort decision stage.

The contrast with the benchmark group-decision case reveals that leadership biases the platforms towards greater greediness (higher share of pocketable benefits) and greater resources expended (wasted) into trying to win command over the budget. Therefore individuals suffer the costs on two counts: (i) they don't obtain the public/private mix they would prefer and (ii) they expend more resources than what they would otherwise do.

These biases vary with group size. The bias towards greediness increases with group size. While in the group decision case —or opportunistic leaders deprived from the use of incentives— the degree of privateness decreases with group size, leaders with access to incentives precipitate a degree of privateness that is increasing in group size.

The second type of bias concerns the over-expending of resources. The unilateral use of incentives will lead to higher win probabilities, but if all

[11] Of course, it must also decrease the win probability of other groups. We shall deal with this issue in the next section.

groups use incentives and hence spend more resources, it cannot be that all the win probabilities increase. Therefore, despite the increased spending of resources some groups will end up with an even lower win probability than under group decision. This is a third dimension of the loses induced by opportunistic leaders. The following proposition identifies who are the losers and who are the winners in terms of success probability resulting from the over-expending of resources.

Proposition 12 *Let the cost function have constant elasticity* $c(r) = \frac{1}{1+\eta}r^{1+\eta}$ *and let* $v(.)$ *be sufficiently close to linearity. There is a threshold level in group size such that the groups with a smaller size will have a lower win probability in the equilibrium with incentives than with an egalitarian distribution and all groups with a larger size will have a higher win probability.*

Proof See appendix.

From this Proposition it follows that if the economy were in an equilibrium with egalitarianism and leaders could compare equilibria, it would be the leaders of the larger groups the ones that would have an interest in unfolding a process of introduction of incentives.[12]

6 Concluding Remark

Free-riding has traditionally been seen as a source of inefficiency in collective action problems. Because of the failure to coordinate and commit to the group optimal action, the total resources contributed are below optimal. In the present paper we point to additional costs on top of this intrinsic inefficiency. They result from a coordination problem so far overlooked by the literature, namely the setting of the group platform, the decision on what

[12] The case of higher degrees of concavity in $v(.)$ remains to be studied. It seems plausible that for high degrees of concavity the result gets reversed and the smaller groups are the main beneficiaries.

To see why observe that there are two counter-vailing forces at work. Incentives eliminate free-rider problems because individuals are rewarded by what they contribute only. On the one hand, free-riding is more severe in larger groups since an individuals deviation has a smaller effect on the win probabilities in bigger groups. Hence larger groups benefit more from the elimination of free-riding. On the other hand, smaller groups can pay higher per unit incentive rates than larger groups. Which effect dominates will depend on the concavity of $v(.)$.

exactly groups are fighting for. Group platforms consist of the specification of the public/private balance and how the private benefits will be distributed within the group.[13]

The economic analysis of collective action has taken the group platform as exogenously given. Under this assumption Esteban and Ray (2001) have proven that the conjecture by Olson (1965) that smaller groups have a higher win probability is incorrect. They obtain that unless the prize is a pure private good and the cost of effort is close to linearity large groups have a higher probability of success. This result is reinforced when group leaders can choose the platforms in view to limit the effects of free-riding. Indeed, we find that both under group decision and under leadership the win probability increases with group size. However, while the group optimal platform entails a degree of publicness that raises with group size, opportunistic leaders choose platforms displaying degrees of publicness that diminish with group size.

Our final comment refers to future research. In this paper we have contrasted the behavior of opportunistic leaders with what would have been chosen by the group if they had been able to coordinate. In the real world we observe the two kinds of organizations (and possibly many in between those extremes). What makes some groups able to reach decisions while others simply follow a leader is an open question. Group size has obviously a role to play[14] and so does the importance of the issue at stake.

7 Appendix

Lemma 1 For each group i there is a unique r_i that satisfies (9) for each individual k.

Proof. For the equalitarian case (9) becomes

$$\frac{1-p_i}{R}\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)-c'(r_{ik})=0$$

It is immediate that this equation has a unique solution $r_{ik}=r_i>0$ for all individuals k of group i.

[13] As Berry (1977) pointed out even the most socially minded public interest groups whose primary purpose is the pursuit of collective goods that will not selectively and materially reward their members have typically lobbied on legislation that affect their tax cuts - surely a private interest matter. See page 10, footnote 11.

[14] But in classical Greece the assembly of citizens, the ultimate decision body, could gather well over 10,000 participants.

For the case with incentives under the relative effort rule we have that

$$p_i \frac{\partial \omega_{ik}}{\partial r_{ik}} = p_i \frac{(1-\lambda_i)\, b(R_i - r_{ik})}{R_i^2} = \frac{(1-\lambda_i)\, b}{R}(1 - \frac{r_{ik}}{R_i}).$$

Using this fact in (9) and rearranging we obtain

$$\frac{\partial u_{ik}}{\partial r_{ik}} = \frac{1}{R}\Big((1-p_i)v(\lambda_i g) + (1-\lambda_i)b\Big) - \Big(\frac{(1-\lambda_i)b)}{R^2} r_{ik} + c'(r_{ik})\Big).$$

Notice that the first brackets is common to all k of group i and that the second brackets is strictly increasing in r_{ik}. Therefore, there is a unique value $r_{ik} = r_i = \frac{R_i}{n_i}$ that solves $\frac{\partial u_{ik}}{\partial r_{ik}} = 0$. ■

LEMMA 2 *In each case —egalitarian and incentives— the unique r_i maximizing u_i is implicitly given by*

$$\frac{1-p_i}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) - c'(r_i) = 0 (egalitarian), or \tag{30}$$

$$\frac{(1-p_i)}{R}(v(\lambda_i g) + \frac{(1-\lambda_i)\, b r_i}{R_i}) + p_i \frac{(1-\lambda_i)\, b(R_i - r_i)}{R_i^2} - c'(r_i) = 0 (incentives). \tag{31}$$

Proof. Differentiating (7) with respect to r_i we obtain the first order condition

$$\frac{\partial u_i}{\partial r_i} = \frac{1-p_i}{R}\omega_i + p_i \frac{\partial \omega_i}{\partial r_i} - c'(r_i) = 0. \tag{32}$$

We start by noticing that under an egalitarian distribution $\frac{\partial \omega_i}{\partial r_i} = \frac{\partial^2 \omega_i}{\partial^2 r_i} = 0$, with incentives $\frac{\partial \omega_i}{\partial r_i} = \frac{(1-\lambda_i) b (R_i - r_i)}{R_i^2}$ and $\frac{\partial^2 \omega_i}{\partial^2 r_i} = -\frac{2(1-\lambda_i) b (R_i - r_i)}{R_i^3}$.

Observe now that $\frac{\partial u_i}{\partial r_i}$ is continuous in r_i for $R > 0$ and that $\lim_{r_i \to 0} \frac{\partial u_i}{\partial r_i} = \frac{\omega_i}{R} > 0$ and that $\lim_{r_i \to \infty} \frac{\partial u_i}{\partial r_i} = -\infty$. Therefore, there must exist at least one $r_i > 0$ for which $\frac{\partial u_i}{\partial r_i} = 0$. Suppose now that $r_i = 0$ and $R = 0$ so that $p_i = \frac{n_i}{n} < 1$. By taking an arbitrarily small $r_i > 0$ we would make $p_i = 1$. Hence in no case $r_i = 0$ can be a best reply for an individual of type i.

We shall now show that there is a unique $r_i > 0$ satisfying the first order condition $\frac{\partial u_i}{\partial r_i} = 0$ and that this condition indeed identifies a maximum.

Differentiating (32) with respect to r_i we obtain

$$\frac{\partial^2 u_i}{\partial r_i^2} = -2\frac{1-p_i}{R^2}\omega_i + 2\frac{1-p_i}{R}\frac{\partial \omega_i}{\partial r_i} + p_i\frac{\partial^2 \omega_i}{\partial r_i^2} - c''(r_i).$$

We can immediately deduce that under egalitarianism u_i is strictly concave in r_i.

For the case with incentives this second derivative becomes

$$\begin{aligned}\frac{\partial^2 u_i}{\partial r_i^2} &= -\frac{2}{R^2}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right) + \frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i - r_i)}{R_i^2} \\ &\quad -p_i\frac{2\,(1-\lambda_i)\, b(R_i-r_i)}{R_i^3} - c''(r_i)\end{aligned}$$

Since the first and the third term on the right hand side are negative, if we can show that the second term plus the fourth term are negative, we are done. Hence, we need to look at

$$\frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i-r_i)}{R_i^2} - c''(r_i)$$

Using now (31) we obtain

$$\begin{aligned}&\frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i-r_i)}{R_i^2} - c''(r_i) \\ =\ &\frac{2}{R}(1-p_i)\left[\frac{c'(r_i)}{p_i} - \frac{1}{R}\frac{(1-p_i)}{p_i}\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right)\right] - c''(r_i) \\ =\ &\frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i) - \frac{2}{R^2}\frac{(1-p_i)^2}{p_i}\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right)\end{aligned}$$

If we can show that $\frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i)$ is negative we are done.

$$\begin{aligned}&\frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i) \\ =\ &\frac{c'(r_i)}{r_i}\left[\frac{2(1-p_i)r_i}{Rp_i} - \eta(r_i)\right] \\ =\ &\frac{c'(r_i)}{r_i}\left[\frac{2(1-p_i)}{n_i} - \eta(r_i)\right]\end{aligned}$$

since $p_i = \frac{n_i r_i}{R}$ in a symmetric equilibrium. But $\frac{2(1-p_i)}{n_i} - \eta(r_i) < 0$ since $n_i \geq 2$ and $\eta(r_i) \geq 1$. Therefore $\frac{\partial^2 u_i}{\partial r_i^2} < 0$. ■

LEMMA 3 $p_i = \psi(R, \lambda_i, n_i, g)$ *is a continuous strictly decreasing function of* R *for all* i.

Proof. Using the implicit definition of p_i in (12) and (13) for the egalitarian and incentives rule we observe that ψ is continuous because both v and c' are continuous. Furthermore, differentiating with respect to R we have:

$$\frac{\partial p_i}{\partial R} = -\frac{\frac{(1-p_i)\omega_i}{R^2} + \frac{p_i c''}{n_j}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < 0 \text{ (egalitarian)}.$$

$$\frac{\partial p_i}{\partial R} = -\frac{\frac{1}{R^2}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{(1-\lambda_i)b(n_i-1)}{R^2} + \frac{p_i}{n_i}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} < 0 \textit{ (incentives)}$$

■

PROPOSITION 1 *For every set of parameters* (λ, n, g) *there exists an equilibrium and it is unique.*

Proof. We have already seen that for each R there is a unique vector of win probabilities defined by (15). The only point that remains to be proven is that there is a unique value of R satisfying (16). By Lemma 3 p_i is a continuous strictly decreasing function of R for all i.

An inspection of the first order conditions (12) and (13) for the egalitarian and the incentives case, respectively, immediately reveals that in both $p \to 1$ as $R \to 0$ and $p \to 0$ as $R \to \infty$. Hence there is a unique R^* satisfying the equilibrium condition (16). ■

PROPOSITION 2 Under an egalitarian division the optimal platform chosen by an altruistic λ_i^{ae} maximizes the equilibrium win payoff. This payoff is implicitly defined by

$$n_i v'(\lambda_i^{ae} b) = 1.$$

Proof. Recall that the first order condition (21) is given by

$$0 = \left(\omega_i - \frac{R}{n_i}c'(p_i\frac{R}{n_i})\right)\frac{\partial p_i}{\partial \lambda_i} + p_i\frac{\partial \omega_i}{\partial \lambda_i} - \frac{p_i}{n_i}c'\left(p_i\frac{R}{n_i}\right)\frac{\partial R}{\partial \lambda_i}. \qquad (33)$$

Now replace $c'\left(p_i \frac{R}{n_i}\right)$ using (9) and use equation (22) for $\frac{\partial R}{\partial \lambda_i}$ and equation (23) for $\frac{\partial p_i}{\partial \lambda_i}$.

Then, the first order condition becomes

$$0 = \frac{\partial \omega_i}{\partial \lambda_i}\left(\frac{\partial \psi_i}{\partial \omega_i}\left(1 - \frac{\frac{\partial \psi_i}{\partial R_i}}{\sum_j \frac{\partial \psi_j}{\partial R}}\right)\omega_i\left(1 - \frac{1-p_i}{n_i}\right) + p_i + \frac{p_i}{R}\frac{1-p_i}{n_i}\omega_i \frac{\frac{\partial \psi_i}{\partial \omega_i}}{\sum_j \frac{\partial \psi_j}{\partial R}}\right)$$

Since $\frac{\partial \psi_i}{\partial \omega_i} > 0$ and $\frac{\partial \psi_j}{\partial R} < 0 \; \forall j$, the first term of the sum in the big braces is positive.

For later use we calculate $\frac{\partial \psi_i}{\partial \omega_i}$ and $\frac{\partial \psi_i}{\partial R}$ explicitly:

$$\frac{\partial \psi_i}{\partial \omega_i} = \frac{\frac{1-p_i}{R}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} \tag{34}$$

$$\frac{\partial \psi_i}{\partial R} = -\frac{\frac{(1-p_i)\omega_i}{R^2} + \frac{p_i c''}{n_i}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < 0. \tag{35}$$

We now show that the sum of the second and third term is also positive. Dividing by p_i and rearranging this is equivalent to showing that

$$\frac{1-p_i}{n_i R}\omega_i \frac{\partial \psi_i}{\partial \omega_i} < -\sum_j \frac{\partial \psi_j}{\partial R}$$

After introducing (34) and (35) and rearranging we see that clearly

$$\frac{1-p_i}{n_i}\frac{\frac{(1-p_i)\omega_i}{R^2}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < \sum_j \frac{\frac{(1-p_j)\omega_j}{R^2}}{\frac{\omega_j}{R} + \frac{Rc''}{n_j}} + \sum_j \frac{\frac{p_j c''}{n_j}}{\frac{\omega_j}{R} + \frac{Rc''}{n_j}}.$$

Thus, the sign of $\frac{du_i}{d\lambda_i}$ is the same as the sign of $\frac{\partial \omega_i}{\partial \lambda_i}$.

Recall that

$$\omega_i = v(\lambda_i b) + (1-\lambda)\frac{b}{n_i}.$$

Since $v(.)$ is concave, it follows that $\omega_i(.)$ is also concave. By differentiation we find that the maximum is attained for $\lambda_i = \lambda_i^{ae}$, as defined in this Proposition. ■

Proposition 5 Under altruistic leadership the win probability p_i is increasing in the group size, n_i.
Proof. We can write

$$\frac{dp_i}{dn_i} = \frac{dp_i}{d\lambda_i}\frac{\partial \lambda_i}{\partial n_i} + \frac{\partial p_i}{\partial n_i}.$$

In Proposition 6 we will prove that λ_i^{ae} maximizes p_i. Hence, the above expression becomes

$$\frac{dp_i}{dn_i} = \frac{\partial p_i}{\partial n_i}.$$

Partially differentiating in (12) we obtain

$$\frac{\partial p_i}{\partial n_i} = \frac{R r_i c^{''}(r_i) - \frac{(1-p_i)(1-\lambda_i)b}{n_i}}{n_i \omega_i + R^2 c''(r_i)}.$$

Using (12) we finally obtain

$$\frac{dp_i}{dn} = \frac{Rc^{'}(r_i)\left(\eta(r_i) - 1\right) + (1-p_i)v(\lambda_i g)}{n_i \omega_i + R^2 c''(r_i)} > 0.$$

■

Proposition 6 Under an egalitarian division the optimal platform chosen by an external opportunistic leader coincides with the group-optimal platform. This platform λ_i^{oe} is implicitly defined by

$$n_i v^{'}(\lambda_i^{oe} b) = 1. \tag{36}$$

Proof. From Lemma 1 we have that the sign of $\frac{dp_i}{d\lambda_i}$is the same as the sign of $\frac{\partial \psi_i}{\partial \lambda_i}$.

In view of (12) and of (15) we can write

$$\frac{\partial \psi_i}{\partial \lambda_i} = \frac{\partial \psi_i}{\partial \omega_i}\frac{\partial \omega_i}{\partial \lambda_i}.$$

We have seen that $\frac{\partial \psi_i}{\partial \omega_i} > 0$. Therefore, p_i is maximal when ω_i is maximal. In Proposition 2 we have demonstrated that this maximum is attained for $\lambda_i^{ae} = \lambda_i^{oe}$. ■

Proposition 8 $\lambda_i^o < \lambda_i^e$. Furthermore, λ_i^o maximizes waste R.

Proof. Recall that λ_i^e satisfies $n_i v'(\lambda_i^e b) = 1$. Let's look again at (28). Notice that

$$n_i v'(\lambda_i b) \leq 1 < \frac{n_i - 1}{1 - p_i} \text{ for } \forall \lambda_i \geq \lambda_i^e$$

Hence, we have

$$\frac{\partial \xi(\lambda_i)}{\partial \lambda_i} = \frac{\frac{(1-p_i)b}{Rn_i}\left[n_i v'(\lambda_i b) - \frac{n_i - p_i}{1-p_i}\right]}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} < 0 \text{ for } \forall \lambda_i \geq \lambda_i^e$$

Therefore $\lambda_i^o < \lambda_i^e$. To see that λ_i^o maximizes waste R recall that by remark 2 R attains a maximum with respect to λ_i when p_i is maximal, i.e. at λ_i^o. ■

PROPOSITION 9 *Let the leader be opportunistic and use incentives to the supply of effort, then the win probability increases with group size and the degree of publicness of the platform decreases with size.*

Proof. We start by performing the differentiation with respect to λ and n in (29). Rearranging we obtain,

$$\frac{d\lambda_i}{dn_i} = -\frac{(1-p_i)^2 v'(\lambda_i b) - \left[(1-p_i) + (n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right]}{(1-p_i)^2 b n_i v''(\lambda_i b)}. \tag{37}$$

Notice that because of the concavity of v the denominator is negative. Hence, the sign of the derivative depends on the sign of the numerator and this in turn depends on the sign and size of $\frac{\partial \xi(\lambda_i)}{\partial n_i}$. We turn now to this.

Let us now partially differentiate p_i with respect to n_i in the first order condition for individual choice to obtain

$$\begin{aligned}
\frac{\partial \xi_i}{\partial n_i} &= \frac{-\frac{1}{R}(1-p_i)\frac{(1-\lambda_i)b}{n_i^2} + \frac{(1-\lambda_i)b}{R}\left(\frac{n_i-(n_i-1)}{n_i^2}\right) + \frac{p_i R}{n_i^2}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} \\
&= \frac{\frac{(1-\lambda_i)b}{Rn_i^2}p_i + \frac{p_i R}{n_i^2}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} > 0.
\end{aligned}$$

So the sign of $\frac{d\lambda_i}{dn_i}$ depends on the sign of

$$\begin{aligned}
& (1-p_i)^2 v'(\lambda_i b) - \left[(1-p_i) + (n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right] \\
= \; & (1-p_i)\left[(1-p_i)v'(\lambda_i b) - 1\right] - \left[(n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right] \\
= \; & (1-p_i)\left[\frac{n_i - p_i}{n_i} - 1\right] - \left[(n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right]
\end{aligned}$$

since in the optimal platform $(1-p_i)v'(\lambda_i b) = \frac{n_i - p_i}{n_i}$. But clearly $\left[\frac{n_i - p_i}{n_i} - 1\right] < 0$, so the sign of $\frac{d\lambda_i}{dn_i}$ is negative. ■

PROPOSITION 10 *Taking the behavior of the other leaders as fixed, all leaders can increase their win probability by introducing incentives.*
Proof. We shall show that even holding the platform λ_i constant the leader will increase the win probability when introducing incentives.

Notice first that holding λ_i constant the equilibrium win payoff will be the same in the two cases, ω_i.

The first order condition for individual effort in the case of incentives is

$$\frac{1-p_i}{R}\omega_i + \delta_i = c^{'}(p_i \frac{R}{n_i}),$$

where $\delta_i \equiv \frac{(1-\lambda_i)b(n_i-1)}{n_i R} > 0$. Notice, further that the first order condition under the egalitarian distribution is the same taking $\delta_i = 0$.

Hence, the result simply follows from the fact that the equilibrium p_i is strictly increasing in δ_i. This can be easily obtained from differentiation and following the same steps as in previous Propositions. ■

PROPOSITION 12 *For constant elasticity cost functions $c(r) = \frac{1}{1+\eta} r^{1+\eta}$ and $v(.)$ sufficiently close to linearity, if we compare the equilibrium with incentives with that with egalitarianism there is a threshold level of groups size such that the groups with a smaller size will have a higher win probability under egalitarianism and all groups with a larger size will have a lower win probability.*
Proof. We shall compute the first order conditions for the two equilibria. We shall use the superindices e and i to denote the equilibrium values under

egalitarianism and under incentives respectively. For a constant elasticity cost function the first order conditions become

$$\begin{aligned}
\frac{1}{R^e}(1-p_j^e)\omega_j^e &= \left(\frac{p_j^e R^e}{n_j}\right)^\eta \text{ and } n_j v'(\lambda_j^e b) = 1 \\
\frac{1}{R^i}(1-p_j^i)\omega_j^i + \frac{(1-\lambda_i)b(n_j-1)}{n_j R} &= \left(\frac{p_j^i R^i}{n_j}\right)^\eta \text{ and } n_j v'(\lambda_j^i b) = \frac{n_j - p_j}{1-p_j^i}
\end{aligned}$$

Rearranging and dividing the first order condition for individuals

$$\begin{aligned}
\left(\frac{R^i}{R^e}\right)^{\eta+1} &= \frac{\frac{1-p_j^i}{p_j^{i\eta}}}{\frac{1-p_j^e}{p_j^{e\eta}}}\frac{\omega_j^i}{\omega_j^e} + \frac{\frac{(1-\lambda_j^i)b}{p_j^{i\eta}}\frac{n_j-1}{n_j}}{\frac{1-p_j^e}{p_j^{e\eta}}\omega_j^e} \\
&= \frac{f(p_j^i)}{f(p_j^e)}\frac{\omega_j^i + \frac{(1-\lambda_j^i)b}{1-p_j^i}\frac{n_j-1}{n_j}}{\omega_j^e} \\
&= \frac{f(p_j^i)}{f(p_j^e)}\frac{v(\lambda_j^i b) + \frac{n_j-p_j}{1-p_j^i}\frac{(1-\lambda_j^i)b}{n_j}}{v(\lambda_j^e b) + \frac{(1-\lambda_j^e)b}{n_j}}
\end{aligned}$$

Introducing the FOC of the leaders we get

$$\left(\frac{R^i}{R^e}\right)^{\eta+1} = \frac{f(p_j^i)}{f(p_j^e)}\frac{v(\lambda_j^i b) + (1-\lambda_j^i)bv'(\lambda_j^i b)}{v(\lambda_j^e b) + (1-\lambda_j^e)bv'(\lambda_j^e b)}$$

Assume that $v(x) = \frac{1}{1-\alpha}x^{1-\alpha}$. Hence $v'(x) = x^{-\alpha}$ and $v(x) = \frac{xv'(x)}{1-\alpha}$. With this v-function we get

$$\begin{aligned}
\left(\frac{R^i}{R^e}\right)^{\eta+1} &= \frac{f(p_j^i)}{f(p_j^e)}\frac{\lambda_j^i + (1-\lambda_j^i)(1-\alpha)}{\lambda_j^e + (1-\lambda_j^e)(1-\alpha)}\frac{bv'(\lambda_j^i g)}{bv'(\lambda_j^e g)} \\
&= \frac{f(p_j^i)}{f(p_j^e)}\frac{\lambda_j^i + (1-\lambda_j^i)(1-\alpha)}{\lambda_j^e + (1-\lambda_j^e)(1-\alpha)}\left(1 + \frac{n_j - 1}{1-p_j^i}\right)
\end{aligned}$$

We assume that $\frac{\lambda_j^i+(1-\lambda_j^i)(1-\alpha)}{\lambda_j^e+(1-\lambda_j^e)(1-\alpha)} \simeq \frac{\lambda_k^i+(1-\lambda_k^i)(1-\alpha)}{\lambda_k^e+(1-\lambda_k^e)(1-\alpha)}$. Note that if $\alpha = 0$ (i.e. v linear) $\frac{\lambda_j^i+(1-\lambda_j^i)(1-\alpha)}{\lambda_j^e+(1-\lambda_j^e)(1-\alpha)} = 1 \forall j$. Hence we have that

$$\frac{f(p_j^i)}{f(p_j^e)}K\left(1 + \frac{n_j - 1}{1-p_j^i}\right) = \frac{f(p_k^i)}{f(p_k^e)}K\left(1 + \frac{n_k - 1}{1-p_k^i}\right)$$

If win probabilities differ when we move from no incentives to incentives there must be at least one group j with $p_j^e > p_j^i$. If there is more than one such group, take the largest one. Since $f(p)$ is decreasing in p we have that $\frac{f(p_j^i)}{f(p_j^e)} > 1$. Since p is increasing in group size we have for $n_k < n_j$

$$1 + \frac{n_k - 1}{1 - p_k^i} < 1 + \frac{n_j - 1}{1 - p_j^i}$$

Therefore for all $n_k < n_j$

$$\frac{f(p_k^i)}{f(p_k^e)} > \frac{f(p_j^i)}{f(p_j^e)} > 1 \Rightarrow p_k^e > p_k^i$$

By construction there is no group larger than n_j for which this inequality holds true.

Similarly, there must be at least one group l such that $p_l^e < p_l^i$. If there is more than one such group take the smallest group. Since $f(p)$ is decreasing in p we now have that we have that $\frac{f(p_l^i)}{f(p_l^e)} < 1$. Since p is increasing in group size we have for $n_h > n_l$

$$1 + \frac{n_h - 1}{1 - p_h^i} > 1 + \frac{n_l - 1}{1 - p_l^i}$$

Therefore for all $n_h > n_l$

$$\frac{f(p_h^i)}{f(p_h^e)} < \frac{f(p_l^i)}{f(p_l^e)} < 1 \Rightarrow p_h^e < p_h^i$$

Consequently, there is a partition of the groups by a threshold size such that their win probabilities with incentives increases or decreases relative to the equilibrium with egalitarian incentives. ■

8 References

Andreoni James, William T. Harbaugh and Lise Vesterlund (2003) "The Carrot or the Stick: Rewards, Punishments, and Cooperation", *American Economic Review* 93, 893-902.

Anesi, Vincent (2007). "Moral Hazard and Free Riding in Collective Action", DeDEx Discussion Paper Series ISSN 1749-3293, University of Nottingham

Bandiera, Oriana, Iwan Barankay and Imran Rasul (2005) "Cooperation in Collective Action", *The Economics of Transition* 13(3), 473-498

Banerjee, Abhijit, Lakshmi Iyer and Rohini Somanathan (2008) in press "Public Action for Public Goods", *Handbook of Development Economics*, Vo. 4, North Holland

Bardhan, Pranab (2000) "Irrigation and Cooperation: An Empirical Analysis of 48 Irrigation Communities in South India", *Economic Development and Cultural Change* 48, 847-65.

Bardhan, Pranab and Nirvikar Singh (2004) "Inequality, Coalitions and Collective Action", UC Santa Cruz Economics Working Paper No. 570

Berry, Jeffrey M. (1977) *Lobbying for the people,* Princeton, Princeton University Press

Cai, Yongshun, (2002) "Peasant community, elite structure, and collective action in rural China" Annual meeting of the American Political Science Association, Boston, Massachusetts. Retrieved 2008-04-21 from http://www.allacademic.com/meta/p65387index.html

Calvert, Randell (1992) "Leadership and Its Basis in Problems of Social Coordination." *International Political Science Review* 13, 7-24.

Colomer, Josep M. (1995) "Leadership Games in Collective Action" *Rationality and Society* 7, 225-246.

Cornes, Richard, and Todd Sandler (1984) "Easy Riders, Joint Production, and Public Goods" *The Economic Journal*, 94, 580-598.

Cornes, Richard, and Todd Sandler (1994) "The comparative static properties of the impure public good model" *Journal of Public Economics* 54, 403-421.

Esteban, Joan, and Debray Ray (2001) "Collective action and the group size paradox", *American Political Science Review* 95, 663-672.

Esteban, Joan, and Debray Ray (2008 forthcoming) “On the Salience of Ethnic Conflict", *American Economic Review*

Fehr, Ernst, and Simon Gächter (2000) “Cooperation and Punishment in Public Goods Experiments", *American Economic Review* 90, 980-94.

Frohlich, Norman, Joe Allan Oppenheimer and Oran Young (1971) *Political Leadership and Collective Goods*, Princeton, Princeton University Press.

Hardin, Garrett (1968) "The Tragedy of the Commons," *Science* 162:1243-1248

Lee, Sanghack (1995) "Endogenous Sharing Rules in Collective-Group Rent-Seeking", *Public Choice* 85, 31-44

Masclet, David, Charles Noussair, Steven Tucker and Marie-Claire Villeval (2003) “Monetary and Nonmonetary Punishment in the Voluntary Contributions Mechanism", *American Economic Review* 93, 366-80.

Mitra, Devashish (1999) "Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Policy Determination", *American Economic Review* 89: 1116-1134

Nitzan, Shmuel (1991) "Collective Rent Dissipation", *Economic Journal* 101(409), 1522-1534

Ueda, Kaoru (2002) "Oligopolization in collective rent-seeking", *Social Choice and Welfare* 19, 613-626

Sandler, Todd (1992), *Collective Action: Theory and Applications*, Ann Arbor, Michigan: University of Michigan Press.

Olson, Mancur (1965) *The Logic of Collective Action.* Cambridge, MA: Harvard University Press.