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Abstract

We study the relation between the number of firms and market power in experimental oligopolies. Price competition under decreasing returns involves a wide interval of pure strategy equilibrium prices. We present results of an experiment in which two, three and four identical firms repeatedly interact in this environment. Less collusion with more firms leads to lower average prices. With more than two firms, the predominant market price is 24. A simple imitation model captures this phenomenon. For the long run, the model predicts that prices converge to the Walrasian outcome, but for the intermediate term the modal price is 24.

Keywords

Laboratory experiments, industrial organisation, oligopoly, price competition, co-ordination games, learning

JEL Classification Codes

C90, C72, D43, D83, L13

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1. Introduction

One of the central themes of the economic analysis of oligopoly is how the number of firms affects prices when there are only few competitors in the market. Theoretical analysis has provided some answers to the above question. The equilibrium proposed by Cournot (1838) for the case of quantity competition yields predictions of a unique price and of a price-cost margin which is decreasing in the number of firms. For the case of price competition Bertrand's and Edgeworth's analysis led to somewhat less natural predictions. Bertrand (1883) formulated a well-known solution for the case of constant returns and no capacity constraints: with more than one firm prices will be equal to marginal cost, independently of the number of firms.

Edgeworth (1925) studied the case of price competition where firms have capacity constraints or can limit the demand that they serve at a given price. His view was that, for this kind of markets, it would be impossible to predict prices, which would oscillate, moving around endlessly. Modern analysis has shown that for this type of market interaction one can guarantee the existence of a unique Nash equilibrium but only in mixed, and not in pure strategies. Under reasonable conditions, the ratio between the average of the price distribution and cost is decreasing in the number of firms.

Economists' stylised view of oligopolistic competition has been complemented by experimental studies on the subject. Numerous studies report results from quantity competition environments. To better reflect the nature of actual oligopolistic interaction most of these studies deal with the case of repeated play in fixed groups and not with a simple one-shot encounter. Huck, Normann and Oechssler (2001) provide a recent survey and synthesis of experimental work on quantity competition. Their conclusion is that duopolists sometimes manage to collude, but that in markets with more than three firms collusion is difficult. With exactly three firms, Offerman, Potters, and Sonnemans (2002) observe that market outcomes depend on the information environment: Firms collude when they are provided with information on individual quantities, but not individual profits. In many instances, total average output exceeds the Nash prediction and furthermore, these deviations are increasing in the number of firms. The price-cost margins found in experimental repeated quantity competition are, hence, qualitatively consistent with the Cournot prediction for the static game.

Dufwenberg and Gneezy (2000) study the effects of the number of firms in a standard Bertrand competition framework with constant marginal cost and inelastic demand. In their experiments, price is *above* marginal cost for the case of two firms but equal to that cost for three and four firms. Their results just modify the theoretical prediction in a simple way: by changing from two to three the number of firms from which on the price can be expected to equal marginal cost.

Davis and Holt (1994), Kruse, Rassenti, Reynolds, and Smith (1994), and Morgan, Orzen, and Sefton (2001) study price competition in environments in which the equilibrium prediction involves randomisation. All three studies find price dispersion distinct but qualitatively similar to those predicted by Nash equilibrium.

In this paper we study experimentally a case in which price competition can lead to positive price-cost margins, but unrelated to the use of mixed strategies; neither zero profits with few firms nor oscillating prices are satisfactory predictions in many instances.

Dastidar (1995) analyses price competition in a model in which firms operate under decreasing returns to scale and have to serve the whole market. The Bertrand assumption that firms *must* supply the demanded quantity regardless of the cost is reasonable in cases in which it its very costly to turn consumers away, as in the case of consumer protection laws. In this setting positive price-cost margins are possible in equilibrium. However, the Nash equilibrium prediction in this model will be strongly indeterminate. There will, in fact, be a whole interval of Nash equilibrium prices with a minimum zero-profits equilibrium price below the Walrasian price and a maximum price above it. The maximum and minimum prices both decrease with the number of firms so that a sufficient increase of the number of firms implies, in a sense, a prediction of lower prices. However, an increase of just one or two firms, as often envisioned in the context of antitrust analysis, is well compatible with equilibrium predictions of decreasing, unaffected, or even increasing prices.

Depending on the demand and cost parameters the maximum equilibrium price may be above the Cournot price and even above the price which yields the joint profit maximum. The indeterminacy is, hence, quite considerable, since it may not even be possible to predict whether this kind of price competition will be weaker or stronger than quantity competition would be.

We present an experimental study on the above price competition setting. We study the interaction over numerous rounds of fixed groups of competitors, i.e. the finite repetition of the stage game described above. We believe that laboratory experiments are well suited for investigating problems of multiplicity of equilibria; they may be able to supply precise predictions on the basis of a variety of behavioural factors. Examples of the use of experiments in contexts with equilibrium multiplicity are Van Huyck et al. (1990), and Cooper, DeJong, Forsythe and Ross (1990).

With our work we wish to contribute to delineating a more complete picture of oligopolistic behaviour from an experimental viewpoint. In particular, we are interested in what price and efficiency levels arise and how they depend on the number of firms, and in finding a rationale for how observed prices are reached. With respect to price levels we are interested in finding out whether prices remain above Walrasian levels with three or four firms. We also study the process by which prices are reached.

Our results show that average prices are always above the Cournot and, hence, the Walrasian level. The resulting prices can be organised according to two principles: The first is the cartel price, which is the modal outcome in n=2, but becomes less frequent with more firms. The second organising principle is a specific price level, 24 in our experimental design, which is the market price in an absolute majority of cases in n=3 and n=4, as well as the second most frequent outcome in n=2. Average prices are decreasing with the number of firms, mainly because there is less collusion and there are more 24-prices as the number of firms increases. The price of 24, which was not predicted by any benchmark theory we were aware of, is characterised by the feature that unilateral undercutting of this price level leads to absolute losses. We propose a simple imitation model, which reproduces this phenomenon and also other regularities in our data quite well.

2. The model and the experimental design

2.1. Demand, cost curves, and stage game equilibria

In our experimental markets the demand function is linear and the cost function is quadratic and common to all firms. The intuitive explanation for the equilibrium price indeterminacy in this case is rather simple and can be easily illustrated with the figures 1 and 2, in which D denotes demand, MC marginal cost and AC/S the average cost and supply curve, for the case of two firms. The figures show the lowest-price and the highest-price equilibrium, respectively. Observe first, in figure 1, the configuration with both firms setting the common price $\underline{p}$. Since the firm with the lowest price serves all the market, any upward deviation leads to zero profits. Any downward deviation implies a price that is below the marginal cost of all the additional units that the deviating firm needs to supply to the market and, hence, every additional unit, leads to a decrease in profits for that firm. At the lowest-price equilibrium profits are zero; a situation with negative profits can naturally not be an equilibrium, since upward deviations lead to zero profits. The lowest-price equilibrium is depicted in figure 1, where the two shaded triangles try to illustrate that at p firms' profits are zero.

A wide range of higher prices can also be supported in equilibrium, leading to positive profits for both firms. The same logic applies: Overcutting leads to zero profits, since the deviating firm does not sell any units. Undercutting the common price, however, leads to lower profits since the firm must sell additional units at excessive marginal costs. There exists, however, a high enough price level, denoted in figure 2 by $\overline{p}$, at which downward deviations lead to zero marginal profits. In the figure the two shaded triangles mean to illustrate how positive and negative additional profits from a deviation will cancel out. Prices above this one are not equilibrium ones any more.

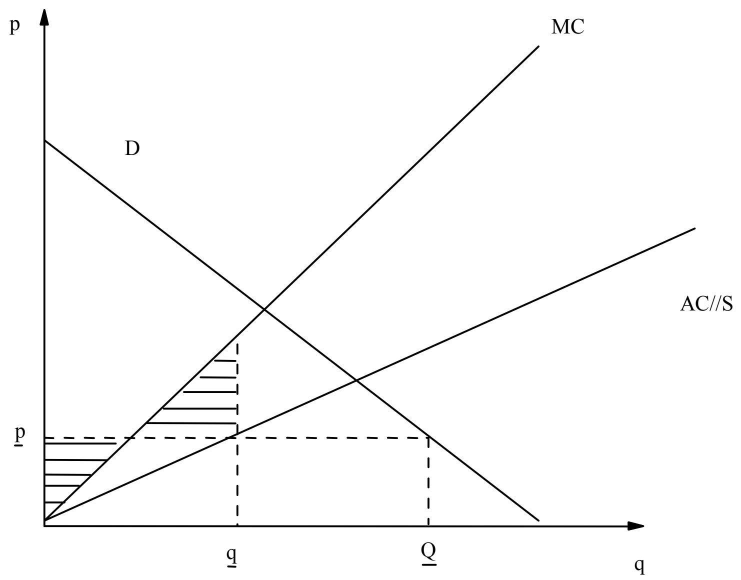

Figure 1

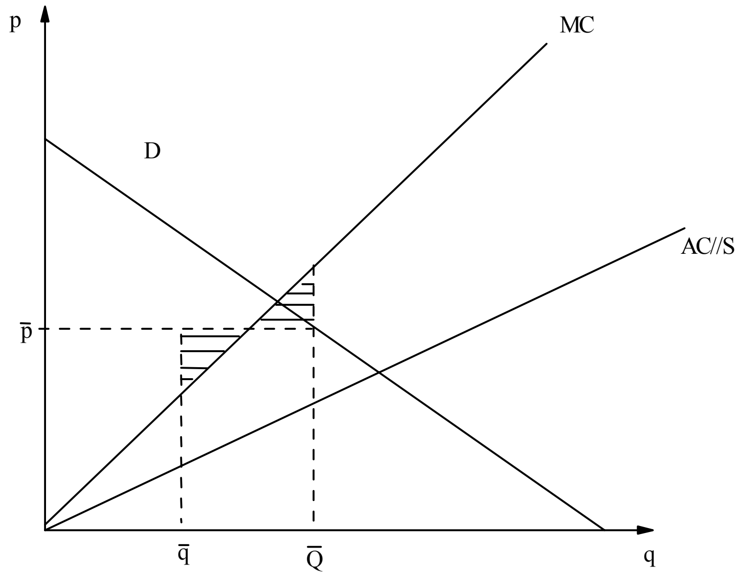

Figure 2

How does the number of firms in the market affect the above argument? Both the minimum and the maximum price decrease with the number of firms; in fact the minimum price converges to zero and the maximum price to a positive lower bound.[1] Due to the fact that marginal costs are increasing a larger number of firms can share the market with zero profits at a lower price level. The decrease in the maximum price is also due

[1] See Vives (1999) for a discussion of this model.

to the fact that marginal costs are increasing. If three firms all set a price equal to $\overline{p}$, individual deviations would be profitable since a firm starting from an output level smaller than $\overline{q}$ will be able to sell many additional low cost units.

2.2. Design choices

In our experiment, we presented the implications of the different pricing decisions in a simplified way. In each round subjects had to individually choose a number on the basis of payoff tables derived from the one shown in table 1. In the treatment with 4 firms, subjects' payoff table was exactly table 1, except that subjects did not see the shaded areas we have inserted to mark several benchmarks. The first column shows all the numbers – prices – that could be chosen.[2] The second column shows, for a given round, a subject's profit in talers (the fictitious experimental currency) if he alone has chosen that number and the other 3 players have chosen higher numbers. The third column shows a subject's profit if he and another player have chosen that number and the other two have chosen higher numbers. Columns 4 and 5 refer to the analogous cases with 3 and 4 subjects and column 6 shows a subject's payoff if he has chosen a higher number than all the other players. For n=3 subjects saw a table without column 5 and for n=2 a table without columns 4 and 5. Appendix B contains the instructions we used.

The payoffs in table 1 are derived from the demand function $D(p)=100–1.5p$ and the common cost function $C(q)=1.5q^2/2$. This particular choice of parameters makes it possible to have the highest equilibrium price be above the price corresponding to the Cournot benchmark. The price grid was chosen in a manner that all possible price choices are equidistant from one another.

2.3. Benchmarks and predictions

It can be seen in table 1 that for n=2 the equilibria are between 13 and 30, for n=3 between 7 and 28, and for n=4 between 3 and 27. For n=2, n=3 and n=4 the Walrasian prices and the Cournot equilibrium prices, which result from the above demand and cost functions, correspond to prices in between those that are eligible. The Walrasian price is between 23 and 24 for n= 2, between 16 and 17 for n=3 and between 12 and 13 for n=4. The Cournot equilibrium price is between 29 and 30 for n= 2, between 22 and 23 for n=3 and between 17 and 18 for n=4.

[2] The numbers for the prices are not those that result from our parameter choices. They have been relabelled to go from 1 to 40 for simplicity.

Table 1. The payoff table for n=4

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
Number cho-sen	Profit if this number is the unique lowest of the four	Profit if this number is the lowest and has been chosen by two players	Profit if this number is the lowest and has been chosen by three players	Profit if this number is the lowest and has been chosen by four players	Profit if this number is not the lowest
40	777	473	334	258	0
39	784	489	348	269	0
38	783	503	360	279	0
37	777	514	370	288	0
36	763	522	379	296	0
35	743	528	387	303	0
34	716	532	393	310	0
33	683	***533***	398	315	0
32	642	532	402	319	0
31	596	528	403	322	0
30	542	522	***404***	323	0
29	482	514	403	324	0
28	415	503	401	***325***	0
27	341	489	397	324	0
26	261	473	392	322	0
25	174	455	385	319	0
24	**81**	**434**	**377**	**315**	0
23	-20	411	367	310	0
22	-126	385	356	304	0
21	-240	357	344	297	0
20	-360	326	330	290	0
19	-487	293	314	281	0
18	-621	257	298	271	0
17	-761	219	279	260	0
16	-908	179	260	248	0
15	-1062	136	239	235	0
14	-1222	90	216	221	0
13	-1389	42	192	206	0
12	-1563	-8	167	189	0
11	-1743	-61	140	172	0
10	-1930	-116	112	154	0
9	-2124	-173	82	135	0
8	-2324	-234	51	115	0
7	-2532	-296	18	94	0
6	-2745	-361	-16	72	0
5	-2966	-429	-52	49	0
4	-3193	-499	-89	25	0
3	-3427	-571	-127	0	0
2	-3667	-646	-167	-26	0
1	-3914	-723	-208	-53	-

Equilibrium

Cournot

Walras

Payoff dom. Equilibrium

Cartel

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Given that the equilibrium range is large, we may look at common selection criteria for tighter predictions for the experiment. Harsanyi and Selten (1988) propose that if a unique *payoff dominant* equilibrium exists, rational players should always select it.[3] An equilibrium is strictly payoff dominant if all players receive higher payoffs in this equilibrium than in any other equilibria. In our set-up, the equilibrium with the highest price is the payoff dominant one, as can easily be seen in table 1. Thus, payoff dominance would predict market prices of p=30 for n=2, p=28 for n=3, and p=27 for n=4. Therefore, if experimental firms behaved according to this prediction, we would indeed observe decreasing prices for an increasing number of firms.

Though not an equilibrium, the cartel price may provide a prediction for the experiment. The cartel price is the price level maximising the joint payoff if set by all firms in the market. It is above the range of stage game equilibrium prices, at p=33 for n=2, p=30 for n=3, and p=28 for n=4, thus decreasing with the number of firms.

The last benchmark highlighted in table 1 is at the price of 24, therefore named "24". When we designed the experiment, we were not aware of a conventional equilibrium selection theory that would predict this particular price level. Its speciality appears to be that unilateral undercutting of this price leads to absolute losses of –20, while prices of 24 or higher can be sustained by a single firm making positive absolute profits.

2.4. The conduct of the experiment

The experiment was conducted in the *Laboratori d'Economia Experimental* (LeeX) of the *Universitat Pompeu Fabra* in Barcelona. The software for the experiment was developed using the *RatImage* programming package (Abbink and Sadrieh (1995)). Subjects were recruited by public advertisement in the Department of Economics and were mostly economics students. Each subject was allowed to participate in only one session.

In our experimental sessions subjects interacted in fixed groups of 2, 3 or 4 for 50 identical rounds, in each of which they had to choose a number between 1 and 40. After each round each subject was informed about the number chosen by each of the other subjects in the group as well as about all subjects' profits. They also received information about the smallest number and the number of subjects having chosen the smallest number. As already mentioned, the same subjects played in the same market throughout the session to reflect the repeated game character of actual oligopoly markets. Subjects were not told with whom of the other session participants they were in the same group.

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[3] A second prominent concept developed by Harsanyi and Selten (1988) is that of risk dominance. The authors propose this criterion only for games without a payoff dominant equilibrium. This concept takes into account the risk attached to co-ordination failure. Risk dominant equilibria are straightforward to find for two-player games with two equilibria. For games with more than two players, the concept in general defies analytical solution, and also numerical computation gets extremely difficult for larger games (Herings and van den Elzen (2002)). Identifying a risk dominant equilibrium for a 40x40x40x40 game with 26 pure strategy equilibria, as our n=4 treatment involves, is practically impossible.

To accommodate some losses, subjects were granted a capital balance of 5000 talers at the outset of each session. The total earnings of a subject from participating in this experiment were equal to his capital balance plus the sum of all the profits he made during the experiment minus the sum of his losses. A session lasted for about 1½ hours (this includes the time spent to read the instructions). At the end of the experiment, subjects were paid their total earnings anonymously in cash, at a conversion rate of one Spanish peseta for eight (n=2), seven (n=3) and six (n=4) talers. On average, subjects earned approximately 3000 Spanish pesetas, which is considerably higher than students' regular wage. 100 pesetas are equivalent to 0.602 euros. At the time of the experiment, the exchange rate to the US dollar was approximately $ 0.55 for 100 pesetas.

We conducted one session with 16 subjects for n=2, two sessions with 15 subjects each for n=3, and two sessions with 16 subjects each for n=4. Since there was no interaction between subjects playing in different groups, each group can be considered as a statistically independent observation. Thus, we gathered 8 independent observations for n=2 and n=4, and 10 independent observations for n=3.

3. Results

3.1. Average prices and the number of firms

The three treatments of our experiment allow us to study the effect of the number of firms on market outcomes. In particular, we can analyse whether an increase in the number of competitors results in lower transaction or market prices. Table 2 indicates that, on average, this is the case. The table shows average market prices, i.e. the lowest of chosen prices, for the different groups over the 50 rounds of the experiment, ordered from the lowest to the highest for each value of n. Average prices are decreasing in the number of firms. Fisher's two-sample randomisation test rejects the null hypothesis of equal average prices at a significance level of α=0.01 (one-sided) for all pairwise comparisons of treatments.

3.2. The distribution of market prices

Looking at aggregate outcomes alone can be informative, but of course it tells only part of the story. We also need to look at the prices that actually have been realised in the experimental markets. Figure 4 shows the overall distribution of market prices in all rounds and all markets, for the three treatments separately. The figure reveals a number of regularities. First, notice that for all three values of n a whole range of low equilibrium price levels is not observed. Second, the payoff dominant equilibrium is rarely played. While we observe a sizeable fraction of market prices of 30 in duopolies, the payoff dominant equilibrium prices hardly ever occur in the treatments with more firms.

Table 2. Average market prices

Group No.	*n*=2	*n*=3	*n*=4
1	25.84	23.78	21.14
2	26.70	23.98	22.80
3	28.66	24.00	23.32
4	29.96	24.54	23.98
5	31.68	24.72	23.98
6	31.90	25.92	24.00
7	32.96	26.52	24.08
8	32.98	28.10	26.38
9		29.14	
10		29.62	
Average	30.09	26.03	23.71
Walrasian price	23-24	16-17	12-13
Cournot benchmark	29-30	22-23	17-18
Payoff dominant equilibrium	30	28	27

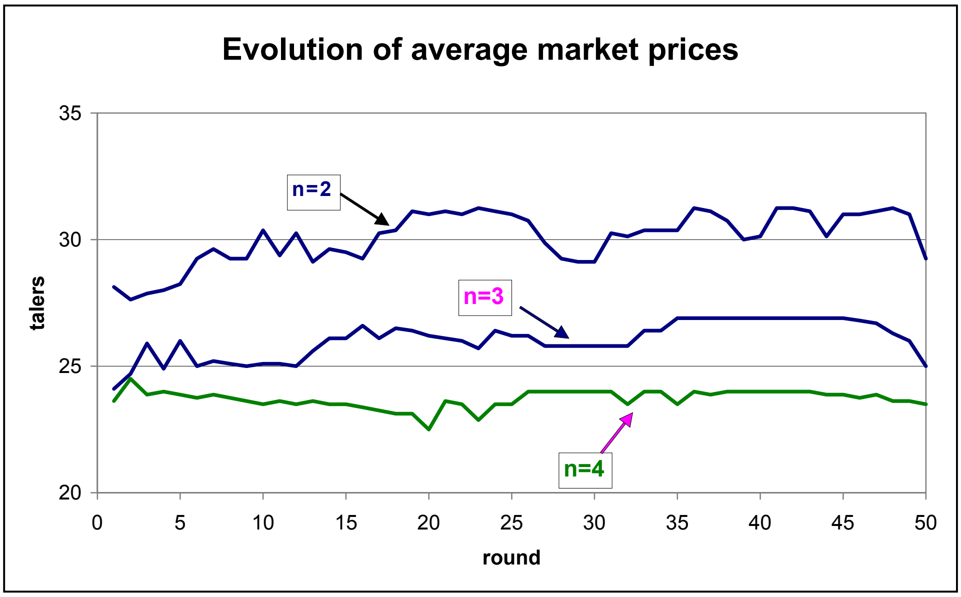

Figure 3

Third, we observe a substantial occurrence of collusion, especially in duopolies. This is not surprising as such, since previous studies have already found that duopolies often manage to collude. However, incentives for collusion seem much lower in our game than in the environments studied before. In the *n*=2 treatment, the collusive payoff is less than 2% higher than that of the payoff dominant equilibrium, and, contrary to the payoff dominant equilibrium, collusion is not self-enforcing in the stage game.[4]

[4] Collusion in earlier rounds can be part of a subgame perfect equilibrium of the finitely repeated super-game. Equilibria exist which involve selecting a low payoff equilibrium in the end rounds if players deviate, and a high payoff equilibrium if the cartel price is set until a certain round. The difference between

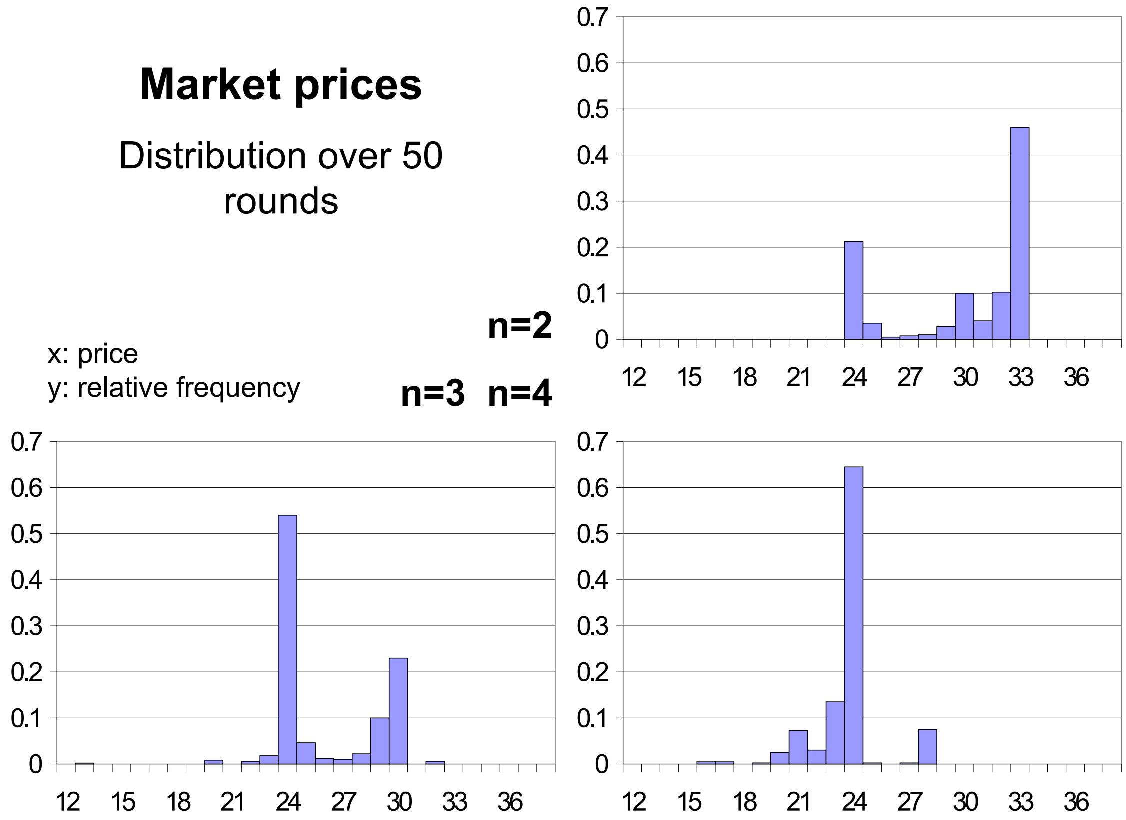

Figure 4

The fourth and perhaps most puzzling result is the strong predominance of the market price of 24. In both the n=3 and the n=4 treatment, 24 seems to be the "magic number" of the experiment, occurring in an absolute majority of cases. Even in n=2 the price of 24 is the most frequent outcome after the collusive one. A look at the individual decisions confirms this result. Figure 5 shows the evolution of individual price choices. Contrary to the previous figures, these plots also include the choices that did not establish the market price. Indeed, two numbers are needed to organise about 80% of the data in each treatment: the collusive price (33 in n=2, 30 in n=3, and 28 in n=4) and the number 24. The decisions in the three treatments differ in the extent to which the 24 is crowded out by the collusive choice.

3.3. Why 24?

What makes the number 24 such a natural choice in the present market environment? It does not match any of the theoretical benchmarks, like the payoff dominant equilibrium, nor is it a round or prominent number. The only apparent speciality of this price – which prevails regardless of the number of firms – is that it is the highest price at which unilat

the end round equilibrium payoffs can be a sufficient incentive to keep the price above the stage game equilibrium price range in the beginning.

eral underbidding is not only disadvantageous compared to equilibrium play, but also unprofitable in absolute terms.[5]

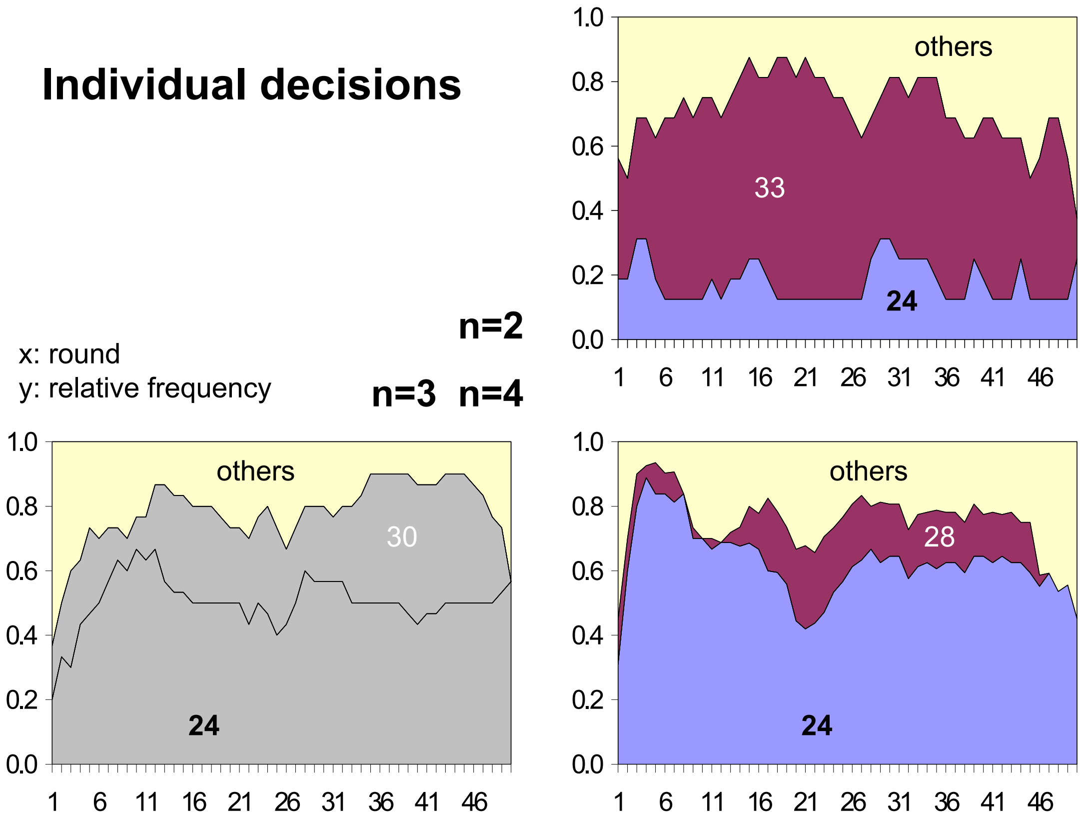

Figure 5

The evolution of play we observe in our experiment suggests a dynamic explanation of this phenomenon, as the predominance of 24 is not yet present at the beginning of the experiment, but rather evolves over time. Figure 5 indicates that in the very beginning of the sessions with n=3 and n=4, the frequency of 24-choices is lower than later on, but rising quickly. This goes along with higher levels of co-ordination on a common price, as figure 6 shows. The figure depicts the relative frequency of markets in which 1, 2, 3, or 4 firms chose the market price. The degree of co-ordination increases quickly in the early rounds of the experiment. Later on, co-ordination reaches a high degree.

Thus, there seems to be some kind of adaptation process taking place, leading to co-ordination on the price of 24; but how is this co-ordination accomplished? Perhaps the simplest behavioural rule that leads to high degree of co-ordination is the one of imitat

[5] This may suggest explanations in terms of loss-aversion (Kahneman and Tversky (1979)). Notice, however, that all equilibria above 24 do not involve the danger of losses either. Therefore, loss-aversion alone may explain why prices typically do not fall beneath 24, but on its own it cannot explain how prices get down to this particular level. However, given there is such a dynamic process, subjects' fear of making losses may reinforce the fact that 24 is a barrier that is hard to break through.

ing the most successful behaviour in previous rounds. This is easy to apply and, as we will see, may well explain why we observe a price level of 24 so frequently.

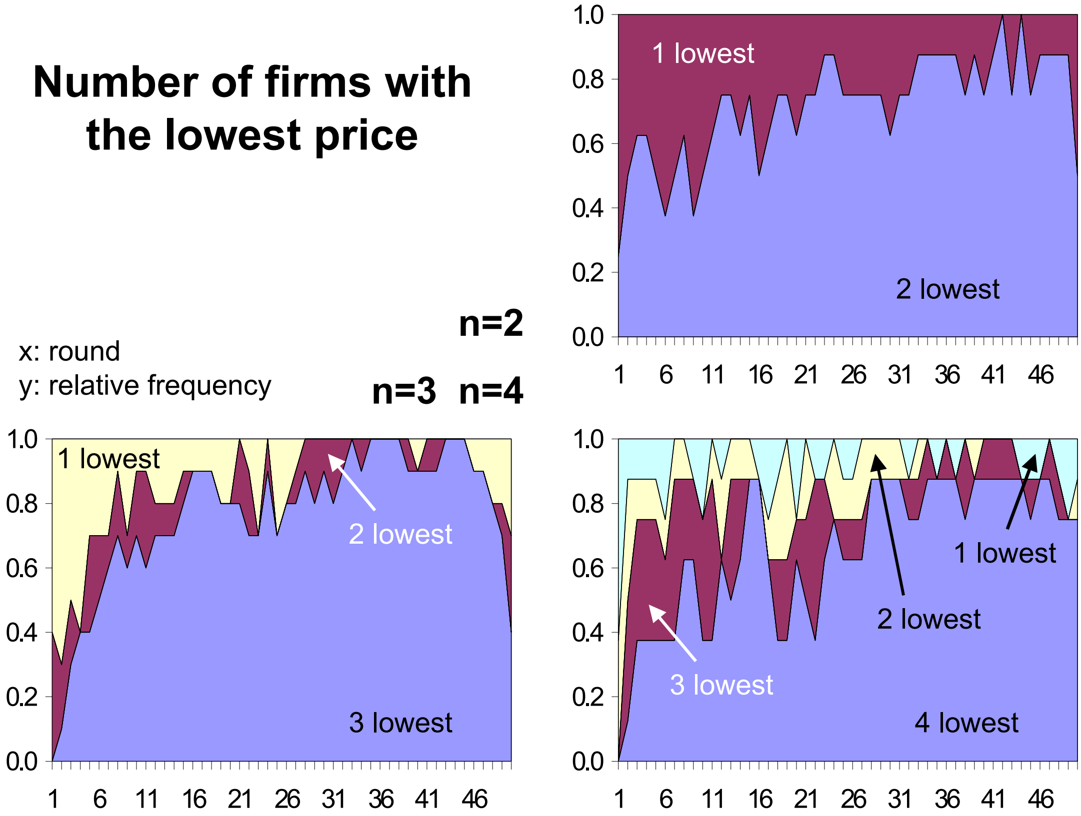

Figure 6

Imitation has recently been used both in theoretical and experimental studies. In a theoretical study, Vega-Redondo (1997) analyses imitation in a quantity competition setting.[6] In that model imitation leads to Walrasian and thus more competitive prices than in the conventional Cournot equilibrium. A number of studies have recently investigated imitation in experimental markets with quantity competition. Huck, Normann and Oechssler (1999, 2000) find evidence favourable to the idea of imitation: more information about competitors yields significantly more competition. Offerman, Potters and Sonnemans (forthcoming) obtain a similar result, while Bosch-Domènech and Vriend (1999) look for, but fail to find support for imitative behaviour.[7]

Our setting has a specific feature making it conducive to imitative behaviour, as doing what others do is essential for earning high profits. Intuitively, one might expect that if

[6] See also Vega-Redondo (1999). For further theoretical insights into the effect of imitation see Schlag (1998), Cubitt and Sugden (1999), and Selten and Ostmann (2001).

[7] A dynamic model describing adaptive behaviour in co-ordination games is provided by Crawford (1995). This model is fit to data on the minimum effort game gathered by Van Huyck, Battalio, and Beil (1990). Our game is reminiscent of this game in that it involves several pareto ranked symmetric equilibria. The payoff structure, however, is rather different.

firms imitate the most successful behaviour, this might well result in price levels of 24. This is because at prices above 24, the most successful price is always the lowest. Below 24, the picture becomes ambiguous: A single firm setting the market price will make losses, and the other firms are better off despite their zero profits. Thus, unilateral undercutting of 24 will not be imitated, while unilateral undercutting of higher prices will be. To examine in more depth how such a dynamic process might work, we now present a specific model of imitation.

3.3.1. A simple imitation model

We propose a simple quantitative dynamic model of imitation in the price competition game. The model assumes that in general, all firms choose the most successful action they observed in the round before with a high probability. With some smaller probability, however, they deviate from this pattern of behaviour and play some other strategy (we refer to this as *experimentation*, in line with Roth and Erev (1995)).[8] We assume that when firms do not imitate, they choose the next higher or next lower price level. It seems reasonable to assume that experimentation takes place gradually rather than over the entire range of choices.

A technical difficulty that arises is that the most successful choice of the round before may not be unique. Different choices may have resulted in the same highest payoff. We could have specified that in this case, firms just pick one of these prices randomly. However, in the spirit of an imitation model it seemed more reasonable to us to introduce a kind of second level imitative reasoning into the tie-breaking rule. Hence, we assume that when firms observe multiple most successful choices, they consider that the less successful behaviour may have been the result of an error and assess what would have been the more successful choice if they had just followed the behavioural standard of imitation. More precisely, they take the most successful price of the previous round as what will be called the *default* price, and compare which of the most successful prices would have been better if the less successful firms had imitated this default price. If there has been more than one most successful price in the previous round, the previous default price is determined using the same tie-breaking rule.[9] Only if this reasoning still does not unambiguously identify a winner, then the firm chooses randomly among all most successful choices of the previous round. This may be necessary because the tie-breaking rule may still leave more than one most successful price, or a default price is not definable for a particular round. For example, if there is a tie in the very first round, there is no earlier round that could be used as a reference.

[8] It is essential for the model to allow some experimentation or error term. This not only seems realistic, it also prevents that only the choices observed in the first round are ever played.

[9] Notice that this establishes a recursive element in determining the default price. Nevertheless, the rule is simple to apply, since firms effectively do not need to trace the complete history, but only need to memorise the winning price of the tie-breaking rule for the following two rounds.

Our simple model is based on the following specifications:

1. In the first round, the probability with which prices are chosen is estimated from the frequencies observed in the first round of the experiment.
2. After every round t, a set of default prices ρ_t is specified.[10] The set Ω is the set of prices that resulted in the highest payoffs in t. Denote by $\boldsymbol{p}_t = (p_{t,1}, .. p_{t,n})$ the vector of prices chosen by the firms $1..n$ in round t. Among the elements of Ω choose a set of default prices $\rho_t \subseteq \Omega$ as follows. If Ω is a singleton, then $\rho_t = \Omega$. If $|\Omega| \geq 2$ and ($\rho_{t-1} = \varnothing$ or $t = 1$), then $\rho_t = \varnothing$. If $|\Omega| \geq 2$ and not ($\rho_{t-1} = \varnothing$ or $t = 1$), then the following tie-breaking rule applies. Denote by Ω' the set of most successful prices for the price vector $\boldsymbol{p}_t'$ with $p_{t,i}' = p_{t,i}$ if $p_{t,i} \in \Omega$ and $p_{t,i}' = \rho_{t-1}$ if $p_{t,i} \notin \Omega$. If Ω' is a singleton, then $\rho_t = \Omega'$, otherwise $\rho_t = \varnothing$.
3. In the following round t+1, each firm sets the auxiliary variable $\phi_{t+1,i} = \rho_t$ if $\rho_t \neq \varnothing$. Otherwise, it selects $\phi_{t+1,i}$ randomly from all elements of Ω, all with equal probability. With probability $\beta > ½$, the firm sets $p_{t+1,i} = \phi_{t+1,i}$. With probability $1-\beta/2$, the firm sets $p_{t+1,i} = \phi_{t+1,i} +1$. With probability $1-\beta/2$, the firm sets $p_{t+1,i} = \phi_{t+1,i} - 1$.

The main ingredients of the model are simple and quite standard. In most cases, firms simply choose the most successful choice of the previous round. Prices that have not been most successful can only be chosen by experimentation. We will now compare the model's implications with our data for the time horizon used in our sessions.

3.3.2. The model's predictions for 50 rounds

We ran simulations with this model letting 100,000 simulated experimental groups play 50 rounds of the game. Figure 7 shows the distribution of the market prices in the simulations. The parameter β for the probability of imitation has been set to $\beta = 0.95$.[11] In order to make them comparable with the data shown in figure 4, we have depicted the overall distributions over all 50 rounds. The figures show that the model captures many qualitative features of the data, especially for n=3 and n=4. As in the experimental data a whole range of lower equilibrium prices does not appear in the simulated results. The modal price is 24, with a tendency to slightly higher prices in n=3 than in n=4. Even quantitatively, the frequency of p=24 choices is similar to the observed one for the treatments with more than two firms.[12] Further, in rounds in which the market price is

[10] Notice that the set ρ is either empty or a singleton. Therefore, we may write that a firm sets $p_i = \rho$, though these are strictly speaking variables of different types.

[11] This choice, which has been made ad hoc, will be relaxed in our sensitivity analysis below.

[12] The quantitative results depend on the particular choice of the parameters. However, the modal price of 24 is very stable for a time horizon of 50 rounds (see section 3.3.4). We have also run simulations with different experimentation patterns (e.g. randomising over all choices) and different tie-breaking rules (e.g. pure randomisation or imitating the average). All of them resulted in modal prices of 24 for 50 rounds and between 2 and 4 firms.

different from 24, these prices tend to be lower with larger n, a phenomenon also apparent in our data (see figure 4). On the other hand, the model naturally does not capture the collusive behaviour present in our data.

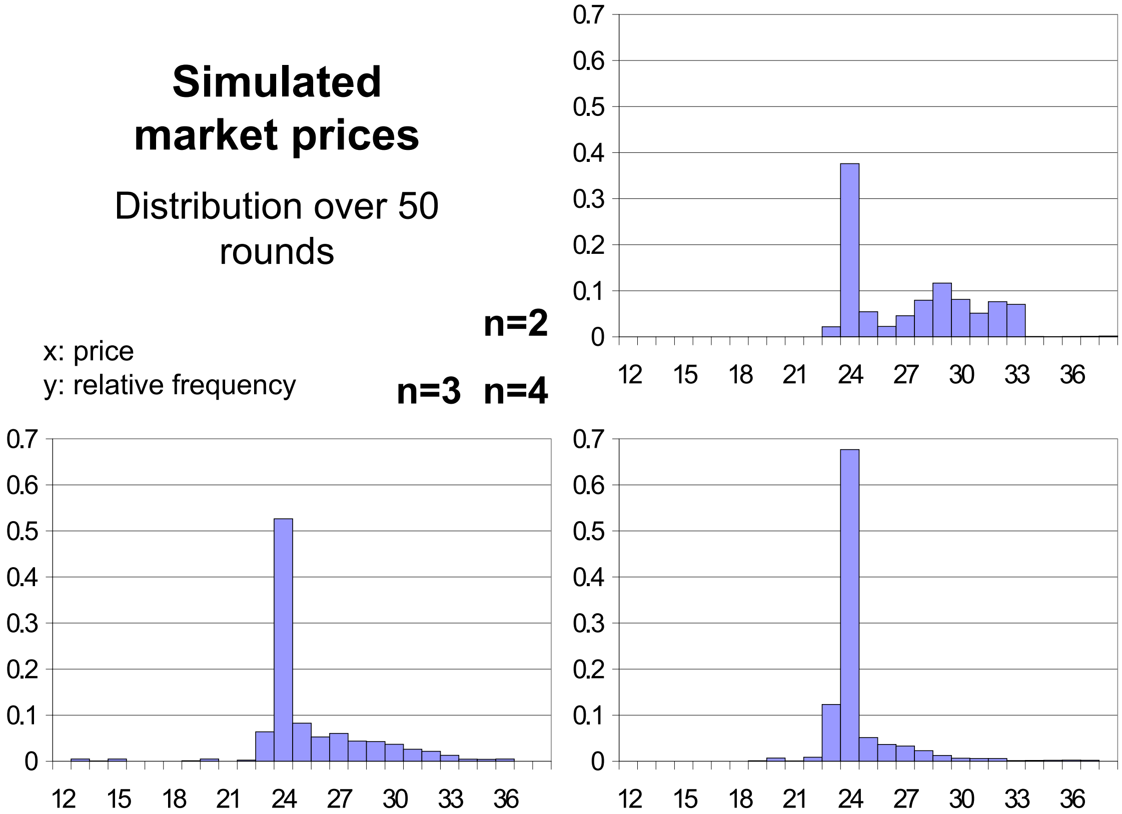

Figure 7

The model results confirm the intuition mentioned earlier. As long as the market price is 24 or higher, the most successful choice is the lowest price. This leads to a downward trend of the market price. If, however, a single firm has set the market price of 23 or lower, it has made a loss and therefore it will not be imitated. Thus, prices will fall below 24 only if – by experimentation – more than one firm has lowered the price in the same round. With a sufficiently high probability of imitation, however, this happens only occasionally, such that the price of 24 is very stable in the intermediate term.

3.3.3. Individual behaviour

The previous analysis suggests that a simple imitation model captures several of the regularities of our data quite well. We now return to our experimental data to examine whether we can directly identify patterns of imitative behaviour in the individual decisions. Table 3 shows how often subjects chose the most successful price of the previous period, and how often they deviated from this price how far. If more than one choice was most successful in the previous round, we applied the tie-breaking rule we used in our model.

Table 3. Frequency of imitations of the default price of the previous round

	n=2		n=3		n=4	
Deviation from default	frequency	per cent	frequency	per cent	frequency	per cent
≥+10	4	0.5	2	0.1	3	0.2
+9	21	2.7	1	0.1	1	0.1
+8	14	1.8	1	0.1	4	0.3
+7	1	0.1	2	0.1	35	2.2
+6	5	0.6	47	3.2	3	0.2
+5	5	0.6	13	0.9	10	0.6
+4	11	1.4	11	0.8	52	3.3
+3	11	1.4	7	0.5	4	0.3
+2	10	1.3	8	0.5	3	0.2
+1	42	5.4	42	2.9	23	1.5
±0	596	76.0	1256	86.0	1354	86.8
–1	40	5.1	50	3.4	47	3.0
–2	9	1.1	5	0.3	1	0.1
–3	6	0.8	4	0.3	0	0.0
–4	0	0.0	7	0.5	4	0.3
–5	3	0.4	4	0.3	5	0.3
–6	0	0.0	0	0.0	0	0.0
–7	2	0.3	0	0.0	10	0.6
–8	4	0.5	0	0.0	1	0.1
–9	0	0.0	0	0.0	0	0.0
≤–10	0	0.0	1	0.1	0	0.0
no default	0		9		8	

The relative frequencies refer to all choices where the default price is well-defined. “No default” means that in 0, 9, and 8 cases a default value was undefined in rounds 2-50.

The table shows that indeed the choice of the most successful price of the previous round is predominant. In all treatments, more than three-quarters of choices are consistent with this criterion. If subjects deviate, two patterns can be identified. Often they stay in the neighbourhood of the default price, as our model prescribes. However, we also observe considerably many upward deviations to much higher prices. These are mainly jumps to the collusive price.

3.3.4. Sensitivity analysis

Table 4 shows the modal price obtained in simulations with different parameterisations of the model. We varied the number of firms ($n \in \{2, 3, 4, 8, 16, 32\}$), the time horizon ($T \in \{50, 100, 200, 500, 1000\}$), and the value of the β parameter ($\beta \in \{0.99, 0.95, 0.90\}$). The simulations for n=2, n=3, and n=4 were initialised with the distributions of choices observed in the respective treatment. For the markets with n=8, n=16, and n=32, for which we have no experimental data, we used the distribution of first round choices in the n=4 treatment. Unlike figure 7, which depicts the distribution over all 50 rounds of the simulation to ensure comparability with the experimental results, the table shows the modal price in the specified round to make the long-term dynamics more visible.

For the simulations, we have extended the grid of the price scale beneath 1, the lowest feasible choice in the experimental sessions. Recall that the choices from 1 to 40 are transformed from the prices in the underlying oligopoly model. In the simulations with many rounds and players, some choices are outside the range feasible in our experiment. These negative choices correspond to positive prices in the original market model.

Table 4. Modal prices in the model simulations

	β = 0.99					β = 0.95					β = 0.90				
T=	50	100	200	500	1000	50	100	200	500	1000	50	100	200	500	1000
n=2	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
n=3	24	24	24	24	24	24	24	24	24	24	24	24	24	23	23
n=4	24	24	24	24	24	24	24	24	23	20	24	23	22	18	13
n=8	24	24	24	24	24	24	23	21	16	13	22	19	13	11	7
n=16	24	24	24	23	22	22	19	13	11	7	13	12	6	5	2
n=32	24	24	23	18	13	12	12	6	4	2	6	2	0	–1	–2

3.3.5. Long-term convergence

To give a more complete picture of the model dynamics, we now examine the evolution of prices in the long run. We have seen in table 4 that price predictions tend to be lower when we repeat the game for more periods. The question arises whether the trend towards lower prices will find a lower rest point and if so, what its properties will be.

Previous studies (Vega-Redondo (1997), Offerman, Potters, and Sonnemans (2002)) have shown that imitative behaviour in quantity competition will lead to Walrasian outcomes. With our model we can find a related result also for price competition. Denote by w^n the Walrasian price of a market in which n firms are active. Further, ρ^1 denotes the first defined default price in the process.

Proposition. *For linear demand functions of the type $p = a - bQ$ and quadratic cost functions $C(q_i) = cq_i^2$ and $\beta > ½$ the following holds. If the number of firms is even, then the default price ρ converges to a price arbitrarily close to w^n. If the number of firms is odd, then ρ converges to a price arbitrarily close to w^{n-1} if $\rho^1 > w^{n-1}$, and to w^{n+1} if $\rho^1 > w^{n+1}$. If $w^{n-1} > \rho^1 > w^{n+1}$, then ρ^1 is stable.*

Proof. See appendix A.

Simply speaking, the result exploits the property of the Walrasian price that it allows $n/2$ firms to produce the market demand (just) profitably, while $n/2–1$ firms would incur a loss. As mentioned earlier, the price is moved upwards or downwards by joint experimentation of a "coalition" of firms. The number of firms needed to move the price downwards increases and the number of firms needed to move the price upwards decreases as the price level moves down. At the Walrasian price of a market with even n, exactly half of the firms are needed to push the price up, as well as half of the firms would sufficient to move the price down. Since upward and downward deviations are equally likely, this is the price at that upward and downward pressure on the price is balanced.[13] For odd numbers, this result changes only in that $n/2$ is not an integer.

As there is downward pressure on prices as long as the price is above the Walrasian level, they will tend to decrease. In addition, the speed at which prices decrease will be higher the more firms there are in the market. However, the process takes very long. Contrary to the quantity competition experiment of Offerman, Potters, and Sonnemans (2002), who obtain convergence within the time span of an experimental session, our model predicts substantial divergence from the convergence point even after 1000 rounds.[14] The pace at which the price decreases further gets slower and slower, as the number of firms jointly deviating downwards to make positive profits increases, making such a joint deviation less and less likely to occur.[15]

3.4. Market efficiency

To conclude the presentation of our experimental data, we shall now look at the implications of our findings for market efficiency. In particular, we are interested in the surplus that would be generated in a market as modelled in our experiment. Total surplus, conventionally defined as the sum of consumer and producer surplus, would be maximised if all firms set the Walrasian price. We derive consumer surplus from the market model underlying our payoff table.[16] Table 5 shows average total surplus for the different groups over the 50 rounds of the experiment, ordered from the lowest to the highest for each value of n. Table 5 shows that efficiency increases substantially with the num

[13] This logic has some flavour of the *impulse balance point* proposed in Selten, Abbink, and Cox (2001). The adaptation process analysed by these authors, however, is different. The impulse balance point is based on adjustments in the direction of what would have been the best choice in the last round.

[14] For n=8 the Walrasian price is between 2 and 3, for n=16 between –5 and –4, and for n=32 between –9 and –8.

[15] The exact duration depends on the parameterisation, i.e. the slope of demand and cost functions and the grid of the price range. Notice that, contrary to the case of quantity competition, the Walrasian price does not automatically maximise efficiency. Experimentation leads to individual deviations from a common price, inducing losses in productive efficiency. These losses are the more severe the lower the price.

[16] In some sense, these calculations are hypothetical, for consumers were not represented by real subjects. If only the payoffs of real subjects are considered, the efficiency maximising outcome is the collusive one.

ber of firms in the market. Fisher's two-sample randomisation test rejects the null hypothesis of equal average total surplus at a significance level of at least α=0.02 (one-sided) for all pairwise comparisons of treatments.

Table 5. Average total surplus in the individual markets

Group No.	n=2	n=3	n=4
1	1123.4	1463.2	1349.1
2	1227.5	1497.2	1663.1
3	1254.9	1502.3	1814.3
4	1314.8	1533.2	1829.6
5	1350.4	1584.4	1871.0
6	1355.6	1613.6	1875.6
7	1399.0	1631.5	1883.5
8	1399.2	1637.8	1947.0
9		1768.0	
10		1803.7	
Average	1303.1	1603.5	1779.2
Maximum relative efficiency	1569.2 83.0%	1904.8 84.2%	2134.2 83.4%

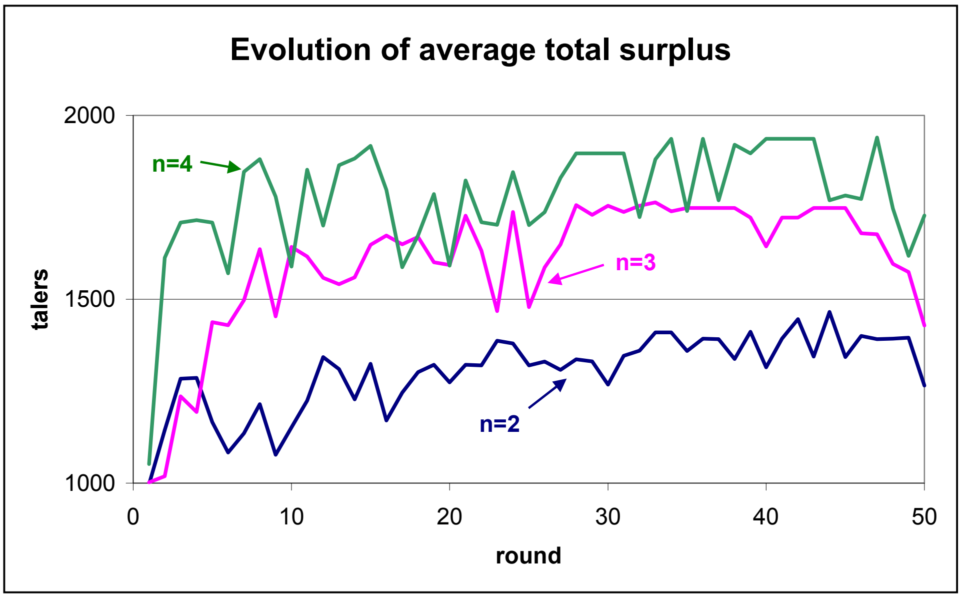

Figure 8

Relative efficiency, i.e. the percentage of actual surplus of the maximal possible surplus, is almost identical over our three treatments. This indicates that the absolute efficiency advantage of more firms is mainly induced by the greater potential of generating surplus. Figure 8 shows the evolution of average total surplus over the fifty rounds of the experiments. We can see that efficiency sharply increases in the early rounds of the experiment, mainly reflecting firms' increasing ability to co-ordinate on a common price.

4. Conclusions

We report an experiment examining price levels and the relation between these levels and of the number of firms in a price competition environment with a strong multiplicity of equilibria. We study the market regularities and give a theoretical explanation for them. Our results show that average market prices are decreasing and that total surplus is increasing in the number of firms. In duopolies, the cartel price is the most frequently observed outcome. With three and four firms, the frequency of collusive outcomes decreases. All this is in line with the results from other oligopoly experiments, as those with quantity competition.

Otherwise, the data left us with a behavioural puzzle. A particular price level equal to 24 is predominant, chosen in an absolute majority of rounds in tripolies and tetrapolies. We propose a dynamic explanation for this phenomenon. What looks like a formidable task – co-ordinating on one equilibrium from a choice of more than 20 – can be remarkably well resolved by a simple heuristic of imitating the most successful player. A straightforward simulation model of such behaviour indeed yields fairly robust modal outcomes of 24 in the time horizon of the experiment and also reproduces some of the other regularities of our experimental data. The heuristic is extremely simple and yet – in the present environment – makes a lot of sense.

Of course, our experiment cannot be more than a starting point for a deeper exploration of price competition under increasing marginal costs. To keep things simple, we started with a very basic model that naturally lacks many of the complexities of real life oligopolies. The assumption that firms have to serve the entire demand is standard in the analysis of such models and justifiable if the costs of declining customers are high, but in many cases we may expect that firms have some choice to decline consumers' demand if serving it is too costly. Thus, future research should also examine markets in which this requirement is relaxed.

Further, the dynamic model we present in this study gives a consistent and, as we believe, plausible explanation of the observed phenomenon. We propose that our environment is particularly conducive to imitative behaviour, because the basic concept of imitation – doing what others do – meets the essential goal of this market model to coordinate on a common price. The dynamic model we propose captures the 24-phenomenon as well as the other regularities quite well, but this is of course subject to the caveat that the model has been formulated ex-post. Thus, we do not claim predictive power for it at this stage. We do believe, however, that the analysis of our model can provide a promising testable prediction for future experiments on oligopolistic competition of many kinds.

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Appendix A. Proof of the proposition

First, it is easy to see that given the linear demand and quadratic cost functions, the minimum number of firms that can profitably serve the entire market demand increases as the price decreases. A single firm's profit as a function of the price p and the number of firms m setting this price as the market price is

$$\pi_i(p,m) = \frac{a-p}{bm}p - c\left(\frac{a-p}{bm}\right)^2 > 0 \quad \Rightarrow \quad m > \frac{c(a-p)}{bp}$$

Thus, as p decreases, the number of firms needed to serve the market profitably increases.

Define a *threshold price* p^m as the lowest price at which

$$m > \frac{c(a-p^m)}{bp^m} \quad \text{and} \quad m < \frac{c(a-(p^m-\Delta))}{b(p^m-\Delta)} \tag{1}$$

where Δ is the distance between p^m and the next lower price on the price grid. We assume that the grid is sufficiently fine such that there is at least one feasible price between any two threshold prices. Therefore,

$$m+1 > \frac{c(a-(p^m-\Delta))}{b(p^m-\Delta)} \tag{2}$$

For convenience, we further assume that no threshold price exactly hits a feasible price on the grid, to avoid cases of indifference. We say the price is at a threshold price (in period t) if $\exists\ m$ such that $\rho_t = p^m$.

We now analyse the properties of the default price set ρ.

Lemma 1. *If* $\rho_t \neq \varnothing$, *then* $\rho_\tau \neq \varnothing\ \ \forall\ \tau > t$.

Proof. If $\rho_t \neq \varnothing$, then only three different prices can be chosen in round t+1, namely $p_{t+1,i} \in \{\rho_t - \Delta, \rho_t, \rho_t + \Delta\}$ for all firms i. Since no threshold price lies exactly on the grid, the firms setting the lowest price will make either positive or negative profit, but not zero profit. Thus, the set of prices having yielded the highest payoff, Ω, is a singleton. If all three price levels have been chosen by at least one firm, then the firms setting the lowest price $\rho_t - \Delta$ make either positive or negative profits. If they make positive profits, then $\rho_{t+1} = \rho_t - \Delta$, because the firms setting the higher price make zero profits. If they make negative profits, then $p_{t,i} = \rho_t - \Delta$ implies $p'_{t,i} = \rho_t$. Thus, $p'_{t,i} \in \{\rho_t, \rho_t + \Delta\}$. $\rho_t + \Delta$ leads to zero profits, ρ_t to either positive or negative profits. Thus, one of them must become the unique default price. Hence, $\rho_{t+1} \neq \varnothing$. By induction, it follows that $\rho_\tau \neq \varnothing\ \ \forall\ \tau > t$. ∎

We now need to show that the probability of never having a non-empty default price (set) is zero.

Lemma 2. prob($\rho_\infty = \varnothing$) = 0.

Proof. Sufficient conditions for $\rho_t \neq \varnothing$ in some round t are that either (1) a positive profit has been made (which implies that Ω is a singleton), or (2) not more than two different prices have been set. The latter is fulfilled e.g. if $p_{t,i} = p_{t,j}\ \forall\ i, j$. The probability for that is $|\Omega|(\beta/|\Omega|)^n$, where β is exogeneous and constant, and $|\Omega|$ is positive and finite because naturally $1 \leq |\Omega| \leq n$. Thus, the probability for that event is strictly positive. ∎

Knowing that a non-empty default price will materialise at some time, and once it is non-empty it will be non-empty forever, we can now restrict the analysis to rounds with a non-empty default price.

Whether or not the firms setting the lowest price make a positive profit depends on the number of firms setting this price. Through experimentation, it may or may not happen that sufficiently many firms set the lowest price to ensure profits. For the further analysis we define the following terms.

Definition. *A* profitable downward deviation *is a deviation of a set M of m firms to* $\rho_t - \Delta$ *such that* $\pi_i > 0$ *for the m downward deviating firms. A* profitable upward deviation *is a deviation of m firms to* $\rho_t + \Delta$ *such that* $\pi_i < 0$ *for all firms* $i \notin M$ and $\forall\ p_{t,i} < \rho_t + \Delta$.

Notice that the definition of profitablity is relative to other firms' ex-post payoffs. Upward deviators typically make zero profits, but this may still be "profitable" compared to the other firms making losses.

Notice, further, that for a profitable upward deviation it is not enough that the actual firm setting the market price has made negative profits, but rather that all firms would have made non-positive profits even if they all had co-ordinated on the best-possible choice.

We can immediately conclude from inequalities (1) and (2) that in between threshold prices p^m and p^{m-1} a downward deviation is profitable if by experimentation at least m firms deviate downwards. An upward deviation is profitable if at least $n-m+1$ firms deviate upwards. Then, $m-1$ firms are left with a lower price, and the best they can do is all setting ρ_t, which still results in negative payoffs. At a threshold price p^m, a downward deviation is profitable if at least $m+1$ firms deviate downwards, while an upward deviation still requires at least $n-m+1$ to be profitable in the sense of the definition.

We now examine how profitability of deviations may influence changes of the price.

Lemma 3. *If $\rho_t \neq \varnothing$, then the following holds. In between price thresholds, $\rho_{t+1} = \rho_t + \Delta$ if and only if there is a profitable upward deviation. $\rho_{t+1} = \rho_t - \Delta$ if and only if there is a profitable downward deviation. Otherwise $\rho_{t+1} = \rho_t$.*

Proof. We need to consider the cases that either (1) all firms experiment, or (2) not all firms experiment.

(1) k firms move down and $n-k$ move up. In between thresholds, either a downward deviation of m firms is needed for $\rho_{t+1} = \rho_t - \Delta$, or an upward devation of $n-m+1$ firms is needed for $\rho_{t+1} = \rho_t + \Delta$. If $k<m$ obviously $n-k \geq n-m+1$, and $k \geq m$ already induces a profitable downward deviation. Since in case of only experimenting firms, only two prices can be chosen, therefore ρ_{t+1} must be one of them.

(2) Not all firms experiment (at least one imitates last round's most successful price). If sufficiently many firms deviate downwards to a price that yields positive profits, then it is immediately clear that their price will be imitated. This is the case of a profitable downward deviation. If no firms deviate downwards, then it is immediately clear that the price moves upwards if and only if there has been a profitable upward deviation. Now consider the case that there have been both upward and downward deviations, but no profitable downward deviation.

If too few firms jointly deviate downwards, they make losses, and $\rho_{t+1} \in \{\rho_t, \rho_t + \Delta\}$. Suppose sufficiently many firms deviate upwards to establish a profitable upward deviation. The tie-breaking rule implies that the firms will regard the downward deviations as if those firms had chosen ρ_t. Because the upward deviation was profitable according to the definition, the firms all setting ρ_t would still have made a loss. Thus, the upward deviators will be imitated. Now suppose too few firms have deviated upwards to establish a profitable upward deviation. Again, the firms will regard the downward deviations as if those firms had chosen ρ_t. But since the upward deviation was not profitable, this means that all other firms setting ρ_t together would have made a positive profit, and thus $\rho_{t+1} = \rho_t$. ■

At the price thresholds, an additional case must be considered.

Lemma 4. *If $\rho_t \neq \varnothing$ and $\rho_t = p^m$, then the following holds. If there is a profitable downward deviation, then $\rho_{t+1} = \rho_t - \Delta$. If there is a profitable upward deviation, then $\rho_{t+1} = \rho_t + \Delta$. If there is no profitable deviation, then $\rho_{t+1} = \rho_t + \Delta$ if exactly m firms deviate downwards and n–m firms deviate upwards. Otherwise $\rho_{t+1} = \rho_t$.*

Proof. Again, consider the cases of all firms experimenting and not all firms experimenting separately. If not all firms experiment, the argumentation is exactly as in the case of lemma 1. Now suppose all firms experiment, where k deviate downwards and $n-k$ deviate upwards. If $k>m$, this is a profitable downward deviation, and the deviators' price will be imitated. If $k<m$, then $k \geq n-m+1$, and therefore we have a profitable upward deviation. The only remaining case possible is $k=m$, in which case the k downward deviators make negative profits, and the $n-k$ upward deviators make zero profits. Thus, $\rho_{t+1} = \rho_t + \Delta$. ■

Lemma 3 states that in between price thresholds, the question whether there is upward or downward pressure on the price is determined by the probability of a profitable upward and downward deviations. If the latter is greater than the former, there will be downward pressure on the prices. Further, a profitable downward deviation is given if at least m firms deviate downwards, the probability for that being $F_B(n, m, (1 - \beta/2))$, where F_B is the upper tail cumulative binomial distribution. Analogously, a profitable upward deviation is given if at least $n-m$-1 firms deviate upwards (leaving $m-1$ firms with a lower price), the probability for that being $F_B(n, n-m-1, (1 - \beta/2))$. Thus, there will be downward pressure if

$$F_B(n, m, (1 - \beta/2)) > F_B(n, n-m-1, (1 - \beta/2))$$

thus $m > n–m–1$. Analogously, there is downward pressure if $m < n–m–1$. For even n, it follows that there is downward pressure if $p > p^{n/2}$ and upward pressure if $p < p^{n/2}$. For odd n, it follows that there is downward pressure if $p < p^{(n-1)/2}$ and upward pressure if $p < p^{(n-1)/2}$.

At the price thresholds, pressure on prices is exerted not only through profitable deviations. Rather, we have to consider four cases. Notice that the number of firms needed to for a profitable downward deviation is now $m+1$, while the number of firms needed for a profitable upward deviation is $n–m–1$ as before.

The following table A.1 describes the four cases and their probabilities. For convenience, we define the variables u, v, and d as listed in the table.

Table A.1. Deviations at the price thresholds and their probabilities

Case	Probability
Profitable upward deviation ⇒ default price moves up	$F_B(n, n–m+1, (1 – \beta/2)) =: u$
m firms move down, $n–m$ firms move up ⇒ default price moves up	$f_T(m, 0, n–m, (1 – \beta/2), \beta, (1 – \beta/2)) =: v$
Other unprofitable deviation ⇒ default price does not change	$1–u–v–d$
Profitable downward deviation ⇒ default price moves down	$F_B(n, m+1, (1 – \beta/2)) =: d$

$f_T(\cdot)$ is the uncumulated trinomial distribution

It can be seen immediately that $u > d \Leftrightarrow m < n/2$, and $u < d \Leftrightarrow m > n/2$. Further, the pressure on prices from profitable deviations is balanced ($u = d$) if $m = n/2$, thus $p = p^{n/2}$. Additionally, there is some upward pressure from the second case at $p = p^{n/2}$.

At this point, we can already conclude that if the price reaches $p = p^{n/2}$, it will rest between that price and the next higher one, as there is upward pressure at prices below, downward pressure at the prices above, and some upward pressure at $p = p^{n/2}$. We have also seen that at all prices above that are not threshold prices there is downward pressure. At all prices below, there is upward pressure, because $u > d \Rightarrow u + v > d$. To show that for even n, the price indeed converges to $p = p^{n/2}$, we have to show that the downward trend for prices approaching from above is not stopped at threshold prices above $p^{n/2}$. Thus, we need to show that $v < d – u$.

Writing out the formulae for the cumulative binomial distribution, we have

$$d = \sum_{i=m+1}^{n}\left(\frac{n!}{i!(n-i)!}\right)\left(\frac{1-\beta}{2}\right)^{i}\left(\beta+\frac{1-\beta}{2}\right)^{n-i} \text{ and } u = \sum_{i=n-m+1}^{n}\left(\frac{n!}{i!(n-i)!}\right)\left(\frac{1-\beta}{2}\right)^{i}\left(\beta+\frac{1-\beta}{2}\right)^{n-i}$$

Thus $d – u$ can be written as

$$d-u = \sum_{i=m+1}^{n-m}\left(\frac{n!}{i!(n-i)!}\right)\left(\frac{1-\beta}{2}\right)^{i}\left(\beta+\frac{1-\beta}{2}\right)^{n-i}$$

which must be greater than v, which is

$$v = \left(\frac{n!}{m!(n-m)!}\right)\left(\frac{1-\beta}{2}\right)^{m}\left(\frac{1-\beta}{2}\right)^{n-m}$$

It is sufficient to show that the first addend of the sum for $d–u$ is greater than v, thus

$$\left(\frac{n!}{m+1!(n-m-1)!}\right)\left(\frac{1-\beta}{2}\right)^{m+1}\left(\beta+\frac{1-\beta}{2}\right)^{n-m-1} > \left(\frac{n!}{m!(n-m)!}\right)\left(\frac{1-\beta}{2}\right)^{m}\left(\frac{1-\beta}{2}\right)^{n-m}$$

After some calculations, one obtains that this is true if

$$\frac{(n-m)\left(\beta+\frac{1-\beta}{2}\right)^{n-m-1}}{(m+1)\left(\frac{1-\beta}{2}\right)^{n-m-1}} > 1$$

Since $m < n/2$, we have $n–m \geq m+1$, and $\beta > ½$ implies $\beta+(1–\beta)/2 > (1–\beta)/2$. Thus, there will be downward pressure at all threshold prices above $p^{n/2}$.

For even n, it remains to show that $p^{n/2}$ is arbitrarily close to the Walrasian price of the market. The Walrasian price, if n firms are on the market, is $a – nab / (2c + nb)$. If $n/2$ firms set this price, each serving an equal share of the demand, each firm will make a profit of

$$\pi_i = pq_i - cq_1^2 = (a - \frac{nab}{2c+nb})q_i - cq_1^2$$

In the Walrasian equilibrium, each of the n firms produces $q_i^W = a / (2c + bn)$. If only n/2 firms produce that quantity, each produces $2q_i^W$, and thus makes a profit of

$$\pi_i = \left(a - \frac{nab}{2c+nb}\right)\left(\frac{2a}{2c+bn}\right) - c\left(\frac{2a}{2c+bn}\right)^2 = \frac{4a^2c}{(2c+bn)^2} - \frac{4a^2c}{(2c+bn)^2} = 0$$

This implies that if the price is at the next feasible price above the Walrasian price, $n/2$ firms will not be sufficient to profitably deviate downwards. At the next higher price level, however, $n/2$ firms make a profit. At the next price below the Walrasian price, an upward deviation of $n/2$ firms is profitable, while a downward deviation of $n/2$ firms is not. Therefore, the price will converge to a price close to the Walrasian equilibrium price if n is even. This price can be arbitrarily close to the Walrasian price, depending on the grid level Δ chosen. ■

The following table summarises the impulses at prices near the Walrasian price. “Slightly up” means that the impulses through profitable deviations are balanced, but a small upward pressure is given through v.

Table A.2. Price impulses for an even number of firms

price level	number of firms needed for profitable downward devation	number of firms needed for profitable upward devation	pressure on price level
$p^{n/2} + \Delta$	$n/2$	$n/2 + 1$	down
$p^{n/2}$	$n/2 + 1$	$n/2 + 1$	slightly up
$p^{n/2} – \Delta$	$n/2 + 1$	$n/2$	up

If n is odd, $n/2$ is not an integer, therefore there is no threshold price of $p^{n/2}$. From the calculations made above we can immediately conclude that $p^{(n–1)/2}$ is the next feasible price above the Walrasian price of the same market in which only $n–1$ firms operate, while $p^{(n+1)/2}$ is the next feasible price above the Walrasian price of the market with $n+1$ firms. Exactly the same argumentation as for even n leads to the result that there is downward pressure whenever $p \geq p^{(n–1)/2}$, and upward pressure whenever $p \leq p^{(n+1)/2}$. Notice that contrary to the case for even n, impulses from profitable deviations are not balanced. Now consider all prices in between $p^{(n–1)/2}$ and $p^{(n+1)/2}$. Lemma 1 implies that pressure on the price is only exerted from profitable deviations. In between $p^{(n–1)/2}$ and $p^{(n+1)/2}$ $(n–1)/2+1$ firms are needed for a profitable downward deviation, while the number of firms needed for a profitable upward deviation is $n–(n–1)/2+1 = (n–1)/2+1$. Thus, for all prices in between $p^{(n–1)/2}$ and $p^{(n+1)/2}$ upward and downward pressure on prices are balanced. The following table summarises the impulses.

Table A.3. Price impulses for an odd number of firms

price level	number of firms needed for profitable downward devation	number of firms needed for profitable upward devation	pressure on price level
$p^{(n-1)/2}$	$(n–1)/2 + 1$	$(n–1)/2 + 2$	down
$p^{(n-1)/2} – \Delta$	$(n–1)/2 + 1$	$(n–1)/2 + 1$	balanced
…	…	…	…
$p^{(n+1)/2} + \Delta$	$(n–1)/2 + 1$	$(n–1)/2 + 1$	balanced
$p^{(n+1)/2}$	$(n–1)/2 + 2$	$(n–1)/2 + 1$	up

Thus, if the default price approaches $p^{(n-1)/2}$ from above, it will converge to $p^{(n-1)/2}$; if it approaches $p^{(n+1)/2}$ from below, it will converge to $p^{(n+1)/2}$ Once this price is reached, there is no systematic pressure on the default price anymore. ■

Notice that although the Walrasian price of the *n* firm market will not be reached if *n* is odd (since it does not constitute a threshold price, it is nevertheless included in the interval in which price pressure is balanced.

Appendix B. Instructions (n=4)

General information: We thank you for coming to the experiment. The purpose of this session is to study how people make decisions in a particular situation. During the session it is not pemitted to talk or communicate with the other participants. If you have a question, please, raise your hand and one of us will come to your table to answer it. During the session you will earn money. At the end of the session the amount you have eraned will be paid to you in cash. Payments are confidential, we will not inform any of the other participants of the amount you have earned. The experiment consists of 50 rounds. In each round you will be paired with three other participants who will be the same during the 50 rounds.

Decisions: In each round you and the three participants that you are paired with will each separately make a decision. This decision will consist in choosing a number between 1 and 40. When you have decided on a number please enter it into the computer.

Earnings: The earnings of each round depend on the four number that you and the participants you are paired with have chosen in that round. Observe now the payoff table that we have given to you (see table 1 in the main text). In it you can see the earnings for each number that you can choose.

- Column 1 shows all possible numbers.
- Column 2 shows your payoff for the case in which the number is the *lowest* of the four chosen numbers.
- Column 3 shows your payoff for the case in which the number has been chosen by you and by one of the three participants with which you are paired and is lower than the other two numbers.
- Column 4 shows your payoff for the case in which the number has been chosen by you and by two of the three participants with which you are paired and is lower than the fourth number.
- Column 5 shows your payoff for the case in which the number has been chosen by you and by the three participants with the three participants you are paired with.
- Column 6 shows your payoff if the number is not the *lowest* of the four chosen numbers.

Information you will receive: At the end of each round you will be able to see the result of that round. In addition, at any moment you will be able to “click” on **History** and see the results of previous rounds.

Payment: The currency used in this experiment is the taler.

- The payoffs shown in the payoff table are in talers.
- At the beginning of the experiment each of you will receive a capital balance of 5000 talers.
- Your total payoff from your participation in the experiment is equal to the sum of all your payoffs and of your capital balance minus your losses.
- At any moment during the experiment you will be able to check on the screen your total payoff in talers.
- At the end of the experiment your total payoff will be converted into pesetas at the exchange rate of 1 peseta for each 6 talers.

If you have a question, please raise your hand and one of us will come to your table to answer it.