Promoting clean technologies under imperfect competition[§]

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Abstract

We develop a general equilibrium multi-sector vintage capital model with energy-saving technological progress and an explicit energy market to study the impact of investment subsidies on investment and output. Energy and capital are assumed to be complementary in the production process. New machines are less energy consuming and scrapping is endogenous. The intermediate inputs sector is modelled *à la* Dixit-Stiglitz (1977). Two polar market structures are considered for the energy market, free entry and natural monopoly. The impact of imperfect competition on the outcomes of the decentralized equilibria are deeply characterized. We identify an original paradox: adoption subsidies may induce a larger investment into cleaner technologies either under free entry or natural monopoly. However, larger diffusion rates do not necessarily mean lower energy consumption at equilibrium, which may explain certain empirical puzzles.

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Key words: Energy-saving technological progress; vintage capital; market imperfections; natural monopoly; investment subsidies

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1 Introduction

The move towards cleaner technologies has become one of the most important policy debates in the recent years. International agreements like the Kyoto Protocol have certainly influenced such a trend, along with the rising discussion on the broader concept of sustainable development (see for example, Arrow et al., 2004). Among possible policy tools to favor the switch to cleaner technologies (i.e. with lower polluting emissions), one can distinguish between quotas and pollution permits (nicely studied by Böhringer and Lange, 2005, for example), and fiscal policies. Fiscal policies include emission taxes designed to limit the use of dirty technologies, investment subsidies in new and cleaner technologies, and scrapping subsidies which favor the dismantlement of the oldest and most polluting techniques. This paper is concerned with fiscal policies designed to promote the switch to clean technologies.

Indeed, a major component of the ongoing debate is about how to save energy consumption, given that the latter is one of the most important sources of pollution. Whether a substantial part of the gains in energy efficiency are due or not to induced-innovation-like mechanisms is not the subject of this paper. Numerous papers have been already devoted to this issue (see for example Jaffe and Stavins, 1995). We are more concerned about the effectiveness of the fiscal instruments outlined above to effectively favor investment in the new and cleaner technologies, and about their impact on GDP. Under a given pace for energy-saving technical progress, do investment (in new capital goods) subsidies and/or scrappage subsidies have ultimately a positive impact on investment and output? This question is far from obvious in a general equilibrium framework where energy suppliers may also react to such policies. This paper highlights the crucial role of market structures in this respect, in particular the energy market.

To make things as realistic as possible, we shall consider a model with a vintage capital structure, newer machines being less energy consuming. Beside realism, there are at least three reasons to work on these models:

1. First of all, in such a setting technological progress is embodied in capital goods so that switching to cleaner technologies amounts to investing in new machines, implying that there is no need to distinguish between technology adoption and investment. In short, investment subsidies can be roughly interpreted as technology adoption subsidies without any additional specifications increasing the size of the model.

2. Second, a nice property of this kind of models (see in particular, Boucekkine et al., 1997 and 1998) is that an investment subsidy does also induce firms to shorten the lifetime of operating capital goods, therefore inducing scrapping of the less

profitable machines. Thus, within such a set-up, there is no need to distinguish between investment subsidies and scrapping subsidies.

3. Last but not least, another sensitive property of this class of models connects the optimal scrapping time with the cost (or price) of the production inputs. A machine or technology is thrown out once its profitability drops to zero, and of course profitability depends on the operation cost of the capital good involved (see the seminal Solow et al., 1966, Malcomson, 1975, and again Boucekkine et al., 1997). Therefore, the efficiency of investment subsidies should tightly depend on the price formation of inputs, like energy, that is on the market structure of the associated inputs.

Few papers have been devoted to analyze the environmental questions outlined above within a vintage structure, probably due to the mathematical sophistication implied by this structure (compared to the homogenous capital structure). Among them, Pérez Barahona and Zou (2006) and Bertinelli et al. (2008) are devoted to the analysis of long-term consequences of exogenous energy-saving technological progress, highlighting some non-standard implications of vintage models. Boucekkine et al. (2008) endogenized energy-saving technological progress under emission quotas. They showed in particular that tighter emission quotas are shown to not prevent firms to grow in the long-run, thanks to endogenous innovation, but they have an inverse effect on the growth rate of profits. In this paper, energy-saving technological progress is exogenously given as in the vast majority of related vintage capital models, but we depart from the standard perfect competition assumptions by introducing imperfect competition in two sectors, the intermediate inputs sector and specially the energy market.

It is nowadays widely admitted that imperfect competition (externalities, barriers, market power, etc.) may explain the observed energy-efficiency gap or the slow diffusion of energy-saving technologies, and that public intervention is a necessary condition for organizing the markets and promoting energy efficiency (see, e.g., Jaffe and Stavins 1994, Stoneman and Diederen 1994, Sutherland 1996, De Almeida 1998, and Brown, 2001). To get useful analytical results, we build on the Leontief vintage capital model popularized by Solow et al. (1966) with complementary inputs, energy and capital. In this framework, we analyze how investment subsidies impact equilibrium investment and output depending on the energy market structure and on the degree of imperfect competition in the intermediate input sector. To model the latter, we take the typical monopolistic competition framework à la Dixit-Stiglitz, and we show somehow straightforwardly how and why diffusion speed of clean technologies is effectively negatively correlated with market power in this sector. Actually, we shall even show that the more we depart from perfect competition in the intermediate inputs sector, the more unlikely balanced growth paths are likely to emerge!

Our main point is however on the energy market, which has received much less attention in the related literature.[1]To highlight the crucial importance of energy market structure in the performance of energy-saving technologies' subsidies, we consider two polar market structures for this market: perfect competition (free entry) and natural monopoly. These two cases are not only interesting for tractability but also to partially assess the recent restructuring and regulatory reforms that have targeted the energy sector, particularly electricity, in the USA and Europe towards more competition in energy markets to achieve a higher energy efficiency. Natural monopoly is a plausible assumption as energy markets generates enormous fixed costs and economies of scale. Water, electricity, and natural gas utilities are typically cited as examples of natural monopolies. In fact, recent deregulation policies observed in several countries (e.g. Argentina, England, New Zealand, Europe, the USA, and Japan) has aimed to encourage a competitive energy generation sector, energy transmission and distribution remaining close to a regulated monopoly situation (Joskow, 1997, Crampes and Moreaux, 2001).[2] Studying the two extreme cases pointed out above, while certainly insufficient to reflect the complexity of actual energy markets, sounds as a desirable benchmark analysis though.

In the environmental literature, the role of subsidies was analyzed in several studies. Based on US data regarding the adoption of thermal insulation technology in new home construction, Jaffe and Stavins (1995) found that technology adoption subsidies have positive effect on the energy efficiency of new homes.[3] De Groot et al. (2001) also observed for a survey of Dutch firms that cost savings are the most important driving force for investing in energy-saving technologies, which suggests an effective role of policy measures like subsidies and fiscal arrangements in promoting for higher energy efficiency. However, possible adverse effects of subsidies were also pointed out. For example, Verhoef and Nijkamp (2003) found in another heterogenous firms modeling that the promotion of energy-efficiency enhancing technologies by means of subsidies may be counter-productive because it could actually increase energy use. The authors also underlined that using energy taxes may reduce the attractiveness of energy-saving technologies. De Groot et al. (2002) suggested that investment subsidies for energy-saving technologies can be also counter-productive as they may favor a lock-in into relatively inferior technologies. Kemp (1997) found for the case of the Netherlands that there was no significant effect of government subsidies on the adoption of thermal insulation by households. Bjørner

[1]For example, Pérez Barahona and Zou (2006) assume an exogenously given energy supply.

[2]It would be also interesting to consider the sector as a network industry with a vertical integrated structure (production, transmission, and distribution) as underlined by, e.g., Tschirhart (1991) and Joskow (1997). Such a modelling would be rather complex and we prefer to postpone it in a further work.

[3]This result was also outlined by Hassett and Metcalf (1995).

and Jensen (2002) found in a panel of Danish industrial firms that subsidies in energy efficiency have no significant effect on energy use. They also found that energy taxes are less effective than voluntary agreements on energy use. Within our theoretical set-up, we shall show that a rise in investment subsidy will increase the price of energy in both cases but while it increases the quantity of energy under free entry, it pushes it down under monopoly. Given the complementarity between energy and capital, the latter effect may end up pushing investment level down in the latter case. Indeed, applied to the debate of promoting energy-saving technologies, our paper sheds light on an original paradox: adoption subsidies may induce a larger investment into cleaner technologies, and such a property can arise either under free entry or under natural monopoly. However, larger diffusion rates do not necessarily mean lower energy consumption at equilibrium, which may explain certain empirical puzzles mentioned juste above.

While the empirical studies provide such discrepant conclusions on the efficiency of investment subsidies in an energy-saving context, there is no paper -to our knowledge- tackling theoretically this issue within the natural vintage setting outlined above. This paper is an attempt to fill this gap while also incorporating market imperfections, and notably the energy market structure, into the discussion. More specifically, the paper contributes to the literature within a new and natural set-up in which the lifetime of capital goods and energy prices are tightly related via the scrapping conditions inherent to vintage models. Indeed, increasing the investment subsidy rate does not only give rise to the typical positive demand effect on investment, it will also launch a supply channel mechanism relying on the scrapping mechanism outlined just above, and which effect on investment depends on the market structure of the energy sector. Under a free entry structure for the energy sector, the latter effect is positive, thus reinforcing the former demand effect, and boosting investment. Under a natural monopoly structure for the energy sector, the supply effect is negative, and can eventually offset the positive demand effect, which is effectively arises under weak enough increasing returns in the production technology in the energy sector.

The paper is organized as follows. Section 2 presents the vintage model with energy-saving technical progress, where we explicitly model the energy sector either as a natural monopoly or a competitive firm with the free entry. Section 3 provides the balanced growth path where all endogenous variables growth at the same constant rate. Section 4 discusses the impacts of investment subsidies on the economy. Section 5 concludes the study.

2 A vintage capital model with energy-saving technical progress

Relying on Boucekkine et al. (1997), we build a decentralized vintage capital model with energy-saving technological progress where the energy sector is either governed by a natural monopoly or under free entry. This model has some salient characteristics. First of all, the production function is linear in vintage capital, following the traditional specification of Solow et al. (1966). Second, to guarantee the existence of a balanced growth path (see Solow et al., 1966, for an illuminating assessment of this question), we will assume that the successive vintages only differ in their (decreasing) energy requirement, and not in their productivity. Thirdly, growth is exogenous. We start by a detailed exposition of the structure of the model and its properties.

2.1 Individual's behavior

Let us assume that the representative household solves a maximization problem with nonlinear instantaneous utility function:

$$\max_{\{c(t),a(t)\}} \int_0^\infty u[c(t)]\, e^{-\rho t}\, \mathrm{d}t, \tag{1}$$

subject to the budget constraint

$$\dot{a}(t) = r(t)a(t) - c(t) - \tau(t),$$

with initial wealth a_0 given; $c(t)$ and $a(t)$ represent per capita consumption and per capita asset holden by household respectively. The interest rate $r(t)$ is taken as given by the household. $\tau(t)$ is per-capita lump-sum taxes. In the model, investment subsidies are entirely financed through this type of taxes. This is the simplest way to disentangle the role of the latter subsidies. For simplification, we shall consider a logarithmic utility function. This optimization problem is very standard, and the corresponding necessary conditions are: $\frac{\dot{c}}{c} = r(t) - \rho$, with $\lim_{t\to\infty} \phi(t)a(t) = 0$, where $\phi(t)$ is the co-state variable associated with the wealth accumulation equation.[4]

2.2 Final good

The final good is produced competitively and the representative final firm solves the following problem

$$\max_{\{y(t)\}} \left\{ y(t) - \int_0^1 p_j(t) y_j(t)\, \mathrm{d}j \right\} \tag{2}$$

[4] We shall abstract hereafter from the transversality conditions involved in the optimization work along the paper, and assume convergence to well-defined balanced growth paths granted. More mathematical literature about this specific issue can be found in Boucekkine et al. (1997, 1998).

where $p_j(t)$ is the market price of the intermediate input j, and the per-capita production $y(t)$ is given by a CES production technology

$$y(t) = \left(\int_0^1 y_j(t)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j \right)^{\frac{\epsilon}{1-\epsilon}} \tag{3}$$

defined over a continuum of inputs $y_j(t)$ with $j \in [0, 1]$. Prices are taken as given by the representative final firm, and elasticity of substitution is such that $\epsilon > 1$. As in the standard monopolistic competition economy (Dixit and Stiglitz, 1977), the corresponding inverse demand function takes the form

$$p_j(t) = \left(\frac{y_j(t)}{y(t)} \right)^{-\frac{1}{\epsilon}}$$

2.3 Input firm

We consider that the technological progress is embodied in the new capital goods acquired by the firm. In any intermediate good sector, there exists a unique monopolistic firm, which solves the problem:

$$\max_{\{p_j(t), y_j(t), \iota_j(t), T_j(t)\}} \int_0^\infty \left[p_j(t) y_j(t) - p_e(t) e_j(t) - (1 - s_q(t)) i_j(t) \right] R(t) \, \mathrm{d}t \tag{4}$$

subject to

$$\begin{aligned} y_j(t) &= b \int_{t-T_j(t)}^t i_j(z) \, \mathrm{d}z && (5) \\ e_j(t) &= \int_{t-T_j(t)}^t q(z) i_j(z) \, \mathrm{d}z && (6) \\ p_j(t) &= \left(\frac{y_j(t)}{y(t)} \right)^{-\frac{1}{\epsilon}} && (7) \\ q(t) &= e^{-\gamma t} && (8) \end{aligned}$$

with initial conditions $i_j(t)$ given $\forall \, t < 0$; $p_e(t)$, $e_j(t)$, and $s_q(t)$ denote energy price, energy consumption and subsidies devoted to the purchase of new equipment respectively. b is a fixed productivity parameters. Equation (5), (6) and (8) describe the technology used at the firm level in the input sector. The production function is Leontief, capital and energy are assumed to be gross complements. It is widely admitted that there exist at least a certain degree of complementarity between these two inputs, so that the Leontief technology used here is in the worse case a worthwhile benchmark case. Moreover, as explained in Boucekkine et al. (1997), such a complementarity is needed to have finite time scrapping at equilibrium. In particular, (6) gives total energy demand at the firm level, which depends on the energy requirements of all active machines. Recall that in this framework, technical progress is assumed to make machines (equipment) less energy-consuming over time. In equation (6) and (8), it is modeled via the variable $q(t)$: a machine of vintage z requires $q(t) = e^{-\gamma t}$ units of energy, $\gamma > 0$ is therefore the given rate of energy-saving technical chance. Finally, government subsidizes the acquisition of new machines via $s_q(t)$ following from taxes $\tau(t)$. For all $t \geq 0$, the tax variables and $p_e(t)$ are taken as given by the monopolist. The discount factor $R(t)$ takes the form:

$$R(t) = e^{-\int_0^t r(z)\,\mathrm{d}z}$$

Following Malcomson (1975), after changing the order of integration and applying some algebra, the problem can be rewritten as

$$\begin{aligned}
\max_{\{y_j(t), i_j(t), J_j(t)\}} \quad & \int_0^\infty \left[y(t)^{\frac{1}{\epsilon}} y_j(t)^{1-\frac{1}{\epsilon}} - \lambda_j(t) y_j(t) - (1 - s_q(t)) i_j(t) \right] R(t)\,\mathrm{d}t \\
+ \quad & \int_0^\infty i_j(t) \int_t^{t+J_j(t)} \left[b\lambda_j(z) - p_e(z) q(t) \right] R(z)\,\mathrm{d}z\,\mathrm{d}t \\
+ \quad & \int_{-T_j(t)}^0 i_j(t) \int_0^{t+J_j(t)} \left[b\lambda_j(z) - p_e(z) q(t) \right] R(z)\,\mathrm{d}z\,\mathrm{d}t
\end{aligned}$$

where $\lambda_j(t)$ denotes the shadow value of $y_j(t)$ and $J_j(t) = T_j(t + J_j(t))$. Notice that $T_j(t) = J_j(t - T_j(t))$. $J(t)$ is the optimal life of machines of vintage t. The first order conditions with respect to $y_j(t)$, $i_j(t)$ and $J_j(t)$ are respectively, $\forall\, t \geq 0$:

$$\begin{aligned}
\lambda_j(t) &= \left(1 - \frac{1}{\epsilon}\right) p_j(t) \\
R(t)(1 - s_q(t)) &= \int_t^{t+J_j(t)} \left[b\lambda_j(z) - p_e(z) q(t) \right] R(z)\,\mathrm{d}z \\
b\lambda_j(t + J_j(t)) &= p_e(t + J_j(t))\, q(t), \quad \forall\, t \geq -T_j(0)
\end{aligned}$$

At the symmetric equilibrium, $p_j(t) = 1$, $y_j(t) = y(t)$, $e_j(t) = e(t)$, $J_j(t) = J(t)$, $T_j(t) = T(t)$, $\lambda_j(t) = \lambda(t)$ and $i_j(t) = i$. In that case, $\forall\, t \geq 0$:

$$\begin{aligned}
\lambda(t) &= \left(1 - \frac{1}{\epsilon}\right) \equiv \mu \\
R(t)(1 - s_q(t)) &= \int_t^{t+J(t)} \left[b\mu - p_e(z)\, e^{-\gamma t} \right] R(z)\,\mathrm{d}z \\
b\mu &= p_e(t)\, e^{-\gamma(t - T(t))}
\end{aligned}$$

where now $q(t) = e^{-\gamma t}$ is explicitly replaced. Notice also that $0 < \mu < 1$, since $\epsilon > 1$. Notice that without imperfect competition, the shadow price $\lambda(t)$ would be equal to 1. The second equation gives the optimal investment rule equalizing the marginal cost of acquiring one unit of (new) capital goods at t and the marginal benefit which amounts to the actualized sum of net benefits over the expected lifetime of the acquired good (that is from t to $t + J(t)$). The last equation is the typical scrapping condition, mentioned repeatedly in the introduction section, it corresponds to the optimality condition with respect to $J(t)$, and can be rewritten as:

$$p_e(t) = b\,\mu\, e^{\gamma(t - T(t))}.$$

This is the counterpart of the classical scrapping condition in Leontief vintage capital models, with energy playing the role of labor in the early vintage models *à la* Solow et al. (1966) and imperfect competition (μ not equal to 1). The marginal value of energy, the price $p_e(t)$ at the decentralized equilibrium, should be equal to the marginal productivity of energy, here equal to $b\,\mu\, e^{\gamma(t-T(t))}$, where $e^{\gamma(t-T(t))}$ is the inverse of the energy requirement of the oldest vintage still in use at t. Therefore, as announced before, the scrapping condition induced by our vintage structures does connect tightly energy price with the optimal lifetime of machines. This connection is key in the main results produced in this paper.

2.4 Energy sector

In the energy sector, we assume that the production function only uses the final good according to:

$$f(h_t) = \left(\frac{h(t)}{A(t)}\right)^{\alpha}, \tag{9}$$

where $h(t)$ denotes the quantity of final goods devoted to energy production, and $A(t)$ is an exogenous variable intended to capture the difficulty or complexity to produce energy. Indeed, the specified production function implies that to produce one unit of energy, $A(t)$ units of the final good are needed, $A(t)$ could be therefore interpreted as a marginal cost. As it will be clear later, our model requires $A(t)$ to be growing over time (or energy to be increasingly difficult to produce) for a regular balanced growth path to arise. In this sense, our specification is close in spirit to the models incorporating complexity to guarantee balanced growth paths (like Segerstrom, 2000). The profit of a firm in the energy sector is:

$$\pi(t) = p_e(t) f(h(t)) - h(t) \tag{10}$$

where we remind that $p_e(t)$ denotes the energy price. We shall distinguish two market structures:

1. **The natural monopoly**: This is the case of decreasing average cost, typically implied by the existence of fixed costs. This structure is obtained when setting $\alpha > 1$. Hereafter we refer to it as the **NM** structure.

2. **Perfect competition**: This is the case of increasing average cost and free entry that is typically obtained under decreasing returns, $\alpha < 1$. We refer to it as the **FE** structure (FE for free entry).[5]

[5]We shall exclude the case $\alpha = 1$ in our study, it will be crystal clear in the next section that a balanced growth path cannot exist under this zero-measure parameterization.

In both cases, the pricing of energy will correspond to the zero profit condition:

$$p_e(t) = h(t)^{1-\alpha}\, A(t)^{\alpha}. \tag{11}$$

While the condition is the same in both cases, it does not cover the same kind of equilibrium concept. In the perfect competition case, it's simply the result of an underlying assumption of free entry. In the natural monopoly case, it corresponds to the well-known second-best Ramsey-Boiteux pricing (see, e.g., Sherman 1989, Carlton and Perloff 2005). This paper will show clearly that the economic implications of investment subsidies strongly depend on the market structure considered for the energy sector.

As briefly mentioned in the introduction, the two market structures considered here are partially or totally supported by numerous studies. For example, Crampes and Moreaux (2001) underlined that transmission and distribution of electricity have common features of natural monopoly while competition may work for generation. This observation is consistent with the empirical results of Christensen and Greene (1976) who found that the U.S. electric power generation sector was governed by scale economies in 1955 while almost all firms were operating in 1970 in the flat area of the average cost curve and a non trivial amount of electricity was generated by a firm with diseconomies of scale. However, in a recent study Hisnanick and Kymn (1999) reached a different conclusion: for them, increasing returns to scale are prevailing in US electric power companies for the period 1957-1987. In the case of Japan, Hosoe (2006) observed that natural monopoly prevails in the electricity industry except the generation sector where there is no definite (or very weak) evidence of scale economy. Burns and Weyman-Jones (1998) found that gas marketing and customer service costs of the British gas sector represent a constant returns to scale when domestic and non-domestic outputs (in terms of British Gas regions) rise by the same proportion. There were however economies of scale when one output is held fixed and the other is kept expending. Isoard and Soria (2001) found that the European emerging renewable energy sector (namely photovoltaic and wind technologies) has a decreasing returns to scale production (the coefficient of returns to scale ranged from 0.8 to 1) in the short run but would not diverge from a constant returns to scale production in the long run.

By studying the two polar market structures, we aim to exemplify some key economic mechanisms which are relevant in the performance of clean energies promotion policies, which would be otherwise hidden in a model with a more complex (realistic) picture for the energy market. To make things even simpler, we focus on the steady state decentralized equilibrium.

2.5 Decentralized equilibrium

From previous sections, the equilibrium of this economy is characterized by the following system, $\forall\, t \geq 0$:

$$
\begin{aligned}
\frac{\dot{c}}{c} &= r - \rho && (12)\\
y(t) &= b \int_{t-T(t)}^{t} i(z)\, \mathrm{d}z && (13)\\
R(t)(1 - s_q(t)) &= \int_{t}^{t+J(t)} \left[b\mu - p_e(z)\, e^{-\gamma t}\right] R(z)\, \mathrm{d}z && (14)\\
b\mu &= p_e(t)\, e^{-\gamma(t-T(t))} && (15)\\
f(h(t)) &= \int_{t-T(t)}^{t} i(z)\, e^{-\gamma z}\, \mathrm{d}z && (16)\\
y(t) &= i(t) + c(t) + h(t) + \tau(t) && (17)\\
J(t) &= T(t + J(t)) && (18)
\end{aligned}
$$

with initial conditions $i(t)$, $\forall\, t \leq 0$ given. Equation (16) represents the equilibrium in the energy market where here $f(h(t))$ denotes the energy supply and where the parameter γ represents (Harrod neutral) technical progress. Equation (17) represents the equilibrium in the goods market. All others equations were previously derived from agents' problems. Equations (12)-(18) allow us to solve the the endogenous variables $y(t)$, $c(t)$, $r(t)$, $i(t)$, $J(t)$, $T(t)$ and $p_e(t)$ given the exogenous technological process.

3 Balanced growth paths

Let us define the environment for balanced growth path (BGP). We assume that at the stationary equilibrium, $c(t) = c\, e^{\gamma t}$, $p_e(t) = p_e\, e^{\gamma t}$, $y(t) = y\, e^{\gamma t}$, $i(t) = i\, e^{\gamma t}$. Accordingly, we set $\tau(t) = \tau\ e^{\gamma t}$ and $A(t) = A\, e^{\gamma t}$, for the BGP to exist.

Definition.- *The BGP equilibrium is a situation where all endogenous variables growth at the same constant rate γ except $J(t) = T(t) = T$.*

We obtain:

$$\begin{aligned}
r &= \gamma + \rho && (19)\\
y &= c + i + h + \tau && (20)\\
y &= b\frac{i}{\gamma}(1 - e^{-\gamma T}) && (21)\\
\frac{1 - s_q}{b\mu} &= \int_t^{t+T} \left[1 - e^{\gamma(z-T)}\, e^{-\gamma t}\right] e^{-r(z-t)}\,\mathrm{d}z && (22)\\
p_e &= b\mu\, e^{-\gamma T} && (23)\\
\int_{t-T}^{t} i(z) e^{-\gamma z}\,\mathrm{d}z &= \left(\frac{h}{A}\right)^{\alpha}, \quad \text{and then } iT = \left(\frac{h}{A}\right)^{\alpha} && (24)\\
p_e &= h^{1-\alpha}\, A^{\alpha} && (25)
\end{aligned}$$

Finally, setting $u = z - t$ we can compute the stationary value for the scrapping age:

$$\frac{1 - s_q}{b\mu} = \int_0^T \left[1 - e^{-\gamma(T-u)}\right] e^{-(\gamma+\rho)u}\,\mathrm{d}u \equiv F(T, \gamma, \rho) \tag{26}$$

which defines function $F(T, \gamma, \rho)$. This integral function can also be rewritten as

$$F(T, \gamma, \rho) = \int_0^T \int_\tau^T \gamma \exp\{-\rho z - \gamma\sigma\}\,\mathrm{d}\sigma\,\mathrm{d}u \tag{27}$$

Along the balanced growth path, the optimal investment rule simplifies to (26). In particular, $F(T, \gamma, \rho)$ provides a measure of the marginal return from investment in the long run. Using (27), we can derive the necessary and sufficient conditions for a balanced growth path (defined above) to exist. Indeed, the stationary system above has a clear recursive structure. This nice configuration is mainly due to the Leontief technology used by the intermediate inputs producers.[6] Once T computed, all the other unknowns can be recovered immediately from the system (19)-(24). For example, equilibrium energy price level can be recovered from (23) given T, and once this price computed, one can use equation (25) to calculate the long-term energy sector input h. And so on. The existence of a long run scrapping age along a balanced growth path is settled in the next proposition.

Proposition 1 *A balanced growth path (BGP) exists if and only if $\rho + \gamma < \frac{b\mu}{1-s_q}$. If γ tends to zero, T tends to infinity. If μ tends to zero, no BGP can exist.*

Proof. Proposition 1 states a necessary and sufficient condition for a unique long-run (positive) scrapping value T to exist, that is such that $F(T,.) = \frac{1-s_q}{b\mu}$. Indeed, by (27), $F(T,.)$ is strictly increasing in T. It should be noticed that $F(T,.)$ is the integral value of a positive function for which the integration support increases with T. Since $F(0, \gamma, \rho) = 0$,

[6] Removing this specification breaks down recursivity and makes the model analytically intractable.

a positive long run value for T exists if and only if $\lim_{T\to\infty} F(T,.) > \frac{1-s_q}{b\mu}$. This limit is computed as:

$$\lim_{T\to\infty}\left(\int_0^{\infty}\int_z^{\infty}\gamma\, e^{-\rho z-\gamma\sigma}\,\mathrm{d}\sigma\,\mathrm{d}u\right)=\frac{1}{\rho+\gamma}$$

which gives the parametric condition of the proposition. Notice that when γ tends to zero (no energy-saving technological progress), the integrand appearing in (27) tends to zero, and T should consequently be infinite for the optimal investment rule to hold. The last claim is trivial.$\square$

It should be already noticed at this stage that the necessary and sufficient condition, $\rho+\gamma < \frac{b\mu}{1-s_q}$, for a BGP to exist does depend on the market power parameter μ: the more we depart from perfect competition in the intermediate inputs sector (that's the lower μ), the more the necessary and sufficient condition above is difficult to fulfill **ceteris paribus**, and the less likely the existence of a BGP. The equilibrium outcomes of the model are therefore strongly affected by the extent of imperfect competition in the intermediate inputs sector. This is not a surprising outcome but our rather complicated model has the noticeable virtue to show this property almost immediately. Comparative statics show more interesting results from the economic point of view. Consistently with Boucekkine et al. (1998), it is effectively possible to say more about the scrapping behavior in terms of the parameters of the problem, using equations (26) and (27).

Proposition 2 *Assuming that conditions in Proposition 1 hold, the following properties hold*:

(i) T *is a decreasing function of* b, μ *and* s_q. *It is increasing in* ρ.

(ii) T *does not depend on the parameters of the energy sector production function,* $f(h)$.

(iii) T *is decreasing with respect to* γ *provided* T *is lower than* $\frac{1}{\gamma}$.

Proof. The proof of (iii) is quite hard given the complicated nature of the integral equation (26). We report its demonstration in the Appendix. The first properties are trivial mathematically speaking.

The depicted properties are mostly easy to get and to understand economically. For example, notice that an increase in μ decreases the left hand side of (26). Hence, $F(T,\gamma,\rho)$ should decrease for the optimal investment rule to be still valid. As function $F(.)$ is strictly increasing in T, the scrapping age should go down to keep on moving on the balanced growth path. In economic terms, this outcome means that as we get closer to perfect competition ($\mu = 1$), the incentives to scrap old capital and to switch to clear technologies become more important. Within our model, the mechanism behind is quite simple: from equation (14), one can see clearly that an increase in μ raises the marginal profitability of new investment, which for given marginal cost, stimulates replacement and scrapping of old capital. Actually an increase in μ operates as an increment in the productivity parameter b in our model, both induce the acceleration of the scrapping of old capital.

The same general argument would a priori apply to γ. However in our model, an increase in γ raises the equilibrium interest rate by equation (19), which diminishes the marginal return from investing. As in Boucekkine et al. (1998), and more recently in Boucekkine et al. (2008), this negative effect is more than compensated by the positive one as long as the interest burden is bounded over the lifetime of machines, for example when $\gamma T \leq 1$ (see the Appendix). Hereafter, we shall assume that we are only considering the parameterizations such that the latter property holds.[7]

Concerning the subsidy variable, the outcomes are rather clear and intuitive as far as scrapping is concerned. For example, an increase in the investment subsidy decreases the marginal cost of acquiring new machines, which accelerates scrapping and boosts new investment. More intriguingly, notice that since equation (26) does not depend neither on the energy production function $f(h)$, the long-term optimal scrapping will neither. Indeed as one can see from (24), a change in $f(h)$ affects the optimal level of investment but not its lifetime. This is a sensitive property of the model, and we shall use it intensively later on.

We now come to a crucial property which is crucial to understand why the energy market structure is so important for the efficiency of subsidies. The following proposition shows up some properties of energy supply and energy price, which are fundamental to understand the mechanisms operating in our model.

Proposition 3 *Assuming that conditions in Proposition 2 hold, the following properties hold*:

(i) $p_e = p_e(\gamma, b, s_q, \mu)$ *decreases with* γ*, but increases with* b*,* s_q *and* μ.

(ii) *Under the NM structure,* $h = h(\gamma, b, s_q, \mu, A)$ *has the opposite comparative statics of the energy price* p_e*, it is increasing in* A.

(iii) *Under the FE structure,* $h = h(\gamma, b, s_q, \mu, A)$ *has the same comparative statics as the energy price* p_e*, it is decreasing in* A.

The proof is trivial. Using (23) and Proposition 2, one gets immediately that p_e is increasing in b and μ directly and via the scrapping variable T which goes down when

[7]Notice that this is the realistic case. For γ around 2.5% per year, we restrict T to be lower than 40 years, which covers by far the typical figures.

each of these parameters increases. More straightforwardly, p_e is an increasing function of the subsidy rate s_q exclusively via the scrapping variable. The effect of a technological acceleration through the rate γ on p_e is much harder to disentangle since p_e is proportional to $e^{-\gamma T}$ in the long-run, and the scrapping time is shortened when γ is raised. The Lemma in the appendix solves the problem. Actually, the product γT is an increasing function of γ, or in other terms T is less than a linear function of γ. This establishes the properties (i) of the Proposition.

Properties (ii) and (iii) are obvious consequences of (i) and the relationship depicted in equation (25), that's:

$$p_e = h^{1-\alpha}\, A^{\alpha},$$

or

$$h = p_e^{\frac{1}{1-\alpha}}\, A^{\frac{\alpha}{\alpha-1}}.$$

As mentioned just above, Proposition 3 is important to get through the mechanisms of the model. In particular, notice that in our model, a rise in investment subsidy does increase the price of energy either under free entry or natural monopoly. This property comes from the scrapping condition (15) (giving rise to the long-run relationship in equation (23)). As the increase in the subsidy rate leads to shorten the scrapping time, the total marginal operation cost of the oldest vintage still in use goes down while the marginal productivity of any vintage is kept constant, equal to $b\mu$. The price of energy on the cost side of equation (15) should go up to re-establish the optimality condition. Broadly speaking, it appears clearly that a scrapping condition like the typical rule in (15) necessarily generates a negative correlation between energy price and scrapping time for any shock which does not affect the productivity parameter, b, or the degree of competition in the intermediate goods sector, μ. Some observations are in order here. First of all, a negative correlation between energy prices and lifetime of capital goods is a fact which has been at the heart of a highly interesting discussion for decades. For example, in Baily (1981), a higher energy price is associated with a shorter capital lifetime, and this argument is quite central in his interpretation of the productivity slowdown. While this view has been challenged in several directions (see for example Gordon, 1981), it is commonly shared, and it can therefore be used to validate the benchmark analysis we are performing in this paper. Second, it is also absolutely clear that this negative correlation property is obtained so clearly here because of the gross complementarity assumed between energy and capital: thanks to the Leontief specification, the price of energy and scrapping time are not simultaneously determined in the long-run, the investment rule (22) determines the scrapping time, and equation (23) determines the energy price given the scrapping time.

With a general production function, things would have been much more complicated, but one can always claim that if energy and capital are close to gross complements,[8] our results should still hold. In any case, the results obtained so far are anyway valuable as a benchmark.

While the subsidy rise increases energy price, its effect on energy supply does depend on the market structure of the energy sector: it raises the quantity of energy under free entry but pushes it down under monopoly by equation (25). The latter generic equation implies a negative correlation between energy price and supply under natural monopoly, while the latter variables move in the same direction under free entry. [9] Therefore, at equilibrium, energy consumption will increase under free entry, and will decrease under natural monopoly. Henceforth, the latter seems to be better adapted to reduce energy use. Nonetheless, given the complementarity between energy and capital, the latter supply effect may be paradoxically accompanied by a slower diffusion of clean technologies under natural monopoly. This is exactly what we will study in the next section.

4 The impact of investment subsidies on investment and output

In this section, we study the effects of subsidies on investment and the output-maximizing subsidies.

4.1 Impact of subsidies on investment level

Let us start with investment response to an increment in the subsidy rate s_q. From (24), one gets:

$$i = \frac{1}{T} \left(\frac{h}{A}\right)^{\alpha}.$$

Notice that an increase in s_q has a priori an ambiguous effect on investment. On one hand, it shortens scrapping (Proposition 2), inducing a more intense investment effort in the new and cleaner technologies (demand effect), but one the other hand, it also affects investment in the energy sector (variable h) and therefore the energy supply (supply effect). By Proposition 3, we know that such an effect dramatically depends on the market structure

[8] Some complementarity is anyway needed to ensure a finite scrapping time as mentioned in earlier sections.

[9] We insist that these relationships are generic, at least for the free entry configuration. Under natural monopoly, our properties derive immediately from the Ramsey-Boiteux pricing. Other pricing rules are possible under natural monopoly but we prefer to focus on the latter pricing for its simplicity and the ease of comparison with the free entry case within our benchmark analysis.

of the energy sector. It follows that the overall effect of larger investment subsidies on the investment level is unclear and mainly depends on whether the energy market is under FE or NM structures.

We can go a step further and bring an analytical solution to the ambiguity problem stated just above. One can use equations (23) and (25) to write i as a function of T. One gets:

$$i = (b\mu)^{\frac{\alpha}{1-\alpha}} A^{\frac{\alpha}{\alpha-1}} \frac{e^{\frac{\alpha\gamma T}{\alpha-1}}}{T}. \tag{28}$$

We shall denote by $\Theta(T)$ the function: $\Theta(T) = \frac{e^{\frac{\alpha\gamma T}{\alpha-1}}}{T}$. Under the structure FE, that is when $\alpha < 1$, function $\Theta(T)$ is decreasing as the product of two positive decreasing functions. Therefore, i investment is boosted by investment subsidies in such a situation since they lower equipment lifetime. Actually, using our interpretation just above, a larger subsidy will yield both positive demand and supply effects in such a case: not only investment is boosted by the typical demand effect inherent to vintage models, it is also stimulated by the rise of energy supply as depicted in Proposition 3, property iii), due to gross complementarity between energy and capital. Therefore under (FE), we get the paradoxical property that subsidizing clean technologies speeds up diffusion as expected but this success is paid at equilibrium by a rise in energy use!

Things are much more complicated in the NM case where the supply effect lowering energy use pushes investment level down, and can offset the positive demand effect induced by the investment subsidy. We show hereafter that the result depends on the strength of the natural monopoly in a very concrete sense.

To clarify the latter concept, let us start with some trivial algebra. Clearly, the impact of subsidies depends algebraically on the properties of functions $\Theta(T)$. Differentiating it yields:

$$\Theta'(T) = \frac{e^{\frac{\alpha\gamma T}{\alpha-1}}}{T^2} \left[\frac{\alpha\gamma T}{\alpha - 1} - 1\right].$$

Suppose $\alpha > 1$ and $\gamma T < 1$. Recall that the latter condition is sufficient to guarantee the realism of the model, and in particular that T is decreasing under technological accelerations. The main trick which allows to be conclusive is the observation that T is independent of α (property (ii) of Proposition 2). Therefore, one can "play" on α without affecting the long-run equilibrium value of T. Since $\frac{\alpha}{\alpha-1}$ is a strictly decreasing function of α, the outcome is clear. For $\alpha > \alpha^0 = \frac{1}{1-\gamma T}$, $\Theta'(T) < 0$, and investment, being a decreasing function of scrapping, is boosted by subsidies. In such a case, the NM structure yields the same prediction as the FE structure. However, when $1 < \alpha < \alpha^0 = \frac{1}{1-\gamma T}$, we get $\Theta'(T) > 0$, and investment gets depressed by subsidies! Therefore, under the NM structure, investment is stimulated by subsidies if and only if the natural monopoly is strong enough in the sense that returns to the production function in the energy sector are large enough (or equivalently, if and only if the average cost in the energy sector is decreasing rapidly enough). Below the α-threshold value, α^0, the reverse happens. We summarize the results in the following proposition:

Proposition 4 *Assuming that conditions in Proposition 1 hold, and provided* $\gamma T < 1$, *the following properties hold*:

(i) *Under the FE structure, an increase in the investment subsidy* s_q *raises the investment level in the long-run.*

(ii) *Under the NM structure, an increase in investment subsidy stimulates long-run investment if and only if returns to the production function in the energy sector are large enough, i.e. if and only if* $\alpha > \alpha^0 = \frac{1}{1-\gamma T}$. *Otherwise, either investment is depressed* ($1 < \alpha < \alpha^0 = \frac{1}{1-\gamma T}$) *or insensitive to fiscal stimulus* ($\alpha = \alpha^0 = \frac{1}{1-\gamma T}$).

Henceforth, our model shows clearly that the market structure of the energy sector does matter as to the efficiency of investment subsidy. The interpretation of the previous proposition is quite neat. As mentioned above, raising the investment subsidy rate s_q definitely stimulates investment, but induces a supply effect which depends on the market structure of the energy sector. Under an FE structure for the energy sector, energy use goes up, thus reinforcing the former demand effect, and boosting investment given complementarity between energy and capital. Under an NM structure for the energy sector, energy use goes down, and can eventually offset the positive demand effect again given complementarity between capital and energy. Proposition 4 shows that this happens under weak enough increasing returns in the production technology in the energy sector. In such a case, one gets the paradoxical property that while investment subsidies lower energy use, they do slowdown investment and therefore the diffusion of clean technologies. Clearly the strength of the supply effect depends on the shape of the (decreasing) average cost in the energy sector: if it is decreasing fast enough, then the supply effect will be limited, energy supply will fall but the magnitude of the drop is limited, and so will be the decline in the investment level involved. The positive demand effect will dominate. Only in such a case, we get the virtuous simultaneous occurrence of lower energy use and faster clean technologies diffusion.

Thus, in general one can see that an increase in investment subsidies generally triggers a higher diffusion of energy-saving technologies as new capital embodies energy-saving technological change. Results described in Proposition 4 seem therefore apparently consistent with the viewpoint of Stoneman and David (1986). However, our analysis of subsidies bring out two important new results. First of all, applied to the debate of promoting energy-saving technologies, our paper sheds light on an original paradox:

adoption subsidies may induce a larger investment into cleaner technologies, and such a property can arise either under free entry or under natural monopoly. However, larger diffusion rates do not necessarily mean lower energy consumption at equilibrium, which may explain certain empirical puzzles mentioned in the introduction section. Second, it could even be the case that adoption subsidies do not induce larger investment into cleaner technologies at all: this is clearly the case under natural monopoly in the energy sector with weakly increasing returns and Ramsey-Boiteux pricing. This new result points at an intermediate energy market configuration which is definitely bad for clean technology diffusion, and therefore "moderate" in a way Stoneman and David's statement, which is certainly more in line with the very contrasted related empirical evidence.

How does this affect output response? Before getting to the algebraic developments, a few comments are in order. By construction, the production function of the final good (which is used for consumption, investment and production of energy) is a vintage capital Leontief technology. It depends on two ingredients: investment and lifetime of machines. The larger investment and the longer the lifetime of machines, the larger output. When the investment subsidy is raised, the lifetime of machines always drops, but not necessarily investment. Under an FE structure in the energy sector, investment does increase, and it is also the case under an NM structure with large enough increasing returns. In these two cases, the overall impact of rising investment subsidies is ambiguous and will be tackled in the next section. Note however that if we retain an NM configuration with low enough increasing returns, the overall effect of subsidies on output is already clear: both the lifetime of machines and investment drop, which unambiguously and markedly depresses output. Henceforth, the latter case is clearly identified as the case against investment subsidies in terms of investment and output impact though it pushes energy consumption down. Let us summarize this property in the following proposition.

Proposition 5 *Assuming that conditions in Proposition 1 hold, and provided $\gamma T < 1$, long-run output level declines in response to rising investment subsidies under the NM structure for the energy sector with low enough increasing returns.*

4.2 Impact of subsidies on output level

Using equations (21) and (28), one can readily express detrended output y as a function of T, precisely:

$$y = \Psi \, \frac{e^{\frac{\alpha\gamma T}{\alpha-1}}}{T} \, \left[1 - e^{-\gamma T}\right], \tag{29}$$

where Ψ is a constant independent of s_q, implying that the impact of investment subsidies on y exclusively depends on the shape of its relationship with T. The first T-function,

$\frac{e^{\frac{\alpha\gamma T}{\alpha-1}}}{T}$, comes from long-term investment level as given in equation (28). It is a decreasing function of T, and notice that it goes to infinity when T goes to zero. The second T-function, $1 - e^{-\gamma T}$, measures directly the impact of capital lifetime on output: a longer lifetime implies a larger output level (since firms will operate a wider range of machines). Notice that this term goes to zero when T tends to infinity. How does output behave when T tends to infinity given that the investment effect goes to infinity and the scrapping time effect goes to zero? A trivial computation leads to the result that output will tend to a constant $\Psi\gamma$ when T goes to zero. This happens when the subsidy rate s_q tends to 1: output is still defined in the limit and equal to a well-identified constant. Nonetheless, such a situation violates the positivity of consumption level in the long-run. By equation (20), since either y and h are finite when T goes to zero while i becomes infinite, consumption must go to $-\infty$. We shall therefore disregard this limit situation as economically irrelevant.

Let us dig deeper. Differentiating output as given by the previous equation with respect to T, one ends up finding that the sign of the derivative depends on the sign of the following difference:

$$e^{-\gamma T} \left[1 - \frac{\gamma T}{\alpha - 1}\right] - \left(1 - \frac{\alpha}{\alpha - 1}\,\gamma T\right),$$

which is by no means trivial and depends, among others, on the position of α with respect to 1. The following proposition states that output level is a monotonic function of the subsidy in both remaining cases: $\alpha < 1$ or $\alpha > \alpha^0$, that is either under the FE or NM structures provided the increasing returns are large enough in the latter configuration. Beside this property, the FE and NM structures produce opposite results, as stated in the next and final proposition:

Proposition 6 *Assuming that conditions in Proposition 1 hold, and provided $\gamma T < 1$:*

(i) *Under the FE structure, long-run output is an increasing function of the subsidy rate, s_q.*

(ii) *Under the NM structure with large enough increasing returns, long-run output is a decreasing function of the subsidy rate, s_q.*

The proof is a bit tricky, we report it in detail in the appendix. Two remarks are in order here. First of all, the mechanisms underlying the properties highlighted just above are clear. Under either an FE or NM structure (with large enough increasing returns), rising the subsidy rate increases investment, which raises output, but lowers the lifetime of machines, which reduces output. Property i) above means that under the FE configuration, the first effect always dominates. In the alternative case, the opposite happens. That is in the NM case, the increase in investment following the rise in the subsidy rate is not large enough to compensate for the output loss due to the larger fraction of capital scrapped. Secondly, the proposition tends to confirm that the NM structure for the energy sector eliminates the potential advantages of investment subsidies in terms of output gains, whatever the extent of increasing returns in that sector. One would conclude from this property that such subsidies would be welfare-worsening under the NM structure as a decrease in output level is likely to induce a drop in consumption level, therefore driving welfare down. This is not that trivial within our benchmark set-up as one ca infer from equation (20), and it is even less in more general frameworks attributing to energy consumption a welfare loss associated to the induced pollution increment.Indeed, one has to keep in mind that while the NM structure may not be the best market structure to raise investment in cleaner technologies, it does allow to reach a lower energy consumption. Ultimately, the arbitrage between NM and FE structures would thus depend on how consumption and pollution (or environmental quality) are weighted in the preferences of the economic agents. This issue is beyond the scope of this paper.

5 Conclusion

In this paper, we develop a general equilibrium vintage capital model with energy saving-technological progress, endogenous scrapping and an explicit energy market. Because of the scrapping condition inherent to vintage capital models, the price of energy is tightly connected with the (optimal) age structure of the operating capital stock. The impact of imperfect competition on the outcomes of the decentralized equilibria are deeply characterized along the paper. In particular, we show that investment subsidies designed to fasten the diffusion of cleaner technologies may not always achieve this objective due a a well-identified general equilibrium effect. Such a result is rather consistent with the highly conflicting related empirical reports. More specifically, increasing investment subsidies do not only generate the typical positive demand effect on investment, often pointed out in partial equilibrium studies, they also affect energy supply and equilibrium energy price, which affects again investment via the scrapping mechanism repeatedly advocated along this paper. Under a free entry structure for the energy sector, the latter effect is positive, thus reinforcing the former demand effect, and boosting investment. Under a natural monopoly structure for the energy sector, the supply effect is negative, and can eventually offset the positive demand effect, which does happen when increasing returns in the production technology in the energy sector are not strong enough. We have got more results on the impact of investment subsidies on output level.

Applied to the debate of promoting energy-saving technologies, an original paradox is pointed out: adoption subsidies may induce a larger investment into cleaner technologies, and such a property can arise either under free entry or under natural monopoly. However, larger diffusion rates do not necessarily mean lower energy consumption at equilibrium, which may explain certain empirical puzzles mentioned in the introduction section. On the other hand, our analysis identifies an intermediate energy market configuration under which adoption subsidies induce a drop in the investment level in cleaner technologies, and therefore it "moderates" in a way Stoneman and David's claim. Again, this outcome is certainly more in line again with the very contrasted related empirical evidence.

Of course, the mechanisms and results identified in this paper deserve further empirical and theoretical analysis. It goes without saying that our results are extracted under linear production functions in the intermediate goods sector, and this linearity simplifies our study to a certain extent. In particular, it allows to solve for the balanced growth paths following a straightforward recursive scheme. Such a scheme, in turns, has allowed for a neat identification of the demand and supply effects described along the paper. We are currently studying another version of the model with a more general production function in the intermediate goods sector, which breaks down partially the above-mentioned recursivity, therefore only allowing for numerical analysis. Another useful complementary study concerns the empirical testing of the theory developed in this paper, which requires in particular an accurate appraisal of the characteristics of energy markets. This looks like a daunting task but it is certainly a necessary step to take to understand the diffusion factors of clean technologies. Finally, our analysis calls for a further investigation on the welfare implications of investment subsidies. As mentioned in the previous section, while the NM structure may not be the best market structure for investment subsidies to speed up diffusion of cleaner technologies, it does lower energy consumption in the long-run equilibrium. Therefore, the welfare implications of our analysis are far from obvious and would deserve a closer appraisal taking into account the welfare loss due to pollution. We are currently working along this line.

Appendix : Proofs

Proof of Proposition 2: As already mentioned, Properties (i) and (ii) are trivial. Let us prove Property (iii). To this end, we need the following Lemma.

Lemma *Assuming that conditions in Proposition 1 hold, the product γT is an increasing function of γ.*

Proof of Lemma. Observe that:

$$\frac{\partial(\gamma T)}{\partial\gamma} = T + \gamma\frac{\partial T}{\partial\gamma} = T - \gamma\frac{\frac{\partial F}{\partial\gamma}}{\frac{\partial F}{\partial T}}$$

which implies

$$\frac{\partial F}{\partial T}\frac{\partial(\gamma T)}{\partial\gamma} = T\frac{\partial F}{\partial T} - \gamma\frac{\partial F}{\partial\gamma}$$

From relation (27), the function F can be rewritten as

$$F(T,\gamma,\rho) = \int_0^T e^{-\rho z}\left(e^{-\gamma z} - e^{-\gamma T}\right)\mathrm{d}u$$

the required partial derivatives can be obtained after some algebraic operations:

$$T\frac{\partial F}{\partial T} - \gamma\frac{\partial F}{\partial\gamma} = \int_0^T \gamma z\, e^{-(\rho+\gamma)z}\mathrm{d}u$$

which is positive. From Proposition 1, we know that $\frac{\partial F}{\partial T} > 0$, we deduce that γT is an increasing function of γ. $\square$

It is now possible to prove Property (iii) of Proposition 2. Consistently with Boucekkine et al. (1998), we will show that a sufficient condition for T to decrease with γ is $T \leq \frac{1}{\gamma}$. The latter property is satisfied if $\rho + \gamma < \frac{b\mu}{4(1-s_q)}$. In fact, the total differentiation of the equation $F(T,\gamma,.) = 1$ leads to

$$\frac{\partial T}{\partial\gamma} = -\frac{\frac{\partial F}{\partial\gamma}}{\frac{\partial F}{\partial T}}$$

As $\frac{\partial F}{\partial T} > 0$ (Proposition 1), T is a decreasing function of γ if and only if the partial derivative of F with respect to γ is positive. Given that

$$\frac{\partial F}{\partial\gamma} = \int_0^T\int_z^T (1+\gamma+\sigma)\, e^{-(\rho z+\gamma\sigma)}\mathrm{d}\sigma\,\mathrm{d}u$$

a sufficient condition for T to decrease when γ rises is the positivity of function $1-\gamma\sigma$ on the integration domain. This is checked if only if the line $\sigma = \frac{1}{\gamma}$ is above the integration domain. This is the case if $T \leq \frac{1}{\gamma}$. Now, note that, using the integral function defined in

(26), the condition $T \leq \frac{1}{\gamma}$ is equivalent to the inequality $\frac{1-s_q}{b\mu} \leq F(\frac{1}{\gamma}, .)$. Computing the integration yields

$$\frac{1-s_q}{b\mu} \leq \frac{e^{-(\frac{\gamma+\rho}{\gamma})}-1}{-(\gamma+\rho)} - \frac{e^{-(\frac{\gamma+\rho}{\gamma})}-e^{-1}}{-\rho}$$

In terms of parameters' expressions of Proposition 1, denote $x = \frac{\rho+\gamma}{b'}$, with $b' = \frac{1-s_q}{b\mu}$. Observe that $x > \gamma' \equiv \frac{\gamma}{b'}$. Elementary algebraic operations allow us to write the following inequality

$$x^2 + \left(e^{-1} - 1 - \gamma'\right) x + \gamma' < \gamma' e^{-\frac{x}{\gamma}}$$

For any fixed γ', one can find the values of x ($x > \gamma'$) such that the above inequality holds. Note that this inequality is very easy to tabulate for function in x and γ' on both sides. In particular, the inequality holds for $\gamma' < x < \frac{1}{4}$. Such a sufficient condition ensures that T is decreasing with respect to γ and is consistent with parameterizations usually adopted in empirical studies. □

Proof of Proposition 6: Recall that the sign of the derivative of output with respect to scrapping time T is the sign of the difference

$$e^{-\gamma T}\left[1 - \frac{\gamma T}{\alpha - 1}\right] - \left(1 - \frac{\alpha}{\alpha-1}\,\gamma T\right),$$

which we may write $\psi_1(T) - \psi_2(T)$ with obvious notations.

Consider the case $\alpha < 1$. We have to study both functions $\psi_1(T)$ and $\psi_2(T)$ for $0 \leq T \leq \frac{1}{\gamma}$. $\psi_2(T)$ is an affine function increasing from 1 to $\frac{1}{1-\alpha}$. Differentiating $\psi_1(T)$ one gets:

$$\psi_1'(T) = \gamma\, e^{-\gamma T}\, \frac{\alpha - \gamma T}{1-\alpha}.$$

Therefore, $\psi(T)$ is increasing on the interval $[0\ \frac{\alpha}{\gamma}]$, from $\psi_1(0) = \psi_2(0) = 1$ to $\psi_1\left(\frac{\alpha}{\gamma}\right)$, then decreasing on the interval $(\frac{\alpha}{\gamma}\ \frac{1}{\gamma}]$. On the other hand, one can readily prove that $\psi_1(T)$ is strictly concave on the whole interval $[0\ \frac{1}{\gamma}]$. Indeed:

$$\psi_1''(T) = \gamma\, e^{-\gamma T}\left[-\frac{2\gamma}{1-\alpha} + \frac{\gamma^2 T}{1-\alpha}\right],$$

and since $T \leq \frac{1}{\gamma}$, we get $\psi_1''(T) < 0$ on the interval $[0\ \frac{1}{\gamma}]$. Notice now that $\psi_1(0) = \psi_2(0) = 1$ and that $\psi_1'(0) = \psi_2'(0) = \frac{\alpha\gamma}{1-\alpha}$. Hence the two functions start at the same point at $T = 0$ and with the same slope (tangency). Since $\psi_1(T)$ is strictly concave while $\psi_2(T)$ is affine increasing, it follows that the two functions can not intersect in the interval $(0\ \frac{\alpha}{\gamma}]$, and $\psi_2(T) > \psi_1(T)$ on this interval. This establishes the first part of Proposition 6.

Let us consider now the case $\alpha > \alpha^0 = \frac{1}{1-\gamma T} > 1$. In such a case, $\psi_2(T)$ is an affine function decreasing from 1 to $\frac{1}{1-\alpha}$. The crucial thing with respect to the case $\alpha < 1$ is that $\psi_1(T)$ is now strictly decreasing and strictly convex on the interval $[0\ \frac{1}{\gamma}]$. It is enough to have a look at the expressions of the first and second order derivatives of this function displayed just above. Further given that $\psi_1(0) = \psi_2(0) = 1$ and that $\psi_1'(0) = \psi_2'(0)$, the two functions cannot intersect, and $\psi_2(T) < \psi_1(T)$ on $(0\ \frac{1}{\gamma}]$. $\square$

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