Estimating Liquidity Using Information on the Multivariate Trading Process

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This Version: March 31, 2006

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Abstract

In this paper we model the dynamic multivariate density of discrete bid and ask quote changes and their associated depths. We account for the contemporaneous relationship between these trading marks by exploiting the concept of copula functions. Thereby we show how to model truncations of the multivariate density in an easy way. A Metropolized-Independence Sampler is applied to draw from the dynamic multivariate density. The samples drawn serve to construct the dynamic density function of the quote slope liquidity measure, which enables us to quantify time varying liquidity risk. We analyze the influence of the decimalization at the NYSE on liquidity.

JEL classification: $G10, F30, C30$

Keywords: Liquidity, Copula Functions, Trading Process, Decimalization, Metropolized-Independence Sampler

1 Introduction

This paper exploits the concept of copula functions to model a conditional truncated multivariate density. We show how to model a conditional multivariate time series density composed of count and continuous variables, and, how to impose certain restrictions on those variables (truncations) in an easy way. Furthermore, we show how to sample from a derived conditional density at every point in time using a metropolized independence sampler (MIS).

We use this approach to derive the conditional density function of a liquidity supply measure for five stocks traded at the New York Stock Exchange (NYSE). The conditional density function of our liquidity measure allows to extract information on the progress of time varying liquidity risk on an intraday basis faced by market participants. We also analyze the impact of the decimalization at the NYSE (29^{th} January 2001) on the shape of conditional density of our liquidity supply measure.

The term "liquidity" is used to describe several aspects of the trading process. Although many people (in particular market participants) have an intuitive feeling about what liquidity means, researchers face a major difficulty in defining the term liquidity appropriately. The following citations should serve as examples:

- Black (1971) "Liquidity seems to have several meanings."
- Kyle (1985): "Market liquidity is a slippery and elusive concept, (...)"
- Engle & Lange (2001): "Liquidity (...) has a variety of definitions and interpretations."
- Danielsson & Payne (2002): "Conceptually, the task of measuring liquidity is challenging due to the fact that there is no generally accepted definition of a 'liquid market'."

However, there is a kind of consensus in the literature that liquidity is the ability to trade a large volume quickly at a low transaction cost and that a mispriced price should quickly return to its fundamental value. But still, these four related dimensions (depth, immediacy, tightness, resilience) are of different importance for market participants and different market participants affect these dimensions in different ways. Furthermore, the researcher needs to quantify and perhaps aggregate these dimensions into a "liquidity measure" which is most suited for his research interest.

In this paper we consider the quote slope liquidity measure which has been introduced by Hasbrouck & Seppi (2001). It is defined as the inside bid-ask spread divided by the sum of the logarithmic bid and ask depths at the best bid and best ask, respectively. Therefore, the quote slope mainly aggregates the depth and the tightness dimension into one figure. Furthermore, the quote slope characterizes the best positions in the limit order book of the stock, but it does not mirror the complete bid and ask sides of the order book. The quote slope can be considered as a liquidity supply measure, since it describes the state of the first (best) buy and sell limit orders, which would be executed against incoming market orders. We consider the quote slope as that liquidity supply measure which extracts maximum information on the supply of liquidity in an easy way from databases such as the Quotes Database of the NYSE. Our analysis is therefore meant to provide insights into the progression of the liquidity supply on an intra day basis. We therefore aim to model the complete density of our liquidity supply measure dynamically at every point in time, since it incorporates all information on the liquidity supply at this particular time. The usefulness of this approach is obvious, since we are then able, beyond describing and making inference about the dynamics of the mean liquidity supply (see e.g. Engle & Lange (2001) for an investigation with the VNET measure and Gomber, Schweickert & Theissen (2005) for an study with the XETRA Liquidity Measure (XLM)), to characterize the dynamics of liquidity (supply) risk, considering for example the change of a certain risk measure like the second moment or a prespecified quantile of the liquidity density over time. This information is of utmost importance for traders since it allows them to optimize their intraday trading and optimal liquidation strategies (see e.g. Bertsimas & Lo (1998), Almgren & Chriss (2000) and Subramanian & Jarrow (2001)). Furthermore, being able to characterize how liquidity risk behaves over time may help to improve models where liquidity risk is priced, such as the liquidity adjusted Capital Asset Pricing Models of Acharya & Pedersen (2004) and Pastor & Stambaugh (2001). Moreover, our empirical observation of a time varying liquidity risk questions several prominent models where liquidity shocks are assumed to have a constant mean and a constant variance (see e.g. Karpoff (1986), Michaely & Vila (1996), Michaely, Vila & Wang (1996) and Fernando (2003)).

From an econometric and computational point of view, constructing the time-varying density of the quote slope liquidity measure is not trivial at all. We rely on the following strategy. First, we model the multivariate dynamic density of the variables involved in the computation of the quote slope, where we especially take the contemporaneous relationship between the variables into account. Second, we draw a sample of length N from this multivariate density at every point in time. Third, using our drawn samples, we compute the value of the quote slope liquidity measure at every point in time N-times. Then, at each point in time, the empirical density function of our N quote slope values is the conditional (on time and on the dynamics of the explanatory variables used in the modelling of the multivariate density) density function of the quote slope liquidity measure.

We model the multivariate density of the best bid and ask quotes' changes and their corresponding bid and ask depths. Thus, we consider a four dimensional density, where we need to account for the fact that the bid and ask quote changes are discrete multiples of the tick size. We model these two count variables with the Integer Count Hurdle (ICH) Model of Liesenfeld, Nolte & Pohlmeier (2006), since it allows us to construct a dynamic count data density with support $\mathbb{Z}$. The bid and ask depths are treated as continuous variables with support $\mathbb{R}^+$, and their dynamic density is modelled with Burr-distributed Autoregressive Conditional Duration (ACD) models of Engle & Russell (1998). The contemporaneous relationship between these four variables is modelled with a copula function, which became popular with the article of Sklar (1959). For the discrete variables, we thereby rely on the concept of continuization of Stevens (1950) and Denuit & Lambert (2005). An important characteristic that we need to account for in the modelling of the dynamic multivariate density is that the bid-ask spread, which is a function of the previous quotes and their corresponding changes, always needs to be positive. This restriction needs to be modelled by truncating the multivariate density correspondingly. We model this truncation using a truncated copula density, which allows us to incorporate the restrictions without imposing restrictions on the marginal processes.

Instead of modelling the density of the quote slope liquidity measure directly, we decided to use the more complex and more complicated modelling approach described above for two reasons. First, we can model the dynamics of each variable involved in the computation of the quote slope separately. This gives a very detailed picture of the reaction of these variables to shocks in the explanatory variables. Furthermore, this allows us to infer how the variables react with each other. Second, we model the discreteness of the bid-ask spread (or the bid and ask quote changes) directly. This discreteness causes humps (several modi) in the density function of the quote slope, which cannot easily be modelled within a parametric framework. In a nutshell, we are able to model the dynamic features of our variables, and therefore of the quote slope, much better than it could be done by considering the aggregated quote slope variable directly.

The paper is organized as follows. In Section 2 we describe the modelling framework in detail. Section 3 contains the descriptive analysis and provides first results for the quote slope liquidity measure. Section 4 presents the estimation results and the analysis of the conditional quote slope desity. Section 7 discusses the results and concludes.

2 Modelling Liquidity

As already mentioned in the introduction, it is not completely clear what liquidity precisely means and how it should be measured, but there is a kind of general consensus that liquidity encompasses at least four properties:

- **Depth:** the ability to trade large volumes, with little influence on the best quotes
- **Immediacy:** the ability to trade quickly at the current quotes
- **Tightness:** low cost of turning over a position at the same time
- **Resiliency:** the recovery speed of the price after an uninformative shock (large trade)

Thereby it is often unclear how these four aspects should be measured exactly. Generally speaking, while accounting for the desired properties, a measure of liquidity (liquidity function) at time t is a function of trading marks that characterize the transaction process. Typical examples are transaction price, traded volume, bid & ask quotes, bid & ask depths, number of transactions and number of quote updates in a specified time period. The outcomes of these marks determine the liquidity of a market or more specifically - the liquidity of a particular stock.

In order to investigate how liquidity evolves over time and how it is affected by changing market conditions of utmost important it is to i) understand how the trading marks interact with each other over time as well as contemporaneously and ii) characterize the conditional density function of a liquidity measure. The latter enables us to quantify liquidity risk in a very elaborate way.

For example, we are able to figure out how the 5% quantile of our liquidity measure changes over time and how it is affected by actions of market participants (e.g. market makers, traders). On the one hand, this is a very important information for a trader, who wants to transact a large position and on the other hand - for a market maker (of an illiquid stock) who usually has to provide liquidity up to a certain degree to ensure smooth trading. Without understanding how the trading marks interact with each other, which means to characterize the joint conditional density function of these marks, it makes no sense to compute the conditional liquidity density function.

2.1 General Model

To formalize the discussion, let Z_t denote the k dimensional vector of trading marks which characterize the transaction process at time t . Let $F_{Z_t}(z|\mathfrak{F}_{t-1})$ be the conditional on $\mathfrak{F}_{t-1}$ cumulative distribution function of Z_t, where $\mathfrak{F}_{t-1}$ denotes the information set at $t-1$. Let

$$L_t(Z_t|\mathfrak{F}_{t-1})$$

be the conditional liquidity function based on Z_t. Then, the conditional distribution of L_t is given by

$$F_{L_t}(l|\mathfrak{F}_{t-1}) = \mathrm{P}(L_t \leq l|\mathfrak{F}_{t-1}) = \int\limits_{L_t(Z_t|\mathfrak{F}_{t-1})\leq l} dF_{Z_t}(z|\mathfrak{F}_{t-1}). \tag{1}$$

One can relate the joint distribution of Z_t to its marginals using copula function $\mathbf{C}$:

$$F_{Z_t} = \mathbf{C}(F_{Z_{1t}}, F_{Z_{2t}}, \ldots, F_{Z_{kt}}). \tag{2}$$

The corresponding joint density of Z_t can be thus given by the product of the marginals and the copula density:

$$\begin{aligned} f_{Z_t} &= f_{Z_{1t}} \cdot f_{Z_{2t}} \cdots f_{Z_{kt}} \cdot \frac{\partial \mathbf{C}(F_{Z_{1t}}, F_{Z_{2t}}, \ldots, F_{Z_{kt}})}{\partial F_{Z_{1t}}, \partial F_{Z_{2t}}, \ldots, \partial F_{Z_{kt}}} \\ &= f_{Z_{1t}} \cdot f_{Z_{2t}} \cdots f_{Z_{kt}} \cdot \mathbf{c}(F_{Z_{1t}}, F_{Z_{2t}}, \ldots, F_{Z_{kt}}), \end{aligned} \tag{3}$$

where $\mathbf{c}$ denotes the density of the copula function. Using this representation the appropriate models for the distribution (density) functions of the marginals and the copula should be specified. Sklar (1959) proved the existence of the copula function $\mathbf{C} : [0,1]^k \rightarrow [0,1]$ in equation (2) and he showed its uniqueness in the case where $Z_{it}, \forall i$ are continuous. Relying on this modelling approach we need to ensure that the marginals and likewise the copula density are correctly specified.

The concept of copula functions is a very flexible tool for modelling the joint density of different variables. As shown in equation (3), it allows to decompose their multivariate density into the marginal distributions of processes to be modelled and the copula function that is responsible for the contemporaneous dependence among them. What makes the copula concept very desirable in econometrics, is its ability to build a true multivariate density when the marginal processes rely on different distributions. It is also possible to apply a copula function to the marginal densities with discrete and real support getting a valid joint distribution function which reflects the dependence between such variables. This special feature makes the concept especially suitable in case of our study.

2.2 Quote Slope

We now consider the quote slope liquidity measure introduced by Hasbrouck & Seppi (2001) in detail. Let $Q_t^b \in \mathbb{N}$ ($Q_t^a \in \mathbb{N}$) denote the bid (ask) quote as multiples of the tick size with corresponding depth $D_t^b \in \mathbb{R}^+$ ($D_t^a \in \mathbb{R}^+$) at time t, summarized in Z_t. Although the depths are recorded in multiples of one hundred shares at the NYSE, due to their large outcome space we consider them to be element of $\mathbb{R}^+$. The quote slope is then given by

$$L_t(Z_t|\mathfrak{F}_{t-1}) \equiv \frac{Q_t^a - Q_t^b}{\ln(D_t^a) + \ln(D_t^b)}.$$

The numerator represents the inside bid-ask spread, whereas the denominator is the sum of the logarithmic depths at the best bid and ask quotes. Thus, a smaller bid-ask spread as well as larger bid and ask depths yield a higher liquidity. This ratio can be seen as an ex ante measure of liquidity or a measure of liquidity supply since it does not involve any information from an executed transaction. What directly affect the quote slope are incoming market orders since the measure characterizes the first stage of the bid and the ask side of the limit-order book as illustrated in Figure 1.

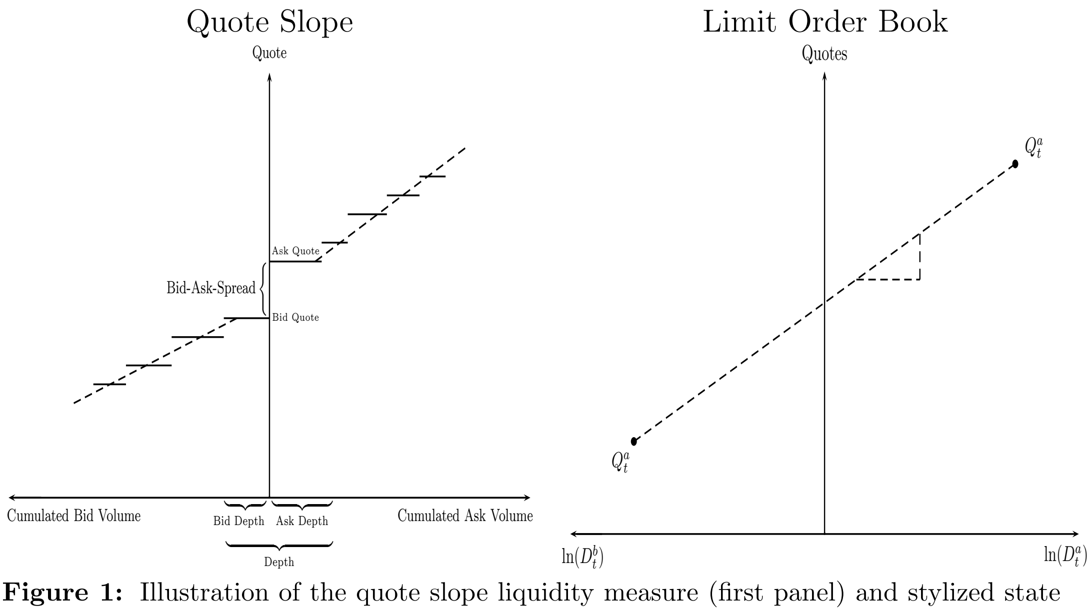

Figure 1: Illustration of the quote slope liquidity measure (first panel) and stylized state of the limit order book at one period (second panel).

The quote slope can be expressed in terms of tick quote changes as

$$L_t(Z_t|\mathfrak{F}_{t-1}) = \frac{Q_t^a - Q_t^b}{\ln(D_t^a) + \ln(D_t^b)} = \frac{Q_{t-1}^a - Q_{t-1}^b + C_t^a - C_t^b}{\ln(D_t^a) + \ln(D_t^b)}, \tag{4}$$

where $C_t^b \in \mathbb{Z}$ ($C_t^a \in \mathbb{Z}$) is the change of the bid (ask) quote from $t-1$ to t. The conditional distribution function of $L_t(Z_t|\mathfrak{F}_{t-1})$ is again given by equation (1) where

$$\int\limits_{L_t(Z_t|\mathfrak{F}_{t-1})\leq l} dF_{Z_t}(z|\mathfrak{F}_{t-1}) = \int\limits_{L_t(Z_t|\mathfrak{F}_{t-1})\leq l} f_{Z_t}(z|\mathfrak{F}_{t-1})dz. \tag{5}$$

Since Q_{t-1}^a and Q_{t-1}^b are measurable with respect to $\mathfrak{F}_{t-1}$ it is sufficient to consider the joint density of the transformed marks $Z_t^* = (C_t^b, C_t^a, D_t^b, D_t^a)$ given by $f_{Z_t^*}(z|\mathfrak{F}_{t-1})$. The great advantage of this representation is that we take into account the discreteness of the quote price changes and therefore of the bid-ask spread. We propose a parametric model for the conditional joint density $f_{Z_t^*}(z|\mathfrak{F}_{t-1})$ which can be expressed as:

$$f_{Z_t^*} = f_{C_t^b} \cdot f_{C_t^a} \cdot f_{D_t^b} \cdot f_{D_t^a} \cdot \mathbf{c}(F_{C_t^b}, F_{C_t^a}, F_{D_t^b}, F_{D_t^a}), \tag{6}$$

Due to this representation of the multivariate density, econometric modelling should involve identification of the marginal distributions as well as the appropriate copula function. In the following we present the parametric models applied to the marginal distributions of the joint density. We rely on the ICH model of Liesenfeld et al. (2006) for the discrete variables (C_t^b, C_t^a), on ACD models for the real positive variables (D_t^b, D_t^a) and on the copula concept to model the contemporaneous relationships between the marks.

Quote Changes

We start with the description of ICH model for quote changes C_t^b. (The exposition is built for bid quote changes C_t^a, ask quote changes are modelled in an analogical way). The ICH model is based on the concept of decomposing the bid-quote change process into two components, a direction process and a size process given that there is a change in the direction of variable movement. Let π_{jt}^b, $j \in \{-1, 0, 1\}$ denote the conditional probability of a decreasing $\mathrm{P}(C_t^b < 0|\mathfrak{F}_{t-1})$, unchanged $\mathrm{P}(C_t^b = 0|\mathfrak{F}_{t-1})$ or increasing bid-quote change $\mathrm{P}(C_t^b > 0|\mathfrak{F}_{t-1})$ at time t. The conditional density of a bid-quote change is then given by

$$f_{C_t^b}(c_t) = {\pi_{-1t}^b}^{\mathbb{1}_{\{C_t^b<0\}}} \cdot {\pi_{0t}^b}^{\mathbb{1}_{\{C_t^b=0\}}} \cdot {\pi_{1t}^b}^{\mathbb{1}_{\{C_t^b>0\}}} \cdot f_{|C_t^b|}(|c_t|\,|C_t^b \neq 0, \mathfrak{F}_{t-1})^{(1-\mathbb{1}_{\{C_t^b=0\}})},$$

where $f_{|C_t^b|}(|c_t|\,|C_t^b \neq 0, \mathfrak{F}_{t-1})$ denotes the conditional density of an absolute bid-quote change, with support $\mathbb{N} \setminus \{0\}$. To get a parsimoniously specified model, we adopt the simplification of Liesenfeld et al. (2006), that the conditional density of an absolute bid-quote change stems from the same distribution irrespectively whether it is an upward or downward bid-quote change.

In order to model the conditional probabilities of a quote direction process, we apply the autoregressive conditional multinomial model (ACM) of Russell & Engle (2002) with a logistic link function, given by

$$\pi^b_{jt} = \frac{\exp(\Lambda^b_{jt})}{\sum_{j=-1}^{1} \exp(\Lambda^b_{jt})}$$

with normalizing constraint $\Lambda^b_{0t} = 0$, $\forall t$. The resulting vector of log-odds ratios $\Lambda^b_t \equiv (\Lambda^b_{-1t}, \Lambda^b_{1t})' = (\ln[\pi^b_{-1t}/\pi^b_{0t}], \ln[\pi^b_{1t}/\pi^b_{0t}])'$ is specified as a multivariate ARMA model:

$$\Lambda^b_t = \sum_{l=0}^{m} G^b_l Z^b_{t-l} + \lambda^b_t \quad \text{with} \quad \lambda^b_t = \mu^b + \sum_{l=1}^{p} B^b_l \lambda^b_{t-l} + \sum_{l=1}^{q} A^b_l \xi^b_{t-l}. \tag{7}$$

The vector Z^b_t contains further explanatory variables, where G^b_l denotes the corresponding coefficient matrix. μ^b denotes the vector of constants, B^b_l and A^b_l denote 2×2 coefficient matrices. The innovation vector of the ARMA model is specified as martingale differences given by

$$\xi^b_t \equiv (\xi^b_{-1t}, \xi^b_{1t})', \quad \text{where} \quad \xi^b_{jt} \equiv \frac{x^b_{jt} - \pi^b_{jt}}{\sqrt{\pi^b_{jt}(1-\pi^b_{jt})}}, \quad j \in \{-1, 1\}, \tag{8}$$

and

$$x^b_t \equiv (x^b_{-1t}, x^b_{1t})' = \begin{cases} (1,0)' & \text{if } C^b_t < 0 \\ (0,0)' & \text{if } C^b_t = 0 \\ (0,1)' & \text{if } C^b_t > 0, \end{cases} \tag{9}$$

Therefore, ξ^b_t represents the standardized state vector x^b_t.

The conditional density of the absolute bid-quote change is modelled with an at-zero-truncated Negative Binomial (Negbin) distribution, given by

$$f_{|C^b_t|}(|c_t|\,|C^b_t \neq 0, \mathfrak{F}_{t-1}) \equiv \frac{\Gamma(\kappa^b + |c_t|)}{\Gamma(\kappa^b)\Gamma(|c_t|+1)} \left(\left[\frac{\kappa^b + \omega^b_t}{\kappa^b}\right]^{\kappa^b} - 1 \right)^{-1} \left(\frac{\omega^b_t}{\omega^b_t + \kappa^b} \right)^{|c_t|},$$

where $|c_t| \in \mathbb{N}\backslash\{0\}$, $\kappa^b > 0$ denotes the dispersion parameter and ω^b_t is parameterized using the exponential link function with a generalized autoregressive moving average model (GLARMA) in the following way:

$$\ln \omega_t^b = \delta^{b'} \tilde{D}_t + \sum_{l=0}^{m} \gamma_l^{b'} \tilde{Z}_{t-l}^b + \tilde{\lambda}_t^b \quad \text{with} \quad \tilde{\lambda}_t^b = \tilde{\mu}^b + S^b(\nu, \tau, K) + \sum_{l=1}^{p} \beta_l^b \tilde{\lambda}_{t-l}^b + \sum_{l=1}^{q} \alpha_l^b \tilde{\xi}_{t-l}^b.$$

where $D_t \in \{-1, 1\}$ indicates a decreasing or an increasing bid-quote change at time t. The corresponding coefficient vector is denoted by δ. $\tilde{Z}_t^b$, with coefficient vector γ_l, contains further explanatory variables. $\tilde{\mu}$ denotes the constant term. $S^b(\nu, \tau, K) \equiv \nu_0 \tau + \sum_{k=1}^{K} \nu_{2k-1} \sin(2\pi(2k-1)\tau) + \nu_{2k} \cos(2\pi(2k)\tau)$ is a fourier flexible form to capture intraday seasonality in the absolute bid-quote changes, where τ is the intraday trading time standardized on $[0, 1]$ and ν is a $2K + 1$ dimensional parameter vector. β_l as well as α_l denote coefficients and $\tilde{\xi}_t^b$ is the innovation term that drives the GLARMA model in λ_t^b. $\tilde{\xi}_t^b$ is constructed as:

$$\tilde{\xi}_t^b \equiv \frac{|C_t^b| - \mathrm{E}(|C_t^b|\,|C_t^b \neq 0, \mathfrak{F}_{t-1})}{\mathrm{V}(|C_t^b|\,|C_t^b \neq 0, \mathfrak{F}_{t-1})^{1/2}},$$

where the conditional moments of the at-zero-truncated Negbin distribution are given by

$$\begin{aligned} \mathrm{E}(|C_t^b|\,|C_t^b \neq 0, \mathfrak{F}_{t-1}) &= \frac{\omega_t^b}{1 - \vartheta_t^b}, \\ \mathrm{V}(|C_t^b|\,|C_t^b \neq 0, \mathfrak{F}_{t-1}) &= \frac{\omega_t^b}{1 - \vartheta_t^b} - \left(\frac{\omega_t^b}{(1 - \vartheta_t^b)} \right)^2 \left(\vartheta_t^b - \frac{1 - \vartheta_t^b}{\kappa^b} \right), \end{aligned}$$

where ϑ_t^b is given by $\vartheta_t^b = [\kappa^b / (\kappa + \omega_t^b)]^{\kappa^b}$.

Depths

In order to cover the dynamic pattern of the depth process we apply ACD models. Our exposition covers only the bid-depth (D_t^b) case and the ask-depth (D_t^a) case follows analogously. The conditional density of the bid-depth is denoted by

$$f_{D_t^b}(d_t | \mathfrak{F}_{t-1}).$$

Engle & Russell (1998) assume that the conditioning filtration $\mathfrak{F}_{t-1}$ enters the conditional density only through the conditional mean function, which we denote by $\varphi_t^b \equiv \varphi_t^b(\theta^b | \mathfrak{F}_{t-1})$, where θ^b denotes the parameter vector. The ACD model incorporates the conditional mean function multiplicatively

$$D_t^b = \varphi_t^b \cdot \varepsilon_t,$$

where the density $f_{\varepsilon_t}(\cdot)$ of ε_t is assumed to have unit mean, a positive support and does not rely on further conditioning information. Applying the transformation theorem, the conditional density of the bid-depth is given by

$$f_{D_t^b}(d_t|\mathfrak{F}_{t-1}) = \frac{1}{\varphi_t^b} f_{\varepsilon_t}\left(\frac{d_t}{\varphi_t^b}\right).$$

We assume that $f_{\varepsilon_t}(\cdot)$ is the Burr density function which is given by

$$f_{\varepsilon_t}(x) = \frac{\breve{\kappa}^b}{\lambda^b}\left(\frac{x}{\lambda^b}\right)^{\breve{\kappa}^b-1}\left[1+\sigma^{2,b}\left(\frac{x}{\lambda^b}\right)^{\breve{\kappa}^b}\right]^{-(1-\sigma^{-2,b})},$$

where $\lambda^b > 0$, $\breve{\kappa}^b > 0$ and $\sigma^{2,b} > 0$. The Burr density is a very flexible specification, since it allows for a non-monotonic shape of the associated hazard function. Furthermore, the Burr density nests the log-logistic density for $\sigma^{2,b} = 1$ and the Weibull density for $\sigma^{2,b} = 0$. The dynamics of the conditional mean function φ_t^b is modelled in the traditional autoregressive way as

$$(1-\breve{\beta}_p^b(L))(\varphi_t^b - \breve{\gamma}^{b\prime}\breve{Z}_t^b) = \breve{\mu}^b + \breve{\alpha}_q^b(L)D_t^b, \tag{10}$$

where $\breve{\mu}^b$ denotes the constant and $\breve{\beta}_p^b(L)$ as well as $\breve{\alpha}_q^b(L)$ denote lag-polynomials of order p and q. $\breve{Z}_t^b$ is the vector of further explanatory variables, with corresponding coefficient vector $\breve{\gamma}^b$.

Copula

Using a copula concept in the context of our study has two main advantages. As mentioned before it allows to model the joint density between the set of discrete (quote changes) and the set of continuous (depths) variables, what enables us in the next step to derive the density function for the liquidity measure. But what is of ultimate importance is that the copula allows to model restrictions (truncations) on the support of the joint density in an easy and elegant way. The restriction we need to impose in our model is that the bid-ask spread must not become negative. In terms of quote changes from $t-1$ to t we need to ensure that the following inequality holds:

$$C_t^a - C_t^b > Q_{t-1}^b - Q_{t-1}^a. \tag{11}$$

We model the copula density $\mathbf{c}(\cdot)$ given in equation (6) with a truncated 4-dimensional Gaussian copula density. The non-truncated 4-dimensional Gaussian copula density is given by:

$$\tilde{\mathbf{c}}(y_{1t}, y_{2t}, y_{3t}, y_{4t}; \tilde{\Sigma}) = \det(\tilde{\Sigma})^{-0.5} \exp\left(\frac{1}{2} q_t'(I_4 - \tilde{\Sigma}^{-1}) q_t\right), \tag{12}$$

where $\tilde{\Sigma}$ denotes the covariance matrix of $q = (q_{1t}, q_{2t}, q_{3t}, q_{4t})'$ with $q_{it} = \Phi^{-1}(y_{it}), i = 1, \ldots, 4$. The truncated 4-dimensional Gaussian copula density, which accounts for the restrictions stated in formula (11) is then given by

$$\mathbf{c}(y_{1t}, y_{2t}, y_{3t}, y_{4t}; \Sigma | C_t^a - C_t^b > Q_{t-1}^b - Q_{t-1}^a) = \frac{\tilde{\mathbf{c}}(y_{1t}, y_{2t}, y_{3t}, y_{4t}; \tilde{\Sigma})}{\mathrm{P}(C_t^a - C_t^b > Q_{t-1}^b - Q_{t-1}^a)}. \tag{13}$$

Note, that $\mathbf{c}(\cdot)$ is a conditional on $\mathfrak{F}_{t-1}$ copula function, since the probability of the truncated region depends on the bid-ask spread at $t-1$. Using this notation $y_{1t} = F_{C_t^b}$, $y_{2t} = F_{C_t^a}$, $y_{3t} = F_{D_t^b}$ and $y_{4t} = F_{D_t^a}$. Assuming that the marginal distributions are correctly specified, for the continuous variables (D_t^b, D_t^a), y_{3t} and y_{4t} are uniformly $U(0,1)$ distributed between zero and one (probability integral transformation). For the discrete variables (C_t^b, C_t^a) this results does not hold, since their cumulative distribution function possesses jump points.

There exist two main approaches to modelling multivariate processes with discrete state space of outcomes applying the concept of copula functions. The first approach is advocated by Cameron, Li, Trivedi & Zimmer (2004) who use the Archimedean copulas to model the bivariate distribution of count variables. They pointed out that it is not possible to obtain the simple canonical representation of copula function out of equation (3) by a differentiation method as the copula function for the count variables is not continuous. In order to get the copula density they use a finite difference approximation of the derivatives. The alternative approach that we follow here relies on using the continuisation method suggested by Stevens (1950) and Denuit & Lambert (2005). The continuisation concept rely on generating artificially continuized variables C_t^{b*}, C_t^{a*} from the discrete count variables C_t^b, C_t^a by adding independent uniformly $U(0,1)$ distributed random variables U_t^b, U_t^a, i.e.

$$C_t^{b/a*} = C_t^{b/a} + (U_t^{b/a} - 1). \tag{14}$$

Their distribution functions are denoted by $F_{C_t^{b*}}$ and $F_{C_t^{a*}}$. The probability integral transformation is then computed on the basis of these continuized distributions, i.e. $y_{1t} = F_{C_t^{b*}}$, $y_{2t} = F_{C_t^{a*}}$, where $F_{C_t^{b*}}$ and $F_{C_t^{a*}}$ can be computed as

$$F_{C_t^{b/a*}}(c_t^{b/a*}) = F_{C_t^{b/a}}(c_t^{b/a} - 1) + U_t^{b/a} \cdot f_{C_t^{b/a}}(c_t^{b/a}). \tag{15}$$

According to Denuit & Lambert (2005) the continuous extension of discrete variables does not influence the concordance between them. Concordance is a measure of dependance, in case of our variables it can be explained as: C_t^b and C_t^a are concordant if high values of C_t^b are associated with the high values of C_t^a, i.e.: $C_t^a > C_t^b \Longleftrightarrow C_t^{a*} > C_t^{b*}$.

Estimation

The parameters of the joint model can be estimated with the Maximum Likelihood (ML) method, where the conditional log likelihood function is given by:

$$\begin{aligned} \ln \mathcal{L} &= \sum_{t=1}^{T} [\ln(f_{C_t^b}) + \ln(f_{C_t^a}) + \ln(f_{D_t^b}) + \ln(f_{D_t^a}) + \\ &\quad + \ln(\mathbf{c}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^b}, F_{D_t^a}))], \end{aligned} \tag{16}$$

Due to the complexity of the model we apply a two step estimation procedure described in Cherubini, Luciano & Vecchiato (2004). In the first step we estimate the parameters of the marginal i.e. ICH and ACD models. Since there are no parameter restrictions across parameter space of the marginal models, the maximization of the first four components of the likelihood function can be performed separately. (For the detailed form of the likelihood function for ICH model please refer to Liesenfeld et al. (2006))

In the second step of the maximization we can obtain consistent estimates of parameters for the gaussian copula function without applying any optimization procedure. The ML estimate of Σ, i.e. the variance-covariance matrix of the multivariate normal distribution with a zero mean is given by:

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^{T} \hat{q}_t \hat{q}_t{}', \tag{17}$$

where $\hat{q}_t = (\Phi^{-1}(\hat{F}_{C_t^{b*}}), \Phi^{-1}(\hat{F}_{C_t^{b*}}), \Phi^{-1}(\hat{F}_{D_t^b}), \Phi^{-1}(\hat{F}_{D_t^b)})'$. Since the unknown Σ is estimated on our empirical data sample, it implicitly accounts for restriction given by equation (11).

3 Empirical Analysis

The empirical analysis is carried out for five stocks with medium and high market capitalizations. The stocks with medium market capitalizations are Black & Decker Corp. (BDK) \$6.60 bn. and HJ Heinz Co. (HNZ) \$ 11.24 bn. The ones with high market capitalizations are Pfizer Inc. (PFE) \$ 182.15 bn, Citigroup Inc. (C) \$ 231.14 bn and Exxon Mobil Corp. (XOM) \$ 376.64 bn. All stocks are traded at the NYSE and the corresponding data stems from the Trades and Quotes (TAQ) Database. We consider two periods of investigation: The first one ranges from the 2^{nd} (Tuesday) to the 26^{th} (Friday) January 2001, which are the four weeks directly before decimalization was introduced. The second period ranges from the 30^{th} (Tuesday) January 2001 to the 23^{rd} (Friday) February 2001, which are the four weeks thereafter. We omitted the 29^{th} January 2001 since it was a Monday and we wanted to compare periods with same daily structure. The data is aggregated to equidistant 5 min data. Since market capitalization can be considered as a rough proxy for liquidity one can consider the stocks chosen to be of medium and high liquidity. Let us recall that the quote slope is given by

$$L_t(Z_t^*|\mathfrak{F}_{t-1}) = \frac{Q_{t-1}^a - Q_{t-1}^b + C_t^a - C_t^b}{\ln(D_t^a) + \ln(D_t^b)},$$

where we need to model the following joint conditional density:

$$f_{Z_t^*} = f_{C_t^b} \cdot f_{C_t^a} \cdot f_{D_t^b} \cdot f_{D_t^a} \cdot \mathbf{c}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^{b*}}, F_{D_t^{a*}}). \tag{18}$$

The descriptive analysis provides a motivation why we model the conditional density functions in equation (18) as proposed in the previous section. Furthermore, the descriptive analysis is meant to give first insights into the consequences of the decimalization at the NYSE. We will not show every result for all stocks but only BDK, the corresponding tables for the other four stocks can be found in the Appendix.

Motivation

Figure 2 shows the histogram for BDK of the bid quote and the ask quote changes (C_t^b and C_t^a) in ticks in January 2001 before the decimalization and in February 2001 after the decimalization. We observe that the histograms have a fairly large support between -10 and 10 ticks in January and an even larger support between -35 and 35 ticks in February. The discreteness of the quote changes combined with the large number of non-zero states justifies the ICH-model approach of Liesenfeld et al. (2006)

for the quote changes, which enables us to construct a conditional discrete density with an integer support. The alternative models to model discrete price changes of Hausman, Lo & MacKinlay (1992) and Russell & Engle (2002) suffer from the drawbacks that they are only capable to model a small finite number of discrete states and that they cannot model states with no observations. Furthermore, the proposed approach is more parsimonious than the decomposition model of Rydberg & Shephard (2003) which also allows to model a conditional discrete density with an integer support.

Table 1 contains the descriptive statistics of the absolute bid and ask quote changes in \$ and not in ticks. We observe that the values of the absolute quote changes at all presented quantiles are, for both bid and ask quotes, smaller in February than in January. This means in particular that, although the distribution of the quote changes has a larger support in terms of ticks in February than in January, the volatility of the quote changes in terms of \$ decreased from January to February.

Figure 3 shows the multivariate autocorrelogram of the vector of the quote direction change as defined in equation (9). There is a certain but no overwhelming dynamic pattern which should be explained by the ACM part of the ICH model. Considering the autocorrelogram of the absolute quote direction changes, which is indeed a proxy for the volatility of the quote direction changes, shows that there is a moderate degree of persistence which should be explained by the GLARMA part of the ICH model. These findings are underpinned by the values of the (Multivariate) Ljung-Box statistics presented in Table 1.

The depths are counted as multiples of 100 shares and range between 100 shares and several 10.000 shares for BDK or even several 100.000 shares for the stocks with a higher market capitalization as shown by the histograms in Figure 5 and by figures in Table 1. The need for the autoregressive modelling structure is affirmed by the autocorrelogram of the depths depicted in Figure 6 and by the values of the Ljung-Box statistics presented in Table 1 .

	January							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.1418	0.27	0.34	2918.58	0.1468	0.31	0.29	4409.04
std. deviation	0.1238	0.44	0.47	4171.38	0.1246	0.46	0.45	5929.37
skewness	3.2220	0.98	0.64	4.15	2.7897	0.78	0.89	3.42
kurtosis	19.4011	1.96	1.41	25.31	14.5013	1.62	1.80	20.48
minimum	0.0625	0	0	100	0.0625	0	0	100
1% Quantile	0.0625	0	0	500	0.0625	0	0	500
5% Quantile	0.0625	0	0	500	0.0625	0	0	500
10% Quantile	0.0625	0	0	500	0.0625	0	0	500
25% Quantile	0.0625	0	0	1000	0.0625	0	0	1000
50% Quantile	0.1250	0	0	1500	0.1250	0	0	2000
75% Quantile	0.1875	1	1	3000	0.1875	1	1	5000
90% Quantile	0.2500	1	1	5500	0.3125	1	1	10000
95% Quantile	0.3750	1	1	10000	0.3750	1	1	15000
99% Quantile	0.6250	1	1	25000	0.6250	1	1	27496
maximum	1.1875	1	1	44400	1.0625	1	1	63000
LB(10)	83.90	110.96		399.03	56.66	161.31		302.30
p-value	0.0000	0.0000		0.0000	0.0000	0.0000		0.0000
LB(20)	105.19	134.19		416.72	63.46	193.27		322.99
p-value	0.0000	0.0001		0.0000	0.0000	0.0000		0.0000
LB(30)	113.20	177.12		421.82	74.57	230.44		341.36
p-value	0.0000	0.0005		0.0000	0.0000	0.0000		0.0000

	February							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.0857	0.32	0.41	1965.59	0.0844	0.43	0.32	2536.03
std. deviation	0.0998	0.46	0.49	4860.14	0.0999	0.49	0.47	4521.80
skewness	3.0160	0.76	0.33	6.42	3.2778	0.26	0.72	4.93
kurtosis	16.4627	1.58	1.11	51.77	20.6080	1.06	1.52	35.95
minimum	0.0100	0	0	100	0.0100	0	0	100
1% Quantile	0.0100	0	0	100	0.0100	0	0	100
5% Quantile	0.0100	0	0	100	0.0100	0	0	100
10% Quantile	0.0100	0	0	200	0.0100	0	0	200
25% Quantile	0.0200	0	0	500	0.0200	0	0	500
50% Quantile	0.0500	0	0	900	0.0500	0	0	1000
75% Quantile	0.1100	1	1	1500	0.1000	1	1	2500
90% Quantile	0.2000	1	1	3500	0.1900	1	1	5300
95% Quantile	0.2700	1	1	6860	0.2800	1	1	10000
99% Quantile	0.4700	1	1	30000	0.4900	1	1	25000
maximum	0.9000	1	1	50000	1.1000	1	1	50000
LB(10)	193.69	167.87		3950.98	219.02	141.63		355.19
p-value	0.0000	0.0000		0.0000	0.0000	0.0000		0.0000
LB(20)	234.69	194.58		4402.90	252.01	181.78		418.77
p-value	0.0000	0.0000		0.0000	0.0000	0.0000		0.0000
LB(30)	242.09	252.58		4409.94	254.37	222.77		460.43
p-value	0.0000	0.0000		0.0000	0.0000	0.0000		0.0000

Table 1: Descriptive statistics of the quotes changes, the quote change direction indicator and the corresponding depths for the bid and ask sides in January and February 2001 for BDK.

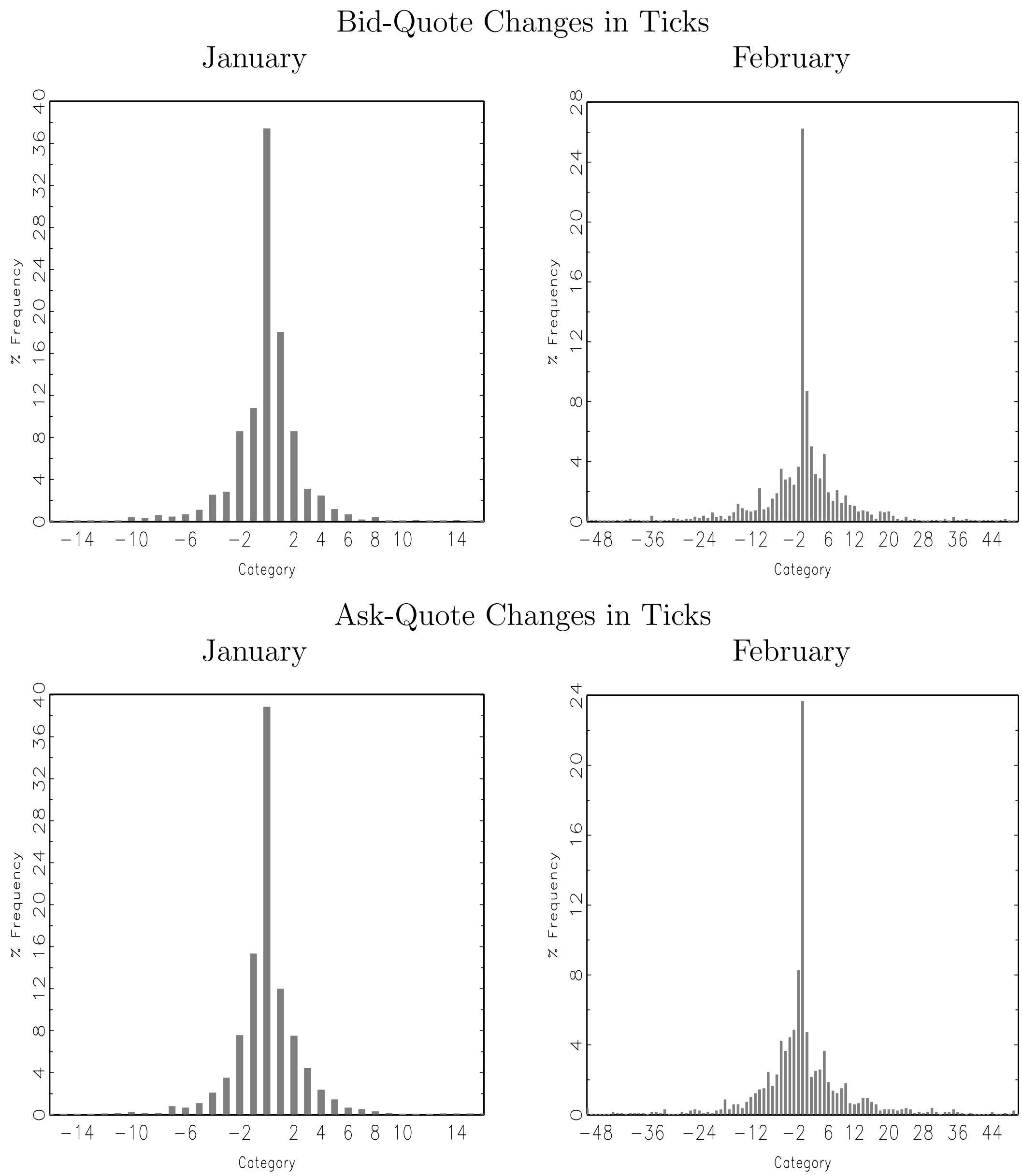

Figure 2: Histograms of the bid-quote changes (upper panels) and ask-quote changes in ticks (lower panels) in January (left panels) and February (right panels) for the BDK stock. The tick size in January is \$1/16 and \$1/100 in February. The quote changes are computed over equidistant 5 min data.

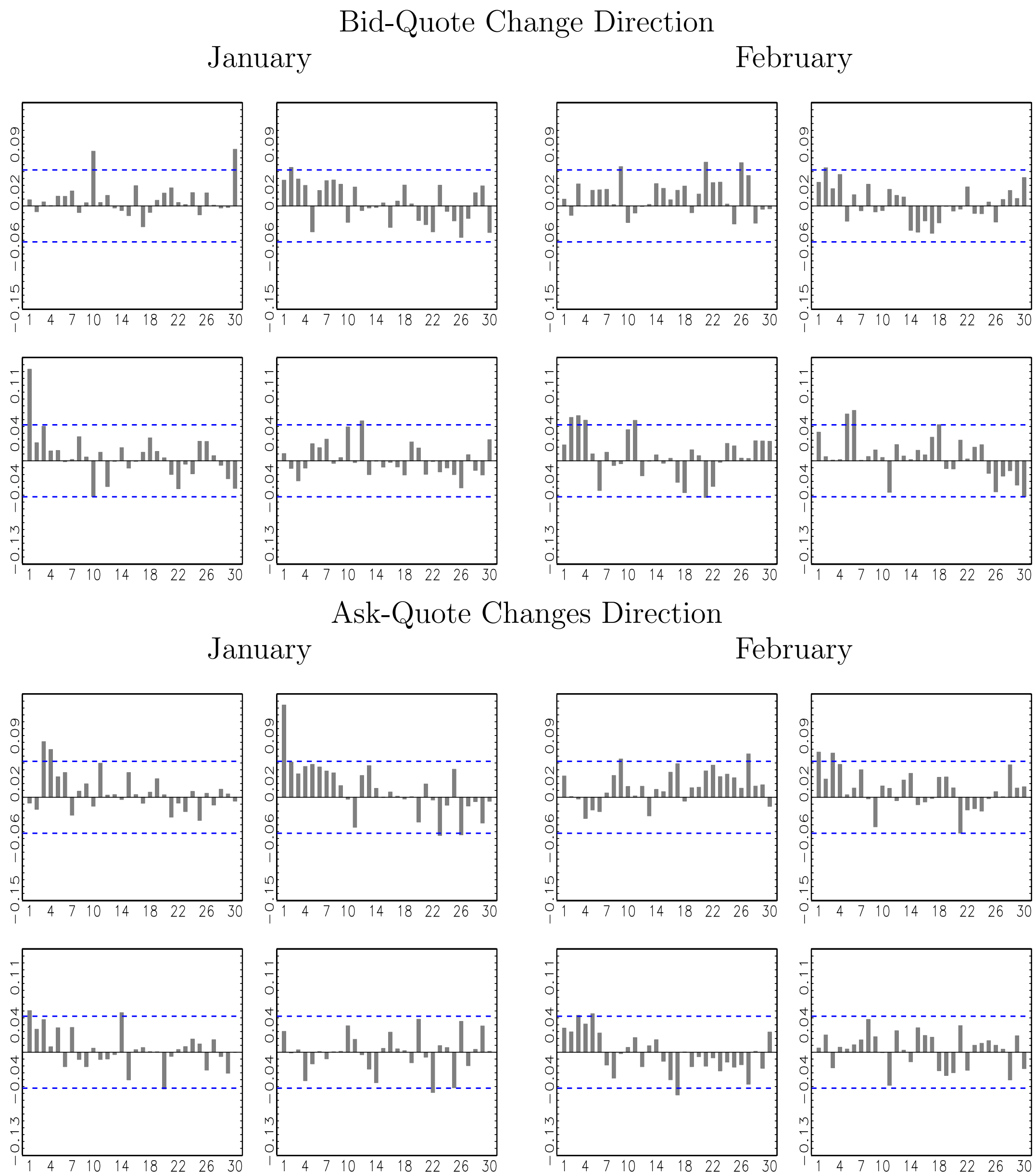

Figure 3: Multivariate-Autocorrelogram of the bid-quote change direction (upper panels) and ask-quote change direction (lower panels) in January (left panels) and February (right panels) for the BDK stock. The dashed lines denote asymptotic 95% confidence bounds.

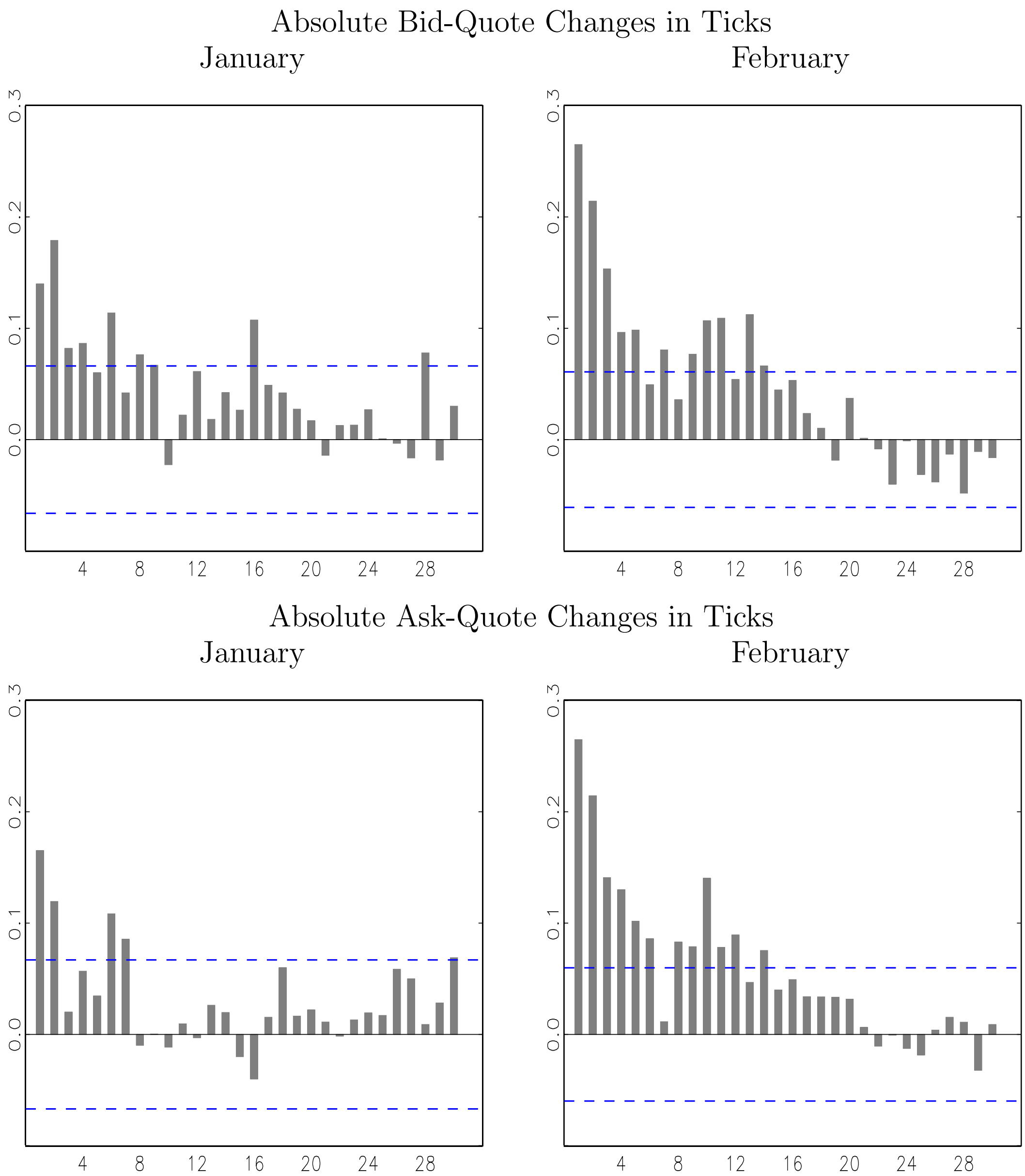

Figure 4: Autocorrelogram of the absolute bid-quote changes (upper panels) and absolute ask-quote changes in ticks (lower panels) in January (left panels) and February (right panels) for the BDK stock. The tick size in January is 1/16$ and 1/100$ in February. The quote changes are computed over equidistant 5 min data. The dashed lines denote asymptotic 95% confidence bounds.

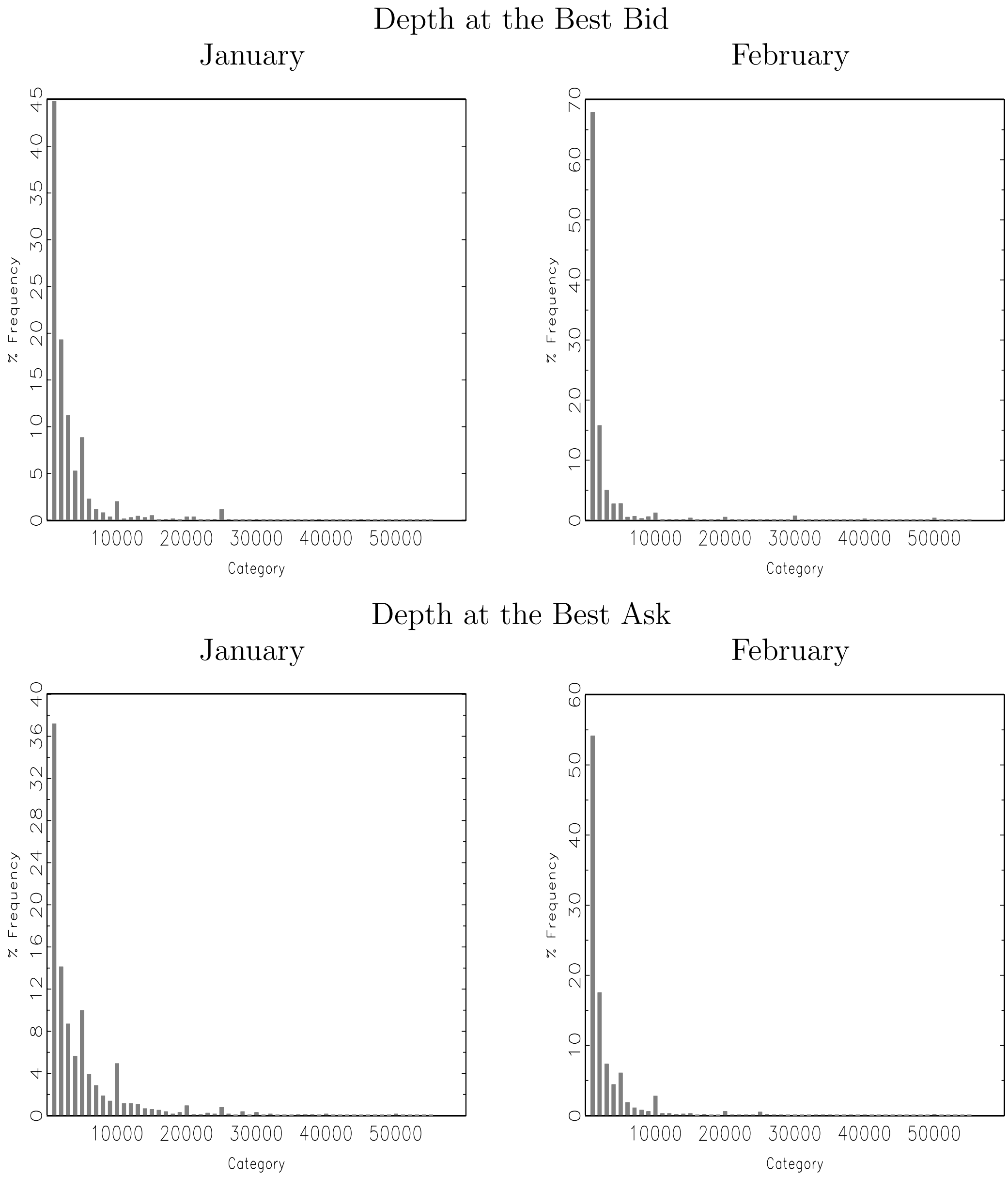

Figure 5: Histograms of the depth at the best bid (upper panels) and depth at the best ask (lower panels) in January (left panels) and February (right panels) for the BDK stock.

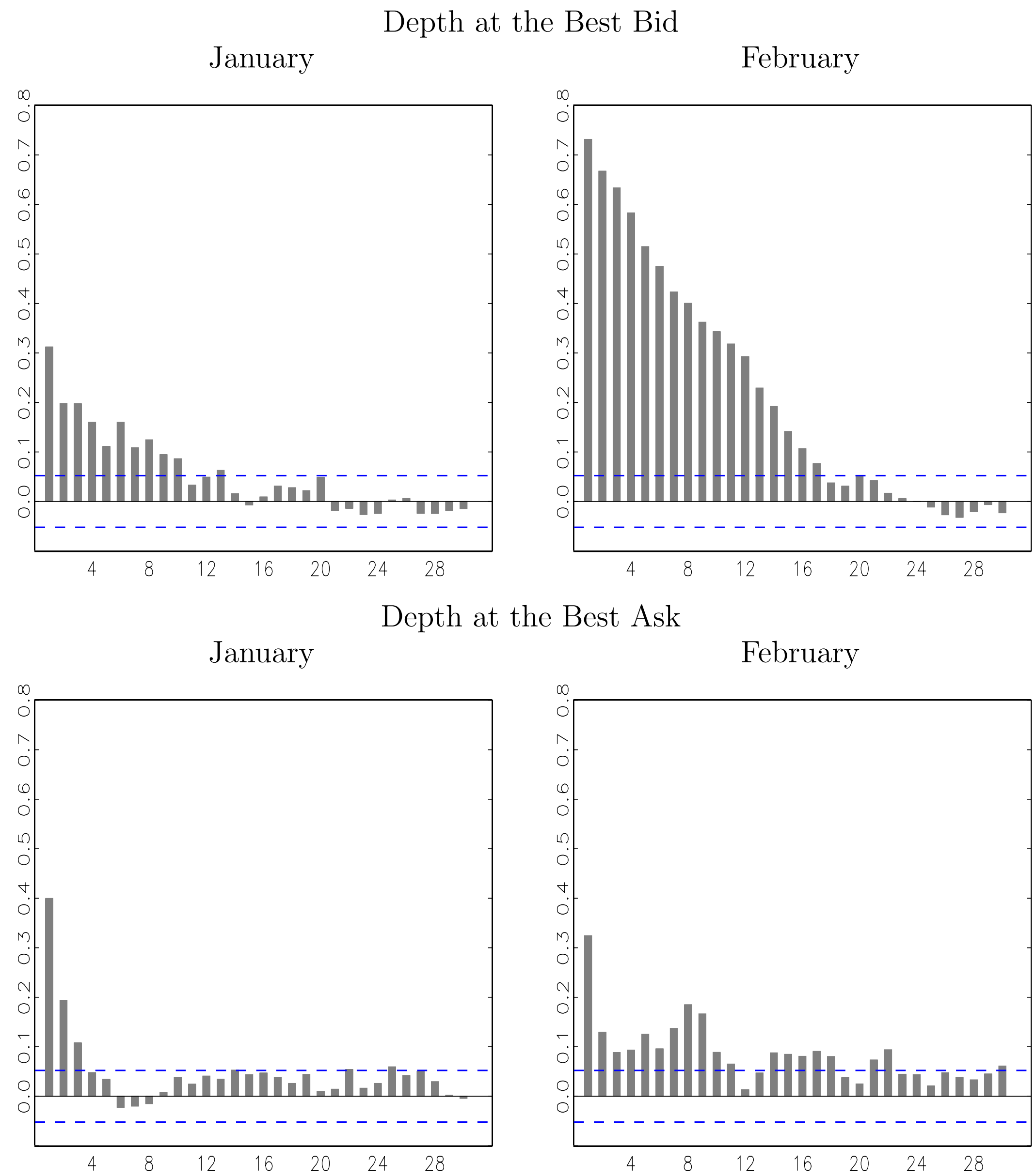

Figure 6: Autocorrelogram of the depth at the best bid (upper panels) and depth at the best ask (lower panels) in January (left panels) and February (right panels) for the BDK stock. The dashed lines denote asymptotic 95% confidence bounds.

Quote Slope

We focus on the descriptive analysis of the quote slope as a measure for liquidity supply, since it aggregates the information contained in the inside stages of the limit-order book. The higher the bid-ask spread and the lower the associated depths the more illiquid is the trading and the higher is the quote slope. Therefore, the (idealized) most liquid case, which is a zero bid-ask spread or infinite bid and ask depths, corresponds to a quote slope of zero. In terms of the density function of the quote slope this means the more mass is closer to zero the more liquid is the trading process. Figure 7 shows the histograms of the quote slope liquidity measure for all five stocks in January and in February 2001. There are two striking observations: i) In comparison to January, the histograms in February are shifted towards zero for all stocks. ii) For February, the histograms do not longer show the humps (several modi), which are visible in January. These humps, which are mainly caused by the large tick size of the bid-ask spread of \$ 1/16 in January, can be interpreted as liquidity supply states. In February, we observe a gradually declining shape of the histogram, where these states are smoothed out. The smooth shape of the histogram in February again represents a mass shift from January to February towards zero, i.e. towards more liquidity supply. These observations can be stressed by considering the quantiles of the empirical quote slope distribution presented in Table 2. The value of the quote slope at the 1% (25%) quantile is about six (two) times higher in January than in February. This observation can be interpreted in the following way: A trader, who would consume (by submitting market orders) 1% (25%) of the liquidity supply would get (in terms of the quote slope) a six (two) times better market condition in February than in January. Of course, this "x times better market condition" needs to be evaluated under the preference function of the trader. For BDK and HNZ, which are the two stocks with the smallest market capitalization, we get smaller values of the quote slope up to the 99% quantile. The same holds for C up to the 75% quantile, for PFE up to the 90% quantile and for XOM up to the 95% quantile. This means a potential trader, who would consume for example 90% of the liquidity supply of C, would get worse market conditions in February than in January. However, such a trader would attract the attention and induce reactions of the other market participants with a higher probability than a trader, who consumes only 1% of the liquidity, since he removes a big piece of the liquidity supply cake.

The Ljung-Box statistics of the quote slope in Table 2 certifies that the quote slope is subject to a high degree of autocorrelation. This dynamic structure is the motivation to model the conditional liquidity density function. The conditional liquidity density function is of utmost importance to figure out how the liquidity changes in certain market conditions and how liquidity reacts to shocks in the trading process. Moreover, our analysis enables us to point out differences in the liquidity reaction before and after the decimalization at the NYSE. Furthermore, it allows to shed light on potential differences between stocks.

	BDK		C		HNZ		PFE		XOM	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	0.0088	0.0060	0.0054	0.0044	0.0070	0.0044	0.0049	0.0035	0.0067	0.0043
std. deviation	0.0043	0.0045	0.0031	0.0041	0.0036	0.0043	0.0024	0.0035	0.0045	0.0054
skewness	0.7971	1.2929	4.9589	3.6648	1.5768	3.7528	3.0961	3.1665	6.5711	5.8062
kurtosis	3.3211	6.7019	53.674	32.525	7.7292	34.750	23.695	17.072	87.812	60.683
minimum	0.0032	0.0005	0.0028	0.0005	0.0030	0.0005	0.0027	0.0005	0.0030	0.0005
1% Quantile	0.0034	0.0006	0.0029	0.0005	0.0032	0.0006	0.0028	0.0005	0.0032	0.0006
5% Quantile	0.0037	0.0007	0.0031	0.0006	0.0034	0.0007	0.0030	0.0006	0.0033	0.0006
10% Quantile	0.0039	0.0009	0.0032	0.0010	0.0036	0.0007	0.0031	0.0007	0.0035	0.0007
25% Quantile	0.0045	0.0024	0.0034	0.0019	0.0040	0.0014	0.0033	0.0013	0.0038	0.0014
50% Quantile	0.0082	0.0052	0.0039	0.0032	0.0066	0.0032	0.0038	0.0026	0.0065	0.0028
75% Quantile	0.0118	0.0087	0.0066	0.0057	0.0087	0.0062	0.0063	0.0043	0.0078	0.0054
90% Quantile	0.0147	0.0120	0.0078	0.0088	0.0119	0.0096	0.0073	0.0070	0.0109	0.0089
95% Quantile	0.0174	0.0143	0.0100	0.0117	0.0141	0.0120	0.0089	0.0096	0.0134	0.0121
99% Quantile	0.0200	0.0193	0.0156	0.0196	0.0185	0.0175	0.0132	0.0198	0.0206	0.0254
maximum	0.0271	0.0394	0.0475	0.0582	0.0362	0.0574	0.0320	0.0288	0.0760	0.0793
LB(10)	171.91	513.46	128.25	302.40	277.85	73.83	226.00	139.88	139.01	45.25
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	183.12	542.13	131.32	458.61	298.06	79.50	274.36	149.96	151.39	53.80
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
LB(30)	188.67	547.88	137.14	566.03	312.73	102.43	292.32	158.44	160.08	69.25
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Table 2: Descriptive statistics of the quote slope liquidity measure for all five stocks.

Table 3 shows the descriptive statistics for the explanatory variables which are used in the estimation of the different models. We use the over 5 minutes aggregated buy and sell volumes as well as the number of buy and sell transactions within the 5 minute interval as explanatory variables. Table 3 shows the figures for BDK, whereas the corresponding tables for the other stocks can be found in the Appendix. The general descriptive result is that there is less trading activity in February than in January for the stocks with a high market capitalization. Here the mean and median trading volumes as well as the mean and median number of transactions decreased. For the stocks with a medium market capitalization there is no obvious difference in the trading activity from January to February.

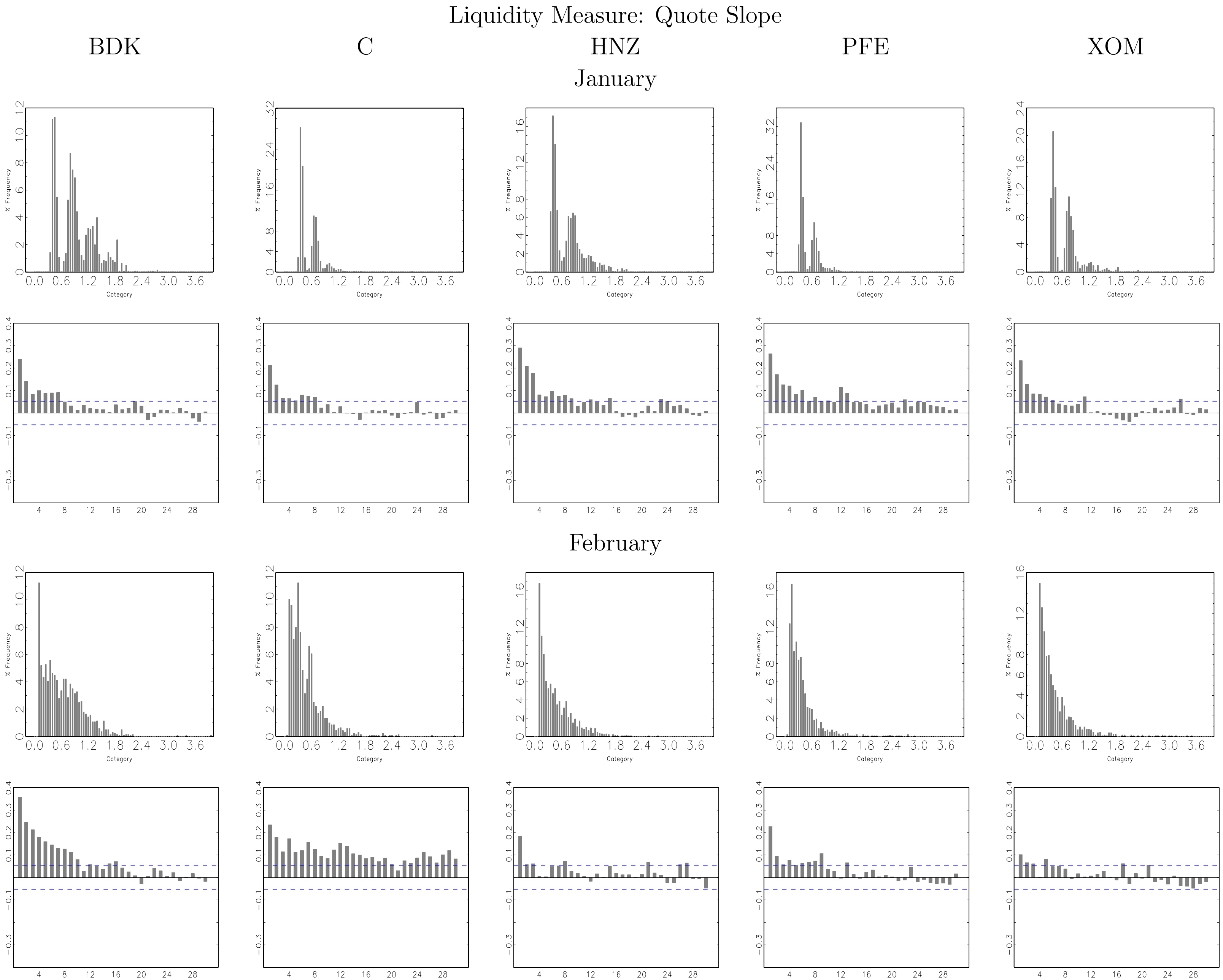

Figure 7: Histograms (first rows) and autocorrelogram (second rows) of the quote slope in January (upper panels) and February (lower panels) for all five stock. The dashed lines represent the asymptotical 95% confidence bounds.

	Buy Volume		Sell Volume		# Buys		# Sells	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	6011.03	3826.49	4406.83	4533.47	3.24	3.46	2.47	2.79
std. deviation	11153.16	8829.25	12795.50	7950.57	3.16	3.18	2.73	3.02
skewness	4.99	5.56	13.91	4.47	1.76	1.61	1.72	1.71
kurtosis	44.65	46.76	315.41	35.97	8.65	6.60	6.83	6.82
minimum	0	0	0	0	0	0	0	0
1% Quantile	0	0	0	0	0	0	0	0
5% Quantile	0	0	0	0	0	0	0	0
10% Quantile	0	0	0	0	0	0	0	0
25% Quantile	300	100	0	500	1	1	0	1
50% Quantile	2200	1000	1100	1800	2	3	2	2
75% Quantile	6600	3600	3700	4800	5	5	4	4
90% Quantile	16720	9300	10900	12000	7	8	6	7
95% Quantile	26000	16480	20000	19080	10	10	8	9
99% Quantile	50296	49976	43696	36272	14	14	12	14
maximum	160400	122200	331300	109100	29	20	19	19
LB(10)	306.91	257.59	45.98	158.99	419.50	613.72	210.28	705.10
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	347.25	317.95	100.16	229.76	466.49	688.77	217.81	935.39
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(30)	350.86	333.70	101.95	236.03	469.94	706.42	229.69	978.00
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3: Descriptive statistics of the explanatory variables for BDK.

4 Estimation Results

Estimation of the presented multivariate model for the supply liquidity measure was performed by the two-step estimation procedure described in Section 2. Once the parameters of the marginal densities for bid and ask quote changes and market depths are given, the copula parameters can be consistently estimated in the second step without applying any optimization procedure (as the MM estimator). As suggested in Liesenfeld et al. (2006) we optimize the likelihood of the ICH model by separately maximizing its two components, i.e ACM and GLARMA likelihood function. Since there are no parameter restrictions across those two components, such a proceeding reduces the computational burden of the estimation phase considerably.

When modelling the four marginal processes that constitue the shape of the conditional liquidity function we decided to use the simplest dynamic specification of the presented models, i.e. ACM-ARMA(1,1), GLARMA(1,1) and ACD(1,1), since these plain models already explained the autocorrelation structure of the modelled processes quite well.

To analyze the influence of shocks in related market microstructure variables on the marginal processes and on the quote slope, we use the following explanatory variables which potentially influence the dynamics of the quote changes and the market depths: cumulative volume and the number of sell and buy initiated transactions - aggregated during time intervals of five minutes. On the one side, the choice of these variables is restricted by the information provided by the TAQ database, on the other side however, we made the quite intuitive assumption that the chosen variables influence on the one hand the probability that the quote moves and on the other hand the size of the quote movement as well as the depth at the best bid and ask quotes. The chosen variables reflect the demand or the consumption of liquidity.

To perform a more comprehensive study of the quote direction processes (ACM submodel) we decided not to put symmetry restrictions on the A_1 matrix as well as on the vectors of coefficients for the microstructure variables, which allows for asymmetric influences of these variables on the probability of respectively upward and downward movement of a quote.

The ML estimation results (based on the Berndt, Hall, Hall & Hausman (1974) algortihm) extended by common diagnostic statistics for the ACM part of the ICH model are summarized in Table 4 and in Tables 17 - 20 in the Appendix and for the GLARMA part of the ICH model in Table 5 and in Tables 21 - 24 in the Appendix. With regards to the estimation results of the quote direction process, the vector of parameters for the explanatory variables (respectively: cumulative volume of buys, cumulative volume of sells, number of buys, number of sells) for an downward movement of the quote is denoted as $(g_{vb1}, g_{vs1}, g_{nb1}, g_{ns1})$, whereas for an upward movement of a quote as $(g_{vb2}, g_{vs2}, g_{nb2}, g_{ns2})$. It turned out that not all the explanatory variables are significant on the 5 percent level. Worth considering are always significant and often high values of the persistency parameter $b_1^{(1)}$. The result shows that if the probability of bid or ask quote changes was high in the previous period, it is also supposed to be considerable high in the next period. The obtained relations $a_{11}^{(1)} < a_{12}^{(1)}$ and $a_{21}^{(1)} > a_{22}^{(1)}$ between the innovation coefficients suggest the existence of some bounce pattern in the evolution of the bid and ask quote process, although the estimates are not always significant especially for the less frequently traded stocks. The dynamic properties of the quote direction processes are reflected by the ACM-ARMA(1,1) models in a satisfactory way. The autocorrelation scheme is considerably lowered when comparing the values of the bivariate Ljung-Box statistic of the standardized residuals with those computed for the raw data series. Only in a few rare cases we still can reject the null of no autocorrelation.

An interesting scheme is to be found in the way the microstructure variables influence the probability of an upward and a downward movement of the quotes. Here, two major observations should be stressed. First, in accordance with a quite intuitive assumption, the volume and the number of buy initiated transactions turn out to have significantly stronger impacts on the probability that ask quote moves up than on the probability that the ask quote moves down. The statement follows from the relations $g_{vb1} < g_{vb2}$ and $g_{nb1} < g_{nb2}$ that are always (except for BDK) fulfilled for ask quotes. Respectively, the volume and the number of sell initiated trades turn out to have stronger impact on the probability that the bid quote moves down than on the probability that it moves up - here the relations $g_{vs1} > g_{vs2}$ and $g_{ns1} > g_{ns2}$ are fulfilled. Therefore, as can be foreseen, transactions initiated by buyers (with market orders) tend to push ask quotes up, whereas those initiated by sellers (with market orders) tend to push bid quotes down.

Secondly, the volume and the number of sells turn out to have significantly stronger positive impact on the probability of the downward movement of the quote, than the upward movement - relations: $g_{vs1} > g_{vs2}$ and $g_{ns1} > g_{ns2}$ are fulfilled. Such a result can be explained by the fact that in addition to the observed sell transactions (sell market orders) there are sell limit orders which improve on the best ask quote. Analogically, for the bid quotes the opposite is true. In addition to the observed buy transactions (buy market orders), the not observed buy limit orders may constitute

a higher best bid quote. This can be see from relations $g_{vb1} < g_{vb2}$ and $g_{nt1} < g_{nt2}$ which show that the probability of an upward movement of the bid quote is higher than the probability of its downward movement, once the number and the volume of buy initiated transactions rises.
The effect of the decimalization is reflected by the following observation. The estimates for the intercepts μ_1 and μ_2 are significantly larger for February than for January. Therefore we can conclude that after the decimalization, the probability of a quote change has increased. This observation is in accordance with an intuitive assumption, since after the decimalization the transactions costs decreased and traders could hit the better place in the limit order book by a lower cost ("tick rule"). Regarding the estimation results for the GLARMA part of the ICH model, it could be observed that the simple GLARMA(1,1) specification is quite successful in explaining the dynamic properties of the process for the quote change sizes - the autocorrelation pattern of the residuals of these models is considerably lower than for the raw series. In all estimated models, the value of the dispersion parameter $\kappa^{-0.5}$ is significantly different from zero, allowing to reject the null hypothesis of an at-zero-truncated Poisson distribution in favor of a Negative Binomial one. Jointly significant coefficients of the seasonal component $S(\nu, \tau, K)$ for all models indicate, that there exists pattern of diurnal seasonality for the absolute bid and ask quote changes. The diurnally seasonalities are depicted in Figure 8. Although either for the January or the February the standard intraday seasonality pattern can be observed (high quote volatility at the beginning of the trading session with a decline afterwards, an increase at lunch time around 12.00 - 13.00 o'clock and a second decline before the end of the trading session), the size of quote changes (measured in number of ticks) heavily increased after the decimalization.

With regards to the impact of the explanatory variables, in the cases, where the estimated coefficients are significant, the following scheme could be observed. First, there is a positive impact of the quote change direction variable D_i on the size of the ask quote change and a negative impact of that variable on the size of the bid quote change. The model forecasts that the upward movement of the ask quote is larger than its downward movement, whereas for the bid quote the opposite is true. Therefore the volatility of the ask quote rises if the ask quote change is positive and the opposite holds for the bid quote change. Positive ask quote change can only be caused by the execution of several market orders as well as cancellations of sell pending limit orders during the five minute interval. During a buy market phase traders either submit buy market orders which consume depths on the ask side of the market or submit buy limit orders extending the bid side depths - which causes a higher bid quote. Therefore in buy market phases we face a decreasing supply on the ask side and an increasing supply on the bid side, which is responsible for a more volatile ask quote and a less volatile bid quote. The inverse explanation holds for sell market phases.

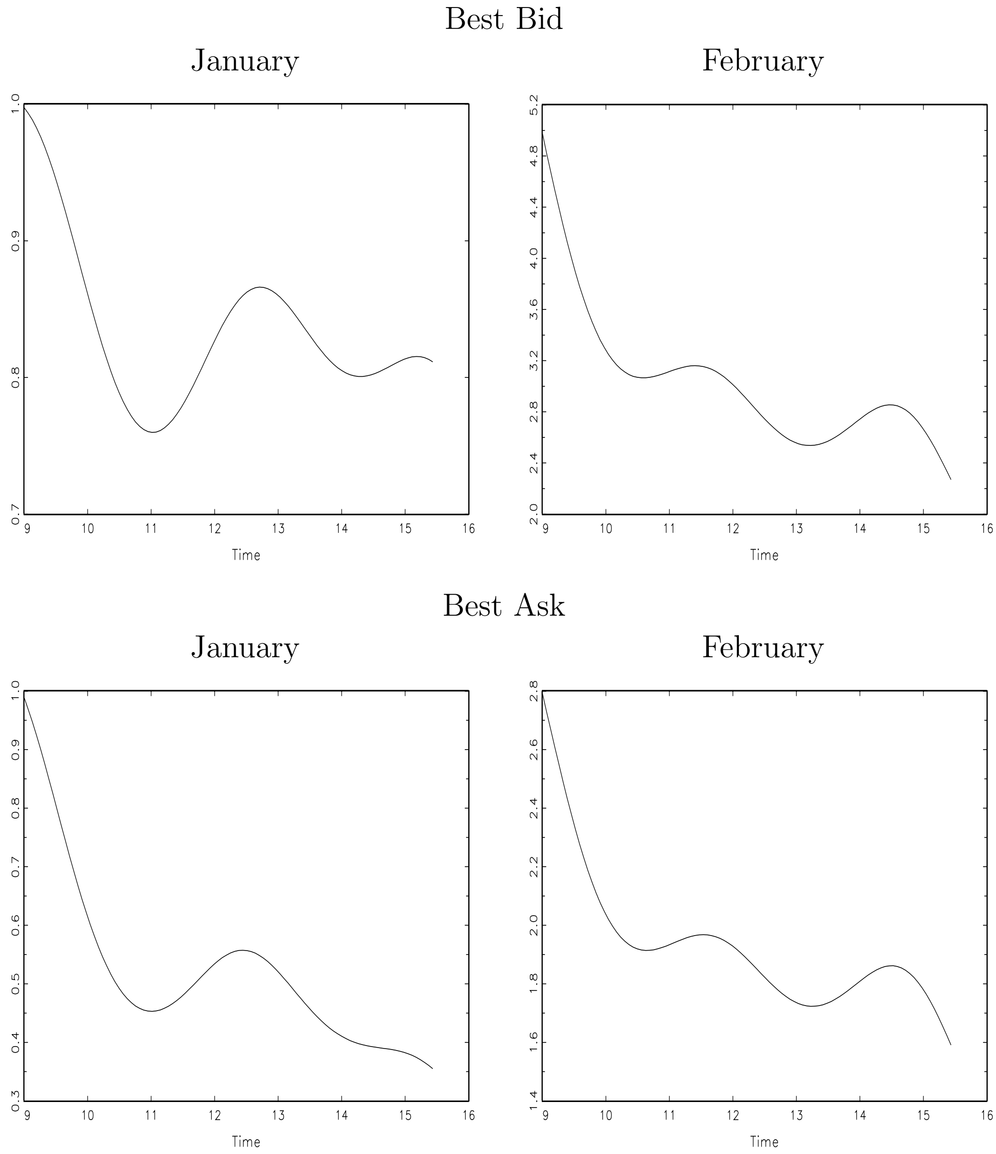

Figure 8: Estimated diurnally seasonality function of the non-zero absolute bid quotes and ask quotes in January and February for the BDK stock.

The observed positive impact of the number and the cumulative volume on the expected size of the bid and ask quote change allows to conclude that the transaction intensity has a positive impact on this potential measure of quote volatility.

The estimation results for the ACD(1,1) models for market depths are summarized in Table 6 and in Tables 25 - 28 in the Appendix. It should be noted, that the two shape parameters $\breve{\kappa}$ and σ^2 are significant at the 5 percent level, which means that neither the Weibull nor the Exponential distribution are a valid alternative to the Burr distribution. The values and the significance of the estimates responsible for the dynamic properties of the depths variables vary across estimated models. In some cases the process is very persistent and nearly integrated (the sum of the $\breve{\alpha}$ and $\breve{\beta}$ is close to one), which accounts for slowly decaying, hyperbolic-shape autocorrelation function of the depth variable, whereas in some other cases the estimate is insignificant.
We cannot find any systematic impact of the explanatory microstructure variables on the depth of the buy and sell side of the market. Coefficients on number of transactions are very often insignificant. Whereas the direction of the impact of significant variables is quite ambiguous. However we can see that there are systematic differences in depths between those two periods. In January the market is considerably deeper which is illustrated in Figure 9, where we plotted the diurnal seasonality for the mean function of the depths at the best quote.
In Table 7 and Tables 29 - 32 in the Appendix we report the contemporaneous correlation matrix of the quantile vector q_t. We can observe strong positive correlation between the quantiles of the conditional cumulative distribution of bid and ask quote changes. The two quotes tend to move simultaneously in the same direction during the five minute long intervals. Furthermore, this dependency measure has decreased after the decimalization was introduced (except for BDK). It seems obvious, since quotes started to fluctuate in wider ranges.

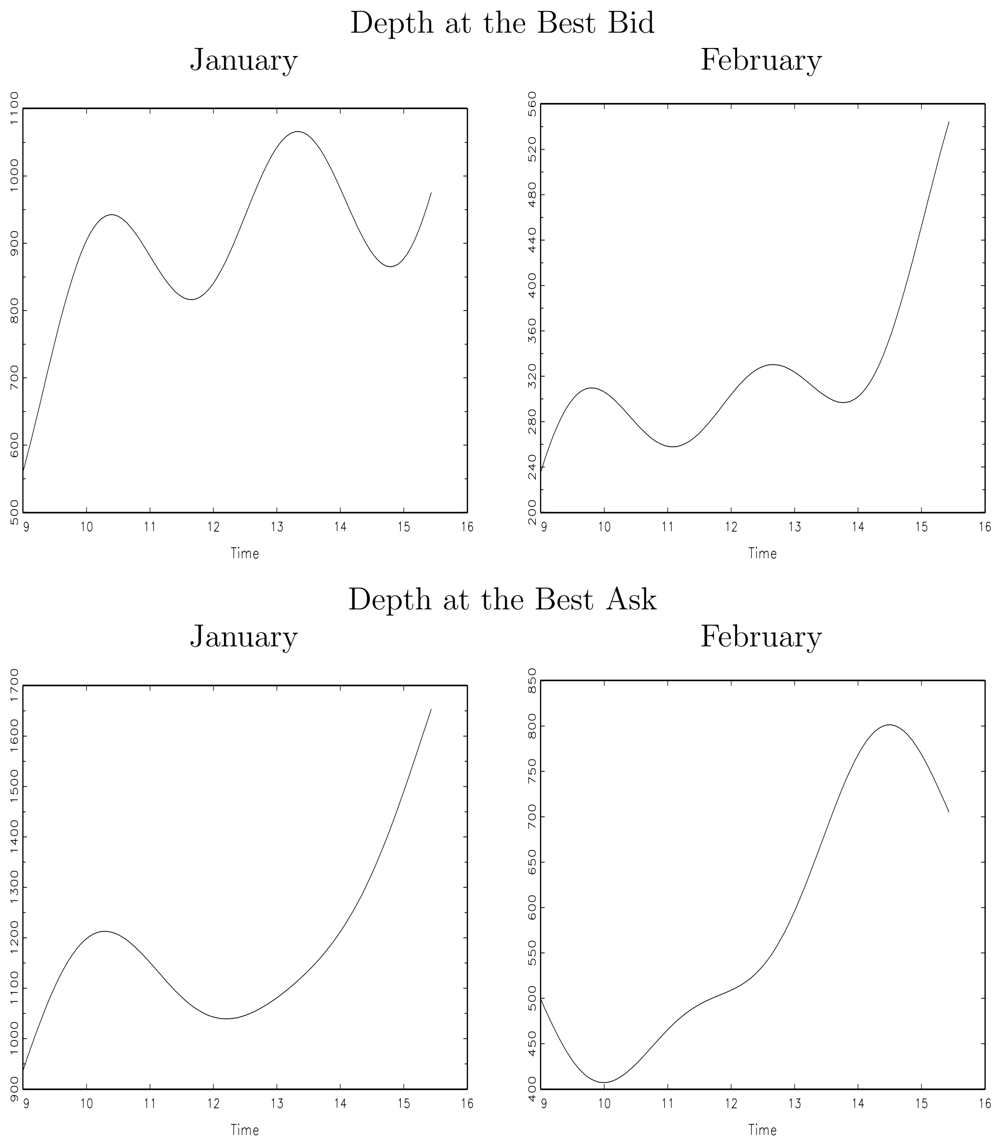

Figure 9: Estimated diurnally seasonality function of the depth at the best bid (upper panels) and depth at the best ask (lower panels) in January (left panels) and February (right panels) for the BDK stock.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
μ_1	-0.3660	0.3115	-1.6220	0.3989	-0.0710	0.0439	-0.1111	0.0424
μ_2	-0.4733	0.3879	-1.1188	0.2920	-0.2456	0.1004	-0.0282	0.0275
$c_1^{(1)}$	0.6974	0.2410	-0.0385	0.2333	0.7800	0.0861	0.8486	0.0435
$a_{11}^{(1)}$	0.1471	0.0678	0.2456	0.0876	0.2474	0.0784	0.2370	0.0530
$a_{12}^{(1)}$	0.2312	0.0981	0.1000	0.0891	0.3123	0.0820	0.2316	0.0551
$a_{21}^{(1)}$	0.1321	0.0733	0.3625	0.0825	0.2883	0.0816	0.2568	0.0571
$a_{22}^{(1)}$	0.1956	0.1205	0.1925	0.0685	0.3043	0.0706	0.2650	0.0496
g_{vb1}	0.0024	0.0011	-0.0054	0.0015	0.0046	0.0022	0.0000	0.0019
g_{vs1}	-0.0058	0.0017	0.0032	0.0018	0.0000	0.0010	0.0009	0.0011
g_{nb1}	0.3256	0.0425	0.1045	0.0386	0.0775	0.0458	0.0328	0.0389
g_{ns1}	0.1259	0.0445	0.4188	0.0477	0.2660	0.0415	0.3083	0.0469
g_{vb2}	-0.0004	0.0014	-0.0003	0.0007	0.0079	0.0021	0.0034	0.0013
g_{vs2}	0.0013	0.0011	-0.0016	0.0022	-0.0012	0.0013	-0.0010	0.0013
g_{nb2}	0.0674	0.0451	0.2707	0.0296	0.2880	0.0462	0.1658	0.0348
g_{ns2}	0.3218	0.0384	0.1064	0.0498	0.1366	0.0452	0.0368	0.0441
log-lik.	-0.892033		-0.893002		-0.908939		-0.931320	
SIC	0.930961		0.931930		0.947867		0.970248	
Q(10)	55.332 (0.001)		38.144 (0.076)		62.389 (0.000)		33.047 (0.196)	
Q(20)	88.988 (0.038)		93.569 (0.018)		108.809 (0.001)		70.942 (0.348)	
Q(30)	124.722 (0.116)		133.821 (0.041)		137.590 (0.025)		116.439 (0.251)	
res. mean	(-0.020,-0.025)		(-0.032, 0.005)		(-0.035, -0.054)		(-0.020, 0.002)	
res. var.	$\begin{pmatrix} 0.975 & 0.244 \\ 0.244 & 1.389 \end{pmatrix}$		$\begin{pmatrix} 0.843 & 0.268 \\ 0.268 & 1.638 \end{pmatrix}$		$\begin{pmatrix} 1.614 & 1.476 \\ 1.476 & 3.160 \end{pmatrix}$		$\begin{pmatrix} 0.874 & 0.464 \\ 0.464 & 1.811 \end{pmatrix}$	

Table 4: ML estimates of the ACM-ARMA part of ICH model. ASK and BID Quote changes in January and February for BDK.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\kappa^{0.5}$	0.6979	0.0710	0.8073	0.0770	0.9832	0.0411	0.9730	0.0408
$\tilde{\mu}$	-0.1641	0.4140	-0.0792	0.0831	0.9407	0.3688	1.4989	1.1184
β_1	-0.5825	0.5653	0.6203	0.2583	0.5153	0.2116	0.1303	0.7409
α_1	0.0568	0.0337	0.0769	0.0254	0.1444	0.0296	0.0669	0.0914
ν_0	-1.0421	0.3246	-0.2096	0.3070	-0.5835	0.2399	-0.8144	0.5334
ν_1	0.0301	0.0906	0.0286	0.0343	0.0519	0.0315	0.0695	0.0660
ν_2	0.1222	0.0972	0.0481	0.0280	0.0339	0.0277	0.0360	0.0463
ν_3	-0.1986	0.1599	-0.0733	0.0959	-0.1116	0.0577	-0.1392	0.0826
ν_4	0.0289	0.1538	0.0246	0.0360	-0.0698	0.0385	-0.1046	0.0775
δ	0.1295	0.0459	-0.1563	0.0467	0.1334	0.0368	-0.1532	0.0376
g_{vb}	0.0008	0.0005	0.0015	0.0006	0.0012	0.0004	0.0014	0.0004
g_{vs}	-0.0001	0.0002	0.0003	0.0002	0.0009	0.0005	0.0011	0.0005
g_{nb}	0.0725	0.0152	0.0866	0.0178	0.0529	0.0107	0.0851	0.0115
g_{ns}	0.1316	0.0152	0.0986	0.0146	0.0882	0.0137	0.0658	0.0151
log-lik.	-0.873340		-0.868164		-2.189387		-2.130800	
SIC	0.909673		0.904497		2.225720		2.167133	
LB(10)	14.360 (0.001)		5.632 (0.060)		22.471 (0.000)		41.941 (0.000)	
LB(20)	19.387 (0.080)		12.465 (0.409)		38.567 (0.000)		58.483 (0.000)	
LB(30)	35.449 (0.035)		26.408 (0.235)		46.041 (0.002)		81.032 (0.000)	
res. mean	-0.009		-0.008		-0.001		-0.000	
res. var.	0.878		0.873		0.957		0.963	

Table 5:ML estimates for the GLARMA part of the ICH model (ASK and BID Quote changes in January and February for BDK).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\breve{\kappa}$	1.7144	0.0808	2.9995	1.1644	1.5273	0.0672	1.5564	0.0663
σ^2	0.9751	0.1205	2.2415	1.5215	0.9401	0.1050	0.7223	0.0845
$\breve{\mu}$	1494.7904	483.8095	1968.8966	1609.0915	1004.9979	377.9930	248.0862	133.2212
$\breve{\alpha}$	0.6080	0.0669	0.2369	0.2565	0.4362	0.0840	0.3852	0.0408
$\breve{\beta}$	0.0947	0.0532	0.1495	0.3075	0.1354	0.1214	0.2472	0.0526
ν_0	1361.6287	818.6938	1257.8651	586.1333	412.7212	558.1108	494.3943	224.4132
ν_1	246.7053	144.7659	-151.5233	154.9453	81.6532	91.8198	59.2648	40.9753
ν_2	-17.9679	111.1106	-197.7178	175.0117	-37.8429	90.1107	56.4739	37.1030
ν_3	346.6133	271.6948	205.4807	223.4757	-227.1569	205.5680	103.0868	76.4462
ν_4	72.0118	154.2225	313.7018	147.9295	-90.8550	113.1423	53.0475	46.9871
g_{vb}	-1.3191	0.3931	0.3967	0.8852	-1.0552	1.2736	1.4277	0.8007
g_{vs}	-0.5518	0.8257	0.5201	0.8536	2.5681	1.4892	2.1415	0.8441
g_{vb}	-22.7808	31.1105	5.2014	34.4017	29.7907	27.0294	0.2101	14.5315
g_{vs}	-11.5943	37.2269	-8.9336	36.5855	22.9978	28.1511	1.4853	15.6220
log-lik.	-9.101555		-8.631609		-8.502439		-8.061327	
SIC	9.1374890		8.667543		8.538372		8.097260	
LB(10)	16.974 (0.000)		65.051 (0.000)		21.913 (0.000)		28.299 (0.000)	
LB(20)	26.667 (0.009)		79.439 (0.000)		33.848 (0.001)		44.607 (0.000)	
LB(30)	29.722 (0.125)		83.934 (0.000)		59.594 (0.000)		61.485 (0.000)	
res. mean	0.908		0.717		0.911		1.002	
res. var.	1.373		0.890		2.644		2.481	

Table 6: ML estimates for the ACD model (ASK and BID Depths in January and February for BDK).

	JANUARY				FEBRUARY			
	Quote changes		Depth		Quote changes		Depth	
	ASK	BID	ASK	BID	ASK	BID	ASK	BID
Quote changes ASK	1.000				1.000			
Quote changes BID	0.595	1.000			0.908	1.000		
Depth ASK	-0.025	-0.061	1.000		-0.051	-0.086	1.000	
Depth BID	0.013	-0.059	0.084	1.000	0.019	-0.033	0.033	1.000

Table 7: Contemporaneous dependence of supply liquidity measure components for BDK.

5 Simulation of the Conditional Liquidity

The estimated conditional multivariate density function of the components of the quote slope allows us to derive the conditional density function of the liquidity measure. Therefore, we are able to verify a hypothesis that not only the conditional mean, but also other different characteristics of its statistical distribution, i.e. higher moments and quantiles of the quote slope are suspect to move according to some dynamic pattern with respect to the past filtration of the process. Moreover, the influence of the microstructure variables on the distributions of our liquidity components could help us to characterize the general impact of these variables not only on the particular components of the quote slope but also on the quote slope itself.
We take advantage of Monte Carlo simulation techniques to receive the conditional density of the quote slope measure. In the first step, we sample from the conditional truncated multivariate (with the restriction given by equation (11)) density ($f_{Z_t^*}$) of the four liquidity components, i.e. bid and ask quote changes and depths which are summarized in Z_t^*. This complete density is conditional on the information set from the past, as it contains the whole history of the marginal processes up to time point $t-1$. Therefore we are able to sample N times from different, conditional with respect to $\mathfrak{F}_{t-1}$ density functions for every time point t in our sample of data. On the basis of a large number of simulated observations for every time point t of our data sample, the N corresponding quote slopes can be obtained and their conditional density can be estimated for example by applying nonparameteric density estimation techniques.
From a statistical point of view it is not straightforward to sample from a multivariate distribution with restrictions across the outcomes of the marginal processes. Therefore we can not use here the standard sampling methods proposed for elliptical copula functions (such as gaussian copulas), presented in detail in Cherubini et al. (2004). In order to sample from our truncated model, we apply a Metropolis-Hastings algorithm, which can be used to obtain draws from any parametric density function, from which it is difficult to draw using standard techniques involving inversions of the cumulative distribution functions. As before $f_{Z_t^*}$ denotes the density function, which we want to obtain, that is the target distribution for the vector of marks $Z_t^* = (C_t^b, C_t^a, D_t^b, D_t^a)$. We decided here to use an Metropolized Independence Sampler (MIS) of Hastings (1970) as we implicitly assumed that a convenient approximation of the target distribution exists. This approximation, the so called proposed density is denoted by $g_{Z_t^*}$. The algorithm of the MIS is summarized in the

following:

Given the current state of vector Z_t^{*i}:

1. Draw Z_t^* from the proposed density $g_{Z_t^*}$ in the following way:

   - compute Cholesky decomposition $\hat{A}$ (4×4) of estimated variance-covariance matrix $\hat{\Sigma}$,
   - simulate $x = (x_1, x_2, x_3, x_4)$ from the 4-dimensional standard normal distribution,
   - set $y = \hat{A}x$,
   - set $u_k = \Phi(y_k), k = 1, ..., 4,$, where Φ denotes the univariate standard normal distribution function,
   - set $Z_t^* = (Z_{t,1}^*, ..., Z_{t,4}^*)$ with $Z_{t,k}^* = F_k^{-1}(u_k), k = 1, ..., 4$ where F_k denotes the marginal cumulative distribution function of the variable corresponding to kth element of the vector Z_t^*.

2. Simulate u from the Uniform[0,1] and let

   - set $Z_t^{*i+1} = Z_t^*$ if $u \leq \min(1, \frac{w(Z_t^*)}{w(Z_t^{*i})})$,
   - set $Z_t^{*i+1} = Z_t^{*i}$ otherwise

   where $w(Z_t^*) = \frac{f_{Z_t^*}}{g_{Z_t^*}}$ is the usual importance sampling weight.

The MIS takes more draws in areas of a high target density and proportionally fewer in areas of a low target probability, by deriving the acceptance probability which is higher for the first and lower for the latter areas. The success of this algorithm depends on how close the candidate density is to the target distribution. In the step 2 of the algorithm, draws Z^* from the proposed density are obtained with a simple sampling method for gaussian copula. We therefore proposed dependent draws from a candidate density equal to the corresponding non-truncated multivariate density whose marginals are given by $F_k, k = 1, ..., 4$ and gaussian copula function with a variance-covariance matrix $\hat{\Sigma}$. The target density $f_{Z_t^*}$ however must account for the truncations on the outcome space, which follow from nonnegative spread bid-ask. As stated in equation (11) we have built in restrictions not into the marginal submodels but into the formula for a copula function. That is the copula component of the canonical representation given by (6) that is truncated. Therefore in the step 4 of the MIS algorithm sketched above we proceed in the following way: for every $Z_{t,i}$ that does not fulfil the restriction (11) $\mathbf{c}(Z_{t,i})$ is equal to zero. In such a framework the task of the MIS can be perceived as a way of to correcting the candidate generating density proposed in the step 2 of the algorithm with respect to distortions resulting from the truncations.

Note, that in case of our model the importance sampling weight w has the simple advantageous form:

$$w(Z_t^*) \quad = \frac{f_{C_t^b} \cdot f_{C_t^a} \cdot f_{D_t^b} \cdot f_{D_t^a} \cdot \mathbf{c}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^{b*}}, F_{D_t^{a*}})}{f_{C_t^b} \cdot f_{C_t^a} \cdot f_{D_t^b} \cdot f_{D_t^a} \cdot \tilde{\mathbf{c}}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^{b*}}, F_{D_t^{a*}})} = \frac{\mathbf{c}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^{b*}}, F_{D_t^{a*}})}{\tilde{\mathbf{c}}(F_{C_t^{b*}}, F_{C_t^{a*}}, F_{D_t^{b*}}, F_{D_t^{a*}})}$$

Before the final simulation we perform many trial samplings from the density using different sample lengths and different tuning parameters used to scale the variance of the proposed density. We deduced that the sample of 100 thousand observations is large enough to get a stationary distribution for the target density function. We repeated the algorithm several times coming to very similar values for the mean and the covariance matrix of the simulated sample. According to Geweke & Tanizaki (2003) the variance of the proposed density should be higher than the one for the target one. To have a more dispersed candidate generating density we multiplied the variance-covariance matrix $\hat{\Sigma}$ by 10. We also performed the diagnostics for monitoring the convergence of the simulation performed. The criteria based on individual PSRF (potential scale reduction factor) and multivariate PSRF plots introduced by Brooks & Gelman (1998) evidenced that after 100 thousand iterations the convergence has been reached.

6 Empirical Findings

There is a large body of market microstructure literature studying the influence of decimalization on different measures of market liquidity. We can meet two main strands focusing on the effect of tick size reduction. On the one hand, according to O'Connell (1997) and Ricker (1998), the liquidity of the market rises due to increased competition between liquidity providers and narrower bid-ask spreads, which yield lower transaction costs. As advocated by Harris (1997) and Harris (1999), the lower tick size reduces the cost of stepping ahead in a limit order book (front running), which enhances the competition between liquidity providers. On the other hand, the studies of Grossman & Miller (1988), Harris (1994) and Harris (1997) suggest that while liquidity demanders profit from a decreasing spread, the liquidity suppliers face higher costs and are therefore discouraged from providing liquidity. In their empirical study for NYSE stocks, Goldstein & Kavajecz (2000) show that after the NYSE reduced the minimum quote variation in 1997, bid-ask spreads and the cumulative depths decreased. Moreover, the lower level of liquidity displayed (smaller depths) in specialist quotes as well as displayed in the limit order book, provided less certainty to liquidity demanders. Chakravarty, Wood & Ness (2004) came to the similar results studying the influence of the decimalization on the quoted and effective bid-ask spreads as well as on depths at the best bid and ask prices. They treat it as an ambiguous result for liquidity, since number of stocks that can be traded at the best prices declined. Our analysis of the quote slope liquidity measure directly addresses this ambiguous result.

As mentioned in the introduction, in opposite to the studies focusing on means of selected one-dimensional liquidity metrics, we encompass the whole distribution of a multidimensional liquidity measure such as the quote slope. In the simulation, we intend to verify whether and how the shape of the conditional density of the quote slope changes while reflecting the whole information on the history of the liquidity process. We aim to compare the different statistics of the derived density for the two periods: before and after the decimalization on the NYSE has been proceeded. In Figures 10 - 14 we plot several time-varying characteristics of the conditional density function obtained with the IMS. Figure 10 and 11 present the line graph of the conditional mean and the conditional standard deviation of the quote slope density function. We concern here three main findings. Firstly, for all stocks under study, the average values of the quote slope are significant lower in February then in January - the mean of the quote slope has declined due to the decimalization. Before

decimalization we can observe distinct negative shocks in the amount of liquidity supplied- the plot of the conditional mean for January indicates much more upward "picks". If one focuses only on the conditional mean of this liquidity measure, the main findings would be, that due to decrease in the tick size the market significantly gained on the level of liquidity provided (the smaller the quote slope, the more liquid is the market). Secondly, average liquidity supply was much more volatile before the decimalization. Thirdly, for the conditional mean and the standard deviation we can observe systematic fluctuations which suggest the existence of an intraday seasonality pattern for these moments. The two first observations agree with the results of descriptive statistics performed for the empirical data and presented in Section 3. Indeed, we have seen there that the two first moments of the empirical liquidity ratio were significantly higher for January than for February.
In the Figures 10 and 11 we can observe the "L"-shape diurnal seasonality patterns for the conditional mean and the standard deviation of the quote slope, obtained with a nonparametric regression (Nadaraya-Watson estimator with the Gaussian kernel and the optimal bandwidth). It is therefore evidenced, that in both periods of our study the market is less liquid after the opening of the trading session. This observation is quite interesting since it corresponds with such well-known market microstructure findings as U-shape pattern for the transaction intensity. The main result, however, is that the mean and standard deviation of the quote slope liquidity function is indeed time-varying, which contradicts several theoretical models, where actions of market participants rely on liquidity shocks with constant mean and variance(Karpoff (1986), Michaely & Vila (1996)).
Comparing presented in Figures 12 and 13 scatter plots of the 10, 25, 50 and 90 percent quantile of the dynamic quote slope density in January and February, we come closer to the most interesting point of our study. As we have seen from the descriptive analysis of the empirical data, due to the coarser grid for the potential quote changes in January, the shape of the liquidity density function evidenced some humps (several modi) - the probability mass of the distribution was concentrated in several states. Those states can be perceived as "liquidity states" - as certain amounts of liquidity supply are much more probable than the others. The analogical finding can be observed in Figures 12 and 13 presenting the scatterplot of the quantiles of the conditional quote slope distribution. Firstly, we can observe there that not only the mean but the whole density of the quote slope is being shifted according to shocks in liquidity supply - the values of the given quantiles fluctuate in time. However, the more important result are the differences between the patterns

according to which the 10, 25 and the 50 percent quantiles in the two subsequent periods of our study fluctuate. In January the quantiles are subject to gravitate to two or three outcome states, while in February those fluctuate more randomly. For example, in January the 25 percent quantile of the conditional density function corresponds to a certain state of the liquidity supply. The amounts of liquidity supply are therefore subject to some abrupt changes, they follow a kind of jump process. According to the number of modi of the conditional density function of liquidity supply, this effect exists at several quantiles. However, it gets smaller for higher quantiles, as illustrated by the 90 percent quantile. In both periods these values fluctuate rather randomly, which could be explained by absence of distinct liquidity states corresponding to very high quote slope values.

The presence of the liquidity states for the medium quote slope values, that is for the medium liquidity state of the limit order book, are supposed to have a distinct impact on the market conditions of the trading process. Traders, who intend to trade very large volumes, because of insider information are not affected by this kind of liquidity supply. But those, who trade for speculative reasons ore who need to trade a moderate volume of a stock are affected by the observed liquidity states. Such a trader consumes a certain part of the liquidity supply, i.e. he consumes liquidity up to a certain quantile (say 10, 50 percent). If this quantile is higher, the liquidity he consumes is more costly. Since we observe jumps in the time-varying quantiles, the trader either does not know the cost he is subject to (and he may be subject to the cost of a high states) or he tries to optimise his trading or liquidation strategy according to it, which creates additional search costs. In both cases the trader suffer from the existence of liquidity states (see e.g. Bertsimas & Lo (1998), Almgren & Chriss (2000) and Subramanian & Jarrow (2001)). After the decimalization, the states do not appear in the conditional liquidity function.

In Figure 15 we present the autocorrelation function of the residuals (defined as the difference between the computed quote slope and the mean of the quote slope density function) and the histograms of integral probability transformations (IPT) for the derived conditional density. We can see that, as there is negative first order autocorrelation in residuals our model is not perfect in explaining the dynamics of the quote slope. However our study should be treated as precursory one, since we do not performed here any model selection procedure for the marginal processes. The inclusion of higher order lags of the explanatory variables as well as absolute innovation terms in the ICH and ACD models is potentially able to improve this result. However, in comparison to the original time series of the quote slope, the autocorrelation pattern in the residuals is considerably lower, as shown in Table 8. The non-uniform shape of the IPT also suggest that the shape of the conditional density could be reflected in a more suitable way. Thus, for a January period of our study we overestimate the low tail of the liquidity function, whereas we underestimate its upper tail. It means that according to the true data generating process amounts of very high liquidity occur more seldom whereas these for very low liquidity - more often. The opposite stands for the February period of our study - we systematically underestimate the probability of a average liquidity level, as our liquidity function is characterized by the too more probability mass on the tails.

	BDK		C		HNZ		PFE		XOM	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	0.878	1.067	0.820	0.825	0.840	1.097	0.831	1.015	0.838	1.002
std. deviation	0.438	0.953	0.405	0.748	0.406	1.362	0.352	0.976	0.477	1.292
LB(10)	78.72	80.62	89.97	96.61	88.98	50.95	96.17	86.97	76.72	62.42
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB(20)	86.10	92.06	96.58	104.84	96.39	53.95	103.13	98.59	91.38	76.23
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB(30)	94.65	100.43	106.18	116.43	111.38	73.77	109.08	112.79	102.69	89.82
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 8: Summary statistics of the constructed residuals for the quote slope liquidity measure.

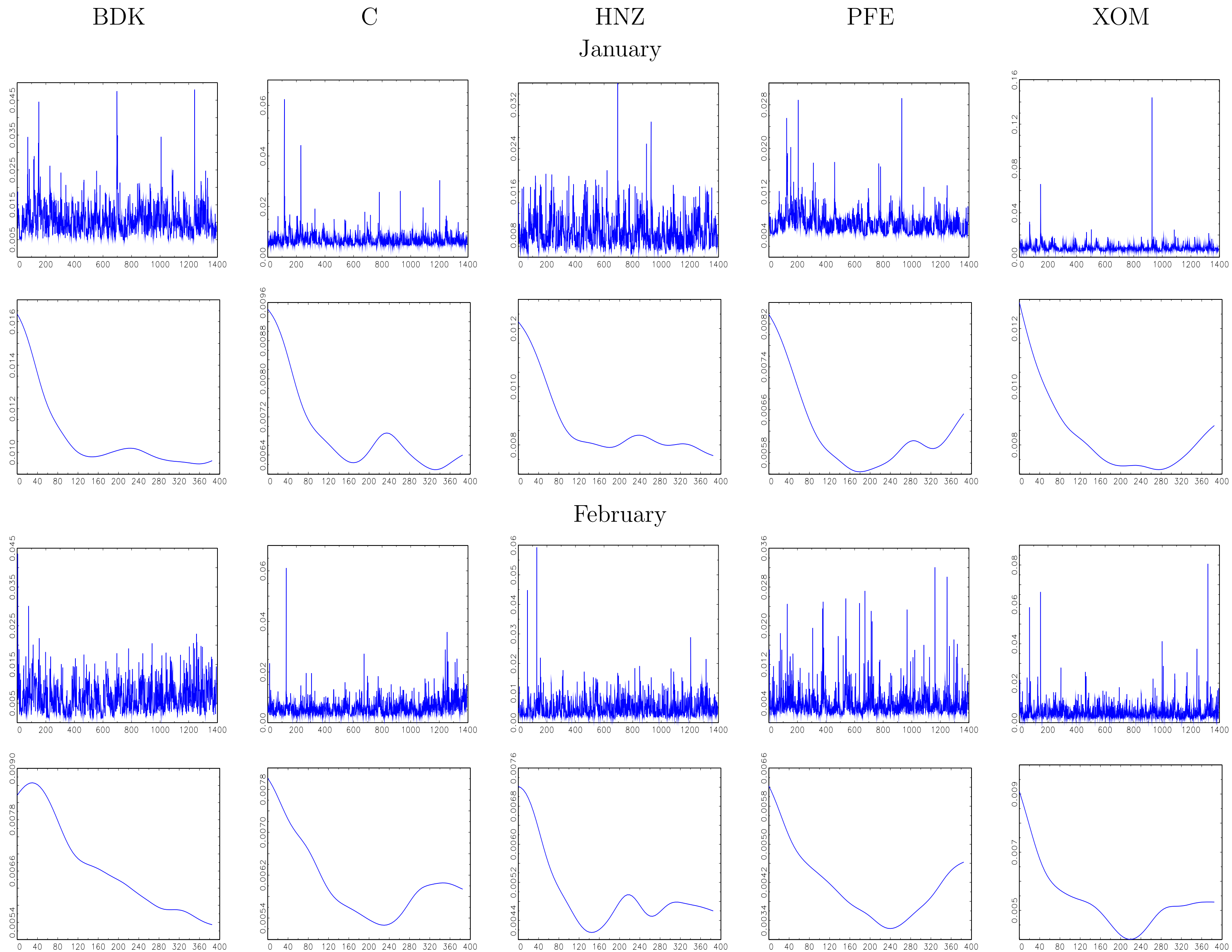

Figure 10: Line Graph of the mean of the conditional quote slope density (MIS Simulation with 100 thousand replications) with corresponding estimated (Nardaraya-Watson with Gaussian Kernel) diurnal seasonality in January (upper panels) and February (lower panels) for all five stock. The x-axis ($t = 1, \ldots, 1404$) is measured in five minutes intervals and corresponds to the time period form the 2^{nd} to the 26^{th} in January 2001 (upper panels, first row) and from the 30^{th} January 2001 to the 23^{rd} February 2001 (lower panels, first row).

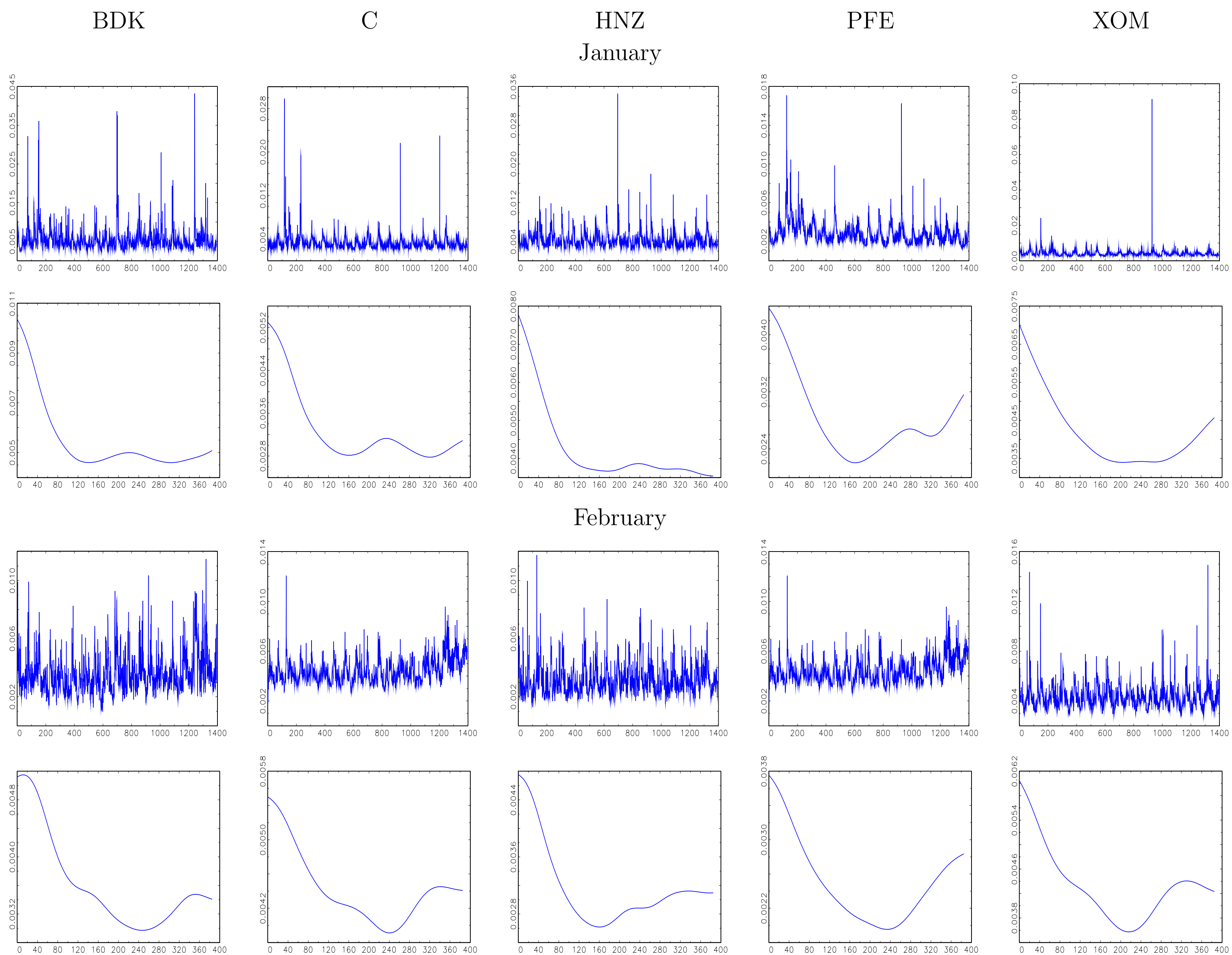

Figure 11: Line Graph of the standard deviation of the conditional quote slope density (MIS Simulation with 100 thousand replications) with corresponding estimated (Nardaraya-Watson with Gaussian Kernel) diurnal seasonality in January (upper panels) and February (lower panels) for all five stock. The x-axis ($t = 1, \ldots, 1404$) is measured in five minutes intervals and corresponds to the time period form the 2^{nd} to the 26^{th} in January 2001 (upper panels, first row) and from the 30^{th} January 2001 to the 23^{rd} February 2001 (lower panels, first row).

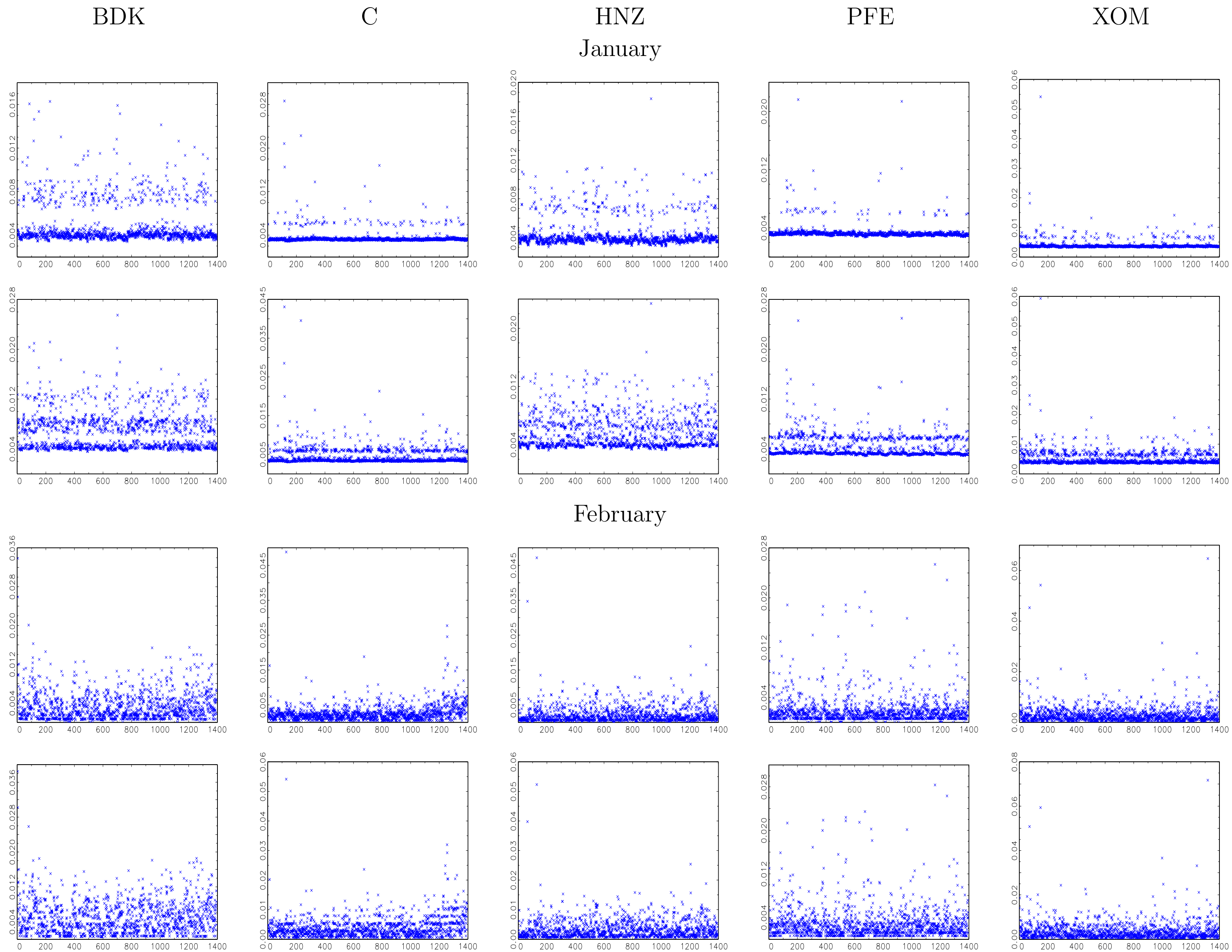

Figure 12: Scatter Plot of the 10% quantile (first row) and the 25% quantile (second row) of the conditional quote slope distribution function (MIS Simulation with 100 thousand replications) in January (upper panels) and February (lower panels) for all five stock. The x-axis ($t = 1, \ldots, 1404$) is measured in five minutes intervals and corresponds to the time period form the 2^{nd} to the 26^{th} in January 2001 (upper panels) and from the 30^{th} January 2001 to the 23^{rd} February 2001 (lower panels).

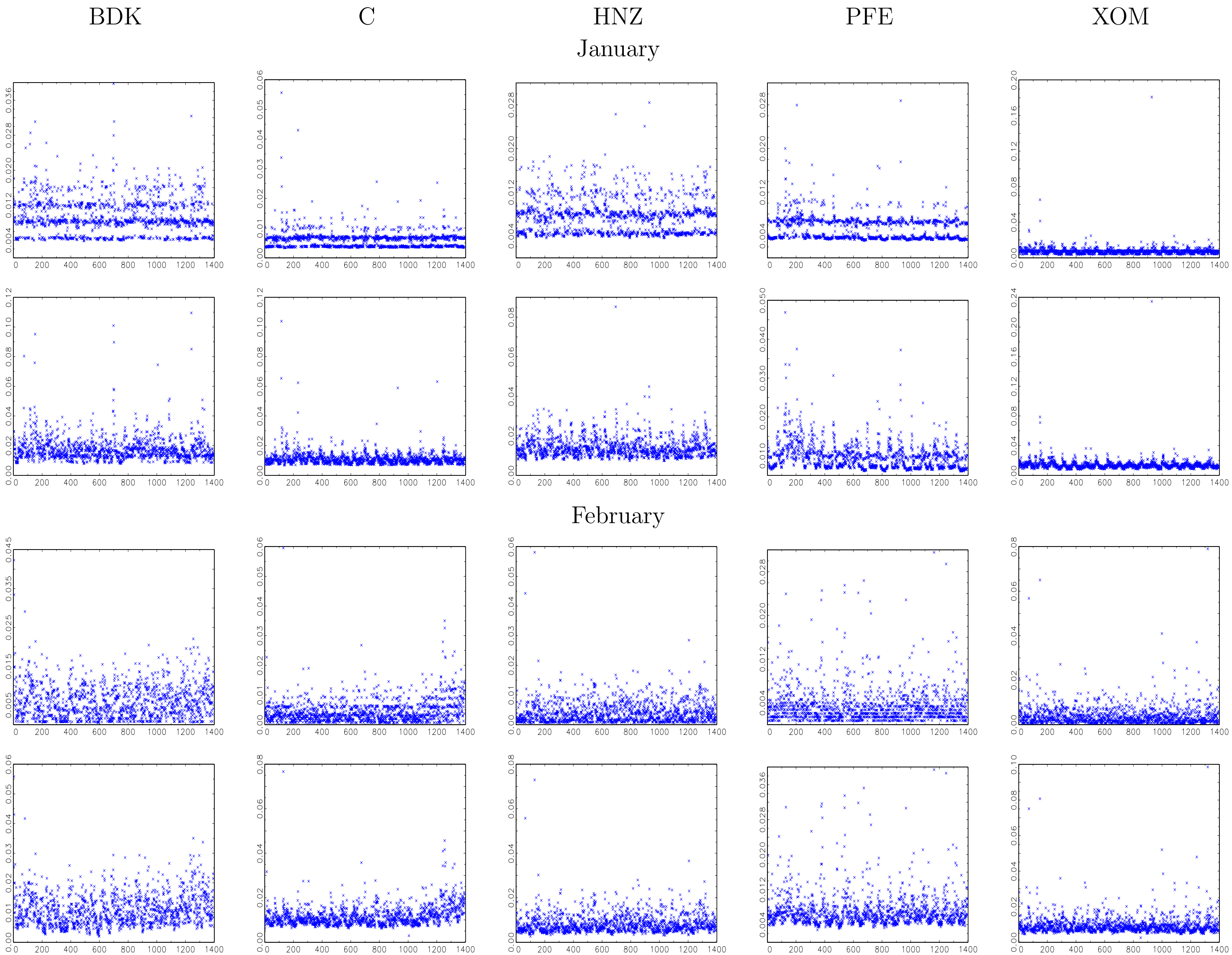

Figure 13: Scatter Plot of the 50% quantile (first row) and the 90% quantile (second row) of the conditional quote slope distribution function (MIS Simulation with 100 thousand replications) in January (upper panels) and February (lower panels) for all five stock. The x-axis ($t = 1, \dots, 1404$) is measured in five minutes intervals and corresponds to the time period form the 2^{nd} to the 26^{th} in January 2001 (upper panels) and from the 30^{th} January 2001 to the 23^{rd} February 2001 (lower panels).

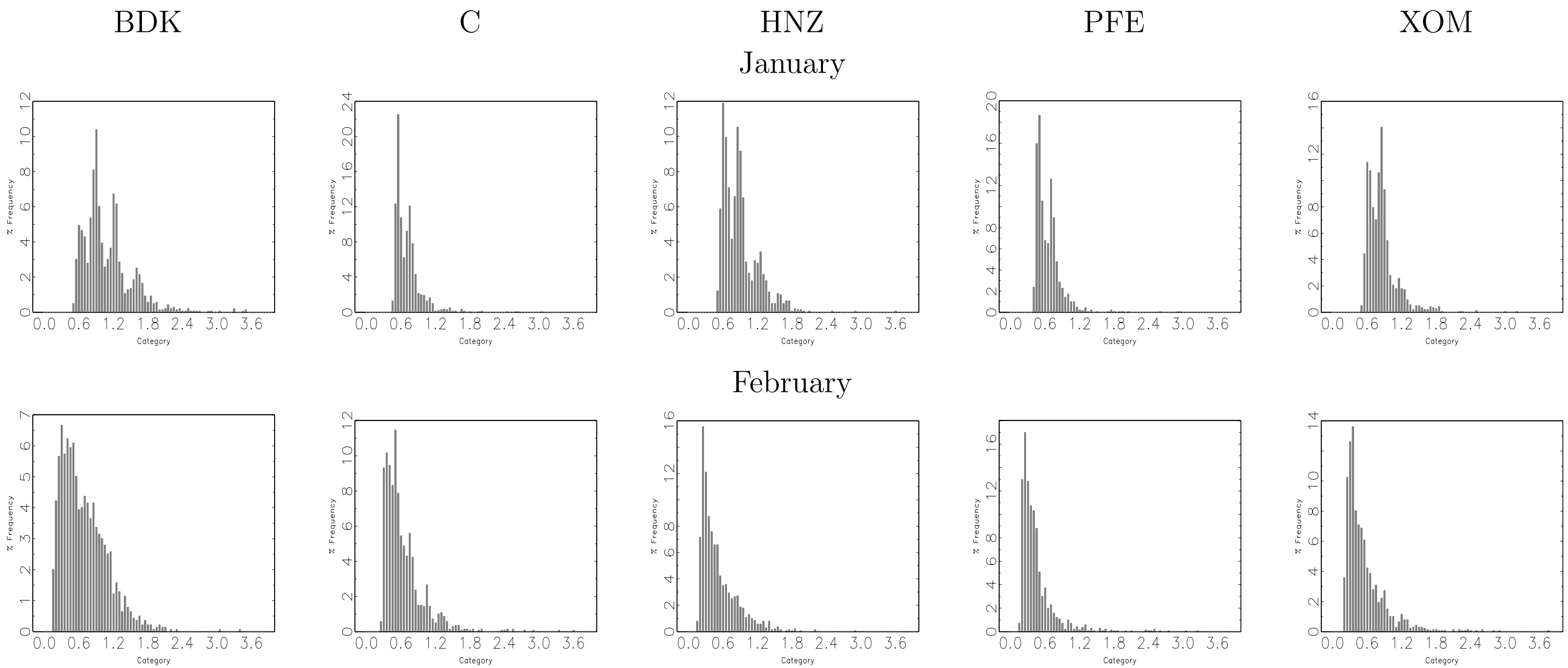

Figure 14: Histograms of the mean quote slope (MIS Simulation with 100 thousand replications) in January (upper panels) and February (lower panels) for all five stock.

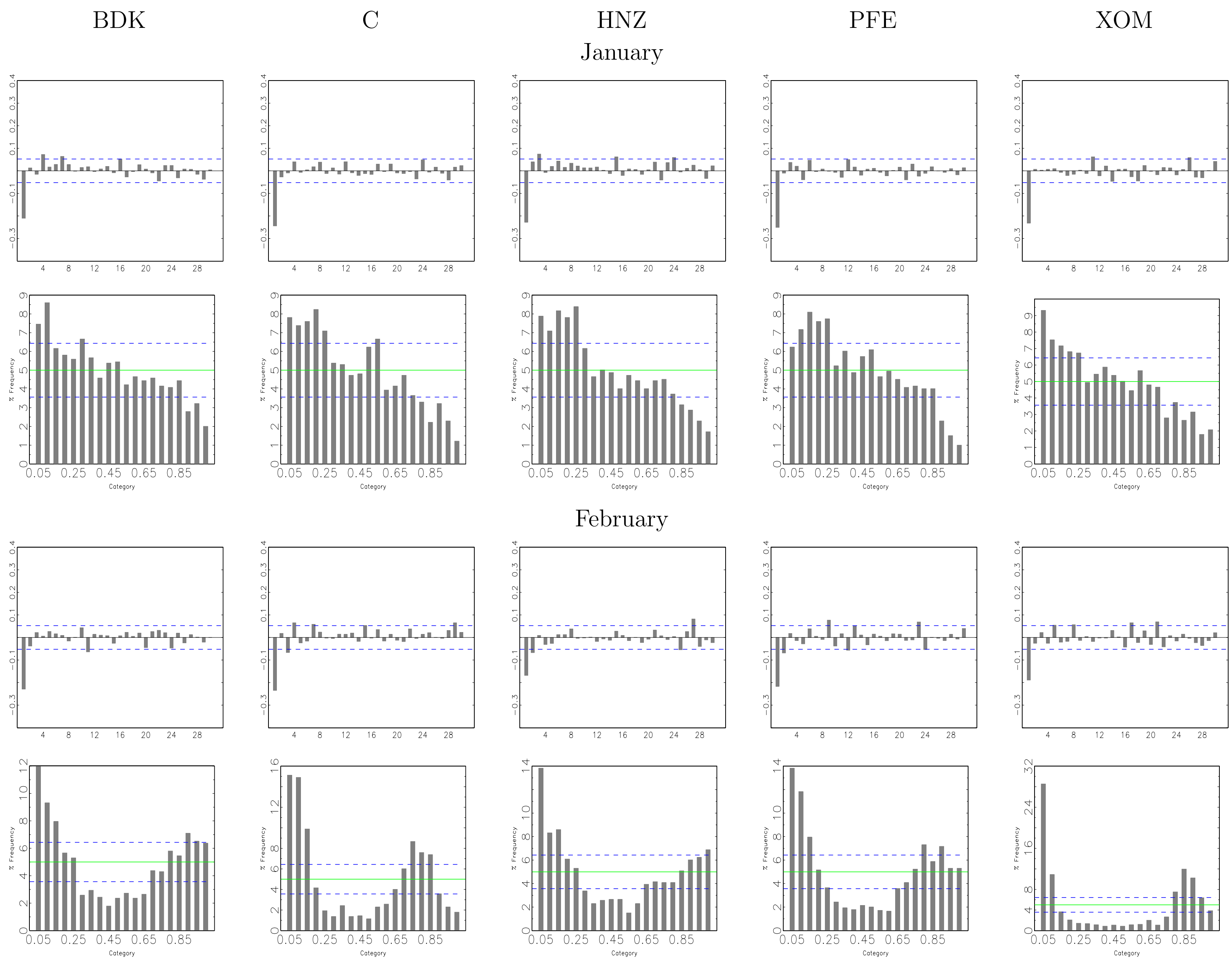

Figure 15: Autocorrelogram of the residuals of the quote slope (first row) and histogram of the values of the probability integral transformation (second row) based on the quote slope distribution function (MIS Simulation with 100 thousand replications) in January (upper panels) and February (lower panels) for all five stock. The dashed lines represent the asymptotic 95% confidence bounds.

7 Conclusion

Exploiting the concept of copula functions we model the dynamic multivariate density of a set of discrete and continuous variables. We show that truncations on the multivariate density can be modelled by imposing the truncations on the copula function. We use this approach to model the dynamic joint density of bid and ask quote changes and their corresponding depths under the restriction that the bid-ask spread must not become zero or negative. Thereby bid and ask quote changes are modelled as discrete variables since they are multiples of the tick size with the help of ICH models of Liesenfeld et al. (2006). Due to the large support of the associated depths, these variables are modelled as continuous variables using Burr distributed ACD models of Engle & Russell (1998). The technique of continuization is applied to model the corresponding copula function.

We construct the dynamic density of the quote slope liquidity measure of Hasbrouck & Seppi (2001), based on samples of the dynamic multivariate density obtained with the Metropolized Independence Sampler of Hastings (1970). This dynamic density is used to analyze how liquidity supply behaves over time and to show the influence of the decimalization at the New York Stock Exchange on the 29^{th} January 2001. We obtain three main results: (i) Mean liquidity supply as well as liquidity supply risk (measured by the standard deviation and by quantiles) is indeed time varying. This observation questions the assumption of liquidity shocks with constant mean and constant variance, made in several theoretical models of investor behavior, e.g. Karpoff (1986), Michaely & Vila (1996), Michaely et al. (1996) and Fernando (2003). (ii) Mean liquidity supply as well as liquidity risk is subject to intraday seasonality. Using information on the intraday seasonality pattern may improve models where optimal trading and optimal liquidation strategies are derived, e.g. Bertsimas & Lo (1998), Almgren & Chriss (2000) and Subramanian & Jarrow (2001). (iii) Before the decimalization, density function of the conditional liquidity is shifted to the right, which corresponds to a smaller liquidity supply, when compared with the liquidity supply after the decimalization. This observation is in line with the findings of Grossman & Miller (1988), Harris (1994) and Harris (1997), who also certify a higher liquidity for liquidity demanders after the decimalization. Furthermore, density function of the conditional liquidity possess several modi, which can be translated into jumps of the conditional quantiles of the liquidity supply density. These modi represent liquidity supply states, where a higher state ultimately relates to higher transaction costs for liquidity demanders. After the decimalization these modi are smoothed out. This observation sheds light on a different aspect of a higher liquidity supply for liquidity demanders as those aspects highlighted by Grossman & Miller (1988), Harris (1994) and Harris (1997) grounding on shifts in mean functions. Our observation states, that after the decimalization, the risk of being in or the cost of to avoid being in an unfavorable liquidity state is diminished, for a specific group a market participants.

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Appendix

	January							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.1123	0.26	0.28	4396.94	0.1111	0.30	0.26	6969.52
std. deviation	0.0980	0.44	0.45	7758.76	0.1037	0.46	0.44	11653.15
skewness	6.8331	1.07	0.97	5.09	8.3124	0.89	1.08	4.83
kurtosis	82.1151	2.14	1.94	40.32	115.295	1.79	2.17	37.72
minimum	0.0625	0	0	100	0.0625	0	0	100
1% Quantile	0.0625	0	0	200	0.0625	0	0	100
5% Quantile	0.0625	0	0	300	0.0625	0	0	300
10% Quantile	0.0625	0	0	500	0.0625	0	0	500
25% Quantile	0.0625	0	0	900	0.0625	0	0	1000
50% Quantile	0.0625	0	0	2000	0.0625	0	0	2900
75% Quantile	0.1250	1	1	5000	0.1250	1	1	8100
90% Quantile	0.1875	1	1	10000	0.1875	1	1	19400
95% Quantile	0.2500	1	1	16380	0.2500	1	1	25240
99% Quantile	0.3750	1	1	42516	0.3750	1	1	50000
maximum	1.4375	1	1	104400	1.7500	1	1	140700
LB(10)	114.56	75.90		900.74	109.88	95.52		644.18
p-value	0.0000	0.0005		0.0000	0.0000	0.0000		0.0000
LB(20)	120.20	121.69		1032.78	117.30	132.55		757.03
p-value	0.0000	0.0018		0.0000	0.0000	0.0002		0.0000
LB(30)	129.62	171.33		1402.22	125.74	161.64		785.56
p-value	0.0000	0.0015		0.0000	0.0000	0.00068		0.0000

	February							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.0590	0.31	0.44	1685.33	0.0624	0.43	0.33	3428.99
std. deviation	0.0605	0.46	0.50	2834.31	0.0713	0.50	0.47	6720.39
skewness	2.1240	0.82	0.23	5.77	3.8046	0.29	0.70	5.90
kurtosis	9.6310	1.67	1.05	53.08	27.2740	1.08	1.48	64.69
minimum	0.0100	0	0	100	0.0100	0	0	100
1% Quantile	0.0100	0	0	100	0.0100	0	0	100
5% Quantile	0.0100	0	0	100	0.0100	0	0	100
10% Quantile	0.0100	0	0	100	0.0100	0	0	200
25% Quantile	0.0200	0	0	400	0.0200	0	0	500
50% Quantile	0.0400	0	0	900	0.0400	0	0	1000
75% Quantile	0.0800	1	1	1800	0.0900	1	1	3200
90% Quantile	0.1400	1	1	4260	0.1400	1	1	9060
95% Quantile	0.1800	1	1	5500	0.1800	1	1	15000
99% Quantile	0.2644	1	1	12676	0.3528	1	1	27676
maximum	0.5000	1	1	34300	0.7600	1	1	112800
LB(10)	121.29	48.20		273.79	91.78	68.00		176.92
p-value	0.0000	0.1752		0.0000	0.0000	0.0038		0.0000
LB(20)	138.79	88.93		275.82	112.21	114.16		224.04
p-value	0.0000	0.2315		0.0000	0.0000	0.0073		0.0000
LB(30)	162.82	106.49		283.98	120.66	150.34		246.16
p-value	0.0000	0.8061		0.0000	0.0000	0.0317		0.0000

Table 9: Descriptive statistics of the quotes changes, the quote change direction indicator and the corresponding depths for the bid and ask sides in January and February 2001 for HNZ.

	January							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.1440	0.39	0.39	16856.20	0.1423	0.41	0.38	20585.40
std. deviation	0.1411	0.49	0.49	22931.07	0.1448	0.49	0.49	23705.32
skewness	9.0114	0.45	0.46	8.18	11.1279	0.38	0.50	2.7650
kurtosis	159.9699	1.20	1.21	146.37	229.1586	1.14	1.25	15.48
minimum	0.0625	0	0	300	0.0625	0	0	100
1% Quantile	0.0625	0	0	1000	0.0625	0	0	1000
5% Quantile	0.0625	0	0	1000	0.0625	0	0	1500
10% Quantile	0.0625	0	0	2000	0.0625	0	0	2500
25% Quantile	0.0625	0	0	5000	0.0625	0	0	5500
50% Quantile	0.1250	0	0	10000	0.1250	0	0	11800
75% Quantile	0.1875	1	1	20000	0.1875	1	1	25000
90% Quantile	0.2500	1	1	40000	0.2500	1	1	50000
95% Quantile	0.3125	1	1	50000	0.3125	1	1	67500
99% Quantile	0.6250	1	1	100000	0.6231	1	1	100000
maximum	3.0000	1	1	500000	3.3750	1	1	250000
LB(10)	107.0946	46.17		117.77	130.80	44.88		140.27
p-value	0.0000	0.2323		0.0000	0.0000	0.2746		0.0000
LB(20)	114.4044	80.81		160.73	136.94	70.21		195.03
p-value	0.0000	0.45		0.0000	0.0000	0.7749		0.0000
LB(30)	121.8002	122.78		211.21	150.67	107.96		237.86
p-value	0.0000	0.4126		0.0000	0.0000	0.7768		0.0000

	February							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.1091	0.46	0.45	8028.92	0.1085	0.49	0.42	9496.15
std. deviation	0.1138	0.50	0.50	13173.02	0.1048	0.50	0.49	17325.66
skewness	3.6289	0.14	0.20	4.92	2.4367	0.05	0.31	7.41
kurtosis	31.6766	1.02	1.04	45.95	11.9358	1.00	1.09	95.75
minimum	0.0100	0	0	100	0.0100	0	0	100
1% Quantile	0.0100	0	0	100	0.0100	0	0	100
5% Quantile	0.0100	0	0	400	0.0100	0	0	400
10% Quantile	0.0200	0	0	700	0.0200	0	0	640
25% Quantile	0.0400	0	0	1100	0.0400	0	0	1300
50% Quantile	0.0800	0	0	3500	0.0800	0	0	4500
75% Quantile	0.1400	1	1	10000	0.1500	1	1	10000
90% Quantile	0.2400	1	1	20000	0.2400	1	1	22160
95% Quantile	0.3175	1	1	30000	0.3000	1	1	30000
99% Quantile	0.5415	1	1	56844	0.5000	1	1	70000
maximum	1.6200	1	1	200000	0.8700	1	1	302200
LB(10)	430.7741	38.42		32.41	400.03	28.46		92.84
p-value	0.0000	0.5414		0.0003	0.0000	0.9137		0.0000
LB(20)	578.9171	74.37		54.64	584.52	65.65		103.06
p-value	0.0000	0.6566		0.0000	0.0000	0.8764		0.0000
LB(30)	658.5780	107.59		68.04	663.11	98.70		106.88
p-value	0.0000	0.7844		0.0001	0.0000	0.9226		0.0000

Table 10: Descriptive statistics of the quotes changes, the quote change direction indicator and the corresponding depths for the bid and ask sides in January and February 2001 for C.

	January							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.1123	0.33	0.33	20404.20	0.1122	0.34	0.33	24943.95
std. deviation	0.0888	0.47	0.47	24684.82	0.0917	0.47	0.47	32447.45
skewness	2.9406	0.73	0.74	3.14	3.2318	0.69	0.72	4.20
kurtosis	15.2992	1.54	1.55	18.48	17.7688	1.48	1.51	35.18
minimum	0.0625	0	0	100	0.0625	0	0	100
1% Quantile	0.0625	0	0	300	0.0625	0	0	300
5% Quantile	0.0625	0	0	1000	0.0625	0	0	1000
10% Quantile	0.0625	0	0	1800	0.0625	0	0	2000
25% Quantile	0.0625	0	0	5000	0.0625	0	0	5800
50% Quantile	0.0625	0	0	12800	0.0625	0	0	14500
75% Quantile	0.1250	1	1	25700	0.1250	1	1	31600
90% Quantile	0.1875	1	1	50000	0.1875	1	1	50000
95% Quantile	0.2500	1	1	61280	0.2500	1	1	99960
99% Quantile	0.4894	1	1	115188	0.5000	1	1	139584
maximum	0.8750	1	1	255400	0.9375	1	1	436500
LB(10)	369.25	127.26		484.77	343.55	126.16		369.00
p-value	0.0000	0.0000		0.0000	0.0000	0.0000		0.0000
LB(20)	387.87	172.81		551.03	368.83	169.64		422.47
p-value	0.0000	0.0001		0.0000	0.0000	0.0000		0.0000
LB(30)	457.66	209.46		554.43	401.99	205.18		436.24
p-value	0.0000	0.0005		0.0000	0.0000	0.0000		0.0000

	February							
	bid-side				ask-side			
	abs. quote change	indicator neg. dir	indicator pos. dir	depths	abs. quote change	indicator neg. dir	indicator pos. dir	depths
mean	0.0695	0.43	0.48	5180.06	0.0695	0.47	0.44	6314.25
std. deviation	0.0744	0.49	0.50	9939.57	0.0752	0.50	0.50	14237.65
skewness	3.0109	0.30	0.10	5.52	2.9593	0.12	0.24	7.69
kurtosis	18.1709	1.09	1.01	44.32	16.8701	1.01	1.06	86.28
minimum	0.0100	0	0	100	0.0100	0	0	100
1% Quantile	0.0100	0	0	100	0.0100	0	0	100
5% Quantile	0.0100	0	0	200	0.0100	0	0	200
10% Quantile	0.0100	0	0	300	0.0100	0	0	400
25% Quantile	0.0200	0	0	900	0.0200	0	0	1000
50% Quantile	0.0400	0	0	2000	0.0500	0	0	2200
75% Quantile	0.0900	1	1	5400	0.0900	1	1	6100
90% Quantile	0.1500	1	1	10560	0.1600	1	1	14700
95% Quantile	0.2100	1	1	20000	0.2100	1	1	23380
99% Quantile	0.3300	1	1	50000	0.3600	1	1	58852
maximum	0.7300	1	1	100000	0.7300	1	1	200000
LB(10)	213.74	38.20		47.14	188.51	51.95		81.32
p-value	0.0000	0.5516		0.0000	0.0000	0.0975		0.0000
LB(20)	220.16	90.69		61.43	201.43	83.33		110.84
p-value	0.0000	0.1942		0.0000	0.0000	0.3775		0.0000
LB(30)	236.99	117.95		70.63	238.06	132.31		127.14
p-value	0.0000	0.54		0.0000	0.0000	0.2085		0.0000

Table 11: Descriptive statistics of the quotes changes, the quote change direction indicator and the corresponding depths for the bid and ask sides in January and February 2001 for PFE.

	January							
	bid-side				ask-side			
	abs. quote change	indicator		depths	abs. quote change	indicator		depths
		neg. dir	pos. dir			neg. dir	pos. dir	
mean	0.1429	0.39	0.37	7176.28	0.1405	0.41	0.35	7791.88
std. deviation	0.1129	0.49	0.48	9293.25	0.1084	0.49	0.48	9992.88
skewness	2.9843	0.45	0.52	3.76	2.4226	0.37	0.61	3.09
kurtosis	19.6754	1.20	1.27	26.79	13.5139	1.14	1.37	17.17
minimum	0.0625	0	0	100	0.0625	0	0	100
1% Quantile	0.0625	0	0	300	0.0625	0	0	500
5% Quantile	0.0625	0	0	1000	0.0625	0	0	1000
10% Quantile	0.0625	0	0	1000	0.0625	0	0	1000
25% Quantile	0.0625	0	0	1500	0.0625	0	0	1500
50% Quantile	0.1250	0	0	4300	0.1250	0	0	4700
75% Quantile	0.1875	1	1	9300	0.1875	1	1	10000
90% Quantile	0.2500	1	1	17920	0.2500	1	1	19060
95% Quantile	0.3750	1	1	24080	0.3750	1	1	29100
99% Quantile	0.5169	1	1	48580	0.5000	1	1	50000
maximum	1.2500	1	1	114900	1.1250	1	1	100000
LB(10)	228.48	104.44		79.21	113.79	56.87		180.78
p-value	0.0000	0.0000		0.0000	0.0000	0.0406		0.0000
LB(20)	237.14	155.34		90.31	139.95	110.47		231.67
p-value	0.0000	0.0000		0.0000	0.0000	0.0136		0.0000
LB(30)	255.13	184.34		101.37	153.82	145.29		238.00
p-value	0.0000	0.0001		0.0000	0.0000	0.0579		0.0000

	February							
	bid-side				ask-side			
	abs. quote change	indicator		depths	abs. quote change	indicator		depths
		neg. dir	pos. dir			neg. dir	pos. dir	
mean	0.0936	0.41	0.49	2864.96	0.09	0.49	0.44	3496.37
std. deviation	0.1076	0.49	0.50	4445.82	0.11	0.50	0.50	6047.77
skewness	3.6830	0.36	0.05	4.10	3.99	0.06	0.25	3.98
kurtosis	25.6795	1.13	1.00	28.74	27.01	1.00	1.06	23.31
minimum	0.0100	0	0	100	0.0100	0	0	100
1% Quantile	0.0100	0	0	100	0.0100	0	0	100
5% Quantile	0.0100	0	0	100	0.0100	0	0	100
10% Quantile	0.0100	0	0	200	0.0100	0	0	200
25% Quantile	0.0300	0	0	500	0.0300	0	0	500
50% Quantile	0.0600	0	0	1100	0.0600	0	0	1300
75% Quantile	0.1200	1	1	3200	0.1200	1	1	4000
90% Quantile	0.2100	1	1	7200	0.2000	1	1	8260
95% Quantile	0.2800	1	1	10680	0.2730	1	1	14200
99% Quantile	0.5137	1	1	20480	0.5912	1	1	30000
maximum	1.2100	1	1	49200	1.1000	1	1	54500
LB(10)	216.32	43.38		38.07	182.43	60.07		58.05
p-value	0.0000	0.33 1		0.0000	0.0000	0.02		0.0000
LB(20)	222.79	69.21		46.79	187.73	99.26		59.10
p-value	0.0000	0.80		0.0006	0.0000	0.07		0.0000
LB(30)	229.26	106.31		49.24	193.02	151.27		73.45
p-value	0.0000	0.81		0.01	0.0000	0.03		0.0000

Table 12: Descriptive statistics of the quotes changes, the quote change direction indicator and the corresponding depths for the bid and ask sides in January and February 2001 for XOM.

	Buy Volume		Sell Volume		# Buys		# Sells	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	6451.00	5754.13	4542.81	4819.87	3.89	4.13	3.43	3.69
std. deviation	11478.48	11194.89	8178.12	8322.69	3.42	3.14	2.92	2.90
skewness	4.48	6.34	4.88	4.45	1.91	1.15	1.31	0.97
kurtosis	30.86	61.51	38.57	30.39	11.02	4.77	5.43	3.92
minimum	0	0	0	0	0	0	0	0
1% Quantile	0	0	0	0	0	0	0	0
5% Quantile	0	0	0	0	0	0	0	0
10% Quantile	0	100	0	0	0	1	0	0
25% Quantile	700	800	400	600	1	2	1	1
50% Quantile	2600	2700	1900	2200	3	4	3	3
75% Quantile	7100	5900	5100	5500	5	6	5	5
90% Quantile	15300	13160	11360	11300	8	8	7	8
95% Quantile	25680	21080	17460	17380	10	10	9	9
99% Quantile	52456	51000	39044	46748	15	14	13	12
maximum	115700	156200	106300	91300	36	22	21	16
LB(10)	114.76	55.33	181.37	84.73	415.29	279.37	216.62	252.51
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	182.06	62.26	197.32	96.99	566.87	371.32	270.84	289.13
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(30)	224.52	68.65	202.37	108.81	600.17	396.69	298.97	300.09
p-value	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 13: Descriptive statistics of the explanatory variables for HNZ.

	Buy Volume		Sell Volume		# Buys		# Sells	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	93537.25	64457.41	76352.28	62904.77	19.75	16.04	16.68	13.55
std. deviation	133638.83	63598.75	99200.57	71115.52	7.94	7.44	7.53	6.64
skewness	5.80	2.78	8.11	4.96	0.45	0.35	0.49	0.66
kurtosis	53.63	14.43	124.94	53.74	3.15	2.85	2.99	3.52
minimum	200	0	0	0	1	0	0	0
1% Quantile	3224	1904	1508	1916	4	1	2	2
5% Quantile	10100	8500	7640	7120	8	5	6	4
10% Quantile	15640	12920	12540	11300	10	7	8	6
25% Quantile	29900	25700	25500	21800	14	11	11	9
50% Quantile	56900	46200	52200	42600	19	16	16	13
75% Quantile	108500	81300	93900	81300	25	21	21	18
90% Quantile	186520	131600	159660	130040	30	26	27	22
95% Quantile	283080	193120	207460	179060	34	29	30.80	26
99% Quantile	666072	326976	384972	336720	40.96	35	36	31
maximum	1778600	574400	2012100	1169700	50	42	45	41
LB(10)	1422.85	827.72	326.00	513.96	234.49	252.74	217.9916	176.37
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	1853.40	1093.76	373.72	744.51	291.42	334.95	248.5169	206.44
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(30)	2147.56	1110.19	397.05	765.78	354.37	342.48	291.5115	224.63
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 14: Descriptive statistics of the explanatory variables for C.

	Buy Volume		Sell Volume		# Buys		# Sells	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	94166.74	70707.55	81068.95	57984.97	21.68	16.60	17.96	13.93
std. deviation	117974.26	96243.99	105833.06	71120.45	9.01	7.10	8.20	6.44
skewness	5.42	7.82	6.34	4.20	0.60	0.27	0.73	0.42
kurtosis	53.68	113.69	85.92	28.32	4.11	3.05	3.86	3.20
minimum	0	0	0	0	0	0	0	0
1% Quantile	2104	904	1604	0	1	1	2	0
5% Quantile	11320	6420	7560	5600	9	5	7	4
10% Quantile	17100	12600	13040	9780	11	8	9	6
25% Quantile	33400	22700	24700	20200	15	12	12	9
50% Quantile	60500	46800	52300	38200	21	16	17	13
75% Quantile	113400	84900	100600	69400	27	21	23	18
90% Quantile	197360	150320	167280	120380	33	26	29	23
95% Quantile	276500	202060	250320	168200	38	29	33	25
99% Quantile	560004	381108	494876	401508	46	34	41	31
maximum	1804800	1880200	1992600	732400	71	45	52	40
LB(10)	356.79	305.81	323.21	286.77	909.80	572.93	370.65	287.41
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	392.71	325.88	424.43	336.78	1132.64	776.73	416.31	341.42
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(30)	396.39	333.53	516.36	356.11	1164.64	789.78	450.44	346.47
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 15: Descriptive statistics of the explanatory variables for PFE.

	Buy Volume		Sell Volume		# Buys		# Sells	
	Jan	Feb	Jan	Feb	Jan	Feb	Jan	Feb
mean	41077.35	28558.69	36442.31	24135.61	11.08	9.87	11.11	9.12
std. deviation	46427.43	35139.22	45764.77	24750.02	6.07	5.16	5.92	4.78
skewness	2.85	6.76	9.83	3.13	0.61	0.60	0.57	0.57
kurtosis	16.55	82.51	206.65	19.55	3.35	3.29	3.28	3.51
minimum	0	0	0	0	0	0	0	0
1% Quantile	0	0	0	0	0	0	0	0
5% Quantile	1700	2820	2300	1720	2	2	2	2
10% Quantile	4240	5100	5000	4000	4	4	4	3
25% Quantile	11800	10700	11900	8400	7	6	7	6
50% Quantile	25300	19900	24100	17200	10	9	10	8
75% Quantile	53900	34700	47300	31800	15	13	15	12
90% Quantile	95060	57020	80920	51060	19	17	19	16
95% Quantile	130560	77220	107080	65660	22	19	22	18
99% Quantile	210532	152884	171384	133532	28	23	27	21.96
maximum	484700	565500	1093800	272500	35	33	33	35
LB(10)	471.45	189.40	232.39	138.04	198.00	226.90	140.08	181.49
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(20)	563.51	198.26	296.47	168.51	224.48	267.37	178.00	215.22
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LB(30)	577.58	212.97	341.56	177.74	243.97	283.21	187.24	219.48
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 16: Descriptive statistics of the explanatory variables for XOM.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
μ_1	-0.1800	0.2325	-0.0246	0.0478	0.5109	0.2291	0.2140	0.2073
μ_2	-0.3678	0.2358	-0.0287	0.0463	0.3600	0.1709	0.1763	0.1696
$c_1^{(1)}$	0.2095	0.1925	0.8641	0.1427	0.7069	0.1253	0.7945	0.1778
$a_{11}^{(1)}$	-0.0560	0.0960	0.0176	0.0510	-0.0842	0.1611	0.0550	0.1865
$a_{12}^{(1)}$	-0.0475	0.0967	0.1010	0.0501	-0.0771	0.1614	0.0948	0.2212
$a_{21}^{(1)}$	0.1032	0.0927	0.0742	0.0569	-0.0380	0.1525	0.0556	0.1762
$a_{22}^{(1)}$	-0.1742	0.0861	0.0635	0.0458	-0.1448	0.1669	0.0380	0.1906
g_{vb1}	0.0002	0.0002	0.0001	0.0001	-0.0004	0.0002	-0.0006	0.0002
g_{vs1}	0.0003	0.0002	0.0003	0.0002	0.0004	0.0002	0.0005	0.0003
g_{nb1}	-0.0559	0.0125	-0.0530	0.0120	-0.0299	0.0168	0.0120	0.0173
g_{ns1}	0.0872	0.0133	0.0799	0.0132	0.0325	0.0166	0.0310	0.0188
g_{vb2}	0.0003	0.0001	0.0004	0.0001	0.0003	0.0002	0.0004	0.0002
g_{vs2}	0.0001	0.0002	-0.0001	0.0002	-0.0001	0.0002	-0.0001	0.0003
g_{nb2}	0.0527	0.0112	0.0493	0.0106	0.0227	0.0165	0.0582	0.0162
g_{ns2}	-0.0317	0.0139	-0.0386	0.0141	-0.0114	0.0166	-0.0304	0.0191
log-lik.	-0.884695		-0.881441		-0.834186		-0.813897	
SIC	0.923623		0.920369		0.873114		0.852825	
Q(10)	54.205 (0.001)		30.547 (0.290)		21.630 (0.756)		41.296 (0.039)	
Q(20)	81.92 (0.104)		68.614 (0.422)		53.860 (0.877)		64.423 (0.567)	
Q(30)	123.834 (0.127)		106.965 (0.483)		94.400 (0.803)		114.410 (0.294)	
res. mean	(-0.013, -0.021)		(0.003, -0.009)		(-0.003, 0.013)		(-0.000, 0.022)	
res. var.	$\begin{pmatrix} 0.963 & 0.592 \\ 0.592 & 2.128 \end{pmatrix}$		$\begin{pmatrix} 0.756 & 0.206 \\ 0.206 & 1.565 \end{pmatrix}$		$\begin{pmatrix} 0.724 & 1.119 \\ 1.119 & 3.433 \end{pmatrix}$		$\begin{pmatrix} 0.818 & 1.284 \\ 1.284 & 3.925 \end{pmatrix}$	

Table 17: ML estimates of the ACM-ARMA part of ICH model. ASK and BID Quote changes in January and February for C.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
μ_1	-0.5500	0.2942	-0.2123	0.0733	-0.0168	0.0932	-0.3882	0.1584
μ_2	-0.7316	0.3684	-0.1704	0.0587	-0.4724	0.2081	-0.0131	0.1230
$c_1^{(1)}$	0.5868	0.2123	0.8509	0.0487	0.4711	0.1971	0.2849	0.1621
$a_{11}^{(1)}$	0.1902	0.0750	0.0588	0.0436	0.2390	0.0788	0.0985	0.0869
$a_{12}^{(1)}$	0.3008	0.1009	0.0891	0.0404	0.2589	0.0792	0.1606	0.0847
$a_{21}^{(1)}$	0.1605	0.0788	0.0853	0.0502	0.2344	0.0995	0.2239	0.0862
$a_{22}^{(1)}$	0.2835	0.0688	0.1082	0.0367	0.2242	0.1013	0.1483	0.0814
g_{vb1}	0.0001	0.0013	0.0008	0.0010	-0.0015	0.0013	-0.0001	0.0014
g_{vs1}	0.0029	0.0009	0.0046	0.0013	0.0012	0.0012	0.0042	0.0016
g_{nb1}	-0.0177	0.0349	-0.0539	0.0311	0.0449	0.0334	0.0157	0.0329
g_{ns1}	0.2292	0.0280	0.2004	0.0302	0.1373	0.0316	0.1315	0.0317
g_{vb2}	0.0040	0.0010	0.0030	0.0008	0.0021	0.0010	0.0018	0.0010
g_{vs2}	-0.0002	0.0014	-0.0026	0.0021	0.0004	0.0014	0.0003	0.0018
g_{nb2}	0.2055	0.0305	0.1334	0.0237	0.1821	0.0306	0.0961	0.0275
g_{ns2}	0.0267	0.0354	-0.0135	0.0351	0.0948	0.0345	0.0258	0.0337
log-lik.	-0.903721		-0.9260685		-0.984456		-0.992658	
SIC	0.942650		0.9649967		1.023384		1.031596	
Q(10)	43.891 (0.021)		36.591 (0.103)		33.448 (0.183)		37.436 (0.087)	
Q(20)	91.458 (0.025)		89.369 (0.035)		68.640 (0.421)		70.946 (0.348)	
Q(30)	132.239 (0.049)		139.606 (0.019)		108.483 (0.442)		90.471 (0.874)	
res. mean	(-0.009, -0.021)		(-0.015, 0.004)		(-0.004, -0.009)		(-0.007, 0.000)	
res. var.	$\begin{pmatrix} 0.908 & 0.110 \\ 0.110 & 1.225 \end{pmatrix}$		$\begin{pmatrix} 0.873 & 0.063 \\ 0.063 & 1.205 \end{pmatrix}$		$\begin{pmatrix} 0.713 & 0.268 \\ 0.268 & 1.023 \end{pmatrix}$		$\begin{pmatrix} 0.713 & 0.310 \\ 0.310 & 1.621 \end{pmatrix}$	

Table 18: ML estimates of the ACM-ARMA part of ICH model. ASK and BID Quote changes in January and February for HNZ.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
μ_1	-0.4935	0.2463	-0.5511	0.2259	1.7459	0.6286	0.1886	0.0842
μ_2	-0.5536	0.2685	-0.5447	0.2239	1.0525	0.5991	0.1782	0.0843
$c_1^{(1)}$	0.5862	0.1639	0.6540	0.1258	-0.5182	0.2179	0.8476	0.0576
$a_{11}^{(1)}$	0.1454	0.0673	0.0977	0.0636	0.4077	0.1281	0.1102	0.0913
$a_{12}^{(1)}$	0.2973	0.0783	0.1219	0.0690	0.4181	0.1274	0.1718	0.0781
$a_{21}^{(1)}$	0.1816	0.0785	0.2783	0.0684	0.5147	0.1431	0.1649	0.0868
$a_{22}^{(1)}$	0.1943	0.0662	0.1153	0.0618	0.5003	0.1435	0.1790	0.0772
g_{vb1}	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-0.0003	0.0002
g_{vs1}	0.0004	0.0001	0.0004	0.0001	0.0007	0.0002	0.0001	0.0002
g_{nb1}	-0.0101	0.0102	-0.0018	0.0102	-0.0254	0.0157	-0.0223	0.0170
g_{ns1}	0.0551	0.0104	0.0609	0.0103	0.0298	0.0180	0.0413	0.0187
g_{vb2}	0.0004	0.0001	0.0004	0.0001	0.0004	0.0002	0.0000	0.0001
g_{vs2}	0.0000	0.0001	0.0002	0.0001	0.0002	0.0003	-0.0004	0.0002
g_{nb2}	0.0466	0.0111	0.0539	0.0103	0.0646	0.0159	0.0743	0.0155
g_{ns2}	-0.0084	0.0117	-0.0088	0.0116	-0.0392	0.0192	-0.0509	0.0197
log-lik.	-0.961045		-0.960572		-0.812708		-0.813106	
SIC	0.999973		0.999501		0.851637		0.852034	
Q(10)	68.006 (0.000)		33.589 (0.178)		37.693 (0.083)		35.949 (0.116)	
Q(20)	97.082 (0.010)		73.581 (0.272)		78.656 (0.156)		86.128 (0.058)	
Q(30)	138.612 (0.022)		107.832 (0.460)		120.818 (0.171)		117.157 (0.236)	
res. mean	(-0.015, -0.029)		(-0.015, -0.016)		(0.001, 0.004)		(0.001, 0.008)	
res. var.	$\begin{pmatrix} 0.958 & 0.504 \\ 0.504 & 2.289 \end{pmatrix}$		$\begin{pmatrix} 0.867 & 0.265 \\ 0.265 & 1.551 \end{pmatrix}$		$\begin{pmatrix} 1.215 & 1.747 \\ 1.747 & 3.937 \end{pmatrix}$		$\begin{pmatrix} 0.715 & 0.854 \\ 0.854 & 2.702 \end{pmatrix}$	

Table 19: ML estimates of the ACM-ARMA part of ICH model. ASK and BID Quote changes in January and February for PFE.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
μ_1	-0.0880	0.1704	-0.4507	0.1828	0.1259	0.0956	0.0366	0.0186
μ_2	-0.3341	0.1969	-0.6421	0.2027	0.0758	0.0627	0.0755	0.0235
$c_1^{(1)}$	0.3189	0.1330	0.3221	0.1027	0.8839	0.0824	0.9414	0.0173
$a_{11}^{(1)}$	0.0674	0.0996	0.0311	0.0864	0.0799	0.1629	-0.1036	0.0613
$a_{12}^{(1)}$	0.1576	0.1009	0.1138	0.0800	-0.0591	0.1235	-0.1556	0.0585
$a_{21}^{(1)}$	0.2268	0.0783	0.3109	0.0867	0.0649	0.1494	-0.1194	0.0628
$a_{22}^{(1)}$	-0.0999	0.0885	-0.0222	0.0869	-0.0193	0.1112	-0.1044	0.0634
g_{vb1}	0.0000	0.0003	-0.0001	0.0003	-0.0003	0.0007	-0.0009	0.0005
g_{vs1}	0.0005	0.0006	0.0014	0.0003	0.0001	0.0005	0.0005	0.0005
g_{nb1}	-0.0500	0.0202	-0.0218	0.0176	-0.0122	0.0279	0.0168	0.0241
g_{ns1}	0.0776	0.0217	0.0741	0.0164	0.0969	0.0288	0.0729	0.0241
g_{vb2}	0.0008	0.0002	0.0008	0.0002	0.0009	0.0006	0.0000	0.0003
g_{vs2}	-0.0009	0.0005	0.0008	0.0004	-0.0011	0.0006	-0.0007	0.0005
g_{nb2}	0.0886	0.0163	0.1106	0.0162	0.1117	0.0271	0.1066	0.0220
g_{ns2}	-0.0364	0.0200	-0.0522	0.0184	-0.0074	0.0304	-0.0708	0.0251
log-lik.	-0.880110		-0.875897		-0.774573		-0.819780	
SIC	0.919038		0.914826		0.813502		0.858709	
Q(10)	44.521 (0.018)		27.297 (0.999)		44.120 (0.020)		27.412 (0.442)	
Q(20)	99.441 (0.006)		56.020 (0.001)		93.870 (0.017)		61.937 (0.652)	
Q(30)	139.109 (0.020)		92.268 (0.027)		121.646 (0.158)		99.856 (0.675)	
res. mean	(-0.004, -0.009)		(0.888, 0.893)		(-0.001, -0.009)		(0.003, 0.000)	
res. var.	$\begin{pmatrix} 0.870 & 0.135 \\ 0.135 & 1.407 \end{pmatrix}$		$\begin{pmatrix} 0.867 & 0.265 \\ 0.265 & 1.551 \end{pmatrix}$		$\begin{pmatrix} 0.941 & 1.406 \\ 1.406 & 3.515 \end{pmatrix}$		$\begin{pmatrix} 0.752 & 0.925 \\ 0.925 & 2.714 \end{pmatrix}$	

Table 20: ML estimates of the ACM-ARMA part of ICH model. ASK and BID Quote changes in January and February for XOM.

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\kappa^{0.5}$	0.6634	0.0650	0.7201	0.0922	0.7604	0.0229	0.7888	0.0245
$\tilde{\mu}$	0.2741	0.1702	0.3590	0.1694	0.4700	0.2508	0.5397	0.3264
β_1	0.6180	0.1798	0.5293	0.2067	0.8335	0.1010	0.8108	0.1231
α_1	0.1399	0.0356	0.0612	0.0343	0.0931	0.0229	0.1106	0.0256
ν_0	-0.2204	0.2298	-0.4777	0.2416	-0.1869	0.0865	-0.2037	0.1230
ν_1	0.0915	0.0448	0.1265	0.0616	0.0282	0.0145	0.0341	0.0191
ν_2	0.0599	0.0252	0.0387	0.0243	0.0019	0.0099	0.0070	0.0119
ν_3	-0.0229	0.0545	-0.0944	0.0562	-0.0504	0.0239	-0.0525	0.0318
ν_4	0.0289	0.0321	0.0017	0.0354	-0.0228	0.0156	-0.0271	0.0191
δ	0.0654	0.0386	0.0046	0.0391	-0.0009	0.0254	-0.0450	0.0269
g_{vb}	0.0002	0.0000	0.0002	0.0001	0.0003	0.0001	0.0003	0.0001
g_{vs}	0.0001	0.0000	0.0002	0.0001	0.0003	0.0001	0.0003	0.0001
g_{nb}	-0.0137	0.0054	-0.0105	0.0059	-0.0124	0.0039	-0.0117	0.0040
g_{ns}	-0.0011	0.0059	-0.0016	0.0062	-0.0109	0.0039	-0.0136	0.0043
log-lik.	-1.113860		-1.125412		-2.872239		-2.879031	
SIC	1.150193		1.161746		2.908572		2.915364	
LB(10)	5.067 (0.080)		14.745 (0.001)		7.701 (0.021)		14.982 (0.001)	
LB(20)	36.390 (0.028)		38.339 (0.017)		41.922 (0.006)		49.720 (0.001)	
LB(30)	64.060 (0.016)		57.948 (0.052)		77.248 (0.001)		69.603 (0.005)	
res. mean	-0.003		-0.005		0.000		0.001	
res. var.	0.863		0.881		0.945		1.005	

Table 21:ML estimates for the GLARMA part of the ICH model (ASK and BID Quote changes in January and February for C).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\kappa^{0.5}$	0.8462	0.2133	0.7878	0.1672	1.1119	0.0640	1.1153	0.0589
$\tilde{\mu}$	-0.1514	0.3445	0.1229	0.2576	1.5419	0.3303	1.4167	0.2360
β_1	-0.5124	0.1358	-0.2362	0.1697	0.0737	0.1678	0.2703	0.1124
α_1	0.0868	0.0279	0.1174	0.0407	0.1957	0.0313	0.1988	0.0297
ν_0	-2.4062	0.6617	-2.0558	0.6741	-0.9053	0.3524	-1.1010	0.2895
ν_1	0.3678	0.1471	0.1921	0.1117	0.2030	0.0680	0.1327	0.0486
ν_2	0.0691	0.1336	0.1351	0.1024	0.0949	0.0581	0.0377	0.0437
ν_3	-0.5425	0.2291	-0.4808	0.2032	-0.2587	0.1243	-0.2927	0.0974
ν_4	-0.0339	0.1428	-0.0668	0.1184	0.0037	0.0661	-0.0617	0.0587
δ	0.0244	0.0614	-0.1946	0.0636	0.1638	0.0409	-0.2494	0.0376
g_{vb}	0.0009	0.0003	0.0006	0.0003	0.0007	0.0003	0.0009	0.0003
g_{vs}	0.0008	0.0004	0.0005	0.0005	0.0020	0.0005	0.0008	0.0004
g_{nb}	0.0218	0.0145	0.0422	0.0147	0.0159	0.0129	0.0485	0.0127
g_{ns}	0.0786	0.0171	0.0573	0.0183	0.0387	0.0154	0.0057	0.0135
log-lik.	-0.601566		-0.597102		-1.980110		-1.904143	
SIC	0.637899		0.633435		2.016443		1.940476	
LB(10)	14.755 (0.001)		24.096 (0.000)		9.888 (0.007)		5.664 (0.059)	
LB(20)	23.356 (0.025)		31.048 (0.002)		36.587 (0.000)		12.397 (0.414)	
LB(30)	29.814 (0.123)		38.130 (0.018)		47.471 (0.001)		23.207 (0.390)	
res. mean	-0.003		-0.002		-0.000		-0.000	
res. var.	1.101		0.999		1.004		0.927	

Table 22:ML estimates for the GLARMA part of the ICH model (ASK and BID Quote changes in January and February for HNZ).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\kappa^{0.5}$	0.8245	0.1282	0.8107	0.1243	0.9392	0.0367	0.9556	0.0407
$\tilde{\mu}$	-0.1349	0.0635	-0.1421	0.0775	0.5363	0.2681	0.9471	0.3636
β_1	0.9065	0.0433	0.8993	0.1245	0.7039	0.1179	0.5181	0.1621
α_1	0.0861	0.0315	0.0752	0.0715	0.1319	0.0241	0.1452	0.0195
ν_0	0.1639	0.1294	0.1299	0.1684	-0.0679	0.1691	-0.2888	0.2136
ν_1	0.0232	0.0195	0.0127	0.0226	0.1340	0.0515	0.1797	0.0682
ν_2	0.0380	0.0154	0.0260	0.0174	0.0072	0.0189	0.0217	0.0309
ν_3	0.0224	0.0401	0.0200	0.0494	-0.0001	0.0402	-0.0367	0.0566
ν_4	0.0520	0.0242	0.0485	0.0249	0.0047	0.0284	-0.0401	0.0410
δ	0.0005	0.0657	-0.0324	0.0576	0.0830	0.0342	-0.0449	0.0356
g_{vb}	0.0001	0.0000	0.0001	0.0000	0.0001	0.0000	0.0001	0.0000
g_{vs}	0.0002	0.0001	0.0002	0.0001	0.0002	0.0000	0.0002	0.0000
g_{nb}	0.0070	0.0103	0.0143	0.0094	-0.0032	0.0052	-0.0021	0.0058
g_{ns}	0.0125	0.0083	0.0089	0.0086	-0.0075	0.0054	-0.0054	0.0060
log-lik.	-0.715621		-0.710645		-2.464914		-2.454040	
SIC	0.751954		0.746978		2.501247		2.490373	
LB(10)	3.828 (0.147)		11.741 (0.003)		7.517 (0.023)		5.737 (0.057)	
LB(20)	10.079 (0.609)		30.198 (0.003)		16.272 (0.179)		14.957 (0.244)	
LB(30)	24.628 (0.315)		41.560 (0.007)		26.643 (0.225)		26.322 (0.238)	
res. mean	-0.005		-0.005		-0.000		-0.000	
res. var.	1.065		1.046		1.023		1.135	

Table 23: ML estimates for the GLARMA part of the ICH model (ASK and BID Quote changes in January and February for PFE).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\kappa^{0.5}$	0.6799	0.0852	0.6527	0.0788	0.9946	0.0359	0.9549	0.0339
$\tilde{\mu}$	-0.0335	0.0725	-0.0104	0.0800	2.6594	0.5888	1.9423	0.6842
β_1	0.7088	0.1429	0.7254	0.0595	-0.0993	0.2339	0.2386	0.2495
α_1	0.0920	0.0261	0.1326	0.0248	0.1184	0.0327	0.1450	0.0262
ν_0	-0.0757	0.1391	-0.0272	0.1314	-1.2662	0.4113	-1.0322	0.4816
ν_1	0.0553	0.0332	0.0620	0.0257	0.3397	0.0891	0.2327	0.0866
ν_2	0.0232	0.0197	0.0041	0.0197	0.0338	0.0491	0.0120	0.0382
ν_3	0.0135	0.0370	0.0243	0.0438	-0.2654	0.1191	-0.2342	0.1247
ν_4	0.0130	0.0267	0.0257	0.0282	-0.1690	0.0756	-0.1251	0.0724
δ	0.0907	0.0439	-0.0739	0.0404	0.0762	0.0374	-0.1083	0.0376
g_{vb}	0.0003	0.0001	0.0004	0.0001	0.0005	0.0001	0.0004	0.0001
g_{vs}	0.0007	0.0001	0.0003	0.0002	0.0004	0.0002	0.0003	0.0001
g_{nb}	0.0114	0.0070	0.0105	0.0071	-0.0085	0.0070	0.0009	0.0069
g_{ns}	0.0113	0.0077	0.0121	0.0093	0.0051	0.0071	0.0003	0.0079
log-lik.	-1.085382		-1.094481		-2.783337		-2.717013	
SIC	1.121715		1.130814		2.819670		2.753346	
LB(10)	4.631 (0.099)		3.475 (0.176)		29.780 (0.000)		16.892 (0.000)	
LB(20)	23.591 (0.023)		12.684 (0.392)		43.900 (0.000)		28.760 (0.004)	
LB(30)	35.246 (0.037)		17.861 (0.714)		56.944 (0.000)		42.964 (0.005)	
res. mean	-0.003		-0.005		-0.000		-0.000	
res. var.	1.021		0.970		0.994		1.046	

Table 24: ML estimates for the GLARMA part of the ICH model (ASK and BID Quote changes in January and February for XOM).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\breve{\kappa}$	1.2477	0.0456	1.3542	0.0484	1.0630	0.0411	1.2216	0.0551
σ^2	0.3410	0.0643	0.4699	0.0708	0.5149	0.0799	0.7600	0.1020
$\breve{\mu}$	3637.5866	3694.8427	149.2934	602.0510	164.1058	745.0565	1378.5237	2061.8022
$\breve{\alpha}$	0.1353	0.0665	0.0427	0.0110	0.0185	0.0115	0.0486	0.0371
$\breve{\beta}$	0.7455	0.1739	0.9387	0.0164	0.9101	0.0490	0.8000	0.2199
ν_0	2645.9912	1485.9435	506.0574	1092.3704	1162.3150	934.4337	1140.9233	1010.1143
ν_1	537.2453	419.4764	98.0804	80.2768	123.9548	87.0491	182.7950	168.6599
ν_2	155.3532	212.6854	32.6720	110.0792	105.0660	77.4693	-155.0393	157.4397
ν_3	795.6364	526.9868	52.6806	348.5421	275.3065	284.3753	458.1371	465.3607
ν_4	577.3692	372.0621	-142.0897	194.1498	210.0334	147.6229	169.7849	167.5052
g_{vb}	2.0105	0.9319	0.8960	1.2411	2.0036	1.1212	1.7227	0.8899
g_{vs}	-0.9846	0.1553	1.0081	1.2067	3.9981	1.1246	2.0794	1.0227
g_{vb}	-423.9547	62.6804	-126.2017	68.1230	-60.0886	41.1922	-230.9233	45.4289
g_{vs}	-137.3289	78.4262	-15.5874	78.1084	-224.3407	44.8365	-62.7379	49.0736
log-lik.	-10.728111		-10.524168		-9.867765		-9.718491	
SIC	10.764045		10.560101		9.903699		9.754425	
LB(10)	12.733 (0.002)		26.182 (0.000)		12.441 (0.002)		7.586 (0.023)	
LB(20)	26.307 (0.010)		34.503 (0.001)		15.618 (0.209)		35.677 (0.000)	
LB(30)	40.061 (0.011)		43.296 (0.004)		20.083 (0.578)		49.692 (0.001)	
res. mean	0.990		0.989		0.930		1.003	
res. var.	1.094		1.366		1.957		2.482	

Table 25: ML estimates for the ACD model (ASK and BID Depths in January and February for C).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\breve{\kappa}$	1.0729	0.0353	1.3570	0.0518	1.1472	0.0573	1.2784	0.0403
σ^2	0.3096	0.0642	0.6087	0.0807	0.7524	0.1220	0.5425	0.0711
$\breve{\mu}$	-107.2918	392.9346	261.3919	354.1579	782.2287	848.5605	32.1001	183.2524
$\breve{\alpha}$	0.3855	0.0682	0.4048	0.0663	0.3226	0.1002	0.3788	0.0538
$\breve{\beta}$	0.4997	0.0967	0.3145	0.1003	0.4160	0.2172	0.2354	0.0719
ν_0	2283.4169	844.0336	2071.9178	619.9100	1153.1301	734.6020	1132.7040	363.1264
ν_1	155.6545	129.6259	122.6115	102.2161	74.1934	129.4176	93.8579	64.2014
ν_2	-98.2901	117.2480	-160.3546	96.6188	99.1996	129.7212	122.5115	52.5749
ν_3	582.5895	284.7249	608.6763	220.4283	264.6824	252.1419	288.3833	110.7908
ν_4	309.8987	143.1080	163.4427	129.1001	119.5453	147.5954	10.0660	64.1947
g_{vb}	-0.6287	0.8494	3.7599	1.7752	0.6970	1.2636	0.2167	0.5468
g_{vs}	2.8158	6.7184	-0.9620	0.6093	-0.8596	1.2488	-0.3035	0.5677
g_{vb}	-9.2654	55.8232	-10.5535	30.1159	46.2116	45.9437	5.1742	14.6962
g_{vs}	-61.0474	87.0347	-39.8773	33.2257	-62.3278	44.6463	30.7505	17.6156
log-lik.	-9.489292		-9.031831		-8.770981		-8.153588	
SIC	9.525226		9.067765		8.806915		8.189522	
LB(10)	22.653 (0.000)		27.308 (0.000)		4.956 (0.084)		47.906 (0.000)	
LB(20)	33.239 (0.001)		40.384 (0.000)		16.786 (0.158)		50.674 (0.000)	
LB(30)	39.162 (0.014)		51.912 (0.000)		19.930 (0.587)		55.291 (0.000)	
res. mean	1.005		0.983		0.832		0.990	
res. var.	2.318		2.166		2.919		2.488	

Table 26: ML estimates for the ACD model (ASK and BID Depths in January and February for HNZ).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\breve{\kappa}$	1.0574	0.0359	1.0265	0.0329	1.1769	0.0497	1.1314	0.0915
σ^2	0.1735	0.0522	0.0486	0.0437	0.8381	0.0962	0.7553	0.2405
$\breve{\mu}$	2216.0872	4022.0374	1635.4147	1395.3030	-446.8148	861.8693	1858.4472	20361.8093
$\breve{\alpha}$	0.1364	0.0660	0.1657	0.0327	0.0572	0.0385	0.0721	0.3889
$\breve{\beta}$	0.7696	0.1560	0.7076	0.0721	0.8306	0.0935	0.5774	2.6565
ν_0	3765.4461	1939.5303	4458.8249	1802.5804	3273.0463	940.5614	1260.7069	4673.7475
ν_1	278.9140	560.5521	211.6184	234.6752	174.5126	135.9883	291.1113	2583.7655
ν_2	290.0288	360.0921	3.8856	21.2341	-115.9292	84.2106	-48.9159	370.1939
ν_3	826.3892	675.9825	1420.5952	608.8783	1017.7633	295.7756	146.2285	2989.7504
ν_4	329.3113	364.6858	413.8309	325.4268	463.2363	174.4009	136.8474	1047.8481
g_{vb}	-0.8645	0.1755	0.0365	0.3315	-0.5575	0.1260	-0.3457	0.3437
g_{vs}	-1.4889	0.2464	0.8939	0.6934	-0.3969	0.5845	-0.1256	0.5205
g_{vb}	-123.5881	74.4629	-196.3755	40.8454	-4.9949	61.5674	9.4202	48.8479
g_{vs}	-175.8359	64.6271	-64.0922	69.9103	-41.3834	51.4765	-20.9788	133.1431
log-lik.	-10.889348		-10.704276		-9.391248		-9.252525	
SIC	10.925282		10.740210		9.427182		9.288457	
LB(10)	18.035 (0.000)		9.174 (0.010)		36.794 (0.000)		12.938 (0.002)	
LB(30)	20.754 (0.054)		14.534 (0.268)		63.827 (0.000)		21.512 (0.043)	
LB(50)	30.528 (0.106)		18.044 (0.703)		78.097 (0.000)		31.894 (0.079)	
res. mean	1.003		1.001		0.775		0.802	
res. var.	1.465		1.050		2.524		2.130	

Table 27: ML estimates for the ACD model (ASK and BID Depths in January and February for PFE).

	JANUARY				FEBRUARY			
	ASK		BID		ASK		BID	
par.	estimate	std. dev	estimate	std. dev	estimate	std. dev	estimate	std. dev
$\breve{\kappa}$	1.2873	0.0515	1.3930	0.0617	1.2031	0.0575	1.2825	0.0600
σ^2	0.5149	0.0956	0.6840	0.1048	0.8510	0.1097	0.8105	0.0990
$\breve{\mu}$	1132.9629	838.1219	1534.3231	655.6646	4291.1001	1366.7403	473.5758	342.5075
$\breve{\alpha}$	0.1779	0.0408	0.0935	0.0387	0.1822	0.0662	0.0956	0.0306
$\breve{\beta}$	0.5068	0.1144	0.7607	0.1182	-0.0227	0.1608	0.6667	0.1087
ν_0	2853.2287	1388.9896	-832.6357	736.2792	89.4258	1186.5859	685.0321	529.9506
ν_1	78.7482	201.8501	-162.7360	143.5914	200.0361	276.7518	36.7594	60.7676
ν_2	193.2589	179.3204	-263.5289	95.7324	357.6511	248.4492	96.2155	67.5272
ν_3	425.5000	413.8853	-326.1001	243.2906	-483.2380	471.0564	240.0031	174.6285
ν_4	315.4891	245.0953	-327.2422	165.9222	64.9887	308.7760	41.3028	92.9858
g_{vb}	0.2190	0.5921	-0.3662	0.2970	-0.9028	0.1950	-0.1994	0.2496
g_{vs}	-0.9620	0.1847	-0.3848	0.7603	0.3954	0.9175	0.2193	0.4274
g_{vb}	-26.5274	39.8423	-1.0112	84.0902	24.0475	34.3836	-5.4484	22.5859
g_{vs}	50.3050	44.2292	48.4942	48.3563	-33.0378	37.6274	19.8489	26.6354
log-lik.	-9.756858		-9.682785		-8.876363		-8.736100	
SIC	9.792792		9.718719		8.912297		8.772033	
LB(10)	16.281 (0.000)		7.768 (0.021)		7.436 (0.024)		3.423 (0.181)	
LB(20)	41.099 (0.000)		14.081 (0.296)		9.311 (0.676)		15.239 (0.229)	
LB(30)	51.549 (0.000)		28.369 (0.173)		21.690 (0.479)		18.710 (0.663)	
res. mean	0.963		0.935		0.752		0.840	
res. var.	1.239		1.413		1.603		1.662	

Table 28: ML estimates for the ACD model (ASK and BID Depths in January and February for XOM).

	JANUARY				FEBRUARY			
	Quote changes		Depth		Quote changes		Depth	
	ASK	BID	ASK	BID	ASK	BID	ASK	BID
Quote changes ASK	1.000				1.000			
Quote changes BID	0.823	1.000			0.681	1.000		
Depth ASK	0.004	-0.051	1.000		-0.059	-0.085	1.000	
Depth BID	0.021	-0.079	0.123	1.000	0.018	0.003	0.124	1.000

Table 29: Contemporaneous dependence of supply liquidity measure components for C.

	JANUARY				FEBRUARY			
	Quote changes		Depth		Quote changes		Depth	
	ASK	BID	ASK	BID	ASK	BID	ASK	BID
Quote changes ASK	1.000				1.000			
Quote changes BID	0.554	1.000			0.274	1.000		
Depth ASK	0.106	-0.009	1.000		-0.101	-0.036	1.000	
Depth BID	0.033	-0.080	0.092	1.000	0.059	-0.023	0.053	1.000

Table 30: Contemporaneous dependence of supply liquidity measure components for HNZ.

	JANUARY				FEBRUARY			
	Quote changes		Depth		Quote changes		Depth	
	ASK	BID	ASK	BID	ASK	BID	ASK	BID
Quote changes ASK	1.000				1.000			
Quote changes BID	0.778	1.000			0.731	1.000		
Depth ASK	0.094	0.015	1.000		0.014	-0.008	1.000	
Depth BID	-0.048	-0.134	0.142	1.000	0.031	0.011	0.036	1.000

Table 31: Contemporaneous dependence of supply liquidity measure components for PFE.

	JANUARY				FEBRUARY			
	Quote changes		Depth		Quote changes		Depth	
	ASK	BID	ASK	BID	ASK	BID	ASK	BID
Quote changes ASK	1.000				1.000	0.693		
Quote changes BID	0.763	1.000			0.693	1.000		
Depth ASK	0.025	0.006	1.000		0.030	0.060	1.000	
Depth BID	-0.014	-0.015	0.156	1.000	0.007	0.031	0.035	1.000

Table 32: Contemporaneous dependence of supply liquidity measure components for XOM.