

Inequality and a Repeated Joint Project*

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Abstract

Agents voluntarily contribute to an infinitely repeated joint project. We investigate the conditions for cooperation to be a renegotiation-proof and coalition-proof equilibrium before examining the influence of output share inequality on the sustainability of cooperation. When shares are not equally distributed, cooperation requires agents to be more patient than under perfect equality. Beyond a certain degree of share inequality, full efficiency cannot be reached without redistribution. This model also explains the coexistence of one cooperating and one free-riding coalition. In this case, increasing inequality can have a positive or negative impact on the aggregate level of effort.

1 Introduction

Agents take part in a joint project to which they voluntarily contribute efforts. The output is distributed among the players according to their share, which can be for instance their relative wealth. In this kind of setting, we know that, if the game is played only once, the first-best optimum will be impossible to sustain¹, as deviation is a dominant strategy and the aggregate level of Nash equilibrium contributions is suboptimal. In contrast, the Folk theorem teaches us that, provided people are not too impatient, new

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¹It can if all the shares are concentrated into the hands of a single individual and efforts are perfectly substitutable.

and more efficient equilibria can be reached thanks to the repetition of the game. In this paper, we want to investigate to what extent inequality of share influences first-best sustainability in an infinitely repeated game. *Does introducing share inequality render cooperation more difficult to support?*

There exist numerous examples of voluntarily provided joint projects in the real world. For instance, one can think of an irrigation scheme where individuals have to put in efforts for building and/or maintaining the infrastructure. Then, the level of their contributions to this collective action depends on their benefit which is directly related to the amount of land they cultivate. This same setup can also apply to many other cases: voluntary provision to local public goods, collective action problems in management of environmental resources (forests, fisheries, pest and weed control), co-operatives, financial lobbying, defence alliances, etc. Industrial Organization also encompasses related issues, i.e. tacit collusion and to a lesser extent moral hazard in teams. Bardhan and Singh (2005) also evoke Velasco and the theoretical literature on the common pool problem in fiscal stabilization policy in Latin America.

Despite most of the literature on these issues uses a static game formalization, these situations are often better described as repeated games. One can indeed easily imagine that people interact repeatedly² and have to find ways to sustain cooperation between them, not knowing when this game is going to end. Our infinitely repeated game framework seems therefore quite realistic.

In one-shot games, Olson (1965), followed by several authors, argued that the effect of inequality was positive on collective action. He claimed that if one single (or a few) agent has a great interest in the collective action, the good is more likely to be provided even if this user is the only one to bear its cost. Olson brought up the two following insights: that contributions are positively related to wealth, which seems plausible, and that great inequality implies a great likelihood of success of collective action. Several papers show that the conditions for the latter result to remain valid are quite demanding. This is put forward, among others, in empirical studies by Bardhan (2000) and Dayton-Johnson (2000) but also put in perspective in more theoretical papers such as Bardhan, Ghatak, Karaivanov (2007) and Ray, Baland, Dagnelie (2007).

Nowadays, it seems that the effect of inequality - not necessarily of wealth - on efficiency or the aggregate level of effort is ambiguous, depending on the cost function (Banerjee et al., 2006). In an empirical paper on collective action in Pakistanese communities, Khwaja (2006) finds a U-shaped relationship between inequality in the distribution of project returns or land ownership and maintenance, the aggregate level of efforts. It also appears that, in many cases, departing from an equalitarian distribution will hurt collective action or efficiency, by increasing the poor's incentives to free-ride or by allowing the rich and powerful to take over rents - as in the case of the Maharashtra

²It fits particularly well the case where people live in a small rural community and share resources.

sugar cooperatives (Banerjee et al., 2001). Then, deepening inequality can increase or decrease further the incentives of the beneficiaries from the redistribution to cooperate³.

With the prospect of future interactions with the same people⁴ arises the possibility of punishing undesired actions, which is conducive to cooperative outcomes. A few papers address the question of dynamic or repeated games. The former refers to a paper by Tarui (2007) investigating the influence of inequality in productivity, access to markets and credit into a dynamic intergenerational game of common property resource use⁵. It takes into account how the resources of the commons endogenously evolve given the harvests by users in previous generations. According to the punishment used and the harvest sharing rule, Tarui shows that first-best sustainability may or not be affected by an increase of inequality.

As to Bardhan and Singh (2005), they explore the influence of wealth inequality on cooperation, sustained by trigger strategies, i.e. Nash reversion. In their model, agents are endowed with private capital which enters, with a complementary input, a constant return to scale Cobb-Douglas production function. To produce this complementary input, agents have to choose between a status quo technology which guarantees some level of output and a cooperative technology the fruits of which can be captured by one or more deviating players. They establish that, in this setting, inequality can affect cooperation and that redistribution can improve the welfare of the rich thanks to the greater possibility of cooperation.

This paper relates to Itaya and Yamada (2003) who investigate the impact of income inequality on a repeated game of private provision of public goods with two players and renegotiation-proof equilibria. They also point out the negative effect of inequality on first-best sustainability.

This research is also very close in spirit to Vasconcelos (2005), a paper on tacit collusion in quantity setting supergames with asymmetric costs. It is quite interesting to transpose Vasconcelos' setting to our model. In both papers, inequality decreases the scope of cooperation sustainability, by increasing the discount factor of interest. Consistently with a large number of historical and empirical evidence, Vasconcelos' market shares are allocated according to the firms' production capacity, which affects marginal costs. In his repeated Cournot model, he shows that the smallest firms are more prone to deviate from the collusive agreement, which fits our framework. As he uses optimal penal codes à la Abreu (the stick and the carrot, 1986, 1988) to sustain cooperation, while

³A comprehensive discussion on this issue can be found in Baland and Platteau (2003) from p. 161.

⁴This renders our framework different from reputation matching games à la Kandori (1992) where relationships are infrequent and "agents change their partners over time and dishonest behaviour against one partner causes sanctions by other members in the society".

⁵Rendering our model dynamic would only make, at each period, the rich, richer and the poor, poorer. It would then lead to a situation where only one player would be rich enough for cooperation to be more desirable than deviation. No one would therefore produce the efficient level of effort.

punishing, the largest firms have the greatest incentives to deviate from the punishment path⁶. Here lies the main difference with our research⁷. As we resort to renegotiation-proof punishments, the cooperative agents do not suffer while punishing. They even profit by carrying out the punishment and are therefore not tempted by deviations.

A common and easy solution considered in the literature to sustain cooperation is Nash reversion (Friedman, 1971) consisting in a permanent return to the Nash equilibrium after a single deviation. We will use it as a benchmark for our analysis. However, one of the main drawbacks of Nash reversion is that, without being the harshest punishment, the punisher suffers from giving a punishment. As stated by Bernheim and Ray (1989) - collective dynamic inconsistency - and Farrell and Maskin (1989) - renegotiation-proofness - who introduced the concept of renegotiation-proofness in the literature⁸, this renders the threat not credible. Everybody indeed anticipates that, ex post, the punishers and the punished will be tempted to renegotiate. Furthermore, after a single deviation, all the agents are stuck for ever in a pareto dominated equilibrium. It would be hard to believe that, in a repeated setting, agents fail to exploit the existing opportunities to reach the pareto frontier. Actually, this is rarely observed on the field as stated in Tarui (2007). The latter quotes Ostrom (1990) who argues that many commons overcame occasional deviations. This indirectly supports the evidence that the punishments are only temporary, allowing a return to cooperation.

This confirms the need to elaborate punishments allowing a return to cooperation after the punishment phase. In this research, we also extend the concept of renegotiation-proofness from 2 to n players, taking into account credible deviations⁹ of coalitions, which, to our knowledge, is a novelty.

These renegotiation-proof and coalition-proof punishments, easy to implement once information is complete, allow to sustain cooperation as long as the agents are not too impatient. For existence of the efficient equilibrium, we check that the discount factor compatible with the different conditions imposed by our punishment is smaller than one.

We also introduce a new explanation as to why inequality can result in suboptimal outcomes. We indeed show that the introduction of inequality under Nash reversion or punishments resisting to renegotiation and deviation by credible coalitions is detrimental to cooperation. In the presence of inequality of shares, the agents have to be more

⁶Abreu proposes symmetric punishments during which everyone suffers. Large firms, having a large market share, suffer more from the price decrease following the breach in the collusive agreement.

⁷We also take into account deviations of credible coalitions whereas it is less relevant in Cournot competition.

⁸Readers interested in this topic could also see the works of van Damme (1989) and Asheim.

⁹In some cases, a certain number of agents is required to make a deviation profitable. Hence, no one would like to deviate unless an agreement is reached. Once such an arrangement is found, it might still be interesting for one or more players to renege on the agreement and deviate further. This would render the deviation not credible, a case which we will not address in this paper.

patient not to deviate than under perfect equality.

In Section 2, we present our simple model giving two options to the agents: cooperating or deviating from the socially optimal contribution. Nash reversion is addressed in Section 3. Then, in Section 4, we propose a renegotiation-proof and coalition-proof punishment scheme and investigate the influence of share inequality on the limit discount factor. We characterize, in Section 5, the lowest share compatible with generalized cooperation, discuss the coexistence of one free-riding and one cooperating coalition and how redistribution can increase or decrease the total amount of effort put in the project. We also examine how redistributing shares from the rich to the poor players can improve the welfare of everybody. Eventually, before concluding in Section 7 as to the negative influence of wealth inequality in this setting, we introduce, in Section 6 outside options. The proofs are collected in an Appendix.

2 Repeated Joint Production with Shares

A group of n agents decide to produce jointly and repeatedly a particular output. All the i agents are identical except for, λ_i , their share in output which can also be a measure of their relative wealth. λ is the vector of shares, $[\lambda_1, \lambda_2, \dots, \lambda_n]$, the sum of which equals 1. Note that $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. All the agents exhibit the same degree of impatience and therefore have the same discount factor, δ ¹⁰.

The collective output is, as in the standard pure public good model, the sum of the (nonnegative) efforts, e_i , of all the agents taking part in the project. This modelling of the output implies that efforts are perfect substitutes what, with respect to efficiency, favours an unequal distribution of shares¹¹. A straightforward illustration of this would be to consider a single agent concentrating all the incentives ($\lambda_i = 1$) to produce the efficient effort. In this case, the first-best optimum is produced at each stage of the game while using a Leontieff production function, where efforts are perfect complements, would result in a suboptimal production of effort. Each agent putting effort in the project has to undergo an isoelastic convex cost expressed in terms of her share¹², $\frac{e_i^\gamma}{\gamma \lambda_i}$ with $\gamma \geq 2$ ¹³. This allows us to release the assumption of strict equality of marginal costs which could have been suspected of driving the results. As the marginal cost of

¹⁰If one introduces inequality in the distribution of discount factors assuming that the poor are less patient than the rich, for instance by endogenizing δ w.r.t. λ , cooperation is even harder to sustain and our conclusions easier to reach.

¹¹For a discussion on the influence of inequality on joint projects when efforts are not perfect substitutes in a one-shot game, see Ray, Baland and Dagnelie (2007).

¹²A similar exercise was run with simply an isoelastic convex cost e_i^γ with $\gamma > 1$ in a previous version of the paper (Dagnelie, 2007). Results are equivalent.

¹³If $\gamma < 2$, a strong inequality implies that the very rich players prefer generalized deviation to generalized cooperation and at $\gamma = 1$, surplus maximization is unbounded.

effort is decreasing in the share endowment¹⁴, the efficient level of effort is smaller for the poor which therefore makes it easier for them to cooperate. γ is also present at the denominator for analytical tractability, without loss of generality.

Collective action individual payoffs, π_i , have therefore the following form:

$$\pi_i = \lambda_i \sum_j e_j - \frac{e_i^\gamma}{\gamma \lambda_i} \quad (1)$$

Maximizing the social surplus provides us with the optimal, cooperative level of effort, e_i^C , equal to $\lambda_i^{\frac{1}{\gamma-1}}$, which depends on the distribution of shares.

If this game is played once, then every agent is going to maximize her own payoff, given the efforts contributed by the other agents.

$$\max_{e_i} \lambda_i (e_i + \sum_{j \neq i} e_j) - \frac{e_i^\gamma}{\gamma \lambda_i} \quad (2)$$

In this case where everyone deviates from the cooperative level of effort and produces $e_i^N \equiv \lambda_i^{\frac{2}{\gamma-1}}$, the outcome is known to be pareto dominated by the first-best optimum¹⁵.

The game described above is similar to a prisoner's dilemma with n agents and continuous strategies. Inequality creates tensions in the sense that the premium from deviating from cooperation is, proportionally to their endowment, higher for the poor which therefore renders deviation more attractive to them.

LEMMA 1 *The agent the most tempted by deviation is always the one with the lowest share.*

When we introduce share inequality, the cooperation payoff decreases faster than the deviation payoff which makes deviation more profitable for the poor. This tends to confirm Olson's hypothesis as to the exploitation by the poor.

If $\pi_i^{C*} < \pi_i^C$ - where π_i^{C*} is the deviation profit¹⁶ which depends on the number of deviating players and π_i^C is the cooperation profit¹⁷ - deviating from the cooperation effort is not profitable. There is therefore no need of punishment in those cases. In this paper, we are not paying attention to those not credible deviations. For deviating to be

¹⁴This could be due to restricted access to credit or scale economies in the production of effort, etc. Note that facilitating cooperation for the poor renders our conclusion harder to reach.

¹⁵It means that $e_i^N = e_i^C \lambda_i^{\frac{1}{\gamma-1}}$ with $e_i^N < e_i^C$ as, if $\lambda_i < 1$, $\lambda_i^{\frac{1}{\gamma-1}} < 1$ for all γ .

¹⁶ $\pi_i^{C*} = \lambda_i (\sum_{n_C} \lambda_j^{\frac{1}{\gamma-1}} + \sum_{n_D} \lambda_j^{\frac{2}{\gamma-1}} - \gamma^{-1} \lambda_i^{\frac{2}{\gamma-1}})$, where $n - n_D = n_C$ cooperate but n_D deviate.

¹⁷ $\pi_i^C = \lambda_i (\sum \lambda_j^{\frac{1}{\gamma-1}} - \gamma^{-1} \lambda_i^{\frac{2}{\gamma-1}})$

interesting, the following condition must be fulfilled:

$$\pi_i^{C^*} > \pi_i^C \equiv \frac{\lambda_i \gamma \sum_{n_D} \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{1}{\gamma-1}})}{\lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}})} < 1$$

with n_D being the number of deviating players. Under a perfectly equalitarian distribution of shares (i.e. $\lambda_i = \frac{1}{n}$), it simplifies to :

$$\gamma \frac{n_D}{n} < \frac{1 - n^{-\frac{\gamma}{\gamma-1}}}{1 - n^{-\frac{1}{\gamma-1}}} \equiv n_D < \frac{n - n^{-\frac{1}{\gamma-1}}}{\gamma(1 - n^{-\frac{1}{\gamma-1}})}$$

For this inequality to be satisfied, the number of deviating players must decrease when γ increases (unless $\gamma \rightarrow +\infty$) and n decreases. When $\gamma = 2$, it is true as long as $n_D < \frac{n+1}{2}$. The latter value is also the higher bound¹⁸ on the number of deviating players when $\gamma > 2$. The proportion of deviators in the group (n_D/n) must decrease with the size of the group (n) for the condition to remain true. It is to be noted that the condition is always satisfied if $n_D = 1$. While cooperating produces a big surplus, if too few cooperate and too many deviate, the small surplus is divided among too many deviating players for deviating to remain profitable.

3 Nash Reversion

In this section, we will temporarily not address the problems posed by renegotiation and consider the following punishment: Once a coalition has deviated, everybody produces the Nash level of effort for ever. For cooperation to be sustainable, the discount factor must respect the following condition:

$$(1 - \delta_N)\pi^{C^*} + \delta_N\pi^N < \pi^C \Rightarrow \delta_N > \frac{\pi^{C^*} - \pi^C}{\pi^{C^*} - \pi^N}$$

where π^N is the Nash profit¹⁹, corresponding to generalized deviation.

If $\frac{\pi^{C^*} - \pi^C}{\pi^{C^*} - \pi^N} < 1$, it is theoretically possible to support cooperation. As long as players have a discount factor greater than δ_N and smaller than 1, Nash reversion can be used as a threat against deviation. Hence, the necessary condition for cooperation to become sustainable is :

$$\pi_i^C > \pi_i^N \equiv \frac{\lambda_i \gamma \sum \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{1}{\gamma-1}})}{\lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}})} > 1 \quad (3)$$

¹⁸As the number of deviating players has to be an integer and the condition is strict, in practice, n_D can never be greater than $n/2$.

¹⁹ $\pi_i^N = \lambda_i (\sum_i \lambda_i^{\frac{2}{\gamma-1}} - \gamma^{-1} \lambda_i^{\frac{2}{\gamma-1}})$

The bigger the difference $\pi_i^C - \pi_i^N$, the smaller δ_N and therefore the easier cooperation can be sustained.

One can see that, if the distribution of shares is perfectly equalitarian, as long as the agents have a discount factor smaller than 1 and greater than δ_N , cooperation can always be sustained by Nash reversion. The condition in equation (3) is indeed always verified and becomes :

$$\gamma > \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \quad (4)$$

PROPOSITION 1 *Introducing inequality among agents renders the condition to sustain cooperation with Nash reversion more difficult to fulfill.*

As cooperation can always be sustained with Nash reversion when the distribution is equalitarian (and $\delta_N < \delta < 1$) and as deviation is a dominant strategy for the agents for which inequality renders $\pi_i^C < \pi_i^N$, one can expect a negative effect of inequality on cooperation. Following the introduction of inequality, the agents losing from the redistribution of shares, i.e. $\lambda_i < n^{-1}$, have to be more patient than before not to enter a deviation phase. All this means that increasing share inequality makes cooperation harder and harder to sustain - as the limit discount factor, δ_N , rises - up to a point where inequality is such that, whatever the impatience degree of the agent with a low share, generalized cooperation is not possible any more.

4 A Renegotiation-Proof and Coalition-Proof Equilibrium

As mentioned before, Nash reversion suffers from several flaws. We therefore want to turn to a renegotiation-proof and coalition-proof equilibrium preventing credible coalitions to deviate from the first-best production level. We also want this equilibrium to allow a return to cooperation after the punishment phase.

As suggested by van Damme (1989) for the 2 players prisoner's dilemma with discrete strategies, cooperation can be sustained if after player 1's deviation, player 2 is allowed to deviate (produce the Nash level of effort) until player 1 cooperates²⁰. Extending renegotiation-proofness from 2 to n players requires taking account of deviations of coalitions. Nonetheless, the original concept with 2 players fits in with the framework set out below.

van Damme's idea, exposed above, is used to devise our punishment scheme. After a deviation of a coalition of one or more players, the cooperative players enter a punishment phase during which they produce a retaliation quantity of effort. They keep playing this level of effort as long as the cheaters - the ones who deviated in the normal

²⁰If player 2 does not cooperate any more, both players are stuck in the Pareto dominated equilibrium.

phase - have not played the punishment level of effort. During this punishment, everybody has the incentives to play accordingly to the scheme - what ensures subgame perfectness - and the punishers get at least as much as when everybody cooperates and produces the first-best optimum - what guarantees renegotiation-proofness.

We can immediately restrict the length of the punishment phase thanks to the following lemma.

LEMMA 2 *Aiming at the smallest discount factor compatible with a renegotiation-proof and coalition-proof punishment limits the length of the punishment phase to one period.*

A multi-period punishment would have two effects, ex ante, it would make deviation less attractive but as the punishment is harsher, ex post, it would increase the incentives to deviate from conforming to the punishment. As will become clearer below, there is a trade-off between these two effects. In this particular case, the latter effect would be dominating as the punishment level of effort is high enough for the punishers to be willing to punish, given the requirement of renegotiation-proofness.

For a punishment to be renegotiation-proof and coalition-proof and to allow a return to cooperation, the discount factor must respect 5 conditions.

Condition 1 *Ex ante, the punishment must be such that deviations are deterred.*

$$(1 - \delta)\pi^{C^*} + \delta(1 - \delta)\pi^P + \delta^2\pi^C < \pi^C$$

$$\Rightarrow \delta_{XA} > \frac{\pi^{C^*} - \pi^C}{\pi^C - \pi^P} \quad (5)$$

δ_{XA} must be smaller than 1 which implies that $2\pi^C - \pi^{C^*} - \pi^P > 0$, with π^P being the payoff obtained by an agent during her punishment. Meeting this condition will prevent a subcoalition of players to alternate between deviating and being punished every other period. If the punishment effort is fixed at the level of the cooperative effort, this condition is always verified as $2\pi^C - \pi^{C^*} - \pi^P > 0$ boils down to equation (4). Hence we know that, if the punishment effort to put in is greater than during cooperation, δ_{XA} decreases.

Condition 2 *The payoff of the punished must be greater when conforming to their punishment than when deviating.*

$$(1 - \delta)\pi^P + \delta\pi^C > (1 - \delta)\pi^{P^*} + \delta(1 - \delta)\pi^{P'} + \delta^2\pi^C$$

$$\Rightarrow \delta_{XP} > \frac{\pi^{P^*} - \pi^P}{\pi^C - \pi^{P'}} \quad (6)$$

with π^{P^*} being the profit obtained when deviating from undergoing the punishment and $\pi^{P'}$, the profit received, while incurring the penalty, by the subcoalition of agents

who deviated from the punishment. When $n_{D^*} = 1$, δ_{XP} reaches its minimum which therefore makes Condition 2 the most easily fulfilled. The benefit from deviating from the punishment is by far outweighed by the burden of the penalty for which the single deviator has to compensate all the cooperating players. If the size of the deviating subcoalition rises, π^{P^*} decreases whereas $\pi^{P'}$ rises much faster which means that δ_{XP} also rises. It is therefore expected that $\arg \max_{n_{D^*} \in (1, n_D)} \delta_{XP} = n_D$.

The following two conditions ensure that all the punishers are willing to conform to the punishment phase.

Condition 3 *The payoff of the punishers must be greater when conforming than when deviating from punishing and then conforming, i.e. $\pi^{1/P} > \pi^{1/P^*}$.*

If $\pi^{1/P}$ and π^{1/P^*} represent respectively the profit from punishing and from deviating from giving the punishment, we get:

$$(1 - \delta)\pi^{1/P} + \delta\pi^C > (1 - \delta)\pi^{1/P^*} + \delta\pi^C \Rightarrow \pi^{1/P} > \pi^{1/P^*}$$

In case of perfect equality of wealth among agents, we have:

$$\gamma \frac{n_{C^*}}{n} (C - C^*) > (C^\gamma - C^{*\gamma}) \quad (7)$$

The idea behind this condition is that it could be more interesting, for the punishing players, to skip the punishment phase and go back directly to cooperation. We are going to check which values of C and C^* are compatible with a renegotiation and coalition-proof punishment. C^* is the level of effort put in when deviating from giving a punishment. Equation (7) must hold for $n_{C^*} = 1$, while if it does not hold when $n_{C^*} > 1$ we have to ensure that all these deviating coalitions are not credible. This is done in Condition 4.

One could imagine that the simplest form of punishment would be that the punishers do not produce for one period while the deviators are constrained to put in such a level of effort that the punishers get at least the cooperative payoff. However, as long as $C < e_i^N$, for very high values of γ , it could be interesting for one punisher to deviate and produce the cooperative level of effort²¹. Once C is fixed at the Nash level of effort, it is never interesting for one punisher²² to deviate from the punishment scheme. Once that C is fixed to e_i^N , it is easy to prove that:

$$\frac{\partial \delta_{XP}}{\partial n_{D^*}} > 0 \quad (8)$$

²¹As showed by $\lim_{\gamma \rightarrow +\infty} \frac{\gamma}{n} (\beta n^{\frac{-1}{\gamma-1}} - 1) - (\beta^\gamma n^{\frac{-\gamma}{\gamma-1}} - 1) < 0$ when $\beta < 1$. For the particular case of $C = 0$, it is easy to check that the condition $C^* > (\frac{\gamma}{n})^{\frac{1}{\gamma-1}}$ is not satisfied for many values of γ, n .

²²A coalition of $n_{C^*} > 1$ players could be tempted to deviate from giving the punishment. This prevents this scheme to be strong Nash.

Hence, we have to focus on the case where Condition 2 is the hardest to satisfy, i.e. $n_{D^*} = n_D$, which gives $\delta_{XP} > \frac{\pi^N - \pi^P}{\pi^C - \pi^P}$. As it is easy to see that $\frac{\partial \delta_{XP}}{\partial \pi^P} < 0$ ²³, we know that the minimum of δ_{XP} is reached when the punishment is also fixed at its minimum.

All that has been said so far allows us to remark that π^N is again a focal point. If the cooperative agents return to putting in the Nash effort, the threat point of the game is infinite repetition of the Nash equilibrium. The deviators can indeed renege for ever on the punishment and also produce the Nash effort. We therefore know that $\pi_i^C > \pi_i^N$ is again to satisfy.

For this setting to be coalition-proof, we must now ensure that no deviating coalition of punishers is credible. For the deviation to be credible, no one should have one's interest in further deviating from the deviating coalition. We hence turn to the next condition.

Condition 4 *For the equilibrium to be coalition-proof, no deviation of punishers should be credible, i.e. $\pi^{1/P^*} < \pi^{1/P^{**}}$.*

If $\lambda_i = \frac{1}{n}$ and $n_{C^{**}} = 1$ ²⁴, it is equivalent to:

$$\frac{\gamma}{n} < \frac{(1 - n^{\frac{-\gamma}{\gamma-1}})}{(1 - n^{\frac{-1}{\gamma-1}})} \quad (9)$$

For all γ and n , this condition is always fulfilled which means it is always more interesting for one player to further deviate from the deviating coalition by producing the Nash level of effort²⁵. Condition 4 is always fulfilled and no deviation of punishers is therefore credible.

It is to be noted that, if all the punished conform to the punishment phase and produce a high level of effort, no coalition of punishers is willing to deviate. A fortiori, if a subcoalition of punished deviates from the punishment phase - and therefore produces less -, no subcoalition of punishers is tempted to skip this phase. Given the punished and the punishers have opposite duties and interests, no deviation of a mixed coalition is credible.

The following condition ensures that the punishers are not going to renegotiate the punishment scheme.

Condition 5 *The payoff from punishing a deviating coalition must be greater or equal than the payoff from generalized cooperation, i.e. $\pi^{1/P} \geq \pi^C$.*

²³ $\frac{\partial \delta_{XP}}{\partial \pi^P} = \frac{\pi^N - \pi^C}{(\pi^C - \pi^P)^2}$ which is negative as long as $\pi^C > \pi^N$.

²⁴ If one player wishes to deviate from the deviating coalition, the latter is not credible.

²⁵ It is not surprising as e_i^N is the best response of player i , being the solution to equation (2).

It means that, if $\lambda_i = \frac{1}{n}$,

$$\underline{P} \geq n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - n_C - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_D} \quad (10)$$

Now that the conditions to satisfy are stated, we want to find the punishment producing the lowest discount factor compatible with generalized cooperation, $\underline{\delta}$. We have therefore to fix the effort level corresponding to the punishment, P , so that the couple $(\delta_{XA}, \delta_{XP})$ is the lowest possible and, in any case, smaller than one. As our punishment scheme has to simultaneously respect the conditions expressed in equations (5) and (6), we have to find the $\max(\delta_{XA}, \delta_{XP})$. To obtain the punishment corresponding to the highest degree of impatience, we have to determine P such that we get $\underline{\delta} \equiv \min \max(\delta_{XA}, \delta_{XP})$.

Comparing δ_{XA} and δ_{XP} boils down to comparing $\pi^{C^*} - \pi^C$ and $\pi^N - \pi^P$, in the limit case where $n_{D^*} = n_D$, as the denominator of these fractions is the same. After simplification,

$$\pi^{C^*} - \pi^C \leq \pi^N - \pi^P \quad (11)$$

becomes:

$$-\gamma^{-1} n^{\frac{-1}{\gamma-1}} \left[(n^{\frac{1}{\gamma-1}} P)^\gamma - 1 \right] + n_D n^{\frac{-\gamma}{\gamma-1}} (n^{\frac{1}{\gamma-1}} P - 1) \leq 0 \quad (12)$$

The root of interest in equation (12) is $P = e^C$. Hence we know that when $P = e^C$, $\underline{\delta} = \delta_{XA} = \delta_{XP}$. We also know that as soon as $P > e^C$, we have to minimize $\pi^N - \pi^P$ and thus P to get $\underline{\delta}$. It means that the minimal punishment compatible with renegotiation-proofness and coalition-proofness is \underline{P} as long as it is greater than the cooperative level of effort. As long as the parameters of equation (10) produce a $\underline{P} < e^C$, resorting to such a punishment effort will not prevent deviations. The penalty incurred by the deviators would be too small to deter them from alternating between deviating and being punished. In this case, the punishment must be to put in the cooperative level of effort. On the other hand, if $P < \underline{P}$, the punishment is not renegotiation-proof.

We have therefore to find $P \equiv \max(e^C, \underline{P})$ and turn to the following equation.

$$\underline{P} \geq e^C \equiv \frac{n_C}{n} \gamma \geq \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \quad (13)$$

Taking into account the condition for deviation to be profitable, i.e. $\pi^{C^*} - \pi^C > 0$, the only case where equation (13) is not verified, with $\gamma \geq 2$, is when $n_C = n_D = \frac{n}{2}$. In the latter case, \underline{P} is too small a punishment and the punished have to put in the cooperative level of effort, exactly as in the two players game put forward by van Damme (1989). Note that this particular case can happen only if n is even.

Considering we took into account the different conditions imposed by our punishment scheme when $\lambda_i = n^{-1}$, we are equipped with the parameters of our punishment,

P and C . As δ_{XP} must be smaller than 1 for cooperation to be a renegotiation-proof and coalition-proof equilibrium, π_i^N must be smaller than π_i^C . It is the same binding constraint as with Nash reversion and we know from equation (4) that it is always true. This allows us to put forward the following proposition:

PROPOSITION 2 *As long as $\delta_{XP} < \delta < 1$, $\lambda_i = n^{-1}$ and the game is infinitely repeated²⁶, it is possible to sustain cooperation with a renegotiation-proof and coalition-proof punishment.*

So far, we have shown that, under a perfectly equalitarian distribution of shares, it is possible to use a renegotiation-proof and coalition-proof punishment scheme to sustain generalized cooperation. One can remark that, with γ known and a perfectly observable and certain output, this equilibrium requires particularly little information. It can be completely decentralized as, after deviation, the punished and the punishers know exactly which level of effort to provide.

We now want to investigate how introducing share inequality influences the way the first-best optimum can be supported. The punishment is, as expected, very similar to the case of perfect share equality. If we introduce inequality, $\pi^{C^*} - \pi^C \leq \pi^N - \pi^P$ becomes:

$$\left[\sum_{j \in n_D} (P_j - \lambda_j^{\frac{1}{\gamma-1}}) \right] - \gamma^{-1} \lambda_j^{\frac{2-\gamma}{\gamma-1}} \left[(P_j \lambda_j^{\frac{-1}{\gamma-1}})^\gamma - 1 \right] \leq 0 \quad (14)$$

in which the only case where the P_j is insufficient to satisfy equation (14), when $\gamma \geq 2$, occurs again when $n_D = \frac{n}{2}$. It is easy to see that for this condition to be respected, the punishment effort has to be greater or equal than the cooperative level of effort, i.e.

$P_j \geq \lambda_i^{\frac{1}{\gamma-1}}$. It means that equation (10) becomes:

$$\underline{P}_j \geq \frac{\lambda_j}{\sum_{j \in n_D} \lambda_j} \left[\sum_{k \in n} \lambda_k^{\frac{1}{\gamma-1}} - \sum_{i \in n_C} \lambda_i^{\frac{2}{\gamma-1}} - \gamma^{-1} \frac{\sum_{i \in n_C} \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}})}{n_C} \right]$$

While, as each punisher must receive $\pi_i^{1/P} = \pi_i^C$, all the deviators have to put in \underline{P}_j so that:

$$\pi_i^{1/P} = \lambda_i \left(\sum_{i \in n_C} \lambda_i^{\frac{2}{\gamma-1}} + \sum_{j \in n_D} \underline{P}_j - \gamma^{-1} \lambda_i^{\frac{2}{\gamma-1}} \right)$$

where $\forall i \in n_C : \sum_{j \in n_D} \underline{P}_j = \sum_{k \in n} \lambda_k^{\frac{1}{\gamma-1}} - \sum_{i \in n_C} \lambda_i^{\frac{2}{\gamma-1}} - \gamma^{-1} \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}})$.

We showed that, under a perfectly equalitarian distribution of shares, our punishment scheme prevents all the agents from deviating from cooperation. We also know that inequality is detrimental to cooperation as it is possible that deviation becomes a

²⁶It remains true if the agents do not know when the game ends.

dominant strategy for poorly endowed agents - i.e. if $\pi_i^C < \pi_i^N$. However, even in less extreme cases, we can state the following proposition.

PROPOSITION 3 *After introducing inequality, the agents losing from the disequalizing change in the distribution of shares have to be more patient than before to produce the efficient level of effort when the punishment is renegotiation-proof and coalition-proof.*

As in the case of Nash reversion, introducing inequality of shares renders first-best efficiency more difficult to support with a renegotiation-proof and coalition-proof punishment.

Now that we have characterized the minimal discount factor with those two kinds of punishment, we can compare them. It is easy to show that $\delta_N \leq \delta_{XP}$ as, after rearranging and symplifying, we get equation (11).

5 Redistribution and Cooperation

In this section, we try to characterize the lower bound of theoretical cooperation, i.e. when δ tends towards 1. Note again that the further $(\pi_i^C - \pi_i^N)$ is from 0, the lower δ , which increases the scope for cooperation. Then, we investigate the issue of redistribution.

5.1 Characterization of the lowest share compatible with generalized cooperation

As the utmost condition to satisfy for cooperation to be sustainable is $\pi_i^C > \pi_i^N$, it is possible to partially characterize the lowest share compatible with generalized cooperation, λ_{min} . To see this, let us define several distributions of shares:

$$\begin{aligned}\underline{\lambda} &\equiv \underline{\lambda}_1 = \dots = \lambda_{n-1} < \lambda_n \\ \tilde{\lambda} &\equiv \tilde{\lambda}_1 = \dots = \lambda_{n-2} < \lambda_{n-1} < \lambda_n \\ \hat{\lambda} &\equiv \hat{\lambda}_1 = \dots = \lambda_{n-2} < \lambda_{n-1} = \lambda_n \\ \bar{\lambda} &\equiv \bar{\lambda}_1 < \lambda_2 = \dots = \lambda_n\end{aligned}$$

The lowest share compatible with cooperation depends on the convexity parameter of the cost term, γ . Therefore, we get:

$$\lambda_{min} \geq \begin{cases} \bar{\lambda}_1 & \text{if } 2 \leq \gamma \leq 4 \\ \bar{\lambda}_1 \text{ or } \underline{\lambda}_1 & \text{if } \gamma > 4 \end{cases} \quad (15)$$

When $\gamma > 4$, the second derivative of the function $\lambda^{\frac{1}{\gamma-1}} - \lambda^{\frac{2}{\gamma-1}}$ is either positive or negative. Hence the latter function is respectively convex or concave which determines

the form of the shares distribution giving the lowest share compatible with generalized cooperation. A high γ favours convexity and therefore inequality while a high n is in favour of concavity and equality of shares.

5.2 Redistribution

We know that, whatever the punishment strategy we use, if $\pi_i^C < \pi_i^N$, the best strategy for player i is to deviate at each period of the game. The first-best optimum cannot be attained any more but cooperation can still be sustained for a subset of players whose cooperation profit is greater than the Nash profit²⁷.

Observation 1 *The only influence of the agents not cooperating because of a low λ_j ²⁸ is to diminish the share of the cooperators.*

Taking into account the poorly endowed agents, equation (3), $\pi_i^C > \pi_i^N$, becomes

$$\frac{\lambda_i \gamma \sum_{i \neq j}^{n_C} \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{1}{\gamma-1}})}{\lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}})} > 1 \quad (16)$$

This inequality is never verified if $n_C = 1$. It means that, if more than one agent gets a positive share, producing the first-best level of effort requires at least two agents to get a big enough share²⁹, i.e. a share such that $\pi_i^C > \pi_i^N$. The number of people whose shares add up to a low λ_{inflim} does not influence cooperation. At the same time, there can coexist one cooperating and one free-riding coalition.

As we know that cooperators put in an effort of $\lambda^{\frac{1}{\gamma-1}}$ and deviators contribute $\lambda^{\frac{2}{\gamma-1}}$, the total level of effort put in the project is $\sum_i e_i = \sum_{n_C} \lambda^{\frac{1}{\gamma-1}} + \sum_{n_D} \lambda^{\frac{2}{\gamma-1}}$. It is therefore possible to compare the different distributions with respect to the total level of effort contributed to the project³⁰.

The only case where the effect of redistribution is unambiguous³¹ is when it lowers the share of a poor player to the benefit of another poor player to the extent that the latter becomes rich enough to cooperate. Then higher inequality increases the total sum

²⁷This can be the case if we suppose that the rich players, knowing that the deviators are so poor that they cannot afford to cooperate, do not turn to punishing the deviating players.

²⁸ $\sum_j \lambda_j = \lambda_{inflim} \Rightarrow \sum_{i \neq j}^{n_C} \lambda_i = 1 - \lambda_{inflim}$

²⁹It implies that, if all the agents but one have a share such that $\pi_i^C < \pi_i^N$, the richest player (whose $\lambda_n < 1$) will produce the deviating level of effort.

³⁰The same exercise done on the social surplus, $\sum_{n_C} \lambda^{\frac{1}{\gamma-1}} (1 - \gamma^{-1}) + \sum_{n_D} \lambda^{\frac{2}{\gamma-1}} (1 - \gamma^{-1} \lambda)$, produces equivalent results.

³¹To the exception of $\gamma = 2$, where redistribution is neutral, disequalizing redistributions between cooperators always lower the aggregate level of effort.

of contributions. In all the other instances of redistribution, the effect on the aggregate level of effort depends on γ which determines the concavity or convexity of the function. Regarding redistribution from/to a deviator to/from a cooperator, its effects are symmetrical. If we redistribute from a deviator to a cooperator (from a cooperator to a deviator), if $\gamma \geq 3$, it decreases (increases) the aggregate level of effort while if $\gamma < 3$ there is a point where redistribution is neutral as to the aggregate level of contributions and beyond this point, increasing inequality will raise (lower) the total level of efforts. As to the redistribution between deviators, the pivotal value of γ is 3. At 3, redistribution does not influence the amount of efforts provided in the project. Below (above) 3, every disequalizing redistribution among the poor decreases (increases) the aggregate level of effort. We therefore show that increasing inequality can have a positive effect on the amount contributed. This means we can observe a U-shaped relationship between inequality and the aggregate level of effort.

This being said, we now want to investigate whether it would be profitable for the rich agents to redistribute part of their share to the less endowed so that the latter can afford to cooperate. If the agents are perfectly patient, once the poor get a share such that their cooperation profit is at least equal to their deviation profit, they are expected to produce the efficient level of effort. The benefits from cooperation are such that, in some cases, they outweigh the loss of welfare suffered by the rich agents redistributing part of their share.

Let us first examine the extreme case where only one individual is rich enough for cooperation to be attractive. In this case, all produce the deviation level of effort. There are therefore large potential gains from cooperation. Simulations on the following system of equations³² teach us that redistributing so that each agent gets a big enough share to cooperate is always interesting, if the number of players is ≥ 3 and $\gamma \geq 3$.

$$\begin{cases} \pi_{\tilde{n}}^C \geq \pi_n^N \text{ with agent } \tilde{n} \text{ being } n \text{ after redistributing part of her share.} \\ \pi_{min}^C = \pi_{min}^N \end{cases}$$

The general form of this system of equations, where cooperation takes place before redistribution, would be:

$$\begin{cases} \pi_{\tilde{n}}^C \geq \pi_n^C \text{ with agent } \tilde{n} \text{ being } n \text{ after redistributing part of her share.} \\ \pi_{min}^C = \pi_{min}^N \end{cases}$$

where we consider the payoff from cooperation for a subcoalition and for the grand coalition under the condition that the poorest agent gets a share such that cooperation is at least as interesting as generalized deviation³³.

³²The detail of this system is presented in Appendix B.

³³We could also examine these conditions when the cooperating subcoalition is enlarged by one player.

As in the case of weakest-link public goods where the aggregator index used to describe the production function is the minimum of all the contributions³⁴, the repetition of the game could induce rich players to redistribute part of their share to render cooperation attractive for the poor. Note that it would not be dictated by altruistic considerations but would pertain to a payoff maximizing behaviour of the richest agents.

6 Outside Option

So far we have constrained the players to produce either a cooperative or a deviation level of effort while preventing them from opting out of the game. This could be the right way to model many joint projects in real life where, once the decision of joining the project has been made, it is not possible to go astern. Nevertheless, the alternative is relevant for other types of project, all the more since we focus on projects with perfect substitutability of efforts. In the presence of outside options, it seems reasonable to consider that, to get the same level of goods as jointly produced by n agents, working solo would require more effort and therefore be costlier than being involved in a joint project. In the case of irrigation schemes, once the decision to participate has been made, the irrigation infrastructure is built. It then becomes difficult to opt out and, unless one is ready to sustain an important cost, the agent's choice is limited to produce the cooperative or the deviation level of maintenance effort. Likewise, regarding tacit collusion in competition à la Cournot, firms, once in the market, produce either the collusive or the Nash quantity. As to defence alliances, countries can decide to leave a coalition, in which case they will probably have to increase their level of effort to benefit from the same level of protection.

Introducing outside options changes the conditions to be fulfilled for cooperation to be sustainable. In the case of Nash reversion, it is necessary to take into account the possibility that the profit from working solo (π_i^S) is higher than the profit from generalized deviation (π_i^N) in which case the condition to be respected becomes:

$$(1 - \delta)\pi^{C^*} + \delta\pi^S < \pi^C \Rightarrow \delta > \frac{\pi^{C^*} - \pi^C}{\pi^{C^*} - \pi^S}$$

Deviating from cooperation then leaving the collective action for working on one's own must indeed be less profitable than producing the cooperative level of effort in the joint project. The ultimate condition for cooperation to be fulfilled is therefore:

$$\pi^C > \max(\pi^N, \pi^S)$$

³⁴See Vicary (1990) for redistributions and weakest-link technology.

One can immediately realize that, if $\pi_i^S > \pi_i^N$, the discount factor is higher than without exit option. Outside options can therefore, as expected, decrease the scope for cooperation.

As to renegotiation-proofness where we have to ensure that $\delta = \min \max(\delta_{XA}, \delta_{XP})$, we have to take into account an additional condition. As the punishment lasts one period and allows a return to cooperation, the payoff received on the punishment path must be higher than the payoff obtained while working solo.

$$(1 - \delta)\pi^P + \delta\pi^C > \pi^S \Rightarrow \delta_S > \frac{\pi^S - \pi^P}{\pi^C - \pi^P}$$

As in the case of Nash reversion, the ultimate condition to be respected, while δ_{XA} and δ_{XP} must be simultaneously verified, becomes:

$$\pi^C > \max(\pi^N, \pi^S)$$

Obviously, as long as the outside option gives nothing more advantageous than the joint project, working solo is not even considered. Once the solo payoff is higher than the payoff from generalized deviation, deviating then working solo becomes a conceivable option which is not chosen unless the player is too impatient.

To summarize the different possibilities created by the introduction of an outside option, let us present the following tables. Note that the outside option is not specified and could differ according to the agent's share (or other factors absent from this model). The first table typically corresponds to the situation of the rich or to an equal to moderately unequal distribution of shares where, for all, the cooperation profit is greater than the Nash profit.

	π_i^{C*}	>	π_i^C	>	π_i^N
π_i^S	\uparrow π_i^S		\uparrow $(1 - \delta)\pi_i^{C*} + \delta\pi_i^S$		\uparrow π_i^C if $\underline{\delta}_S < \delta$
					\uparrow π_i^C if $\underline{\delta} < \delta$

According to the level of their solo payoff (π_i^S), agents are going to make different decisions as to their participation in the project. If the payoff from working alone is even greater than deviating from the project for agent i , the latter is going to work individually and will never participate in the project. If the solo payoff is greater than the cooperation payoff but smaller than the payoff from deviating, one can expect this agent to deviate in the first period of her participation in the collective action and then to opt out and work solo. In this case, it is likely that the cooperative players decide, if possible, to restrict access to the project.³⁵ While if working solo is less profitable than

³⁵They could also introduce a membership fee or a commitment device for discouraging such strategic

cooperating but more than generalized deviation, as long as the player's discount factor is greater than the limit discount factor taking account of the outside option, the player is going to cooperate. That, she will also do if working solo is less interesting than generalized deviation. Whatever, the discount factor in this latter case, she is taking part in the joint project. Whether she cooperates or deviates will then depend on the player's and the limit discount factors.

It is interesting to note that once some players opt out, the condition for cooperation to be sustainable among the remaining players boils down to equation (16). Even if the payoff of the rich decreases when poor and thus deviators leave the joint project, the departure of the latter does not render cooperation harder to sustain for the rich. Outside options might therefore increase the scope for feasible redistribution. It could indeed be interesting for the rich to redistribute so that the poor get $\pi_i^N > \pi_i^S$ and, even though deviating, keep participating in the project³⁶.

The second table exhibits the case where there is enough inequality for the Nash profit, typically of the poorly endowed agents, to be greater than the cooperation profit.

	$\pi_i^{C^*}$	>	π_i^N	>	π_i^C
π_i^S	\uparrow π_i^S		\uparrow $(1 - \delta)\pi_i^{C^*} + \delta\pi_i^S$		\uparrow $(1 - \delta)\pi_i^{C^*} + \delta\pi_i^N$

In those cases where cooperation is never a conceivable option, the agent either works directly on her own if her solo payoff is greater than her deviation payoff or first deviates then leaves the joint project if her solo payoff is comprised between her deviation and Nash payoff. Eventually, if the payoff from generalized deviation is greater than the solo payoff, at each period of the game, this player is going to produce the deviation level of effort.

Again, in some cases, the rich players knowing the situation of the poorest could let them deviate while a subcoalition could keep cooperating.

As an illustration, we present the following solo payoff function: $\pi_i^S = \alpha e_i - \frac{e_i^\gamma}{\gamma \lambda_i}$ which gives an optimal level of effort to put in being $e_i^S = (\alpha \lambda_i)^{\frac{1}{\gamma-1}}$ while the payoff becomes $\pi_i^{S^{opt}} = \alpha^{\frac{\gamma}{\gamma-1}} \lambda_i^{\frac{1}{\gamma-1}} (1 - \gamma^{-1})$. If $\alpha < 1$, it is clear that the optimal level of effort to put in is smaller when working solo than in a joint project. The conditions on α as to the different cases detailed above are presented in Appendix C.

behaviours. Otherwise, they could let the solo agent deviates instead of collectively deviate anticipating that they would be stuck in the pareto dominated equilibrium or that the solo player would join the project once they have returned to cooperation.

³⁶A larger redistribution such that $\pi_i^C > \pi_i^N$ for all i is also to be considered in this case.

7 Conclusion

We showed that, in this particular model, cooperation can be supported under Nash reversion or a renegotiation-proof and coalition-proof punishment. We also demonstrated that introducing inequality of share among players increases the discount factor compatible with sustainable cooperation, reducing the scope for cooperation. Once inequality has been introduced, the poorer agents involved in the repeated project have to be more patient than before to keep cooperating. Hence, we demonstrated that inequality is, in this game, detrimental to generalized cooperation, the efficient outcome. In this respect, this paper puts forward new insights as to the relationship between inequality and efficiency.³⁷

Inequality can be such that some agents cannot afford to produce the efficient level of effort while others may keep cooperating. Our model can therefore also explain the coexistence of well endowed players providing a high level of effort and poor agents only putting in the Nash level of effort. This, in a way, complies with Olson's hypothesis that contributions are positively related to wealth.

Comparing several distributions of share where cooperators and deviators coexist shows that increasing share inequality can have a positive or negative impact on the aggregate level of effort (or social surplus) depending on the cost parameter, γ , (and the number of cooperators). Hence, share inequality can have a U-shaped relationship with the aggregate level of effort (or social surplus), as regularly seen in case studies.

We also mentioned that, in some cases, it can be profitable for the rich agents to redistribute part of their share to the poor players so that the latter can afford to cooperate at each period of the game. Repetition of the game can therefore enlarge the scope for redistribution.

Eventually, we put forward that outside options could restrict the scope for cooperation and increase the scope for redistribution.

³⁷Several other explanations have been proposed so far in the vast literature exploring the link between inequality and efficiency. Missing or imperfect capital markets have been shown to prevent poor individuals to develop their full potential leading to inefficiency from the viewpoint of the social surplus (Loury, 1981). Inequality implying high redistribution would generate inefficiency through tax induced distortions on resource allocation (Alesina and Rodrik, 1994, Persson and Tabellini, 1994). As modelled by Esteban and Ray (2006), agents lobbying for government support put in effort along two dimensions: productivity of the project and wealth. This blurs the signal received by the government which leads it to make inefficient decisions in the presence of wealth inequality.

APPENDIX

A Proofs

Proof of Lemma 1:

The relative premium from deviating in terms of her own share is the following:

$$\frac{\pi_i^{C^*} - \pi_i^C}{\lambda_i} = \left[\sum_{n_D} \lambda^{\frac{1}{\gamma-1}} (\lambda^{\frac{1}{\gamma-1}} - 1) \right] - \gamma^{-1} \lambda^{\frac{2-\gamma}{\gamma-1}} (\lambda^{\frac{\gamma}{\gamma-1}} - 1)$$

If the derivative with respect to the share is negative, a wealth decrease renders the deviation more attractive.

$$\frac{\partial(\pi_i^{C^*} - \pi_i^C)/\lambda_i}{\partial \lambda_i} \equiv \frac{1}{\gamma-1} \lambda_i^{\frac{-\gamma+2}{\gamma-1}} \left[(\lambda_i^{\frac{1}{\gamma-1}} - 1) + \frac{2-\gamma}{\gamma} (\lambda_i^{-1} - \lambda_i^{\frac{1}{\gamma-1}}) \right]$$

As the derivative is always negative when $\gamma \geq 2$, the relative premium from deviating rises when the share declines. ■

Proof of Equation (4)

After rearranging equation 4, we get:

$$(\gamma - 1) - n^{\frac{-1}{\gamma-1}} (\gamma - n^{-1}) > 0$$

Taking alternatively the limit of γ towards 2 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

Proof of Proposition 1

We are going to compare two discount factors compatible with cooperation, first under a perfectly equalitarian distribution of shares, then after introducing a disequalizing change in the distribution. As discount factors and ease to sustain cooperation vary in opposite directions, we are done if we can prove that introducing inequality in the distribution of shares makes the discount factor rise.

As long as $\delta_N < 1$, it is theoretically possible to sustain cooperation. We know that the bigger $\pi_i^C - \pi_i^N$, the lower δ_N .

$$\pi_i^C - \pi_i^N = \lambda_i \left[\left(\sum \lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}}) \right) - \gamma^{-1} \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \gamma^{\frac{\gamma}{\gamma-1}}) \right]$$

As $\frac{\partial(\pi_i^C - \pi_i^N)}{\partial \lambda_i} > 0$, a decrease in λ_i lowers the gain from cooperation and therefore increases the incentives to deviate and the limit discount factor, δ_N . ■

Proof of Lemma 2

Let us assume the length of the punishment is t periods with $t \in [1, \dots, T]$. If we compare the different δ corresponding to equations (5) and (6), we get:

- ex ante, no one wishes to deviate

$$\sum_{t=1}^T \delta_{XA}^t > \frac{\pi^{C^*} - \pi^C}{\pi^C - \pi^P} \Rightarrow \delta_{XA-1} > \dots > \delta_{XA-T}$$

- The payoff of the punished must be greater when conforming to their punishment than when deviating.

$$\delta_{XP-t} > \left(\frac{\pi^{P^*} - \pi^P}{\pi^C - \pi^P} \right)^{\frac{1}{t}} \Rightarrow \delta_{XP-1} < \dots < \delta_{XP-T}$$

To get the equilibrium compatible with the biggest impatience of the agents, we have to find the lowest δ , $\underline{\delta} \equiv \min \max(\delta_{XA-t}, \delta_{XP-t})$ with t being the number of periods of the punishment.

If $\delta_{XA-1} \leq \delta_{XP-1}$, we are done as this couple would be smaller than any other one in the following general ordering: $\delta_{XA-t} < \dots < \delta_{XA-1} \leq \delta_{XP-1} < \dots < \delta_{XP-t}$.

It therefore boils down to prove that $\pi^{C^*} - \pi^C \leq \pi^{P^*} - \pi^P$ which is the case when $P \geq e_i^C$ as explained after equation (12). ■

Proof of Equation (8)

Taking into account that $\delta_{XP} < 1$ and that $\pi^C - \pi^{P'} > 0$, and rearranging, we get $\pi^{P^*} + \pi^{P'} - \pi^C - \pi^P < 0$. Our strategy is to focus on the terms varying with n_{D^*} to get the simplest expression of the derivative.

Considering the punishment level of effort of equation (10), we get respectively for P and P' :

$$\begin{aligned} P &= n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - (n - n_D) - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_D} \\ P' &= n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - (n - n_{D^*}) - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_{D^*}} \end{aligned}$$

Deriving $\pi^{P^*} + \pi^{P'} - \pi^C - \pi^P$ with respect to n_{D^*} , we get:

$$n^{-1} \left[-P + P' + n_{D^*} \frac{\partial P'}{\partial n_{D^*}} - n^2 P'^{\gamma-1} \frac{\partial P'}{\partial n_{D^*}} \right]$$

Substituting in the latter expression $\frac{\partial P'}{\partial n_{D^*}} = \frac{1}{n_{D^*}} [-P' + n^{\frac{-2}{\gamma-1}}]$, we have the following sum which we can sign:

$$n^{-1} \left[-(P - n^{\frac{-2}{\gamma-1}}) + \frac{n^2}{n_{D^*}} P'^{\gamma-1} (P' - n^{\frac{-2}{\gamma-1}}) \right]$$

It is easy to prove that the second term is greater than the first one if $n_{D^*} = 1$.

As we know that $P \leq P'$ with equality when $n_D = n_{D^*}$, we are done if we can show that the derivative is positive when all the punished deviate from the prescribed punishment. In this case, the derivative becomes:

$$n^{-1} \left[\left(P - n^{\frac{-2}{\gamma-1}} \right) \left(P'^{\gamma-1} \frac{n^2}{n_D} - 1 \right) \right]$$

Regarding the first difference, $P - n^{\frac{-2}{\gamma-1}}$, the condition for it to be positive boils down to:

$$\gamma > \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}}$$

which we know is always true.

While the second difference, $P'^{\gamma-1} \frac{n^2}{n_D} - 1$, is positive if $n^{\frac{\gamma}{\gamma-1}} - n - n_D^{\frac{\gamma}{\gamma-1}} + n_D - \gamma^{-1} (n^{\frac{\gamma}{\gamma-1}} - 1) > 0$. Since the higher bound on n_D is $\frac{n}{2}$ and the lower bound on γ is 2, it is always respected.

Those two differences being positive, we know that $\frac{\partial \delta_{XP}}{\partial n_{D^*}} > 0$. ■

Proof of Equation (9)

The scheme of this proof follows the proof of equation 4, what gives:

$$(\gamma - n) - n^{\frac{-1}{\gamma-1}} (\gamma - 1) > 0$$

Taking alternatively the limit of γ towards 2 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

Proof of Proposition 3

As, in the limit case where $n_{D^*} = n_D$, the condition for cooperation to be sustainable becomes $\pi_i^C > \pi_i^N$, the proof of Proposition 3 boils down to the proof of Proposition 1. ■

Proof of Equation (15):

As the condition for one agent to cooperate is $\pi_i^C - \pi_i^N > 0$, the bigger the positive difference, the easier the cooperation. We can therefore compare equation (3)³⁸, with two different distributions of λ to see which one will be more favourable to cooperation. If λ_1 is compatible with cooperation such that equation (3) is verified then all the 'less compatible with cooperation' distributions of shares will produce a smaller result.

³⁸ $\pi_i^C > \pi_i^N \equiv \sum \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{1}{\gamma-1}}) - \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}}) > 0$

This being said, we can compare the different distributions of shares.

We compare equation (3) with $\underline{\lambda}$ and $\tilde{\lambda}$:

$$\begin{aligned} & (n-x)(\lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}}) + x \left[\left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{2}{\gamma-1}} \right] - \gamma^{-1} \lambda_1^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_1^{\frac{\gamma}{\gamma-1}}) \geq \\ & (n-x-1)(\lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}}) + (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} \\ & + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] - \gamma^{-1} \lambda_1^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_1^{\frac{\gamma}{\gamma-1}}) \end{aligned}$$

It simplifies to:

$$\begin{aligned} & \lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{2}{\gamma-1}} \right] \geq \\ & (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] \end{aligned}$$

As the second derivative of $\lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}})$ with respect to λ is equal to:

$$\frac{1}{(x-1)^2} \lambda^{\frac{3-2\gamma}{\gamma-1}} [2 - \gamma - 2(3-\gamma)\lambda^{\frac{1}{\gamma-1}}]$$

this function is concave if $2 - \gamma - 2(3-\gamma)\lambda^{\frac{1}{\gamma-1}} < 0$ (convex if > 0).

As $\lambda_1 < \lambda_2 < \dots < \lambda_n \Rightarrow \lambda_1 < \frac{1}{n}$ we get:

$$\frac{1 - (n-x)\lambda_1}{x} > \frac{1 - (n-x)\lambda_1 - \varepsilon}{x} > \lambda_1$$

This point and the concavity of the function $\lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}})$ when $2 \leq \gamma \leq 4$ allow us to state that, if $x = 1$, $\underline{\lambda}_1 < \tilde{\lambda}_1$. If ε grows and becomes $= \frac{1-n\lambda_1}{x+1}$,

$$\begin{aligned} & (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] = \\ & (x+1) \left[\left(\frac{1-(n-(x+1))\lambda_1}{x+1} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-(x+1))\lambda_1}{x+1} \right)^{\frac{2}{\gamma-1}} \right] \end{aligned}$$

The same reasoning shows therefore that $\underline{\lambda}_1 < \tilde{\lambda}_1 < \hat{\lambda}_1$. Then applying the same reasoning with $x = [2, \dots, n-1]$ gives our result when $2 \leq \gamma \leq 4$.

The reverse is true when $\gamma > 4$, and the function $\lambda^{\frac{1}{\gamma-1}} - \lambda^{\frac{2}{\gamma-1}}$ is convex, this being dependent on the shares (and therefore the number of players). ■

B Systems of equations

Extreme case:

$$(n-1)\lambda_{min}^{\frac{1}{\gamma-1}}(1-\lambda_{min}^{\frac{1}{\gamma-1}}) + (1-(n-1)\lambda_{min})^{\frac{1}{\gamma-1}}[1-(1-(n-1)\lambda_{min})^{\frac{1}{\gamma-1}}] - \gamma^{-1}\lambda_{min}^{\frac{\gamma-2}{\gamma-1}}(1-\lambda_{min}^{\frac{\gamma}{\gamma-1}}) > 0$$

$$\lambda_n(\sum \lambda^{\frac{2}{\gamma-1}} - \gamma^{-1}\lambda_n^{\frac{2}{\gamma-1}}) > (1-(n-1)\lambda_{min})[(1-(n-1)\lambda_{min})^{\frac{1}{\gamma-1}} + (n-1)\lambda_{min}^{\frac{1}{\gamma-1}} - \gamma^{-1}(1-(n-1)\lambda_{min})^{\frac{\gamma-2}{\gamma-1}}(1-(1-(n-1)\lambda_{min})^{\frac{\gamma}{\gamma-1}})]$$

General form:

$$\lambda_n(\sum_{n_C} \lambda^{\frac{1}{\gamma-1}} + \sum_{n_D} \lambda^{\frac{2}{\gamma-1}} - \gamma^{-1}\lambda^{\frac{2-\gamma}{\gamma-1}}) < (\lambda_n - \varepsilon) \left[\sum_{min, \dots, n \in n_C} \tilde{\lambda}^{\frac{1}{\gamma-1}} + \sum_{1, \dots, x \in n_D} \lambda^{\frac{2}{\gamma-1}} - \gamma^{-1}(\lambda_n - \varepsilon)^{\frac{2}{\gamma-1}} \right]$$

$$\lambda_{min} \equiv \pi^C = \pi^N \Rightarrow \sum_{n_C} \lambda^{\frac{1}{\gamma-1}}(1-\lambda^{\frac{1}{\gamma-1}}) - \gamma^{-1}\lambda_{min}^{\frac{\gamma-2}{\gamma-1}}(1-\lambda_{min}^{\frac{\gamma}{\gamma-1}}) = 0$$

C Conditions on α

In the general case:

$$\pi^C > \pi^S \Rightarrow \alpha < \left[\frac{\lambda_i^{\frac{\gamma-2}{\gamma-1}} \sum \lambda_i^{\frac{1}{\gamma-1}} - \gamma^{-1}}{1 - \gamma^{-1}} \right]^{\frac{\gamma-1}{\gamma}}$$

The latter expression is smaller than 1 if $\lambda_i < n^{-1}$ and $\gamma \geq 2$. It means that for cooperation to be attractive for a poor α has to be smaller than for a rich. If $\lambda_i > n^{-1}$, the cooperation payoff is always greater than the Nash payoff and, even if $\alpha > 1$, cooperation can, in some cases, be preferred to working individually.

The following condition is easier to satisfy than the previous one.

$$\pi^{C*} > \pi^S \Rightarrow \alpha < \left[\frac{\lambda_i^{\frac{\gamma-2}{\gamma-1}} \left(\sum_{n_C} \lambda_i^{\frac{1}{\gamma-1}} + \sum_{n_D} \lambda_i^{\frac{2}{\gamma-1}} - \gamma^{-1}\lambda_i^{\frac{2}{\gamma-1}} \right)}{1 - \gamma^{-1}} \right]^{\frac{\gamma-1}{\gamma}}$$

As expected, the following condition is more demanding than the previous one.

$$\pi^N > \pi^S \Rightarrow \alpha < \left[\frac{\lambda_i^{\frac{\gamma-2}{\gamma-1}} \left(\sum \lambda_i^{\frac{2}{\gamma-1}} - \gamma^{-1} \lambda_i^{\frac{2}{\gamma-1}} \right)}{1 - \gamma^{-1}} \right]^{\frac{\gamma-1}{\gamma}}$$

In the case of an equalitarian distribution, the three previous equations become:

$$\pi^C > \pi^S \Rightarrow \alpha < 1$$

$$\pi^{C*} > \pi^S \Rightarrow \alpha < \left[\frac{n_C n^{-1} + n^{\frac{-\gamma}{\gamma-1}} (n_D - \gamma^{-1})}{1 - \gamma^{-1}} \right]^{\frac{\gamma-1}{\gamma}}$$

$$\pi^N > \pi^S \Rightarrow \alpha < n^{\frac{-1}{\gamma}} \left[\frac{1 - \gamma^{-1} n^{-1}}{1 - \gamma^{-1}} \right]^{\frac{\gamma-1}{\gamma}}$$

$$\frac{\partial rhs}{\partial n} < 0, \frac{\partial rhs}{\partial \gamma} > 0$$

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