Can Social Norms Affect the International Allocation of Innovation?

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Abstract

If economic agents coordinate on social norms more oriented towards the protection of national industries, an asymmetric international specialization in the research and development (R&D) arises even in a tariff free world with no a priori differences across countries in endowments, demography or technology.

This paper exploits the indifference in the composition of R&D expenditure across sectors of the typical multi-sector Schumpeterian framework (forward-looking decisions, CRS R&D technology and free entry) to construct a theory of the international allocation of innovation and education based on sunspot equilibrium. A role for industrial policies as mere coordination devices emerges in an international Schumpeterian framework. The implications for the relationships between inequality and growth are examined.

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1 Introduction

Can social norms determine the international division of innovative labor? Why are some countries innovating in more sectors than other countries, after controlling for market fundamentals? For example the US and Continental Europe are two regions of the world that have similar potentialities for research and development (R&D) and comparably efficient education systems[1]. Yet the US economy is active in a larger number of technological sectors - most notably information and communication technologies, and biotechnology[2] - have a larger percentage of educated workers as well as higher wage inequality, and innovate relatively more in the aggregate. Using standard Schumpeterian theory this paper claims that these features may partly be a consequence of the coordination of US investors on a social norm more oriented towards the defense of the national interest in a larger number of industries than economic actors in Europe.

Historical and geographical explanations may be given to argue that national cohesion has been more strictly enforced in the US than in a similarly sized subcontinent such as Europe. This paper shows that such behavior

[1] OCDE surveys (e.g. PISA 2000) on the performances in maths, science and languages of students under 15 years of age in Europe and in the USA shows no significant difference (a slightly higher mean in Europe with more dispersion).

[2] "The sector distribution of U.S. industrial R&D performance is among the most widespread and diverse among OECD members.... United States have...become globally competitive in numerous industries rather than just a few industries or niche technologies. In 2000 no U.S. industrial sector accounted for more than the 13 percent of total industrial R&D concentrated in the electronic equipment manufacturing sector. In comparison, most of the other countries displayed somewhat higher sector concentrations. For example, 20 percent or more of industrial R&D was concentrated in electronic equipment manufacturing in Finland (at 49 percent of its industry total), South Korea (37 percent), Canada (29 percent), and Sweden (23 percent). Indeed, the electronic equipment sector was among the largest performers of industrial R&D in 7 of the 11 countries shown and was the second largest performer of industrial R&D for the entire European Union. Among other manufacturing sectors, motor vehicles in Germany and pharmaceuticals in the United Kingdom accounted for 20 percent or more of total R&D performance, which was consistent with general economic production patterns...One of the more significant trends in both U.S. and international industrial R&D activity has been the growth of R&D in the service (nonmanufacturing) sector. According to the internationally harmonized data in table 4-20 , this sector accounted for 34 percent of total industrial R&D performance in the United States in 2000". (National Science Board, *Science and Engineering Indicators*, 2004, Ch. 4, available at http://www.nsf.gov/sbe/srs/seind04/c4/c4s4.htm).

does not violate the assumptions of individual rationality.

It is common in new trade theory to predict perfectly symmetric long run outcomes for countries that share the same demographic structures, same preferences, same technologies, and same institutional features. Factor price equalization (FPE) and identical performance are typically predicted by the Schumpeterian theory of trade and growth (see e.g. Grossman and Helpman 1991 ch.7, and Aghion and Howitt 1998 ch.10). The seminal paper by Dinopoulos and Segerstrom (1999) - that extends to international economics two important theories of technological evolution that rule out the "strong scale effect" (Jones 2003) which flawed the early generation growth and trade models - is no exception in this respect. I show here that in the same framework otherwise identical countries with no trade impediments could coordinate on each of at least a continuum of different equilibria, with only one delivering the celebrated symmetric outcome. Indeterminacy cannot be ruled out by a concentration on the steady state analysis, because of the existence a *continuum of steady states*. I argue that, far from being a problem for the explanatory ability of Schumpeterian theory, indeterminacy enriches its possibility of incorporating interesting social norms.

This multiplicity of equilibria is due to the forward looking nature of the Schumpeterian "creative destruction" mechanism formalized in the single country-single industry path breaking Aghion and Howitt (1992). Private firms undertake R&D activities aimed at developing better versions of an already existing product because the winner of the patent race will enjoy monopoly profits; but such profits are temporary because new R&D firms are always trying to make the monopolist's product obsolete. Hence the expectation of more R&D in the future discourages current R&D. Similarly to the overlapping generations models with money, indeterminacy can emerge because there is no way to force yet unfounded future firms to a particular path of R&D investment, and depending on the different expectations about the future levels of R&D current R&D investment may differ.

While in Aghion and Howitt's (1992) economy indeterminacy was confined to non-stationary equilibria with the interior steady state being locally unique, our multisector economy admits indeterminacy of stationary equilibria, each of them thereby qualifying as a stationary rational expectations equilibrium. In any of the asymmetric equilibria we will study the different countries specialize endogenously in the R&D of a subset of the existing consumer goods. However such specialization is not the result of exogenously given resource or technology endowments, but of the self-fulfilling expectation of a "punishment" against the firm that deviates from such behavior: if a country A firm invades one of country B's monopolized sectors country B's research units will try to regain monopoly in that market by investing more in R&D. The above mentioned behavior is perfectly rational and requires only a one period expected "punishment" of deviators. The main consequence is that in the continuum of asymmetric steady states we will construct countries differ in the range of goods in which they undertake R&D and dominate production worldwide. The logic of my argument goes as follows:

1. The reward to innovation in a given industry depends negatively on expected future innovation in the industry: future innovations render current innovations obsolete.

2. Expectations about future R&D is conditional on the nationality of the next innovator. A public signal can coordinate atomless national R&D investors to an aggregate behavior that ends up "punishing" the foreign R&D firms innovate in a subset of industries.

3. In equilibrium these punishments are credible, and a country can capture a disproportionate part of the innovative sectors. This raises the demand for skilled workers, increases the wage premium and therefore inequality.

Our "discriminatory equilibria" are an admittedly highly abstract metaphors susceptible to real world interpretations. In particular, they capture all the cases in which a country's authorities, media, and trade organizations are concerned with defending their country's international competitiveness in some sectors in which they traditionally held primacy. When a high technology sector is threatened by foreign competition it is not uncommon to see private and public economic forces mobilize and try to defend it. Does such mobilization violate the logic of perfectly competing R&D and financial markets in general equilibrium? This paper proves that it need not be so. In fact, what industrial associations, government agencies, mass media, etc. may be doing is to give a public signal that trigger a R&D over-investment by multitude of private economic agents. Hence they are playing the role of a "sunspot" (Farmer 1998) incarnated by the monitoring of the publicly visible untouched industrial excellence in a range of sectors[3]. A different ex-

[3]A striking example is Japanese MITI's "Visions" in shaping national R&D by coordinating the expectations of the business community. According to Watanabi (2004), "Japans ratio of governmental R&D support to overall industry R&D investment is just 3% (1997). This ratio is extremely small compared to 15% in the USA (1998), 24% in Germany (1997), 13% in France (1995) and 10% in the UK (1997). This observation implies the ability of Japans R&D policy system to effectively stimulate industry R&D with

pected reaction by the private R&D investors to a public complaint about a sector's lost primacy is part of a different sunspot equilibrium. The existence of sunspot equilibria found in this class of models implies that the industrial policies of nations can play the role of natural coordination devices of large volumes of private resources, with dramatic international consequences.

The paper is organized as follows. Section 2 sets up the model. Section 3 derives the main results and discusses their sunspot equilibrium interpretation. Section 4 explains the intuitions for the macroeconomic consequences of the asymmetric steady states. Section 5 concludes.

2 The Model

2.1 Households

Assume a two country world, in which population, preferences, technologies, and institutions are identical in both countries A and B. Time is continuous and unbounded and expectations are rational. There are no tariffs, there is perfect mobility of goods, capital, ideas, and technology, but there is international immobility of labor.

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only limited financial resources. Despite a limited financial role, MITI (Japans Ministry of International Trade & Industry), which is responsible for industrial technology policy, has developed other governance systems which permit it to play a leading role in the stimulation of industry R&D...A report on science and governing issues published in 1963 by the advisory council to the Minister of MITI became Japans first so-called "Vision", and in response, MITI established the Large-Scale R&D Project in 1966. This programme focused on strategic R&D initiated by R&D consortia run on government initiative, and laid the groundwork for MITI's long-lasting national R&D programme.

It is clear that MITIs "Visions" programme in the 1960s, 1970s and 1980s played an important role in shaping the respective decades. In this policy system, the "Visions" concept played a fundamental role. This approach provided a vehicle for synchronizing possible, expected and preferred futures by perceiving future directions, identifying long-term goals, creating consensus, instilling confidence, and establishing the respective sharing of responsibilities among the broad sectors concerned. In Japan, this approach proved to be a vehicle for effective governance by creating the conditions for a virtuous cycle in which technological progress and socio-economic development reinforced one another."

"Visions" in Co-Evolution: A Japanese Perspective on Science and Governance", Chihiro Watanabi, IPTS Report 2004.

As in Dinopoulos and Segerstrom (1999) within each country the households differ in the ability - uniformly distributed over the unit interval - of their individual members to become skilled workers, while all individuals have identical intertemporally additively separable and unit elastic preferences for an infinite set of consumption goods indexed by $\omega \in [0,1]$, and are endowed with a unit of labor/study time endowment whose supply entails no disutility. Dropping country indexes for notational simplicity, we assume that households choose their optimal consumption bundle for each date by solving the following optimization problem:

$$\max \int_0^\infty N_0 e^{-(\rho-n)s} \log u_\theta(s) ds \tag{1}$$

subject to

$$\log u_\theta(s) \equiv \int_0^1 \log \left[\sum_{j=0}^{j^{\max}(\omega,s)} \lambda^j q_\theta(j,\omega,s) \right] d\omega$$

$$c_\theta(s) \equiv \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,s)} p(j,\omega,s) q_\theta(j,\omega,s) \right] d\omega$$

$$W_\theta(t) + Z_\theta(t) = \int_t^\infty N_0 e^{-\int_t^s (r(\tau)-n)d\tau} c_\theta(s) ds, \text{ for all } t \geq 0,$$

where N_0 is the initial population in both countries and n is its common growth rate, ρ is the common rate of time preference - with $\rho > n$ - and $r(s)$ is the market interest rate. $q_\theta(j,\omega,s)$ is ability $\theta \in [0,1]$ household's per-member consumption flow of quality $j \in \{0,1,2,...\}$ of good $\omega \in [0,1]$ at time $s \geq 0$ - $p(j,\omega,s)$ being the price of good ω of quality j at time s - $c_\theta(s)$ is nominal individual expenditure, $W_\theta(t)$ and $Z_\theta(t)$ are human and non-human wealth levels. A new vintage of a good delivers $\lambda > 1$ more quality services than its previous version. Different versions of the same good ω are regarded by consumers as perfect substitutes after adjusting for their quality ratios, and $j^{\max}(\omega,s)$ denotes the time s top quality of good ω. As is common in quality ladders models I will assume price competition[4] at all dates, with the

[4] Dinopoulos and Segerstrom (1999) assume quantity competition instead of price competition. In this paper I prefer to follow the traditional Bertrand competition assumption because of its notational simplicity: however all qualitative results would hold under quantity competition.

implication that in equilibrium only the top quality product will be produced and consumed in positive amounts.

Individuals are finitely lived members of infinitely lived households, being continuously born at rate β, and dying at rate δ, with $\beta - \delta = n > 0$; $D > 0$ denotes the exogenously given duration of their life[5]. Demographics is identical in each country. People are altruistic in that they care about their household's total discounted utility according to the intertemporally additive functional shown in (1). They choose to train and become skilled at the beginning of their lives, and the (positive) duration of their training period - in which the individual cannot work - is exogenously fixed at $T < D$.

Hence an individual with ability θ decides to train if and only if the following is satisfied:

$$\int_t^{t+D} e^{-\int_t^s r(\tau)d\tau} w_L(s)ds < \int_{t+T}^{t+D} e^{-\int_t^s r(\tau)d\tau} \max\left(\theta - \gamma, 0\right) w_H(s)ds,$$

with $0 < \gamma < 1/2$. Notice that an individual of ability $\theta > \gamma$ is postulated able to accumulate skill (human capital) $\theta - \gamma$ after training, while individuals with too low ability ($\theta < \gamma$) never get any skill from schooling.

As Dinopoulos and Segerstrom (1999) I will focus on the steady state (balanced growth) analysis, in which all variables grow at constant rates and w_L, w_H, and c_θ are all constant. It easily follows that $r(s) = \rho$ at all dates, and that the individual will train if and only if her ability is higher than

$$\theta_0 = \left[\left(1 - e^{-\rho D}\right) / \left(e^{-\rho T} - e^{-\rho D}\right)\right] \frac{w_L}{w_H} + \gamma \equiv \sigma \frac{w_L}{w_H} + \gamma. \tag{2}$$

We will prove later that in our economy factor price equalization (FPE) does not generically hold, and hence we have to label wages for the country where the corresponding labor is hired. The supply of unskilled labor in country $K = A, B$ at time t is

$$L^K(t) \equiv \theta_0^K N^K(t) = \left(\sigma \frac{w_L^K}{w_H^K} + \gamma\right) N^K(t) \tag{3}$$

Notice that $N^A(t) = N^B(t) \equiv N(t)$ by assumption.

[5] As in Dinopoulos and Segerstrom (1999: 454) it is easy to show that the above parameters cannot be chosen independently, but that they must satisfy $\delta = \frac{n}{e^{nD}-1}$ and $\beta = \frac{ne^{nD}}{e^{nD}-1}$ in order for the number of births at time t to match the number of deaths at $t + D$.

Following the same steps as Dinopoulos and Segerstrom (1999: 456) the reader can easily verify that the supply of skilled labor in country $k = A, B$ at time t is

$$H^K(t) = \left(\theta_0^K + 1 - 2\gamma\right)\left(1 - \theta_0^K\right)\phi N(t)/2, \tag{4}$$

with $0 < \phi < 1$. Clearly in any steady state the growth rate of $L^A(t)$, $L^B(t)$, $H^A(t)$, and $H^B(t)$ is equal to n.

2.2 Manufacture

In each country firms can hire unskilled workers to produce any consumption good $\omega \in [0, 1]$ of the second best quality under a constant returns to scale (CRS) technology described by the simple unit cost function aw_L^A for country A and aw_L^B for country B, with $a > 0$ common in all industries. However in each industry the top quality product can be manufactured only by the firm that has discovered it - whose rights are protected by a patent - in the whole world. This assumption allows for multinational companies establishing subsidiaries in low wage countries to carry out the manufacturing of their leading-edge products: hence in equilibrium unskilled labor prices will equalize, and we will chose it as numeraire, that is: $w_L^A = w_L^B = 1$.

As usual in Schumpeterian models with vertical innovation (see e.g. Grossman and Helpman 1991 and Aghion and Howitt 1998) the next quality of a given good is invented by the R&D performed by challenger firms in order to earn monopoly profits that will be destroyed by the next innovator. During each temporary monopoly the patent holder can sell the product worldwide at prices higher than the unit cost. As Dinopoulos and Segerstrom (1999) we assume that the patent expires when further innovation occurs in the industry. Hence the monopolist rents are destroyed not only by obsolescence but also because a competitive fringe can copy the product using the same CRS technology.

The unit elastic demand structure[6] encourages the monopolist to set the highest possible price to maximize profits, but the existence of a competitive fringe worldwide sets a ceiling to it equal to the world lowest unit cost of

[6] Any CES utility index with elasticity of substitution not greater than one would imply this result. Were the elasticity of substitution greater than one, if the quality jump λ were high enough, we could have drastic innovations, with unconstrained profit maximizing top quality produceres that wipe out sales of the second best quality consumption good (produced by unskilled workers). Our asymmetric specialization results would not change in any meaningful way.

the previous quality product. This allows us to conclude that the price $p\left(j^{\max}(\omega,s),\omega,s\right)$ of every top quality good is:

$$p\left(j^{\max}(\omega,s),\omega,s\right)=\lambda a, \text{ for all } \omega\in[0,1] \text{ and } s\geq 0. \tag{5}$$

From the assumed household's preference structures, we can immediately conclude that each consumer θ in country $K=A$, B will buy quantity $c_\theta^K/\lambda a$ of every product. Hence the world demand for each product ω is:

$$\frac{N(s)\int_0^1 c_\theta^A d\theta+N(s)\int_0^1 c_\theta^B d\theta}{\lambda a}\equiv\frac{\left(c^A+c^B\right)N(s)}{\lambda a}\equiv q, \tag{6}$$

and in equilibrium this will coincide with the production of every consumption good by the firm that monopolizes it.

It follows that the stream of monopoly profits accruing to the monopolist which produces a state-of-the-art quality product in country $K=A,B$ will be equal to:

$$\pi^K(\omega,s)=q\left(\lambda-1\right)a=\left(c^A+c^B\right)N(s)\left(1-\frac{1}{\lambda}\right). \tag{7}$$

Hence a firm that produces good ω in country $K=A,B$ has an expected discounted value that satisfies the following

$$v^K(\omega,s)=\frac{\pi^K(s)}{\rho+I(\omega,s)-\frac{\dot{v}^K(\omega,s)}{v^K(\omega,s)}}=\frac{q(s)\left(\lambda-1\right)a}{\rho+I(\omega,s)-\frac{\dot{v}^K(\omega,s)}{v^K(\omega,s)}},$$

where $I(\omega,s)$ denotes the worldwide Poisson arrival rate of an innovation that will destroy the monopolist's profits in industry ω.

In a steady state where per-capita variables all grow at a constant rate, it is easy to prove that $\frac{\dot{v}^A(\omega,s)}{v^A(\omega,s)}=\frac{\dot{v}^B(\omega,s)}{v^B(\omega,s)}=n$. Hence the previous equations become

$$v^K(\omega,s)=\frac{q(s)\left(\lambda-1\right)a}{\rho+I(\omega,s)-n} \tag{8}$$

$K=A,B$. In what follows it is important to notice that within each country firms in all industries have the same value if and only if obsolescence - i.e. their corresponding value for $I(\omega,s)$ - is the same. In the event[7] obsolescence is identical across countries as well, firms will have the same stock market value.

[7] Which, as we shall see, corresponds to the steady state in the so called "TEG" specification of the research technology.

2.3 R&D Races

In each industry the world leaders are challenged by the R&D firms that employ skilled workers and produce a probability intensity of inventing the next version of their products. The arrival rate of innovation in industry ω at time s is $I(\omega,s) \equiv I^{A}(\omega,s)+I^{B}(\omega,s)$, and it is the world aggregate summation of the Poisson arrival rate of innovation produced by all R&D firms targeting product ω.

Every R&D firm can produce a Poisson arrival rate of innovation in the product line it targets by use of a CRS technology characterized by unit cost function $bw_{H}^{A}X(\omega,s)$ in country A and $bw_{H}^{B}X(\omega,s)$ in country B, with $b>0$ common in all industries and $X(\omega,s)>0$ measuring the degree of complexity in the invention of the next quality product in industry ω. Hence R&D is formally equivalent to buying a lottery ticket that confers to its owner the exclusive right to the corresponding innovation profits, with the aggregate rate of innovation proportional to the "number of tickets" purchased. The Poisson specification of the innovative process implies that the individual contribution to R&D by each skilled labor unit gives an independent (additive) contribution to the worldwide instantaneous probability of innovation: hence R&D productivity is the same if each research worker undertakes R&D by self-employing herself as if they are co-working in large firms. In this paper we will work under the assumption that regardless of the firm sizes the nationality of the labor unit responsible for a successful innovation is public information.

The technological complexity index $X(\omega,s)$ has been introduced into endogenous growth theory after Charles Jones' (1995) empirical criticism of R&D based growth models generating scale effects in the steady state per-capita growth rate, and it is standard to assign it two alternative laws of motion. According to Segerstrom's (1998) interpretation of Jones' (1995) solution to the "strong scale effect" problem (Jones 2003), it is increasing in the accumulated stock of effective R&D, as:

$$\frac{\dot{X}(\omega,s)}{X(\omega,s)}=\mu I(\omega,s), \tag{TEG}$$

with positive μ, thus formalizing the idea that early discoveries fish out the easier inventions first, leaving the most difficult ones for the future.

Alternatively, Dinopoulos and Thompson (1998) suggest that

$$X(\omega, s) = \frac{k}{2} N(t), \tag{PEG}$$

with positive k, thereby formalizing the idea that it is more difficult to introduce a new product in a more crowded market.

Both formulations, as well as similar others rule out implausible "scale effects", but they have different long run implications: the first one - TEG[8] - implies that more and more difficult inventions will eventually make percapita GDP growth vanish over time unless more and more resources are invested in R&D, thereby requiring a growing educated population; instead the PEG[9] formulation allows for sustained growth without population growth. In the present framework with quality improving consumer goods "growth" is interpreted as the increase over time of the representative consumer utility level.

Dinopoulos and Segerstrom (1999) carry out their analysis for both specifications of the difficulty index, and so will this paper do.

For industries targeted by positive R&D in each country $K = A, B$ the constant returns to R&D and free entry and exit imply the no arbitrage condition

$$v^K(\omega, s) = bw_H^K X(\omega, s) \text{ for all } \omega \in [0,1] \text{ and } s \geq 0. \tag{9}$$

3 Self-Fulfilling World Disparities

3.1 Asymmetric R&D Races

From (8) and (9) follows that not all industries $\omega \in [0, 1]$ will necessarily be targeted by R&D in country A. In fact assume that there exists a proper non-zero measure subset of industries $\xi_B \subset [0, 1]$ such that all firms expect that after an R&D firm in A discovers the next version of a product $\omega \in \xi_B$ more R&D will be undertaken worldwide in industry ω than in the other industries

[8] Acronym "TEG" refers to the "temporary effects on growth" of policy measures such as R&D subsidies and tariffs: they cannot alter the steady state percapita growth rate, which is instead pinned down by the population growth rate, the negative dynamic externality parameter μ, and the number of countries where R&D is active.

[9] Acronym "PEG" refers to the "permanent effects on growth" of policy measures such as R&D subsidies and tariffs: they can alter the steady state percapita growth rate.

$\omega' \in [0,1] - \xi_B$, assumed of (however small but) positive measure. It follows that no country A R&D firm will ever target product $\omega \in \xi_B$, because they expect that after the next quality jump obsolescence will be higher than for product $\omega' \in [0,1] - \xi_B$, thus generating the same flow of profits (7) for a shorter expected time. By applying the same logic to country B R&D firms targeting products $\omega \in \xi_A = [0,1] - \xi_B$, we can partition the unit interval into two nonzero-measure subsets $\xi_A \cup \xi_B = [0,1]$, $\xi_A \cap \xi_B = \emptyset$, such that all indexes $\omega \in \xi_A$ correspond to industries in which only R&D firms in country A are active, while all indexes $\omega \in \xi_B$ correspond to industries in which only R&D firms in country B are active. As a consequence in the long run each country will maintain leadership in its corresponding set of industries. In what follows labels are such that $m(\xi_A) \geq 1/2 \geq m(\xi_B)$. Propositions 1 and 2 will show that in any steady state the more diversified country has higher skilled labor wage.

The above beliefs can be part of an equilibrium because (9) impose indifference across sectors where positive R&D is undertaken, and since the CRS R&D technology leaves indeterminate the composition of the R&D efforts across the active sectors in each country, it allows the market participants to coordinate on *any* belief about it provided that after the subsequent quality jump the symmetric allocation is recovered. This is a building block of the new kind of sunspot equilibria used in this paper, which is formally proved in Section 3.5.

In real life terms, the beliefs sustaining such a discriminatory equilibrium involve an expected backlash by R&D firms in country B motivated by the will to regain a lost leadership. According to this interpretation, after losing the leading edge in a sector previously dominated by country B the R&D firms of this country will hire in that sector more difficulty index-adjusted R&D labor than hired by country A's firms in each sector $\omega \in \xi_A$: the expectation of this reaction dissuades country A's firms from undertaking R&D in that sector and drives to zero the probability of conquering leadership. At the same time, the expectation that from the next quality jump the *status quo* will be re-established anywhere renders the country B's firms indifferent - from a pecuniary point of view - between investing in that sector and investing in all the other ξ_B's product lines, thereby justifying the higher investment in the sector by extrinsic factors. The precise R&D investment levels that justify these equilibria are explicitly obtained analytically in Sections 3.5.1 and 3.5.2.

Notice that the kind of leadership we are analyzing is due to entirely strategic reasons and that, moreover, these strategies are not set by individual monopolists, because free entry and CRS would annihilate their dissuasion power. Such strategies are "sunspot" strategies, that is a coordination of large numbers of entrant R&D firms on a stochastically generated public signal that triggers their aggregate response. Hence in this paper we will see how "sunspots matter" (Cass and Shell 1983) in a standard Schumpeterian international economy. The multitude of private R&D firms and their financiers will accept to follow the public signal to defend the threatened competitiveness of a sector because - due to the R&D arbitrage conditions - it costs nothing to them. Hence the arbitrage condition and constant returns by implying indeterminate R&D sector sizes leaves room for the emergence of a richer set of real world features.

We will work under the assumption that in the active sectors within each country R&D intensity and arrival rates are identical and we will assume identical initial conditions. Therefore

$$I(\omega, s) = I^K(\omega, s) + 0 \equiv I^K,\ \omega \in \xi_K,\ K = A, B.$$

Similarly we will label complexity measures as follows:

$$X(\omega, s) \equiv X^K,\ \omega \in \xi_K,\ K = A, B,$$

where the reader must keep in mind that both in TEG and in PEG specifications complexity regards technology *worldwide*, and therefore the above superscript $K = A, B$ only indicates that the corresponding good is currently being produced and targeted by R&D firms located in the corresponding country, though there is no technological necessity for this. Also, I have dropped time indexes s for notational convenience.

3.1.1 The "Partition" of the Product Space is Not Necessary

It is worthwhile to emphasize that though in the rest of the paper we will analyze only the benchmark case in which the two subsets of industries in which the two countries undertake R&D do not overlap, cases with $\xi_A \cap \xi_B \neq \emptyset$ and/or $\xi_A \cup \xi_B \neq [0, 1]$ can be analyzed formally along similar lines. This is easily seen in the PEG case[10]. In such cases, the beliefs sustaining the asymmetric steady states should be amended in the following way: for

[10] Similar results hold for the TEG case, but the full derivation is more cumbersome.

every product $\omega \in \xi_A \cap \xi_B \neq \emptyset$ the firms expect that after a country A R&D firm's success in discovering its next version worldwide R&D investment will be $I^A(\omega, s) + I^B(\omega, s) = I^A(\omega', s)$, $\omega' \in \xi_A - \{\xi_A \cap \xi_B\}$, whereas they expect that after a country B R&D firm's success in discovering its next version worldwide R&D investment will be $I^A(\omega, s) + I^B(\omega, s) = I^B(\omega', s)$, $\omega' \in \xi_B - \{\xi_A \cap \xi_B\}$. These R&D investment levels would be observed with probability one along the equilibrium path. The off-equilibrium punishments for country B firms investing in $\omega' \in \xi_A - \{\xi_A \cap \xi_B\}$ and country A firms investing in $\omega' \in \xi_B - \{\xi_A \cap \xi_B\}$ are characterized by the same features as the punishment strategies we will describe for the case of a partition. Of course these punishment actions are never observed in equilibrium.

Instead for every product $\omega \in [0, 1] - \{\xi_A \cup \xi_B\} \neq \emptyset$ the firms expect that after any R&D firm's success in discovering its next version worldwide R&D investment will become $I^A(\omega, s) + I^B(\omega, s) > \max\{I^A(\omega', s), I^B(\omega', s)\}$, $\omega' \in \xi_A \cup \xi_B$. This condition states that in sectors that do not belong to $\xi_A \cup \xi_B$ no R&D firm would ever find it convenient to invest, regardless of its nationality. In fact, in case one firm succeeds in inventing a better version of a good $\omega' \in [0, 1] - \{\xi_A \cup \xi_B\}$, there would be a flood of follower innovators for a (stochastically long) period. However, at the end of that period, sector ω' will becomes an inactive sector.

3.2 Asymmetric Balanced Growth Paths

Let us remind that in this hypothetical world economy there is perfect international mobility of goods and capital, perfect mobility of manufacturing methods and perfect spillover of ideas, but international immobility of labor. Moreover, we do not assume different R&D technologies despite the fact that our behavioral assumption of non-overlapping and exhaustive active R&D industries makes the results observational equivalent to a case of international specialization of R&D technologies. However, for the reader who feels more comfortable with an interpretation of international specialization in terms of more "fundamental" factors rather than "self-fulfilling prophecies", it is worthwhile to remark that the derivations here and in Sections 3.3 and 3.4 maintain their validity under the alternative assumption that country $K = A, B$ firms have an exogenously given relative competitive advantage in the R&D of products $\omega \in \xi_K$.

We are now in a position to analyze the general equilibrium implications of the previous setting. As in Dinopoulos and Segerstrom (1999) we will focus on the steady state properties of the model.

Eq.s (8) and (9) imply that

$$q = bw_H^K X^K \frac{\rho + I^K - n}{a(\lambda - 1)}. \tag{10}$$

Recall that each final good monopolist employs unskilled labor worldwide to manufacture the same amount of each commodity to sell worldwide. Therefore the unskilled labor market equilibrium at the world level is

$$N(\theta_0^A + \theta_0^B) = aq. \tag{11}$$

Unskilled labor market equilibrium (11) and arbitrage condition (10) yield:

$$\theta_0^A + \theta_0^B = bw_H^K x^K \frac{\rho + I^K - n}{\lambda - 1}, \tag{12}$$

where $x^K \equiv \frac{X^K}{N}$ denotes the population-adjusted degrees of complexity for products produced in country $K = A, B$.

Similarly,eq..(4) and skilled labor market equilibrium in both countries imply:

$$\left(\theta_0^K + 1 - 2\gamma\right)\left(1 - \theta_0^K\right)\phi/2 = bI^K x^K m(\xi_K), \tag{13}$$

where $m(\xi_K)$ denotes the measure of industries where country K does R&D.

3.3 Steady State Properties of the TEG Specification

In this section we will adopt the TEG specification of technological evolution and leave the analysis of the PEG specification to the next section.

In a steady state all per-capita variables are constant and therefore $\frac{\dot{X}(\omega,s)}{X(\omega,s)} = n$. Hence (TEG) implies: $I^A = I^B = n/\mu$. As usual in growth models with increasing complexity the steady state arrival rate of innovation in every industry is a linear increasing function of the population growth rate. Hence, using eq. (2), we can rewrite (12) and (13) as follows:

$$\theta_0^A + \theta_0^B = \frac{bx^K \sigma (\rho + n/\mu - n)}{\left(\theta_0^K - \gamma\right)(\lambda - 1)} \text{ and} \tag{14}$$

$$\left(\theta_0^K + 1 - 2\gamma\right)\left(1 - \theta_0^K\right)\phi/2 = bx^K m(\xi_K) n/\mu,\ K = A, B. \tag{15}$$

Proposition 1 *Under the TEG specification, if* $\gamma \in (0, .5)$ *a steady state exists for every* $m(\xi_A) \in [1/2, 1)$*. At each steady state in which* $m(\xi_A) \in (1/2, 1)$ *the following properties hold:*

a. $\theta_0^A < \theta_0^B$ *and* $w_H^A > w_H^B$*.*

b. $x^A < x^B$ *and* $x^A m(\xi_A) > x^B m(\xi_B)$*.*

c. Country A's innovation rate - $I^A m(\xi_A)$ *- is higher than country B's innovation rate -* $I^B m(\xi_B)$*.*

d. The world growth rate is invariant.

e. The world level of human capital is lower in any asymmetric steady state than in any symmetric steady state.

Proof. In the Appendix.

The economic reasons for our results can be described. Country A's R&D specializes in a larger number of sectors than country B's R&D: therefore in country A human capital would be diluted in a wider range of sectors, ξ_A, than in country B. This induces a lower probability of innovation per unit time in each country A's sector - i.e. lower per-sector expected obsolescence - which, in the TEG case, implies a slower evolution of the R&D complexity indexes in such sectors. Since profits are equal worldwide, due to the presence of multinational manufacturing firms and consumer preferences symmetric across varieties, and interest rates are equalized in the global capital market, the stock market value of each country A firm is higher than the value of each country B firm. As a consequence, perfect competition in the R&D sectors in both countries requires a higher skilled/unskilled wage ratio in country A than in country B, in order to annihilate expected R&D profits. In turn, the higher skill premium in country A would induce a higher proportion of the population to get skilled, because educational choices respond to the relative wage incentives. The higher human capital in country A however does not suffice to equalize the skill premium worldwide, because individual educational abilities differ, and the additional students are less and less able to accumulate human capital. The assumed heterogeneous learning abilities imply that asymmetric equilibrium unequal cross-country labor force education attainments are inefficient from a worldwide perspective: since country A's marginal student is less able to accumulate human capital than country B's marginal student, the worldwide human capital level would increase following a hypothetical marginal decrease education in country A in exchange for an equal marginal increase in education in country B, while leaving the worldwide manufacturing production unaffected. Of course, more human capital worldwide translate into more steady state innovation rates (in the PEG case) and population-adjusted technological levels (in the TEG case).

3.4 Steady State Properties of the PEG Specification

We now turn our attention to the analysis of the consequences of the PEG specification of technology. From eq. (PEG) it obviously follows that in both countries and for all industries the population-adjusted difficulty index is always equal to a constant $k/2$. Therefore we can rewrite eq.s (12) and (13) as:

$$\theta_0^A + \theta_0^B = \frac{bk\sigma\left(\rho + I^K - n\right)}{2\left(\theta_0^K - \gamma\right)(\lambda - 1)} \text{ and} \tag{16}$$

$$\left(\theta_0^K + 1 - 2\gamma\right)\left(1 - \theta_0^K\right)\frac{\phi}{2} = b\frac{k}{2}m(\xi_K)I^K,\ K = A, B. \tag{17}$$

Then we derive:

Proposition 2 *Under the PEG specification, for all parameter values such that*

$$\frac{bk\sigma(\rho - n)}{2(\lambda - 1)} < 1 - \gamma^2$$

a steady state exists for every $m(\xi_A) \in [1/2, 1)$. At each steady state in which $m(\xi_A) \in (1/2, 1)$ the following properties hold:

a. $\theta_0^A < \theta_0^B$ and $w_H^A > w_H^B$.

b. $I^A < I^B$ and $I^A m(\xi_A) > I^B m(\xi_B)$.

c. The world innovation rate is lower in any asymmetric steady state than in any symmetric steady state.

Proof. In the Appendix.

Remark. In the PEG specification the international division of innovative - and correspondingly non-innovative as well - labor is cause of inequality both within and between countries and of inefficient growth of everybody's utility. Such international division of labor can be self-fulfilling in our model, but the same consequences would apply if it was exogenously generated. At the heart of this inefficiency result is the endogenous skill formation by rational individuals with heterogeneous abilities.

Like in the previous section, the economic reasons for our results can be described. Country A's R&D specializes in a larger number of sectors than country B's R&D: therefore in country A human capital would be diluted in a wider range of sectors, ξ_A, than in country B. This induces a lower probability of innovation per unit time in each country A's sector, that is a lower expected obsolescence of country$A$ monopolies. Since profits are equal worldwide, due to the presence of multinational manufacturing firms and symmetric consumer preferences, and interest rates are equalized in the global capital market, the stock market value of each country A firm is higher than the value of each country B firm. As a consequence, perfect competition in the R&D sectors in both countries requires a higher skilled/unskilled wage ratio in country A than in country B, in order to annihilate expected R&D profits. In turn, the higher skill premium in country A would induce a higher proportion of the population to get skilled, because educational choices respond to the relative wage incentives. The higher aggregate human capital in country A implies, in the PEG specification, a more intense aggregate flow of innovations in country A than in country B. However, it does not suffice to equalize the skill premium worldwide, because individual educational abilities differ, and the additional students are less able to accumulate human capital. As in the TEG case, the assumed heterogeneous learning abilities imply that unequal cross-country labor force education attainments are inefficient: since country A's marginal student is less able to accumulate human capital than country B's marginal student, the worldwide human capital level would increase following a hypothetical a marginal decrease education in country A in exchange for an equal marginal increase in education in country B, while leaving the worldwide manufacturing production unaffected.

3.5 Off Equilibrium Paths

There are two likely problems that we have to solve. The first one is that the equilibria the we focus on (asymmetric outcomes for symmetric countries) need to survive the further requirement that out-of-equilibrium beliefs be credible. What are the individually rational incentives for negligible domestic investors to punish the foreign innovators who challenge the national industries? The second problem is that a full treatment of the problem cannot be done by assuming that the economy is in the steady state. Some of the deviations from equilibrium considered are likely to put the economy in the transition towards a new steady state.

In this section we will explicit off equilibrium path strategies in a way that rationalizes each element of the previous set of asymmetric equilibria. Let us remind that the value of a monopolist firm is:

$$\pi^K(\omega,s) = q(\lambda-1)\,a = \left(c^A + c^B\right) N(s)\left(1-\frac{1}{\lambda}\right). \tag{18}$$

Hence a firm that produces good ω in country $K = A, B$ has an expected discounted value that satisfies the following

$$v^K(\omega,s) = \frac{\left(c^A+c^B\right)N(s)\left(1-\frac{1}{\lambda}\right)}{\rho + I(\omega,s) - \frac{\dot{v}^K(\omega,s)}{v^K(\omega,s)}} \equiv \frac{\pi N(s)}{\rho + I(\omega,s) - \frac{\dot{v}^K(\omega,s)}{v^K(\omega,s)}}.$$

Along the equilibrium path no country A firm will ever challenge the incumbent of a product $\omega \in \xi_B$ and viceversa.

However, should at time $t_0 \geq 0$ a country A R&D firm decide to be active in a sector $\omega \in \xi_B$, in the hypothetical history after its success in discovering a better quality $j(\omega,t_0)+1$ and in displacing the former country B monopolist producing quality $j(\omega,t_0)$, sector ω would be targeted by punisher country B R&D firms. Punisher country B firms' supernormal investment - which we will compute shortly for both TEG and PEG cases - entails[11] $I_f(\omega,t) > I_A(\omega',t)$, $\omega' \in \xi_A$, would only last during the "punishment phase", i.e. until quality $j(\omega,t_0)+2$ is discovered, after which R&D investment in sector ω would become equal to all the remaining sectors in ξ_B.

I construct $I_f(\omega,t)$ such that during the "punishment phase", that is

$$\text{for all } t \in R_+ : j(\omega,t) = j(\omega,t_0)+1,$$

the value of the patent

$$v_f(\omega,t) = \frac{\pi N(s)}{\rho + I_f(\omega,t) - \frac{\dot{v}(\omega,t)}{v(\omega,t)}} \tag{19}$$

satisfies the following equality:

$$v_f(\omega,t) = bw_H^A X(\omega,t)(1-\phi) < bw_H^A X(\omega,t), \tag{20}$$

[11] To facilitate the reader, we will be using the f deponent to denote the values of the firm and of obsolescence (by convention, not of the difficulty index) expected during the punishment phase of the game between the would-be challenger and the other future potential outsider R&D firms.

with[12] $\phi \in]0,1[$.

It is interesting to see that now we departed from Dinopoulos and Segerstrom's (1999) restriction on the agent's expectation of smooth time profiles for the stock market values of the top quality firms in each sector. According to (20), potential R&D investors from country A expect that if one them succeeds in developing the next version of good $\omega \in \xi_B$, the value of the idea patented by the new quality leader will be strictly lower than its research and development cost. For this reason R&D firms will not find it convenient to invest in that sector.

Interestingly, given the generality of the possible ϕs, we are characterizing a continuum of off equilibrium strategies whose expectation is consistent with the previously described equilibrium strategies.

In what follows it is important to note that we are only considering deviations from equilibrium in one sector, ω. To fix the ideas we will focus on $\omega \in \xi_B$, but the symmetric case $\omega \in \xi_A$ can be treated along the same lines.

3.5.1 The TEG Case

In this section we will consider off-equilibrium one-sector deviations from the steady state equilibrium, with resulting punishment that lasts until the next quality jump in that sector. It is important to notice that punishment, by implying a super-normal R&D investment in a sector, will force that sector out of the steady state. This renders an explicit study of the transitional dynamics around the steady state necessary. Fortunately, since each sector is zero measure, the one-sector deviation from the equilibrium by leaving all the aggregate variables unaltered, will not change the rest of the economy, which will keeps following a balanced growth path. However, by driving the sectorial R&D difficulty index at a faster speed, will cast that sector out of

[12] As the reader will note, what we are assuming here is more than enough for subgame perfection, that would just require inequality (20) to hold only at one instant, i.e. at $t = t^F$, with

$$t^F \equiv \min \{ t \in R_+ : j(\omega, t) = j(\omega, t_0) + 1 \} \ .$$

However, (20) allows a neat analytical expression of a possible set of off-equilibrium strategies which render our asymmetric steady state equilibria subgame perfect. Morever, in the TEG case this will allow a precise study of the transitional dynamics of the off-equilibrium sector difficulty index.

the steady state. Putting eq.s (20) and (TEG) together we have:

$$\frac{\dot{v}_f(\omega,t)}{v_f(\omega,t)} = \frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu I_f(\omega,t), \tag{21}$$

and therefore

$$\frac{\pi N(t)}{\rho + I_f(\omega,t)(1-\mu)} = (1-\phi) b w_H^A X(\omega,t). \tag{22}$$

From now on we will assume $\mu \in]0,1[$.

Solving (22) for $I_f(\omega,t)$ yields:

$$I_f(\omega,t) = \left[\frac{A\pi N(t)}{(1-\phi)\, b w_H^A X(\omega,t)} - \rho\right] \frac{1}{1-\mu}. \tag{23}$$

Therefore eq. (23) gives at all dates the value of country B punishers' investment sufficient to dissuade the entry in sector ω by country A R&D firms. I will prove next that this threat will in turn be credible.

Eq. (23) yields the following law of motion of sector ω's R&D difficulty index:

$$\dot{X}(\omega,t) = \left[\frac{\pi N(t)}{(1-\phi)\, b w_H^A} - \rho X(\omega,t)\right] \frac{\mu}{1-\mu}$$

for all $t \in R_+ : j(\omega,t) = j(\omega,t_0) + 1$. In per-capita terms:

$$\dot{x}(\omega,t) = \frac{\mu}{1-\mu} \frac{1}{(1-\phi)\, b w_H^A} \pi - \frac{(\rho-n)\mu + n}{1-\mu} x(\omega,t). \tag{24}$$

Since $\rho > n$ by assumption, this law of motion of the population-adjusted difficulty index of the sector that unilaterally deviated from the equilibrium path implies monotonic convergence to

$$x(\omega) \equiv \frac{\mu\pi}{(1-\phi)\, b w_H^A \left[(\rho-n)\mu + n\right]},$$

which is a finite value above the steady state value, x^A, of population adjusted difficulty indexes.

Given R&D investment (23), the corresponding Poisson process implies that with probability one, at some finite date a new quality jump will occur that will be appropriated by country B's R&D firms. In order for punishers'

R&D investment $I_f(\omega, t)$ given by (23) to be part of a best response by country B R&D firms, each country B R&D investor shall not expect additional gains or losses from investing in sector ω, that is the usual R&D arbitrage condition must hold for the further future R&D activity. Hence:

$$v(\omega, t) = \frac{\pi N(t)}{\rho + I(\omega, t)(1-\mu)} = bw_H^B X(\omega, t), \tag{25}$$
$$\text{for all } t \in R_+ : j(\omega, t) \geq j(\omega, t_0) + 2.$$

This in turn implies a requirement on the expected future value of $I(\omega, t)$ after the following quality jump[13] in sector ω. Consistently, we set:

$$\text{for all } t \in R_+ : j(\omega, t) \geq j(\omega, t_0) + 2$$

$$I(\omega, t) = \left[\frac{\pi N(t)}{bw_H^B X(\omega, t)} - \rho\right] \frac{1}{1-\mu} \tag{26}$$

and

$$\dot{X}(\omega, t) = \left[\frac{1}{bw_H^B} \pi N(t) - \rho X(\omega, t)\right] \frac{\mu}{1-\mu}.$$

In per-capita terms:

$$\dot{x}(\omega, t) = \frac{\mu}{1-\mu} \frac{1}{bw_H^B} \pi - \frac{(\rho - n)\mu + n}{1-\mu} x(\omega, t). \tag{27}$$

Since $\rho > n$ by assumption, after the end of the - previous - "punishment phase", the deviated sector returns an ordinary ξ_B sector, and this law of motion of the population-adjusted difficulty index of the sector that unilaterally deviated from the equilibrium path implies monotonic convergence to

$$x^B = \frac{\mu\pi}{bw_H^B \left[(\rho - n)\mu + n\right]}.$$

To sum up, this section has shown that the asymmetric equilibria previously described do not entail any deviation from agent optimizing behavior in their actions and in their beliefs. Moreover, even for off-equilibrium unilateral deviations from the steady state, the implied transitional dynamics has been treated.

[13] Notice that I have now dropped deponent f, to denote the obsolescence expected for dates after the end of the punishment phase.

3.5.2 The PEG Case

In this case, eq. (PEG) dictates $X(\omega,t) \equiv \frac{k}{2}N(t)$. Therefore the evolution of the difficulty index is independent from the system being in the steady state. However, even evaluating hypothetical deviations from the equilibrium of our game from a steady state standpoint, the resulting off-equilibrium punishment in any given sector implies that in the sector the flow of R&D investment - and the resulting probability intensity of the Poisson process for innovation - will differ from its steady state level during the stochastic period of punishment following a detected deviation[14]. Therefore we will have to consider both the exact punishment actions (over-investment in R&D) and the requirement on expected future (after the end of the punishment period) investment needed to make punishment worthwhile. Hence, we will need to compute the counterparts of eq.s (23) and (26). This will be the main task of the subsection.

Putting eq.s (20) and (PEG) together we have:

$$\frac{\dot{v}_f(\omega,t)}{v_f(\omega,t)} = \frac{\dot{X}(\omega,t)}{X(\omega,t)} = n, \tag{28}$$

and therefore

$$\frac{\pi N(t)}{\rho + I_f(\omega,t) - n} = (1-\phi) b w_H^A \frac{2kN(t)}{A}. \tag{29}$$

Solving (22) for $I_f(\omega,t)$ yields:

$$I_f(\omega,t) = \left[\frac{2\pi}{(1-\phi)\, b w_H^A k} - \rho\right] \frac{1}{1-\mu}. \tag{30}$$

Therefore eq. (23) gives at all dates the value of country B punishers' investment sufficient to dissuade the entry in sector ω by country A R&D firms. I will prove next that this threat will in turn be credible.

Given R&D investment (30), the corresponding Poisson process implies that with probability one, at some finite date there will be a new quality

[14] Notice that in all our equilibria, deviations are assumed to be detected after a stochastically long period needed to observe their effect (innovation by the new leader), i.e. only in case the deviator is successful in its R&D activity. This does not require more information structure than usually assumed in quality ladders models, as Dinopoulos and Segerstrom's (1999) model.

jump appropriated by country B's R&D firms. In order for punishers' R&D investment $I_f(\omega,t)$ given by (30) to be part of a best response by country B R&D firms, each country B R&D investor shall not expect additional gains or losses from investing in sector ω, that is the usual R&D arbitrage condition must hold for the further future R&D activity. Hence:

$$v(\omega,t) = \frac{\pi N(t)}{\rho + I(\omega,t)(1-\mu)} = bw_H^B \frac{kN(t)}{2}, \qquad (31)$$
$$\text{for all } t \in R_+ : j(\omega,t) \geq j(\omega,t_0) + 2.$$

This in turn implies a requirement on the expected future value of $I(\omega,t)$ after the following quality jump[15] in sector ω. Consistently, we set:

$$\text{for all } t \in R_+ : j(\omega,t) \geq j(\omega,t_0) + 2$$

$$I(\omega,t) = \left[\frac{2\pi}{bw_H^B k} - \rho\right]\frac{1}{1-\mu} \qquad (32)$$

To sum up, this section has shown that the previously described asymmetric equilibria do not entail any deviation from agent optimizing behavior in their actions and in their beliefs.

It is interesting to note that the previous derivation did not hinge on symmetry. We could have heterogenous profits ($\pi(\omega)$s) and difficulties ($k(\omega)$s) and yet use the previous formulas. Viceversa, exploiting the structurally symmetric fundamentals, we can view the same derivation in terms of value functions:

1. At any instant $\tau > 0$ *after* a first successful deviation from the equilibrium path in industry $\omega \in \xi_B$ the punishment of the country A's deviator is worth pursuing because the contingent future is rationally expected according to eq. (32) - predicting $I(\omega,s) = I(\omega'',s)$, $\omega,\omega'' \in \xi_B$, for all $s \geq \min\{t : j^{\max}(\omega,t) = j^{\max}(\omega,\tau)+1\}$ - and eq. (31), predicting $v(\omega,s) = v(\omega'',s)$, for all $s \geq \min\{t : j^{\max}(\omega,t) = j^{\max}(\omega,\tau)+1\}$.

2. At any instant $\tau > 0$ after a first successful deviation from the equilibrium path in industry $\omega \in \xi_B$, ϕ-parameterized punishment (30), though harsh - $I_f(\omega,\tau) > I(\omega',\tau)$, $\omega' \in \xi_A$ - is feasible because each sector's R&D is

[15] As in the previous subsection, I have now dropped deponent f, to denote the obsolescence expected for dates after the end of the punishment phase.

a zero measure subset of the mass of active sectors ξ_B and hence $I(\omega, \tau)$ can be made arbitrarily larger without affecting total country B's R&D $\int_{\omega'' \in \xi_B} .I(\omega'', \tau) d\omega''$.

3. Expecting the just described response, at any instant $\tau \geq 0$ *before* a first successful deviation from the equilibrium path in industry $\omega \in \xi_B$ firms in country A always know that in case of their success in R&D in industry ω it will follow: $v(\omega, s) < v(\omega'', s)$, for all $s \geq \min\{t : j^{\max}(\omega, t) = j^{\max}(\omega, \tau) + 1\}$.

3.6 Sunspot Equilibrium Interpretation

It is important to clarify that the equilibrium strategies used in this paper are not those of a game of entry deterrence: they belong to a special case of sunspot equilibrium.

In each perfectly competitive R&D market economic agents take the price vector as given. Each firm is assumed to be so small that changes or deviations in its individual choices do not affect prices[16]. In the present paper, one may puzzle by thinking that we have negligible and competitive R&D firms that nonetheless expect that their individual choices affect the choices of other negligible and competitive firms. In particular, if they do R&D in a "foreign sector" they will be punished by foreign firms. How could this be? If the R&D market is competitive, the free-entry conditions are fine as stated but out-of-equilibrium beliefs should play no role in any argument since R&D firms take prices and length of monopoly power as given[17]. In order to fully understand the multiple-equilibria argument in our competitive R&D setting, the reader shall realize that in our stylized economy there

[16] In a Nash equilibrium individuals take the actions of other individuals as given. Each individual is assumed to think that changes in his/her individual choices do not affect the actions of other players. This seems a reasonable assumption for games with a large number of players, but not for games with a small number of players. In the latter context, it seems more reasonable to assume that individuals think that changes in their choices will elicit changes in the choices of other agents. After all, individuals are large in the sense that changes in their individual choices affect the environment faced by other agents. In such a context, beliefs about what these further changes are become crucial and this is exactly what concepts of equilibrium such as subgame perfection focus upon.

[17] If the R&D market were oligopolistic, then there might be a role for out-of equilibrium beliefs in an argument but the free-entry conditions in the paper are no longer relevant, which is not the market structure we have in mind. In fact it is not acceptable to assume that firms are in fact small and yet they believe to be large in the sense that their individual choices elicit some response by other firms.

are two different sets of prices, lengths of market power and international allocations of R&D such that firms maximize their values taking those prices as given. Actually, the previous sections have shown that there are two different sets of prices, lengths of market power and international allocations of R&D such that firms maximize their values assuming that other firms react in some specific way to changes in the national primacy in a sub-set of sectors. Without complicating notation even more, let us just notice that we could define a continuous time sunspot variable, $\sigma(\omega, t)$, "nationality at time $t \geq 0$ of the patent holder in any sector $\omega \in [0, 1]$" that take on values A and B. Strategies would be contingent on the realisations of $\sigma(\cdot, t)$, prescribing that the mass of R&D firms in countries A and B react differently to each $\{\sigma(\omega, t)\}_{\omega \in [0,1]}$ according to their equilibrium decision rules.

Unlike in a game of entry deterrence, here the R&D firms of each country only react to a public signal: the nationality of the patent race winner in any given sector. This public signal is distributed according to a Poisson process whose intensity depends on the R&D expenditures in each sector. If a country A firm undertakes positive R&D in a sector $\omega \in \xi_B$ it may even alter the probability distribution of the ω's component of the sunspot variable by a vanishing small amount. However, its victory in the patent race - which will occur randomly - would be associated with a change in the value of the public signal that would trigger R&D firms B's disproportionate investment in sector ω. Country B R&D firms are equally better off "overinvesting" in sector ω because the R&D arbitrage condition equalizes their expected returns accross sectors. Therefore any country A firm, when undertaking R&D in sector $\omega \in \xi_B$ is buying an asset whose returns are perfectly correlated with the off-equilibrium-path sunspot signal. Therefore the value of the amount it will expect to gain in case it is successful is the value of the patent associated with high country B R&D investment.

To sum up, the potentially deviating country A R&D firm would not find it convenient to undertake R&D in sector $\omega \in \xi_B$ not because it expects to alter substantially[18] the probability of triggering a punishment, but because it expects to be trying to appropriate a patent whose value is the stock market price of a new monopoly facing an abnormally fierce patent race in sector $\omega \in \xi_B$. The prospective "punishers" only respond to a public signal incorporating sunspot value $\sigma(\omega, t) = A$. Hence, R&D firm A's success is

[18] There may be more firms attempting to deviate from the equilibrium and the chance of each of them of winning the race can be negligible anyway.

associated with public signal - sunspot value - $\sigma(\omega, t) = A$, which imposes low patent values in sector ω.

Atomless R&D industries consist of a density of competitive firms which take sunspot contingent prices as given and formulate sunspot contingent strategies. The probability of sunspot variables taking on different values in this model are endogenous and depend on the national composition of the mass of R&D expenditures undertaken by the zero measure R&D firms in perfectly competitive R&D industries. From the definition of atomless industries, each single country A R&D firm investing in a sector $\omega \in \xi_B$ at time t cannot modify the probability of $\sigma(\omega, t') = A$ for any $t' \geq t$, nor can it modify the industry's probability intensity of the next innovation - determined by the total mass of country B's R&D firms in the industry. It only determines the amount of state contingent patent values it buys, but neither their price nor the probability of the events on which payoff is contingent.

4 Intuition and Implications

The economic intuition for Proposition 1 and 2 is simple. Due to discriminating social norms country A's R&D specializes in a larger number of sectors than country B's. Human capital will be larger in country A because profits are equalized worldwide due to the presence of multinational manufacturing firms and interest rates are the same: therefore perfect competition in the R&D sector requires a higher skilled wage in order to compensate for the lower obsolescence per-sector. Higher country A's skill premium does not suffice to equalize obsolescence rates because the average ability of the researchers lowers as more people are induced to educate themselves. In the long run TEG specification of technology implies that the difficulty index grows less for products invented in country A. Under PEG specification the difficulty index remains the same. In both cases the average quality of the R&D workers in country A remains lower than in country B, but their increased mass suffices to produce a larger aggregate flow of innovations per unit time. Higher functional inequality in country A may seem responsible for the larger proportion of educated population and for the better innovative performance, however the ultimate source of cross country differences is the different range of sectors targeted by R&D.

As common in quality ladders models, the free entry condition into research in each industry implies R&D investors' indifference across sectors, and therefore it does not suffice to determine the overall level of investment in research that will occur. Given that expected profits return to normal for the domestic firms after they have punished a foreign entrant, it is rational for the domestic firms to "over-invest" in R&D during the punishment phase. Nothing in the model requires higher returns during the punishing phase, so the reader can immediately see what would justify higher levels of domestic investment in R&D at that time. Notice that social norms exempt the government from providing economic incentives to the firms to undertake additional R&D: this kind of social norms can be described as rational.

Asymmetric international specialization in innovation could be interpreted as exogenous with no change in our formal results. Hence this paper's conclusions do not necessarily depend on the indeterminacy of equilibria. However it has shown that this cause of cross-country growth performance and income inequality could be generated by self-enforcing social norms whose possibility is disclosed by the perfect functioning of R&D and capital markets. As in every case of multiplicity of equilibria the natural interpretation is the necessity to look also at extra-economic factors to completely explain diverging economic performances. Hence what is emphasized is the possibility that in a Schumpeterian world variables publicly affecting market psychology - mass media science and technology histories, R&D promotion declarations, organizations' cultures and rituals, geopolitical debates, etc. - may induce economic agents to coordinate their expectations on a particular equilibrium thereby having real effects.

It is natural for the reader to wonder if the symmetric steady state has some additional virtues that make it more likely to be obtained. In particular it would be desirable to have a complete transitional dynamics analysis. Unfortunately, as Dinopoulos and Segerstrom (1999) remark, out of the steady state the model would be "intractable". However, the structure of the model allows a brief heuristic discussion of the stability of each steady state characterized in the previous sections. For example, let us imagine that after having been following the symmetric steady state expectations unexpectedly changed in the way consistent with one of our asymmetric steady states. Given the existing equal levels of human capital in both countries, country A will dilute it over a larger mass of sectors than country B. Given the same level of the difficulty indexes in all sectors and in both countries, the demand for skilled labor will increase in country A and it will decrease in country B. Given the assumed schooling technology the human capital levels will remain constant for a period of length T implying that the real wage of human capital will increase in country A and decrease in country B, whereas of course unskilled wage will remain equal due to the presence of multinational final good firms in each sector. Responding to the wage premium incentives more people of lower ability to learn will gradually start educating themselves in country A whereas fewer people in country B will get skilled. After T time units the human capital level will start increasing in country A and decreasing in country B, thereby pushing real wages in the opposite directions, though not to their original values (associated with the initial stocks of human capital).

Hence, intuitively, we expect inequality in country A to increase first and then to partially decrease, and the reverse to occur in country B. Country A's larger number of active R&D sectors implies fewer people undertaking R&D. In the PEG specification of technology we expect this process to be continuing pushing the system in the direction of the new steady state.

Under the TEG specification an additional set of state variables should be considered. In fact, in country A the dynamic decreasing returns to R&D will set in at a lower speed relative to population growth, and hence population-adjusted complexity indexes will be increasing at a lower speed than in every research line active in country B [19]. In turn less complexity of R&D in country A continues to spur R&D and education. Higher R&D employment in country A than in country B seems therefore likely both in the medium and in the long run. Lower complexity also will tend to allow a smaller mass of human capital per sector to carry out the same number of inventions as do the more concentrated research mass in country B. Finally, the long run per-capita innovation rates in each country will tend to be proportional to the number of active industries, because of TEG. However due to the inefficient allocation of human capital across countries the per-capita level of technology worldwide - indexed by $x^A m(\xi_A) + x^B m(\xi_B)$ - will be asymptotically lower than in the symmetric steady state.

Our main results do not strictly hinge on Dinopoulos and Segerstrom's (1999) assumption of endogenous skill formation. If we made the assumption that the endowments of skilled and unskilled labor were independent of the

[19] In fact, solving differential equation (TEG) implies that $\frac{X}{N} \rightarrow \frac{\mu s}{N_0}$, where s is the share - constant in the steady state - of the labor force that gets skilled. Therefore a higher s implies no effect on the growth rate of the population-adjusted difficulty index but a positive level effect on its steady state amount.

relative wage of skilled labor the skill premium would likely remain higher in country A, as there would be no variation in the proportion of the population that gets skilled. The different ratios of human capital per-sector would affect the international skilled wage differentials and - in the TEG case - the complexity indexes evolution.

If labor was imperfectly mobile, the main results of endogenous asymmetric speciation would still hold. In particular, at however small migration cost, the unskilled labor would not migrate because the unskilled wage is the same in both countries due to the presence of multinationals. Instead, the different skill premium would induce some country B workers to migrate to country A, until the current value of the differential wage flow is equal to the migration cost. As a consequence, despite a lower cross-country difference in the skill premium, country A's larger fraction of the labor force that is skilled would be even larger, with amplified consequences on the innovation rates and the difficulty indexes. Scenarios with lower mobility frictions would lead us to asymmetric equilibria in which the wage inequality asymmetries are weaker and quantity asymmetries are stronger, whereas the inefficient use of human capital would be less sever.

A few theoretical considerations are in order. In this paper I have used the word "sunspot" equilibria to refer to public signals that trigger the "punishing" behavior by R&D investors depending on the nationality of the innovators. Like in a correlated equilibrium (Mas-Colell et al., 1995, p. 252), the strategies of the agents prescribe actions as functions of a common public signal. Notice that, though this economy admits more Nash equilibria, the sunspot is not used to coordinate agents to a particular equilibrium: in each equilibrium the sunspot basically "signals" to all agents what other agents shall do. Like in a game of entry deterrence, out-of-equilibrium beliefs on "punishments" are crucial to the argument, but in our general equilibrium framework with perfectly competitive R&D the punishment is not necessarily carried out by the challenged incumbent, but by a multitude of other domestic R&D investors whose actions are taken after observing the realization of the public signal. Out-of-equilibrium beliefs play an important role in the functioning of each sunspot equilibrium, because they contemplate the expectation that the sunspot will signal to all R&D investors the loss of an innovative sector and the time to invest more intensively there. The range of sectors on which the agents believe the public signal - i.e. the sunspot - will trigger the punishers' response to a deviation differs in each of our equilibria. For this reason, what we have here is a multiplicity of sunspot equilibria.

5 Final Remarks

This paper has shown that in a Schumpeterian world though national innovative performance and international trade may seem to be correlated with both technologies and endowments a major role can be played by social norms that are consistent with perfectly rational individualistic behavior. According to this theoretical approach, national systems of innovations can be interpreted as collective coordination devises of large masses of economic resources - such as labor and savings - on the enforcement of an equilibrium in which countries acquire persistent technological excellence in particular industries. The role of policies as coordination devices - "sunspots" - is emphasized in a technologically competitive international environment.

This paper does not deny realistic path dependence affecting fundamental variables such as firm-specific tacit knowledge, corporate technological trajectories, etc. However, it emphasizes that in a world where ideas are increasingly codified and communicated, the education industry is increasingly standardized to the best practices available, and goods and capital are increasingly mobile, a fundamental role of the international constellation of the national actors of innovations may be strategic.

Propositions 1 and 2 proved that in our stylized two-country world economy there exist a continuum of steady states in which one country (country A) maintains a leadership in a larger set of final products and has a more educated labor force than the other country (country B). This kind of indeterminacy differs from Benhabib and Perli (1994) and related literature (see Benhabib and Farmer 1999 for a review) because it hinges on the basic insight of Schumpeterian "creative destruction": the similarity between the forward looking character of R&D and that of the overlapping generation models with money (as Samuelson 1958 and Shell 1971).

It is worthwhile to remark that, as shown in section 3.1.1, our sunspot equilibria do not dictate that countries cannot simultaneously carry out R&D in the same industries, but only that countries do not carry out R&D *exactly* in the same number of industries. If countries or continents differ - no matter how little - in the number of sectors in which they innovate our qualitative analysis applies. To keep the results more transparent I have focussed on the extreme case of full specialization.

Moreover, though we concentrated on steady states, our analysis does not imply that from time to time countries cannot change their social norms.

For example, the government and the media in the US in the '80s and in the 90s signalled a major intention to gain competitive supremacy in the high technology sectors, thereby implicitly or explicitly stimulating a surge in private R&D, not matched by a similar increase in public R&D expenditures. Along with other factors, I suggest that also a change in social norms may have affected the real economy. The simultaneous increase in the skill premium and in education attainments seem to give indirect support to this interpretation.

A few decades earlier, the role of nationally oriented social norms in the break of US and European supremacy in the car industry by the firms of Japan and Korean[20] can have been non-negligible.

Somewhat similarly, the recent European enthusiasm for the advent of the euro and the attempts to coordinate efforts at the continental level may improve the interpretation of some recent high technology successes by the old continent: for example, twenty years ago, the civil aviation industry was dominated by the US firms, while the now leader Airbus was an industry disaster.[21].

To summarize the consequences of the self-fulfilling international division of innovation, let us remind that in every asymmetric steady state the more R&D diversified country, A, is characterized by a larger fraction of the population employed in the R&D sector, more wage inequality, same manufacturing profits, better stock market performance of its manufacturing firms[22], same unskilled labor wage and a larger set of industries that are undertaking R&D.

These characteristics do not depend on the effect of growth on the complexity of R&D. Hence both TEG and PEG yield the same predictions for the variables described above. However their predictions diverge in the fol-

[20] Where there were virtually no domestically made cars until after the Korean war.

[21] This is an example where governments provided huge subsidies and so does not really follow our example of a pure coordination issue. It is a case where both countries decided to defend that particular industry, by mobilizing both private and public R&D resources. However, it has been quoted to emphasize an effect of the old continent's equilibrium change - in the direction of a more European oriented social norm - on an industrial sector at least partially consistent with the predictions of our model.

It is intriguing to note that, by introducing some degree of public R&D in our hypothetical economy, as well as the presence of overlapping sectors (the non-partition case), it would in principle be possible to explicitly account for such a case.

[22] Due to lower sectoral obsolescence, permanent in the PEG case and at least temporary in the TEG case.

lowing aspects of the asymmetric steady state. Under the TEG specification, per-capita innovation rate is higher in country A, per sector innovation rate is the same in both countries, and R&D is less difficult in country A and hence its fewer R&D workers per sector are more productive. Under the PEG specification, per-capita innovation rate is higher in country A, per sector innovation rate is higher in country B, R&D is equally difficult in both countries.

Therefore both specifications predict that more industrial diversification generates a higher skill premium, more educated workers, less intense but more spread R&D, more per-capita innovation and growth in the long run. Hence both predict a positive cross country relationship between education and growth (as in Barro and Sala-I-Martin 1995) and a positive cross-country correlation between inequality and growth (as found by Forbes 2000).

According to TEG, the world steady state utility growth rate does not depend on the distribution of education and R&D activities across countries, but there is a negative worldwide level effect of the asymmetry. PEG predicts that as a result of a self-fulfilling inter-country inequality (asymmetry) the country with the higher inequality is innovating more than the more equal country, but the growth rate of individual utility worldwide is strictly lower than in the symmetric equilibrium. Hence, this paper suggests some care in interpreting a positive cross-country inequality-growth relationship as an indicator that inequality is good for growth: the consideration of endogenous inter- and intra-country inequality can reverse the judgement from mere cross-country data. Therefore this theoretical exercise has identified at the same time a new source of potential inefficiency in a Schumpeterian world economy and a new negative relationship between inequality and growth.

Our assumption of immobile labor certainly played a role in the derivation of international wage differentials. It would be interesting to link the channel highlighted by this paper with the new economic geographical models on (at least) this important aspect. However, considering imperfectly mobile skilled workers would probably reinforce the asymmetric result, but certainly not the decisive "divergence" force. Indeed, in the US case for example, there has been a deteriorating skill composition of the immigrants flow. Roughly the same skill composition as the native workers in the 1970s, and a significantly higher mean educational attainment for the latter in 1990: 13.2 versus 11.6 years (Borjas, 1994).

In a sense our efficient or inefficient use of human capital argument depends on the number of high ability individuals in a country, so some notion of scale does appear to matter here. The worldwide equilibrium conditions imply that domestic R&D be large enough. If scale were completely irrelevant then it would seem that within individual European countries there be greater cohesion than within the US. Only when looking at Europe as a whole would this argument for less cohesion seem relevant. So our choice of comparing the US to Europe rather than the US to Germany is important. It would be interesting to extend this theory to the case of heterogeneously sized countries.

6 Appendix

Proof of Proposition 1. Eq.s (14)-(15) is a second order equation system in θ_0^A, θ_0^B, x^A, and x^B. Solving the two equations in (15) for x^A, and x^B, and then substituting into the two equations in (14) yields the following two equation system:

$$\begin{aligned} \left(\theta_0^A+\theta_0^B\right)\left(\theta_0^B-\gamma\right) &= C_B\left(\theta_0^B+1-2\gamma\right)\left(1-\theta_0^B\right) \qquad (33)\\ \theta_0^B\left(\theta_0^A-\gamma\right) &= -\left(\theta_0^A\right)^2(1+C_A)+\theta_0^A\gamma\,(1+2C_A)+C_A(1-2\gamma),\\ \text{where } C_K &\equiv \frac{\sigma\left(\rho+n/\mu-n\right)\phi\mu}{2\left(\lambda-1\right)m(\xi_K)n},\ K=A,B. \end{aligned}$$

Since the right hand side of the first eq. (33) is a strictly concave quadratic polynomial with roots $-(1-2\gamma)$ and 1, there exists one and only one real and positive solution $\theta_0^B\in(\gamma,1)$ for each value of θ_0^A and it is a continuous function $\varphi\left(\theta_0^A\right)$. Since the right hand side of the second eq. (33) is a strictly concave quadratic with two roots, one negative and one larger than γ but less than 1, it follows that there exists one and only one real and positive solution $\theta_0^A\in(\gamma,1)$ for each value of θ_0^B, and it is a continuous function $\chi\left(\theta_0^B\right)$. Therefore

$$(\chi,\varphi):[\gamma,1]^2\to[\gamma,1]^2$$

admits a fixed point $\left(\theta_0^A,\theta_0^B\right)=\left(\chi\left(\theta_0^B\right),\varphi\left(\theta_0^A\right)\right)$ by Brouwer's theorem, which is in the interior of $[\gamma,1]^2$, as seen by direct substitution. Plugging this into (15) and solving for x^K gives the steady state vector $\theta_0^A\in(\gamma,1)$, $\theta_0^B\in(\gamma,1)$,

$x^A \in R_{++}$, $x^B \in R_{++}$. This derivation can be repeated for each partition of measures $m(\xi_K)$, $K = A, B$.

The comparative statics properties of such solutions are those mentioned in the statement of the Proposition, as easily checked. In fact, from eq.s (14) follows that $\frac{\theta_0^A-\gamma}{\theta_0^B-\gamma} = \frac{x^A}{x^B}$. Hence if $x^A \geq x^B$ then $\theta_0^A \geq \theta_0^B$. But then since the left hand sides of eq.s (15) are decreasing functions of θ_0^A and θ_0^B this would imply $x^A < x^B$- because $m(\xi_A) > m(\xi_B)$ - which is a contradiction. Hence properties a, b and c follow. Point d holds because the growth rate is $(m(\xi_A)n/\mu + m(\xi_B)n/\mu) \log \lambda = (n/\mu) \log \lambda$ as in the symmetric steady state. For point e notice that in any of the two symmetric steady states (our case of $m(\xi_A) = m(\xi_B) = 1/2$ and Dinopoulos and Segerstrom's case with $m(\xi_A) = m(\xi_B) = 1$) $\theta_0^A = \theta_0^B \equiv \widetilde{\theta}_0$ are the same, as immediately seen by direct substitution, after remembering to replace $bx^K m(\xi_K)n/2\mu$ for the right sides of eq. (15). Instead in all steady states in which $m(\xi_A) = 1 - m(\xi_B) > 1/2$ the total stock of human capital per-capita in our two country world is given by:

$$\frac{\sum_{K=A,B} H(\theta_0^K)}{N} \equiv \sum_{K=A,B} h(\theta_0^K) = \sum_{K=A,B} \left(\theta_0^A + \theta_0^B\right)\left(\theta_0^K - \gamma\right) cm(\xi_K),$$

where $c \equiv \frac{n(\lambda-1)}{\sigma(\rho+n/\mu-n)\mu}$. We know from a that $\theta_0^A < \theta_0^B$. If $\theta_0^A + \theta_0^B < 2\widetilde{\theta}_0$ then we can use the previous equation to write:

$$\begin{aligned} \sum_{K=A,B} h(\theta_0^K) &< 2\widetilde{\theta}_0 \left[\theta_0^A m(\xi_A) + \theta_0^B m(\xi_B) - \gamma\right] c \\ &< 2\widetilde{\theta}_0 \left[\theta_0^A m(\xi_A) + \left(2\widetilde{\theta}_0 - \theta_0^A\right) m(\xi_B) - \gamma\right] c \\ &\leq 2\widetilde{\theta}_0 \left(\widetilde{\theta}_0 - \gamma\right) c = 2h(\widetilde{\theta}_0). \end{aligned}$$

where in the second inequality we have used $\theta_0^B < 2\widetilde{\theta}_0 - \theta_0^A$ and in the third inequality $m(\xi_A) \equiv 1 - m(\xi_B)$ and $m(\xi_B) < 1/2$ were used.

If instead $\theta_0^A + \theta_0^B \geq 2\widetilde{\theta}_0$ then

$$\sum_{K=A,B} h(\theta_0^K) < 2hH(\widetilde{\theta}_0)$$

is a direct consequence of the strict concavity of the sum of the left sides of eq. (15). Q.E.D.

Proof of Proposition 2. Eq.s (16)-(17) is a second order equation system in θ_0^A, θ_0^B, x^A, and x^B. By substitution, they can be rewritten as:

$$\left(\theta_0^A + \theta_0^B\right)\left(\theta_0^K - \gamma\right) = \frac{bk\sigma}{2\left(\lambda - 1\right)}\left[\frac{\left(\theta_0^K + 1 - 2\gamma\right)\left(1 - \theta_0^K\right)\phi}{2bkm(\xi_K)} + \rho - n\right] \text{ , } K = A, B. \tag{34}$$

The right sides of (34) are strictly concave quadratic functions with one root smaller than γ and one larger than 1. Hence the existence of a positive solution vector $\theta_0^A \in (\gamma, 1)$, $\theta_0^B \in (\gamma, 1)$, $I^A \in R_{++}$, $I^B \in R_{++}$ for all partitions of measures $m(\xi_K)$, $K = A, B$ is guaranteed by Brouwer's theorem if the left sides of (34) are smaller than the value of their corresponding right sides when $\theta_0^K = 1$ and $\theta_0^K = \gamma$. Since the left sides of (34) are strictly increasing functions of θ_0^K and the right hand sides are decreasing uniqueness is guaranteed too. The comparative statics properties of each steady state are easily checked. In fact, from eq.s (16) follows that $\frac{\theta_0^A - \gamma}{\theta_0^B - \gamma} = \frac{\rho + I^A - n}{\rho + I^B - n}$. Hence if $I^A \geq I^B$ then $\theta_0^A \geq \theta_0^B$. But then since the left sides of eq.s 17) are decreasing functions of θ_0^A and θ_0^B this would imply $I^A < I^B$ - because $m(\xi_A) > m(\xi_B)$ - which is a contradiction. Hence the stated properties a and b follow In particular, eq. (2) is used to derive our results about wages.

For point c notice that in any of the two symmetric steady states (our case of $m(\xi_A) = m(\xi_B) = 1/2$ and Dinopoulos and Segerstrom's case with $m(\xi_A) = m(\xi_B) = 1$) $\theta_0^A = \theta_0^B \equiv \widetilde{\theta}_0$ are the same, as immediately seen by direct substitution, after remembering to replace I^K for $I^A + I^B$ in the right sides of eq. (16). Instead in all steady states in which $m(\xi_A) = 1 - m(\xi_B) > 1/2$ the total stock of human capital per-capita in our two country world is given from eq.s (4) and (34) by:

$$\sum_{K=A,B} h(\theta_0^K) = \sum_{K=A,B} \left[\left(\theta_0^A + \theta_0^B\right)\left(\theta_0^K - \gamma\right)c - d\right] m(\xi_K),$$

where $c \equiv \frac{2(\lambda-1)}{\sigma}$ and $d \equiv bk\left(\rho - n\right)$. We know from point a that $\theta_0^A < \theta_0^B$.

If $\theta_0^A + \theta_0^B < 2\widetilde{\theta}_0$ then we can use the previous equation to write:

$$\begin{aligned}\sum_{K=A,B} h(\theta_0^K) &< \sum_{K=A,B} \left[2\widetilde{\theta}_0 \left(\theta_0^K - \gamma\right) c - d\right] m(\xi_K) \\ &\leq 2\widetilde{\theta}_0 \theta_0^A cm(\xi_A) + 2\widetilde{\theta}_0 \left(2\widetilde{\theta}_0 - \theta_0^A\right) cm(\xi_B) - 2\widetilde{\theta}_0 \gamma - d \\ &\leq 2\widetilde{\theta}_0 \left(\widetilde{\theta}_0 - \gamma\right) c - d = 2h(\widetilde{\theta}_0).\end{aligned}$$

where in the second inequality we have used $\theta_0^B < 2\widetilde{\theta}_0 - \theta_0^A$. If instead $\theta_0^A + \theta_0^B \geq 2\widetilde{\theta}_0$ then

$$\sum_{K=A,B} h(\theta_0^K) < 2h(\widetilde{\theta}_0)$$

is a direct consequence of the strict concavity of the sum of the left sides of eq. (17). Q.E.D.

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