ARCH in the G7 Equity Markets: A Speculative Explanation

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Abstract

This paper explores whether speculative activity can, in practice, generate the ARCH-type behavior found in financial time series. Specifically, G7 equity market indices are examined for evidence of a dynamic whereby speculative interest is self-sustaining – that is, markets can become "hot". A straightforward model, taken from Faruqee and Redding [9], generates some testable implications of the idea. Tests of the model on the data show that not only does the model offer an explanation for volatility clustering, but also can be considered a statistical improvement on standard GARCH representations.

JEL Classifications: G12, F30, G15

1 Introduction

Recent years have seen crises in Asian and other developing markets, as well as what appears as of this writing to be a bursting bubble in NASDAQ shares on the US market. Such events have increased interest in understanding not only the phenomenon of the volatility of asset prices, but also the importance of speculative dynamics in determining the fluctuations of asset prices.

This paper presents an empirical argument that the two questions are importantly related. Following on a model from Faruqee and Redding [9], we look to test the hypothesis that conditional volatility patterns can be understood from a microfoundation of speculative dynamics. We then ask whether this provides a better description of market behavior than a straightforward GARCH specification.

The empirical strategy addresses the above question via a Monte Carlo technique. A model is presented which describes speculative dynamics in a way which straightforwardly indicates

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the sort of volatility clustering found in GARCH specifications. Monte Carlos are then used to compare G7 equity price behavior with random walks. These random walks are calibrated to each of the G7 markets, and the speculative dynamics model fits the actual data better than the GARCH random walks. In this way, we suggest that our speculative dynamics model offers an interpretation of volatility in the financial markets which not only incorporates the GARCH specification but also improves on it.

The outline of the paper is straightforward. Section two reviews the model from Faruqee and Redding [9]. Section three then presents the empirical strategy, and results for the equity markets are given in section four. Section five concludes.

2 Model

2.1 Model Description

The model to be used in this paper seeks to describe speculative dynamics by a simple "hot markets" characteristic. This characteristic is one whereby markets which are already "hot" – that is, currently holding a large amount of speculative attention – attract media and public attention. The innovation to the level of speculative activity in period $t+1$ therefore has a high variance when the absolute level of such activity is high in period t. It might therefore be thought of as related to the positive-feedback model of DeLong et al [6], except that it is the variance, rather than the level, of the innovation to speculative interest that is driven by previous price activity.

The model therefore assumes a self-sustaining nature to speculative interest, and in that way draws a clear parallel to ARCH-style models, in which the innovation in period $t+1$ has a high variance when the absolute value of the *innovation* is high in period t. An empirical test for our "hot markets" specification will produce spuriously positive results if the true specification is ARCH, and vice versa. If, on the other hand, the model presented here is driving volatility, tests for ARCH properties will clearly test positive, but the "hot markets" phenomenon should be present to a greater degree in the actual price time series than it is in calibrated random walks generated to have ARCH properties. The first part of this implication – that financial markets have ARCH statistica properties – has been thoroughly verified in the existing literature. We therefore focus on the second part – looking to see whether the hot markets phenomenon is more important in actual data than in (G)ARCH random walks.

2.2 Model Specification

This section outlines a model of the behavior described above. The model is of the market for a financial asset, and can be found in more detail in Faruqee and Redding [9]. The infintely divisible asset exists in unit supply and is a claim on a (known) discrete cash flow stream $\{C_t\}$, which in the present context we would likely take to be dividend payments.

Speculative interest in the model is represented by uninformed noise traders. Their net demand may be negative and is perfectly inelastic. There are informed but risk averse "smart money" investors who act competitively and will be willing to accommodate this net demand for a positive expected return.

The "hot markets" phenomenon is represented by taking the noise trader demand N_t (shown in currency terms) to be mean-reverting but with a zero-mean innovation whose variance is increasing in the absolute level of N_{t-1}:

$$N_t = \alpha N_{t-1} + \delta_t |N_{t-1}| \tag{1}$$

where $0 < \alpha < 1$ and δ_t is normally distributed with zero mean and variance σ_δ^2. This representation will generate ARCH-type activity since a large shock to speculative activity in period t will tend to produce a high value of N_t and thus cause a large shock to speculative activity in period $t + 1$.

The aggregate demand for stock (in shares) of "smart money" investors depends only on the expected return. For simplicity, it is taken to be linear in this return, as in Shiller [14]:

$$S_t = \frac{(E_t R_t - \rho)}{\varphi} \tag{2}$$

where ρ can be interpreted as a normal rate of return (the shadow cost of capital to the smart money traders) and φ a measure[1] of the risk aversion of the smart money[2].

Expected return is, as usual, this period's cash flow plus the expected capital gain:

$$E_t R_t = \frac{C_{t+1} + E_t P_{t+1} - P_t}{P_t} \tag{3}$$

Since the asset exists in unit supply, the equilibrium condition that demand adds up to one unit is given by:

$$1 = S_t + \frac{N_t}{P_t} \tag{4}$$

There are, of course, an infinite number of solutions to the time series P_t characterized by bubbles, but imposing the requirement that the present discounted value of P_t has an upper bound yields (after some algebra) the unique solution:

$$P_t = \sum_{i=0}^{\infty} \frac{C_{t+i+1} + \varphi E_t N_{t+i}}{(1+\rho+\varphi)^{i+1}} \tag{5}$$

This price can be usefully decomposed into a "fundamental value" V_t representing the discounted value of the cash flow and a residual term reflecting the price distortions caused by noise trading:

$$P_t = \sum_{i=0}^{\infty} \frac{C_{t+i+1}}{(1+\rho+\varphi)^{i+1}} + \varphi \sum_{i=0}^{\infty} \frac{E_t N_{t+i}}{(1+\rho+\varphi)^{i+1}} \tag{6}$$

[1]The linear response of demand to expected return is a well-known property of constant absolute risk aversion combined with normally distributed wealth. However, in this case, φ is not either of the standard Arrow-Pratt measures of risk aversion, since wealth is not normally distributed due to the "hot markets" effect.

[2]To traders with finite horizons, the noise traders create price risk even if the asset has no fundamental risk.

Computing expectations of N_{t+i} using (1), we can rewrite (6) as:

$$P_t = V_t + \frac{\varphi N_t}{(1+\rho+\varphi-\alpha)} \tag{7}$$

Put another way, the deviation of price can be seen as a linear function of the level of noise trading:

$$(P_t - V_t) = N_t \frac{\varphi}{1+\rho+\varphi-\alpha} \tag{8}$$

where as would be expected, the price effects of a given level of noise trading N_t are greater the more persistent noise trading is (α) and the more reluctant smart money is (φ) to accommodate the noise demand.

Computing the variance of P_{t+1} using equation (7) and then using (8) shows the empirical implication that we wish to test:

$$Var_t P_{t+1} = \frac{1+\rho+\varphi-\alpha}{\varphi} \sigma_\delta^2 (P_t - V_t)^2 \tag{9}$$

This shows that a large (in absolute value) deviation from fundamentals in one period will cause greater volatility going forward.

3 Empirical Methods

3.1 Data Description

We will test the above empirical implication using weekly equity index time series. The Morgan Staley Capital Indices were collected for each Friday close from 1980 through 1998, in domestic currencies for each of the G7 countries (Canada, France, (the Federal Republic of) Germany, Italy, Japan, the United Kingdom, and the United States). In each case, logarithms have been taken of the nominal exchange rate series.

3.2 Two Empirical Challenges

Equation (9) predicts that a large deviation from fundamentals will cause an increase in volatility. There are therefore two empirical challenges to address. First, the concept of "fundamental value" must be empirically estimated, and second, a working definition of "volatility" must be formulated.

We have chosen to estimate fundamental value by the means of statistical filters. The likely alternative – using a macroeconomic model, perhaps in a VAR context – would be problematic at reasonably high frequencies. Further, it raises concerns about the joint test of two economic hypotheses – the macroeconomic model and the "hot markets" hypothesis. Robustness of the empirical technique is assessed by applying a second filter to the data and comparing results.

To estimate fundamentals, we have first used a Hodrick-Prescott [13] filter, and also used a low-pass filter as suggested by Baxter and King [1], and the modification to this filter suggested by Woitek [15]. The results shown have a the Hodrick-Prescott smoothing parameter λ set to 57,600, and the Baxter-King filter threshhold set to 26 weeks. However, robustness tests on each of these parameters (not shown) indicates that these parameters can be altered without affecting the qualitative results.

The second challenge (estimating volatility) has been addressed by fitting the data to three GARCH (conditional heteroskedasticity) models which provide an estimate of the time series of volatility. The model to be estimated for the innovations to the asset price will be an AR(2) model with GARCH(p,q) errors. Specifying the time series as s_t, this implies that the process is given by:

$$\Delta s_t = a_0 + a_1 \Delta s_{t-1} + a_2 \Delta s_{t-2} + \epsilon_t \tag{10}$$

where the variance h_t of ϵ_t is given by:

$$h_t = c + p_1 h_{t-1} + p_2 h_{t-2} + \ldots p_p h_{t-p} + q_1 \epsilon_{t-1}^2 + q_2 \epsilon_{t-2}^2 + \ldots + q_q \epsilon_{t-q}^2 \tag{11}$$

An ARCH(q) model lacks the lagged variance terms of the GARCH model and can therefore be thought of as a GARCH(0,q) specification. We have estimated conditional variance using ARCH(2), ARCH(3), and GARCH(2,3) processes. Naturally, the entire data set is used to estimate the parameters in equation (11). However, once these parameters have been chosen, the fitted variance for each period depends only on lagged disturbance realizations. The present model predicts that deviations from fundamentals impact future volatility. This effect therefore will not appear fully in the ARCH and GARCH specifications for several periods. A lag of 4 periods has been chosen to test whether the deviation $(P_{t-4} - V_{t-4})$ (which will be squared in the estimation to fit equation (9) and eliminate the sign) influences volatility. Shorter lags than this actually improve the results, but this is somewhat spurious – with an ARCH(3) specification a shock in one period automatically increases measured volatility for the next three periods, so if the lag is three or less, even i.i.d. white noise would provide some apparent support for the "hot markets" phenomenon.

3.3 Monte Carlo Estimation

As described above, we will seek to determine whether a large deviation from fundamentals causes a subsequent increase in volatility. This has the potential to provide strong support for the hot markets interpretation of conditional volatility, but it must be shown that the tests we perform would not spuriously produce the same results given white noise innovations to financial asset prices.

In particular, as described above, if asset prices are inherently GARCH for some reason other than the present hypothesis, we will still likely find that a large deviation from measured fundamentals in one period appears to cause a high volatility a few periods forward. To control for this possibility, we run Monte Carlo simulations whereby the GARCH parameters are estimated from the actual data, and these parameters are used to generate calibrated GARCH random walks. By performing the same analysis that was done with the actual data on these random walks, we can discover whether (as the hypothesis requires) the "hot markets" phenomenon of deviations from fundamentals increasing volatility is a more prominent feature of the actual data than it is of the random walks. This will be done, as equation (9) dictates, by regressing measured volatility on the (lagged) square of deviations from fundamentals. The resulting coefficient β should be significantly positive if the model is correct. Further, we would look for the estimated β from looking at actual financial market data to be greater than the mean value β_{MC} generated by running regressions on calibrated random walks.

4 Empirical Results

As stated above, the goal is to test equation (9) by regressing variance on lagged deviations from fundamentals. To show robustness, differing GARCH specifications for fitted variance and differing filter specifications for fundamentals are used. In each case, the results are compared with the results of a Monte Carlo simulation with 1,000 repetitions run to generate a mean value β_{MC} against which the regression coefficient from the actual data can be compared.

The first conditional variance specification, ARCH(2), has results presented in Table 1. The top section of this table shows the Hodrick-Prescott results. The Hodrick-Prescott filter separates the series into a permanent and transitory component, and we therefore use this transitory component as a measure of the deviation from fundamentals. As (9) requires, this deviation is then squared before being used as an independent variable in a regression of fitted variance. The fitted variance is generated by fitting the innovations of each series to an ARCH(2) process, which generates ARCH parameters q_1 and q_2 as in equation (11). These parameters are shown in the first two rows of Table 1 and can be used to generate a fitted conditional variance series as required. To eliminate serial correlation, twelve lagged values of the fitted conditional variance are included in the regression but to save space the coefficients are not reported. The regression coefficent β on the deviation from fundamentals is then shown for each country. In each case it can be seen that the coefficient is significantly positive at the 1% level.

Monte Carlos are then run to test an alternative, necessarily imprecise hypothesis: that some other force is generating ARCH properties in the data, which then can provide spurious support for our test of equation (9). To control for this hypothesis, the random walks have been generated with the same ARCH parameters found for the actual data and then each random walk is subjected to the same treatment as the actual data. The mean coefficient on deviation from "fundamentals" in these random walks is presented in Table 1 as β_{MC}. As the Table shows, in each case the evidence for hot markets is greater than what would be expected from simple ARCH data, and in five of the seven cases this is significant at the 1% level.

The next two panels of Table 1 re-examine the ARCH(2) specification where a Baxter-King low-pass filter. Dynamics having a frequency higher than 26 weeks are separated from those having a lower frequency, toa gain enable the asset price series to be separated into its transitory and fundamental components.

Again, the data provides coefficients that are consistently significantly greater than zero, generally at well above the 1% level and always at least the 5% level. When testing against the Monte Carlo random walks, five of the seven countries still show significantly (at 1%) stronger hot markets effects than ARCH alone would suggest, although the case of Japan is somewhat problematic, having a very statistically insignificant difference between the β and β_{MC}. These results are largely repeated when Woitek's modifications to the Baxter-King filter are included.

Tables 2 and 3 test the robustness of the above results by using ARCH(3) and GARCH(2,3) specifications to provide estimates for conditional volatility. The results are qualitatively similar to the ARCH(2) case, and perhaps even more supportive of the model than the results in Table 1. For each of the filters and for each of the seven countries, the data generates a value of β which is significantly positive at the 0.1% level of significance. Further, again for each of the filters and each of the G7 countries, the β generated from the data is greater than the β_{MC} which would result from straightforward (G)ARCH behavior. For each filter used, six of the seven countries show this difference as significant at the 5% level, and more often than not at the 1% level.

5 Conclusion

This paper has further evaluated the "hot markets" hypothesis introduced in Faruqee and Redding [9] – that speculative activity is self-sustaining because it draws attention of other speculators to the market. This method of describing speculative activity offers an opportunity to provide an economic interpretation for the conditional volatility patterns observed in financial markets.

Such an economic interpretation requires empirical testing, of course, and while Faruqee and Redding [9] tested this hypothesis on foreign exchange data, the present paper has shown that the results are robust to equity index data in the G7 countries. Indeed, a comparison of the statistical significance of the results between the two papers suggests that the model fits equity price data even better than it fits exchange rate data. This is consistent with an idea that speculative activity is more important for share prices than it is for foreign exchange where trade flows may have a moderating influence.

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	ARCH(2) Parameters						
	Canada	Germany	France	Italy	Japan	Britain	USA
Q_1	0.2151	0.1148	0.0893	0.1992	0.1751	0.2454	0.1742
Q_2	0.2031	0.3187	0.2309	0.1345	0.1280	0.0229	0.1538
	Hodrick-Prescott Filter						
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0142	0.0132	0.0093	0.0090	0.0059	0.0499	0.0175
	(0.0023)	(0.0019)	(0.0028)	(0.0020)	(0.0019)	(0.0048)	(0.0020)
$p(\beta = 0)$	0.000	0.000	0.001	0.0000	0.002	0.000	0.000
β_{MC}	0.0051	0.0074	0.0044	0.0035	0.0032	0.0042	0.0035
$p(\beta = \beta_{MC})$	0.000	0.002	0.080	0.006	0.162	0.000	0.000
	Baxter-King Filter						
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0649	0.0572	0.0378	0.0376	0.0124	0.2338	0.0657
	(0.0081)	(0.0068)	(0.0098)	(0.0063)	(0.0057)	(0.0142)	(0.0059)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.030	0.000	0.000
β_{MC}	0.0190	0.0277	0.0187	0.0145	0.0136	0.0180	0.0142
$p(\beta = \beta_{MC})$	0.000	0.000	0.051	0.000	*0.840*	0.000	0.000
	Baxter-King-Woitek Filter						
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0687	0.0696	0.0381	0.0282	0.0125	0.2480	0.0693
	(0.0085)	(0.0072)	(0.0104)	(0.0066)	(0.0060)	(0.0149)	(0.0062
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.037	0.000	0.000
β_{MC}	0.0197	0.0389	0.0196	0.0151	0.0142	0.0189	0.0148
$p(\beta = \beta_{MC})$	0.000	0.000	0.075	0.046	*0.783*	0.000	0.000

Table 1: ARCH(2) Results

	Canada	Germany	France	Italy	Japan	Britain	USA
ARCH(3) Parameters							
Q_1	0.2214	0.1065	0.1063	0.1738	0.1437	0.2473	0.1656
Q_2	0.1831	0.3068	0.1620	0.0713	0.0505	0.0221	0.1177
Q_3	0.1342	0.0880	0.2063	0.2345	0.2233	0.0604	0.0753
Hodrick-Prescott Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0125	0.0113	0.0094	0.0148	0.0108	0.0502	0.0162
	(0.0023)	(0.0016)	(0.0026)	(0.0023)	(0.0024)	(0.0048)	(0.0019)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0054	0.0069	0.0046	0.0052	0.0046	0.0044	0.0031
$p(\beta = \beta_{MC})$	0.002	0.007	0.062	0.000	0.012	0.000	0.000
Baxter-King Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0552	0.0486	0.0800	0.0569	0.0370	0.2355	0.0601
	(0.0082)	(0.0058)	(0.0086)	(0.0069)	(0.0072)	(0.0142)	(0.0055)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0178	0.0246	0.0187	0.0226	0.0222	0.0178	0.0122
$p(\beta = \beta_{MC})$	0.000	0.000	0.000	0.000	0.038	0.000	0.000
Baxter-King-Woitek Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0583	0.0515	0.0857	0.0593	0.0390	0.2499	0.0634
	(0.0086)	(0.0061)	(0.0091)	(0.0072)	(0.0075)	(0.0150)	(0.0058)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0184	0.0257	0.0197	0.0238	0.0235	0.0185	0.0127
$p(\beta = \beta_{MC})$	0.000	0.000	0.000	0.000	0.039	0.000	0.000

Table 2: ARCH(3) Results

GARCH(2,3) Parameters							
	Canada	Germany	France	Italy	Japan	Britain	USA
Q_1	0.1932	0.0397	0.0990	0.1148	0.0488	0.2279	0.1581
Q_2	0.0433	0.1420	0.1432	0.0000	0.0125	0.0560	0.0000
Q_3	0.0000	0.0000	0.0387	0.0721	0.1578	0.0000	0.0047
P_1	0.4693	0.0000	0.0000	0.3513	0.5785	0.0000	0.6010
P_2	0.1971	0.6555	0.5279	0.4090	0.0954	0.3247	0.1404
Hodrick-Prescott Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0109	0.0045	0.0047	0.0054	0.0101	0.0466	0.0153
	(0.0020)	(0.0007)	(0.0013)	(0.0012)	(0.0017)	(0.0044)	(0.0018)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0042	0.0026	0.0023	0.0026	0.0037	0.0043	0.0029
$p(\beta = \beta_{MC})$	0.001	0.010	0.064	0.018	0.000	0.000	0.000
Baxter-King Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0478	0.0189	0.0278	0.0161	0.0423	0.2176	0.0575
	(0.0071)	(0.0026)	(0.0046)	(0.0037)	(0.0048)	(0.0131)	(0.0053)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0140	0.0107	0.0083	0.0099	0.0201	0.0168	0.0105
$p(\beta = \beta_{MC})$	0.000	0.002	0.000	0.093	0.000	0.000	0.000
Baxter-King-Woitek Filter							
	Canada	Germany	France	Italy	Japan	Britain	USA
β	0.0505	0.0200	0.0282	0.0165	0.0452	0.2307	0.0606
	(0.0075)	(0.0028)	(0.0049)	(0.0038)	(0.0051)	(0.0137)	(0.0055)
$p(\beta = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{MC}	0.0145	0.0113	0.0086	0.0104	0.0217	0.0175	0.0109
$p(\beta = \beta_{MC})$	0.000	0.002	0.000	0.108	0.000	0.000	0.000

Table 3: GARCH(2,3) Results