# Stata Technical Bulletin

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A publication to promote communication among Stata users

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Content

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## an1.1 STB categories and insert codes

Inserts in the STB are presently categorized as follows:

Gene	ral Categories:		
an	announcements	ip	instruction on programming
сс	communications & letters	OS	operating system, hardware, &
dm	data management		interprogram communication
dt	data sets	qs	questions and suggestions
gr	graphics	tt	teaching
in	instruction	ZZ	not elsewhere classified
Statis	stical Categories:		
sbe	biostatistics & epidemiology	srd	robust methods & statistical diagnostics
sed	exploratory data analysis	ssa	survival analysis
sg	general statistics	ssi	simulation & random numbers
smv	multivariate analysis	SSS	social science & psychometrics
snp	nonparametric methods	sts	time-series, econometrics
sqc	quality control	sxd	experimental design
sqv	analysis of qualitative variables	SZZ	not elsewhere classified

In addition, we have granted one other prefix, crc, to the manufacturers of Stata for their exclusive use.

an23	CPS labor extracts available

Daniel Feenberg, National Bureau of Economic Research, feenberg@nber.harvard.edu.

The National Bureau of Economic Research has recently prepared a CD-ROM diskette including a series of extracts covering 13 years (1979 through 1991) from the Current Population Survey Outgoing Rotation Group Annual Merge Files. These are presented as Stata binary files. The annual files include interviews for everyone in a CPS outgoing rotation group during a single calendar year, or about 300,000 observations per year. To keep file sizes within reason for Stata users, each year of data is divided into three files. Only a few minutes are required to load each file, however. It is possible to leave a do-file running that will process all 13 years of data in a few hours, without operator intervention!

The extracts contain information for respondents who were 16 or older. The 50 or so variables selected for the extracts relate to employment: hours worked, earnings, industry, occupation, education, unionization. The extracts also contain many background variables: age, sex, race, ethnicity, geographic location, etc. Every effort has been made to keep the variables consistent over all the years. Users should note, however, that unionization variables are available for 1983 and after, student enrollment status is available for 1984 and after, metropolitan/central city variables undergo several changes in 1985 (e.g., SMSA status becomes metropolitan status) and unedited variables may contain spurious data for not-in-universe observations.

Also included on the CD-ROM is the complete 1991 Merged Outgoing Rotation Group file, as supplied by the BLS, but converted to ASCII and with DOS end-of-line characters after each record. This file is 292,590,662 bytes.

To read the extracts, a user will need a computer capable of reading ISO 9660 disks (any drive for IBM-PC, Mac or Unix workstation should be fine), 16 megabytes of memory and, of course, a 32-bit Stata. The diskette itself is available for \$100 from:

Publications Department National Bureau of Economic Research 1050 Mass Ave Cambridge, MA 02138 Tel: 617-868-3900 Fax: 617-868-2754

Questions should be directed to feenberg@nber.harvard.edu. The 1991 extract is available for anonymous ftp from that address.

Variable list for 1991 extract:

Month in sample	Age
State	Marital status
Central city status	Race
MSA/PMSA FIPS code	Major activity last week
PMSA ranking	How many hours last week?
CMSA/MSA ranking	Reason $\leq = 35$ hours last week
MSA/CMSA size	Why absent from work last week?
CMSA code	3-digit industry code (1980)
Metropolitan status code	3-digit occupation code (1980)
Individual central city code	Class of worker 1
Household ID	Usual hours
Sex	Paid by the hour
Veteran	Union member
Highest grade attended	Ethnicity
Whether completed highest grade	Labor force status recode
What was doing most of last week	Full-time or part-time status
How many hours last all jobs	Detailed industry code
Usually works $>= 35$ hrs at this job	Detailed occupation code
Why not at least 35 hours last week	(Earnings) eligibility flag
Class of worker	Class of worker 2
Usual hours	Earnings per hour
Paid by the hour	Earnings per week
Earnings per hour	Final weight
Usual earnings per week	Earnings weight for all races
Union member	Usual hours (I25a) allocation flag
Covered by a union contract	Paid by hour (I25b) allocation flag
Enrolled student full/part time	Earnings/hr (I25c) allocation flag
Relationship to reference person	Usl Earn/hr (I25d) allocation flag

crc17 Bug fixes
-----------------

For the most up-to-date list of new features and bug fixes, type 'help whatsnew' after installing the CRC updates. Also remember, CRC updates are cumulative—if you have not installed past updates, it does not matter. The most recent updates include all past updates.

Fixed as of STB-7, but not reported due to publication deadlines, were the following:

- 1. (cc, cs, mcc in [5s] epitab.) Reported results were (obviously) incorrect when frequency weights were specified, some of the frequencies were 0, and the sum of the frequencies was 0 for one or more cells.
- 2. (kwallis in [5s] kwallis.) The if exp was ignored.

Fixed this time (or things to note) are

- 3. (qreg in [5s] qreg.) Despite the syntax diagram, qreg does not allow the no constant option.
- (codebook in crc13 of STB-8.) A variable containing only zeros and missing values would cause codebook to loop endlessly (until Break was pressed).

	crc18	Important difference between regular and Intercooled Stata	
--	-------	--	--

codebook (*crc13* in STB-8) emphasizes an important difference between the regular and Intercooled (which includes Unix) versions of Stata. The maximum number of variables that can be specified, either explicitly or implicitly, with codebook (and all other ado-commands) is 28 when running the regular version of Stata and 255 when running the Intercooled version. These maximums apply only to commands written as ado-files, not to Stata's built-in commands, and actually, both maximums are probably larger than 28 and 255. The difference between the two versions, however, is always present.

Let us explain: What is true is that the maximum length of a macro is 255 characters in regular Stata and 2,296 characters in Intercooled Stata. Among other things, ado-files use macros to store the names of the variables you specify or imply. The names are unabbreviated and a single blank is inserted between the names. The maximum length of a variable name in Stata is 8 characters, meaning that if all variable names were 8-characters long, a single macro could hold  $\lfloor 255/9 \rfloor = 28$  names in regular Stata and  $\lfloor 2296/9 \rfloor = 255$  names in Intercooled. ( $\lfloor x \rfloor$  refers to the largest integer  $k \le x$  and is called the floor of x.) It is unlikely that all variables are exactly 8-characters long. If variable names averaged, say, 4 characters, a macro could hold  $\lfloor 255/5 \rfloor = 51$  in regular Stata and  $\lfloor 2296/5 \rfloor = 459$  in Intercooled.

Now consider what happens when you type 'codebook' with no arguments, which is the same, as far as Stata is concerned, as typing 'codebook' followed by every variable in the data. If you are using the regular version of Stata, all those variables names, with the intervening blank, must fit into a single macro—which is to say, 255 characters. If they do not, you will get a too-many-variables error. For Intercooled Stata users, the rule is less restrictive but still present—the variable names must fit into 2,296 characters.

In either case, if you get the too-many-variables error, you can avoid the problem by explicitly specifying a subset of the variables and issuing the command multiple times. In the case of codebook, however, if you do run into this difficulty, you must also not specify the mv option. The mv option searches the data for patterns of missing values and, in the process, must make a list of all the variable names it is to search over. Thus, typing 'codebook myvar, mv' can also result in the too-many-variables error.

Regular's Stata 255-character maximum is arguably too small, but there is nothing we can do about it; regular Stata is designed to run on small computers with little memory. For Intercooled Stata users, we admit that even the 2,296-character maximum is unfortunately small; it will be increased in the next release.

crc19 Nonlinear regression command
------------------------------------

Royston's nonlinear regression command nl (Royston 1992a, b) is now an official part of the CRC updates. This means (1) the command and help files will automatically be installed when you install these updates or any future updates; (2) nl will be a part of the next release of Stata with documentation printed in the *Reference Manual*; and (3) nl is now supported by us. In the meantime, for printed documentation on this command, see Royston (1992a).

#### References

Royston, P. 1992a. sg1.2: Nonlinear regression command. Stata Technical Bulletin 7: 11-18.

-----. 1992b. sg1.3: Nonlinear regression command: bug fix. Stata Technical Bulletin 8: 12.

dm10	Infiling data: Automatic dictionary creation	
		William Gould, CRC, FAX 310-393-7551

Reading raw data into Stata (or any statistical package) is probably one of the most difficult tasks facing a researcher. On the STB-9 media, I provide a utility to examine formatted, raw data and automatically create a Stata .dct dictionary for reading it and, moreover, for reading it efficiently in that variables are given appropriate storage types.

This utility is in the form of an .exe file for DOS users but, for all users, source code—including a makefile—is provided. Unix users can easily create their own executable by typing make. Details of installation are explained at the end of this insert.

## creatdct

creatdct is a command you issue from your operating system, not Stata. Typing creatdct without arguments presents a syntax diagram:

```
C:\XMPL> creatdct
creatdct: usage: creatdct [/b /d /t] infile > outfile
          Reads data with variable names on top and writes to standard out
          a Stata .dct dictionary that can read the data.
          Options:
            /b treat blank lines as significant;
            /d write only the dictionary, referring to rather than including
                the raw data in the output;
            /t infile contains data only; no titles.
          Limits:
            Maximum width of input file:
                                            1000
            Maximum number of variables:
                                             254
          References:
            STB-9: dm10 (which includes source code)
```

(Under Unix, options are preceded by a '-' rather than '/' and, if run from Unix, the syntax diagram would reflect that fact.)

To understand what creatdct does, consider the following (small) data set:

mydta.raw						
Make and Model	Price	MPG	Weight	Repair		
AMC Concord	4099	2.2e+1	2930	average		
"BMW 320"	9735	25	2650	good		
Honda Civic	4499	28	1760	exc		
Volvo 260	11995	17	3170	exc		
end of file						

In this data, each line represents an observation and a title line appears above the data. The file could also contain blank lines (although this one does not). To create a dictionary for this data, type

C:\XMPL> creatdct mydta.raw > mydta.dct

The new file mydta.dct contains

----- mydta.dct ----dictionary { \* This file was created by creatdct from mydta.raw. \* Type "infile using <this file>" to read the data. \_column(1) str11 make\_an %11s "Make and Model" \_column(25) int price byte mpg %5f "Price" %6f "MPG" \_column(33) \_column(41) int weight %4f "Weight" \_column(49) str7 repair %7s "Repair" } AMC Concord 4099 2.2e+1 2930 average 9735 "BMW 320" 25 2650 good Honda Civic 4499 28 1760 exc 3170 Volvo 260 11995 17 exc ----- end of file -----

Thus, the entire Stata session to read the data might be:

. !creatdct mydta.raw > mydta.dct

. infile using mydta.dct

And, to prove that the data read correctly:

. describe							
Contains data							
Obs: 4 (max	= 5119)						
Vars: 5 (max							
Width: 23 (max	= 200)						
1. make_an	str11	%11s		Make	and Model		
<ol><li>price</li></ol>	int	%8.0g		Price			
3. mpg	byte	%8.0g	g MPG				
4. weight	int	%8.0g Weight					
5. repair	str7	%9s		Repair			
Sorted by:							
. list	. list						
make_an	pric	е	mpg	weight	repair		
1. AMC Concord	409	9	22	2930	average		
2. BMW 320	973	5	25	2650	good		
3. Honda Civic	449	9	28	1760	exc		
4. Volvo 260	1199	5	17	3170	exc		

## Variations

By default, creatdct creates a new file containing the dictionary and the data, but you can prevent creatdct from making a second copy of the data by specifying the /d (-d, Unix) option:

C:\XMPL> creatdct /d mydta.raw > dctonly.dct

The new file dctonly.dct contains

```
----- dctonly.dct ------
dictionary using mydta.raw {
* This file was created by creatdct from mydta.raw.
* Type "infile using <this file> in 2/1" to read the data.
                str11 make_an %11s "Make and Model"
 _column(1)
                       price
                               %5f "Price"
 _column(25)
                int
                                %6f "MPG"
 _column(33)
                byte
                       mpg
                                %4f "Weight"
  column(41)
                int
                       weight
  _column(49)
                               %7s "Repair"
                str7
                       repair
}
                          ----- end of file -----
```

As the comment at the top of the file instructs, typing 'infile dctonly in 2/1' will read the data set. The /d option is most useful when dealing with large amounts of data. (Do not assume that you always type the modifier "in 2/1"; creatdct modifies the recommended command according to the actual line containing the first observation of the data. From inside Stata, you can use the type command to display the dictionary on your screen and so read the recommendation.)

#### **Basic logic**

The above two examples illustrate how creatdct is typically used. In particular, creatdct assumes:

- 1. Blank lines can occur any place in the file; their presence is irrelevant.
- 2. The first non-blank line is a title line; it is from that line that variable names and labels are to be derived.
- 3. After the title lines, remaining nonblank lines contain the data. Each line contains all the data for a single observation.

Assumptions 1 and 2 can be relaxed by specifying options, but assumption 3 must be true. The /b (-b, Unix) option relaxes the first assumption, but there is never any reason to specify it. The /t (-t, Unix) option relaxes the second assumption.

#### Data without titles

Small ASCII data sets often include a title line, but large data sets generally do not. Large data sets, however, at least tend to be formatted. creatdct can be used to automatically construct a dictionary for such data sets assuming (1) each observation occupies one line and (2) the values recorded in the data do not "run together"—there is white space separating the columns of the data. Consider mydta.raw without the title line:

mydta2.raw						
AMC Concord	4099	2.2e+1	2930	average		
"BMW 320"	9735	25	2650	good		
Honda Civic	4499	28	1760	exc		
Volvo 260	11995	17	3170	exc		
		end o	f file			

Typing 'creatdct /t mydta2.raw > mydta2.dct'—the /t indicates that the data does not include a title line—results in the file:

```
----- mydta2.dct -----
dictionary {
* This file was created by creatdct from mydta2.raw.
*
 Type "infile using <this file>" to read the data.
  _column(1)
                 str11 v1
                                %11s
  _column(25)
                 int
                       v2
                                %5f
                                %6f
  _column(33)
                byte
                       v3
  column(41)
                int
                       v4
                                %4f
  _column(49)
                str7
                       v5
                                %7s
3
AMC Concord
                       4099
                              2.2e+1
                                     2930
                                             average
"BMW 320"
                       9735
                                     2650
                              25
                                             good
                       4499
                              28
                                     1760
Honda Civic
                                             exc
Volvo 260
                      11995
                              17
                                     3170
                                             exc
                      ----- end of file -----
```

When there is no title line, creatdct uses names like v1, v2, etc. The /d option can be combined with the /t option; typing 'creatdct /t /d mydta2.raw > mydta2.dct' results in a file that references, rather than contains, the underlying data. This can be especially useful when the data is large because then you can edit the dictionary and change the names to more reasonable ones.

### Data with "errors"

creatdct tries to deal with problems that may crop up in real applications. Consider:

mydta3.raw							
Make and Model	Price	MPG	Weight	Repair	Repair		
AMC Concord	4099	2.2e+1	2930	average	fair		
BMW 320	9735	25	2650	good	good		
Honda Civic	4499	28	1760	exc	good		
Volvo 260	11995	17	3170	exc	average		
end of file							

In this data, the title Repair occurs twice (the first time for repair record in 1978 and the second for 1977, although there is no way you could know that from the data). Typing 'creatdct /d mydta3.raw > mydta3.dct' results in

```
----- mydta3.dct -----
dictionary using mydta3.raw {
* This file was created by creatdct from mydta3.raw.
* Type "infile using <this file> in 2/1" to read the data.
 _column(1)
              str11 make_an %11s "Make and Model"
              int price
                           %5f "Price"
 column(25)
                           %6f "MPG"
 _column(33)
            byte
                   mpg
 _column(41)
              int
                   weight
                           %4f "Weight"
 _column(49)
                           %7s "Repair"
              str7
                   v 5
                           %7s "Repair"
 _column(60)
              str7 repair
}
           ------ end of file ------
```

creatdct changed one of the duplicate names to v5. creatdct would also change variable names to names like v5 if the column header was not a legal Stata variable name—for instance, if the column header were "78 repair".

One of the hardest problems for creatdct is to match the column header to the data—much of its code is dedicated to that problem and still, it is not always successful. creatdct, however, knows when it has problems and tries to behave reasonably if not as elegantly. Consider the raw data:

			mydt	a4.raw -		
Make and Model	Х	Price	MPG	Weight	Repair	Repair
AMC Concord		4099	2.2e+1	2930	average	fair
BMW 320		9735	25	2650	good	good
Honda Civic		4499	28	1760	exc	good
Volvo 260		11995	17	3170	exc	average
			end o	f file -		

Notice the extra "X" in the title line. Is it part of "Make and Model"; is it part of "Price"; or is it all by itself, representing a variable for which there is no data? Typing 'creatdct /d mydta4.raw > mydta4.dct' creates

```
----- mydta4.raw -----
dictionary using mydta4.raw {
* This file was created by creatdct from mydta4.raw.
* Type "infile using <this file> in 2/1" to read the data.
* Note from creatdct: I had trouble lining up the titles against the data.
* Due to blanks in the titles, it appeared that there were more columns than
* columns in the data. I finally gave up and just used the parts of the
* headers that were directly above data. Sorry.
               str11 make_an %11s "Make and Mo"
 _column(1)
                             %5f "Price"
 _column(25)
               int
                     price
                             %6f "MPG"
 _column(33)
               byte
                     mpg
                             %4f "Weig"
 _column(41)
               int
                     weig
                             %7s "Repair"
 _column(49)
               str7
                     v 5
                     repair
                             %7s "Repair"
 _column(60)
               str7
'}
         ----- end of file -----
```

All of creatdct's complicated logic for extending the titles around the data columns failed it, so it resorted to a dumber rule—taking just the part of the titles that lie directly above the data. Notice that the weight variable is now simply called weig.

#### Installing creatdct

For DOS users, simply copy creatdct.exe to a directory that is in your DOS path such as c:\dos or c:\stata. Although c:\stata seems more appealing, c:\dos is probably a better idea. If you put it in c:\stata, the next time we update Stata, you will have to reinstall creatdct.exe.

For Unix users, note that the C source code is provided. Copy the source to any temporary directory and, with the temporary directory as your current directory, type 'make creatdct'. This will produce the executable creatdct which you can copy to some place along your path.

Actually, the C source code is provided for all users, although it is setup to compile under Unix, which means only that the option switch character has been set to '-' rather than '/'. DOS users can edit the file machdep.h and reset the manifest ALTSWITCHAR—the change is obvious once you are looking at the file. The limits of a maximum file width of 1,000 characters and a maximum number of variables of 254 are also set in machdep.h. You can change these limits—much larger limits are no problem under Unix. DOS users are warned, however, that the size of buffers is determined by these limits and that buffers are allocated on the stack.

#### **Request for comments**

If you have a data set on which creatdct fails or, while in the spirit of the files processed by creatdct, does not exactly match the rules, please contact me. Our goal is to further refine creatdct through the STB and then, once it is well debugged, to include its logic in the Stata infile command, offering users a way of reading formatted data without the bothersome step of creating a dictionary.

Users interested in further automating the infile process might consider typing in the following ado-file and saving it in their ado directory (c:\ado under DOS or "/ado under Unix):

Typing 'autoin mydta.raw', for instance, would then read the data in the first example.

I do not include this ado-file among the creatdct materials because I cannot believe that creatdct is sufficiently robust that you should use it without at least looking at the dictionary it produces first, but you are welcome to use the autoin command if you wish. At some future date, creatdct should be sufficiently robust that a more elegant version of autoin will be justified.

gr11 Using CorelDraw as a Stata graphics editor
---

Marc Jacobs, Dept. of Sociology, Univ. of Utrecht, The Netherlands, FAX (011)-31-30 534405

After some work I have discovered a way to import Stata graphics into CorelDraw. It is still necessary to put some work into it.

1. A Stata graphic (boosted up by Stage or not) is translated into a Lotus pic file. For this, I use a DOS BAT file containing the single line:

c:\stata\gphpen %1 /DC:\stata\pic.pen /oc:\data\%1.pic /n %2 %3 %4 %5

Calling this file pic.bat, I can type 'pic filename' to create the file filename.pic in my \data directory.

- CorelDraw is started and the pic file is imported. ALT-F(ile), I(mport), choose LOTUS PIC file format and pick the proper file. The pic file is imported, but anything, including text, is seen as several objects in one group. There can easily be 512 objects in one group.
- 3. Ungroup the objects first: ALT-A(rrange), U(ngroup). In most cases the graphic is too small for comfort, or the conversion of the lines is not satisfying. Before editing, change the line size to 1 mm and black: Choose the PENCIL icon, then the 1 MM LINE icon, and finally the BLACK icon, while the group object still is chosen.

Stata text is made of plotter symbols. It is better to remove the text and replace them by the fancier fonts of CorelDraw. The only way to do this is to delete the group of objects forming one word of sentence and to replace them by the CorelDraw text possibilities.

As general advise: Put guide lines (blue lines pulled out of the rulers) first, where the old Stata text is, before deleting it. Put snap to guide lines on by choosing ALT-D(isplay), G(uide lines).

Suggestions for improvement are welcome.

ip2 A keyboard shortcut
-------------------------

Peter A. Lachenbruch, Dept. of Biostatistics, UCLA

I am providing a neat trick that other Stata users may already know, but if not it's a stroke saver. Since the missing value code '.' is greater than any nonmissing value, we can use that to exclude cases in a simple manner. Suppose I want to compute the means for X for those cases in which Y is not missing. The usual way is to write

summ X if Y~=.

I find it easier to write

summ X if Y<.

The '<' key is easier for me to reach, and only takes one stroke.

sbe7 Hyperbolic regression analysis in biomedical applications
--

Paul Geiger, USC School of Medicine, pgeiger@vm.usc.edu

Hyperbolic regression fits a curve to experimentally obtained data points for many analyses used in biochemistry and molecular biology (Studnicka 1987, 1991). These areas include enzyme kinetics, agarose gel electrophoresis of DNA fragments, SDS-polyacrylamide gel electrophoresis of proteins, enzyme-linked immunosorbent assays (ELISA), radioimmunoassays (RIA), Bradford assays (protein), and the much used and cited Lowry protein assay.

Hyperbolic regression fits curves without the biases linear regression introduces to an equation transformed to double reciprocal coordinates (Lineweaver and Burk 1934). More complicated transforms like the logit-log or four-parameter methods (Rodbard et al. 1987; Geiger 1991, 1992) may also be unnecessary when a hyperbolic relationship between variables exists. Hyperbolic regression, like linear regression, has the additional advantage of yielding one simple equation with its estimated constants. This equation can model actual chemical and biological processes.

Fitting curves with the cubic spline or the model-free approach of Guardabasso et al. (1987) produces no single equation to describe the scientific phenomena meaningfully. Results can also differ from analysis to analysis.

Studnicka (1987) implemented an algorithm for hyperbolic regression on a Digital Equipment Corporation PDP-11 series computer and later (1991) designed a spreadsheet to carry out the process. An outline of the mathematics follows.

A general equation for a hyperbola can be written as

$$(Y - Y_o)(X - X_o) = C_o$$

where  $(X_o, Y_o)$  is the mathematical origin displaced from (0, 0) and  $C_o$  is a constant. Rearranging the equation by multiplying and collecting the constants and defining a new constant,  $W_o = C_o - X_o Y_o$ , gives:

$$XY = X_oY + XY_o + W_o$$

Following Bevington (1969) least squares was satisfied by minimizing:

$$\sum_{i=1}^{N} (X_i Y_i - X_o Y_i - X_i Y_o - W_o)^2$$

This can be solved using Stata's linear regression where  $X_o$ ,  $Y_o$ , and  $W_o$  are the coefficients.

The above method of fitting works only if curvature really exists. A perfectly linear data set is unacceptable as the determinants of the matrices all go to zero. Also Studnicka (1987) remarks that if each data point consists of a number of experimental estimates, the mean must be calculated and used in the equations to get the best fit. This characteristic is a result of the matrix algebra. I have not observed this behavior.

The above equations have been incorporated into a Stata program, hbolic.ado, that calculates the necessary  $X_o$ ,  $Y_o$  and  $C_o$  and the fitted curve, hat. This program, together with a help file and an example data set, dnafrag2.dta, are provided on the STB diskette.

The syntax of hbolic is

hbolic depvar indepvar

It should be noted that Xo, Yo, and Co are calculated constants that can be renamed according to the application. For instance, name Km for Xo, Vmax for Yo, and KmVmax for Co if you are concerned with Michaelis-Menten kinetics. The variable Yhat now denotes the fitted regression line and resids the residuals. The program displays the regression curve fitted to the original data points. It also displays the residuals plotted against Yhat. These graphs are saved for printing as gph1 and gph2. Finally, the contents of the completed file are shown on the screen with describe followed by list Yhat depvar indepvar Xo Yo Co and a message inviting the user to view the saved graphs.

Unlike Studnicka's program devised for the DEC PDP-11 (1987), I found hbolic.ado works with several Y values (replicates) for each X value (standard). A mean for the Ys need not be computed before running hbolic.

Three examples of the use of hyperbolic regression with hbolic follow.

**Example 1.** Michaelis-Menten kinetics model many enzymic and other biological processes. In studies of an enzyme, the two desired characteristic parameters are the maximum velocity,  $V_m$ , and the Michaelis constant,  $K_m$ .  $V_m$  reflects the maximum velocity of the reaction, usually in micromoles per minute, at substrate saturation.  $K_m$  is the binding constant for substrate with enzyme in units of micromoles. The Michaelis-Menten equation is usually written:

$$V_o = \frac{V_m[S]}{K_m + [S]}$$

Initial velocity is half-maximal when  $[S] = K_m$ . The Lineweaver-Burk (1934) rearrangement is the so-called double reciprocal form, and  $1/V_o$  was plotted against 1/[S] in a graphical analysis approach necessary before the advent of computers. But the original equation is easily rearranged to make its hyperbolic form evident:  $(V_o - V_m)([S] + K_m) = -K_m V_m$ .

Studnicka (1987) used the values in the file michment.dta (taken from his paper and supplied on the disk) to show strengths and weaknesses in the hyperbolic solution versus the double reciprocal plot to which he applied linear regression. The file contains the model values from his demonstration.

Estimation errors are less likely to occur using the hyperbolic form. This form is most sensitive to changes at high [S] values where measurement errors of initial velocity and [S] occur less often. You can see how small changes to  $V_o$  or [S] affect estimated  $V_m$  and  $K_m$  values ( $Y_o$  and  $X_o$  produced by hbolic). Simply run hbolic after changing one value of X or Y. If the highest value of  $V_o$  changes by  $\pm 10\%$ ,  $K_m$  changes a whopping  $\pm 50\%$  and the  $V_m$  by about  $\pm 15\%$ .

**Example 2.** Studnicka (1987) also applied hyperbolic regression to DNA fragment analysis. Data taken from Figure 2 in the 1987 paper referring to an agarose gel electrophoresis experiment are in the file dnafrag.dta supplied on the disk. hbolic generates the constants,  $X_o$ ,  $Y_o$ ,  $C_o$ , after entering the values for X, the known DNA fragment sizes in kilobasepairs (kB), and Y, the measured migration distances in mm.

Measuring the migration distances accurately in this kind of experiment is the critical aspect and most subject to error. Compute the sizes of your own unknowns in kB by giving the command 'generate sizes=(Co/(unk-Yo))+Xo', solving the hyperbolic equation for X ("unk" not provided here). In this equation "unk" stands for the migration distances of unknowns, measured in mm. Alternatively, use infile X Y unk to import a complete ASCII data file containing X, Y and "unk". If you use hbolic.ado frequently for many such analyses, append the necessary command to solve for sizes of fragments.

**Example 3.** This sample radioimmunoassay (RIA) data comes from Sundqvist et al. (1989) and appears in Geiger (1991, 1992). Geiger (1991) demonstrates the overworked logit-log analysis and Geiger (1992) the four-parameter model solved with Danuso's nonlinear regression program (Danuso 1991), an iterative procedure.

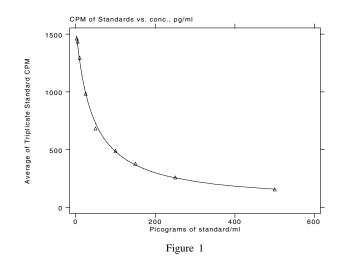
Application of hyperbolic regression to radioimmunoassay data seems every bit as effective as these more complicated methods. Furthermore, the blank or nonspecific binding cpms need not be subtracted as when performing logit-log analysis. The included file, hypria.dta, reproduces the RIA data from the previous publications.

The original triplicate cpm data were averaged with the command 'egen Y=rmean(c1 c2 c3)'. hbolic then generated the necessary constants. Applying the equation in Example 2 above provided the results (pg/ml) from the cpms of the unknowns after correcting for volume as in the previous reports.

With the file hypria.dta in Stata's memory, type 'describe' to see the labeled variables. Enter 'list smpl ans \_2ans' to see Sundqvist's original sample identification numbers and the logit-log result compared to the hyperbolic regression result, \_2answer. The comparison is

	Result (pg/ml)	Result (pg/ml)
Sample ID	logit-log calc.	Hyperbolic regression
11772	1567.16	1557.26
11772	1544.25	1534.88
11772	1781.31	1788.14
11772	1767.13	1774.17
11773	1332.42	1344.80
11773	1300.03	1295.99
11774	1074.86	1075.22
11774	1167.37	1181.08
11774	1193.03	1206.56
11774	1032.11	1033.22
11775	1517.34	1508.58
11775	1557.94	1548.26
11775	1748.40	1755.71
11775	1805.22	1811.69
11776	1292.76	1288.87
11776	1343.17	1355.45
11777	1029.33	1030.49
11777	1209.31	1222.73
11778	1486.53	1497.30
11778	1512.91	1504.25
11779	1415.52	1409.01
11779	1360.10	1354.80
11779	1506.45	1516.98
11779	1526.62	1536.91
11780	927.03	929.89
11780	953.58	968.24
11780	986.39	1000.97

Of course, the hyperbolic regression technique still requires investigators to obtain good duplicate and preferably triplicate observations of their experimental data.



#### References

Bevington, P. R. 1969. Data Reduction and Error Analysis for the Physical Sciences. New York: McGraw-Hill, 92-163.

Danuso, F. 1991. sg1: Nonlinear regression command. Stata Technical Bulletin 1: 17-19.

Geiger, P. J. 1991. sbe3: Biomedical analysis with Stata: radioimmunoassay calculations. Stata Technical Bulletin 3: 12-15.

-----. 1992. sbe4: Further aspects of RIA analysis. Stata Technical Bulletin 5: 7-10.

Guardabasso, V., D. Rodbard, and P. J. Munson. 1987. A model-free approach to estimation of relative potency in dose-response curve analysis. Am. J. Physiol 252(15): E357-E364.

Lineweaver, H. and D. Burk. 1934. The determination of enzyme dissociation constants. J. Am. Chem. Soc. 56: 658-666.

Rodbard, D., et al. 1987. Statistical Aspects of Radioimmunoassay. In Radioimmunoassay in Basic and Clinical Pharmacology, vol. 82 of Handbook of Experimental Pharmacology, ed. C. Patrono and B. A. Peskar, ch. 8. New York: Springer-Verlag.

Studnicka, G. M. 1987. Hyperbolic regression analysis for kinetics, electrophoresis, ELISA, RIA, Bradford, Lowry, and other applications. CABIOS 3: 9–16.

-----. 1991. ELISA assay optimization using hyperbolic regression. CABIOS 7: 217-224.

Sundqvist, C., et al. 1989. A radioimmunoassay program for Lotus 1-2-3. Comput. Biol. Med. 19: 145-150.

sbe8 Left-censored survival data
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#### William Rogers, CRC, FAX 310-393-7551

Left-censored survival data arise when a subject comes under observation some time after the event that starts the "clock." For example, suppose you are studying factors that predict whether a woman will give birth to a child. You sample women, give them a baseline survey, and then follow them annually for 6 years thereafter. At the start of the study, each woman has been at risk since her last birth, or since menarche.

Suppose that woman A in this sample had her last child 3.2 years ago, and she has a child 2.4 years after your baseline survey. You would probably want to analyze the birth interval of 5.6 years. The hitch is that you could not have observed an interval of 5.6 years for this woman unless she had gone at least 3.2 years from her last birth without having another child. In other words, you want to analyze the probability of surviving from 3.2 years to 5.6 years and then giving birth.

Actually, most analysts are not interested in the probability per se, but rather the relative risk of giving birth for a woman with one set of characteristics as opposed to another. For example, what is the relative risk of giving birth for an unmarried woman as opposed to a married one?

There are several ways to handle this kind of problem in Stata, of which I will discuss two here.

**Method 1:** Use exponential instead of Cox regression. In exponential regression, the likelihood is completely specified and the hazard does not depend on previous time at risk. There is, however, a price. It now becomes your responsibility to verify that the likelihood does not depend on time since last birth or to specify the dependence if there is one.

In this example, there is a dependence. It is almost impossible to have another child within 11 months, and the likelihood is low immediately after that. So you might break the time into intervals and introduce variables for 11–16 months, 17–24 months 25–36 months, etc., and replicate the observation on each woman up to the point she has another child or no longer appears in the data. This replication, resulting in a dataset where the same woman appears more than once, could potentially result in a dataset where the outcomes are dependent on each other. A solution to this problem is "Huber" exponential regression.

Method 2: Use time-varying Cox regression and introduce a variable that describes the left-censoring period.

The following example shows how to do the data preparation for each method. They are both tricky. Let me begin by concocting a data set:

	input	t id	age	previous	tbirth	isbirth
		1	24	10	5	1
		2	19	2	2	1
		3	42	2	6	0
		4	36	6	6	0
		5	30	2	4	1
		6	32	18	6	1
		7	38	14	6	0
		8	28	6	3	1
		9	22	3	1	1
er	ıd					
•	save	bir	th, r	eplace		

previous records the number of years at the time of the baseline survey since the previous birth (or menarche). tbirth is the time from the baseline survey to the next birth or end of the survey period. isbirth records 1 if the woman had another (or first) birth during the survey period and 0 otherwise.

The exponential-regression solution relies on the fact that the probability distribution of tbirth is the same as the distribution of tbirth + previous given previous:

```
. use birth, clear
. expand 2
. sort id
. qui by id: gen record = _n
. gen over5 = record==2
. drop if record==1 & previous>=5
. gen atrisk = min(tbirth,5-previous) if record==1
. replace isbirth=0 if record==1 & atrisk<tbirth
. drop if record==2 & isbirth[_n-1]==1 & id==id[_n-1]
. replace atrisk = tbirth-atrisk[_n-1] if record==2 & id==id[_n-1]
. replace atrisk = tbirth if record==2 & id~=id[_n-1]
. hereg atrisk age over5, dead(isbirth) group(id) hr
```

Each woman in the original data potentially becomes two observations, the first reflecting the first five years (from the last child or menarche) and the second the remaining period. Since the same woman may appear in the data more than once, we use hereg (see sg8.1 in this issue) to correct for the violation of independent-observations assumption.

The second approach, using Cox regression, is easier. tbirth + previous is used as the time-to-birth variable, but we must create a censored segment for the times between 0 and previous:

. use birth, clear . expand 2 . sort id . qui by id: gen record = \_n . gen atrisk = tbirth+previous if record==2 . replace atrisk = previous if record==1 . gen left = (record==1) . replace isbirth = 0 if record==1 . cox atrisk age left, dead(isbirth) tvid(id) hr

The output produced by the two estimation steps is

```
. hereg atrisk age over5, dead(isbirth) group(id) hr
Iteration 0: Log Likelihood = -12.037857
Iteration 1: Log Likelihood = -10.206155
Iteration 2: Log Likelihood = -8.9897404
Iteration 3: Log Likelihood = -8.8922672
Iteration 4: Log Likelihood = -8.8907967
Exponential regression (log relative hazard form) Number of obs
                                                      =
                                                            11
Log Likelihood
                     = -8.891
                                         Pseudo R2
                                                      = 0.2614
Grouping variable: id
                                  _____
 atrisk | Hz. Ratio Std. Err. t P>|t| [95% Conf. Interval]
-----
  age | .8403259 .0352805 -4.144 0.003
over5 | 1.514651 1.063751 0.591 0.571
                                         .7627832
                                                       .9257515
                                             .2998902 7.650031
    _____
             _____
                      _____
```

. cox atrisk age left, dead(isbirth) tvid(id) hr

Iteration 0: Log Likelihood =-9.0484095
Iteration 1: Log Likelihood =-2.8854591
Iteration 2: Log Likelihood =-2.1052036
 (output omitted)
Iteration 44: Log Likelihood =-1.7583729
Iteration 45: Log Likelihood =-1.7583729

Cox regres	ssion				Number of obs chi2(2)	= 14.58
Log Likel:	ihood = -1.758	33729			Prob > chi2 Pseudo R2	= 0.0007 = 0.8057
	Haz. Ratio		t	P> t	[95% Conf.	Interval]
age left	.7223358 3.37e-21	.1759505	-1.335	0.199	.4320623	1.207624

For the exponential regression, the nonsignificant coefficient for over5 suggests that previous time exposed is not an important factor in the hazard rate.

For the Cox regression, the impact of the variable left is huge. Since this regression is presented in hazard form, the hazard ratio of 3.37e–21 reflects the fact that we observed no failures in the left-censoring period. The long iteration log is characteristic of a parameter that is being driven to infinity. Stata recognizes that something is wrong with this variable and prints a '.' for the standard error. In some cases you will get a zero t-statistic, which is also a sign of trouble.

Although both hazard ratios for age are well below 1, the two methods give slightly different hazard ratios. More importantly, the t-statistics are quite different. There are two reasons. First, the parametric information typically improves precision. Second, the hereg estimates are based on asymptotic results and overstate accuracy in very small samples.

sg8.1	Huber exponential regression

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After my insert on probability weighting (Rogers 1992), I have received a number of questions asking what to do about design problems (probability weighting and clustering) with survival analysis, most often in reference to Cox regression.

Unfortunately, the Huber method that worked for regress and logit (yielding the Stata hreg and hlogit commands) does not work well in the case of Cox regression because the conditional likelihood derivatives for each observation are a complicated function of the entire dataset. It is, however, easy to apply the Huber method to exponential regression, which is a close cousin of Cox proportional hazards model.

hereg has the syntax:

hereg [depvar [varlist] [weight] [if exp] [in range] ] [, hazard hr tr dead(varname) level(#) group(varname) maximize\_options ]

See [5s] huber for a description of the treatment of the optional weight and group() option; see [5s] ereg for a description of the remaining options.

In survival analysis, violations of assumptions can arise both at the sampling stage and be purposefully induced by the researcher at the analysis stage. In *sbe8* of this issue, I demonstrate a purposeful induction to estimate a model with both left and right censoring. hereg handles the problems associated with the violations of assumptions.

## References

Rogers, W. H. 1992. Probability weighting. Stata Technical Bulletin 8: 15-17.

sg9	Similarity coefficients for 2 x 2 binary data
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Binary similarity measures estimate the proximity between two 1/0 binary variables. Three types of measures are provided: ones that can be thought of as similar to a correlation coefficient, others that can be interpreted as conditional probabilities, and lastly those that are predictability measures. I have provided a program called similari which displays twelve such similarity coefficients. The program is called similari and not similar because it is an immediate command (see [4] immediate); one need only input the tabulated summary data on the command line. The syntax of similari is

similari  $A_{0,0} B_{0,1} C_{1,0} D_{1,1}$ 

Hence, you may directly type in the summary data from the Stata tabulate command.

The following statistics are provided:

- 1. Czekanowski (Dice): A matching coefficients measure in which double weights are given to matches (1,1).
- 2. Dispersion: A similarity measure that ranges from -1 to 1.
- 3. Jaccard: A similarity ratio in which 0,0 is excluded from the equation.
- 4. Match percent: The ratio of total matches to the total population.
- 5. Ochiai: A similarity measure in cosine form.
- 6. Phi 4-point: A binary form of the Pearson product correlation coefficient.
- 7. Russell & Rao: a binary dot product; 1,1 matches to total population.
- 8. Hamann: A conditional probability measure ranging in value from -1 to 1.
- 9. Anderberg's D: A predictability measure indicating the reduction in the probability of error when an item is used to predict another.
- 10. Goodman and Kruskal's Lambda: Indicates the proportional reduction in the probability of error when one item is used to predict another when the prediction directions are equal. The predictability of the value of one item given the value of another.
- 11. Yule's Q: A binary version of the Gamma test ranging from -1 to 1.
- 12. Yule's Y: A coefficient of colligation ranging from -1 to 1.

All coefficients range from 0 to 1 unless otherwise indicated. An example program run follows:

. use lbw					
. tab smoke low	r				
smoked  bi during	rth weight<	2500g			
pregnancy	0	1			
0	86	29	115		
	44		74		
Total	130	59	189		
. similari 86 2 Simi	larity coef		or 2 X 2 bi	nary data	
_		Controls			
Cases	0 +		1	Total	
0	86		29	115	
1	44		30	74	
Total	. 130		59	189	
Proximity m	easures		Conditi	onal probability measu	re
Czekanowski =	0.4511		Hamann	= 0.22	75
Dispersion =					
Jaccard =			Pred	lictability measures	
Match % =				g's D = 0.00	
Ochiai =				umbda = 0.00	
Phi 4-point =				= 0.33	
Russell & Rao =	0.1587		Yules Y	(colligation)= 0.17	42

Each listed statistic accords with that produced by the proximity command in SPSS for Windows.

## References

Anderberg, M. R. 1973. Cluster Analysis for Applications. New York: Academic Press.

Romesburg, H. C. 1984. *Cluster Analysis for Researchers*. Belmont, CA: Lifetime Learning Publications. SPSS. 1991. *SPSS Statistical Algorithms*. 2d ed. Chicago: SPSS.

sg10	Confidence limits in bivariate linear regression
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[The method discussed in this insert is often described as the "calibration method."-Ed.]

Many analytical methods in chemistry and biology provide a linear relationship between the response variable, Y, and the amount of material determined, X. We construct a standard curve using known amounts of X and observe the response in Y. Samples containing unknown amounts of X are treated along with the known quantities. From the standard curve given by linear regression of Y on X, the unknown X's can be estimated. Examples that come readily to mind from my own work are the modified Fisk and Subbarow colorimetric assay for inorganic phosphate, spectrophotometric and fluorometric assays for various enzymes and their metabolic products (Lowry and Passonneau 1972; Bergmeyer 1983–1986) and scintillation counting of radioactive element-tagged compounds.

In planning further experiments or making decisions based on values of X computed from the standard curve, we may wish to know the confidence limits of X. Also, in developing a new method, we want to know if the analysis is trustworthy within reasonable limits.

I followed the equations given in Snedecor and Cochran (1989, 170–172) in writing the program confx.ado. The program is furnished along with a help file on the STB diskette. It gave correct answers for the problems listed in Snedecor and Cochran as well as for example 14.7 from Sokal and Rohlf (1981, 498). These exercises are also supplied on the disk.

#### References

Bergmeyer, H. U., editor in chief. 1983-1986. Methods of Enzymatic Analysis. 3d ed. Deerfield Beach, FL: Verlag Chemie.

Lowry, O. H. and J. V. Passonneau. 1972. A Flexible System of Enzymatic Analysis. New York: Academic Press.

Snedecor, G. W. and W. G. Cochran. 1989. Statistical Methods. 8th ed. Ames, IA: University of Iowa Press.

Sokal, R. R. and F. J. Rohlf. 1981. Biometry: The Principles and Practice of Statistics in Biological Research. 2d ed. San Francisco: Freeman.

sg11	Quantile regression standard errors
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Obtaining standard errors for the coefficients in quantile regression (qreg) is a difficult problem and one for which the literature provides only sketchy guidance. In the case of linear regression, one can prove that the standard errors of the coefficients are given by the diagonal of  $s^2(\mathbf{X}'\mathbf{X})^{-1}$ , at least if the residuals are i.i.d. (independently and identically distributed) N(0,  $\sigma^2$ ) ( $s^2$  is the estimate of  $\sigma^2$  and formulas can similarly be derived). A considerable literature on "robustness" and accompanying algorithms is available when assumptions fail (e.g., rreg, hreg).

What is worth understanding is that there are no clear-cut answers for quantile regression even when assumptions are met. There are some existing proofs (Koenker and Bassett 1978, 1982) that suggest estimates for the asymptotic parameter variances and covariances. These estimates form the basis of the Stata calculations. Of course, all proofs depend on assumptions and asymptotic estimates are not guaranteed to apply in small samples. The question of how good these estimates really are in small samples is open.

A primary purpose of quantile regression is to escape assumptions. One particular noteworthy assumption in the Koenker and Bassett formulation is that the error distribution is *homoscedastic*, that is it does not depend on  $\mathbf{x}$ , the vector of independent variables. Indeed, a major use of quantile regression is to calculate quantiles in situations where the quantiles are not parallel.

Bootstrapping (Efron 1982) provides an alternative way of obtaining standard errors without assumptions, but at the cost of computer time. In bootstrapping, we replicate the sampling experiment that created the sample by drawing from the sample as if it were a population. The idea that the sample is a close approximation to the population is the fundamental idea behind sampling. (A companion insert (*sg11.1*) provides a bsqreg command for constructing such estimates.) However, the fact that bootstrapping involves resampling makes it nonverifiable because the answers depend on the vagaries of a random number generator. In order to minimize the randomness, many replications should be taken.

In this insert, I explore the quality of the standard errors produced by the qreg command and those produced by bootstrapping. I find that the reported standard errors are quite satisfactory except in the case of heteroscedastic errors, in which case they substantially understate the true standard error. Since quantile regression is often used to estimate with precisely this kind of data, in such cases, bootstrap standard errors are preferable.

#### Analytic standard errors for quantile regression

Before comparing the standard errors produced by qreg to bootstrap standard errors, let me quickly review how qreg calculates the standard errors it reports. The Stata manual is somewhat terse on the subject and, worse, there is an error. In [5s] qreg, the manual says the variance-covariance matrix is estimated by  $\mathbf{R}_2^{-1}\mathbf{R}_1\mathbf{R}_2^{-1}$ . What it does not say, but should, is that  $\mathbf{R}_1$  is estimated as  $\mathbf{X'WW'X}$ , where W is a  $n \times n$  diagonal matrix with elements

$$W_{ii} = \begin{cases} q/f_{\text{errors}}(0) & \text{if } r > 0\\ (1-q)/f_{\text{errors}}(0) & \text{if } r < 0\\ 0 & \text{otherwise} \end{cases}$$

and  $\mathbf{R}_2$  is the design matrix  $\mathbf{X}'\mathbf{X}$ . This is derived from formula 3.11 in Koenker and Bassett (1982), although their notation is much different.  $f_{\text{errors}}()$  refers to the density of the true residuals.

There are many things that Koenker and Bassett leave unspecified, including how one should obtain a density estimate for the errors in real data. For example, a side effect of quantile regression is that if there are k parameters, at least k residuals must be exactly zero. Also, while their formula recognizes that heteroscedasticity may alter the standard errors, the factor increase is not computable unless one goes back to the assumption that the errors are i.i.d.

Below, we will explore the quality of the standard errors produced by qreg and its boot-strapped variant bsqreg in:

- 1. A simple bivariate model  $y = \alpha + \beta_1 x + u$ .
- 2. A trivariate model  $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + u$ .
- 3. Various models in which assumptions or tacitly assumed conditions fail to be true, including (a) heteroscedastic errors and (b) skewed covariates.

#### Monte Carlo Experiment 1: A simple bivariate model

We wish to consider estimation of various quantiles for models of the form  $y = \beta_0 + \beta_1 x + u$ . A typical experiment in this group worked like this:

```
. drop _all
. set obs 1000
. gen x = invnorm(uniform())
. gen y = invnorm(uniform())
. qreg y x, quant(0.4)
. display _b[x]/_se[x]
. display (_b[_cons]-invnorm(0.4))/_se[_cons]
```

If the standard errors produced by qreg are accurate, the results displayed by the last two display statements should each be normally distributed with mean 0 and variance 1. This example is actually more general than one might first think, since the quantile regression function is a linear function of the independent variables. The quantile-regression estimator is not a linear estimator, but it does have certain properties that are worth appreciating, and which we will call "transparency." Given the model  $y = \mathbf{bx} + u$ , where the vector x may include an element equal to 1 corresponding to the constant, the transparency properties are

- 1.  $y \rightarrow cy \Rightarrow \mathbf{b} \rightarrow c\mathbf{b}, s_{\mathbf{b}} \rightarrow cs_{\mathbf{b}}$ . Multiplying the dependent variable by any constant results in new coefficient estimates that are the constant multiplied by the original coefficients. Standard errors are similarly multiplied by the constant; t-statistics are unchanged. This statement holds for c > 0. If c < 0, it holds if the quantile fraction is reversed.
- 2.  $\mathbf{x} \rightarrow \mathbf{A}\mathbf{x} \Rightarrow \mathbf{b} \rightarrow \mathbf{b}\mathbf{A}^{-1}$ . Transforming the independent variables in some way comes out entirely in the coefficient estimates and does not affect the errors.
- 3.  $y \rightarrow y + g\mathbf{x} \Rightarrow \mathbf{b} \rightarrow \mathbf{b} + \mathbf{g}$ . Adding a multiple of  $\mathbf{x}$  to the dependent variable comes out entirely in the estimated coefficients and does not essentially change the solution.

You can add to the Stata experiment shown above to verify these properties:

. gen w = 2\*y
. qreg w x, quant(.4)
. gen z = - x
. qreg y z, quant(.4)
. gen v = y + 10\*x
. qreg v x, quant(.4)

As a result of these transparency properties, it is not necessary to consider a wide range of multivariate problems. Complex multivariate problems can be transformed to look like univariate ones. For example, I do not need to consider problems where the **x**'s are highly correlated.

I do consider problems over a range of sample sizes. In order to gain a complete understanding of the most common situation (median regression), I ran this simulation 5,000 times with a variety of sample sizes ranging from 20 to 5,000 (this took 240 hours on an unloaded DECstation 3100).

Sample size does not appear to be a major consideration (there was no relationship of either the formula or the bootstrap performance based on sample size).

		va	$\mathbf{r}(t_x)$	var	$(t_{ m cons})$
Quantile	Replications	Formula	Bootstrap	Formula	Bootstrap
Median	5000	1.02	1.13	0.98	1.11
.4	219	1.00	1.09	1.02	1.19
.3	1250	1.11	1.11	0.98	1.13
.2	1250	0.93	1.10	0.95	1.22
.1	200	0.97	1.29	1.09	1.33
.05  n > 100	236	1.40	0.91	1.10	1.40
.05 $n \le 100$	1014	38.15	1.66	23.34	2.53

The desired answer for all of the entries on the right-hand side of the table is 1.00, meaning that both the formula answer and the bootstrap answer produce asymptotic normal t-statistics.

For the median, the variance of the bootstrap answer is slightly higher (e.g., 1.13 instead of 1.02) because the bootstrap standard error is computed from 20 replications. Since this estimated standard error becomes the denominator of a t-statistic, we must evaluate that t-statistic against a t-distribution with 20 degrees of freedom. The match is perfect; the probability of exceeding the 5% cutoff values for a t-distribution with 20 degrees of freedom is 5.1% for both the coefficient of x and \_cons. The performance of the formula answers is consistent with the asymptotic theory for a z-statistic.

The performance of the formula holds up well down to about the 10th percentile and does not depend on sample size. Below that, small-sample phenomena begin to take over. At the 5th percentile, both the formula and the bootstrap break down, but for different reasons. The formula breaks down because the density estimate breaks down. In the group with sample size 100 and less, the 10th percentile was 3.8 observations from the bottom edge of the dataset on the average. The density estimate is based on the nearest  $\sqrt{n}$  observations, which covers a thicker part of the distribution. The bootstrap is much better than the formula, but still underestimates the variance.

#### Monte Carlo Experiment 2: A trivariate model

The second experiment is a trivariate regression. For the reasons of the previous discussion (which I confirmed in simulations not reported), there is nothing to be learned from an experiment with two normally distributed x's. So I simulated one x (say  $x^2$ ) that distributed with one observation at +1, one at -1, and 198 at zero.

I was interested in the effect that  $x_2$  might have on the other x, which was normally distributed. But this  $x_2$  pointed out an interesting dilemma faced by quantile regression theory. It is easy to construct examples where there are two quite different answers with exactly the same minimum weighted sum of deviations, and this particular design happens to be one such instance. Regardless of the quantile, one point seems to be ignored completely, while the coefficient of  $x_2$  fits the other point exactly.

The answer to our original question—which standard error is better—now becomes clear. Although the formula is stressed, it is in the ballpark. The bootstrap frequently misses both of the high-leverage points and comes up with a much different solution. For the normally distributed x, the answers are pretty good for both methods, but for x2 and \_cons, the bootstrap is not a competitor.

### Monte Carlo Experiment 3: Other not-so-nice problems

The following kinds of problems might be suspected to cause some problems for the theory:

a. Problems where there are heteroscedastic errors.

I simulated a case similar to the one-variable simulation above, but made sd(y) = exp(x). This implies that the sd(y) is about 50 times higher on one end of the x-distribution compared with the other one. This would be noticeable in a graph!

b. Problems with skewed covariates.

These simulations were all done with median regression with 1,250 replications of a sample size of 200:

	va	$\mathbf{r}(t_x)$	var(	$(t_{ m cons})$
Problem	Formula	Bootstrap	Formula	Bootstrap
Heteroscedastic Errors	2.89	0.88	3.93	1.13
Skewed Covariates	1.23	1.18	0.92	1.07

Heteroscedastic errors turn out to be a major problem for the standard error formula, consistent with the theory suggested by Koenker and Bassett (1982). They have roughly the same kind of impact as they would have in regular regression. The actual standard deviations of the t-statistics are well above 1. Standard errors of coefficients are underestimated, and so results are non-conservative.

The bootstrap is better, but slightly conservative for the coefficient of x. With the 20 degree-of-freedom approximation, we would like a variance of 1.11 in this column.

For a skewed covariate (implying a few points with very high leverage), the bootstrap performed well, but the formula was slightly high for the variance of x and low for \_cons.

#### Conclusions

Quantile regression is becoming a favorite technique for identifying and exploiting naturally heteroscedastic phenomena, such as income inequality. These results suggest that standard errors produced in this kind of problem should be looked at suspiciously. We suggest that the bootstrap standard errors be used in cases where heteroscedasticity is suspected.

Quantile regression has also been suggested as a form of robust regression—a method that can be trusted without looking into the impact of outliers in y or x. These results suggest that such complacency is premature.

Finally, do not assume that either answers are good for extreme quantiles; for example, if n < 5/q or n < 5/(1-q).

#### References

Efron, B. 1982. The Jackknife, the Bootstrap and Other Resampling Plans. Philadelphia: Society for Industrial and Applied Mathematics.

Koenker, R., and G. Bassett. 1978. Asymptotic theory of least absolute error regression, *Journal of the American Statistical Association* 73: 618–622. ——. 1982. Robust tests for heteroscedasticity based on regression quantiles. *Econometrica* 50: 43–61.

Ruppert, D., and R. J. Carroll. 1980. Robust regression by trimmed least squares. Journal of the American Statistical Association 75: 828-838.

	sg11.1	Quantile regression with bootstrapped standard errors	
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Rogers in sg11 argues that the formula-based Koenker and Bassett standard errors used by qreg are not satisfactory when heteroscedasticity of the residuals is suspected and suggests the substitution of bootstrap standard errors. Bootstrapping allows one to obtain standard errors for any statistic even when an analytical formula is not available and a large literature on this subject is available (see, for example, Efron 1982 and Wu 1986). The method is procedurally simple but computationally expensive. One has a data set containing N observations and an estimator which, when applied to the data, produces certain statistics (such as the coefficients produced by qreg). One draws, with replacement, N observations from the N observation data set. In this random drawing, some of the original observations will appear once, some more than once, and some not at all. Using that data set, one applies the estimator and estimates the statistics. One then does it again, drawing a new random sample and reestimating, and again, and keeps track of the estimated statistics at each step of the way (called a replication).

Thus, one builds a data set of the estimated statistics. From this data, one calculates the standard deviation of the statistic using the standard formula  $(\sqrt{\sum [b_i - \bar{b}]^2 / [K - 1]})$ , where K is the number of replications). That number is your estimate of the standard error of the statistic. Note that, while the average value of the observed statistic  $(\bar{b})$  is used in the calculation of the standard deviation, it is not used as the estimated value of the statistic itself. The statistic is obtained in the normal way using the original N observations. (Many researchers new to bootstrapping think that  $\bar{b}$  is somehow a better estimate of the statistic than the statistic obtained in normal ways. That is not quite true. What is true is, that if the statistic is biased in some way,  $\bar{b}$  exaggerates the bias. Denoting b as the statistic calculated in the normal way, the amount of the bias can be estimated as  $\bar{b} - b$  and, in fact, an unbiased statistic would be  $b - (\bar{b} - b) = 2b - \bar{b}$ . This adjustment, however, should only be applied if there are strong theoretical reasons to believe the statistic is biased.)

The syntax of bsqreg is

That is, the syntax is identical to qreg (see [5s] qreg) except (1) weights are not allowed and (2) the option reps() is added. The reps(#) option controls the number of bootstrap replications used for calculating the standard errors and defaults to 20 if not specified.

In [5s] qreg, the following model was reported as an example:

. qreg price weight len Iteration 1: WLS sum	0 0	deviations	= 1140	43.7	
Iteration 1: sum of ab	s. weighted	deviations	= 11499	8.67	
Iteration 2: sum of ab (output omitted)	s. weighted	deviations	= 1117	86.7	
Iteration 9: sum of ab	s. weighted	deviations	= 10882	2.59	
Median Regression			N	umber of obs	= 74
Raw sum of deviations	142205 (a	bout 4934)			
Min sum of deviations	108822.6		Р	seudo R2	= 0.2347
price   Coef.				[95% Conf	. Interval]
weight   3.933588	.8602185	4.573	0.000	2.217936	5.64924
length -41.25191	28.86931	-1.429	0.157	-98.82993	16.3261
foreign   3377.771	577.3392	5.850	0.000	2226.304	4529.238
_cons   344.6494	3260.245	0.106	0.916	-6157.704	6847.002

This model is an excellent candidate for bootstrapping as one might suspect that a model of price (as opposed to, say, log of price) would suffer from heteroscedasticity. Estimating this model with bsgreg:

(estimatin	orice weight long base model) oping	0	0			
	gression, boot of deviations	-			Number of obs =	74
Min sum	of deviations	108822.6			Pseudo R2 =	0.2347
·	Coef.				[95% Conf.	Interval]
weight	3.933588	2.837216	1.386	0.170	-1.725061	9.592236
length	-41.25191	73.45879	-0.562	0.576	-187.7608	105.257
foreign	3377.771	1164.462	2.901	0.005	1055.325	5700.217
_cons	344.6494	6587.404	0.052	0.958	-12793.51	13482.81

As we would expect based on Rogers' findings, the standard errors are larger (the ratios vary from 2.02 to 3.3).

The standard errors produced by the bootstrap technique are only approximations. Estimating the same model again produces different estimates:

(estimatir	price weight long base model) pping	0	0			
Median Reg	gression, boot	strap(20)	SEs		Number of obs =	74
	of deviations of deviations		(about 4934)		Pseudo R2 =	0.2347
price			t	P> t	[95% Conf.	Interval]
weight	3.933588	3.042472	1.293	0.200	-2.134431	10.00161
length	-41.25191	84.80841	-0.486	0.628	-210.397	127.8931
foreign	3377.771	974.9878	3.464	0.001	1433.219	5322.323
_cons	344.6494	7482.968	0.046	0.963	-14579.66	15268.96

The accuracy of the approximation increases with the number of replications, but it is worth noting that, even at a moderate 20 replications, results are not substantively different.

Anytime one works with a computer, it is important to be able to reproduce results. Although the standard errors produced by bsqreg have a random component, they can be mechanically reproduced by resetting the random number seed (see [5d] generate) assuming one knows the seed prior to estimation. For instance, the following two commands will always produce the same results:

- . set seed 573998311 . bsqreg price weight length foreign
- (output omitted)

One could produce more accurate estimates of the standard errors by including the rep() option: 'bsqreg price weight length foreign, rep(50)'.

bsqreg is not as well integrated into Stata as the other estimation commands. While \_b[] does contain the parameter estimates, and predict can be used to obtain predicted values and residuals, \_se[], test, and correlate cannot be used after estimation.

## References

Efron, B. 1982. The Jackknife, the Bootstrap and Other Resampling Plans. Philadelphia: Society for Industrial and Applied Mathematics.

Wu, C. F. J. 1986. Jackknife, bootstrap and other resampling methods in regression analysis. Annals of Statistics 14: 1261–1350 (including comments and reply).

snp4	Non-parametric test for trend across ordered groups	

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The syntax of the nptrend command is

nptrend varlist [if exp] [in range], by(groupvar) [trend(scorevar)]

nptrend performs a non-parametric test for trend across ordered groups. This test, developed by Cuzick (1985), is an extension of the Wilcoxon rank-sum test and is a useful adjunct to the Kruskal-Wallis test (kwallis). Formula for the test statistic is given by Cuzick (1985) and Altman (1991). Correction for ties is incorporated into the formula.

groupvar is a grouping variable, and scorevar defines scores for groups. When trend() is not specified, the values of groupvar are used as the scores.

## Example

Consider the following data (Altman 1991):

Group	Transmission of visible light	Ocular exposure to ultraviolet radiation
1	< 25%	1.4 1.4 1.4 1.6 2.3 2.3
2	25 to 35%	0.9 1.0 1.1 1.1 1.2 1.2 1.5 1.9 2.2 2.6 2.6
		2.6 2.8 2.8 3.2 3.5 4.3 5.1
3	> 35%	0.8 1.7 1.7 1.7 3.4 7.1 8.9 13.5

We can use nptrend to test for a trend of increasing exposure across the three groups. When we do not specify scores for groups, they are defined by the grouping variable groupvar:

. nptrend exp, by(group)				
Test: Trend	across	groups		
gr	_Sco	re	_Obs	_RankSum
1		1	6	76.00
2		2	18	290.00
3		3	8	162.00
z		1.519		
probability	= (	0.1288		

When the groups are given any equally spaced scores—such as -1, 0, 1—we obtain the same answer as above. To illustrate the effect of changing scores, an analysis of these data with scores 1, 2, 5 (admittedly not very sensible in this case) gives

```
. nptrend exp, by(group) tr(score)
Test: Trend across groups
    group
              _Score
                             _Obs
                                    _RankSum
                               6
                                       76.00
        1
                   1
        2
                   2
                               18
                                      290.00
                                      162.00
        3
                   5
                                8
                  1.464
          z =
probability =
                  0.1432
```

This example suggests that the analysis is not all that sensitive to the scores chosen.

#### References

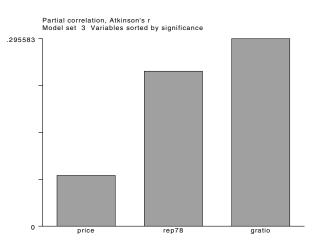
Altman, D. G. 1991. *Practical Statistics for Medical Research*. London: Chapman and Hall, 215–217. Cuzick, J. 1985. A Wilcoxon-type test for trend. *Statistics in Medicine* 4: 87–90.

sqv3.1	Graphical display of Atkinson's R values
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Building on the lwald command supplied in STB-7 (Hilbe 1992), I have written a program that graphically displays Atkinson's R values in ascending order. Moreover, the program saves the graph for future printing. It can be located in the  $c:\ado$  directory as pc\_n.gph, where n is the number of variables in the model. Histograms are not provided for R values of 0.

The program, called atrgph, is used like logit. An example partial output is shown below.

```
. atrgph foreign price rep78 gratio
/* Normal logistic regression output not shown */
Wald Statistics and Partial Correlations (Atkinson's R)
                         Prob(Chi)
No. Var
               Wald
                                      Partial Cor
                 2.575
                            0.109
                                          0.080
1
     price
    rep78
                 7.376
                            0.007
2
                                          0.244
3
     gratio
                 9.869
                            0.002
                                          0.296
```



### References

Hilbe, J. 1992. sqv3: Wald and Atkinson's R extension to logistic. Stata Technical Bulletin 7: 18.

sqv4.1	Correction to Idev command output

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Users of the ldev extension to the logistic command may have noticed that in some cases the screen display indicating the number of observations in the data set is incorrect. The deviance and  $\chi^2$  values are not affected. A fix has been made to ldev; simply replace the old program with the new one found on the STB-9 diskette.

sqv5 Univariate log-likelihood tests for model identification
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Comparing the log-likelihood of a logistic regression model containing only the intercept with that of a model having a single predictor provides *prima facie* evidence of whether the predictor in fact contributes to the model. The comparison statistic is provided by the likelihood-ratio test. At the early model building stage, a *p* value of .25 or less can be considered adequate for inclusion of a variable as a main effects predictor. However, this does not mean that transformation or collapsing value levels may not prove to later enhance the contribution a variable may make to the full model. What we are really looking for at this

stage are high p values. If we find one, at say .80, we can probably exclude it from subsequent analyses. But a caveat—if the variable is likely to serve as a factor in a significant interaction—we may need to retain it regardless of its p value.

I provide a program called unilogit. It calculates, for each variable listed after the response variable, the coefficient, standard error, log-likelihood, chi-square (LL ratio statistic), and significance. The intercept-only model log-likelihood is also provided for comparison. The default is one degree of freedom.

The intercept-only log-likelihood value can be calculated directly from the distribution of the response variable. Let  $n_0$  and  $n_1$  represent the respective number of observations having 0's or 1's and let N be the total number of nonmissing observations. The log-likelihood can be determined by

$$LL = n_0 ln(n_0) + n_1 ln(n_1) - N ln(N)$$

After the logit command, one can also obtain the LL statistic directly from Stata by

 $LLo = -(\_result(6)+(-2*\_result(2)))/2$ 

where \_result(6) is the  $\chi^2$  and \_result(2) is the log-likelihood of the model with predictor(s).

The likelihood-ratio test evaluates the hypothesis that the slope coefficient is zero. Given  $LL_1$  as the log-likelihood of the model with the predictor, and  $LL_0$  as the intercept log-likelihood, the ratio is determined by  $\chi^2 = 2(LL_1 - LL_0)$ .

### Example

```
. use lbw
. describe
Contains data from lbw.dta
 Obs: 189 (max= 166927)
Vars:
         12 (max=
                      99)
Width:
          14 (max=
                     200)
                         %8.0g
  1. id
                  int
                                            identification code
  2. low
                         %8.0g
                                            birth weight<2500g
                  byte
                         %8.0g
                                            age of mother
  3. age
                  byte
  4. lwt
                  int
                         %8.0g
                                            weight at last menstrual period
  5. smoke
                  byte
                         %8.0g
                                            smoked during pregnancy
                         %8.0g
  6. ptl
                  byte
                                            premature labor history (count)
  7. ht
                  byte
                         %8.0g
                                            has history of hypertension
                         %8.0g
  8. ui
                  byte
                                            presence, uterine irritability
                         %8.0g
                                            race==white
 9. race1
                  byte
10. race2
                                            race==black
                  byte
                         %8.0g
                         %8.0g
11. race3
                  byte
                                            race==other
12. ftv
                  byte
                         %8.0g
                                            1st trimester M.D. visits
Sorted by:
. unilogit low age lwt smoke ptl ht ui ftv
                   Univariate Logistic Regression Models
                           1 Degrees of Freedom
Intercept LL = -117.3360
Variable
             Coeff
                         St Error
                                            LL
                                                          Chi2
                                                                       Prob
-----
           -0.0512
                           0.0315
                                       -115.9560
                                                          2.7600
                                                                       0.0966
age
lwt
           -0.0141
                           0.0062
                                        -114.3453
                                                          5.9813
                                                                       0.0145
                           0.3196
            0.7041
                                        -114.9023
                                                          4.8674
                                                                       0.0274
smoke
            0.8018
                           0.3172
                                       -113.9463
                                                          6.7794
                                                                       0.0092
ptl
ht
            1.2135
                           0.6083
                                       -115.3249
                                                          4.0221
                                                                       0.0449
                                        -114.7979
                                                                       0.0243
шi
            0.9469
                           0.4168
                                                          5.0761
           -0.1351
                           0.1567
                                        -116.9494
                                                          0.7731
                                                                       0.3792
ftv
```

#### References

Hosmer, D. W. and S. Lemeshow. 1989. Applied Logistic Regression. New York: John Wiley & Sons.

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The syntax of maxr2 is

#### maxr2

This program can only be used after the fit estimation command. No options are allowed; the routine will automatically ensure that the same cases that are in your fit are used here.

How does one determine the "goodness-of-fit" (GOF) of an ordinary least squares regression? Although many people use R-squared as a summary GOF measure, it is not a good measure for a number of reasons; thus, most users of regression supplement R-squared with a number of more specific, and limited, measures and graphs. Many of these are included in Stata.

There is a situation in which (1) it is possible to obtain a simple summary GOF measure, and (2) in which R-squared is a particularly bad GOF measure. This occurs when one's data set includes "replicates": cases that are tied on every independent variable but that may differ in their values on the dependent variable. In particular, if the replicates do differ in their Y values, it is impossible to obtain an R-squared of 1.0 making the usual use of R-squared questionable (at best).

For instance, we want to predict a person's weight based on their height and have the following data:

Weight	Height
130	66
135	67
140	68
145	69
150	70
155	71
160	72
165	73
170	74
175	75
180	76
185	77

In the above data, we have a perfect relation: for every additional inch of height we have 5 more pounds of weight. Following is the regression:

						ht height	. fit weigh
12	Number of obs =		MS		df	SS	Source
•	F(1, 10) =						+-
•	Prob > F =		3575.00	3	1	3575.00	Model
1.0000	R-square =	0.00			10	0.00	Residual
1.0000	Adj R-square =						+-
0.00	Root MSE =		325.00		11	3575.00	Total
nterval]	[95% Conf. Ir	P> t	t	Err.	Std.		weight
				0		5	height
•	•	•		0		-200	_cons
1.000 1.000 0.0	F( 1, 10) = Prob > F = R-square = Adj R-square = Root MSE = [95% Conf. In	•	3575.00 0.00 325.00 t	Err.	1 10 	3575.00 0.00 3575.00 Coef. 5	Model   Residual   + Total    weight   + height

R-squared is 1.0 and everything else is missing since all measures of variance are equal to 0. Now let's add a case with height equal to 70 but weight equal to 145; we also show the results of the maxr2 routine:

. fit weigh	t height				
Source	SS	df	MS		Number of obs = 13
Model   Residual	3696.48422 22.7465536	1 369 11 2.	6.48422 0678685		F( 1, 11) = 1787.58 Prob > F = 0.0000 R-square = 0.9939 Adj R-square = 0.9933
Total	3719.23077	12 309	.935897		Root MSE = $1.438$
0 .	Coef.		t	P> t	
height	5.04772 -203.7911	.1193884 8.531825		0.000	4.784948 5.310492 -222.5695 -185.0127

Note that not only is R-squared no longer equal to 1.0 (and variances are now positive so we have p-values), but that the maximum possible R-squared is less than 1.0 (an R-squared of 1.0 implies that the regression line goes through every point; this is not possible if two points have the same X value(s) but different Y values). The "relative R-square" is the R-square reported by Stata (here, .9939) divided by the maximum possible R-square (.9939/.9966) and similarly for "Rel. Adj. R-square" (except for possible rounding errors). Thus, the "relative" values are again proportions of 1.0.

Following the R-squared information is a pure error lack-of-fit F test, if the p-value is "small," then the fit is not good. This test examines the sums of squares within "replicates." If this test is not significant, and thus the fit is "good," you must still examine your regression for failures of assumptions, influential points, etc.

The final block of information just gives a count of the number of covariate patterns and the ratio of the number of covariate patterns to N, the number of observations.

Note that if we change the 13th data point so that the weight becomes more discrepant, the R-squared information changes, sometimes drastically, but the GOF test does not change. The following two examples demonstrate this. In the first, the weight for the 13th data point is changed to 130 (from 145) and the height is left at 70; in the second, the weight is changed to 110 and the height is again unchanged.

```
. fit weight height
 Source SS
                   df
                           MS
                                            Number of obs =
                                                              13
  _____
                                            F(1, 11) = 118.15
  Model | 3909.13207 1 3909.13207
                                           Prob > F = 0.0000
R-square = 0.9148
Residual | 363.944857 11 33.0858961
   Adj R-square = 0.9071
                                           Root MSE
                                                      = 5.752
  Total | 4273.07692 12 356.089744
_____
                                              -----
 weight | Coef. Std. Err. t P>|t| [95% Conf. Interval]
   -
-----•

        height
        5.19088
        .4775538
        10.870
        0.000
        4.139791
        6.241969

        _cons
        -215.1644
        34.1273
        -6.305
        0.000
        -290.278
        -140.0507

          _____
. maxr2
max R-square
             = 0.9532
relative R-square = 0.9597
Rel. Adj. R-square= 0.9561
SSLF (df) = 163.94486 (10) MSLF = 16.394486
SSPE (df) = 200 (1) MSPE = 200
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) =
                                         0.0820 (10,1)
                              prob > F =
                                          0.9942
number of covariate patterns = 12
 as ratio of observations: 0.923
. fit weight height
                    df
 Source SS
                                             Number of obs =
                            MS
                                                              13
                    _____
                                            F(1, 11) = 31.75
Model | 4201.91288 1 4201.91288
Residual | 1455.77943 11 132.343584
                                                    = 0.0002
= 0.7427
                                            Prob > F
                                            R-square
                                            \operatorname{Adj} \operatorname{R-square} = 0.7193
  Total | 5657.69231 12 471.474359
                                           Root MSE
                                                       = 11.504
_____
 weight | Coef. Std. Err. t P>|t| [95% Conf. Interval]
  _____
 height5.38176.95510765.6350.0003.2795837.483938_cons-230.328768.2546-3.3750.006-380.5561-80.10138
  _____
```

This comparison helps give a better idea of what is going on: the GOF test is not affected by how much different the one case is, but the maximum R-square is. The test and measure are discussed in Draper and Smith (1981, 33–42); the test is also discussed in Neter, Wasserman and Kutner (1989, 131–140) and Weisberg (1985, 89–95). Each book gives a bivariate regression example; the data for these three are on the disk and the examples follow:

```
. use ds38
(Draper & Smith example, p. 38)
. fit y x
 Source
            SS
                   df
                         MS
                                          Number of obs =
                                                          24
F(1, 22) =
                                                         6.57
                                          Prob > F = 0.0178
R-square = 0.2298
 Model 6.32466658 1 6.32466658
Residual | 21.1936683 22 .963348559
        _____
                                          Adj R-square = 0.1948
  Total | 27.5183349 23 1.19644934
                                         Root MSE = .9815
 _____
                                         _____
     y | Coef. Std. Err. t P>|t| [95% Conf. Interval]
______
  x | .3378862 .1318692 2.562 0.018
_cons | 1.436396 .5900072 2.435 0.023
                                         .0644062
                                                      .6113662
                                            .2127955
                                                      2.659996
     _____
. maxr2
            = 0.5468
max R-square
relative R-square = 0.4203
Rel. Adj. R-square= 0.3939
SSLF (df) = 8.7236679 (11) MSLF = .79306072
SSPE (df) = 12.47 (11) MSPE = 1.1336364
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) =
                                       0.6996 (11,11)
                             prob > F =
                                       0.7183
number of covariate patterns = 13
 as ratio of observations: 0.542
. use neter132
(Neter, Wasserman & Kutner example, p. 132)
. fit numnew minimum
 Source SS
                         MS
                   df
                                          Number of obs =
                                                          11
                                          Model | 5141.33841 1 5141.33841
Residual | 14741.5707 9 1637.9523
                _____
                                          Adj R-square = 0.1762
  Total | 19882.9091 10 1988.29091
                                                   = 40.472
                                          Root MSE
                                   _____
numnew | Coef. Std. Err. t P>|t| [95% Conf. Interval]
- ..
-------

        minimum
        .4867016
        .2747105
        1.772
        0.110

        _cons
        50.72251
        39.39791
        1.287
        0.230

                                         -.1347368
                                                      1.10814
                                           -38.40176
                                                      139.8468
     -----
               _____
. maxr2
max R-square = 0.9423
relative R-square = 0.2744
Rel. Adj. R-square= 0.1938
SSLF (df) = 13593.571 (4) MSLF = 3398.3927
SSPE (df) = 1148 (5) MSPE = 229.6
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) = 14.8014 (4,5)
                                      0.0056
                             prob > F =
number of covariate patterns = 6
 as ratio of observations: 0.545
```

. use wberg (Weisberg e	90 xample, p. 9	0)						
. fit y x								
Source	SS	df	М	S		Number of obs		
Model	4.56925017		4.5692	5017		F( 1, 8) Prob > F		
	4.21663901					R-square	=	0.5201
+- Total	8.78588918	9	.97620	9909		Adj R-square Root MSE		
у   	Coef.	Std.	Err.	t	P> t	[95% Conf.	Int	erval]
						.1154935 .8458762		
relative R-	re = 0.7 square = 0.7 -square= 0.6	109						
	: 1.8582478 ( : 2.3583912 (	-						
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) = $2.3638$ (2,6) prob > F = $0.1750$								
number of covariate patterns = 4 as ratio of observations: 0.400								

The final two examples use the Stata auto.dta for multiple regression-first with three variables and then with two variables and an if expression.

. use auto (1978 Auto	o omobile Data)								
. fit mpg	weight w2 for	eign							
	SS			4S		Number of obs F(3, 70)			
Model Residual	1689.15372 754.30574	3 70	563.0 10.77	57963		Prob > F R-square Adj R-square	= 0.0000 = 0.6913		
Total	2443.45946	73	33.472	20474		Root MSE			
mpg	Coef.	Std.	Err.	t	P> t	[95% Conf.	Interval]		
weight	0165729	.0039	692	-4.175	0.000	0244892	0086567		
w2	1.59e-06	6.25e	-07	2.546	0.013	3.45e-07	2.84e-06		
						-4.3161			
_cons	56.53884	6.197	383	9.123	0.000	44.17855	68.89913		
relative F	are = 0.9 R-square = 0.7 R-square= 0.6	049							
SSLF (df) = 707.30574 (65) MSLF = 10.881627 SSPE (df) = 47 (5) MSPE = 9.4									
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) = $1.1576$ (65,5) prob > F = $0.4897$									
	number of covariate patterns = 69 as ratio of observations: 0.932								

. fit mpg	weight w2 if	foreig	n==0				
	SS			5		Number of obs	
Model	905.395466	2	452.697733			F( 2, 49) Prob > F	= 0.0000
	242.046842					R-square Adj R-square	
Total	1147.44231	51	22.4988	3688		Root MSE	
	Coef.		Err.	t	P> t	[95% Conf.	Interval]
			307	-4.077	0.000	0196642	0066794
w2	1.11e-06	4.95e	-07	2.249	0.029	1.19e-07	2.11e-06
_cons	50.74551	5.162	014	9.831	0.000	40.37205	61.11896
<pre>. maxr2 max R-square = 0.9590 relative R-square = 0.8228 Rel. Adj. R-square = 0.8155 SSLF (df) = 195.04684 (44) MSLF = 4.4328828 SSPE (df) = 47 (5) MSPE = 9.4</pre>							
F (dfn, dfd) for lack-of-fit test (MSLF/MSPE) = 0.4716 (44,5) prob > F = 0.9193							
	covariate pat o of observati						

I know of no other software that provides the maximum R-square measure for multiple regression. MINITAB does provide the pure error GOF test, and I used MINITAB for the two regressions using the auto data—both matched to the precision shown by the two packages. MINITAB also provides, and Draper and Smith and others (see, e.g., Christensen, 1989), a GOF test based on "near-replicates," but I have not found this very useful due to problems in defining what is "near" and have not implemented it.

Finally, weighting is not allowed in the regression, although such an adjustment could probably be added fairly easily (at least for frequency weighting).

## References

Christensen, R. 1989. Lack-of-fit tests based on near or exact replicates. The Annals of Statistics 17: 673-83.

Draper, N. and H. Smith. 1981. Applied Regression Analysis. 2d ed. New York: John Wiley & Sons.

Neter, J., W. Wasserman, and M. H. Kutner. 1989. Applied Linear Regression Models. 2d ed. Homewood, IL: Irwin.

Weisberg, S. 1985. Applied Linear Regression. 2d ed. New York: John Wiley & Sons.