Electoral Uncertainty and the Deficit Bias in a New Keynesian Economy

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First Draft - May 2008
This Draft - February 2009

Abstract: Recent attempts to incorporate optimal fiscal policy into New Keynesian models subject to nominal inertia, have tended to assume that policy makers are benevolent and have access to a commitment technology. A separate literature, on the New Political Economy, has focused on real economies where there is strategic use of policy instruments in a world of political conflict. In this paper we combine these literatures and assume that policy is set in a New Keynesian economy by one of two policy makers facing electoral uncertainty (in terms of infrequent elections and an endogenous voting mechanism). The policy makers generally share the social welfare function, but differ in their preferences over fiscal expenditure (in its size and/or composition). Given the environment, policy shall be realistically constrained to be time-consistent. In a sticky-price economy, such heterogeneity gives rise to the possibility of one policy maker utilising (nominal) debt strategically to tie the hands of the other party, and influence the outcome of any future elections. This can give rise to a deficit bias, implying a sub-optimally high level of steady-state debt, and can also imply a sub-optimal response to shocks. The steady-state distortions and inflation bias this generates, combined with the volatility induced by the electoral cycle in a sticky-price environment, can significantly raise the costs of having a less than fully benevolent policy maker.

JEL Codes: E62, E63

Keywords: New Keynesian Model; Government Debt; Monetary Policy; Fiscal Policy, Electoral Uncertainty, Time Consistency.

*We would like to thank Francesca Flamini for very helpful discussions in the process of drafting this paper. We are also grateful for comments from seminar participants at the Universities of Cambridge, Dortmund, Glasgow, Oxford, Reading and St Andrews. All errors remain our own. We are also grateful to the ESRC, Grant No.RES-156-25-003 and Grant No. RES-062-23-1436, for financial assistance. Address for correspondence: Campbell Leith, Department of Economics, University of Glasgow, Adam Smith Building, Glasgow G12 8RT. E-mail c.b.leith@lbss.gla.ac.uk.

1 Overview

There has been a wealth of recent work deriving optimal monetary policy utilising New Keynesian models. Such models introduce a stabilisation role for monetary policy by assuming a nominal friction, often in the form of overlapping price contracts of the Calvo (1983) type. However, such models usually only introduce fiscal policy as a convenient device through which to ensure the steady-state is efficient, ignoring the impact of monetary policy on the government's finances. Recent papers which relax this assumption by introducing distortionary taxes include Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004), where they examine optimal monetary and fiscal policies under commitment and assess the extent to which ignoring the fiscal consequences of monetary policy affects the usual description of optimal monetary policy. A key result from this literature is that, under commitment, the steady-state level of debt should follow a random walk - in an economy subject to nominal inertia, the cost of undoing the fiscal consequences of shocks outweighs the benefits of doing so. Leith and Wren-Lewis (2006) extend this analysis to the case of multiple fiscal policy instruments and discretionary policy making. They find that the need for policy to be time-consistent will imply that policy makers do offset the fiscal consequences of shocks, and that this 'debt stabilisation bias' can be particularly damaging in welfare terms.

While this literature has made the description of fiscal policy richer, the assumption that the policy maker is infinitely lived and seeks to maximise social welfare hides much of the interesting political game play highlighted by the New Political Economy literature. In particular, the existence of an electoral cycle and partisan politicians could reasonably be expected to influence policy setting in ways which may overturn the desirability of using fiscal policy as a stabilisation device when it is under the control of an infinitely lived benevolent policy maker.

To explore this possibility we extend the model of Leith and Wren-Lewis (2006) in which consumers supply labour to imperfectly competitive firms who are only able to change prices at random intervals of time. Workers' labour income is taxed and the policy maker can choose tax rates, levels of government spending and interest rates to maximise their objectives. We then assume that households can be split into two types, according to their preferences for two baskets of government expenditure. Probabilistic voting as in Lindbeck and Weibull (1987) then implies that there can be random, but endogenous, shifts in the median voter resulting in election of parties representing one or other of the two groups. When in power a particular party will seek to implement policy which maximises the welfare of their electoral base, but in doing so will consider the influence their actions have on the policies adopted by the other party should they happen to be elected in the future. They can influence these actions by affecting the stock of debt inherited by the other party in the future, as well as influencing the chances of electoral success. Therefore the political economy elements we add encompass those of Alesina and Tabellini (1990), Persson and Svensson (1989) and Aghion and Bolton (1990).

There are a number of papers building on this earlier literature and exploring time-consistent fiscal policy in a setting where there is political conflict. This often takes place in the context of real models featuring distributional conflict in an overlapping generations framework (see, for example, Song et al (2007) and Hassler et al. (2005)), or a heterogeneous initial wealth distribution in a neoclassical growth model (Krusell et al. (1997)). Debertoli and Nunes (2007) explore the impact on government debt of exogenous political turnover in a real infinite horizon model with endogenous government spending.

In contrast our model is an infinite-horizon sticky-price New Keynesian economy with nominal government debt where policy makers utilise monetary and fiscal policy instruments (tax and spending policy are endogenous) to maximise their micro-founded objectives in the context of an electoral game with an endogenous voting mechanism The introduction of sticky prices to a general equilibrium model of political conflict is crucial in three respects. Firstly, in the absence of sticky-prices it is costless to use a surprise inflation to deflate the real value of nominal debt, (see Lucas and Stokey (1983)) and nominal debt could not be used strategically as a state variable. Secondly, the introduction of sticky prices implies that policy makers will trade-off the use of instruments for business cycle stabilisation purposes against the strategic use of policy to both tie the hands of their political opponents and influence the endogenous outcome of future elections. Thirdly, any movement in policy instruments for strategic reasons will have repercussions on other welfare-relevant endogenous variables, particularly inflation, in a sticky-price economy, and this will affect the game-play between the political opponents.

We find that the combination of sticky prices, nominal debt, political heterogeneity and an electoral cycle gives rise to a deficit bias which typically pushes the desired level of debt significantly above its efficient level. Debt is raised as an incumbent party wishes to tie the hands of future governments who may not share its preferences - this leads to a suboptimally low level of government spending. In a sticky price environment, it also tends to imply a negative inflation bias, as, otherwise, incumbent parties would wish to use negative inflation shocks to further increase the real value of debt to tie the hands of their opponents further. Given the spillovers from policy instruments to other variables in our New Keynesian economy, the policy mix used to achieve the policy maker's strategic objectives depends crucially on the degree of price stickiness.

Finally we assess the welfare costs of electoral shocks relative to more standard shocks (technology and cost push shocks) and find that the biases introduced by considering realistic relaxations in the assumptions that policy makers are fully benevolent are significantly more costly than the costs of standard shock processes in the benchmark New Keynesian model.

The plan of the paper is as follows. In Section 2 we outline our model in which households supply labour to imperfectly competitive firms who are only able to change prices at random intervals of time. Workers' labour income is taxed. Households are split into two groups with different preferences for the composition and/or size of government spending. In Section 3 we derive a second-order approximation to welfare for these consumers and contrast that with social welfare. In Section 4, we describe the nature of the electoral game and optimal time-consistent policy in the face of this electoral uncertainty, where political parties represent the interests of one of the two types of household. This then informs the simulation results in section 5, which reveal that political parties operating in a New Keynesian economy, facing electoral uncertainty and constrained to be time consistent in their policy actions, will suboptimally raise the level of debt to tie the hands of their opponents and influence the outcome of the electoral game.

2 The Model

This section outlines our economy. Excluding the political economy aspects of our analysis[1], the model is a standard New Keynesian model, but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or borrowing. This basic set-up is similar to that in Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) but with some differences. Firstly, we split government spending into two types and allow policy makers to vary such spending in a way which they find optimal, rather than simply treating government spending as an exogenous flow which must be financed. Secondly, we allow for probabilistic voting (Lindbeck and Weibull, 1987) which leads to endogenous fluctuations in the median voter such that political parties representing two types of consumer preferences over government spending alternate in power. This gives rise to the possibility of policy makers using debt strategically, and the generation of electorally driven economic fluctuations. We examine the households' problem initially, before turning to the firms' problem.

2.1 Households

There are a continuum of households of size one. However, the households split evenly into two types depending on their tastes for government spending. We shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint. As a result the typical household will seek to maximise the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, G_t^A; G_t^B) \tag{1}$$

where C, G^A, G^B and N are a consumption aggregate, two types of public goods aggregate, and labour supply respectively.

The consumption aggregate is defined as[2]

$$C = (\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

[1] Section 4 below, describes the political aspects of the policy problem.

[2] We drop the time subscript when all variables in an expression are dated in the same period and there is no possibility of confusion.

where j denotes the good's type or variety. The public goods aggregates take the same form[3]

$$
\begin{aligned}
G^A &= (\int_0^1 G^A(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}, \text{ and,} \\
G^B &= (\int_0^1 G^B(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}
$$

The elasticity of substitution between varieties $\epsilon_t > 1$ is assumed to time varying as we wish to allow for iid cost-push/mark-up shocks.

The budget constraint at time t is given by

$$\int_0^1 P_t(j)C_t(j)dj + E_t\{Q_{t,t+1}D_{t+1}\} = \Pi_t + D_t + W_t N_t(1-\tau_t) - T_t$$

where $P_t(j)$ is the price of variety j , D_{t+1} is the nominal payoff of the portfolio held at the end of period t, Π is the representative household's share of profits in the imperfectly competitive firms, W are wages, τ is an wage income tax rate, and T are lump sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead pay-offs.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. Optimisation of expenditure for any individual good implies the demand function given below,

$$C(j) = (\frac{P(j)}{P})^{-\epsilon} C$$

where we have a price index given by

$$P = (\int_0^1 P(j)^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$$

The budget constraint can therefore be rewritten as

$$P_t C_t + E_t\{Q_{t,t+1}D_{t+1}\} = D_t + W_t N_t(1-\tau_t) - T_t \tag{3}$$

where $\int_0^1 P(j)C(j)dj = PC$.

2.1.1 Households' Intertemporal Consumption Problem

The first of the households intertemporal problems involves allocating consumption expenditure across time. For tractability assume that (1) takes the specific

[3] The assumption that private and public consumption baskets take the same form is common in the literature, since it makes aggregation of demand for individual firms' products across the private and public sectors straightforward. We make the same assumption in further separating public consumption into two types for the same reason.

form for household type i, $i = 1, 2$.

$$E_0 \sum_{t=0}^{\infty} \beta^t (\frac{C_t^{1-\sigma}}{1-\sigma} + \chi_i^A \frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \chi_i^B \frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{N_t{}^{1+\varphi}}{1+\varphi}) \tag{4}$$

It is differences in the χ_i^A and χ_i^B parameters that will be the source of political conflict to the extent that these differ across the two household types and the two parties that represent them. We leave the exact nature of that heterogeneity unspecified such that we can consider differences in preferences over the composition of government spending as in Alesina and Tabellini (1990) or government size as in Persson and Svennson (1989).

We can then maximise utility subject to the budget constraint (3) to obtain the optimal allocation of consumption across time,

$$\beta(\frac{C_t}{C_{t+1}})^{\sigma}(\frac{P_t}{P_{t+1}}) = Q_{t,t+1}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\{(\frac{C_t}{C_{t+1}})^{\sigma}(\frac{P_t}{P_{t+1}})\} = 1 \tag{5}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one period bond paying off a unit of currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

A log-linearised version of (5) can be written as

$$\widehat{C}_t = E_t\{\widehat{C}_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}) \tag{6}$$

where hatted variables denote percentage deviations from steady-state, $r_t = R_t - \rho$ where $\rho = \frac{1}{\beta} - 1$, and $\pi_t = p_t - p_{t-1}$ is price inflation.

The second foc relates to their labour supply decision and is given by,

$$(1-\tau)\left(\frac{W}{P}\right) = N^{\varphi}C^{\sigma}$$

Log-linearising implies,

$$-\frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau} + \widehat{w} = \varphi\widehat{N} + \sigma\widehat{C}$$

2.2 Allocation of Government Spending

The allocation of government spending across goods is determined by minimising total costs, $\int_0^1 P(j)(G^A(j) + G^B(j))dj$. Given the form of the baskets of public goods this implies,

$$\begin{aligned} G^A(j) &= (\frac{P(j)}{P})^{-\epsilon} G^A \\ G^B(j) &= (\frac{P(j)}{P})^{-\epsilon} G^B \\ G &= G^A + G^B \end{aligned}$$

2.3 Firms

The production function is linear, so for firm j

$$Y(j) = AN(j) \tag{7}$$

where $a = \ln(A)$ is time varying and stochastic. While the demand curve they face is given by,

$$Y(j) = (\frac{P(j)}{P})^{-\epsilon} Y$$

where $Y = \left[\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. The objective function of the firm is given by,

$$\sum_{s=0}^{\infty} (\theta_p)^s Q_{t,t+s} \left[\frac{P(j)_t}{P_{t+s}} Y(j)_{t+s} - \frac{W_{t+s}}{P_{t+s}} \frac{Y(j)_{t+s}(1-\varkappa)}{A}\right] \tag{8}$$

where θ_p is the probability that the firm is unable to change its price in a particular period and $\varkappa$ is a time-invariant employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition and distortionary income taxes. Profit maximisation then implies that firms that are able to change price in period t will select the following price,

$$P_t^* = \frac{\sum_{s=0}^{\infty} (\theta_p)^s Q_{t,t+s} \left[\epsilon_t \frac{W_{t+s}}{P_{t+s}} P_{t+s}^{\epsilon_t} \frac{Y_{t+s}}{A_{t+s}}\right]}{\sum_{s=0}^{\infty} (\theta_p)^s Q_{t,t+s} \left[(\epsilon_t - 1) P_{t+s}^{-1} P_{t+s}^{\epsilon_t} Y_{t+s}(1-\varkappa)\right]}$$

Leith and Wren-Lewis (2006) demonstrate that log-linearisation of this pricing behaviour implies a New Keynesian Phillips curve for price inflation which is given by,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(\widehat{mc}_t + \widehat{\mu}_t)$$

where $\lambda = \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}$, $\widehat{mc} = -a + \widehat{w}$ are the real log-linearised marginal costs of production, and $\widehat{\mu}_t = \ln(\frac{\epsilon_t}{\epsilon_t - 1}) - \ln(\frac{\overline{\epsilon}}{\overline{\epsilon} - 1})$ is a mark-up shock representing the temporary deviation of the desired mark-up from its steady-state value.

2.4 Equilibrium

Goods market clearing requires, for each good j,

$$Y(j) = C(j) + G^A(j) + G^B(j) \tag{9}$$

which allows us to write,

$$Y = C + G$$

where aggregate output is defined as, $Y = [\int_0^1 Y(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj]^{\frac{\epsilon_t}{\epsilon_t - 1}}$. Log-linearising implies

$$\begin{aligned} \widehat{Y} &= \theta \widehat{C} + (1-\theta)\widehat{G} \\ \widehat{G} &= \gamma \widehat{G}^A + (1-\gamma)\widehat{G}^B \end{aligned}$$

where we define $\theta = \frac{\overline{C}}{\overline{Y}}$ and $\gamma = \frac{\overline{G^A}}{\overline{G}}$. These steady-state shares will be related to deeper preference parameters below.

2.5 Government Budget Constraint

Combining the series of the representative consumer's flow budget constraints, (3), with borrowing constraints that rule out Ponzi-schemes, gives the intertemporal budget constraint (see Woodford, 2003, chapter 2, page 69),

$$\sum_{T=t}^{\infty} E_t[P_T C_T] \leq D_t + \sum_{T=t}^{\infty} E_t[Q_{t,T}(\Pi_T + W_T N_T (1 - \tau_T) - T_T)]$$

Noting the equivalence between factor incomes and national output,

$$PY = WN + \Pi - \varkappa WN$$

and the definition of aggregate demand, we can rewrite the private sector's budget constraint as,

$$D_t = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_T G_T - W_T N_T(\tau_T - \varkappa) - T_T)]$$

In order to focus on any deficit bias and time-inconsistency problems associated with the introduction of electoral uncertainty in a New Keynesian model with debt and distortionary taxation we follow Rotemberg and Woodford (1997) and later authors and introduce a steady-state subsidy. This subsidy offsets, in a benchmark steady-state, the distortions caused by distortionary taxation and imperfect competition in price setting, and removes the usual desire on the part of policy makers to raise output above its natural level to compensate for these distortions. In other words, this subsidy ensures that our benchmark steady state is efficient, such that any desire to deviate from that steady-state is solely being driven by the biases and distortions caused by heterogeneity in preferences over government spending and the electoral cycle. The steady state subsidy is financed by lump-sum taxation.[4] We shall assume that both the level of the

[4] An alternative means of ensuring the steady-state was efficient with a positive stock of government debt, but without recourse to a lump sum tax would be to allow the policy maker to apply a time invariant distortionary tax to leisure - see, for example, Bilbiie et al (2008).

subsidy and the associated level of lump-sum taxation cannot be altered from this steady state level, so that any changes in the government's budget constraint have to be financed by changes in distortionary taxation or government spending. This implies that $W_T N_T \varkappa = T_T$ in our economy at all points in time, allowing us to simplify the budget constraint to,

$$D_t = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_T G_T - W_T N_T \tau_T)]$$

i.e. distortionary taxation and spending adjustments are required to service government debt as well as stabilise the economy. Rewriting in real terms and noting that government debt is dated at the beginning of the period,

$$\frac{B_t}{P_{t-1}}\frac{P_{t-1}}{P_t} = \sum_{T=t}^{\infty} E_t[\beta^{T-t}(\frac{C_t}{C_T})^{\sigma}(w_T N_T \tau_T - G_T)]$$

where real debt is defined as, $b_t \equiv \frac{B_t}{P_{t-1}}$ and its initial steady-state is given by,

$$\overline{b} = \frac{\overline{w}\overline{N}\overline{\tau} - \overline{G}}{1-\beta}$$

Log-linearising around this steady-state,

$$\begin{aligned} \widehat{b}_t - \pi_t - \sigma(\widehat{C}_t) &= \beta\widehat{b}_{t+1} - E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &\quad +[-\sigma(1-\beta)(\widehat{C}_t) + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t] \end{aligned} \tag{10}$$

Appendix 1 defines the steady-state ratios contained in this log-linearisation as a function of model parameters and the initial steady-state debt-GDP ratio.

3 Policy Makers' Objectives

In order to derive a objective functions for policy analysis we proceed in the following manner. Firstly, we consider the social planner's problem. We then contrast this with the outcome under flexible prices in order to determine the level of the steady-state subsidy required to ensure the model's initial steady-state is socially optimal when the government implements a plan for government expenditure consistent with that chosen by the social planner. We then construct a quadratic approximation to utility in our sticky-price/distortionary tax economy which assesses the extent to which endogenous variables differ from the efficient equilibrium due to the nominal inertia, tax distortions and electoral uncertainty present in the model. We contrast this social objective function with the objective function adopted by either policy maker when they represent their constituencies. This will imply that a given policy maker has a different target for each expenditure gap and that the weight attached to that target differs across policy makers. Finally, we recast our model in terms of the 'gap' variables contained within our social welfare metric.

3.1 The Social Planner's Problem

The social planner is not constrained by the price mechanism and simply maximises the average of the two households' utilities, (4), subject to the technology, (7), and resource constraints, (9). Therefore the social planner's objective function is given by,

$$\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi^A \frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \chi^B \frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \tag{11}$$

where $\chi^A = \frac{1}{2}(\chi_i^A + \chi_j^A)$ and $\chi^B = \frac{1}{2}(\chi_i^B + \chi_j^B)$ are the average household preference weights attached to the two types of government spending basket.

This yields the following first order conditions,

$$(C_t^*)^{-\sigma} = \frac{1}{2}(\chi_i^A + \chi_j^A)(G_t^{A*})^{-\sigma}$$
$$(C_t^*)^{-\sigma} = \frac{1}{2}(\chi_i^B + \chi_j^B)(G_t^{B*})^{-\sigma}$$
$$(C_t^*)^{-\sigma} - Y_t^{*\varphi} A_t^{-(1+\varphi)} = 0$$

where we introduce the '*' superscript to denote the efficient level of that variable. These can be log-linearised around the deterministic efficient steady-state, and given the national accounting identity we obtain,

$$\widehat{Y}_t^* = \left(\frac{1+\varphi}{\sigma+\varphi}\right) a_t$$

and,

$$\widehat{Y}_t^* = \widehat{C}_t^* = \widehat{G}_t^* = \widehat{G}_t^{A*} = \widehat{G}_t^{B*}$$

3.2 Flexible Price Equilibrium

Appendix 1 derives the subsidy, $\varkappa$, required for the flexible price equilibrium to reproduce the efficient steady state. If any government implements its spending plans in line with the social planner's problem in steady-state then the flex price steady-state conditional on the initial fiscal position is the same as the efficient output level. Appendix 1 also defines the steady-state ratios contained in the log-linearised budget constraint, (10), as a function of model parameters and the initial steady-state debt-GDP ratio.

3.3 Social Welfare

Appendix 1(2) derives the quadratic approximation to utility

$$\begin{aligned}\Gamma = & -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \sigma(1-\theta)\gamma(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 \\ & +\sigma(1-\theta)(1-\gamma)(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 + \varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\lambda}\pi_t^2\} + tip + O[2]\end{aligned}$$

where *tip* refers to 'terms independent of policy' and $O[2]$ captures terms of order greater than 2. It contains quadratic terms in price inflation reflecting the costs of price dispersion induced by inflation in the presence of nominal inertia, as well as terms in the consumption, government spending and output gaps i.e. the difference between the actual value of the variable and its optimal value. The weights attached to each element are a function of deep model parameters.

However, we wish to consider how policy is affected by political parties having welfare functions which differ from the social optimum due to heterogeneity in preferences over government spending. Therefore, we need to define the equivalent welfare measures adopted by our political parties when they solely represent the interests of one of the two types of household present in our economy. The difference between social welfare and the objective function adopted by party i is given by,

$$\begin{aligned}\Gamma_i &= \Gamma + (\chi_i^A - \chi^A)(\overline{G}^A)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{2\widehat{G}_t^A + (1-\sigma)(\widehat{G}_t^A)^2\} \\ &\quad + (\chi_i^B - \chi^B)(\overline{G}^B)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{2\widehat{G}_t^B + (1-\sigma)(\widehat{G}_t^B)^2\}\end{aligned}$$

where $\chi^A = \frac{\chi_1^A + \chi_1^A}{2}$ and, $\chi^B = \frac{\chi_1^B + \chi_2^B}{2}$ are the average weights across the two household types. Using the steady-state relationship,

$$\begin{aligned}\chi^A(\overline{G}^A)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)\gamma \\ \chi^B(\overline{G}^B)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)(1-\gamma)\end{aligned}$$

this can be rewritten as,

$$\begin{aligned}\Gamma_i &= -\frac{1}{2}\overline{N}^{1+\varphi}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \Omega_i^A(\widehat{G}_t^A - \widehat{G}_t^{A*} - \widehat{G}_t^{ATi})^2 \\ &\quad + \Omega_i^B(\widehat{G}_t^B - \widehat{G}_t^{B*} - \widehat{G}_t^{BTi})^2 + \varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\gamma}\pi_t^2\} \qquad (12)\end{aligned}$$

where $\Omega_i^A = (1-\theta)\gamma(\sigma - \frac{\chi_i^A - \chi^A}{\chi^A}(1-\sigma))$ and $\Omega_i^B = (1-\theta)(1-\gamma)(\sigma - \frac{\chi_i^B - \chi^B}{\chi^B}(1-\sigma))$ are the transformed weights on the government spending gaps for policy maker i and,

$$\begin{aligned}\widehat{G}_t^{ATi} &= \frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} + \frac{(\chi_i^A - \chi^A)(1-\sigma)}{\chi^A - \chi_i^A(1-\sigma)}\widehat{G}_t^{A*} \\ \widehat{G}_t^{BTi} &= \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} + \frac{(\chi_i^B - \chi^B)(1-\sigma)}{\chi^B - \chi_i^B(1-\sigma)}\widehat{G}_t^{B*}\end{aligned}$$

are the policy maker specific targets. They reflect a preference driven constant desire to move away from the socially optimal level of spending as well as an element reflecting the desire to alter that target in the face of shocks.

Since these party specific targets will drive the incentives facing governments to utilise debt strategically it is helpful to explore how different types of heterogeneity affect these targets. Assume that $\chi^A = \chi^B$ so that the social planner would choose the same level of spending of each basket of goods, implying $\gamma = 1/2$, and that these average taste parameters are held constant. Consider the constant elements first, which capture the desire to deviate from the socially optimal government spending plan in the absence of shocks. If we introduce a symmetrical composition heterogeneity in party tastes then $\chi_1^A - \chi^A = \chi^A - \chi_2^A > 0$ and $\chi_2^B - \chi^B = \chi^B - \chi_1^B > 0$. That is party 1 prefers basket A to basket B and party 2 prefers the opposite. This implies the following inequalities,

$$\begin{aligned} \frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} &< (>) - \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} \\ \text{when } \sigma &> (<)1,\ \chi_i^A - \chi^A > 0 \text{ and } \chi_i^B - \chi^B < 0 \end{aligned}$$

In other words, when $\sigma > 1$ each party wishes to cut spending on its least preferred basket by more than it wishes to increase spending on its preferred basket, while when $\sigma < 1$ the opposite is true. The σ parameter is identical to the concavity index, $-H_{GG}(G)/(H_G(G))^2$ for each government spending utility felicity, $H(G) = \frac{(G_t)^{1-\sigma}}{1-\sigma}$. A value of $\sigma > 1$ implies that the concavity of the felicity is increasing in G, such that the different government consumption baskets become greater substitutes as the overall level of spending is increased. Accordingly, parties implement more similar policies when resources are scarce, and indulge in political conflict when resources are abundant. Therefore, when $\sigma > 1$ we find a general desire to reduce government spending and achieve a minimal provision of both types of government spending. This drives the deficit bias we shall observe below. Persson and Tabellini (2000) find that a similar condition emerges in the analysis of a variety of political conflicts.

Similarly when we consider a size heterogeneity such that $\chi_1^A - \chi^A = \chi^A - \chi_2^A > 0$ and $\chi_1^B - \chi^B = \chi^B - \chi_2^B > 0$, then the following results hold,

$$\frac{(\chi_1^C - \chi^C)}{\chi^C - \chi_1^C(1-\sigma)} < (>) - \frac{(\chi_2^C - \chi^C)}{\chi^C - \chi_2^C(1-\sigma)} \text{ when } \sigma > (<)1 \text{ where } C = A, B$$

This implies that the party preferring 'small' government wishes to cut spending by more than the 'large' government party wishes to increase it whenever $\sigma > 1$.

Finally, the desired response to shocks is also affected by any heterogeneity, such that,

$$\frac{(\chi_i^C - \chi^C)(1-\sigma)}{\chi^C - \chi_i^C(1-\sigma)} < (>)0 \text{ when } \sigma > 1,\ \chi_i^C > (<)\chi^C \text{ and } C = A, B$$

and, when $\sigma > 1$, parties utilise their least preferred instruments to respond to shocks.

It is helpful to rewrite the per-period loss function for party i in the following form before computing the time consistent solution to the policy problem,

$$\begin{aligned}
&\sigma\theta(\widehat{C}_t^i-\widehat{C}_t^*)^2+\Omega_i^A(\widehat{G}_t^{Ai}-\widehat{G}_t^{A*}-\widehat{G}_t^{ATi})^2+\Omega_i^B(\widehat{G}_t^{Bi}-\widehat{G}_t^{B*}-\widehat{G}_t^{BTi})^2\\
&+\varphi(\widehat{Y}_t^i-\widehat{Y}_t^*)^2+\frac{\epsilon}{\gamma}(\pi_t^i)^2+tip\\
=\;&\boldsymbol{\pi}_t^i\mathbf{R}\boldsymbol{\pi}_t^i+(\mathbf{u}_t^i-\mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^i-\mathbf{u}_t^{i*})+tip+O[2]
\end{aligned}$$

$$\begin{aligned}
\text{where }\mathbf{R} &= \left[\frac{\varepsilon}{\lambda}\right]\text{ and }\mathbf{Q}^i=\begin{bmatrix}0&0&0&0\\0&Z_i^C&Z_i^{CA}&Z_i^{CB}\\0&0&Z_i^A&Z_i^{AB}\\0&0&0&Z_i^B\end{bmatrix}\\
Z_i^C &= \sigma\theta+\varphi\theta^2\\
Z_i^A &= \Omega_i^A+\varphi(1-\theta)^2\gamma^2\\
Z_i^B &= \Omega_i^B+\varphi(1-\theta)^2(1-\gamma)^2\\
Z_i^{CA} &= 2\varphi\theta(1-\theta)\gamma\\
Z_i^{CB} &= 2\varphi\theta(1-\theta)(1-\gamma)\\
Z_i^{AB} &= 2\varphi(1-\theta)^2\gamma(1-\gamma)
\end{aligned}$$

and the vector of controls and targets are given by,

$$\mathbf{u}_t^i=\begin{bmatrix}\widehat{\tau}_t^i-\widehat{\tau}_t^*\\\widehat{C}_t^i-\widehat{C}_t^*\\\widehat{G}_t^{Ai}-\widehat{G}_t^{A*}\\\widehat{G}_t^{Bi}-\widehat{G}_t^{B*}\end{bmatrix}\text{ and }\mathbf{u}_t^{i*}=\overline{u}^{i*}+\mathbf{K1}^i\mathbf{S}_{t-1}+\mathbf{K3}^i\xi_t$$

where $\overline{u}^{i*}$, $\mathbf{K1}^i$ and $\mathbf{K3}^i$ are defined in Appendix 2(1).

3.4 Gap variables

We have derived welfare based on various gaps, so we now proceed to rewrite our model in terms of the same gap variables to facilitate derivation of optimal policy. The consumption Euler equation can be written in gap form as,

$$(\widehat{C}_t-\widehat{C}_t^*)=E_t\{(\widehat{C}_{t+1}-\widehat{C}_{t+1}^*)\}-\frac{1}{\sigma}((r_t-r_t^*)-E_t\{\pi_{t+1}\})$$

where $r_t^*=\sigma\frac{1+\varphi}{\sigma+\varphi}(E_t\{a_{t+1}\}-a_t)$ is the natural/efficient rate of interest. (This comes from the fact that $\widehat{C}_t^*=\widehat{Y}_t^*$ and the definition of the efficient level of output).

While the NKPC can be written in gap form as,

$$\pi_t=\beta E_t\pi_{t+1}+\lambda(\varphi(\widehat{Y}_t-\widehat{Y}_t^*)+\sigma(\widehat{C}_t-\widehat{C}_t^*)+\frac{\overline{\tau}}{1-\overline{\tau}}(\widehat{\tau}_t-\widehat{\tau}_t^*))$$

where, following Benigno and Woodford (2003) we define, $\frac{\overline{\tau}}{1-\overline{\tau}}\,\widehat{\tau}_t^*=\widehat{\mu}_t$. In other words we are defining our 'efficient' tax rate as the tax rate required to perfectly offset the impact of a cost-push shock.[5] If we had access to a lump-sum tax to finance the budget deficit then this would be the optimal tax rate. However, given the need to finance the government liabilities through distortionary taxation, actual tax rates are likely to deviate from the level required to perfectly offset shocks. Appendix 1 rewrites the budget constraint in gap form as,

$$\widehat{b}_t-\pi_t-\sigma(\widehat{C}_t-\widehat{C}_t^*) = \beta\widehat{b}_{t+1}-\beta E_t\{\pi_{t+1}+\sigma(\widehat{C}_{t+1}-\widehat{C}_{t+1}^*)\}+ps_t-f_t-\sigma(1-\beta)(\widehat{C}_t-\widehat{C}_t^*)$$

with the primary surplus defined as,

$$ps_t = \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}[(1+\varphi)(\widehat{Y}_t-\widehat{Y}_t^*)+\frac{1}{1-\overline{\tau}}(\widehat{\tau}_t-\widehat{\tau}_t^*)+\sigma(\widehat{C}_t-\widehat{C}_t^*)]-\frac{\overline{G}}{\overline{b}}(\widehat{G}_t-\widehat{G}_t^*) \quad (13)$$

and

$$f_t = -(\sigma(1-\beta\rho_a)+(1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi}a_t-\frac{\overline{w}\overline{N}}{\overline{b}}\widehat{\mu}_t$$

capturing the extent to which the various shocks hitting our model have fiscal consequences. Also recall the composition of government spending,

$$\widehat{G}_t-\widehat{G}_t^* = \gamma(\widehat{G}_t^A-\widehat{G}_t^{A*})+(1-\gamma)(\widehat{G}_t^B-\widehat{G}_t^{B*})$$

4 The Electoral Game and Time Consistent Policy

We now examine the time-consistent solution to the policy problem under electoral uncertainty. We assume that elections occur after a random interval of time, such that the probability of observing an election in a given time period is a constant, e. This is different from most of the literature which assumes that there is an election in every period. There are several reasons why we allow for a random election probability. The first is that we have developed a sticky-price business cycle model where the natural interpretation of a time period is one quarter year, and it is clearly unrealistic to assume that elections occur at this frequency. Secondly, adopting an election probability rather than assuming fixed term elections is more tractable. Under the assumption that there is a constant probability of facing an election, economic agents forecasts of future economic policies will be conditional on who happens to be the incumbent. If we were to adopt a fixed term election structure, economic agents forecasts of the future would not only be conditional on who was incumbent, but also on how many periods we were from the next election.

Accordingly, we can define the probability of party i being in power in the next period as follows,

$$q(i \mid j)_t = eq(i)_t \text{ for } i\neq j,\ i,j=1,2$$
$$q(i \mid i)_t = (1-e)+eq(i)_t \text{ for } i=1,2$$

[5] It should be noted that we could define the tax 'gap' as being the actual tax rate relative to any benchmark tax rate we choose, such as, for example, the initial steady-state tax rate. However, it is convenient to define the gap relative to the tax rate which offsets the impact of a cost-push shock on inflation.

where $q(i \mid j)_t$ reflects the probability of party i obtaining power, given that party j is the current incumbent, and $q(i \mid i)_t$ gives the probability of party i obtaining power given that i is incumbent. Since there is not an election every period, there is a clear advantage from being in power. $q(i)_t$ then captures the probability that, given an election has been called in period t, party i wins that election. $q(j)_t = 1 - q(i)_t$ is the complementary probability that party j wins the election. These election victory probabilities shall be endogenously determined through a process of probabilistic voting which shall be outlined below. However, it is worth noting that the numerical simulations conducted below reveal that the quantitative importance of the endogeneity of electoral success is relatively small, and that the main mechanisms operating within the model can also be understood by assuming a constant exogenous value for $q(i)_t$. Before detailing the probabilistic voting mechanism it is helpful to outline the policy problem facing policy maker i.

Policymaker i's policy problem can be described as follows,

$$V^i(\mathbf{S}_{t-1};\xi_t) = \underset{\mathbf{u}_t^i}{Min}(\boldsymbol{\pi}_t^i\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})) \quad (14)$$

subject to,

$$\boldsymbol{\pi}_t^i = \mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t$$
$$\mathbf{S}_t^i = \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t$$

where $\mathbf{CJ}^i$ and $\mathbf{DJ}^i$**,**with $J = 1, 2, 3$, are the coefficient matrices defined in Appendix 2 after exploiting the linear-quadratic form of the problem to eliminate expectations. The i superscript denotes who is the incumbent. Aside from affecting the choice of control variables, $\mathbf{u}_t^i$, the coefficient matrices are also indexed by i since who is incumbent affects economic agents' forecasts of the future and these forecasts are embedded in the coefficient matrices. $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$, $\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ and $\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K}^i\mathbf{S}_t = \overline{u}^{i*} + \mathbf{K1}^i\mathbf{S}_{t-1} + \mathbf{K3}^i\boldsymbol{\xi}_t$ are the vectors of state, control and policy maker specific target variables respectively, while $\boldsymbol{\xi}_t$ is a vector of iid innovations to the model's shock processes.

The value of the continuation game depends on whether or not party i is re-elected in the next period,

$$E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1}) = q(i \mid i)_{t+1} E_t V^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1}) + q(j \mid i)_{t+1} E_t W^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})$$

where the expected pay-offs when party i is out of power are given by,

$$W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) = \boldsymbol{\pi}_t^j\mathbf{R}\boldsymbol{\pi}_t^j + (\mathbf{u}_t^j - \mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^j - \mathbf{u}_t^{i*}) + \beta E_t C^i(\mathbf{S}_t^j;\boldsymbol{\xi}_{t+1})$$

Note that the latter expression computes the flow benefits to party i given the policies implemented by party j, and values the continuation game conditional on the level of debt left by party j highlighting the scope for using debt strategically. Further strategic considerations are introduced by endogenising the election victory probability, $q(i)_t$.

4.1 Voting Behaviour

Parties are assumed to represent the economic interests of the households they represent. That is party i possesses an economic objective function which reflects the economic preferences of household i. However, in order to introduce probabilistic voting behaviour, we also assume that individual members of an particular household also care about other non-economic factors, such that for individual k of household i they receive a reduction in losses of $(\sigma^{ik} + \delta)$ when party 1 is elected, and zero otherwise. σ^{ik} is a zero mean random variable uniformly distributed with density Ψ^i, $i = 1, 2$. δ is also uniformly distributed with mean 0 and density Λ.

The timing of events is repetitive and can be summarised as in Figure 1, beginning from the point at which economic agents form their expectations of the next period, t, conditional on knowing who is incumbent and the policies they have implemented in period t-1. We then enter period t and there is a draw from the distribution which determines whether or not there will be an election. Upon that signal being positive, with probability e, there is a draw of the voter preference shocks which affects the outcome of the probabilistic voting, namely the general preference towards party i across all voters, δ, and the individual specific preferences towards party 1 of voters of households 1 and 2, respectively, σ^{ik} . These preference parameters remain in place until there is another signal to hold an election. Accordingly, $\frac{(\sigma^{ik}+\delta)}{1-\beta(1-e)}$captures the expected discounted non-economic benefit to voter k of household i of party 1 winning the election. Since these variables are redrawn at each election, their expected value prior to an election is zero. Therefore, even if a particular draw of voter preference shocks has meant that voters were very content to see party i elected, this will not affect future election outcomes and will not affect incumbent behaviour.

Voters then decide how to vote, comparing the expected economic and non-economic benefits of electing either party, such that voter k of household 1 will vote for party 1 whenever,

$$E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - \frac{(\sigma^{1k} + \delta)}{1 - \beta(1 - e)} < E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$$

In other words the voter compares the expected discounted losses associated with party 1 being in power, $E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, after adjusting for the discounted idiosyncratic, σ^{1k}, and general, δ, non-economic benefits of party 1 being in power, with the losses he would experience if party 1 was out of power, $E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$. Since, the voting takes place prior to the realisation of the innovations to the economic shock processes in period t, the expected value of these losses is based on information available at the end of period t-1.[6]

Given these trade-offs facing voters, the swing voter of household 1 is defined as,

$$\sigma^1 = (1-\beta(1-e))\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) - \delta$$

While the swing voter of household 2 is given by,

$$\sigma^2 = (1-\beta(1-e))\left(E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) - \delta$$

As a result party 1's vote share is given by,

$$v^1 = \frac{1}{2}\Psi^1(\frac{1}{2\Psi^1} - \sigma^1) + \frac{1}{2}\Psi^2(\frac{1}{2\Psi^2} - \sigma^2)$$

The first element captures the votes from household 1 and the second from household 2. Note that since δ is a random variable σ^1 and σ^2 are also random.

Party 1's probability of winning becomes,

$$\begin{aligned}
q(1)_t &= \Pr[\frac{1}{2}\Psi^1(\frac{1}{2\Psi^1} - \sigma^1) + \frac{1}{2}\Psi^2(\frac{1}{2\Psi^2} - \sigma^2) \geqslant \frac{1}{2}] \\
&= \Pr[\frac{21-\beta(1-e)}{\Psi^1+\Psi^2}\begin{pmatrix} -\frac{1}{2}\Psi^1\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) \\ -\frac{1}{2}\Psi^2\left(E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right)\end{pmatrix} \geqslant \delta] \\
&= \frac{1}{2} - z^1\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) \\
&\quad + z^2\left(E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right)
\end{aligned}$$

where $z^i = \frac{\Lambda(1-\beta(1-e))}{\Psi^i+\Psi^j}\Psi^i$, $i = 1, 2$. Therefore the probability of winning the election depends upon the expected relative costs to voters of their natural party being out of power. As the expected losses of household i when party i is out of power, $E_{t-1}W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, rise relative to the losses they expect to experience when party i is in power, $E_{t-1}V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, fewer voters within household i will be tempted to switch allegiance to party j for a given draw of voter preference shocks. Furthermore, the weights attached to the different expected losses when a particular party is in or out of power, z_i , are dependent upon the relative densities of the voter preference shocks within each household, Ψ^i, $i = 1, 2$. As the density is increased voters are more homogeneous within each household. Therefore, when $z_i > z_j$, the voters within household i are more homogeneous than those within household j. This implies that there are fewer swing voters in household i and policy makers will find it relatively easy to tempt voters from household j to switch allegiance to party i creating an electoral bias towards party i, cet. par.

There are several differences between this set-up and that which would emerge in the absence of electoral uncertainty and heterogeneity in preferences

[6] An alternative timing assumption would be to allow voters to observe period t's economic shocks prior to voting. This would not affect the strategic behaviour of the policy maker in the previous period, t-1, since he expects these shocks to be, on average, zero. However, it would add additional volatility to the electoral outcome as economic shocks affect voter choices.

across households. Firstly, the heterogeneity gives rise to targets for control variables which capture the desire to deviate from the socially optimal level of such variables. Secondly, the value of the continuation game must take account of the likely behaviour of one's political rival to the extent that he/she is likely to be elected. The possibility that one's rival is elected and their behaviour if elected are affected by the actions of the incumbent policy maker since the debt passed from one period to the next affects the trade-offs facing voters and constrains the actions of any subsequent policy-maker. Debt can therefore be used as a strategic tool to both affect the likely outcome of elections and tie the hands of future policy makers.

The solution proceeds by 'guessing' the form of the pay-offs when each party is in or out of power and using this to solve the policy problem conditional on these undetermined coefficients, as well as those involved in relating expectations to state-variables. Substituting this solution into the model and the Bellman equation allows us to solve for the undetermined coefficients and complete the description of policy for both parties. Full details of this procedure are given in Appendix 3.[7]

5 Numerical Results

In this section we explore the nature of any biases introduced by heterogeneity in party preferences over the composition and size of government spending in the face of electoral uncertainty, as well as any macroeconomic fluctuations induced by the electoral cycle. We conclude by comparing the welfare costs of the biases and election-induced fluctuations with those arising from more conventional shock processes (namely, technology and mark-up shocks).

Following the econometric estimates in Leith and Malley (2005) we adopt the following parameter set, $\varphi = 1$, $\sigma = 2, \mu = 1.2$, $\overline{\epsilon} = 6$, $\beta = 0.99$. We assume $\chi^A = \chi^B = 4/9$ which implies, in line with Gali (1994), that the share of government consumption in GDP, $1 - \theta = 0.25$ and the optimal size of the two government spending baskets is the same, $\gamma = 1/2$. In our benchmark simulations we assume a degree of price stickiness of $\theta_p = 0.75$, which implies that an average contract length of one year, and an initial debt-GDP ratio of 60%. However, we also explore the implications of alternative assumptions regarding the degree of price stickiness below.

As noted above, our assumption that $\sigma > 1$ implies that the utility felicities for private and public consumption are increasingly concave in their arguments, and this is crucial in creating the incentives to move the economy towards a

[7] For the special case of $e = 1$ and $z_1 = z_2 = 0$, it is possible to solve the policy problem analytically using the symbolic maths package Maple 10. Outside of this special case, the policy problem is solved using the non-linear numerical solution algorithms of Matlab. When applicable, both approaches yield the same results. Furthermore, solving the model without conflict using these approaches yields identical results to the discretionary solution of the same model using the Matlab code of Soderlind (1999).

deficit bias. While this parameter can be difficult to estimate using single equation instrumental variable approaches (see, for example, Yogo(2004)), in the context of systems estimation of general equilibrium policy analysis models it tends to have an estimated value in excess of one (an intertemporal elasticity of substitution less than 1) since the impact of interest rate changes on output is too great otherwise. For example, our assumed value of 2 is in line with GMM-systems estimation of a New Keynesian description of the US and Euro-area economies in Leith and Malley (2005) and the Bayesian posterior distributions of Smets and Wouters (2005).

5.1 The Deficit Bias and the Electoral Cycle

We begin by considering the nature of the electoral cycle when parties differ in their preferences over the composition of government spending. Specifically, we consider the following pattern of preferences, $\chi_1^A = \chi_2^B = \frac{5}{9}$ and $\chi_2^A = \chi_1^B = \frac{3}{9}$ such that party 1 (2) prefers basket A (B) to basket B (A). Figure 2 details the fluctuations in the economy in the absence of shocks other than electoral shocks. This figure also assumes that the probability of election victory for party i is exogenous, $q(i)$, is 1/2 which implies that both parties will behave the same in aggregate with only the composition of government spending varying across parties. The solid blue line details the impulse response when, in expectation, there is a four year electoral cycle, $e = 1/16$, and the dashed green line when there is an election every quarter year, $e = 1$. In both cases, while there is a clear saw-tooth pattern in the spending on the respective government spending baskets as the party in power alternates, this does not induce any aggregate fluctuations.[8] However, there is a significant deficit bias and both parties will raise debt above its efficient level. The reason for this is that since $\sigma > 1$ each party wishes to cut expenditure on its least preferred basket by more than it wishes to increase spending on its preferred basket. Therefore, each party i wishes to issue more debt to tie the hands of its opponent party j $(i \neq j)$ and reduce their spending on party i's least preferred basket, even although this will reduce the spending on party i's preferred basket too. The higher debt is supported by both reduced government spending and higher taxes. The more frequent are elections the greater the magnitude of these effects as an incumbent's time-horizon is effectively shortened. The sign of the impact on aggregate consumption is ambiguous, as higher taxes reduce output but reduced government spending reduces the crowding out of private consumption. The steady-state rate of inflation is negative reflecting the nature of the inflationary-bias problem facing policy makers - without negative inflation they would be tempted to induce surprise reductions in inflation in order to increase the real

[8] A key to this result is that the each basket is comprised of the same goods. If we were to assume that each basket drew on goods from different sectors of the economy, each subject to sticky prices, then fluctuations in the composition of government spending would lead to aggregate fluctuations.

value of debt inherited by their opponent.[9]

In Figure 3 we endogenise the probability of election victory, $q(1)$ by assuming $z_1 = 0.9$ and $z_2 = 0.1$, while maintaining the assumption that elections are typically held on a four year cycle, $e = 1/16$. This means that voters in household 1 are more homogenous than the voters in household 2, which makes the latter more prone to shift allegiance to the other party and tends to bias the electoral success probability towards party 1. This asymmetric implementation is contrasted to the symmetric case where $z_1 = z_2 = 0.5$. In the latter case there are no aggregate fluctuations since, in expectation, party 1's voters are just as likely to defect to party 2, as the other way round and the election success probability remains $q(1) = 1/2$. In the asymmetric case the election success probability is biased towards party 1, which means that party 2 is more interested in disciplining party 1 than vice versa. As a result, party 2 issues more debt than party 1 and we now observe fluctuations in aggregate variables. Despite these fluctuations, the average debt level is lower than under the symmetric case. Additionally, in the asymmetric case, there are fluctuations in the election success probability, such that $q(1)$ is higher when party 1 is in power, and vice versa. However, these endogenous fluctuations in the election success probability are insignificant, relative to the levels shift and similar pictures could have been drawn with an exogenous electoral success probability.

The fluctuations in economic aggregates, largely reflect changes in policy when the incumbent loses office to the opposition. In this case, the electorally strong party 1, upon winning an election, relaxes monetary policy which raises consumption and inflation, reducing the real value of government debt. At the same time, the party cuts tax rates moderating the inflationary impact of the relaxation in monetary policy, but the lower tax rates are not sufficient to reverse the lower debt implied by reduced debt service costs.

Figure 4 then outlines the stochastic steady-state as we vary the political homogeneity of voters within each household, such that the average electoral success probability for party 1, $q(1)$, varies between 0 and 1. Here the deficit bias is at a maximum when $q(1) = 1/2$ since this maximises electoral turnover. As either parties' probability of electoral success increases the need to tie the hands of the other party falls. However, the desire to increase the steady-state debt stock above the efficient level chosen by the social planner remains as the objective function of party 1 implies that they wish to reduce the consumption of basket B more than they wish to increase the consumption of basket A such that he wishes to reduce overall government spending. This means that even when there is no strategic interaction between parties the wish to reduce aggregate government spending is consistent with a higher debt stock. The pattern of higher consumption and lower government spending, but without variation in

[9] It is important to note that here monetary and fiscal policy are used jointly to achieve the strategic and stabilisation goals of the policy makers. An interesting extension to the present analysis would be to allow monetary policy to be set by an unelected policy maker with an objective function consistent with social welfare, while fiscal policy remained in the hands of heterogeneous political parties. This may affect the nature of the inflationary bias observed as an equilibrium in this model.

tax rates and inflation, is sufficient to ensure this steady-state is time consistent.

Figure 4 also plots the steady-state levels of variables around which each party would fluctuate if in power. These are calculated by imagining a party i being permanently in power, but expecting to lose/regain power with probability $q(i)$ should an election be called. The figure reveals that debt is slightly higher for the low electoral success party, which also tends to pursue lower consumption and inflation as policy is tightened to raise debt. However, the effect of very large variations in the electoral success probability on debt is relatively small, possibly explaining the empirical results of Lambertini (2009) who fails to find that opinion poll data Granger causes fiscal deficits.

Figure 5 explores the same composition heterogeneity as a function of the degree of price stickiness. Here there is still the desire to constrain the behaviour of the other policy maker through increasing debt levels, but the nature of the time inconsistency problem this induces changes. At low levels of price stickiness surprise inflation is the most effective means of achieving a desired debt level with limited impact on other variables, such that the use of debt strategically declines. In the limit, as we tend towards a flexible price economy, it would no longer be possible to tie the hands of one's opponent through the issuance of nominal debt since surprise inflation could costlessly undo any such constraint. At intermediate levels of price stickiness and assumed levels of steady-state debt, manipulating real interest rates rather than fiscal instruments are more effective in controlling debt. Finally, when prices are highly sticky control of debt returns to fiscal instruments as the output costs of varying interest rates become too great.

We now turn to consider size heterogeneity in party preferences such that $\chi_i^A = \chi_i^B = \frac{5}{9}$ and $\chi_j^A = \chi_j^B = \frac{3}{9}$ i.e. party 1 desires 'large' government, while party 2 prefers 'small'. Figure 6 details the nature of the electoral cycle this implies, with an exogenous election success probability, $q(1) = 1/2$ and frequent,$e = 1$ and infrequent elections, $e = 1/16$, respectively. When elections are frequent, the large government party runs a relatively lower debt than the other party although it is still sub-optimally large. In order to reduce the debt, party 1 relaxes monetary policy and increases taxes which leads to a surprise jump in inflation thereby deflating the debt. The higher government spending crowds out private consumption. When elections are less frequent, the size of the fluctuations are greater, and the large government party actually reduces government debt below its initial efficient level.

Figure 7, endogenises the election success probability by setting $z_1 = 0.444$ and $z_2 = 0.556$ which implies that the average election success probability is $\overline{q}(1) = 0.5$. Here, unlike the case of composition heterogeneity, there are significant fluctuations in the election success probability. However, since elections are relatively infrequent, this does not significantly affect behaviour and the aggregate fluctuations are similar to the case when the election success probability is exogenous, $q(1) = 1/2$. Again, this is consistent with the empirical evidence in Lambertini (2009).

Figure 8 fixes the size distortion at $\chi_1^A = \chi_1^B = \frac{5}{9}$ and $\chi_2^A = \chi_2^B = \frac{3}{9}$ and varies the homogeneity of voters within each household such that the electoral victory probability, $q(1)$ varies between 0 and 1. Here if we increase the probability of party 1's victory we get to a situation where steady-state debt is actually reduced. Why? Party 1 desires large government which requires additional resources. By reducing debt there is lower debt service costs allowing the party to increase government spending. It is important to note that this surplus doesn't emerge at $q(1) = 1/2$ as party 2's desire to reduce spending is greater than party 1's desire to increase it. Notice, from Figure 8 which plots the steady-state around which each party would fluctuate, the 'large' government party 1 would also support the increased debt for electoral probabilities $q(1) < 0.45$. Party 1 never wants to issue as much debt as 2, but doesn't attempt to completely reverse the policies of 2. There are two constraints on party 1 preventing them achieving the lower level of debt they would choose if permanently in power - firstly, their current policy choices will depend upon inflation and consumption expectations which partly depend upon the expected actions of their opponents and, secondly, in a sticky-price environment, any attempt by their opponent to undo the strategic use of debt will give rise to aggregate fluctuations which are costly to the incumbent. This explains why the fluctuations in debt across the parties are small relative to the overall deficit/surplus bias.

Figure 9 plots the stochastic steady-state of the economy with size heterogeneity in party preferences for government spending, against the degree of price stickiness. There is an expected 4-year electoral cycle, e=1/16 and the densities of the political preference shocks imply, $z_1 = z_2 = 0.5$. Here we can see that when prices are near flexible steady-state deflation is very high to minimise the further use of suprise inflation/deflation to manipulate the level of debt. While when prices are very sticky, steady-state inflation is close to zero since the costs of using significant inflation surprises to manipulate debt are simply too great. - it is preferable to generate changes in the tax base to affect fiscal revenues. Here we can see that at intermediate levels of price stickiness, where real interest rates are an effective tool for manipulating government debt, the average size of the debt stock and the size of the endogenous fluctuations in the election success probability are greatest.

5.2 The Welfare Costs of the Deficit Bias, Electoral Cycle and Shocks

Finally, we assess the welfare costs of introducing these electoral cycles to the New Keynesian model. To do so we add productivity and mark-up shocks. The productivity shock follows the following pattern,

$$a_t = \rho_a a_{t-1} + \xi_t$$

where we adopt a degree of persistence in the productivity shock of $\rho_a = 0.99$ and a standard deviation of the productivity shock of $\sigma_\xi = 0.01$. This is similar to the productivity process in Ireland (2004), although he adopts a unit-root process in technology which we render stationary as in Smets and Wouters (2005). We adopt an iid mark-up shock with a standard deviation of 0.0175

, where we have rescaled the persistent shock in Ireland (2004) to match the unconditional variance of the mark-up process.

We then consider plausible degrees of composition and size heterogeneity to assess the extent to which these generate costs in the New Keynesian model. The evidence on such effects is mixed - see, for example, the meta-analysis of Imbeau *et al.* (2001) of studies which explore the link between the party composition of government and policy outcomes. Therefore, in order to measure the possible size of composition and size heterogeneities in a manner which would allow us to calibrate our model, we examined data from US federal government spending[10] broken down into spending components between 1970 and 2007, and scaled by GDP. We then quadratically detrended this data and regressed the residuals on a dummy based on the outcome of the presidential elections. The results are given in Table 1. Here there are negligible party effects on all categories except for Defense and Total Health, where Republicans typically raise defence spending by 0.785% of GDP relative to the level chosen by the Democrats, while, in the case of health spending, Democrats tend to spend 0.24% of GDP more than the Republicans.

[10] The data was taken from Table 3.1 of the Historical Tables of the US Budget for the Fiscal Year 2009 and refers to 'Outlays by Superfunction and Function 1940-2013', where the data beyond 2007 are estimates.

Table 1 - Party Differences in Categories of Government Spending.

Category	Coefficient on Democrat Dummy	t-ratio
Defense	-0.7854	-3.3348*
Education	0.0522	1.0412
Total Health	0.2370	5.2125*
Transport	-0.0032544	-0.163
Other	-0.05841	-0.98071

While these results suggest that there is both a composition and size component to heterogeneity in party preferences, given the wide range of conflicting empirical evidence on this issue it is difficult to reach a clear conclusion on the exact size of these effects. In light of this, we consider composition and size heterogeneity separately, with moderate measures of heterogeneity in both cases. We adopt the following parameters when consider the composition heterogeneity, $\chi_1^A = \chi_2^B = \frac{416}{900}$ and $\chi_2^A = \chi_1^B = \frac{384}{900}$. This implies that party 1 will drive spending on basket A 0.5% of GDP higher than party 2. Similarly, party 2 will attempt to drive spending on basket B 0.5% of GDP higher than party 1. While in the case of size heterogeneity we assume $\chi_1^A = \chi_1^B = \frac{422}{900}$ and $\chi_2^A = \chi_2^B = \frac{378}{900}$. This then implies that party 1 will drive spending on baskets A and B 0.5% of GDP higher than party 2.

We also set the weights in the election success probability equation equal to $z_1 = z_2 = 0.5$. This implies an election success probability of roughly 0.5 under both forms of political conflict, with a slight bias towards the 'small' government party when considering size heterogeneity. This is in line with the average relative popular vote shares of the Republican and Democrat candidates in US presidential elections where, between 1948 and 2008, the relative vote shares are 51% and 49% for the Republican and Democratic candidates, respectively.

We noted above that the introduction of heterogeneity in party preferences over government spending would tend to imply that parties utilised their least preferred government spending basket more actively in responding to shocks. Figure 10 gives the impulse response to a 1% technology shock with party 1 and party 2 in power, respectively, where they differ in their preferences over the composition of government spending. The figure reveals that party 1 does move spending on basket A by more in its initial response to the shock. However, the size of the effect is small. The response in the absence of political conflict is not very different. Figure 11 does the same, but with a size heterogeneity. In this case, there are some differences across parties in terms of the impulse response of all variables, but, again, these are not large.

These results are then reflected in Table 2 which gives the welfare costs of shocks with and without different types of political conflict. These costs are measured as a percentage of steady-state consumption, which implies that the costs of the technology and mark-up shocks in the absence of political conflict are 0.092% of consumption. In contrast the costs of the electoral cycle alone is 1% for a composition heterogeneity and 1.74% for size heterogeneity, when elections occur every period and the election success probability is exogenously fixed at $q(1) = 1/2$. The relative size of these two effects, reflects the fact that composition heterogeneity, when $q(1) = 1/2$, only induces fluctuations in the composition of government spending, while the size heterogeneity induces additional fluctuations in other welfare relevant variables. However, in both cases the dominant effect is the levels shift induced by the deficit bias, the implied inflation bias and the changes in policy variables to support the sub-optimally high level of government debt. As could be seen from the impulse responses to shocks, the sub-optimal response to shocks with political heterogeneity is quantitatively small and has a negative, but negligible impact on welfare.

If we then allow for the fact that elections do not occur every quarter, but occur on a typical four year cycle, then the costs of symmetrical composition heterogeneity are slightly reduced, while the costs of the size heterogeneity rise from 1.74% to 2.1% of steady-state consumption. This reflects the greater size of fluctuations induced by the size heterogeneity when elections are infrequent, as each party feels less constrained by the possibility that their opponent will undo their policies at some subsequent date, since that possibility is likely to be some time off.

Endogenising the election success probability does not significantly affect the welfare costs of the electoral cycle. It might be thought, that this was because the parties may be reluctant to invest in electoral success when the probability of there being an election in the next period was relatively small, $e = 1/16$. However, if we assume that elections are held every period, then this does not significantly affect the costs of the electoral cycle under the size heterogeneity (it would have no effect under the composition heterogeneity). For this reason we conjecture that introducing fixed term elections would not significantly affect our results.

Table 2 - Welfare Costs of Deficit Biases, Electoral Cycles and Economic Shocks[11]

		Economic Shocks	No Economic Shocks
No Conflict		0.092%	0%
$e = 1$, Exogenous $q(i) = 1/2$	Composition Heterogeneity	1.103%	1.001%
	Size Heterogeneity	1.839%	1.737%
$e = 1/16$, Exogenous $q(i) = 1/2$	Composition Heterogeneity	1.100%	1.001%
	Size Heterogeneity	2.151%	2.055%
$e = 1/16$, Endogenous $q(1)$:	Composition Heterogeneity	1.100%	1.001%
$z_1 = z_2 = 1/2$	Size Heterogeneity	2.159%	2.050%

[11] Welfare is expressed as a percentage of steady-state consumption.

6 Conclusions

In this paper we have combined two separate literatures - firstly, the New Keynesian analysis of monetary and fiscal policy which has typically assumed the existence of a single benevolent policy maker, and secondly, the New Political Economy analysis of political conflict over fiscal policy which has usually taken place in the context of real economies. In combining these literatures we created an environment where policy makers trade-off the use of instruments for business cycle stabilisation purposes against the strategic use of policy to both tie the hands of their political opponents and influence the endogenous outcome of future elections.

Our first key result is that a significant and costly deficit bias can be generated in a sticky price New Keynesian economy with nominal debt. Previous analysis of the deficit bias problem was based on the manipulation of real debt in a real economy, since increases in nominal debt can be costlessly offset by surprise inflation in a flexible price economy. We demonstrate that empirically plausible degrees of price stickiness are sufficient to ensure that nominal debt can also be an effective strategic variable.

Our second main result is that the electoral cycle that such behaviour generates gives rise to significant welfare costs, which greatly exceed those associated with the technology and mark-up shocks traditionally used to drive business cycles in New Keynesian models. Moreover, much of these costs, for example the implied inflationary bias and the endogenous fluctuations in aggregate variables, are directly related to the existence of nominal inertia within an economy subject to political conflict.

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Appendix 1 - Deriving Policy Objectives

(1) Flexible Price Equilibrium

Profit-maximising behaviour implies that firms will operate at the point at which marginal costs equal marginal revenues,

$$\begin{aligned} -\ln(\mu_t) &= mc_t \\ \left(1-\frac{1}{\epsilon_t}\right) &= \frac{(1-\varkappa)}{(1-\tau_t)}(N_t^n)^{(\varphi)}A_t^{-1}(C_t^n)^{\sigma}\xi_t^N \end{aligned}$$

In the initial steady-state this reduces to,

$$\left(1-\frac{1}{\overline{\epsilon}}\right) = \frac{(1-\varkappa)}{(1-\overline{\tau})}(\overline{N}^n)^{\varphi}(\overline{C}^n)^{\sigma}$$

If the subsidy $\varkappa$ is given by

$$(1-\varkappa) = (1-\frac{1}{\overline{\epsilon}})(1-\overline{\tau})$$

then

$$(\overline{C}^n)^{-\sigma} = (\overline{N}^n)^{\varphi}$$

which is identical to the optimal level of employment in the efficient steady-state. Given the steady-state government spending rule,

$$\frac{\overline{G}}{\overline{Y}} = (1+(\chi^A)^{-\frac{1}{\sigma}}+(\chi^B)^{-\frac{1}{\sigma}})^{-1}$$

and,

$$\frac{\overline{G}^A}{\overline{G}} = \gamma = [\left(\frac{\chi^A}{\chi^B}\right)^{-\frac{1}{\sigma}}+1]^{-1}$$

the steady-state level of output is given by,

$$\overline{Y} = \overline{N} = (1+(\chi^A)^{\frac{1}{\sigma}}+(\chi^B)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma+\varphi}}$$

and, if the subsidy is in place, then the steady-state real wage is given by,

$$\overline{w} = \frac{1}{1-\overline{\tau}}$$

The steady-state tax rate required to support a given debt to GDP ratio is given by,

$$\overline{\tau} = \frac{(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}}{1+(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}}$$

and the tax revenues relative to debt this implies are given by,

$$\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} = \frac{\frac{\overline{\tau}}{1-\overline{\tau}}}{\frac{\overline{B}}{\overline{Y}}}$$

This is enough to define all log-linearised relationships dependent on model parameters and the initial debt to GDP ratio.

(2) Derivation of Welfare

Average household utility in period t is

$$(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{2}(\chi_i^A + \chi_j^A)\frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \frac{1}{2}(\chi_i^B + \chi_j^B)\frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}\xi_t^N}{1+\varphi})$$

Before considering the elements of the utility function we need to note the following general result relating to second order approximations,

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln(\frac{Y_t}{Y})$, $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}(\frac{C_t - \overline{C}}{\overline{C}}) - \frac{\sigma}{2}\overline{C}^{1-\sigma}(\frac{C_t - \overline{C}}{\overline{C}})^2 + tip + O[2]$$

where tip represents 'terms independent of policy'. Using the results above this can be rewritten in terms of hatted variables,

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}\{\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2\} + tip + O[2]$$

Similarly for the terms in government spending,

$$\chi^A\frac{(G_t^A)^{1-\sigma}}{1-\sigma} = \chi^A(\overline{G}^A)^{1-\sigma}\{\widehat{G}_t^A + \frac{1}{2}(1-\sigma)(\widehat{G}_t^A)^2\} + tip + O[2]$$

$$\chi^B\frac{(G_t^B)^{1-\sigma}}{1-\sigma} = \chi^B(\overline{G}^B)^{1-\sigma}\{\widehat{G}_t^B + \frac{1}{2}(1-\sigma)(\widehat{G}_t^B)^2\} + tip + O[2]$$

The final term in labour supply can be written as,

$$\frac{N_t^{1+\varphi}\xi_t^N}{1+\varphi} = \overline{N}^{1+\varphi}\{\widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2 + \widehat{N}_t\widehat{\xi}_t^N\} + tip + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms' demand for labour yields,

$$N = (\frac{Y}{A})\int_0^1 (\frac{P_H(i)}{P_H})^{-\epsilon_t} di$$

It can be shown (see Woodford, 2003, Chapter 6) that

$$\begin{aligned}
\widehat{N} &= \widehat{Y} - a + \ln[\int_0^1 (\frac{P(i)}{P})^{-\epsilon_t} di] \\
&= \widehat{Y} - a + \frac{\epsilon}{2} var_i\{p(i)\} + O[2]
\end{aligned}$$

so we can write

$$\begin{aligned}
\frac{N_t^{1+\varphi}}{1+\varphi} &= \overline{N}^{1+\varphi}\{\widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2} var_i\{p_t(i)\}\} \\
&\quad + tip + O[2]
\end{aligned}$$

Using these expansions, individual utility can be written as

$$\begin{aligned}
\Gamma_t &= \overline{C}^{1-\sigma}\{\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2\} \\
&\quad + \chi^A(\overline{G}^A)^{1-\sigma}\{\widehat{G}_t^A + \frac{1}{2}(1-\sigma)(\widehat{G}_t^A)^2\} \\
&\quad + \chi^B(\overline{G}^B)^{1-\sigma}\{\widehat{G}_t^B + \frac{1}{2}(1-\sigma)(\widehat{G}_t^B)^2\} \\
&\quad - \overline{N}^{1+\varphi}\{\widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t \\
&\quad + \frac{\epsilon}{2} var_i\{p_t(i)\}\} \\
&\quad + tip + O[2]
\end{aligned}$$

Using second order approximation to the national accounting identity,

$$\theta\widehat{C}_t = \widehat{Y}_t - (1-\theta)\widehat{G}_t - \frac{1}{2}\theta\widehat{C}_t^2 - \frac{1}{2}(1-\theta)\widehat{G}_t^2 + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

and,

$$\gamma\widehat{G}_t^A = \widehat{G}_t - (1-\gamma)\widehat{G}_t^B - \frac{1}{2}\gamma(\widehat{G}_t^A)^2 - \frac{1}{2}(1-\gamma)(\widehat{G}_t^B)^2 + \frac{1}{2}\widehat{G}_t^2 + O[2]$$

With the steady-state subsidy in place and government spending chosen optimally, the following conditions hold in the initial steady-state,

$$\overline{C}^{1-\sigma} = \overline{N}^{1+\varphi}\theta$$

and,

$$\begin{aligned}
\chi^A(\overline{G}^A)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)\gamma \\
\chi^B(\overline{G}^B)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)(1-\gamma)
\end{aligned}$$

Which allows us to eliminate the levels terms and rewrite welfare as,

$$\begin{aligned}\Gamma_t &= \overline{C}^{1-\sigma}\{-\frac{1}{2}\sigma\widehat{C}_{tt}^2\} + \chi^A(\overline{G}^A)^{1-\sigma}\{-\frac{1}{2}\sigma(\widehat{G}_t^A)^2\} + \chi^B(\overline{G}^B)^{1-\sigma}\{-\frac{1}{2}\sigma(\widehat{G}_t^B)^2\} \\ &\quad -\overline{N}^{1+\varphi}\{\frac{1}{2}\varphi\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}var_i\{p_t(i)\}\} \\ &\quad +tip + O[2]\end{aligned}$$

We now need to rewrite this in gap form using the focs for the social planner to eliminate the term in the technology shock,

$$\begin{aligned}\Gamma_t &= -\overline{N}^{1+\varphi}\frac{1}{2}\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \sigma(1-\theta)\gamma(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 + \sigma(1-\theta)(1-\gamma)(\widehat{G}_t^B - \widehat{G}_t^{B*})^2 \\ &\quad +\varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \epsilon var_i\{p_t(i)\}\} + tip + O[2]\end{aligned}$$

Using the result from Woodford (2003) that

$$\sum_{t=0}^{\infty}\beta^t var_i\{p_t(i)\} = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + O[2]$$

we can write the discounted sum of utility as,

$$\begin{aligned}\Gamma = -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 \quad &+\sigma(1-\theta)\gamma(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 + \sigma(1-\theta)(1-\gamma)(\widehat{G}_t^B - \widehat{G}_t^{B*})^2 \\ +\varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\lambda}\pi_t^2\} \quad &+tip + O[2]\end{aligned}$$

(3) The budget constraint using gap variables

The log-linearised budget constraint is given by,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma\widehat{C}_t &= \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &\quad +\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t - \sigma(1-\beta)\widehat{C}_t\end{aligned}$$

Using the labour supply function to eliminate real wages and the definition of efficient output to eliminate the technology shock,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma\widehat{C}_t &= \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &\quad -\sigma(1-\beta)\widehat{C}_t - \frac{\overline{G}}{\overline{b}}\widehat{G}_t \\ &\quad +\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}\widehat{\tau}_t + \sigma(\widehat{C}_t - \widehat{C}_t^*) + \widehat{Y}_t^*)\end{aligned}$$

Gapping the remaining variables and combining shock terms,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t - \widehat{C}_t^*) &= \beta E_t\{\widehat{b}_{t+1} - \pi_{t+1} - \sigma(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} - f_t \\ &\quad -\sigma(1-\beta)(\widehat{C}_t - \widehat{C}_t^*) - \frac{\overline{G}}{\overline{b}}(\widehat{G}_t - \widehat{G}_t^*) \\ &\quad +\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t - \widehat{\tau}_t^*) + \sigma(\widehat{C}_t - \widehat{C}_t^*))\end{aligned}$$

where

$$f_t = -(\sigma(1-\beta\rho_a) + (1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi}a_t - \frac{\overline{w}\overline{N}}{\overline{b}}\mu_t$$

captures the fiscal consequences of the various shocks hitting the economy.

Appendix 2 - Time Consistent Policy with Targets and Electoral Uncertainty

(1) Deriving the Bellman equation

The first problem we face is in formulating a recursive problem when our model contains expectations of the future value of variables, in particular consumption, $E_t c^g_{t+1}$ and inflation, $E_t \pi_{t+1}$. However, since we have a linear-quadratic form for our problem we can hypothesize a solution for these endogenous variables of the form, conditional on party i being the incumbent,

$$E_{t-1}(c^g_t \mid i) = \mathbf{G0}^i + \mathbf{G1}^i \mathbf{S}_{t-1}$$
$$E_{t-1}(\pi_t \mid i) = \mathbf{F0}^i + \mathbf{F1}^i \mathbf{S}_{t-1} \tag{15}$$

where $\mathbf{G0}^i = [g0^i]$, $\mathbf{G1}^i = \begin{bmatrix} g1^i & g2^i & g3^i \end{bmatrix}$ and $\mathbf{F0}^i = [f0^i]$ and $\mathbf{F1}^i = \begin{bmatrix} f1^i & f2^i & f3^i \end{bmatrix}$ are two $1x3$ vectors of undefined constants and $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$ is the vectors of state variables. Note that in forming these expectations economic agents do not know who is going to be in power, however they do know who the incumbent is, the exogenous probability that there will be an election, e, and the probability that party i will win that election, $q(i)$ and the corresponding probability that party j, $i \neq j$, will win, $q(j) = 1 - q(i)$. Unless elections occur in every period, there is an electoral advantage to being an incumbent such that we must condition the expectations on who is incumbent at the time the expectations are formed. The constants reflect the influence of the parties' targets on expectations of future policy independent of the current state of the economy, but conditional on who is the incumbent at the time the expectations are formed.

Using the former of these we can write the equations describing the evolution of the state variables[12] as conditional on the assumption that party i is currently in power,

$$\mathbf{B0S}^i_t = \mathbf{B1S}_{t-1} + \mathbf{B2u}^i_t + \mathbf{B3}\boldsymbol{\xi}_t + \mathbf{B4}^i$$

$$\mathbf{B0}^i = \begin{bmatrix} B0^i_{1,1} & B0^i_{1,2} & B0^i_{1,3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $B0^i_{1,1} = \beta - \sigma\beta g1^i$, $B0^i_{1,2} = (\sigma(1-\rho_a\beta) - (\sigma-1)(1-\beta))\dfrac{(1+\varphi)}{\sigma+\varphi} - \sigma\beta g2^i$

and $B0^i_{1,3} = \dfrac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} - \sigma\beta g3^i$.

[12] In this section we make the empirically plausible assumption that debt is denominated in nominal terms.

$$\mathbf{B1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}, \mathbf{B2} = \begin{bmatrix} B2_{1,1} & B2_{1,1} & B2_{1,3} & B2_{1,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{B4}^i = \begin{bmatrix} \sigma\beta g0^i \\ 0 \\ 0 \end{bmatrix}$$

$$\text{where } B2_{1,1} = \frac{(\varphi\theta+\sigma)(-(1+\lambda)\frac{\overline{\overline{B}}}{\overline{Y}} - (1-\theta)) - \theta(1-\theta) - (1-\beta)\theta\frac{\overline{\overline{B}}}{\overline{Y}} + \beta\varphi\theta\frac{\overline{\overline{B}}}{\overline{Y}}}{\frac{\overline{\overline{B}}}{\overline{Y}}},$$

$$B2_{1,2} = \frac{(-(2-\theta) - (1-\beta+\lambda)\frac{\overline{\overline{B}}}{\overline{Y}})((1-\beta)\frac{\overline{\overline{B}}}{\overline{Y}} + (1-\theta))}{\frac{\overline{\overline{B}}}{\overline{Y}}} \text{ and,}$$

$$B2_{1,3} = \frac{((1+\varphi)(\theta-(1-\beta)\frac{\overline{\overline{B}}}{\overline{Y}}) - \varphi(1+\lambda\frac{\overline{\overline{B}}}{\overline{Y}}))(1-\theta)\gamma}{\frac{\overline{\overline{B}}}{\overline{Y}}}$$

$$B2_{1,4} = \frac{((1+\varphi)(\theta-(1-\beta)\frac{\overline{\overline{B}}}{\overline{Y}}) - \varphi(1+\lambda\frac{\overline{\overline{B}}}{\overline{Y}}))(1-\theta)(1-\gamma)}{\frac{\overline{\overline{B}}}{\overline{Y}}}$$

$\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ is the vector of control variables in gap form and $\boldsymbol{\xi}_t$ is a vector of iid shocks to our shock processes. This allows us to rewrite the equation of motion for the state variables as,

$$\mathbf{S}_t^i = \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t \tag{16}$$

where

$$\begin{aligned} \mathbf{D0}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B4}^i \\ \mathbf{D1}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B1} \\ \mathbf{D2}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B2} \\ \mathbf{D3}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B3} \end{aligned}$$

Similarly we can write the evolution of inflation as follows,

$$E_t\boldsymbol{\pi}_{t+1} = \mathbf{A1}\boldsymbol{\pi}_t^i + \mathbf{A2}\mathbf{u}_t^i \tag{17}$$

where

$$\mathbf{A1} = \left[\frac{1}{\beta}\right] \text{ and,}$$

$$\mathbf{A2} = \begin{bmatrix} -\frac{\lambda(\varphi\theta+\sigma)}{\beta} & -\frac{\lambda((1-\beta)\frac{\overline{\overline{B}}}{\overline{Y}}+(1-\theta))}{\beta} & -\frac{\lambda\varphi(1-\theta)\gamma}{\beta} & -\frac{\lambda\varphi(1-\theta)(1-\gamma)}{\beta} \end{bmatrix}$$

Leading equation (15) forward one period and utilising the equation describing the evolution of the state variables, we can write,

$$\mathbf{F0}^i + \mathbf{F1}^i\mathbf{D0}^i + \mathbf{F1}^i\mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{F1}^i\mathbf{D2}^i\mathbf{u}_t^i + \mathbf{F1}^i\mathbf{D3}^i\boldsymbol{\xi}_t = \mathbf{A1}\boldsymbol{\pi}_t^i + \mathbf{A2}\mathbf{u}_t^i$$

Solving for inflation,

$$\boldsymbol{\pi}_t^i = \mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t$$

where

$$\begin{aligned}
\mathbf{C0}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D0}^i + \mathbf{F0}^i] \\
\mathbf{C1}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D1}^i] \\
\mathbf{C2}^i &= -[\mathbf{A1}]^{-1}[\mathbf{A2} - \mathbf{F1}^i\mathbf{D2}^i] \\
\mathbf{C3}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D3}^i]
\end{aligned}$$

These allow us to derive the optimisation of policymaker i as follows,

$$V^i(\mathbf{S}_{t-1};\xi_t) = \underset{\mathbf{u}_t^i}{Min}(\boldsymbol{\pi}_t^i\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})) \quad (18)$$

subject to,

$$\begin{aligned}
\boldsymbol{\pi}_t^i &= \mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t \\
\mathbf{S}_t^i &= \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t
\end{aligned}$$

and if party j is out of power,

$$W^j(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) = (\boldsymbol{\pi}_t^i\mathbf{R}^j\boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{j*})'\mathbf{Q}^j(\mathbf{u}_t^i - \mathbf{u}_t^{j*})) + \beta E_t C^j(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})$$

where $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$, $\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ and $\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K}^i\mathbf{S}_t$ are the vectors of state and control variables respectively. While $\boldsymbol{\xi}_t$ is a vector of iid innovations to the model's shock processes and $\mathbf{u}_t^{i*}$ is the vector of party-specific target variables defined as,

$$\overline{u}^{i*} = \begin{bmatrix} 0 \\ \overline{C}^{Ti} \\ \overline{G}^{ATi} \\ \overline{G}^{BTi} \end{bmatrix} \text{ and } \begin{bmatrix} \overline{C}^{Ti} \\ \overline{G}^{ATi} \\ \overline{G}^{BTi} \end{bmatrix} = \begin{bmatrix} 2Z_i^C & Z_i^{CA} & Z_i^{CB} \\ Z_i^{CA} & 2Z_i^A & Z_i^{AB} \\ Z_i^{CB} & Z_i^{AB} & 2Z_i^B \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2\Omega_i^A \frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} \\ 2\Omega_i^B \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} \end{bmatrix}$$

$$\begin{aligned}
\mathbf{K}^i &= \begin{bmatrix} 0 \quad 0 \quad 0 \\ \widetilde{K}^i \end{bmatrix} \\
\widetilde{\mathbf{K}}^i &= \begin{bmatrix} 2Z_i^C & Z_i^{CA} & Z_i^{CB} \\ Z_i^{CA} & 2Z_i^A & Z_i^{AB} \\ Z_i^{CB} & Z_i^{AB} & 2Z_i^B \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\Omega_i^A \frac{(\chi_i^A - \chi^A)(1-\sigma)}{\chi^A - \chi_i^A(1-\sigma)} \frac{1+\varphi}{\sigma+\varphi} & 0 \\ 0 & 2\Omega_i^B \frac{(\chi_i^B - \chi^B)(1-\sigma)}{\chi^B - \chi_i^B(1-\sigma)} \frac{1+\varphi}{\sigma+\varphi} & 0 \end{bmatrix}
\end{aligned}$$

Notice that the targets are given by,

$$\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K}^i\mathbf{S}_t$$

however, the K^i matrix only implies a dependence of the target variables on the exogenous shock processes. Accordingly we can rewrite the targets as,

$$\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K1}^i\mathbf{S}_{t-1} + \mathbf{K3}^i\xi_t$$

where $\mathbf{K1}^i = \mathbf{K}^i \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}$ and $\mathbf{K3}^i = \mathbf{K}^i$.

(2)The Form of the Continuation Game

Given that the problem is linear-quadratic the value function for player i, $V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$ and the discounted pay-offs if he is out of power, $W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$ are given by,

$$\begin{aligned} V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \mathbf{\Phi0}^i + \mathbf{\Phi1}^i\mathbf{S}_{t-1} + \mathbf{S}_{t-1}'\mathbf{\Phi2}^i\mathbf{S}_{t-1} + \mathbf{\Phi3}^i\boldsymbol{\xi}_t + \mathbf{S}_{t-1}'\mathbf{\Phi4}^i\boldsymbol{\xi}_t + \boldsymbol{\xi}_t'\mathbf{\Phi5}^i\boldsymbol{\xi}_t \\ W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \boldsymbol{\mu}\mathbf{0}^i + \boldsymbol{\mu}\mathbf{1}^i\mathbf{S}_{t-1} + \mathbf{S}_{t-1}'\boldsymbol{\mu}\mathbf{2}^i\mathbf{S}_{t-1} + \boldsymbol{\mu}\mathbf{3}^i\boldsymbol{\xi}_t + \mathbf{S}_{t-1}'\boldsymbol{\mu}\mathbf{4}^i\boldsymbol{\xi}_t + \boldsymbol{\xi}_t'\boldsymbol{\mu}\mathbf{5}^i\boldsymbol{\xi}_t \end{aligned}$$

where $\mathbf{\Phi J}^i$ and $\boldsymbol{\mu}\mathbf{J}^i$ with $J = 0, 1, 2$ are matrices of unknown coefficients for player i's value and payoff functions. However, the value of the continuation game, not only directly depends on these payoffs, but also indirectly through their impact on the probability of election victory. Since we are focusing on linear strategies we need only consider a 2nd order approximation to the continuation game,

$$E_tC^i(\mathbf{S}_t^j;\boldsymbol{\xi}_{t+1}) = \boldsymbol{\beta}\mathbf{0}^{i|j} + \boldsymbol{\beta}\mathbf{1}^{i|j}\mathbf{S}_t + \mathbf{S}_t'\boldsymbol{\beta}\mathbf{2}^{i|j}\mathbf{S}_t + O[2]$$

where i indicates from which party's perspective we are evaluating the continuation game, j denotes who is setting policy, $O[2]$ refers to terms which are greater than second order and,

$$\begin{aligned} \boldsymbol{\beta}\mathbf{0}^{i|j} &= \frac{1}{2}e\widetilde{\mathbf{\Phi0}}^i + (1-\frac{1}{2}e)\widetilde{\boldsymbol{\mu}\mathbf{0}}^i \\ &+e(-z^i\left(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\mathbf{\Phi0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\beta}\mathbf{0}^{i|i} &= (1-\frac{1}{2}e)\widetilde{\mathbf{\Phi0}}^i + \frac{1}{2}e\widetilde{\boldsymbol{\mu}\mathbf{0}}^i \\ &+e(-z^i\left(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\mathbf{\Phi0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\beta}\mathbf{1}^{i|j} &= \frac{1}{2}e\mathbf{\Phi1}^i + (1-\frac{1}{2}e)\boldsymbol{\mu}\mathbf{1}^i \\ &+e(-z^i\left(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\mathbf{\Phi0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\mathbf{\Phi1}}^i - \widetilde{\boldsymbol{\mu}\mathbf{1}}^i) \\ &+e(-z^i\left(\mathbf{\Phi1}^i - \boldsymbol{\mu}\mathbf{1}^i\right) + z^j\left(\mathbf{\Phi1}^j - \boldsymbol{\mu}\mathbf{1}^j\right))(\widetilde{\mathbf{\Phi0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i) \end{aligned}$$

$$\begin{aligned}
\boldsymbol{\beta}\mathbf{1}^{i|i} &= (1-\frac{1}{2}e)\boldsymbol{\Phi}\mathbf{1}^i + \frac{1}{2}e\boldsymbol{\mu}\mathbf{1}^i \\
&+e(-z^i\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{1}}^i - \widetilde{\boldsymbol{\mu}\mathbf{1}}^i) \\
&+e(-z^i\left(\boldsymbol{\Phi}\mathbf{1}^i - \boldsymbol{\mu}\mathbf{1}^i\right) + z^j\left(\boldsymbol{\Phi}\mathbf{1}^j - \boldsymbol{\mu}\mathbf{1}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\beta}\mathbf{2}^{i|j} &= \frac{1}{2}e\boldsymbol{\Phi}\mathbf{2}^i + (1-\frac{1}{2}e)\boldsymbol{\mu}\mathbf{2}^i \\
&+e(-z^i\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{2}}^i - \widetilde{\boldsymbol{\mu}\mathbf{2}}^i) \\
&+e(-z^i\left(\boldsymbol{\Phi}\mathbf{2}^i - \boldsymbol{\mu}\mathbf{2}^i\right) + z^j\left(\boldsymbol{\Phi}\mathbf{2}^j - \boldsymbol{\mu}\mathbf{2}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i) \\
&+e(-z^i\left(\boldsymbol{\Phi}\mathbf{1}^i - \boldsymbol{\mu}\mathbf{1}^i\right)' + z^j\left(\boldsymbol{\Phi}\mathbf{1}^j - \boldsymbol{\mu}\mathbf{1}^j\right)')(\boldsymbol{\Phi}\mathbf{1}^i - \boldsymbol{\mu}\mathbf{1}^i)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\beta}\mathbf{2}^{i|i} &= (1-\frac{1}{2}e)\boldsymbol{\Phi}\mathbf{2}^i + \frac{1}{2}e\boldsymbol{\mu}\mathbf{2}^i \\
&+e(-z^i\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^j - \widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{2}}^i - \widetilde{\boldsymbol{\mu}\mathbf{2}}^i) \\
&+e(-z^i\left(\boldsymbol{\Phi}\mathbf{2}^i - \boldsymbol{\mu}\mathbf{2}^i\right) + z^j\left(\boldsymbol{\Phi}\mathbf{2}^j - \boldsymbol{\mu}\mathbf{2}^j\right))(\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i - \widetilde{\boldsymbol{\mu}\mathbf{0}}^i) \\
&+e(-z^i\left(\boldsymbol{\Phi}\mathbf{1}^i - \boldsymbol{\mu}\mathbf{1}^i\right)' + z^j\left(\boldsymbol{\Phi}\mathbf{1}^j - \boldsymbol{\mu}\mathbf{1}^j\right)')(\boldsymbol{\Phi}\mathbf{1}^i - \boldsymbol{\mu}\mathbf{1}^i)
\end{aligned}$$

where $\widetilde{\boldsymbol{\Phi}\mathbf{0}}^i = \boldsymbol{\Phi}\mathbf{0}^i + tr[\boldsymbol{\Sigma}\boldsymbol{\Phi}\mathbf{5}^i]$ and $\widetilde{\boldsymbol{\mu}\mathbf{0}}^i = \boldsymbol{\mu}\mathbf{0}^i + tr[\boldsymbol{\Sigma}\boldsymbol{\mu}\mathbf{5}^i]$. In other words, the second order approximation to the continuation game takes account of the impact of debt (and other state variables) on both the expected pay-offs to each party when in and out of power, but also factors in the repercussions of this for the probability of electoral success.

The incumbent policymaker therefore takes account of the impact a marginal increase in debt has on the value of the continuation game,

$$\frac{\partial E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})}{\partial \mathbf{S}_t^i} = \boldsymbol{\beta}\mathbf{1}^{i|i\prime} + (\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{S}_t^i$$

where the debt affects not only the payoffs when each party is in or out of power, but, as a consequence, also the probability of each party winning any election that takes place.

Appendix 3 - Solving the Bellman equation

(1)The first-order conditions conditional on 'guesses'

The first-order conditions with respect to the control variables chosen by policy maker i from solving (18) are then given by,

$$2\mathbf{C2}^{i\prime}\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta\mathbf{D2}^{i\prime}\frac{\partial E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})}{\partial \mathbf{S}_t^i} = 0$$

Using the definition of the targets for player i, $\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K1}^i\mathbf{S}_{t-1} + \mathbf{K3}^i\boldsymbol{\xi}_t$ we can write this as,

$$2\mathbf{C2}^{i\prime}\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \overline{u}^{i*} - \mathbf{K1}^i\mathbf{S}_{t-1} - \mathbf{K3}^i\boldsymbol{\xi}_t) + \beta\mathbf{D2}^{i\prime}\frac{\partial E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})}{\partial \mathbf{S}_t^i} = 0$$

Using the equation of motion the vector of state variables, the relationship between inflation and state variables and the 'guess' for the value of the continuation game, this can be written as,

$$\begin{aligned}
& 2\mathbf{C2}^{i\prime}\mathbf{R}[\mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t] \\
& +(\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \overline{u}^{i*} - \mathbf{K1}^i\mathbf{S}_{t-1} - \mathbf{K3}^i\boldsymbol{\xi}_t]) \\
& +\beta\mathbf{D2}^{i\prime}[\boldsymbol{\beta}\mathbf{1}^{i|i\prime} + (\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})[\mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t]] \\
= \; & 0
\end{aligned}$$

and solved for control variables,

$$\mathbf{u_t^i} = -[\mathbf{U2}^i]^{-1}\mathbf{U0}^i - [\mathbf{U2}^i]^{-1}\mathbf{U1}^i\mathbf{S}_{t-1} - [\mathbf{U2}^i]^{-1}\mathbf{U3}^i\boldsymbol{\xi}_\mathbf{t}$$

where

$$\begin{aligned}
\mathbf{U0}^i &= 2\mathbf{C2}^{i\prime}\mathbf{R}\mathbf{C0}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\overline{u}^{i*} + \beta\mathbf{D2}^{i\prime}[\boldsymbol{\beta}\mathbf{1}^{i|i\prime} + (\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D0}^i] \\
\mathbf{U1}^i &= [2\mathbf{C2}^{i\prime}\mathbf{R}\mathbf{C1}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\mathbf{K1}^i + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D1}^i] \\
\mathbf{U2}^i &= [2\mathbf{C2}^{i\prime}\mathbf{R}\mathbf{C2}^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime}) + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D2}^i] \\
\mathbf{U3}^i &= [2\mathbf{C2}^{i\prime}\mathbf{R}\mathbf{C3}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\mathbf{K3}^i + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D3}^i]
\end{aligned}$$

The solution for inflation is now given as,

$$\boldsymbol{\pi}_t^i = \mathbf{P0}^i + \mathbf{P1}^i\mathbf{S}_{t-1} + \mathbf{P3}^i\boldsymbol{\xi}_\mathbf{t}$$

where,

$$\begin{aligned}
\mathbf{P0}^i &= \mathbf{C0}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-1}\mathbf{U0}^i \\
\mathbf{P1}^i &= \mathbf{C1}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-\mathbf{1}}\mathbf{U1}^i \\
\mathbf{P3}^i &= \mathbf{C3}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-\mathbf{1}}\mathbf{U3}^i
\end{aligned}$$

(2)Equating the Undetermined Coefficients in the Forecasting Equations

We are now in a position to obtain our first set of equations to solve for the forecast guess parameters. Taking expectations of the optimal relationship between the controls and state variables, conditional on party i being in power,

$$\begin{aligned} E_{t-1}(\mathbf{u}_t \quad &| \quad i) = q(i \mid i)_t E_{t-1}\mathbf{u}_t^i + q(j \mid i)_t E_{t-1}\mathbf{u}_t^j \\ &= \widetilde{\mathbf{G0}}^i + \widetilde{\mathbf{G1}}^i \mathbf{S}_{t-1} \end{aligned}$$

where,

$$\begin{aligned} \widetilde{\mathbf{G0}}^i \quad = \quad & -[1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^i]^{-1}\mathbf{U0}^i \\ & -[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^j]^{-1}\mathbf{U0}^j \end{aligned}$$

and

$$\begin{aligned} \widetilde{\mathbf{G1}}^i \quad = \quad & -[1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^i]^{-1}\mathbf{U1}^i \\ & -[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^j]^{-1}\mathbf{U1}^j \\ & -e([\mathbf{U2}^i]^{-1}\mathbf{U0}^i-[\mathbf{U2}^j]^{-1}\mathbf{U0}^j)[-z^i\left(\mathbf{\Phi1}^i-\boldsymbol{\mu}\mathbf{1}^i\right)+z^j\left(\mathbf{\Phi1}^j-\boldsymbol{\mu}\mathbf{1}^j\right)] \end{aligned}$$

The second row of this relationship can then be equated to the guess parameters in

$$E_{t-1}(c_t^g \mid i) = \mathbf{G0}^i + \mathbf{G1}^i \mathbf{S}_{t-1}$$

Similarly, we can take expectations of the expression for inflation to obtain,

$$\begin{aligned} E_{t-1}[\boldsymbol{\pi}_t \quad &| \quad i] = q(i \mid i)_t E_{t-1}\boldsymbol{\pi}_t^i + q(j \mid i)_t E_{t-1}\boldsymbol{\pi}_t^j \\ &= \widetilde{\mathbf{P0}}^i + \widetilde{\mathbf{P1}}^i \mathbf{S}_{t-1} \end{aligned}$$

where

$$\begin{aligned} \widetilde{\mathbf{P0}}^i \quad = \quad & [1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]P\mathbf{0}^i \\ & +[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]\mathbf{P0}^j \end{aligned}$$

and

$$\begin{aligned} \widetilde{\mathbf{P1}}^i \quad = \quad & [1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]P\mathbf{1}^i \\ & +[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]\mathbf{P1}^j \\ & +e(P\mathbf{0}^i-P\mathbf{0}^j)[-z^i\left(\mathbf{\Phi1}^i-\boldsymbol{\mu}\mathbf{1}^i\right)+z^j\left(\mathbf{\Phi1}^j-\boldsymbol{\mu}\mathbf{1}^j\right))] \end{aligned}$$

which can then be equated with the elements in,

$$E_{t-1}(\pi_t \mid i) = \mathbf{F0}^i + \mathbf{F1}^i \mathbf{S}_{t-1}$$

(3)Equating the Undetermined Coefficients in the Continuation Game

It is helpful to write the evolution of the vector of state variables.

$$\begin{aligned} \mathbf{S}_t^i &= \mathbf{D0}^i + \mathbf{D1}^i \mathbf{S}_{t-1} + \mathbf{D2}^i \mathbf{u}_t^i + \mathbf{D3}^i \boldsymbol{\xi}_t \\ &= \mathbf{J0}^i + \mathbf{J1}^i \mathbf{S}_{t-1} + \mathbf{J3}^i \boldsymbol{\xi}_t \end{aligned}$$

where

$$\begin{aligned} \mathbf{J0}^i &= \mathbf{D0}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U0}^i \\ \mathbf{J1}^i &= \mathbf{D1}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U1}^i \\ \mathbf{J3}^i &= \mathbf{D3}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U3}^i \end{aligned}$$

Using the definition,

$$u_t^{i*} = \overline{u}^{i*} + \mathbf{K1}^i \mathbf{S}_{t-1} + \mathbf{K3}^i \xi_t$$

It is convenient to define,

$$\mathbf{u_t^i} - \mathbf{u_t^{j*}} = \widetilde{\mathbf{U0}}^{ij} + \widetilde{\mathbf{U1}}^{ij} \mathbf{S}_{t-1} + \widetilde{\mathbf{U3}}^{ij} \boldsymbol{\xi}_t$$

where

$$\begin{aligned} \widetilde{\mathbf{U0}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U0}^i + \overline{\mathbf{u}}^{\mathbf{j}*}] \\ \widetilde{\mathbf{U1}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U1}^i + \mathbf{K1}^{\mathbf{j}}] \\ \widetilde{\mathbf{U3}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U3}^i + \mathbf{K3}^{\mathbf{j}}] \end{aligned}$$

We now need to find expressions to solve for the 'guessed' parameterisation of the continuation game.

$$\begin{aligned} E_t C^i(\mathbf{S}_t^i; \boldsymbol{\xi}_{t+1}) &= \boldsymbol{\beta 0}^{i|i} + \boldsymbol{\beta 1}^{i|i} \mathbf{S}_t^j + \mathbf{S}_t^{j\prime} \boldsymbol{\beta 2}^{i|i} \mathbf{S}_t^j \\ &= \beta 0^{i|i} + \beta 1^{i|i} \mathbf{J0}^i + \mathbf{J0}^{i\prime} \beta 2^{i|i} \mathbf{J0}^i \\ &\quad [\boldsymbol{\beta 1}^{i|i} \mathbf{J1}^i + \mathbf{J0}^{i\prime} (\beta 2^{i|i} + \beta 2^{i|i^\prime}) \mathbf{J1}^i] \mathbf{S}_{t-1} \\ &\quad + \mathbf{S}_{t-1}^\prime \mathbf{J1}^{i\prime} \beta 2^{i|i} \mathbf{J1}^i \mathbf{S}_{t-1} + tis + O[2] \end{aligned}$$

where tis denotes non-constant terms independent of the vector of state variables. Similarly we can define the value of the continuation game for party i given that party j has been in power,

$$\begin{aligned} E_t C^i(\mathbf{S}_t^j; \boldsymbol{\xi}_{t+1}) &= \boldsymbol{\beta 0}^{i|j} + \boldsymbol{\beta 1}^{i|j} \mathbf{S}_t^j + \mathbf{S}_t^{j\prime} \boldsymbol{\beta 2}^{i|j} \mathbf{S}_t^j \\ &= \beta 0^{i|j} + \beta 1^{i|j} \mathbf{J0}^j + \mathbf{J0}^{j\prime} \beta 2^{i|j} \mathbf{J0}^j \\ &\quad [\boldsymbol{\beta 1}^{i|j} \mathbf{J1}^j + \mathbf{J0}^{j\prime} (\boldsymbol{\beta 2}^{i|j} + \boldsymbol{\beta 2}^{i|j\prime}) \mathbf{J1}^j] \mathbf{S}_{t-1} \\ &\quad + \mathbf{S}_{t-1}^\prime \mathbf{J1}^{j\prime} \boldsymbol{\beta 2}^{i|j} \mathbf{J1}^j \mathbf{S}_{t-1} + tis + O[2] \end{aligned}$$

Using these results the Bellman equation can be rewritten as,

$$\begin{aligned} V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \mathbf{P0}^{i'}\mathbf{RP0}^i + \widetilde{\mathbf{U0}}^{ii\prime}\mathbf{Q}^i\widetilde{\mathbf{U0}}^{ii} + \beta(\beta 1^{i|i} + \beta 1^{i|i}\mathbf{J0}^i + \mathbf{J0}^{i\prime}\beta 2^{i|i}\mathbf{J0}^i) \\ &\quad [\mathbf{P0}^{i\prime}(\mathbf{R}+\mathbf{R}')\mathbf{P1}^i]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\mathbf{P1}^{i\prime}\mathbf{RP1}^i\mathbf{S}_{t-1} \\ &\quad +[\widetilde{\mathbf{U0}}^{ii\prime}(\mathbf{Q}^i+\mathbf{Q}^{i\prime})\widetilde{\mathbf{U0}}^{ii}]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\widetilde{\mathbf{U1}}^{ii\prime}\mathbf{Q}^i\widetilde{\mathbf{U1}}^{ii}\mathbf{S}_{t-1} \\ &\quad +\beta[\boldsymbol{\beta}\mathbf{1}^{i|i}\mathbf{J1}^i + \mathbf{J0}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i} + \boldsymbol{\beta}\mathbf{2}^{i|i\prime})\mathbf{J1}^i]\mathbf{S}_{t-1} + \beta\mathbf{S}'_{t-1}\mathbf{J1}^{i\prime}\boldsymbol{\beta}\mathbf{2}^{i|i}\mathbf{J1}^i\mathbf{S}_{t-1} \\ &\quad +tis + +O[2] \end{aligned}$$

The pay-offs for a party out of power can be written as,

$$\begin{aligned} W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \mathbf{P0}^{j'}\mathbf{RP0}^j + \widetilde{\mathbf{U0}}^{ji\prime}\mathbf{Q}^i\widetilde{\mathbf{U0}}^{ji} + \beta(\beta 0^{i|j} + \beta 1^{i|j}\mathbf{J0}^j + \mathbf{J0}^{j\prime}\beta 2^{i|j}\mathbf{J0}^j) \\ &\quad [\mathbf{P0}^{j\prime}(\mathbf{R}+\mathbf{R}')\mathbf{P1}^j]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\mathbf{P1}^{j\prime}\mathbf{RP1}^j\mathbf{S}_{t-1} \\ &\quad +[\widetilde{\mathbf{U0}}^{ji\prime}(\mathbf{Q}^i+\mathbf{Q}^{i\prime})\widetilde{\mathbf{U1}}^{ji}]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\widetilde{\mathbf{U1}}^{ji\prime}\mathbf{Q}^i\widetilde{\mathbf{U1}}^{ji}\mathbf{S}_{t-1} \\ &\quad +\beta[\boldsymbol{\beta}\mathbf{1}^{i|j}\mathbf{J1}^j + \mathbf{J0}^{j\prime}(\boldsymbol{\beta}\mathbf{2}^{i|j} + \boldsymbol{\beta}\mathbf{2}^{i|j\prime})\mathbf{J1}^j]\mathbf{S}_{t-1} + \beta\mathbf{S}'_{t-1}\mathbf{J1}^{j\prime}\boldsymbol{\beta}\mathbf{2}^{i|j}\mathbf{J1}^j\mathbf{S}_{t-1} \\ &\quad +tis + O[2] \end{aligned}$$

These can then be equated with the corresponding terms from the value function guesses,

$$\begin{aligned} V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \widetilde{\boldsymbol{\Phi}\mathbf{0}}^i + \boldsymbol{\Phi}\mathbf{1}^i\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\boldsymbol{\Phi}\mathbf{2}^i\mathbf{S}_{t-1} + tis \\ W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \widetilde{\boldsymbol{\mu}\mathbf{0}}^i + \boldsymbol{\mu}\mathbf{1}^i\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\boldsymbol{\mu}\mathbf{2}^i\mathbf{S}_{t-1} + tis \text{ where } i = 1,2 \end{aligned}$$

which completes the description of optimal policy for both parties.

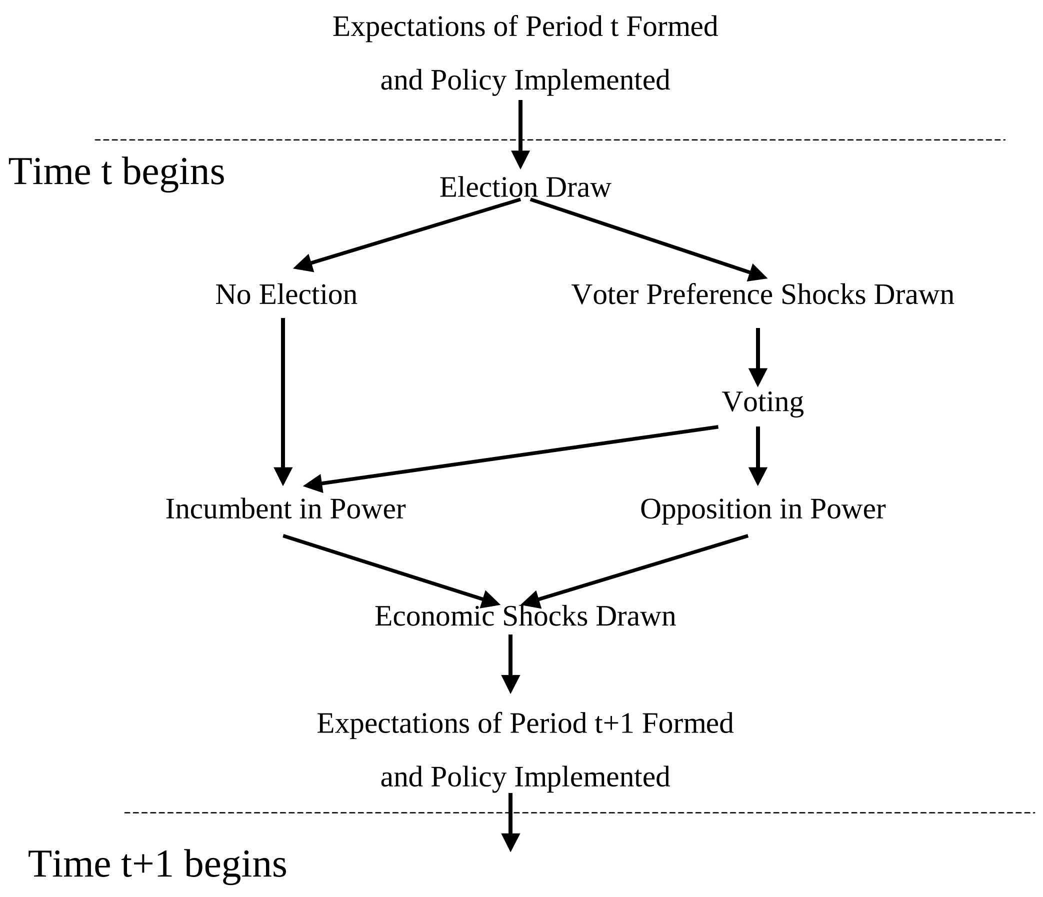

Figure 1: Timing of Events

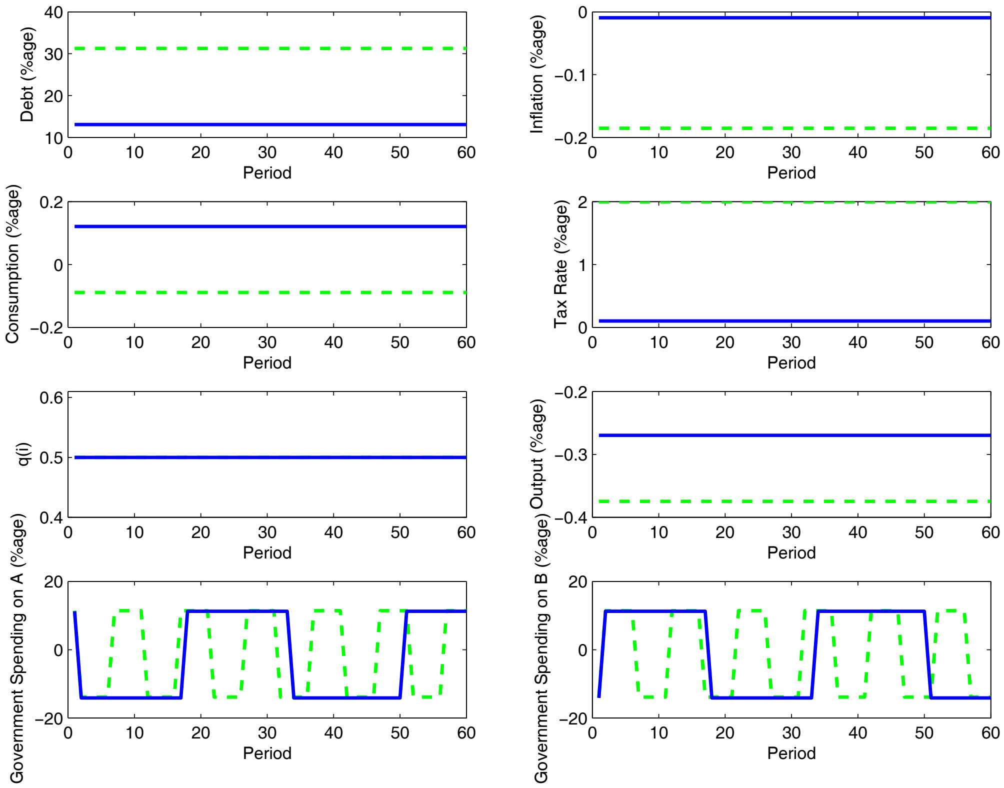

Figure 2: Impulse responses under composition heterogeneity and an electoral cycle.

Notes to Figure - solid blue line - election probability e=1/16; green dashed line - election probability e=1.

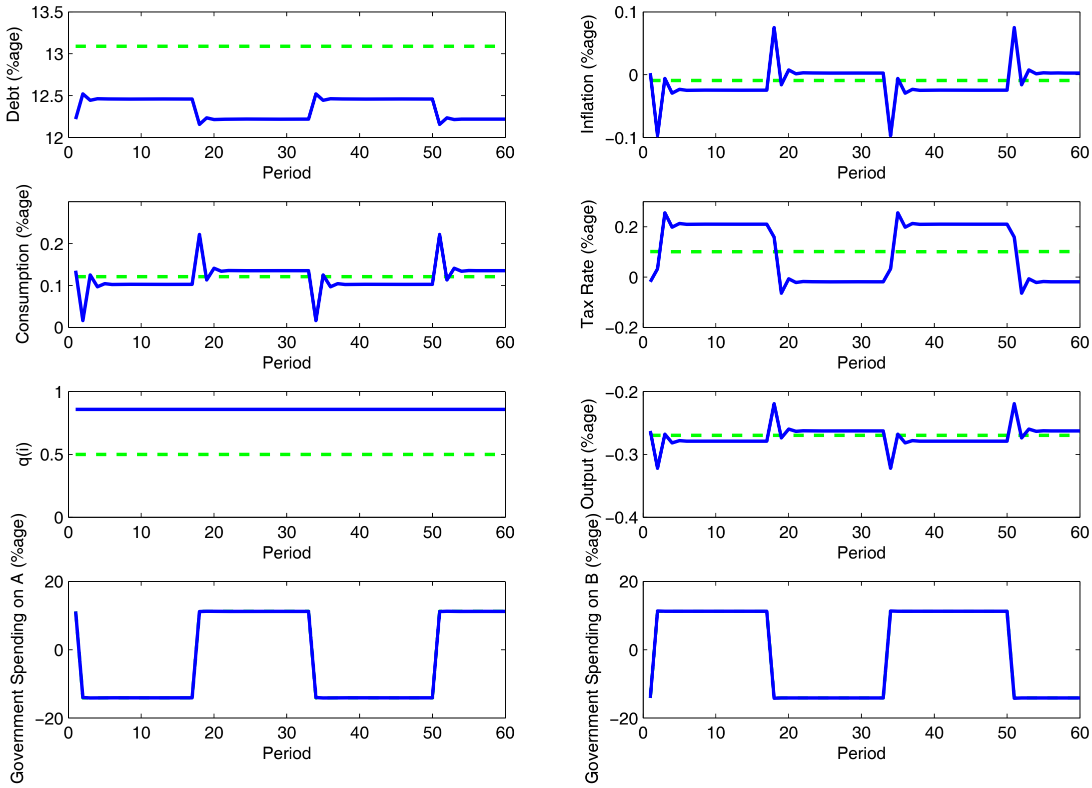

Figure 3: Impulse response with composition heterogeneity and an electoral cycle.

Notes to Figure - solid blue line, asymmetrical probabilistic voting, $z_1 = 0.9$, $z_2 = 0.1$; green dashed line, symmetrical probabilistic voting, $z_1 = z_2 = 0.5$.

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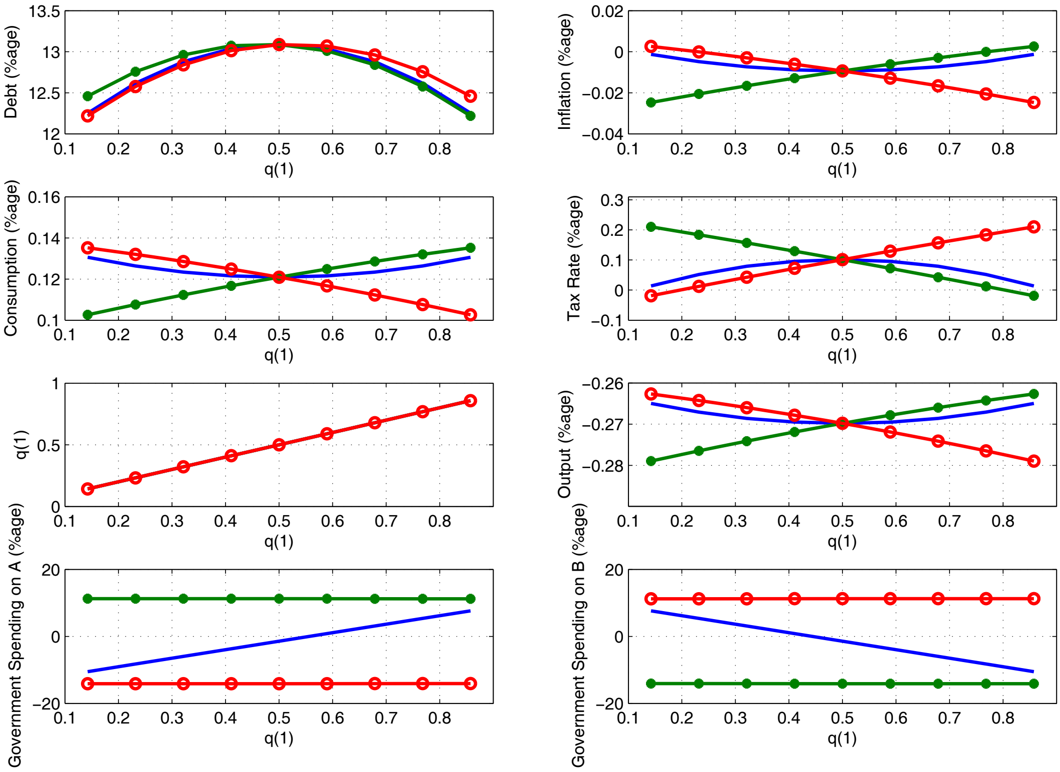

Figure 4: Steady-State with composition heterogeneity as function of (endogenous) election probability.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

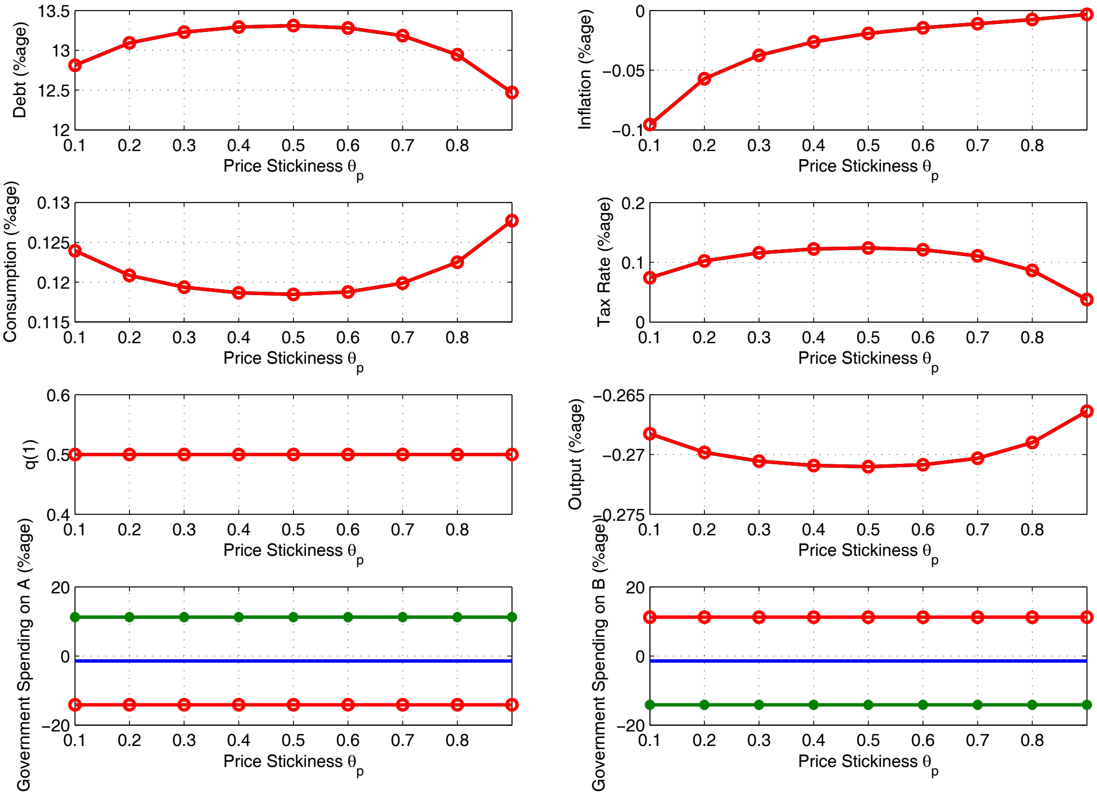

Figure 5: Steady-state with composition heterogeneity as a function of degree of price stickiness.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

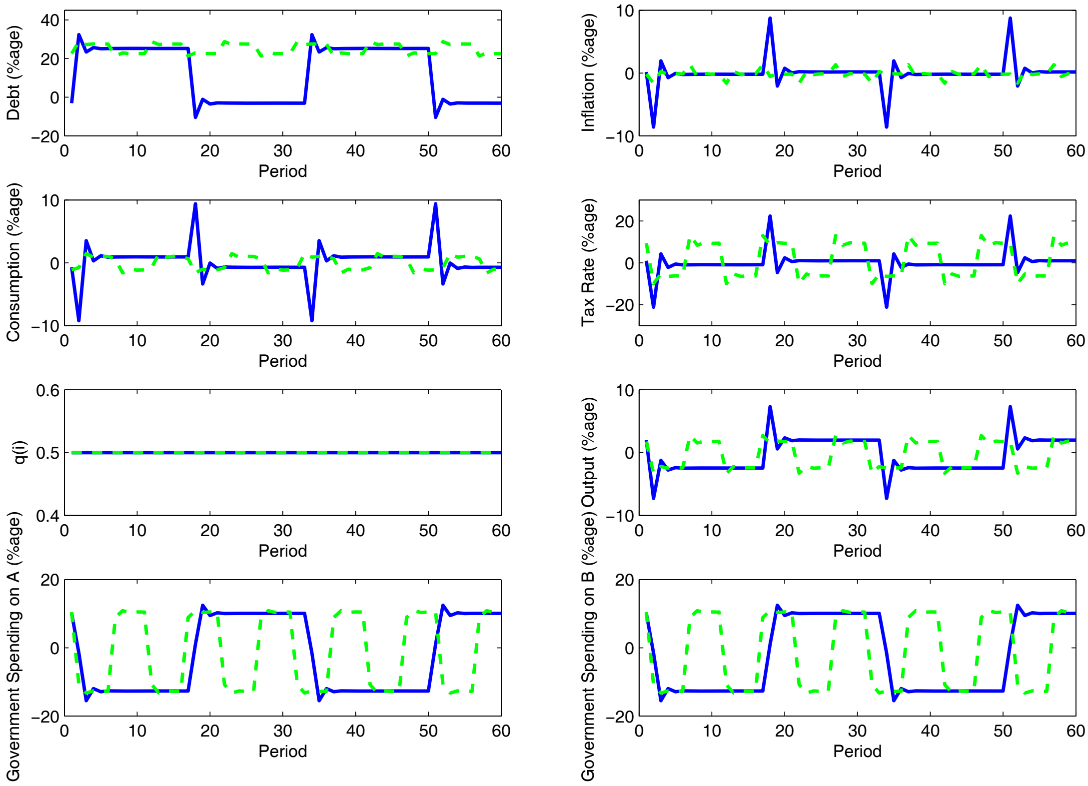

Figure 6: Impulse responses under size heterogeneity and an electoral cycle.

Notes to Figure - solid blue line, election probability $e = 1/16$; green dashhed line, election probability $e = 1$.

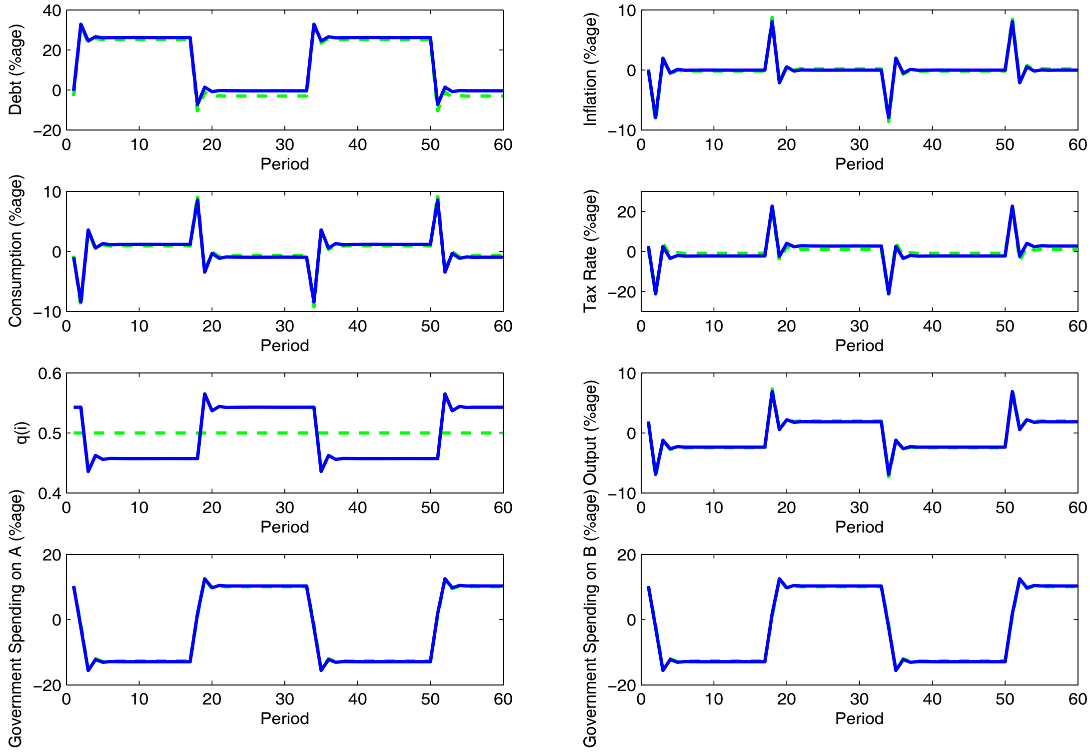

Figure 7: Impulse responses with size heterogeneity and an electoral cycle.

Notes to Figure - solid blue line, endogenous election victory probability; green dashhed line, exogenous election victory probability, q(i)=1/2.

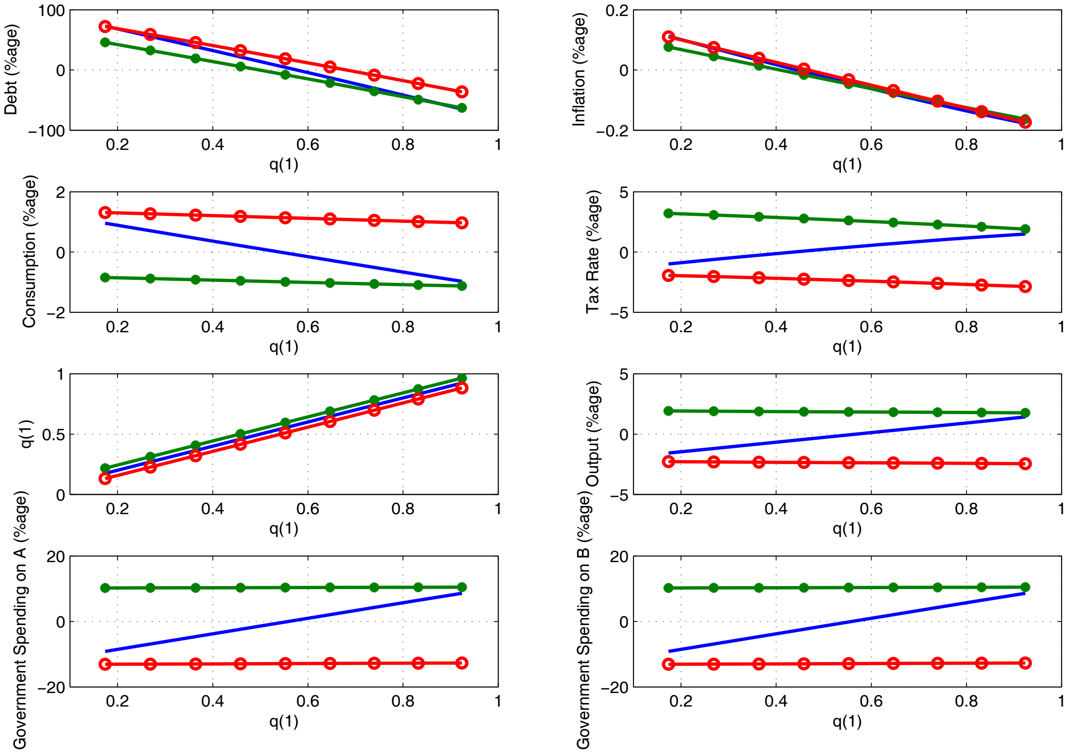

Figure 8: Steady-state with size heterogeneity as function of the (endogenous) electoral probability.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

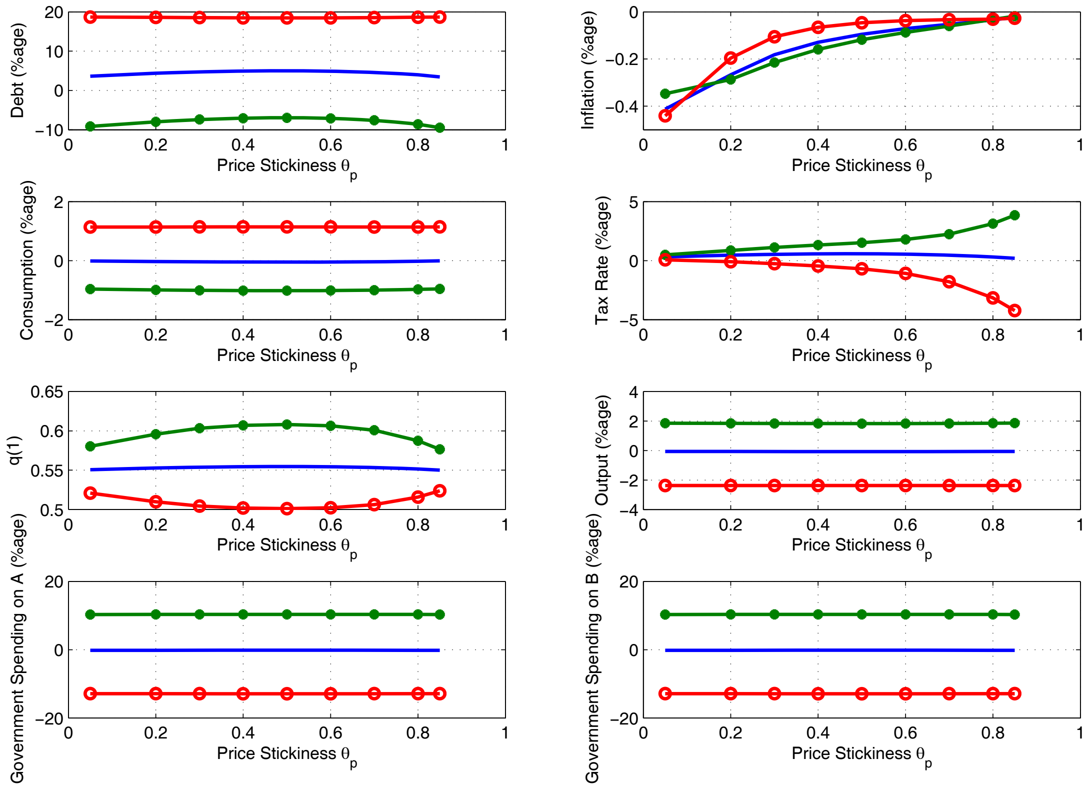

Figure 9: Steady-state with size heterogeneity as a function of the degree of price stickiness.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

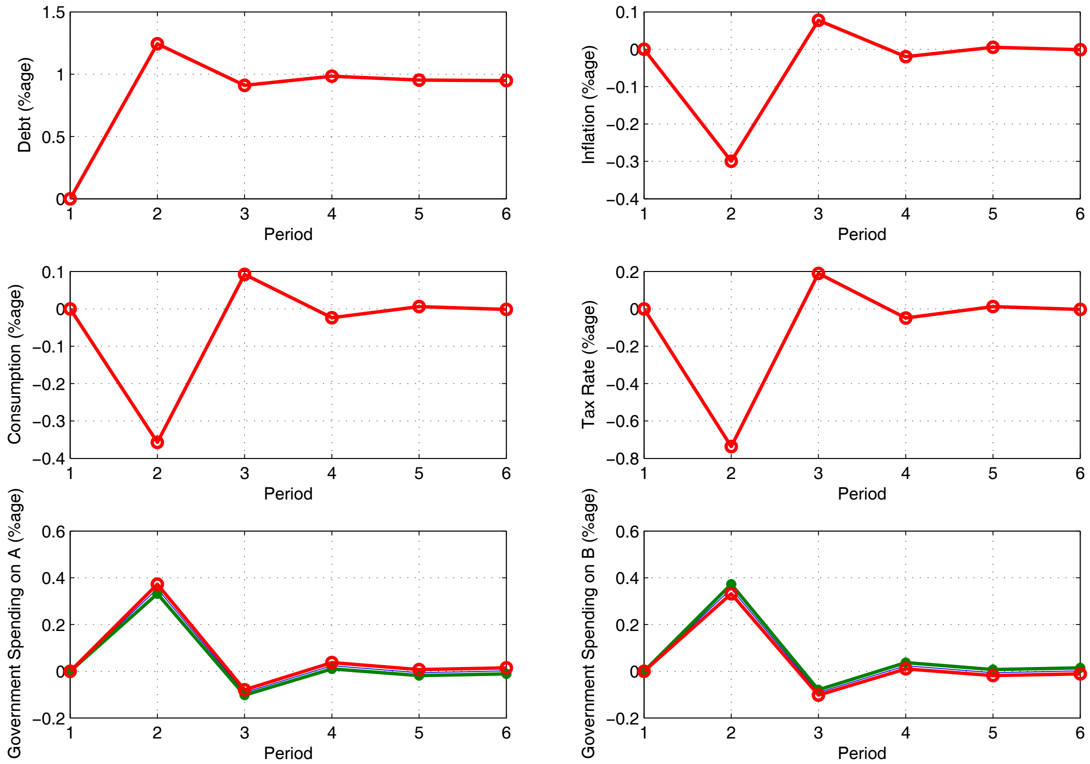

Figure 10: Impulse response to 1% technology shock with compositional heterogeneity.

Notes to Figure - green line with stars party 1's response, red line with hollow circles i party 2's response.

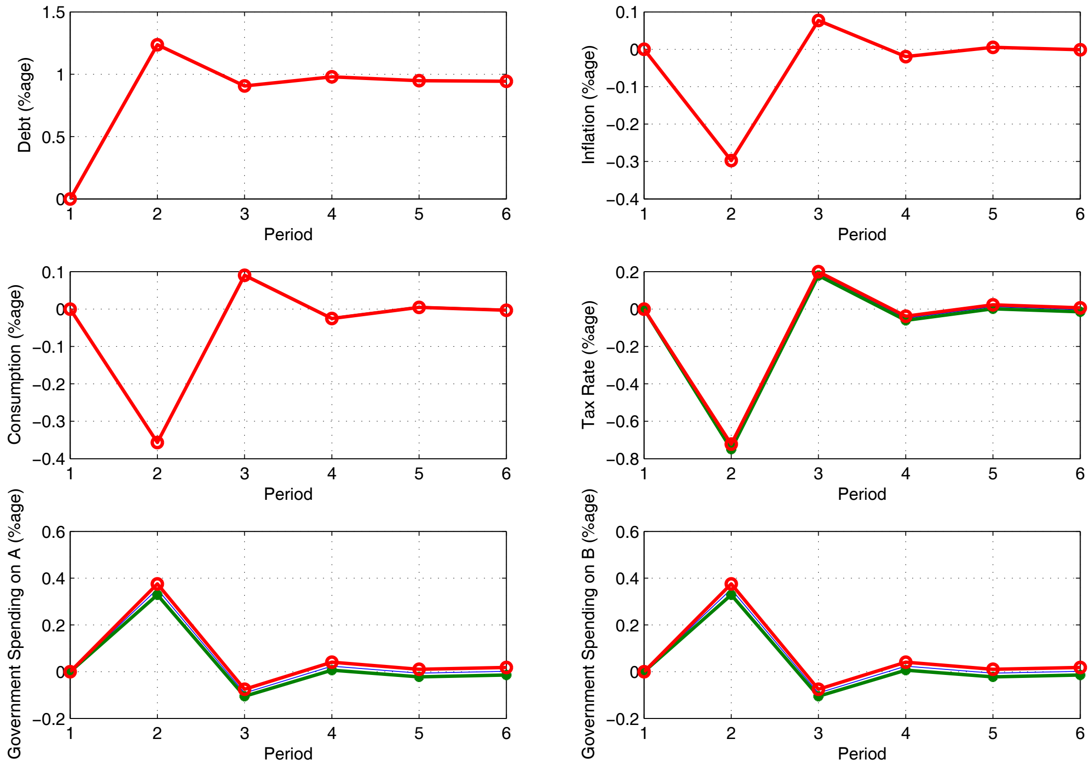

Figure 11: Impulse response to 1% technology shock with size heterogeneity.

Notes to Figure - green line with stars party 1's response, red line with hollow circles party 2's response.