CUBIC SPLINE POPULATION DENSITY FUNCTIONS AND SUBCENTRE DELIMITATION

The case of Barcelona

Ivan Muñiz, Anna Galindo and Miguel Ángel García* †

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Summary. The presence of subcentres cannot be captured by an exponential function. Cubic spline functions seem more appropriate to depict the polycentricity pattern of modern urban systems. Using data from Barcelona Metropolitan Region, two possible population subcentre delimitation procedures are discussed. One, taking an estimated derivative equal to zero, the other, a density gradient equal to zero. It is argued that, in using a cubic spline function, a delimitation strategy based on derivatives is more appropriate than one based on gradients because the estimated density can be negative in sections with very low densities and few observations, leading to sudden changes in estimated gradients. It is also argued that using as a criteria for subcentre delimitation a second derivative with value zero allow us to capture a more restricted subcentre area than using as a criteria a first derivative zero. This methodology can also be used for intermediate ring delimitation.

* Corresponding author:
Ivan Muñiz, Anna Galindo, and Miguel Ángel García,
Department of Applied Economics, Universitat Autònoma de Barcelona, Edifici B Facultat CCEE,
08193 Bellaterra (Cerdanyola del V.), Spain, Tel: 34 935812288 Fax: 34 935812292
imuniz@volcano.uab.es magarcia@selene.uab.es ana.galindo@uab.es
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1. INTRODUCTION

A residential density function relates gross or net population density to distance to the city centre. The estimated equation allows one to predict for any distance from the city centre the population density at that distance. A residential density function is a useful device for describing the metropolitan urban structure and the spatial pattern of population distribution. Since Clark's (1951) seminal study on residential density patterns, most theoretical and empirical work on urban spatial structure has adopted the negative exponential density function, by which that population density declines in a smooth fashion as distance from the city centre increases. While such analysis traditionally assumed monocentricity, several recent studies have demonstrated the presence of population and employment subcentres, peripheral massive housing neighbourhoods, discontinuities in the form of open spaces such as parks, mountains and green belts and the relative lack of residential land use at the centre of the urban area. These studies suggest that a more complex pattern in population density requires a more flexible function form. Cubic spline density functions have demonstrated to be able to depict that complexity.

The basic structure of the paper is to use cubic spline estimates in subcentre boundaries definition. Two possible population subcentre delimitation procedures are discussed. One, taking an estimated derivative equal to zero, the other, a density gradient equal to zero. It is argued that, in using a cubic spline function, a delimitation strategy based on derivatives is more appropriate than one based on gradients because the estimated density can be negative in sections with very low densities and few observations, leading to sudden changes in estimated gradients.

This paper is structured in the following way. Section 2 discusses why the exponential function can not explain the population density pattern of the discontinuous, polycentric and relatively dispersed contemporary city; Section 3 discusses alternative methodologies for subcentre delimitation; Section 4 characterises the Barcelona Metropolitan Region; Section 5 presents the alternative functional forms and the results of OLS estimations applied to the Barcelona Metropolitan Region; Section 6 reports the results of Barcelona subcentres delimitation; and Section 7 gives the conclusions of the research.

2. WHY THE EXPONENTIAL DENSITY FUNCTION CANNOT EXPLAIN THE RESIDENTIAL DENSITY PATTERN OF MODERN URBAN REGIONS

2.1. The Monocentric City Model and the Exponential Density function

One of the main conclusions of the Monocentric City Model is that the relationship between residential density and accessibility is seen as a reflection of a more basic relationship between land rent and accessibility. Residential density declines with distance to the city centre because bid rent declines to compensate for commuting costs. Equation (1) represents the standard residential density function

$$DEN(x) = D_0 e^{-gx} \qquad (1)$$

where *DEN(x)* is residential density at distance *x* from the city centre, D_0 is the theoretical density in the central district, and γ is the density gradient. The population density gradient measures the proportional decline in residential density per unit of distance. The estimated exponential function enables the density level at any city centre distance to be predicted. The value of the gradient is in turn related to the suburbanisation level. The greater the level of suburbanisation the flatter the estimated gradient.

2.2 What the exponential function cannot depict

a) The CBD

Many European and American cities present low density centres due to both the presence of economic activities that can offer higher bids, as well as to the population's suburbanisation toward the periphery in search of homogeneous neighbourhoods and ecological amenities. The Monocentric City Model establishes that the residential density function is discontinuous. From the city centre until the edge of the CBD, residential density is low and relatively constant, followed by a slightly decrease in density levels as we move away from the city centre. However, density variation may not be so sudden. In such a case, a function that allowed to capture the increasing evolution of density in the CBD until arriving to a maximum point from which the residential density decreases, would capture the density pattern better than a discontinuous exponential function with a constant tract corresponding to the CBD radius. (1)

b) Massive housing neighbourhoods

Under the influence of the modern movement (Le Corbusier, 1977), massive housing neighbourhoods, urbanised by means of high residential blocks for low income families, were built between 1950 and 1970 at the edge of the traditional city in many European cities and also in some Anglo-Saxon cities. Their presence implies that residential density doesn't fall gradually from the limits of the CBD, but rather it continues increasing up to a distance that can oscillate between 5 and 15 km.

c) Green belts and metropolitan parks.

Anti-sprawl policies carried out in many urban regions under the influence of *Garden-city* movements (Howard, 1898) and *Regional Planning* (Geddes, 1915), have created a discontinuous urban region fragmented by green belts and metropolitan parks. Again, the exponential function is not able to capture this pattern since it doesn't incorporate the possibility of a local minimum associated to the rural or ecological metropolitan spaces.

d) Subcentres

Large urban regions usually include employment and residential subcentres. By means of some corrections, polycentricity can be included in the Monocentric City Model. However, the residential density function can no longer be the exponential one, rather a more flexible function able to incorporate a local maximum.Subcentres can be a modern urban phenomena, as in the case of the edge cities in a number of North American cities, or the result of an increasing commuting integration of previous settlements, as in many European and Asian cities (Garreau, 1991; Giuliano and Small, 1991; Cervero and Wu, 1996; Dieleman and Faludi, 1998; Lambooy, 1998; Champion, 2001).

2.3. Cubic spline functions

The presence of “density craters” in city centres, dense peripheries, subcentres, and green belts cannot be captured by an exponential function, therefore cubic spline functions have been used in research applied to European cities (Goffette-Nagot and Schmitt, 1999), Asian cities (Zheng, 1991) and North American cities (Anderson, 1982, 1985; McDonald, 1989). Spline functions are a “device for approximating the shape of a curvilinear stochastic function without the necessity of pre-specifying the mathematical form of the function” (Suits *et al*., 1978, p. 132).

It is convenient in cubic spline estimation to consider segments with the same length. Considering three segments divided by points (knots) x_0, x_1, x_2 and x_3, the relationship between density and distance would be:

$$\begin{aligned} DEN(x) &= \left[a_1 + b_1(x-x_0) + c_1(x-x_0)^2 + d_1(x-x_0)^3\right]D_1 \\ &+ \left[a_2 + b_2(x-x_1) + c_2(x-x_1)^2 + d_2(x-x_1)^3\right]D_2 \\ &+ \left[a_3 + b_3(x-x_2) + c_3(x-x_2)^2 + d_3(x-x_2)^3\right]D_3 + u \end{aligned} \tag{2}$$

Cubic spline estimation only allows for identifying central theoretical densities by the coefficients a_1, a_2 and a_3, the gradients have to be obtained indirectly. In order to guarantee the continuity of the function, as well as the first and second derivative, equation (2) must be restricted to:

a) Equal value in the right and the left of adjacent segments

$$a_{i+1} = a_i + b_i(x_i - x_{i-1}) + c_i(x_i - x_{i-1})^2 + d_i(x_i - x_{i-1})^3 \tag{3a}$$

b) Equal slope in the right and the left of knots corresponding to adjacent segments

$$b_{i+1} = b_i + 2c_i(x_i - x_{i-1}) + 3d_i(x_i - x_{i-1})^2 \tag{3b}$$

c) Equal second derivative in the right and the left of knots corresponding to adjacent segments

$$c_{i+1} = c_i + 3d_i(x_i - x_{i-1}) \tag{3c}$$

Substituting (3a), (3b) and (3c) into (2) and considering equal segments $[(x_1 - x_0) = (x_2 - x_1) = (x_3 - x_2) = (x_{i+1} - x_i)]$ the cubic spline reduced form can be obtained.

$$\begin{aligned} DEN(x) &= a_1 + b_1(x - x_0) + c_1(x - x_0)^2 + d_1(x - x_0)^3 \\ &+ (d_2 - d_1)(x - x_1)^3 D_1^* + (d_3 - d_2)(x - x_2)^3 D_2^* + u \end{aligned} \tag{4}$$

where $D_i^* = \begin{cases} 1 \text{ if } x \geq x_i \\ 0 \text{ otherwise} \end{cases}$

The generalised form for *k+1* segments is

$$\begin{aligned} DEN(x) &= a_1 + b_1(x - x_0) + c_1(x - x_0)^2 + d_1(x - x_0)^3 \\ &+ \sum_{i=1}^{k} (d_{i+1} - d_i)(x - x_i)^3 D_i^* + u \end{aligned} \tag{5}$$

3. VARIABLE CUBIC SPLINE DENSITY GRADIENTS AND SUBCENTRE DELIMITATION

3.1. Variable cubic spline density gradients

In exponential functions estimations, the gradient is by definition constant for each distance, whereas in cubic spline estimation the gradient is variable and must be obtained by taking the derivative $dDEN/dx$ divided by $-DEN$, and evaluating it at various distances between the initial and terminal knots (Anderson, 1985).

The density gradient is an empirical instrument which has no correspondence to any economic theoretical concept. The density gradient is not a slope, nor is it an elasticity, it is the proportional density variation per unit of distance. Why do we use such an usual indicator? The answer is that it remains constant for each distance in the case of the exponential function, whereas the slope and the elasticity do not. Therefore, we can describe the density performance, that is, urban spatial structure and/or city suburbanisation level, by only one number which is easily comparable between diverse cities and periods.

Contrary to the constant gradient at all distances given by the negative-exponential function, the cubic spline density function has a variable gradient in sign and magnitude. What is the economic significance of such variation beyond the obvious reflection of a flexible functional form? It is hard to say, because the gradient loses its main property as it acquires other functional forms. That must be the reason why Anderson (1982) just describes the density gradient variability in applying a cubic spline function for the city of Detroit. In taking functional forms alternative to the exponential one, density gradient does not present any advantage relative to other indicators like the slope value or the elasticity value. In spite of its important limitation related to the lack of a theoretical explanation of gradient variability, variable cubic spline density gradients could be used for population subcentres delimitation.

3.2. Subcentre delimitation

The North American empirical literature uses to identify employment subcentres, not population subcentres, the former being the key variable to explain the influence of a zone over its surrounding area according to the monocentric city model. Polycentric urban models only adapt that assumption for a number of employment centres. It is a reasonable assumption in the case of American cities, but not in Mediterranean cities, where subcentres correspond to medium-sized mixed cities, previously developed separately from the main centre, which combine population and employment. Therefore, urban subcentres identification is not really a problem, because the preeminence of such areas is so obvious that no empirical framework is needed. Nevertheless, subcentre boundaries delimitation requires some criteria because the impact on population density sprawls beyond the administrative limits.

The empirical literature provides a variety of definitions of employment subcentres. Dunphy (1982) defines subcentres by using a long list of local data. In Giuliano and Small (1991) and Small and Song (1994) an employment subcentre is defined as a contiguous set of zones, each with a employment density above some cut-off that contains total employment above some other cut-off. The problem of such methodologies is that they require subjective criteria. In contrast with these informal approaches, McDonald and Prather (1994) depart from the classical monocentric analysis, and define subcentres as locations with significant positive residuals. In McMillen and McDonald (1997) Chicago employment subcentres are identified using a nonparametric analysis and locally weighted regressors. McMillen and McDonald (1998) propose a two-steps method combining Giuliano and Small (1991) thresholds and a formal model on employment probability and employment density.

Two recent papers have improved the identification procedure in two directions. McMillen (2001) propose a two-stage non-parametric procedure. The first step uses a non-parametric estimator, locally weighted regression, to smooth employment density (McMillen, 2001, p. 449). In the second step, a semi-parametric regression is used. The non-parametric part of the regression captures the effect of distance from the CBD using a flexible Fourier form. The parametric part of the second regression accounts for the effects of subcentre proximity on employment density (McMillen, 2001, p. 450). On the other hand, Craig and Ng (2001) use employment density quantile splines. The method developed by Craig and Ng (2001), like McMillen and McDonald (1997) and McMillen (2001), and different from Giuliano and Small (1991), allow employment densities to be conditioned on distance to the CBD. Also, quantile splines allows to investigate real densities rather than to infer peaks.

Departing from Mc Millen (2001) and Craig and Ng (2001), it is posible to identify population subcentres boundaries by taking CBD distances where variable cubic spline gradient or the estimated slope is zero. Three points capture the beginning of the increasing density section, the secondary density peak, and estimated CBD distance where density stops falling.

The problem with gradients in cubic spline estimation is that the estimated density –the denominator of the function- can be negative in sections with very low densities and few observations, leading to sudden changes in estimated gradients (Figure 1). Using Quantile Smoothing Splines, this would not pose a problem because real data is used, therefore, no negative value is computed. Taking as a criteria a zero derivative works better than a zero density gradient if a cubic spline function is used.

[Fig. 1]

Taking as a criterion for subcentre delimitation a first derivative with value zero still poses an important problem. We are implicitly supposing that the boundary between the influence area of the main centre and the influence area of the subcentre is only one point. But we know that there are large areas where the density influence of the main centre and subcentres is very low. Taking as a criterion a second derivative with value zero, which captures the inflexion of the curve, allows us to define an intermediate area not significantly influenced by the effect of the main centre and the subcentre, as well as a more restrictive subcentre criteria delimitation.

4. BARCELONA: A POLYNUCLEATED URBAN REGION

4.1. Polynucleated urban regions in Europe

The pattern of urbanisation in Europe is dominated by polynucleated cities. In Western Europe many cities have between 200 000 and one million inhabitants (Dieleman and faludi, 1998, p. 365), particularly in Netherlands (Randstad), Belgium (Flemish Diamond), and Germany (Rhin-Ruhr Metropolitan Region). Neither of these three metropolitan regions contains a primate city of more than one and a half million inhabitants. Yet, altogether they concentrate large populations of more than five million people (Dieleman and Faludi, 1998, p.365). Champion has characterised the way in which these urban regions have emerged as a *fusion mode*, that is, “the urban region emerges from the fusion of several previously independent centres of similar size as a result of their own separate growth both in overall size and lateral extent and particularly because of the improvement of transport links between them” (Champion, 2001, p. 664).

Mediterranean polycentric cities (Barcelona, Florence, Bologna, etc.), differ from Western Europe urban networks in the presence of a large primate city, but also from monocentric cities that have recently evolved towards a polycentric pattern by the formation of new edge cities. City growth does not come from a *centrifugal mo*de (American cities, Paris, London), nor does it come from a *fusion mode* (Northwest urban networks), but from an *incorporation mode*. Cities like Barcelona or Florence are becoming polycentric urban regions because the large urban centre is expanding its commuting area, so that it incorporates medium-sized cities that had previously been self-sufficient in terms of employment and services (Champion, 2001, p. 664). These polycentric urban regions have integrated a Christallerian pre-industrial urban system (Hohenberg and Lees, 1985) due to the overlapping of two different processes, population suburbanisation, and the expansion of the commuting area of the central city and the subcentres.

4.2 The historical process of Barcelona metropolitan growth

Barcelona 1900-1920: The boundaries of the municipality of Barcelona are stabilised. As in many European cities, the classical structure of Barcelona begins with the old central city within walls, the *ensanche*(2), that is, a large area urbanised following a grill pattern, and the annexation of old villages near the central city with a vast network of transport inspired in Haussmann (3).

Barcelona 1920-1960: First and second ring urbanisation. The first ring has been urbanised with massive housing blocks becoming a very dense environment following the modern movement criteria. The second ring concentrates second residences used only at weekends and during vacations.

Barcelona 1960-1985: Second ring integration into the commuting area of the central zone due to an increasing process of utilisation of second residences as primary residences. Medium-sized cities suburbanisation toward their contiguous municipalities.

Barcelona 1985-2000: Subcentres integration and expansion of the metropolitan region including extensive areas beyond subcentres which combine residential and rural uses, the metropolitan corridors. Following the North American methodology for MSA delimitation, Clusa and Roca (1997) have estimated that the Barcelona Metropolitan Region was conformed by 62 municipalities and 3,4 million inhabitants in 1981; 94 municipalities and 3,7 million inhabitants in 1986; 145 municipalities and 4,2 million inhabitants in 1991 and 162 municipalities, the population remaining at 1991 levels, in 1996 (Pacte Industrial Metropolità, 2001).

At the present time, Barcelona is a conurbation with a large, diverse, and compact centre (the municipality of Barcelona), an extremely dense first metropolitan ring urbanised by massive housing blocks, discontinuities in the form of agricultural land and metropolitan parks, seven activity and residential subcentres and an extensive area that combines rural and low density residential uses. Five subcentres are historically medium sized cities which endogenously developed in the past beyond the impulsion and attraction of the municipality of Barcelona, whereas 2 subcentres have recently developed under the influence of the Barcelona dynamism. The transportation network is radial. All subcentres and corridors are connected to the city centre through diverse railroad lines and Metropolitan highways. The BMR is a complex, diverse, discontinuous, polycentric and also partly dispersed metropolitan region. A city of cities with more than 160 municipalities that occupies nearly 4000 km^2 in a radius of approximately 50 km (**Table 1**).

The spatial dynamism of the Barcelona metropolitan region during the last 10 years is characterised by population and employment suburbanisation from the densest areas, that is, Barcelona, its first ring and subcentres, towards lower density settlements, the second ring, the commuting influence area of subcentres and the metropolitan corridors. Simultaneously, there is an increasing metropolitan integration of the surrounding areas, not only driven by suburbanisation, but also by previous settlements' integration due to improvements in transportation infrastructures. Both phenomena imply an important redistribution of densities inside the metropolitan area.

[Table 1]

[Fig. 2]

5. BARCELONA DENSITY FUNCTIONS ESTIMATES

Data for 3481 census tracts were obtained from the 1996 population census report: total population and the area of each tract. A Geographic Information System (GIS) was used to provide coordinates for the census tract centroids. These coordinates are used to measure distance to the CBD, an air distance.

Tables 2, 3 reports the estimates for the whole BMR and each one of its 6 axes. **Table 4** summarises total Barcelona Metropolitan Region optimal results for each function. Cubic spline functions have been estimated by considering two, three and four interior knots to provide three, four and five equal-distance intervals respectively from the minimum distance observed to the maximum. More knots could be computed, but as it can be seen in **Table 3**, in most cases additional knots does not significantly decreases the standard error of regression. The optimal number of knots can be determined using minimum standard error of regression (Anderson, 1982, 1985), maximum R^2, the statistical significance of the coefficients estimated (Zheng, 1991), or a combined method. We have followed Anderson adopting the minimum standard error criteria. In the cubic spline axis estimation, the number of knots varies depending on each axis. 2 knots have been chosen in two axes, 3 knots are chosen in 2 other axes, and 4 knots in the remaining two axes.

In general, the cubic spline function fits better than the exponential function. The estimates provide strong evidence that the negative exponential function is not an appropriate form to use in estimating urban-density functions in the case of the Barcelona Metropolitan Region.

[Table 2]

[Table 3]

[Table 4]

From cubic spline of **Figure 3** it can be seen that, on average, the maximum height of the density function is not at the centre, but about 5 km distance from it. Beyond distance 5 km density declines until distance 24 km, where a local minimum is obtained. Beyond that point, density increases reaching a maximum at distance 38 km, where, on average, a subcentre is located.

[Fig. 3]

In **Figure 4**, the metropolitan axes have been grouped into 3 categories: axes that include a historical subcentre; coastline axes, which also include a historical subcentre; and modern subcentre axes.

[Fig. 4]

Figure 4 reveals that massive housing peripheral municipalities are located at distance 7-9 km from the city centre, that is, beyond the administrative boundaries of the municipality of Barcelona, in the axes Vilanova, Granollers, Vilafranca, and Mataró. The maximum of the density function in the axis Terrassa-Sabadell corresponds to a peripheral district of the Municipality of Barcelona (Sarrià-St Gervasi), while in the Vilanova and Martorell axis population density falls from the city centre. It is worth noting that while obviously there is only one real value corresponding to population density at the city centre, each axis presents different theoretical densities at the city centre. Another interesting pattern is that, in comparing cubic spline functions in the coastline axes, Mataró and Vilanova, we find that between distance 5 and 42 km population density level is higher in the Mataró axis. This result is consistent with a previous historical investment in transport infrastructures in the Mataró axis compared to the Vilanova axis, mostly due to its plain relief.

6. BARCELONA SUBCENTRES DELIMITATION

Table 5 and **Figure 5** reports the results from applying the first and second derivative criteria discussed in section 3.2. We correct for distinctive topographical and historical features which conform a non-symmetrical subcentre pattern around the CBD by dividing the total area in six wedges where the methodology is replicated. This procedure requires neither “visual inspection” nor previous local knowledge. Therefore it is an objective approach which is easily reproducible to a variety of cities.

The first derivative methodology provides a subcentre radius with values between 15.04 km, Vilanova, and 25.23 km, Mataró. In applying the second derivative criterion we find a much more restricted area in radius distance and number of census tracts. Vilafranca has the larger radius (16.58 km), and Martorell the lower (7.08 km). Terrassa-Sabadell includes the higher number of census tracts, 258, and Martorell the lower, 22. The intermediate ring between the main centre and subcentres oscillates between 9 km (Martorell and Vilanova), and 16 km (Vilafranca).

[Fig. 5]

[Table 5]

7. CONCLUDING REMARKS

In the case of the Barcelona Metropolitan Region, the inappropriateness of the negative exponential functional form of the density function has been demonstrated. A more flexible function is needed to depict a polycentric, and fragmented city region. Peak densities occur some distance from the city centre, on average, about 5 km, which corresponds to massive housing peripheral municipalities, and 38 km, which corresponds to traditional or modern subcentres localisation. The intuitive appropriateness of the cubic spline function is particularly convincing as it provides a more realistic urban density pattern. Cubic spline density function can be also used in population subcentre delimitation. We have argued that neither a gradient with value zero nor the first derivative are appropriate criteria. We propose in urban subcentre delimitation to take as a criterion a second derivative with value zero when a cubic spline function is used.

FOOTNOTES

(1) Latham and Yeates (1970) and Newling (1969) provided a polynomial exponential function in order to capture the notion that population density may have a “crater” at the CBD (McDonald, 1989).

(2) The Barcelona ensanche was designed by Ildefons Cerdà in 1850.

(3) The French urbanist Leon Jaussely designed *El Plan de Enlaces* of Barcelona in 1907.

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TABLES

Table1: The Barcelona Metropolitan Region

Metropolitan rings	***Number of municipalities***	***Average distance from the city centre***	***Net density residential levels (population/ Ha)***	***Percentage of public transport commuting trips***	***Percentage of residential units in buildings with more than 3 floors***	***Average population***
Barcelona	1	2,5	366	41	94	1,6 millions
First ring	10	12,2	378	29	86	88230
Second ring	23	20,3	241	19	69	23289
Subcentres	7	38,1	169	15	68	85283
Subcentres commuting area	20	41,3	54	13	33	5391
Metropolitan Corridors	101	41,2	69	16	46	5830

Table 2: Exponential Density Function Estimates 1996

	D_0	***g***	***Obs.***	***S.E.***	***R^2***
BMR total	597.19* (50.35)	-0.05* (-23.28)	3481	232.23	0.2570
GRANOLLERS	566.68* (30.89)	-0.03* (-10.77)	1159	243.71	0.1806
MARTORELL	604.98* (23.15)	-0.05* (-7.85)	927	264.87	0.1434
MATARÓ	531.31* (24.41)	-0.03* (-8.98)	775	245.52	0.1836
TERRASSA—SABADELL	523.31* (37.569	-0.05* (-15.63)	1296	192.78	0.2504
VILAFRANCA.	587.26* (24.13)	-0.04* (-6.80)	855	256.78	0.1684
VILANOVA	549.21* (23.55)	-0.05* (-8.42)	500	232.77	0.2467

t-values are in brackets
(*) statistically significant variable

Table 3: Cubic Spline Estimates 1996

	a_1	b_1	c_1	d_1	d_2-d_1	d_3-d_2	d_4-d_3	d_5-d_4	Knots	Obs	S.E.	R^2
BMR total	408.42* (16.19)	25.13* (3.72)	-3.55* (-7.12)	0.08* (7.81)	-0.13* (-7.69)	0.15* (4.44)	-	-	20.62 41.24	3481	229.92	0.2718
	269.5* (8.21)	**79.10* (7.59)**	**-9.32* (-9.73)**	**0.25* (9.92)**	**-0.33* (-9.43)**	**0.13* (5.00)**	**-0.17 (-1.80)**	-	15.36 30.92 46.38		**228.62**	**0.2800**
	241.32* (5.62)	95.34* (6.02)	-11.84* (-6.82)	0.36* (6.44)	-0.40* (-5.38)	0.00 (0.04)	0.10 (1.53)	-0.26 (-1.11)	12.37 24.74 37.11 49.48		229.28	0.2758
GRANOLLERS	152.18* (3.21)	118.36* (7.19)	-11.91* (-7.50)	0.29* (7.06)	-0.39* (-6.14)	0.23* (2.53)	-	-	16.82 33.64	1159	233.65	0.2468
	129.75 (1.86)	131.55* (4.61)	-14.02* (-4.35)	0.39* (3.74)	-0.38* (-2.70)	-0.09 (-1.01)	0.29 (1.27)	-	12.61 25.23 37.84		234.56	0.2409
	332* (3.68)	**32.59 (0.79)**	**-0.65 (-0.12)**	**-0.14 (-0.69)**	**0.49 (1.92)**	**-0.70* (-5.28)**	**0.69* (3.79)**	**-0.92 (-1.89)**	10.09 20.18 30.27 40.36		**233.20**	**0.2497**
MARTORELL	88.49 (1.16)	165.09* (5.29)	-19.85* (-5.32)	0.61* (4.68)	-0.67* (-3.38)	-0.05 (-0.16)	-	-	12.60 25.20	927	255.60	0.2023
	124.75 (1.17)	147.72* (2.90)	-17.56* (-2.38)	0.53 (1.64)	-0.24 (-0.55)	-0.64* (-2.04)	0.81 (1.03)	-	9.45 18.90 28.35		255.66	0.2019
	277.67* (1.98)	**55.98 (0.73)**	**-1.13 (-0.09)**	**-0.36 (-0.54)**	**1.08 (1.27)**	**-1.11* (-2.38)**	**0.47 (0.68)**	**-0.17 (-0.09)**	7.56 15.12 22.68 30.24		**255.51**	**0.2028**
MATARÓ	290.8* (5.99)	54.31* (4.01)	-5.33* (-4.92)	0.11* (4.86)	-0.17* (-4.55)	0.16* (2.98)	-	-	20.62 41.24	775	241.32	0.2113
	152.12* (2.38)	**115.17* (5.22)**	**-12.15* (-5.65)**	**0.32* (5.52)**	**-0.42* (-5.21)**	**0.17* (3.05)**	**-0.21 (-1.41)**	-	15.46 30.92 46.38		**240.12**	**0.2191**
	284.7* (3.48)	57.54 (1.76)	-5.73 (-1.55)	0.13 (1.06)	-0.02 (-0.18)	-0.28* (-2.89)	0.43* (3.31)	-0.80* (-2.36)	12.37 24.74 37.11 49.48		240.96	0.2137
TERRASSA—SABADELL	**299* (5.96)**	**73.15* (3.75)**	**-11.75* (-5.23)**	**0.41* (5.49)**	**-0.60* (-5.30)**	**0.38* (2.43)**	-	-	12.76 25.52	1296	**190.20**	**0.2703**
	286.6* (3.71)	83.37* (2.26)	-13.97* (-2.64)	0.55* (2.47)	-0.55 (-1.92)	-0.24 (-1.51)	0.98* (2.12)	-	9.56 19.24 28.71		190.51	0.2680
	420.6* (4.05)	6.47 (0.11)	-1.07 (-0.11)	-0.08 (-0.18)	0.43 (0.77)	-0.67* (-3.11)	0.34 (1.16)	0.69 (0.64)	7.65 15.30 22.95 30.60		190.37	0.2690
VILAFRANCA	165.80* (3.15)	121.60* (6.90)	-12.77* (-7.99)	0.30* (7.76)	-0.43* (-6.69)	0.24* (2.34)	-	-	18.72 37.44	855	245.90	0.2373
	58.45 (0.85)	**173.39* (6.45)**	**-19.79* (-6.45)**	**0.56* (5.72)**	**-0.66* (-4.4)**	**0.07 (0.59)**	**0.10 (0.40)**	-	14.04 28.08 42.12		**245.48**	**0.2399**
	33.95 (0.37)	188.68* (4.69)	-22.53* (-4.19)	0.70* (3.40)	-0.66* (-2.24)	-0.15 (-0.79)	0.13 (0.52)	0.08 (0.13)	11.23 22.46 33.70 44.93		245.96	0.2370
VILANOVA	499.2* (7.71)	-0.49 (-0.35)	-0.72 (-0.26)	0.00 (0.11)	0.05 (0.46)	-0.27 (-1.72)	-	-	15.26 30.52	500	231.13	0.2573
	444.1* (5.12)	19.03 (0.46)	-4.51 (-0.88)	0.15 (0.80)	-0.17 (-0.64)	0.10 (0.58)	-0.50 (-1.53)	-	11.44 22.89 34.33		231.20	0.2568
	341.9* (3.18)	**81.88 (1.43)**	**-15.33 (-1.79)**	**0.68 (1.77)**	**-0.91 (-1.73)**	**0.54 (1.42)**	**-0.59 (-1.12)**	**0.10 (0.12)**	9.15 18.31 27.46 36.62		**230.79**	**0.2594**

t-values are in brackets
(*) statistically significant variable

Table 4: BMR estimates 1996

Function	***Coefficients***	***t-stad.***	***Knots***	***Obs.***	***S.E.***	***R^2***
Exponential	D_0= 597.19	50.35	-	3481	232.23	0.2570
	g = -0.05	-23.38				
Cubic Spline	a_1= 269.5	8.21		3481	228.62	0.2800
	b_1= 79.10	7.59				
	c_1= -9.32	-9.73	15.36			
	d_1= 0.25	9.92	30.92			
	d_2-d_1= -0.33	-9.43	46.38			
	d_3-d_2= 0.13	5.00				
	d_4-d_3= -0.17	-1.80				

Table 5: Barcelona Population Subcentre Delimitation

	FIRST DERIVATIVE METHODOLOGY			**SECOND DERIVATIVE METHODOLOGY**				
	Subcenter Interval Distance	***Subcenter center CBD Distance***	***Total Subcenter Census Tracts***	***Subcenter Interval Distance***	***Subcenter center CBD Distance***	***Total Subcenter Census Tracts***	***Intermediate Ring Interval Distance***	***Total Intermediate Ring Census Tracts***
Granollers	24.24	29.71	143	8.96	29.71	75	10.84	67
Martorell	18.73	24.18	60	7.08	24.18	22	9.57	64
Mataró	25.23	29.52	134	11.76	29.51	89	13.34	87
Terrassa – Sabadell	16.88	23.96	361	10.07	23.96	258	9.59	181
Vilafranca	32.64	38.63	58	16.58	38.43	48	16.31	42
Vilanova	15.04	40.48	59	11.59	40.36	50	9.65	12

FIGURES

Figure 1: Subcentre Delimitation: Gradient problems

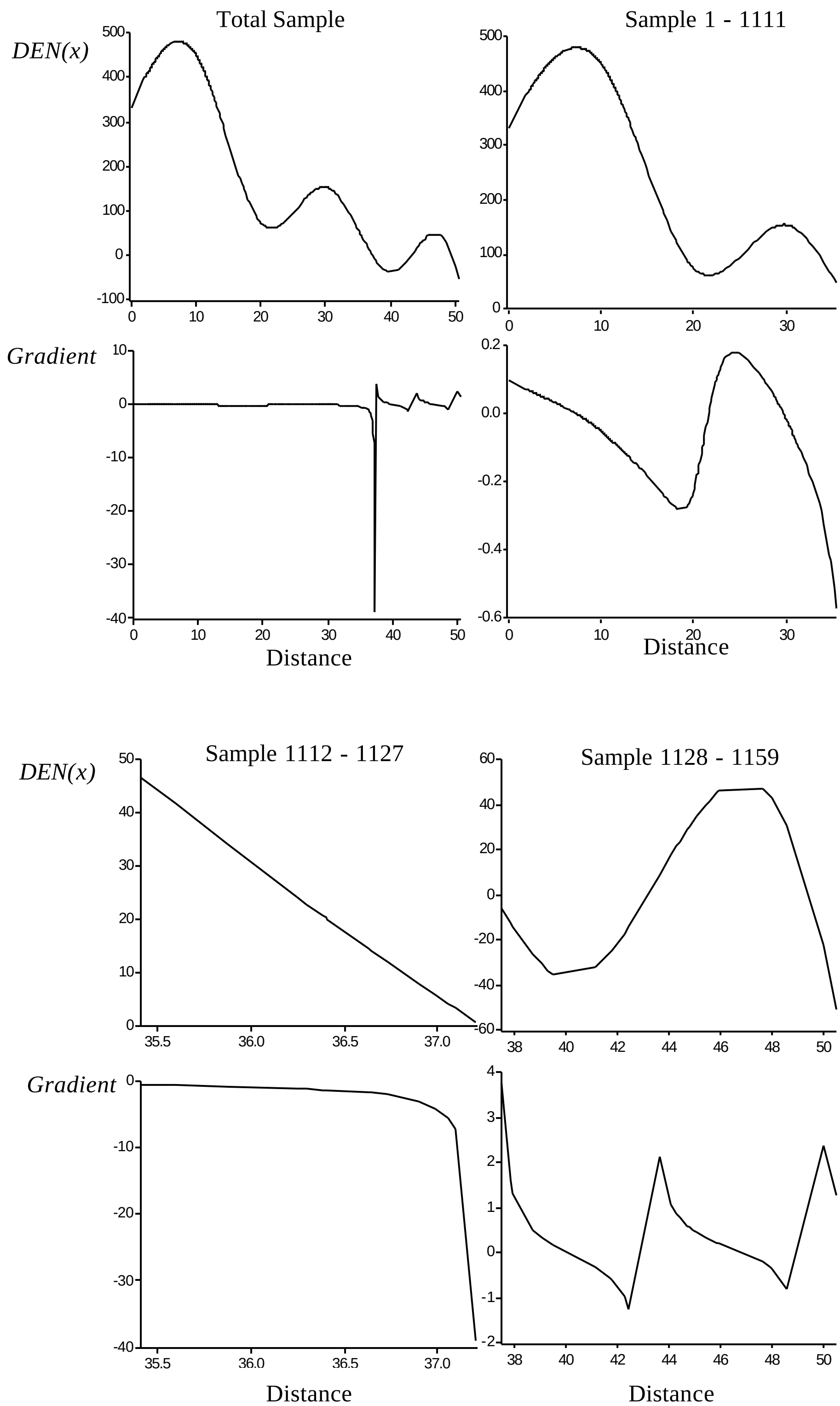

Figure 2: The BMR evolution

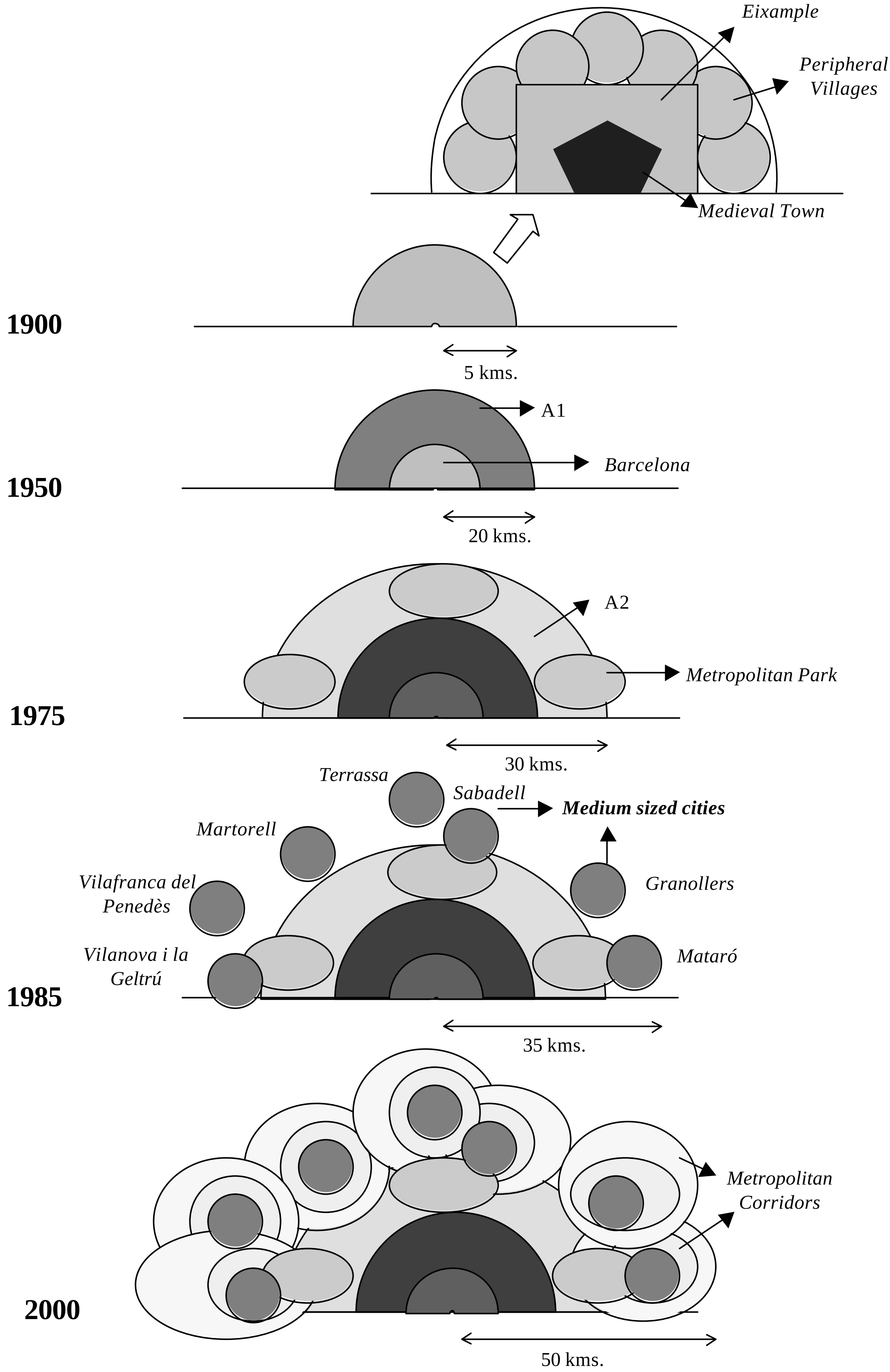

Figure 3: Exponential and Cubic Spline Density Functions Estimates: Total BMR

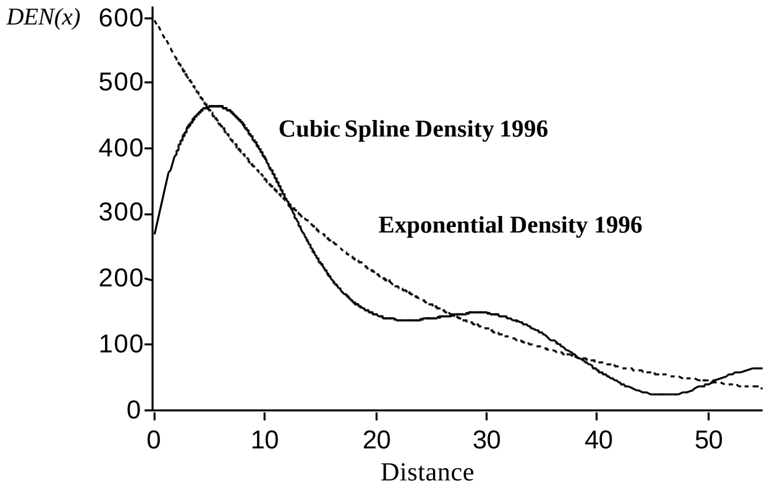

Figure 4: Cubic Spline Estimates: axes

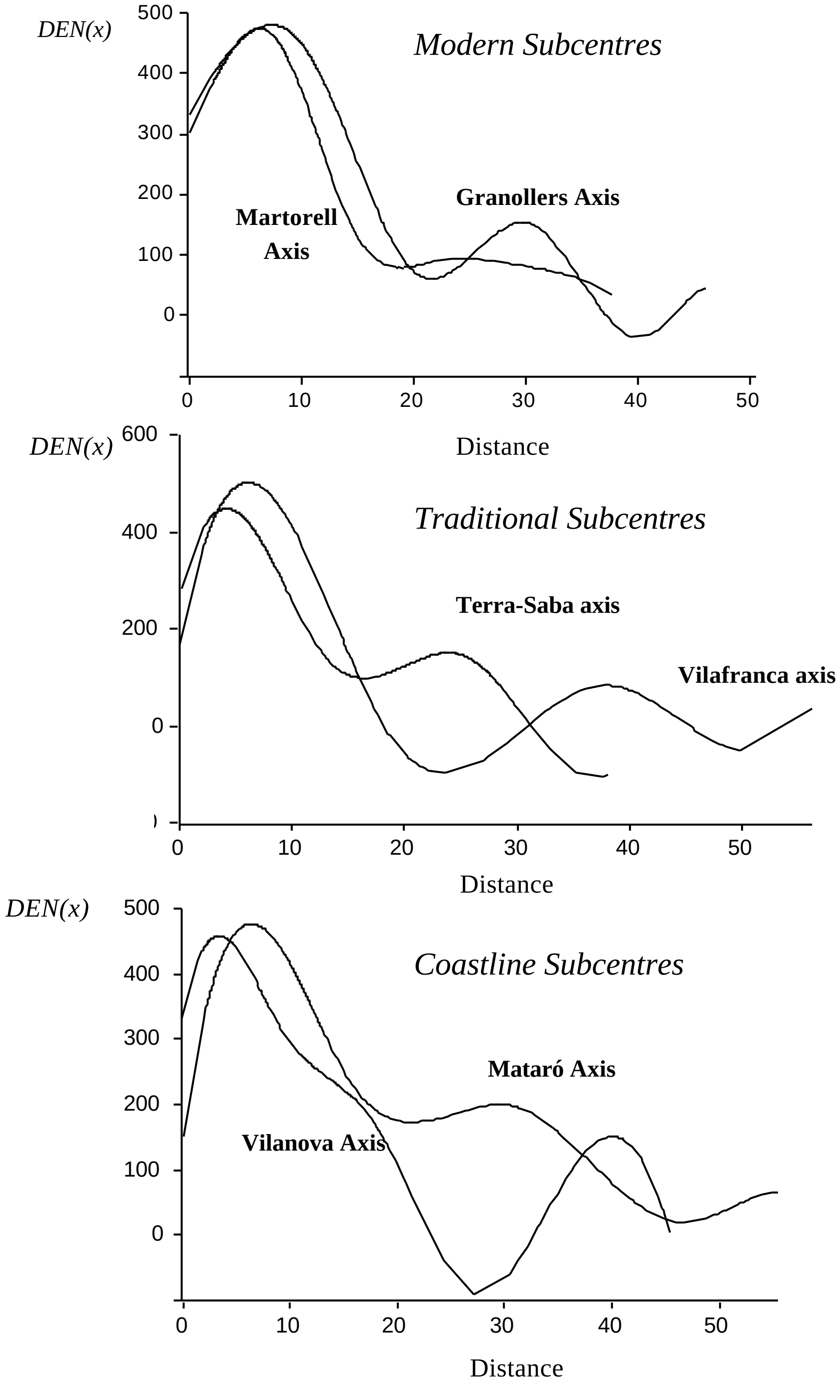

Figure 5: Barcelona Subcentre Delimitation