The Role of Central Bank Operating Procedures in an Economy with Productive Government Spending*

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December 3, 2001

Abstract

The choice of either the rate of monetary growth or the nominal interest rate as the instrument controlled by monetary authorities has both positive and normative implications for economic performance. We reexamine some of the issues related to the choice of the monetary policy instrument in a dynamic general equilibrium model exhibiting endogenous growth in which a fraction of productive government spending is financed by means of issuing currency. When we evaluate the performance of the two monetary instruments attending to the fluctuations of endogenous variables, we find that the inflation rate is less volatile under nominal interest rate targeting. Concerning the fluctuations of consumption and of the growth rate, both monetary policy instruments lead to statistically equivalent volatilities. Finally, we show that none of these two targeting procedures displays unambiguously higher welfare levels.

Keywords: monetary policy targets, productive government spending.
JEL Classification: E52, E62

* This paper has benefited from suggestions and comments from Cruz Angel Echevarria, Luisa Fuster, Tim Kehoe, Hugo Rodriguez, and participants in the FEDEA seminar, the Meeting of the Society of Economic Dynamics in Costa Rica, the 25th Symposium of Economic Analysis in Barcelona, and the 5th Workshop on Dynamic Macroeconomics at the Universidade de Vigo. The usual disclaimer applies. Financial support from the European Commission's Phare ACE Program, the Spanish Ministry of Science and Technology through grant SEC2000-0684, and from the Generalitat of Catalonia through grant SGR2000-00054 is gratefully acknowledged.

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1. Introduction

It is widely accepted that the central bank operating procedures could affect the fluctuations of macroeconomic variables. Monetary authorities may control either monetary aggregates or nominal interest rates, but not both independently. Therefore, a monetary authority should decide whether to use the growth rate of the money supply or the nominal interest rate as the policy instrument, taking into account that one monetary policy instrument could lead to a superior performance than the other. In this paper we want to reexamine some of the issues related to the choice of the monetary policy instrument in a general equilibrium model exhibiting endogenous growth, where the productive government spending is partially financed by means of currency printing. We evaluate the performance of the two monetary instruments from the point of view of both fluctuations of endogenous variables and welfare.

As it was already pointed out by Poole (1970) in an IS-LM framework, the problem of finding the optimal policy instrument is irrelevant when we are dealing with a non-stochastic economy. The question becomes however relevant under uncertainty. Poole evaluated the performance of both instruments by just looking at output fluctuations. He found that, when the origin of disturbances comes mainly from money demand shocks, to target the nominal interest rate is the best policy in terms of output stabilization, whereas to target the rate of monetary growth is the most stabilizing policy when the origin of disturbances comes mainly from real shocks. Concerning price fluctuations, he recommends nominal interest rate targeting as a more price stabilizing policy.

After Poole's contribution the question of the optimal choice of the monetary policy instruments has been analyzed by several authors through more sophisticated frameworks. For example, Carlstrom and Fuerst (1995) consider a cash-in-advance model with portfolio rigidity in the households' cash-saving choice, and find that interest rate targeting is the instrument that outperforms monetary aggregate targeting in terms of welfare even if the former delivers more volatile output. Collard, Dellas and Ertz (1998) evaluate the two targeting procedures in a growth model with labor augmenting technological progress, and consider the effects of technology, money demand, and fiscal shocks. They conclude that nominal interest rate targeting results in higher welfare and lower volatility of both output and inflation rate regardless of the origin of the shocks generating the disturbances. Canzoneri and Dellas (1998) study the effect of the choice of the monetary policy instrument on the level of the risk premium in a cash-in-advance economy without capital where labor contracts induce rigidities. They find that, under nominal interest rate targeting, the average level of the real interest rate is higher and prices are less volatile. Nevertheless, they conclude that it is not clear which policy performs better in terms of welfare.

In this paper we deal with an endogenous growth model in which two kinds of shocks are present: technology and money demand shocks. Technology shocks enter directly into the production function. Money demand shocks are introduced in the form of a modified cash-in-advance constraint in the spirit of Woodford (1991). We evaluate the influence of both shocks on the rate of economic growth, on the inflation rate, and on consumption when the monetary authority follows either monetary aggregate targeting or nominal interest rate targeting.

We consider a production function with government spending like in Barro (1990). Barro assumes in his model that government spending is entirely financed by means of a flat rate income tax, while Blackburn and Hung (1996) assume instead that the government finances its spending through seignorage, that is, by printing money. The latter assumption allows the model to create a new link between monetary shocks and output, since the inflationary revenues obtained by the government are transformed into productive spending. However, we will assume that the government obtains revenues both from taxes and from seignorage. If the revenues from income taxes were disregarded, the model would display unrealistic values both of the money growth rate and of the nominal interest rate. Therefore, we combine both forms of financing as in Palivos and Yip (1995). Finally, let us mention that many countries subject to large tax evasion have used seignorage to obtain easy inflationary revenues (see Roubini and Sala-i-Martin, 1992 and 1995).

When we evaluate the performance of the two monetary instruments from the point of view of fluctuations of endogenous variables, we find that the inflation rate is less volatile under nominal interest rate targeting. Concerning the fluctuations of consumption and of the growth rate, both monetary policy instruments lead to statistically equivalent volatilities. Considering the welfare implications, none of the two targeting procedures gives rise to unambiguously higher welfare levels. Even if recently many central banks have been reoriented to nominal interest rate targeting, our analysis suggests that this will not affect neither welfare nor economic stability in terms of consumption and of growth rates.

The remainder of the paper is organized as follows. The model is described in section 2. We present the solution technique and calibrate the model in section 3. We perform the steady state analysis for an non-stochastic economy in section 4. The evaluation of both targeting procedures in a stochastic setup is presented in section 5. Section 6 concludes the paper.

2. The Model

2.1. The Households

Let us consider an economy populated by infinitely lived identical households. The preferences of a representative household at time t are given by the following utility function defined over the random stream of consumption $\{c_j\}_{j=t}^{\infty}$:

$$E_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\left(\frac{c_j^{1-\theta}-1}{1-\theta}\right)\right], \tag{2.1}$$

where $\beta \in (0,1)$ is the discount factor and the parameter $\theta > 0$ is the inverse of the elasticity of intertemporal substitution.

The sequence of events within each period is the following:

1) Individuals enter a given period t with a certain amount of monetary balances M_t and of real assets z_t.

2) Individuals learn the state of the economy (A_t, s_t) in the current period, where A_t is a technology shock and s_t is a money demand shock. We assume that the two shocks are mutually independent.

3) Individuals supply inelastically a unit of labor and production takes place.

4) Individuals receive their real income in the form of a wage w_t and a return to asset holdings $r_t z_t$.

5) Income is taxed by the government at the rate τ_t.

6) The goods market opens and individuals purchase consumption using their money balances and a fraction of their after-tax income.

7) The financial market opens and agents choose both their nominal money holdings M_{t+1} and their assets holdings z_{t+1} for period $t+1$.

The budget constraint for period t is thus the following:

$$c_t + (z_{t+1} - z_t) + \frac{M_{t+1} - M_t}{p_t} \leq (1 - \tau_t)\left[w_t + r_t z_t\right]. \tag{2.2}$$

In strict cash-in-advance models with uncertainty and a single consumption good, consumers must purchase such a good by using only currency, and the income earned in the current period cannot be converted into money until the next financial exchange. However, following Woodford (1991), we will assume here that a fraction of the t period after-tax income can be used for current period purchases. Moreover, like in Canzoneri and Dellas (1998), we allow this fraction to fluctuate randomly. Therefore, the cash-in-advance constraint becomes

$$c_t \leq \frac{M_t}{p_t} + s_t(1 - \tau_t)\left[w_t + r_t z_t\right], \tag{2.3}$$

where s_t is the money demand shock. Money demand shocks are assumed to be lognormally distributed and to follow an autoregressive process,

$$\ln s_{t+1} = (1 - \rho_s)\ln \bar{s} + \rho_s \ln s_t + \varepsilon_{s,t+1},$$

where $\rho_s \in (0,1)$, $\ln \bar{s}$ is the unconditional expected value of the logarithm of the money demand shock, and the variables $\varepsilon_{s,t}$ are identically and independently distributed with $\varepsilon_{s,t} \sim N(0, \sigma_s^2)$.[1]

[1] Note that $E(\ln s_t) = \ln \bar{s}_t$, whereas the conditional expectation is $E_t(\ln s_{t+1}) = (1 - \rho_s)\ln \bar{s}_t + \rho_s \ln s_t$. Therefore, $\bar{s}_t = \exp\left[E(\ln s_t)\right]$.

The shock s_t can be viewed as a measure of the efficiency of the payment system. Depending on both the after-tax income $(1-\tau_t)[w_t + r_t z_t]$ and the realization of the shock s_t in the current period, agents know how severe is their cash-in-advance constraint. If we set $s_t = 0$ for all t, we would obtain the cash-in-advance constraint usually found in standard monetary models. In this case, only the currency held at the end of the previous financial exchange could be used to purchase goods. The more general cash-in-advance constraint (2.3) allows a fraction s_t of the after-tax income in period t to be spent immediately. Thus, if the value of s_t is high, then the cash-in-advance constraint would not be so tight. Clearly, a high value of s_t means that less income has to be converted into a non-interest bearing asset (money) in order to get a given level of consumption. Thus a higher value of s_t corresponds to a more efficient payment system and, it is obviously associated with a higher velocity of money.

A representative household chooses the stochastic vector sequence $\{c_t, M_{t+1}, z_{t+1}\}_{t=1}^{\infty}$ in order to maximize the expected discounted sum of instantaneous utilities (2.1) subject to the budget constraint (2.2) and the cash in advance constraint (2.3).

2.2. The Firms

In this economy there are identical firms, and each of them produces the single good of this economy according to the technology represented by the gross production function

$$y_t = A_t k_t^{\alpha} g_t^{1-\alpha}, \tag{2.4}$$

where y_t is the output per worker, A_t is a random variable that represents the technology shock, k_t is the stock of capital per worker, g_t is the government expenditure per capita and $\alpha \in (0,1)$ is the elasticity of output with respect to capital. Our formulation follows thus that of Barro (1990), according to which the flow of government spending raises the factors productivity. The rate of depreciation of private capital is δ. Technology shocks are also assumed to be lognormally distributed and to follow an autoregressive process,

$$\ln A_{t+1} = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_t + \varepsilon_{A,t+1},$$

where $\rho_A \in (0,1)$, $\ln \bar{A}$ is the unconditional expected value of the logarithm of the technology shock, and the variables $\varepsilon_{A,t}$ are identically and independently distributed with $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$. We normalize the number of firms so that there is a firm per consumer. Firms do not pay any fee for the use of public services associated with the flow of government spending. Consumers lend both capital and labor to the firms. Both the rental price r_t of capital and the real wage w_t are set competitively so that firms end up getting zero profits in equilibrium. Therefore,

$$r_t = \alpha A_t k_t^{\alpha-1} g_t^{1-\alpha} - \delta \tag{2.5}$$

and

$$w_t = (1-\alpha) A_t k_t^{\alpha} g_t^{1-\alpha}. \tag{2.6}$$

Obviously, in equilibrium $k_t = z_t$, and we obtain that the net income per capita turns out to be equal to the net domestic product per capita,

$$w_t + r_t k_t = A_t k_t^{\alpha} g_t^{1-\alpha} - \delta k_t. \tag{2.7}$$

2.3. The Government

The government of this economy sets a public spending to GDP ratio and controls the values of the monetary policy parameters. We will consider that the government spends a constant fraction G of GDP in each period,

$$\frac{g_t}{y_t} = G, \tag{2.8}$$

for all t. Concerning how the monetary policy is conducted, government has at its disposal two monetary instruments: it can regulate either the monetary aggregate or the nominal interest rate. When the amount of money is targeted, the nominal interest rate is determined endogenously, whereas when the nominal interest rate is used as the monetary policy instrument, it is the quantity of money to be printed that accommodates the demand for monetary balances.

We assume that government expenditures are financed by a flat-rate income tax and by printing money (seignorage). The tax rate on net income τ_t is endogenous and it is set in every period so as to fulfil the following government budget constraint:

$$g_t = \tau_t (y_t - \delta k_t) + \frac{M_{t+1} - M_t}{p_t}, \tag{2.9}$$

where $M_{t+1} - M_t$ is the nominal money injected into the economy in period t. In order to finance the flow of its productive spending, the government borrows the amount g_t per capita at the beginning of period t. Immediately after production has taken place, the government pays some of its spending by taxing income. Moreover, after consumption has taken place, money is issued to finance the rest of the current period spending. The government does not pay interest on the amount borrowed as both the spending and the corresponding payment occur within the same period.

2.4. Equilibrium

Let λ_t and η_t be the non-negative Lagrange multipliers associated with the budget constraint (2.2) and the cash-in-advance constraint (2.3), respectively. Given the competitive nature of our economy, the solution of the problem faced by a consumer is characterized by the following equations, which are obtained from replacing the net income $w_t + r_t z_t$ by the net production (see equation (2.7)), and by taking derivatives of the corresponding Lagrangian with respect to consumption, money, and capital:

$$c_t^{-\theta} = \lambda_t + \eta_t, \tag{2.10}$$

$$\frac{\lambda_t}{p_t} = \beta E_t \left(\frac{\lambda_{t+1} + \eta_{t+1}}{p_{t+1}} \right), \tag{2.11}$$

$$\begin{aligned} \lambda_t &= \beta E_t \left(\lambda_{t+1} \left\{ 1 + (1 - \tau_{t+1}) \left[\alpha A_{t+1} \left(\frac{g_{t+1}}{k_{t+1}} \right)^{1-\alpha} - \delta \right] \right\} \right) + \\ &\quad \beta E_t \left(\eta_{t+1} s_{t+1} (1 - \tau_{t+1}) \left[\alpha A_{t+1} \left(\frac{g_{t+1}}{k_{t+1}} \right)^{1-\alpha} - \delta \right] \right). \end{aligned} \tag{2.12}$$

Moreover, the following transversality condition must hold:

$$\lim_{j \to \infty} E_t \left(\beta^{t+j} \lambda_{t+j} k_{t+j+1} \right) = 0. \tag{2.13}$$

The Lagrange multipliers associated with the budget and cash-in-advance constraints, λ_t and η_t, can be interpreted as the marginal utility of wealth and the marginal utility of real balances, respectively. The first order condition on consumption (2.10) tells us that the existence of binding liquidity constraint drives a wedge between the marginal utility of wealth and the marginal utility of consumption, since wealth cannot be used instantaneously to buy consumption goods. The left hand side of the first order condition on nominal balances (2.11) can be interpreted as the loss of utility due to the acquisition of an extra unit of money. At the margin this amount must be equal to the value of the liquidity services provided by such a unit of money plus the discounted expected utility increase due to capital gains resulting from price level changes. Condition (2.12) combines the costs and expected gains of investing one marginal unit of wealth into capital.

To define the nominal interest rate i_{t+1} from t to $t+1$, we follow Svensson (1985). Note that if a nominal bond were available, the discounted expected utility of investing a monetary unit in this bond would be

$$\beta E_t \left(\frac{\lambda_{t+1}(1 + i_{t+1})}{p_{t+1}} \right). \tag{2.14}$$

The discounted expected utility of holding a monetary unit as cash at the end of period t is

$$\beta E_t \left(\frac{\lambda_{t+1} + \eta_{t+1}}{p_{t+1}} \right). \tag{2.15}$$

By arbitrage, the expected marginal utilities (2.14) and (2.15) must be equated. Hence, taking into account that the nominal interest i_{t+1} is known at t, we have that

$$i_{t+1} = \frac{E_t \left(\frac{\eta_{t+1}}{p_{t+1}} \right)}{E_t \left(\frac{\lambda_{t+1}}{p_{t+1}} \right)}. \tag{2.16}$$

The nominal interest rate is thus the ratio between the expected marginal utility of the liquidity services of money and the expected marginal utility of money wealth.

Definition: Given the set of initial conditions k_1, M_1 and p_0, an equilibrium is a vector of stochastic processes $\{c_t, k_{t+1}, M_{t+1}, \mu_{t+1}, i_{t+1}, p_{t+1}, g_t, \tau_t\}_{t=0}^{\infty}$ such that

(a) The representative household is maximizing the discounted expected utility (2.1) subject to the budget constraint (2.2) and the cash-in-advance constraint (2.3);

(b) Markets for goods and money clear in every period,

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + g_t, \tag{2.17}$$

where $y_t = A_t k_t^{\alpha} g_t^{1-\alpha}$, and

$$\mu_{t+1} = \frac{M_{t+1}}{M_t}; \tag{2.18}$$

(c) The government budget constraint (2.9) holds and government spending satisfies (2.8) for an exogenously given real number $G > 0$;

(d) The following relationship between the growth rate of money supply μ_{t+1} and the nominal interest rate i_{t+1} holds:

$$\frac{\mu_{t+1}}{1 + i_{t+1}} = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{p_t}{p_{t+1}} \frac{M_{t+1}}{M_t} \right); \tag{2.19}$$

(e.1) If the government pegs the rate of monetary growth, then $\mu_{t+1} = \mu$ for $t = 0, 1, ...,$ where μ is exogenously given, or

(e.2) If the government pegs the nominal interest rate, then $i_{t+1} = i$ for $t = 0, 1, ...,$ where i is exogenously given.

Obviously, the asset market also clears in equilibrium, $z_t = k_t$, as follows from Walras' law. Equation (2.19) linking the rate of monetary growth to the nominal interest rate is immediately obtained by combining (2.16), (2.11) and (2.18).

3. Solution Method and Calibration of the Model

3.1. The Transformed Model

In order to analyze the equilibrium of the economy we have to solve simultaneously the equilibrium equations we have stated at the end of the previous section. Since we are dealing with an endogenous growth model, many variables are non-stationary. Therefore, we will work instead with variables expressed in ratios. We define $\hat{c}_t$ as the consumption to current period capital ratio and γ_{t+1} as the gross rate of growth of capital per capita,

$$\hat{c}_t = \frac{c_t}{k_t} \text{ and } \gamma_{t+1} = \frac{k_{t+1}}{k_t}.$$

We redefine correspondingly the other variables as

$$\hat{y}_t = \frac{y_t}{k_t}, \; \hat{g}_t = \frac{g_t}{k_t}, \; \hat{m}_t = \frac{M_t}{p_t \; k_t}, \; f_{t+1} = \frac{p_{t+1}}{p_t}, \; \hat{\lambda}_t = \lambda_t k_t^{\theta}, \text{ and } \hat{\eta}_t = \eta_t k_t^{\theta},$$

where $\hat{y}_t$, $\hat{g}_t$, $\hat{m}_t$, f_{t+1}, $\hat{\lambda}_t$ and $\hat{\eta}_t$ are the output to capital ratio, government spending to capital ratio, real balances to capital ratio, inflation rate, transformed marginal utility of wealth, and transformed marginal utility of real balances, respectively. After such a transformation, the equilibrium conditions (2.17), (2.4), (2.3), (2.9), (2.18), (2.10), (2.11), (2.12), (2.19), and (2.8) become

$$\hat{c}_t + \gamma_t - (1-\delta) + \hat{g}_t = \hat{y}_t, \tag{3.1}$$

$$\hat{y}_t = A_t \hat{g}_t^{1-\alpha}, \tag{3.2}$$

$$\hat{c}_t = \hat{m}_t + s_t \left(1-\tau_t\right)\left(\hat{y}_t - \delta\right), \tag{3.3}$$

$$\hat{g}_t = \tau_t \left(\hat{y}_t - \delta\right) + (\mu_{t+1} - 1)\hat{m}_t, \tag{3.4}$$

$$\mu_{t+1} = \frac{\hat{m}_{t+1}}{\hat{m}_t}\gamma_{t+1} f_{t+1}, \tag{3.5}$$

$$\hat{c}_t^{-\theta} = \hat{\lambda}_t + \hat{\eta}_t, \tag{3.6}$$

$$\hat{\lambda}_t = \beta E_t \left(\frac{\hat{\lambda}_{t+1} + \hat{\eta}_{t+1}}{f_{t+1}} \gamma_{t+1}^{-\theta} \right), \tag{3.7}$$

$$\hat{\lambda}_t = \beta E_t \left\{ \hat{\lambda}_{t+1}\gamma_{t+1}^{-\theta} \left\{1 + (1-\tau_{t+1})\left[\alpha \hat{y}_{t+1} - \delta\right]\right\} + \hat{\eta}_{t+1}\gamma_{t+1}^{-\theta} s_{t+1}\left(1-\tau_{t+1}\right)\left[\alpha\hat{y}_{t+1} - \delta\right] \right\}, \tag{3.8}$$

$$\frac{\hat{\lambda}_t}{1+i_{t+1}} = \beta E_t \left(\frac{\hat{\lambda}_{t+1}}{f_{t+1}} \gamma_{t+1}^{-\theta} \right), \tag{3.9}$$

and

$$\hat{g}_t = \left(A_t G\right)^{1/\alpha}, \tag{3.10}$$

respectively. Due to the timing of events in our model, the cash-in-advance constraint (3.3) could become non-binding for some realizations of the shocks. Several studies that take into account analogous timing conclude that the cash-in-advance constraint is binding for almost all shock realizations (see for example Hodrick, Kocherlakota and Lucas, 1991; or Hromcová, 2000). Therefore, we write the cash-in-advance constraint with equality. However, we have checked in all our simulations that the Lagrange multiplier η_t takes in fact always strictly positive values, which means that the cash-in-advance is actually binding.

3.2. Solution Method

The previous model does not admit a closed form solution and, therefore, we will use a numerical solution method. The solution technique we will apply will be the one of Uhlig (1997), which is based on the log-linearization of the necessary equations characterizing the equilibrium around the steady state. Uhlig's method uses the Euler equation to solve for the law of motion of the recursive equilibrium. That law of motion is found by means of the undetermined coefficients method. The solution procedure is described in detail in the Appendix.

3.3. Calibration

We calibrate the model to match the quarterly US data for the period 1979:1-1997:2. We choose this particular period, because it exhibits a stationary income velocity of M1, which allows us to assume safely that money demand shocks are stationary .

We use a *bar* to denote the steady state value of a variable of the transformed model (3.1) – (3.10). The stationary value $\bar{\gamma}$ of the gross rate of growth is set to 1.00375.[2] The value we choose for the growth rate of the money supply is $\bar{\mu} = 1.015$, which coincides with the empirical growth rate of M1. The nominal interest rate $\bar{i}$ that corresponds to this growth rate of the money supply is 0.02, which is computed using the equilibrium conditions at the steady state. The share of capital in the production function is set equal to $\alpha = 0.36$. We set the value of the depreciation rate $\delta = 0.0085$ in order to match the empirical averages of the ratios of consumption to output, investment to output, and government spending to output. Taking into account that on average the government spending represents 20% of the GDP, we choose for the steady state ratio $\bar{g}$ of government spending to capital a value that generates $g_t/\,y_t = 0.2$. The resulting average level of the income tax rate is the one that satisfies the government budget constraint, which turns out to be $\bar{\tau} = 0.232$. The discount factor β is adjusted depending on the value of the parameter characterizing the inverse of the elasticity of intertemporal substitution θ. For the benchmark model we take $\theta = 1.2$ and the corresponding discount factor is $\beta = 0.995$. To calibrate the stochastic process of technology shocks, we calculate the Solow residuals using an analogous procedure to that described in Cooley (1997). Thus, the technology parameters take the values $\bar{A} = 1$, $\rho_A = 0.9977$, and $\sigma_A = 0.00923$. The parameter values for the efficiency of the payment system are determined in a similar way. We use data of both consumption as a fraction of GDP and money velocity in order to generate series for s_t using the cash-in-advance constraint. Thus, we set $\bar{s} = 0.66$, $\rho_s = 0.975$, and $\sigma_s = 0.0103$. The benchmark parameter values are summarized in Table 1.

[2]This value corresponds to the growth rate of GDP per worker.

$\bar{\gamma} = 1.00375$
$\bar{\mu} = 1.015$ or $1 + \bar{i} = 1.02$ $(\Longrightarrow \bar{\tau} = 0.232)$
$\alpha = 0.36$
$\delta = 0.0085$
$\theta = 1.2$, $\beta = 0.995$
$\bar{A} = 1$, $\rho_A = 0.9977$, $\sigma_A = 0.00923$
$\bar{s} = 0.66$, $\rho_s = 0.975$, $\sigma_s = 0.0103$

Table 1: Benchmark values of the parameters.

Using these parameters the model displays the following characteristics:

averages for:	data	calibration
government spending/output	0.2	0.2
consumption/output	0.65	0.59
investment/output	0.15	0.21
income velocity	6.79	6.5

Table 2: Comparison between empirical and calibrated values of some variables.

4. Steady State Analysis

We will analyze in this section the effects of permanent shocks on the stationary equilibrium of the non-stochastic version of the model for both targeting policies. To this end, we set $\sigma_A = \sigma_s = 0$.

4.1. Steady State Effects of both the Rate of Monetary Growth and the Nominal Interest Rate

We will first analyze the effect of permanent changes of the rate of growth of money supply on the stationary equilibrium of the economy. An increase in the growth rate of money translates into more seignorage. This means that in order to keep the government spending to output ratio constant, the tax rate on income decreases. Higher disposable income can be distributed optimally between more consumption and more capital accumulation. Therefore, in our model, an increase in the growth rate of money is growth enhancing. Note that equations (3.5) and (3.9) become at a non-stochastic steady state

$$\bar{\mu} = \bar{\gamma}\bar{f}, \tag{4.1}$$

and

$$\frac{1}{1+\bar{i}} = \frac{\beta\,(\bar{\gamma})^{-\theta}}{\bar{f}},$$

respectively. Combining the last two equations we obtain

$$\bar{\mu} = (\bar{\gamma})^{1-\theta} (1 + \bar{i}). \tag{4.2}$$

Taking into account that the growth rate $\bar{\gamma}$ reacts positively to an increase in $\bar{\mu}$ and that $\theta > 1$, we immediately obtain that the nominal interest rate increases with the rate of monetary growth.[3] Therefore, the effects of an increase in the rate $\bar{\mu}$ of monetary growth are equivalent to those of an increase in the nominal interest rate $\bar{i}$.

The effect of an increase in the rate of monetary growth $\bar{\mu}$ (or of an equivalent increase in the nominal interest rate $\bar{i}$) on the stationary inflation rate $\bar{f}$ depends on the size of the effect on the growth rate $\bar{\gamma}$. If the increase in $\bar{\gamma}$ does not outweigh the increase in $\bar{\mu}$, then inflation increases (see (4.1)). We can see in Figure 1 that this is the case for our calibrated model. The reactions of both $\bar{\gamma}$ and $\bar{f}$ to changes in $\bar{\mu}$ and in $\bar{i}$ are plotted in Figure 1.

(Insert Figure 1)

4.2. Steady State Effects of Technology Changes

We want to characterize now the effects of a change in the steady state level of the technology $\bar{A}$ on the stationary values both of the growth rate and of the inflation rate. When the steady state level of the technology increases, it has a positive effect on the growth rate because a higher value of $\bar{A}$ corresponds to higher total factor productivity. The associated higher income level permits to lower the tax rate, which implies in turn an increase in consumption. Agents increase their demand for real balances and, therefore, we observe a decrease in the inflation rate for both monetary policy instruments. These results are plotted in Figure 2.

(Insert Figure 2)

4.3. Steady State Effects of Changes in the Efficiency of the Payment System

When the steady state level $\bar{s}$ of the efficiency of the payment system increases, a higher fraction of the individuals' current period income can be used to purchase consumption goods. Therefore, the relative demand for real monetary balances goes down, that is, $\bar{m}$ decreases. This induces a decrease in seignorage and, in order to fulfill the government budget constraint, the tax rate must increase. Lower disposable income implies a decrease in the growth rate of consumption relative to capital, that is, $\bar{c}$ decreases. Combining equations (3.1), (3.2) and (3.10) we obtain the following equation evaluated at the steady state:

$$\bar{c} + \bar{\gamma} - (1 - \delta) + (\bar{A}G)^{1/\alpha} = \bar{A}^{1/\alpha} (G)^{(1-\alpha)/\alpha}, \tag{4.3}$$

[3] The positive association between the rate of monetary growth and the nominal interest rate is also found in the standard model of capital accumulation with money in the utility function (Sidrauski, 1967). Note that if $\theta < 1$, the sign of the previous relation would be reversed.

which tells us that the sum $\bar{c} + \bar{\gamma}$ must remain constant. Therefore, as $\bar{c}$ decreases, the rate of economic growth $\bar{\gamma}$ must increase.

Concerning the change in prices, the money market equilibrium condition (4.1) implies that, when the growth rate $\bar{\mu}$ of money is fixed and the rate $\bar{\gamma}$ of economic growth increases, inflation must drop. Moreover, under nominal interest rate targeting, the relationship between the stationary values of the nominal interest rate $\bar{i}$ and the growth rate $\bar{\mu}$ of money supply, given in equation (4.2), implies analogously that the inflation rate must decrease.

The behavior of the stationary growth rate $\bar{\gamma}$ and the inflation rate $\bar{f}$ for both monetary policy instruments is plotted in Figure 3. Notice that the effects turn out to be quantitatively very small.

(Insert Figure 3)

5. Effects of the Two Targeting Procedures on the Stochastic Equilibrium

5.1. Monetary Aggregate Targeting: Impulse-Responses

We consider here that the monetary policy consists of pegging a constant monetary growth rate, $\mu_t = \mu$ for all periods. We solve the system (3.1)-(3.10) using the numerical technique described in section 3.2 and in the Appendix. The log-linearized equations and the matrices of endogenous state variables, control variables, and exogenous state variables can be found in the Appendix.

To see the effect of each shock we will consider a reaction of the economy to a technology shock and to a money demand shock separately. We will assume that the economy is in the non-stochastic steady state at time $t = 0$. At time $t = 1$ the perturbation $\varepsilon_{A,t}$ ($\varepsilon_{s,t}$) is selected in such a way that the technology shock (the money demand shock) experiences a 1% deviation from its steady state. These perturbations become $\varepsilon_{A,t} = 0$ and $\varepsilon_{s,t} = 0$, respectively, for all $t > 1$. In Figures 4 and 5 we plot the impulse-responses to a technology and to a money demand shock, respectively, for the following variables: growth rate of capital, output to capital ratio, consumption to capital ratio, real balances to capital ratio, the inflation rate, the nominal interest rate, and the tax rate. We concentrate our analysis on the effects on economic growth and inflation rates.

(Insert Figures 4 and 5)

A positive transitory technology shock increases output directly. Taxable income increases and hence the tax rate can be reduced. This results in higher disposable income for both consumption purchases and capital accumulation so that the growth rate γ_t of the economy increases in the short run. Since consumption also increases, the demand for real balances increases too. However, since the growth rate of nominal balances is fixed, we observe a temporary decrease of the inflation rate in order to clear the money market.

A positive transitory money demand shock relaxes the liquidity constraint and, therefore, the value of money will be lower. This implies lower seignorage and a higher tax rate. However, as the payment system will become more inefficient, future tax rates will be lower. This means that the sequence of future growth rates must increase over time, which is only possible if the rate of growth γ_t goes down in the short run. Since the rate of monetary growth is fixed, the reduction in the (adjusted) monetary balances is achieved exclusively through a change in prices. This amounts to a reduction in the rate f_t of inflation in the short run.

5.2. Nominal Interest Rate Targeting: Impulse-Responses

We assume now that the monetary policy consists on pegging a constant nominal interest rate, $i_t = i$ for all periods. In order to keep fixed the nominal interest rate, the government must let the money supply respond to both technology and money demand shocks. The log-linearized equations and the vectors of state and control variables are similar to those corresponding to the monetary aggregate targeting case. These equations can be found in the Appendix. To see the effect of each shock separately on the growth rate and on the inflation rate, we also analyze the corresponding impulse-responses.

A positive transitory technology shock increases current output and, consequently, it also increases consumption and capital accumulation. Such an increase of the growth rate γ_t must be accompanied by an instantaneous reduction in the inflation rate f_t so as to increase real monetary balances. This increase of real balances is required to purchase a larger amount of consumption.

A positive transitory money demand shock implies a decrease in the demand for real balances. The supply of money reacts endogenously in such a way that seignorage actually increases. Therefore, the current tax rate turns out to be lower than future tax rates. This means that the sequence of future growth rates must decrease over time, which is only possible if the rate γ_t of growth goes up in the short run. As the payment system will become less efficient in the future, future prices will be lower than the current ones in relative terms, and we observe thus a reduction in the inflation rate f_t in the short run.

Impulse-responses of the different variables of our model to technology and money demand shocks are plotted in Figures 4 and 5.

5.3. Comparison of the Two Targeting Procedures

In this section we compare the performance of the two targeting procedures from the point of view of the volatilities of some relevant endogenous variables. We also make some welfare considerations.

5.3.1. Fluctuations

Technology and money demand shocks affect the behavior of all variables through different channels under either monetary aggregate targeting or nominal interest rate targeting. We will analyze the induced volatilities on consumption, on the economic growth rate, and on the inflation rate. Moreover, we will evaluate the contribution of technology and money demand shocks to the total volatility of these variables. We perform the analysis for different values of the elasticity of intertemporal substitution. We define the volatility σ_h of a variable h_t as the standard error of the second moment of the logarithm of the Hodrick-Prescott filtered series. The simulated series have a length of 150 quarters and the volatilities σ_h reported in our tables are obtained by averaging over 500 shock realizations. We evaluate three cases: when the origin of disturbances are both technology and money demand shocks, only technology shocks, and only money demand shocks, respectively.

As can be seen from Table 3, the rate of economic growth γ_t is less volatile under nominal interest rate targeting for values of $\theta < 1$, and under monetary aggregate targeting for $\theta > 1$. The same holds for consumption volatility when only technology shocks or both kinds of shocks operate. When the only source of disturbances comes from money demand shocks, consumption is less volatile under nominal interest rate targeting for all values of θ under consideration. Nevertheless, the differences are very small and the confidence intervals overlap. This means that we cannot clearly say which of the two targeting procedures delivers a less volatile consumption or a less volatile growth rate. The situation is different when we look at the inflation rate f_t, since this rate is clearly less volatile under nominal interest rate targeting whenever only money demand shocks or both kinds of shocks operate.

The reason for which we do not observe large differences in the fluctuations of consumption and of the growth rate lies in the fact that their volatility is mostly driven by technology shocks. As we have seen in our impulse-response analysis, technology shocks affect the economy in a very similar manner under both targeting procedures.

When we consider the volatility of the inflation rate, we have to look at the behavior of real balances. Even if real balances react to shocks in a similar way under both targeting procedures, the adjustment of nominal balances, and thus of prices, is quite different. We observe that under monetary aggregate targeting, money demand shocks are the ones that contribute more significantly to the volatility of the inflation rate. However, under nominal interest rate targeting, fluctuations of the inflation rate are mostly driven by technology shocks. Under monetary aggregate targeting all the adjustment of real balances to shocks comes through changes in prices. Under nominal interest rate targeting, nominal balances react to a technology shock in an opposite direction than real balances, and hence the inflation rate turns out to be more volatile. However, nominal and real balances adjust in the same direction when a money demand shock occurs, and we observe almost negligible fluctuations of the inflation rate.

5.3.2. Welfare

When the government cares about the welfare of the representative agent, its objective should be to maximize the following welfare function:

$$W = E_t \left(\sum_{j=t}^{\infty} \beta^{j-t} \, \frac{c_j^{1-\theta} - 1}{1-\theta} \right). \tag{5.1}$$

To ensure that the expectation in (5.1) is finite, we write the consumption as

$$c_t = c_t^* \bar{\gamma}^t,$$

where c_t^* is the detrended consumption, and rewrite the welfare function as

$$W = E_t \left(\sum_{j=t}^{\infty} \left[(\beta^*)^{j-t} \; \frac{\left(c_j^*\right)^{1-\theta}}{1-\theta} - \frac{\beta^{j-t}}{1-\theta} \right] \right)$$

where $\beta^* = \beta \bar{\gamma}^{1-\theta}$ be strictly less than 1 so as to ensure the desired convergence. Such an inequality is satisfied by our calibrated model. We calculate the welfare as an empirical mean of 500 shock realizations of time series with a horizon of 800 periods.

We again let the origin of disturbances be both technology shocks and money demand shocks, only technology shocks, and only money demand shocks. As can be seen from Table 4, in most of the cases the average level of welfare is higher under nominal interest rate targeting. Notice, that the confidence intervals under the two targeting procedures almost completely overlap. Therefore, we are unable to choose the monetary policy instrument that would clearly lead to a welfare improvement. This result confirms the fact that the differences in fluctuations of consumption under the two targeting procedures are too small to generate some significant differences in the associated welfare levels.

6. Conclusion

In this paper we have analyzed the effects of two targeting procedures of monetary policy. We have performed two kinds of analyses: the one concerning the non-stochastic version of the economy and the one concerning to its stochastic counterpart. In a non-stochastic economy we have seen the effects of permanent changes in technology and in the efficiency of the payment system (the money demand) on the stationary values of growth and inflation rates. In a stochastic economy we have studied how the endogenous variables react to unexpected transitory shocks. Moreover, we have shown the contribution of those two particular shocks to the fluctuations of the growth rate, of the inflation rate, and of consumption.

The non-stochastic economy and the stochastic economy (both in the short and in the long run) behave similarly when taking into account the shocks in the technology regardless of whether they are permanent or transitory. This is not the case when we consider changes in the efficiency of the payment system. The basic difference we observe is that a transitory positive money demand shock makes the rate of economic growth fall in the short run, whereas this rate is permanently increased by a permanent positive money demand shock. This discrepancy is caused by the different future value of money when changes are transitory or permanent. When a positive transitory shock affects the efficiency of the payment system, the value of money experiences a decrease and it comes back to its original level. Nevertheless, when a permanent increase in the efficiency of the payment system occurs, the future value of money decreases permanently.

Concerning the comparison of the two targeting procedures with respect to fluctuations, we find that the inflation rate is less volatile under nominal interest rate targeting. Concerning the volatility of consumption and the growth rate, our model delivers statistically equivalent results for both targeting procedures. The same holds for our welfare analysis: none of the two targets is clearly superior if the goal of the government is to maximize the individuals' expected lifetime utility. Therefore, our results coincide with those of Poole (1970) and Collard, Dellas and Ertz (1998) when we analyze the performance of nominal interest rate targeting as a price stabilizing policy. The welfare analysis delivers similar results as Canzoneri and Dellas (1998), who conclude that it is not clear which policy performs better in terms of welfare. Therefore, our model suggests that the welfare and the stability of either consumption or growth rates do not depend much on the choice of the monetary policy targeting procedure. If the goal of the monetary authority is to stabilize the inflation rate, the choice of the nominal interest rate as the monetary policy instrument seems appropriate.

We have studied the behavior of the economy for passive monetary policies.[4] An analysis that remains to be done is to evaluate the performance of the economy under active monetary policies. This amounts to analyzing how should the monetary authorities react to current disturbances in both the technology and the efficiency of the payment system in order to stabilize the growth rate or to achieve higher welfare, when they face the choice of controlling either the monetary aggregates or the nominal interest rates.

[4] A passive monetary policy is understood as the one that follows a target that is fixed for all periods, i.e., either constant money growth rate or constant nominal interest rate. An active monetary policy would be a function of past disturbance realizations.

Appendix

Applying the Solution Method

To solve the system of equilibrium equations (3.1)-(3.10) we proceed as follows:

1) We assign values to parameters and calculate the steady state (expressed in ratios). To this end, we must first rewrite the equilibrium equations of the transformed model (3.1)-(3.10) in a non-stochastic steady state (with $\sigma_A = \sigma_s = 0$) as follows:

$$\bar{c} + \bar{\gamma} - (1-\delta) + \bar{g} = \bar{y},$$

$$\bar{y} = \bar{A}\bar{g}^{1-\alpha},$$

$$\bar{c} = \bar{m} + \bar{s}(1-\bar{\tau})\left(\bar{y} - \delta\right),$$

$$\bar{g} = \bar{\tau}\left(\bar{y} - \delta\right) + (\bar{\mu} - 1)\bar{m},$$

$$\bar{\mu} = \bar{\gamma}\bar{f},$$

$$(\bar{c})^{-\theta} = \bar{\lambda} + \bar{\eta}$$

$$\bar{\lambda} = \frac{\beta\bar{\gamma}^{-\theta}}{\bar{f}}\left(\bar{\lambda} + \bar{\eta}\right),$$

$$1 = \beta\bar{\gamma}^{-\theta}\left\{(1-\bar{\tau})\left(\alpha\bar{y} - \delta\right)\left[1 + \bar{s}\frac{\bar{\eta}}{\bar{\lambda}}\right] + 1\right\},$$

$$\frac{1}{1+\bar{i}} = \frac{\beta\bar{\gamma}^{-\theta}}{\bar{f}},$$

$$\bar{g} = \left(\bar{A}G\right)^{1/\alpha}.$$

where the variables with a *bar* denote their steady state values.

2) We log-linearize the system of equilibrium equations (3.1)-(3.10) around its steady state. A variable with a *tilde* denotes the log-deviation of such a variable from its steady state, that is,

$$\tilde{h}_t = \ln\hat{h}_t - \ln\bar{h},$$

where $\hat{h}_t$ is the original variable and $\bar{h}$ is its steady state value. The log-linearization of the system (3.1)-(3.10) can be written in the following way:[5]

$$-\bar{c}\tilde{c}_t - \bar{\gamma}\tilde{\gamma}_{t+1} - \bar{g}\tilde{g}_t + \bar{y}\tilde{y}_t = 0,$$

$$-\tilde{y}_t + \tilde{A}_t + (1-\alpha)\,\tilde{g}_t = 0,$$

$$-\bar{c}\tilde{c}_t + \bar{m}\tilde{m}_t + \bar{s}\,(\bar{y}-\delta)\,(1-\bar{\tau})\,\tilde{s}_t + \bar{s}\bar{y}\,(1-\bar{\tau})\,\tilde{y}_t - \bar{s}\bar{\tau}\,(\bar{y}-\delta)\,\tilde{\tau}_t = 0,$$

$$-\bar{g}\tilde{g}_t + \bar{\tau}\,(\bar{y}-\delta)\,\tilde{\tau}_t + \bar{\tau}\bar{y}\tilde{y}_t + (\bar{\mu}-1)\,\bar{m}\tilde{m}_t + \bar{\mu}\bar{m}\tilde{\mu}_{t+1} = 0,$$

$$E_t\left(-\tilde{m}_{t+1} - \tilde{\gamma}_{t+1} - \tilde{f}_{t+1} + \tilde{\mu}_{t+1} + \tilde{m}_t\right) = 0,$$

$$\theta\bar{c}^{-\theta}\tilde{c}_t + \bar{\lambda}\tilde{\lambda}_t + \bar{\eta}\tilde{\eta}_t = 0,$$

$$E_t\left(\frac{\beta\bar{\gamma}^{-\theta}}{\bar{f}}\tilde{\lambda}_{t+1} + \frac{\beta\bar{\gamma}^{-\theta}}{\bar{f}}\frac{\bar{\eta}}{\bar{\lambda}}\tilde{\eta}_{t+1} - \theta\tilde{\gamma}_{t+1} - \tilde{\lambda}_t - \tilde{f}_{t+1}\right) = 0,$$

$$E_t\left(\beta\bar{\gamma}^{-\theta}\left[1+(1-\bar{\tau})\,(\alpha\bar{y}-\delta)\right]\tilde{\lambda}_{t+1} + \beta\bar{\gamma}^{-\theta}\left[(1-\bar{\tau})\,\alpha\bar{y}\left(1+\bar{s}\frac{\bar{\eta}}{\bar{\lambda}}\right)\right]\tilde{y}_{t+1} + \right.$$
$$\left.\beta\bar{\gamma}^{-\theta}\frac{\bar{\eta}}{\bar{\lambda}}\bar{s}\,(1-\bar{\tau})\,(\alpha\bar{y}-\delta)\left\{\tilde{\eta}_{t+1}+\tilde{s}_{t+1}\right\} - \beta\bar{\gamma}^{-\theta}\bar{\tau}\,(\alpha\bar{y}-\delta)\left(1+\bar{s}\frac{\bar{\eta}}{\bar{\lambda}}\right)\tilde{\tau}_{t+1} - \theta\tilde{\gamma}_{t+1} - \tilde{\lambda}_t\right) = 0,$$

$$E_t\left(\tilde{\lambda}_{t+1} - \theta\tilde{\gamma}_{t+1} - \tilde{\lambda}_t - \tilde{f}_{t+1} + \frac{\beta\bar{\gamma}^{-\theta}}{\bar{f}}\bar{i}\tilde{i}_{t+1}\right) = 0,$$

$$-\tilde{g}_t + \frac{1}{\alpha}\tilde{A}_t = 0.$$

It is now convenient to write the log-linearized system of equilibrium equations in a matrix form. To do so, we define $\tilde{x}_t$ as a $q-$dimensional vector of endogenous state variables, $\tilde{u}_t$ as a $n-$dimensional vector of control variables, and $\tilde{e}_t$ as a $d-$dimensional vector of exogenous state

[5]When the monetary aggregate targeting is employed, $\mu_{t+1} = \mu$ for all t, and thus $\tilde{\mu}_{t+1} = 0$. On the other hand, when the nominal interet rate is targeted, $i_{t+1} = i$ for all t and thus $\tilde{\imath}_{t+1} = 0$.

variables. We can then write

$$A\tilde{x}_t + B\tilde{x}_{t-1} + C\tilde{u}_t + D\tilde{e}_t = 0,$$

$$E_t\left(F\tilde{x}_{t+1} + G\tilde{x}_t + H\tilde{x}_{t-1} + J\tilde{u}_{t+1} + K\tilde{u}_t + L\tilde{e}_{t+1} + M\tilde{e}_t\right) = 0,$$

$$\tilde{e}_{t+1} = N\tilde{e}_t + \varepsilon_{t+1}, \qquad \text{with} \qquad E_t(\varepsilon_{t+1}) = 0,$$

where it is assumed that C is of dimension $l \times n$, with $l \geq n$ and $rank(C) = n$, l is the number of deterministic equations, F is of dimension $(q + n - l) \times q$, and N is of dimension $d \times d$.

If we consider our original model, there are two endogenous state variables, k_t, and M_t, eight control variables, c_t, λ_t, η_t, y_t, p_t, τ_t, g_t, and i_{t+1} or μ_{t+1}, depending on the monetary policy, and two exogenous state variables, A_t and s_t. We thus have $l = 6$ deterministic equations and $n = 8$ control variables. However, the matrix C is properly defined for $l \geq n$. Therefore, we reduce n by redefining some endogenous control variables as state variables in order to have $n = l = 6$. Obviously, we have that $q = 4$ and $d = 2$. For the monetary aggregate targeting policy we will consider the following vector of endogenous state variables:

$$\tilde{x}_t = \begin{pmatrix} \tilde{\gamma}_{t+1}, & \tilde{i}_{t+1}, & \tilde{\eta}_t, & \tilde{f}_{t+1} \end{pmatrix}'.$$

For the nominal interest rate targeting policy the vector $\tilde{x}_t$ is instead

$$\tilde{x}_t = \begin{pmatrix} \tilde{\gamma}_{t+1}, & \tilde{\mu}_{t+1}, & \tilde{\eta}_t, & \tilde{f}_{t+1} \end{pmatrix}'.$$

The vectors of control and exogenous state variables are the same for both targeting procedures

$$\tilde{u}_t = \begin{pmatrix} \tilde{c}_t, & \tilde{\lambda}_t, & \tilde{y}_t, & \tilde{m}_t, & \tilde{\tau}_t, & \tilde{g}_t \end{pmatrix}',$$

and

$$\tilde{e}_t = \begin{pmatrix} \tilde{A}_t, & \tilde{s}_t \end{pmatrix}'.$$

3) We obtain the recursive equilibrium law of motion in the form

$$\tilde{x}_t = \mathbf{P}\tilde{x}_{t-1} + \mathbf{Q}\tilde{e}_t,$$

$$\tilde{u}_t = \mathbf{R}\tilde{x}_{t-1} + \mathbf{S}\tilde{e}_t,$$

where the algorithm looks for matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ so that the equilibrium described by these equations is stable. Uhlig (1997) proves that $\mathbf{P}$ is the solution of the following quadratic matrix equation

$$\Psi\mathbf{P}^2 - \Gamma\mathbf{P} - \Theta = 0,$$

where

$$\begin{aligned}\Psi &= F - JC^{-1}A, \\ \Gamma &= JC^{-1}B - G + KC^{-1}A, \\ \Theta &= KC^{-1}B - H.\end{aligned}$$

The matrix $\mathbf{R}$ is given by

$$\mathbf{R} = -C^{-1}\left(A\mathbf{P} + B\right),$$

The matrix $\mathbf{Q}$ satisfies

$$\Big(N' \otimes \Big(F - JC^{-1}A\Big) + I_r \otimes \Big(J\mathbf{R} + F\mathbf{P} + G - KC^{-1}A\Big)\Big)\ vec\left(\mathbf{Q}\right) =$$

$$vec\left(\Big(JC^{-1}D - L\Big)N + KC^{-1}D - M\right),$$

where $\otimes$ is the Kronecker product, I_d is the identity matrix of size $d \times d$, and $vec(\cdot)$ denotes columnwise vectorization. Finally, $\mathbf{S}$ is given by

$$\mathbf{S} = -C^{-1}\left(A\mathbf{Q} + D\right).$$

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	θ	$\mu_t = \mu$ for all t			$i_t = i$ for all t		
		(A, s) $\times$ $1e-3$	(A) $\times$ $1e-3$	(s) $\times$ $1e-3$	(A, s) $\times$ $1e-3$	(A) $\times$ $1e-3$	(s) $\times$ $1e-3$
σ_c	0.5	25.3 (20.7,29.9)	23.9 (20.6,27.2)	5.11 (1.6,8.6)	24.8 (24.8,29.3)	23.5 (20.2,26.8)	4.87 (1.3,8.4)
	0.9	39.95 (32.5,47.3)	37.73 (32.4,43)	8.02 (2.2,13.8)	39.91 (32.5,47.2)	37.7 (32.4,43)	7.98 (2.1,13.8)
	1	41.82 (34.1,49.5)	39.497 (33.9,45)	8.4 (2.3,14.5)	41.81 (34.1,49.5)	39.493 (33.9,45)	8.36 (2.2,14.5)
	1.2	44.65 (36.3,52.9)	42.16 (36.4,52.9)	8.96 (2.4,15.5)	44.67 (36.4,52.9)	42.18 (36.2,48.1)	8.95 (38.5,51.2)
	1.5	47.4 (38.6,56.3)	44.83 (38.5,51.2)	9.538 (2.5,16.5)	47.5 (38.7,56.3)	44.88 (38.5,51.2)	9.534 (2.5,16.5)
σ_γ	0.5	29.847 (23.1,36.6)	27.54 (23.2,31.8)	7.197 (1.9,12.5)	29.839 (23.1,36.6)	27.53 (23.2,31.8)	7.194 (1.9,12.5)
	0.9	30.102 (23.2,36.9)	27.7573 (23.3,32.1)	7.2954 (1.9,12.6)	30.101 (23.2,36.9)	27.7569 (23.3,32.1)	7.2953 (1.9,12.6)
	1	30.1351 (23.2,37)	27.78 (23.3,32.2)	7.3079 (1.9,12.6)	30.1348 (23.2,37)	27.78 (23.3,32.2)	7.3079 (1.9,12.6)
	1.2	30.1845 (23.2,37)	27.8265 (23.4,32.24)	7.3266 (1.9,12.7)	30.1848 (23.2,37)	27.8269 (23.4,32.24)	7.3268 (1.9,12.7)
	1.5	30.234 (23.3,37)	27.868 (23.4,32.3)	7.3454 (1.9,12.7)	30.235 (23.3,37)	27.869 (23.4,32.3)	7.3458 (1.9,12.7)
σ_f	0.5	1.57 (1.3,1.8)	1.25 (1,1.4)	0.904 (0.7,1)	0.6050 (0.49,0.72)	0.5710 (0.489,0.65)	0.1218 (0.032,0.21)
	0.9	1.15 (0.98,1.3)	0.678 (0.56,0.79)	0.913 (0.79,1)	0.6055 (0.49,0.72)	0.5715 (0.489,0.65)	0.1219 (0.032,0.21)
	1	1.1 (0.95,1.2)	0.6 (0.5,0.7)	0.917 (0.79,1)	0.6055 (0.49,0.72)	0.5716 (0.489,0.65)	0.1219 (0.032,0.21)
	1.2	1.05 (0.91,1.2)	0.5 (0.4,0.6)	0.923 (0.8,1)	0.6056 (0.49,0.72)	0.5716 (0.489,0.65)	0.1219 (0.032,0.21)
	1.5	0.98 (0.85,1.1)	0.491 (0.41,0.56)	0.855 (0.73,0.97)	0.6057 (0.49,0.72)	0.5717 (0.489,0.65)	0.1219 (0.032,0.21)

Table 3: Volatility of consumption, of the growth rate, and of the inflation rate for the two targeting procedures (confidence intervals in brackets);[6] μ = monetary growth rate, i = nominal interest rate, θ = inverse of the elasticity of intertemporal substitution, (A, s) = both shocks operate, (A) = only technology shocks operate, (s) = only money demand shocks operate, σ_c = volatility of consumption, σ_γ = volatility of the rate of economic growth, σ_f = volatility of the inflation rate.

[6]All reported volatilities must be multiplied by $1e - 3$.

θ	$\mu_t = \mu$ for all t			$i_t = i$ for all t		
	(A,s)	(A)	(s)	(A,s)	(A)	(s)
0.5	147.4 $(-526{,}821)$	147.3 $(-524{,}819)$	-62.58 $(-63.1,-62.0)$	156 $(-555{,}868)$	156 $(-555{,}868)$	-62.55 $(-62.6,-62.5)$
0.9	-55.42 $(-179{,}68.4)$	-55.39 $(-179{,}68.5)$	-77.27 $(-77.7,-76.7)$	-55.35 $(-179{,}68.6)$	-55.36 $(-179{,}68.6)$	-77.24 $(-77.3,-77.2)$
1	-69.55 $(-197{,}58.6)$	-69.526 $(-197{,}58.6)$	-81.27 $(-81.78,-80.76)$	-69.52 $(-197{,}58.6)$	-69.524 $(-197{,}58.6)$	-81.24 $(-81.28,-81.19)$
1.2	-98.16 $(-253{,}57)$	-98.13 $(-253{,}57)$	-89.46 $(-90,-88.9)$	-98.09 $(-253{,}57)$	-98.09 $(-253{,}57)$	-89.43 $(-89.48,-89.38)$
1.5	-157.16 $(-429{,}115)$	-157.12 $(-429{,}115)$	-101.42 $(-102,-100.8)$	-156.82 $(-428{,}114)$	-156.82 $(-428{,}114)$	-101.38 $(-101,-101.3)$

Table 4: Welfare levels achieved under the two targeting procedures (confidence intervals in brackets); μ = monetary growth rate, i = nominal interest rate, θ = inverse of the elasticity of intertemporal substitution, (A,s) = both shocks operate, (A) = only technology shocks operate, (s) = only money demand shocks operate.

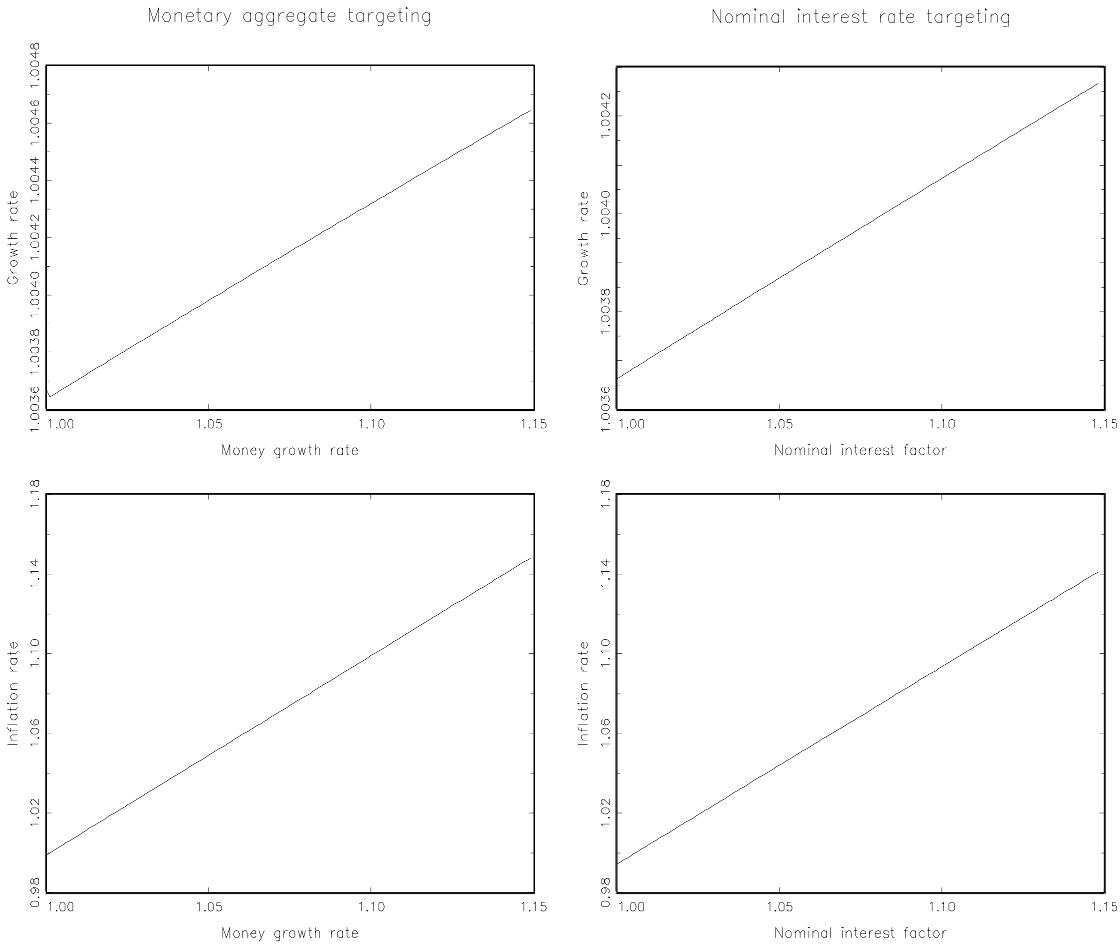

Figure 1: Steady state effects of the growth rate of money supply on the rate of economic growth and on the inflation rate.

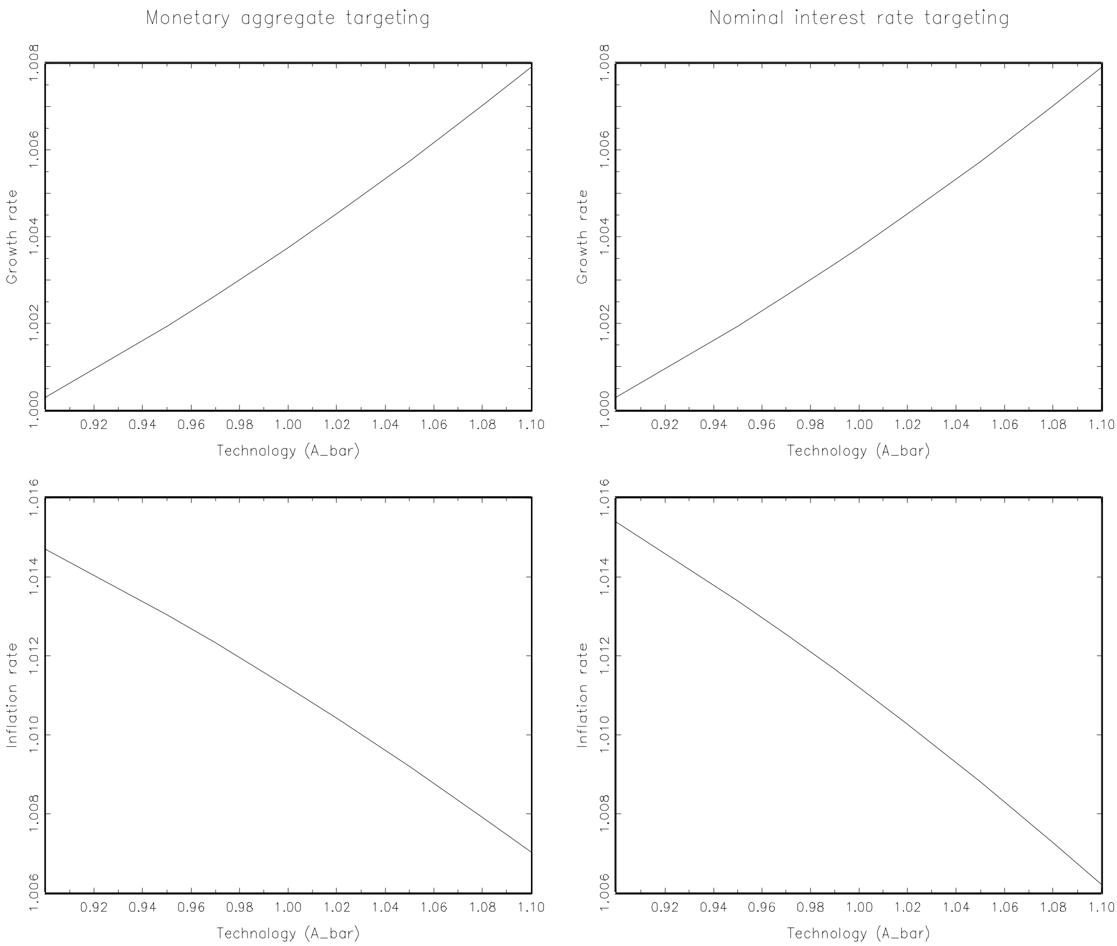

Figure 2: Steady state effects of the technology on the rate of economic growth and on the inflation rate.

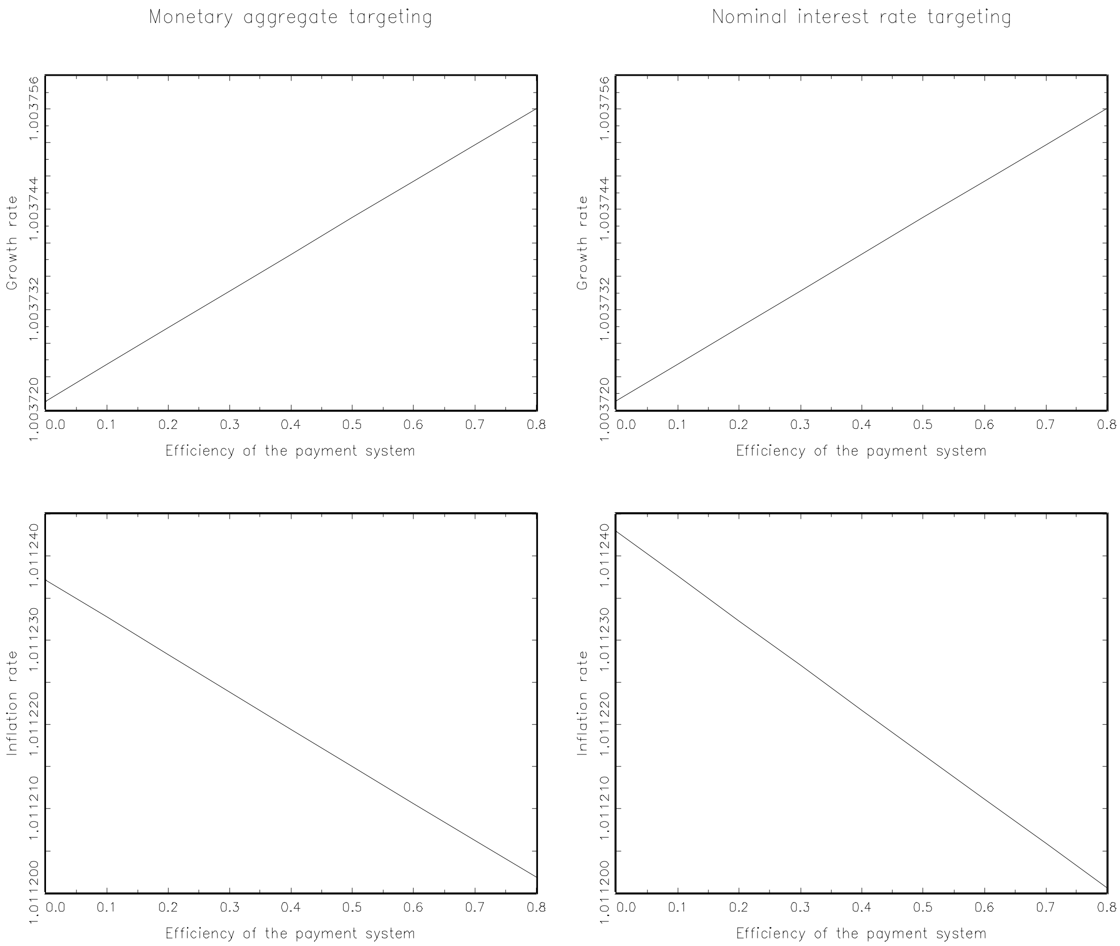

Figure 3: Steady state effects of the efficiency of the payment system on the rate of economic growth and the inflation rate.

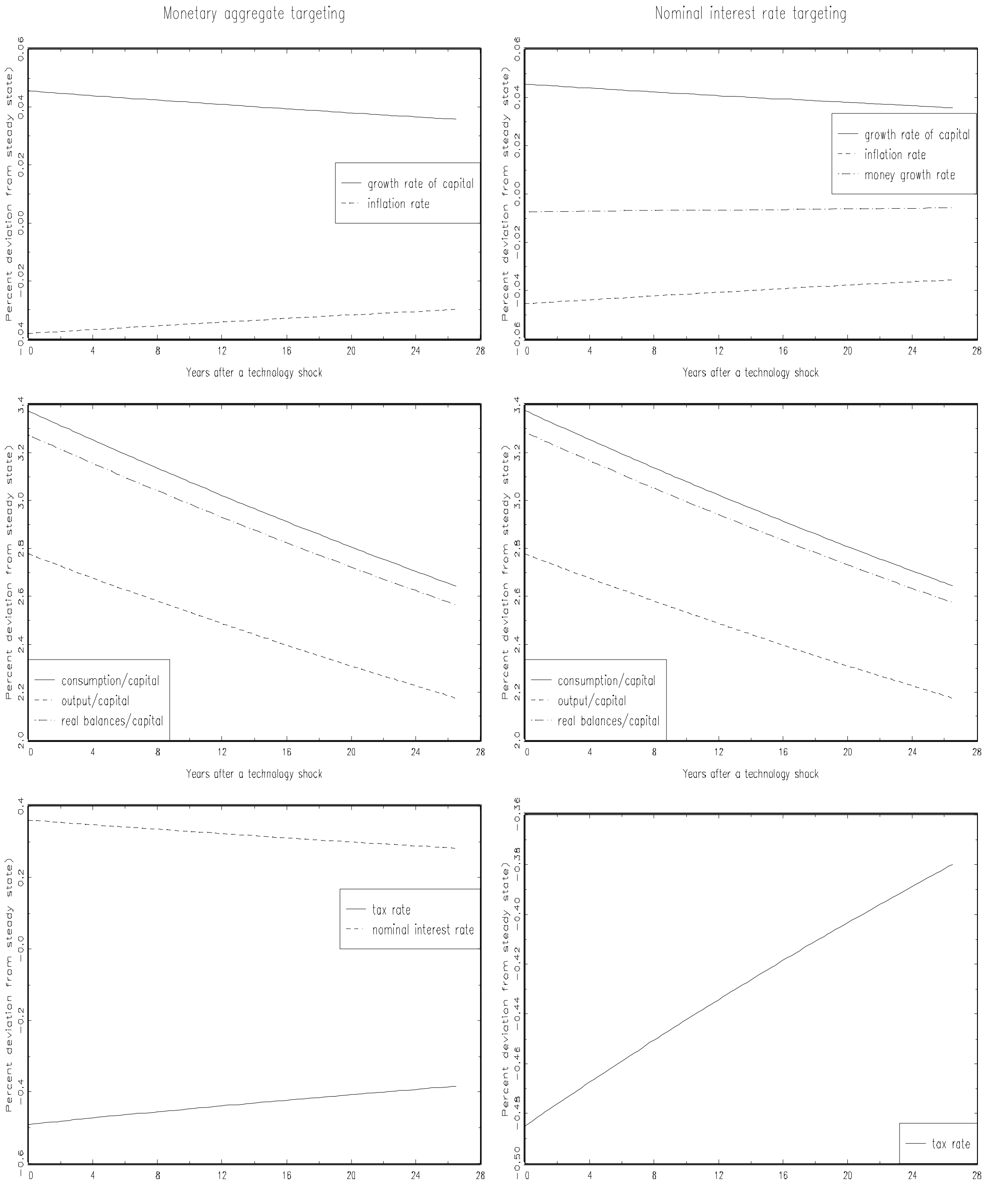

Figure 4: Impulse responses of several variables to a shock in technology, under monetary aggregate targeting (left column), and under nominal interest rate targeting (right column).

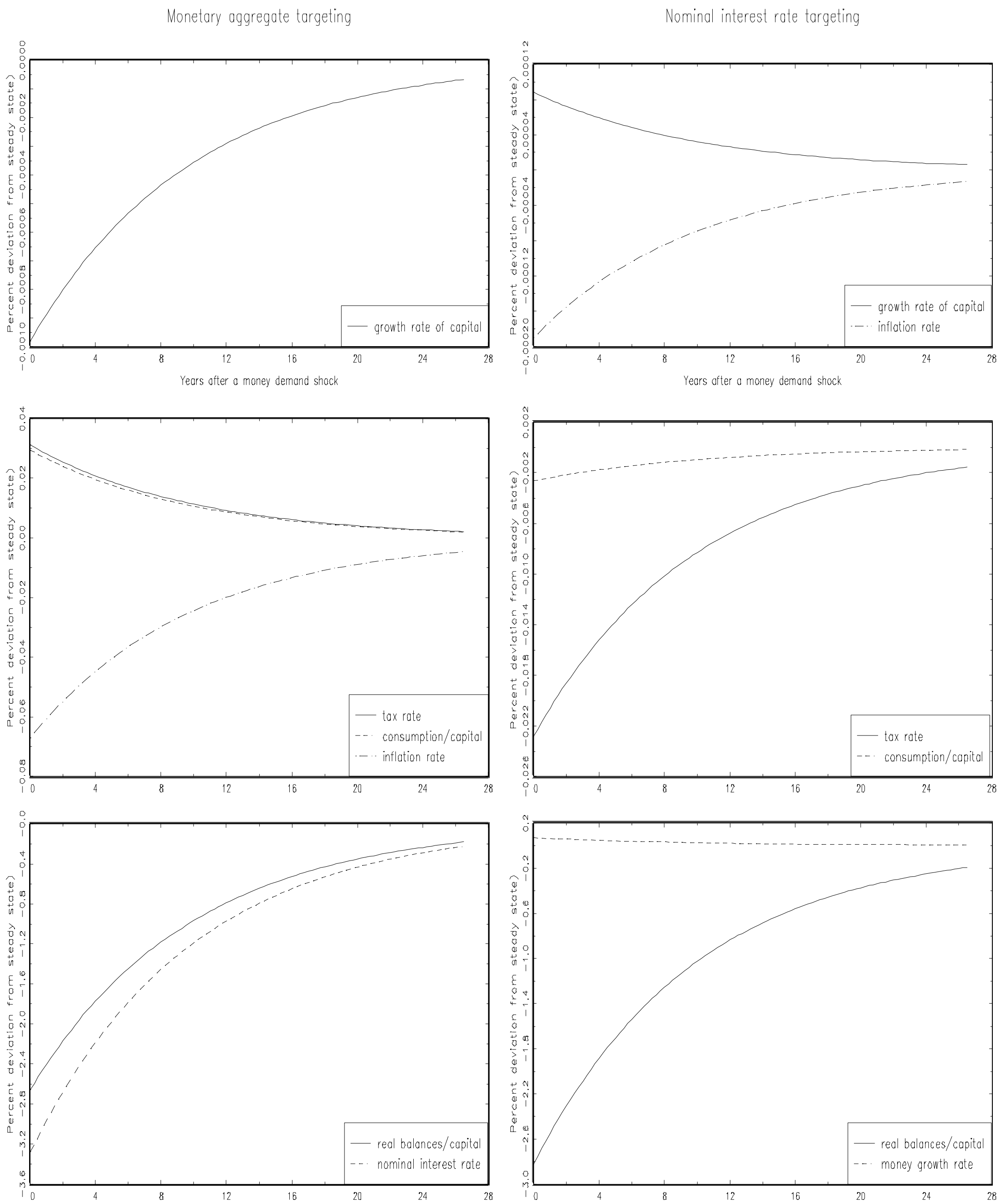

Figure 5: Impulse responses of several variables to a shock in money demand, under monetary aggregate targeting (left column), and under nominal interest rate targeting (right column).