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ON INSURANCE CONTRACT DESIGN FOR LOW PROBABILITY EVENTS

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Abstract:
This paper extends the analysis of insurance contracts design to the case of "low probability events", when there is a probability mass on the "no accident-zero loss"-event. The optimality of the deductible clause is discussed both at the theoretical and empirical levels.

Keywords : Optimal insurance design, low probability events.

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1 Introduction

Since Arrow (1963) is is well known that efficient insurance policies involve deductibles. Raviv (1979) has shown that the result still holds under various assumptions on the shape of the insurer's cost. Karni (1992), Machina (1995) and Carlier, Dana and Shahidi (2003) have shown that it can be extended to several non-expected utility models of choice under risk. The scope of Arrow's theorem has been enlarged by Gollier and Schlesinger (1996) and Vergnaud (1997), who have used stochastic dominance arguments in order to establish the superiority of deductible policies for a broader class of the insured's preferences (see Gollier (2000) for a survey). Finally, the result has also been extended in contexts including multiple risks by Cummins and Mahul (2003), Mahul (1999, 2000a,b), Mahul and Wright (2004), and is the starting point of several studies on the determination of the optimal deductible levels (Schlesinger (1981), Eeckhoudt and Gollier (1999)).

This paper aims first at assessing the robustness of deductible clause to the relaxation of the smoothness assumption of the loss distribution (section 3). Second, it studies the sensibility of the contract to the probability of loss (section 4) and to the other parameters (risk-aversion, insurer's loading factor) of the model. I show, considering that there is a probability-mass on the no accident-no loss state as it is the case for small probability accidental events, that the existence of a variable cost in insurance is still necessary but not sufficient to obtain a positive deductible in the expected utility model. In Arrow's case of constant returns to scale in insurance, sufficiency requires large values for the insurer's marginal cost - *i.e.* a marginal cost higher than a threshold which is increasing in the probability of no accident - which are empirically implausible. Finally, a simple calibration of the model also shows that the optimal deductible displays a lack of sensibility to the probability (mass) of accident, and that large deductibles are still efficient, unless high values for the risk-aversion index are introduced. The next section first describes the model.

2 Model and assumptions

Assume that the initial wealth w_0 of an individual is subject to a loss which is supposed to be a perfectly observable random variable X with a known probability distribution. I introduce a mixed mass and density representation for the distribution of X, whose realizations are taking values on $[0,M]$, with $\Pr ob(X=0)=p_0>0$, and I denote $(1-p_0)f(t)>0$ the density on $[0,M]$ such that $\forall x\in[0,M]$:

$$\Pr ob(0\le X\le x)=(1-p_0)\int_0^x f(t)dt$$

and $\int_0^M f(t)dt=1$.

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The optimal insurance contract is a vector of transfers $(P;\{I(x),\ \text{for all}\ \ x\in[0,M]\})$ with P the insurance premium and $I(x)$ the indemnity contingent on the value of the damage, defined as the solution to the maximization problem of the expected utility index of the insured:

$$EU = p_0 u(w_0 - P)$$
$$+(1-p_0)\int_0^M u(w_0 - P + I(x) - x) f(x)dx$$

given the condition of premium (the insurer's participation constraint):

$$P \geq (1-p_0)\int_0^M (I(x) + c[I(x)]) f(x)dx$$

and taking into account for the non-negativity constraints on the indemnity schedule:

$$I(x) \geq 0, \forall x \in [0, M]$$

where u is the insured's utility, with $u' > 0$ and $u'' < 0$; $c[I(x)]$ is the insurer's cost, with: $c(0) = c_0$, $c'(I) \geq 0$ and $c''(I) \geq 0$.

3 Analysis

The first result is the analogue to theorem 1 in Raviv (1979).

Proposition 1 : *Any efficient indemnity schedule is characterized by a* $D \in [0, M]$ *such that:*

$$(C1):\ I^*(x)\begin{cases} >0 & \text{if } x > D \\ =0 & \text{otherwise} \end{cases}$$

with a marginal coverage for all $x > D$ *given by:*

$$(C2):\ I'^*(x) = \left[1 + T^u(w)\frac{c''(I^*(x))}{1 + c'[I^*(x)]}\right]^{-1} \leq 1$$

where: $T^u(w) = -\frac{u'(w)}{u''(w)}$ *is the insured's index of absolute risk tolerance, evaluated at* $w = w_0 - P + I^*(x) - x$.

Proof : Note that due to the monotony assumption on the preferences of both the insurer and the insured, any efficient contract requires a binding participation constraint for the insurer. Denote $\lambda > 0$ its associated shadow price. Denote $\mu(x) \geq 0$ the Lagrange multiplier associated to a constraint of type (3). For any given fixed $P > 0$, the problem may be solved state by state *i.e.* for each value of $x \in [0, M]$; the necessary and sufficient conditions (all functions are well behaved) for optimization are:

$$u'(w_0 - P + I(x) - x) - \lambda(1 + c'[I(x)]) = -\mu(x)$$

with $\mu(x) = 0$ if $I(x) > 0,$ but $\mu(x) \geq 0$ otherwise. Since the LHS in (4) is an increasing and continuous function of x , there exists a unique $D \geq 0$ which is defined by:

$$u'(w_0 - P - D) = \lambda(1 + c'(0))$$

and such that the optimal indemnity schedule is the one in (C1). Differentiating (4) in x in the range where $I(x) > 0$, and rearranging leads to the expression for the marginal coverage (C2). ■

Second order conditions are satisfied since every function is well behaved. The following proposition focuses on the existence of a non trivial deductible.

Proposition 2 : *i) Assume* $p_0 > 0$ *; then: a)* $c' > 0$ *is necessary for efficient contracts to involve a* $D > 0$ *; b) if* $c' = 0$ *, then* $D = 0$ *and the optimal contract provides full insurance of each loss.*

ii) Assume $p_0 = 0$ *; then, any efficient contract contains a strictly positive deductible if and only if* $c' > 0$.

Proof : Integrating condition (4) leads to:

$$\int_0^M \left(u'(w) - \lambda(1 + c'[I^*(x)])\right) dF(x)$$
$$= -\int_0^M \mu(x) dF(x)$$

The maximization of (1) under (2) with respect to P gives:

$$p_0 u'(w_0 - P) + (1 - p_0) \int_0^M u'(w) dF(x) = \lambda$$

Hence, condition (5) may also be written as:

$$\int_0^M \mu(x) dF(x)$$
$$= p_0 \left[u'(w_0 - P) - \int_0^M u'(w) dF(x) \right]$$
$$+ \lambda \int_0^M c'[I^*(x)] dF(x)$$

i) Assume that the optimal policy contains a strict deductible $D > 0$ with a coinsurance arrangement above D such that according to proposition 1: $I(x) - x < 0$, and with $\mu(x) \geq 0$ for x smaller than D. By concavity of u, we thus have for all $x \in]0, M]$:

$$u'(w_0 - P) \leq u'(w_0 - P + I(x) - x)$$

and integrating both sides yields:

$$\left[u'(w_0 - P) - \int_0^M u'(w) dF(x) \right] < 0$$

a) As a result, it must be that $c' > 0$ in order that the RHS of (7) be positive. However, this is not sufficient: sufficiency requires either that the insurance premium entails a cost in terms of welfare high enough (see $\lambda > 0$ in (6)) or that the (expected) marginal cost for the insurer be high enough. Otherwise, the RHS is negative, contradicting that $\mu(x) \geq 0$ at least for some x .

b) On the other hand, assume that $c' = 0$; then $I(x) - x < 0$ at least for some x cannot be optimal since it implies that the LHS of (7) takes a negative sign, contradicting that $\mu(x) \geq 0$. In contrast, a policy paying $I(x) = x$ for all x (zero deductible and full reimbursement of all losses) implies that the bracketed term is nil, and thus $\mu(x) = 0$ for all x , such that (7) holds.

ii) is the standard result (see Raviv (1979)), which is straightforward from (7) setting $p_0 = 0$. ■

The following corollary focuses on the specific case of Arrow (1963).

Corollary 3: *Assume that* $c' = \lambda = cons\tan t$. *There exists a probability-threshold* $\hat{p} \equiv \frac{\lambda}{1+\lambda} \in]0,1[$ *such that:*

i) if p_0 *is small enough in the sense that* $p_0 \leq \hat{p}$, *then the optimal deductible is strictly positive.*

ii) if p_0 *is large enough in the sense that* $p_0 > \hat{p}$ *, then the optimal deductible may be nil.*

Proof : Using condition (6), condition (5) also writes:

$$(1 - p_0)\int_0^M \mu(x)dF(x) = p_0 u'(w_0 - P)$$
$$+ \lambda\left[(1 - p_0)\int_0^M c'[I^*(x)]dF(x) - p_0\right]$$

and when $c' = \lambda = cons\tan t$, the bracketed term reduces to $(1 - p_0)\lambda - p_0 = \lambda - p_0(1+\lambda)$.

i) Assume that $\lambda - p_0(1+\lambda) \geq 0 \Leftrightarrow p_0 \leq \hat{p}$; the RHS in (8) is strictly positive, implying that the LHS in (8) must also be positive at equilibrium: as a result, it exists some values of x for which $\mu(x) > 0$; hence the result that efficient contracts contain a non trivial deductible.

ii) Conversely, assume that:

$$\lambda - p_0(1+\lambda) < 0 \Leftrightarrow p_0 > \hat{p}$$

Then the RHS in (8) may be either positive or negative, depending on the various parameters of the model and/or the shape of the insured's utility function, and in some cases the deductible may trivially be close to $D = 0$. ■

The intuition of corollary 3 is that p_0 is associated to the no-accident/no-loss event: thus, when p_0 is small enough, the probability to pay the premium and being not compensated by the insurer is small. According to (5), the cost in terms of welfare due to the premium charged is spread among all the states of the nature; moreover, it is easily compensated by the insurance policy even when the coverage is concentrated on the states of the nature where the damage is the higher, since the marginal utility of wealth is the larger in those states. Thus, it is optimal for the insured to accept a positive deductible in order to lower the effective premium and obtain full compensation for the infra-marginal losses over the deductible[1]. In contrast, as p_0 increases, the cost in terms of welfare is focused on the no-accident event and sometimes it may not be compensated by insurance reimbursements unless the damage is paid back to the insured in each state of the nature. Providing almost full insurance would be optimal in such a situation. The argument is close to the one developed

[1] According to (C2), we have $I'^*(x) = 1$ if $x > D$.

by Johnson and alii (1993). However, it is not clear whether this occurs for a reasonable numerical simulation of the expected-utility model, as we will show now.

4 Discussion

Let us focus on the case $c' = \lambda = cons\tan t$.

In this case, Arrow's famous theorem establishes that " if a insurance company is willing to offer an insurance policy against loss desired by the (expected-utility) buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by the risk-averting buyer will take the form of of 100 percent coverage above a deductible minimum" (Arrow (1971)). Basically, it is usually recognized that the deductible clause reflects the best possible trade-off between two conflicting objectives implicit to insurance contracting: on the one hand, the promise for the risk-averse insured to obtain, at a reasonable price, the highest possible coverage for the most severe losses he may be facing; on the other, the willingness of the insurer to minimize the transaction costs incurred in its activity, since these costs represent a dead-weight loss on any contract. As a result, risk-averse consumers never purchase insurance against small losses for which the benefits obtained are smaller than the transaction costs incurred to fill these claims. From a practical point of view, the problem of insuring any risk for any risk-averse consumer becomes a simple one, whatever the characteristics (nature) of this risk: the selection of an optimal deductible level. Our results is consistent with this view, but cast some doubt about whether high deductibles are desirable for insuring low probability events, and/or how the deductible is sensible to the probability (mass) of accident.

The first issue is the extent to which $p_0 \le \hat{p}$ may appear as a stringent sufficient condition. Due to a lack of information about the costs structure in different insurance lines, the sufficient condition in corollary 3ii) may be equivalently stated in terms of a threshold value for the marginal cost in insurance: for values of λ above a threshold $\tilde{\lambda} = \frac{p_0}{1-p_0}$, the deductible policy is still efficient. Sufficiency says now that the larger the probability of no loss the larger the marginal cost incurred by insurer required to obtain a deductible clause. In practice individuals are exposed to very small probability events during their lifetime. The most frequent risks such as affecting both their human and non-human wealth correspond to values for $1-p_0$ smaller than 10^{-3} (in annual rate). It is straightforward to see that for deductible policies to be optimal, it must be that the loading factor be closed to huge values; for an example, take $1-p_0 = 1/4000$, then $\tilde{\lambda} = 3999$. Such huge values are empirically unlikely.

Finally, let us consider as an illustrative example the following calibration of the model. Assume that X is uniformly distributed on $[0, M]$ with $f(t) = \frac{1}{M}$ and suppose that the insured/consumer displays constant relative risk aversion, with $u(w) = -\frac{1}{1-\theta} w^{1-\theta}$. We assume that $w = 300000$ and $M = 250000$.

In the next tables, we display the results of the simulation for the sensibility of both the premium (based on the variable cost of the insurer[2] *i.e.* up to the fixed-cost of the insurer, due to the lack of information on c_0) and the deductible to: the probability p_0, the risk-aversion parameter θ and marginal cost (loading factor) λ.

To begin with, we consider the influence of the probability of no accident.

[2] The value of the premium charged by the insurer is given by:
$$P = (1+\lambda)\left(\frac{1-p_0}{M}\right)\left(\frac{M^2+D^2}{2} - M.D\right) + c_0.$$

Table 1. Sensibility to p_0
($\theta = 2; \lambda = 10\%$)

p_0	$P - c_0$	D
$1-\frac{1}{400}$	$306,38$	13982
$1-\frac{1}{500}$	$245,11$	13978
$1-\frac{1}{600}$	$204,26$	13975
$1-\frac{1}{4000}$	$30,642$	13963
$1-\frac{1}{5000}$	$24,514$	13963
$1-\frac{1}{6000}$	$20,428$	13963

Table 1 shows that the premium charged by the insurer is far more sensible to the risk of accident (P decreases with p_0) than the deductible: D decreases with p_0 but as the probability of no-accident becomes enough large, D is almost constant.

The value $\theta = 2$ is generally seen as a reasonable one for the relative risk-aversion index. However, several studies (see for example Mehra and Prescott (1985), Kocherlakota (1990)) have provided arguments that larger values of θ may be plausible, at least useful, to provide the solutions to several empirical puzzles in the area of consumer's behavior on financial markets. Table 2 considers cases where $\theta \geq 2$:

Table 2. Sensibility to θ
($p_0 = 1-\frac{1}{4000}; \lambda = 10\%$)

θ	$P - c_0$	D
2	$30,642$	13963
3	$31,843$	9382
4	$32,460$	7064
10	$33,596$	2849

Table 2 shows now in contrast to table 1, that the deductible is more sensible to the insured's risk-aversion index than the premium. P increases with θ and D decreases with θ - however, the increase in the premium is smaller than the decrease in the deductible, in the sense than doubling θ allows to divide the deductible by almost a factor 2 , while the increase in the premium is quite moderate. Moreover, it appears that the expected-utility model cannot explain that small deductibles may be desirable, unless we consider the opportunity of a large risk-aversion index.

Table 3. Sensibility to λ
($p_0 = 1-\frac{1}{4000}; \theta = 2$)

λ	$P - c_0$	D
10%	$30,642$	13963
20%	$30,067$	26143
30%	$29,521$	36888

The last issue is the influence of the loading factor, which has a key role in Arrow's analysis. The last table focuses on the relationship between the insurer's (constant) marginal cost and the optimal insurance contract. Table 3 shows that the increase in the loading factor (insurer's marginal cost) affects more the deductible than the premium charged by the insurer. Multiplying the loading factor requires almost doubling the deductible level.

5 Conclusion

Despite the attractiveness of Arrow's theorem and the generalizations afforded, it is well known that it does not match so easily empirical findings. Both experimental evidences and data on effective insurance purchases show that consumers do not like (large) deductibles. Johnson and ali (1993) argued that the even assumption according to which the insurance premium is perceived as a segregate loss actually implies that expected utility-based models are not able to explain why consumers actually reject deductible. The argument is as follows. Consider a risk with a small probability of occurrence and a large probability of no loss; for the consumer, the no accident-no loss state is perceived as a segregate state: it is associated to a segregate cost, the insurance premium, implying a high loss of welfare since it is not compensated by the payment of an indemnity. Hence, to compensate this cost, the insured will accept any contract which yields sufficiently high expected benefits in case of loss through the payment of the indemnity. It can be the case that it is obtained only through coinsurance contracts, associated to an admissible premium - based on expected costs which are not excessive for the insurer *as compared* to the small probability of loss. In words, the efficient design of insurance contracts for low probability events reflects a trade-off between two dead weight losses: the premium paid by the insured and the transactions costs incured by the insurer. On the other hand, Chichilnisky (2000) argued that the expected utility functional displays insensitivity to small-probabilities events, and thus is not an appropriate tool to analyze decision problems with small probability events (emerging from environmental risks or more generally from catastrophic risks).

To summarize, our findings are in some sense more conservative: the expected-utility model predicts that the deductible displays a weak sensibility to the probability (mass) of accident but a more significant sensibility to the loading factor in insurance and/or to the risk-aversion index of the insured. We also find that in order to rationalize small deductible levels, we need large values of the (relative) risk-aversion index, a result consistent with previous findings on financial markets.

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