

# Work Schedules, Wages, and Employment in a General Equilibrium Model with Team Production 

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December 6, 1996

I am indebted to Edward Prescott and Fernando Alvarez for their comments and advice. I would also like to thank Richard Rogerson and seminar participants at the Cleveland and Kansas City Federal Reserve Banks, Cornell University, the State University of New York at Stonybrook, the University of California at Santa Barbara, and the University of Western Ontario. This paper is drawn from the first chapter of my doctoral dissertation from the University of Minnesota.


#### Abstract

This paper provides a general equilibrium framework in which the number of working hours and the employment levels of heterogeneous workers is endogenously determined. This is done in an environment where production requires coordinating the work schedules of different worker types, a characteristic I refer to as team production. In particular, I assume that all workers on a production team must work the same hours. Output is produced by a large number of teams, where different teams may operate different numbers of hours. The model economy has two types of people, who differ in their preferences over leisure and in the labor services they provide. Firms offer tied wage-hours packages to workers, who choose among these packages. An interesting aspect of this economy is that wages are not linear in the number of working hours, although prices are linear over the traded commodities. I show that an employment tax on high-wage workers has substantially different effects on the employment and wages of low-wage and high-wage workers when team production is explicitly modeled compared to a case when it is not.


## 1. Introduction

In evaluating the consequences of many labor market policies and regulations, it is crucial to understand the effect of these policies on firms' decisions about working hours and employment. In particular, policymakers would like to know whether and by how much firms respond to various policies by adjusting employment levels and/or by changing hours per worker, how these adjustments vary across workers with different skills, and how these policies influence wage levels and wage disparities among workers. Examples of policies that may affect the hours and employment decisions of firms include employment subsidies and taxes.

Existing general equilibrium models are poorly suited to evaluate such consequences of labor market policies. Virtually all of these models use a neoclassical production function in which the labor input is simply total hours of each type of labor, and thus makes no distinction between employment and hours per worker in production.

In this paper, I develop a general equilibrium framework for evaluating labor market policies which affect firms' decisions about work schedules and employment. A key feature of this framework is that the labor services of workers are coordinated in a specific manner, a characteristic I call team production. Since labor market policies typically affect groups of workers differently, modeling the interaction of heterogeneous workers in production may be important in analyzing the consequences of these policies.

In fact, labor economists have long recognized that team production plays a key role in firms' decisions about work schedules and employment. In the Handbook of Labor Economics (Ashenfelter and Layard, 1986), Sherwin Rosen notes the potential importance of team production, and characterizes it as follows:

Team production necessarily requires that hours decisions are closely coordinated among all members, and this cannot be done if each member
makes a unilateral decision of how many hours to work and how to distribute them across the working day. ${ }^{8}$

The goal of this paper is to develop a tractable general equilibrium framework with team production in which the work schedules, employment, and wages of heterogeneous workers are endogenously determined. After presenting the framework, I illustrate the potential importance of explicitly modeling team production in examining the consequences of labor market policies using a simple example.

Team production is modeled in the following manner. Output is produced by a large number of production teams, each consisting of a group of heterogeneous workers working with a given amount of capital for a fixed number of hours. For simplicity, I assume that hours worked by team members are "perfectly coordinated" in the sense that all members of that team must work the same hours. Different teams can operate different numbers of hours. The team production function distinguishes between the stock of workers and machines and the number of hours each stock is used. The model economy focuses on firm-level decisions as to how long to operate its teams, and how many workers of each type to employ.

A crucial aspect of this framework is that working hours are indivisible goods. Labor market commodities are differentiated by the type of labor and the number of working hours. Firms offer tied wage-hours packages to workers. W orkers decide which production team to work on by choosing among the packages offered, and then must work the hours which that team operates. People can work on one team at most, and can randomize over different packages. A $n$ interesting feature of this framework is that wages are generally not linear in the number of working hours, but prices are linear over the commodities traded. This is a result of the commodity space used.

This framework is consistent with a striking feature of work schedules in the

[^0]U.S. economy that is ignored in standard labor supply theory and in most of the applied general equilibrium literature: M ost workers are not free to unilaterally determine their working hours at a given wage rate, but rather they must work the hours set by their factory or office. ${ }^{9}$ I conjecture that capturing this feature of reality is important for understanding the determination of work schedules and employment.

As a preliminary step in exploring the potential importance of modeling team production explicitly when analyzing labor market policies, I present an illustrative policy experiment. In this example, I examine the consequences of an employment tax on high-wage workers in two model economies that are identical except for the specification of the technologies. The first economy uses individual production, or teams with one worker, and workers produce independently. The second economy uses team production in which high- and low-wage workers must work together. I find that modeling team production leads to significantly different predictions for the consequences of this simple policy.

The model constructed here builds on the work of Hornstein and Prescott (1993), who developed a general equilibrium construct in which work schedules are endogenously determined. In their construct, production is done by homogeneous workers. An extension of the Hornstein and Prescott setup that includes heterogeneous workers is used as a reference point in illustrating the potential importance of team production in analyzing labor market policies.

In focusing on the role of technology in determining work schedules and defining the labor market commodities to be tied wage-hours packages, I am building on the seminal work of Lewis (1969). Several prominent papers by Rosen (1974, 1977, 1986) further develop these ideas and provide a framework for analyzing market

[^1]equilibria where tied wage-hours packages are traded. Kinoshita (1987) uses the hedonic framework developed in Rosen (1974) to study the structure of working hours and wages in a framew ork where working hours are indivisible goods. However, none of these papers provides a general equilibrium framework.

In focusing on team production, I am following the work of Deardorff and Stafford (1976), who explore the consequences of a coordination requirement on the work schedules and wages of heterogeneous workers in a partial equilibrium framework. M ore recently, W eiss (1996) presents a model in which the work schedules of workers with disparate preferences for leisure may be synchronized when there are gains to working together.

The present paper is intended as a first step in developing general equilibrium models useful for addressing the issues outlined above. I have therefore tried to develop a tractable framework that captures team production in a simple manner. In addressing a specific question one would generally want to modify this structure along dimensions important for that question.

The remainder of this paper is organized as follows: Section 2 describes the team production economy. Section 3 provides the definition of a competitive equilibrium and two properties of an equilibrium. Section 4 examines how preferences and technology interact in this framework to determine work schedules and wages. Section 5 presents an illustrative policy experiment demonstrating the potential importance of team production in policy analysis. Section 6 contains some concluding remarks.

## 2. The Team Production E conomy

The team production economy has one period and is populated by a continuum of people with measure 1 . There are two types of people with measure $\lambda_{1}$ and $\lambda_{2}$ respectively, where $\lambda_{1}+\lambda_{2}=1$. E ach person has a time endowment of 1 each period
that can be allocated to either work, h, or leisure, I= 1-h. Type 1 and type 2 people are endowed with $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$ units of capital respectively. People types are distinguished by the following properties. First, type 1 and type 2 Iabor services are different factors of production. Second, types may have different preferences over leisure.

A useful starting point when laying out the model economy is to describe the team production technology and show that this technology gives rise to an aggregate production possibility set which is a convex cone, a standard feature of general equilibrium analysis. In deriving the production possibility set, it is first necessary to define an aggregate team production function and describe the commodity space used.

## A. Team Production Function

There are two types of workers in the economy, type 1 and type 2. A production team is a group of $d_{1}$ type 1 workers and $d_{2}$ type 2 workers, working $h$ hours with $k$ units of capital. Thus, a production team type can be characterized by a four-tuple $\left(h, k, d_{1}, d_{2}\right)$. The output of a type $\left(h, k, d_{1}, d_{2}\right)$ production team is given by the team production function

$$
\begin{array}{rll}
f\left(h, k, d_{1}, d_{2}\right)= & h k^{\theta_{k}} d_{1}^{\theta_{1}} d_{2}^{\theta_{2}} & \text { if } d_{1} \in[0, \underline{N}]  \tag{1}\\
& h k^{\theta_{k}} \underline{N}^{\theta_{1}} d_{2}^{\theta_{2}} & \text { if } d_{1}>\underline{N},
\end{array}
$$

where $\theta_{k}+\theta_{1}+\theta_{2}>1, \theta_{k}+\theta_{2}<1$, and $\theta_{k}, \theta_{1}, \theta_{2}>0$. Restricting the number of type 1 workers that are productive on a team is a convenient way of assuring that there will be an optimal team type or types in equilibrium.

The output of a team is proportional to how long the team operates. ${ }^{10}$ Given the assumption on the productivity of type 1 workers, there are decreasing returns to additional workers and machines above some level, which is assumed to be small relative to the size of the economy. I refer to teams operating $h$ hours as having a workweek of length h. Notice that the utilization of the capital stock varies with how long a team operates. I assume that each unit of capital can be used by only one team of workers.

Each worker on a team must work the hours that the team operates. Different teams may operate different number of hours. The formation of teams is costless to firms. Parameters are chosen so that teams are small relative to the size of the economy. Although the team technology may appear to introduce a nonconvexity in the aggregate production possibility set, we will see that, as in standard neoclassical models, this set is a convex cone. However, unlike typical neoclassical models, it cannot be characterized by a neoclassical aggregate production function with capital and aggregate hours of each of the labor types as inputs.

## B. Aggregate Team Production Function

Next, I derive an aggregate team production function for teams operating $h$ hours. This function gives the maximum output given K units of capital, $\mathrm{D}_{1}$ type 1 workers, and $D_{2}$ type 2 workers if all teams must operate $h$ hours. The specification of the team production function implies that, for a given $h$, output will be maximized by having only one team type operating, with that team type having $d_{1}$ equal to $\underline{N}$. Thus, output is maximized by having a measure $M$ equal to $D_{1} / \underline{N}$ of team production types ( $\mathrm{h}, \mathrm{K} / \mathrm{M}, \mathrm{N}, \mathrm{D}_{2} / \mathrm{M}$ ) operating. A little algebra produces the aggregate team production

[^2]function for teams operating h hours:
(2) $F\left(h, K, D_{1}, D_{2}\right)=A h K^{\theta_{k}} D^{-\theta_{k}-\theta_{2}} D_{2}$,
where $A=\underline{N}^{-\left(1-\theta_{k}-\theta_{1}-\theta_{2}\right)}$.
It will be convenient to define an aggregate production team as a group of $D_{1}$ type 1 workers and $D_{2}$ type 2 workers, each working $h$ hours, and $K$ total units of capital, with workers and capital assigned to production teams so that total output is maximized, as given by (2). As with a production team, an aggregate production team type can be characterized by a four-tuple ( $\mathrm{h}, \mathrm{K}, \mathrm{D}_{1}, \mathrm{D}_{2}$ ).

## C. Traded Commodities

Before defining the aggregate production possibility set, we must specify the commodities that are traded and priced in this economy. The key feature of the commodity space is that workweeks of different lengths and types are different commodities. Thus, there is a continuum of different labor inputs of each type. ${ }^{11}$ Introducing workweeks of different lengths as different commodities creates an indivisibility. People cannot work $2 / 3$ of a 40 -hour workweek and $1 / 3$ of a 30 -hour workweek. A $n$ analytically useful strategy for working with economies that have indivisibilities is to allow people to randomize over workweeks of different lengths using lotteries (following Prescott and Townsend [1984] and Rogerson [1988]). With the introduction of lotteries, people can work a 40-hour workweek with probability $2 / 3$ and a 30 -hour workweek with probability $1 / 3$.

The set of feasible workweek lengths is denoted by H , where $\mathrm{H} \subset[0,1]$. The exposition of the model economy is simplified by assuming that there is a large but

[^3]finite number of possible workweek lengths. For this case, $H \equiv\left\{h_{1}, h_{2}, \ldots, h_{N_{h}}\right\}$, with $h_{1}=0$, and $h_{N_{h}}=1$. Type 1 people have access to lotteries over type 1 labor workweeks of lengths $h \in H$, and type 2 people have access to lotteries over type 2 labor workweeks. People can only supply workweeks of their own type, and may work on only one team. A person cannot work a 20-hour workweek with one team and a 30hour workweek with a second team, but can choose to work on a team working 50 hours.

The commodity space, denoted by $L$, is $R^{2} \times M(H) \times M(H)$, where $M(H)$ denotes the set of signed measures on the Borel sigma algebra of H . An element of L is given by $\left(c, k, n_{1}, n_{2}\right)$, where $c$ is the consumption good, $k$ is the services of the capital stock, $n_{1}$ is a measure over type 1 labor workweeks, and $n_{2}$ is a measure over type 2 labor workweeks. One unit of capital produces one unit of capital services. W hen H is a finite set, $n_{1}$ is a vector and $n_{1}(h)$ is the measure of type 1 workweeks of length $h$. Similarly, $n_{2}$ is a vector and $n_{2}(h)$ is the measure of type 2 workweeks of length $h$.

## D. Aggregate Production Possibility Set

Before defining the aggregate production possibility set, I first define an aggregate production plan. Let J denote the set of feasible aggregate production team types, so that $J \subset R$. A gain for expositional clarity assume that J is a finite set, so that we can index possible aggregate team types by $j=\left\{1,2, \ldots, N_{\jmath}\right\}$, where $N_{J}$ is the number of possible aggregate team types. Next, let $m_{j}$ denote the measure of aggregate teams of type j operated, where $\mathrm{m}_{\mathrm{j}} \geq 0$. A $n$ aggregate production plan is a vector of $\mathrm{N}_{\mathrm{J}}$ numbers, $\left\{m_{1}, m_{2}, \ldots, m_{N}\right\}$, that describes how the aggregate inputs are distributed across teams of different types. The aggregate production possibility set, $Y$, is defined as follows.
$Y \equiv\left\{\left\{C, K, N_{1}, N_{2}\right\}\right.$ : there exists a production plan $\left.m \in R\right\lrcorner$ such that

$$
\begin{aligned}
& C \quad \leq \Sigma_{j} m_{j} A h_{j} K_{j}{ }_{k} D_{1 j} D^{1-\theta_{k}-\theta_{2}} D_{2 j}{ }^{\theta_{2}} \\
& \Sigma_{j} m_{j} K_{j} \quad \leq K \\
& \left.\Sigma_{j j} h_{j}=h\right\} m_{j} D_{1 j} \leq N_{1}(h) \quad \text { all } h \in H \\
& \Sigma_{\left.j j: h_{j}=h\right\}} m_{j} D_{2 j} \leq N_{2}(h) \quad \text { all } h \in H .
\end{aligned}
$$

The first constraint says that the total amount of the consumption good is less than or equal to the total output produced by all team types. The second constraint states that the capital allocated across all team types is less than or equal to the total capital available. The third and fourth constraints state that the amount of type 1 (type 2) workweeks of length $h$ allocated across all team types is less than or equal to the total amount of type 1 (type 2 ) workweeks of length $h$ available. It is immediate that $Y$ is a convex cone. As with models with neoclassical aggregate production functions, the number of firms is indeterminate. Thus, for convenience, we may act as if there is one firm.

## E. Preferences

The specification of this economy is completed by presenting people's preference ordering and their feasible consumption bundles. The utility of a type i person, $i \in\{1,2\}$, choosing the commodity point $x=\left(c, k, n_{1}, n_{2}\right)$ is given by

$$
\begin{equation*}
U_{i}(x) \equiv u(c)-\Sigma_{h} n_{i}(h) v_{i}(h), \tag{3}
\end{equation*}
$$

where $v_{i}: R_{+} \rightarrow R_{+}$and $u: R_{+} \rightarrow R, v_{i}(0)=0$, and $\lim _{c \rightarrow 0} u^{\prime}(c)=\infty$. N otice that $\sum_{h}$ $n_{i}(h) v_{i}(h)$ is the expected disutility of working for type $i$.

The consumption possibility set of a type i person is given by
(4) $\quad X_{i}\left(k_{i}\right) \equiv\left\{\left(c, k, n_{1}, n_{2}\right): k \leq k_{i}, \sum_{h_{\epsilon} H} n_{i}(h)=1, n_{i}(h) \geq 0\right.$ and $n_{j}(h)=0$ all $h \in H, j \neq i$

$$
c \geq 0, k \geq 0\},
$$

where $k_{i}$ is type i's capital stock endowment. The consumption possibility set of a type i person, $\mathrm{X}_{\mathrm{i}}\left(\mathrm{k}_{\mathrm{i}}\right)$, contains the following conditions: capital services are restricted by the capital stock endowment; $\mathrm{n}_{\mathrm{i}}$ is a probability measure; type i people cannot work a type j workweek for $j \neq i$; and the standard nonnegativity constraints apply.

## 3. Competitive Equilibrium

Next, I describe the decision problems faced by the firm and by individuals in this economy, provide the definition of a competitive equilibrium, and establish some equilibrium properties. The commodities traded are given by $x=\left(c, k, n_{1}, n_{2}\right)$. Prices are in terms of the consumption good. The rental price of capital is given by r . Next, $W_{1}$ and $w_{2}$ are pricing functions mapping signed measures into $R_{+}$. With a finite set of possible workweeks, $w_{1}$ is a vector of prices, where $w_{1}(h)$ is the price of a type 1 workweek of length h. That is, if a type 1 person works a type 1 workweek of length h with probability one, then that worker receives $\mathrm{w}_{1}(\mathrm{~h})$ units of the consumption good. Similarly, $\mathrm{w}_{2}$ is a vector of prices for type 2 workweeks.

## A. The Firm's Decision Problem

The firm in this economy rents capital, employs type 1 and type 2 workers to work weeks of various lengths, and decides how to allocate these resources across teams of different types. In hiring type 1 workers, the firm buys lottery contracts from type 1 people. These lottery contracts specify the probability of working type 1 workweeks of different lengths, possibly including a workweek of length 0 . The firm then uses a randomizing technology to determine which workweek each type 1 person will work. However, the firm, which sells a large number of contracts, faces no
uncertainty as to the number of type 1 workers who will work different workweek lengths. The hiring of type 2 workers occurs in the same fashion. One interpretation of this contract is that the firm is providing full insurance to workers against the uncertainty of the length of their weekly hours.

Given prices $\left(r, W_{1}, W_{2}\right)$, the firm solves the following optimization problem:

$$
\begin{align*}
& \max C-r K-\sum_{h} W_{1}(h) N_{1}(h)-\sum_{h} W_{2}(h) N_{2}(h)  \tag{5}\\
& \quad \text { subject to } \\
& \left(C, K, N_{1}, N_{2}\right) \in Y,
\end{align*}
$$

where $N_{1}(\mathrm{~h})$ and $\mathrm{N}_{2}(\mathrm{~h})$ are the total number (or, more precisely, measure) of type 1 and type 2 workweeks of length $h$.

## B. Individuals' Decision Problems

Individuals in this economy purchase the consumption good and sell capital and labor services to firms. In selling labor services, an individual sells a lottery contract that specifies the probability of working different workweek lengths. The amount an individual receives for a given lottery contract does not depend on the lottery's outcome, that is, on the length of the workweek the individual works ex post. ${ }^{12}$

A type $i$ person, $i \in\{1,2\}$, solves the problem

[^4]\[

$$
\begin{align*}
& \max \left\{u(c)-\sum_{h} n_{i}(h) v_{i}(h)\right\}  \tag{7}\\
& \text { subject to } \\
& \left(c, k, n_{1}, n_{2}\right) \in X_{i}\left(k_{i}\right)  \tag{8}\\
& c \leq r k+\sum_{h} w_{i}(h) n_{i}(h)
\end{align*}
$$
\]

## C. Definition of Equilibrium

A competitive equilibrium for the model economy is an allocation ( $x_{1}{ }^{*}, x_{2}{ }^{*}, y^{*}$ ) and a price system ( $r, w_{1}, w_{2}$ ) such that
i) $\quad \mathrm{X}_{\mathrm{i}}{ }^{*}$ maximizes $\mathrm{U}_{\mathrm{i}}(\mathrm{x})$ subject to $\mathrm{x} \in \mathrm{X}_{\mathrm{i}}\left(\mathrm{k}_{\mathrm{i}}\right)$, and the budget constraint (9), $\mathrm{i} \in\{1,2\}$
ii) $\quad y^{*}$ maximizes (3) subject to $y \in Y$, and
iii) $\quad \lambda_{1} x_{1}{ }^{*}+\lambda_{2} x_{2}{ }^{*}=y^{*}$.

I restrict my attention to anonymous equilibrium allocations having the property that all people of the same type choose the same commodity point. ${ }^{13}$ This does not imply that all people of the same type work the same workweek length, since the allocation will generally involve randomizing over different workweek lengths.

When the set of feasible workweeks H is finite, the commodity space is finite dimensional and it is straightforward to show that the first and second welfare theorems hold, and that there exists an equilibrium. ${ }^{14}$ Therefore, we can study the properties of the anonymous Pareto optima of this economy to establish properties of competitive equilibrium allocations.

[^5]The set of anonymous Pareto optima is the closure of the set of solutions to the weighted social planner's problems with positive weights. The $\Psi$-weighted social planner's problem, where $0<\Psi<1$, is to maximize

## (10) $\Psi \lambda_{1} U_{1}\left(x_{1}\right)+(1-\Psi) \lambda_{2} U_{2}\left(x_{2}\right)$

subject to $x_{1} \in X_{1}(K), x_{2} \in X_{2}(K), y \in Y$, and $\lambda_{1} x_{1}+\lambda_{2} x_{2}=y$. A solution to this problem exists, given that the $U_{i}$ functions are continuous and the constraint set is compact. ${ }^{15}$

The characteristics of Pareto optimal allocations can be derived by analyzing a much simpler equivalent problem. For this problem, we no longer distinguish between the organization of production ( m ) and the supply of work schedules ( $\mathrm{n}_{\mathrm{i}}$ ). Let the event j now be characterized by a triplet ( $\mathrm{h}, \mathrm{k}, \mathrm{d}$ ), where an event is interpreted as an aggregate team working $h$ hours with $k$ units of capital per type 1 worker and $d$ type 2 workers per type 1 worker. A ssume there is a finite number of possible ( $\mathrm{h}, \mathrm{k}, \mathrm{d}$ ) triplets indexed by j . The set of Pareto optimal allocations comprises the solutions to the following problem:
(11) $\max _{c_{1}, c_{2}, m_{2}} \Psi \lambda_{1}\left\{u\left(c_{1}\right)-\Sigma_{j} m_{j} v_{1}\left(h_{j}\right)\right\}+(1-\Psi) \lambda_{2}\left\{u\left(c_{2}\right)-\Sigma_{j} m_{j} d_{j} v_{2}\left(h_{j}\right)\right\}$

$$
\begin{array}{ll}
\lambda_{1} C_{1}+\lambda_{2} C_{2} & \leq \Sigma_{j} m_{j} A h_{j} k_{j}^{\theta_{k}} d_{j}^{\theta_{2}} \\
\Sigma_{j} m_{j} k_{j} & \leq K \\
\Sigma_{j} m_{j} & =\lambda_{1} \\
\Sigma_{j} m_{j} d_{j} & =\lambda_{2} . \tag{15}
\end{array}
$$

[^6]The solution to this problem has the same measure of each person type working $h$ hours on teams with $k$ units of capital per type 1 worker and d type 2 workers per type 1 worker, and the same $c_{1}$ and $c_{2}$ as the solution to the original problem has.

The following proposition states that any Pareto optimal allocation, and hence any equilibrium allocation, will have people working at most four different workweek lengths. The remaining workweek lengths will not be traded.

PROPOSITION 1. For any $\Psi \in(0,1)$, the solution to maximizing (11) subject to (12) - (15) is unique and the number of triplets ( $h, k, d$ ) receiving strictly positive mass is less than or equal to 4 . ${ }^{16}$

Proof. First, define $W\left(c_{1}, c_{2}\right)$ as the solution to maximizing (11) with respect to $m$, given $C_{1}$ and $C_{2}$. Results from linear programming guarantee that the solution is unique and places strictly positive mass on 4 points at most. Furthermore, $W\left(c_{1}, c_{2}\right)$ is a concave function. Next, solve $\max _{c 1, c 2 \geq 0} \Psi \lambda_{1} u\left(c_{1}\right)+(1-\Psi) \lambda_{2} u\left(c_{2}\right)+W\left(c_{1}, c_{2}\right)$, which is a strictly concave function and, given that there is an upper bound on total consumption given by feasibility, has a unique solution.

For each initial distribution of capital, the next proposition guarantees that the equilibrium allocation will be unique.

PROPOSITION 2. Given an initial capital distribution ( $k_{1}, k_{2}$ ), there is a unique competitive equilibrium allocation.

Proof. Since the First W elfare Theorem holds for this economy, a competitive

[^7]equilibrium allocation solves the Pareto problem for some weight $\Psi$. F or each $\Psi$ there is a unique allocation $x_{1}(\Psi), x_{2}(\Psi)$ that solves the Pareto problem and a unique initial capital allocation $\mathrm{k}_{1}(\Psi), \mathrm{k}_{2}(\Psi)$ that corresponds to a competitive equilibrium without transfers. Given total capital $K \equiv \lambda_{1} k_{1}+\lambda_{2} k_{2}$, it can be shown that $k_{1}(\Psi)$ is a strictly increasing function of $\Psi$. This implies that there is at most one weight $\Psi$ for which ( $\mathrm{k}_{1}$ , $\mathrm{k}_{2}$ ) is a competitive equilibrium without transfers, and the unique solution to this Pareto problem is the unique equilibrium allocation.

When the set of feasible workweeks H is no longer a finite set but instead is the interval from 0 to 1 , and the set of feasible aggregate plant types J is a rectangular subset of $R$, then the summations in the various optimization problems are replaced by integrals over measures. In this case, the commodity space is infinite dimensional and the Second W elfare Theorem is much more difficult to prove. However, this theorem has been proved for economies with similar constructs to that of the economy presented here, and I conjecture that propositions 1 and 2 continue to hold for this limiting economy. ${ }^{17}$ For the remainder of this paper, I assume that there is a continuum of feasible workw eek lengths, $H \equiv[0,1]$, and the set of feasible aggregate production team types, $J$, is a rectangle in $R^{4}$.

## 4. Determination of Work Schedules

In this section, I examine more closely how the team technology and individuals' preferences interact to determine work schedules, wages, and employment in this environment. It is useful to begin by looking at the firm's problem in more detail. Necessary conditions for a solution to the firm's optimization problem are

[^8]\[

$$
\begin{align*}
A h_{j} K_{j}{ }_{k}^{\theta} D_{1 j}{ }^{1-\theta_{k}-\theta_{2}} D_{2 j}{ }_{2}^{\theta}-r K_{j}-W_{1}\left(h_{j}\right) D_{1 j}-W_{2}\left(h_{j}\right) D_{2 j} & \leq 0  \tag{16}\\
& \text { for all team types } j \in J, \\
= & 0 \quad \text { for all types } j \text { with } m_{j}>0 .
\end{align*}
$$
\]

These conditions state that no aggregate team type earns strictly positive profits in equilibrium, and the aggregate team types that are operated generate zero profit. Since the left-hand side of (16) is homogeneous of degree zero in ( $K, D_{1}, D_{2}$ ), only the ratios $K / D_{1}$ and $D_{2} / D_{1}$ are determined for aggregate production team types with zero profit in equilibrium. ${ }^{18}$

Next, define the profit function

$$
\begin{equation*}
\pi\left(h, K, D_{2} ; p\right)=A h K^{\theta_{k}} D_{2}^{\theta_{2}}-r K-w_{1}(h)-w_{2}(h) D_{2} \tag{17}
\end{equation*}
$$

where $p \equiv\left(r, w_{1}, w_{2}\right)$. A ny aggregate team type $\left(h^{*}, K^{*}, D_{1}{ }^{*}, D_{2}{ }^{*}\right)$ that is operated in equilibrium must satisfy

$$
\begin{equation*}
\left\{h^{*}, K^{*} / D_{1}{ }^{*}, D_{2}^{*} / D_{1}{ }^{*}\right\} \in \operatorname{argmax} \pi\left(h, K, D_{2} ; p\right) \text { s.t } h \leq 1 \text { and nonnegativity. } \tag{18}
\end{equation*}
$$

A marginal condition for an equilibrium workweek length, assuming an interior solution, is given by setting the derivative of the profit function $\pi\left(h, K, D_{2} ; p\right)$ with respect to $h$ equal to zero, which is written

$$
\begin{equation*}
A K^{\theta}{ }^{\theta} D_{2}{ }^{\theta}=w(h)+w(h) D_{2}, \tag{19}
\end{equation*}
$$

${ }^{18}$ Recall that for a given $h$ and output level there is a unique production team type that is operated in equilibrium.
where $w(h)$ and $w(h)$ denote the derivatives of $w_{1}(h)$ and $w_{2}(h)$ with respect to $h$. Equation (19) states that, at an interior equilibrium workweek $h$, the additional output from increasing $h$ must equal the increase in wage payments. It is clear that the shape of the wage functions is critical in determining the workweek lengths operated in equilibrium.

Notice that conditional on $h, \pi(h, \cdots ; p)$ is a strictly concave function in $K$ and $D_{2}$. The solution to the problem of maximizing $\pi\left(h, K, D_{2} ; p\right)$ by choice of $K$ and $D_{2}$ is uniquely solved by

$$
\begin{align*}
& K^{*}(h ; p)=B_{k} r^{-\left(1-\theta_{2}\right) / \theta_{1}} h^{1 / \theta_{1}} W_{2}(h)^{-\theta_{2} / \theta_{1}},  \tag{20}\\
& D_{2}^{*}(h ; p)=B_{d} r^{-\left(\theta_{k}\right) / \theta_{1}} h^{1 / \theta_{1}} W_{2}(h)^{\left(\theta_{k}-1 / \theta_{1}\right.}
\end{align*}
$$

where $B_{k}$ and $B_{d}$ are positive constants that depend on $\theta_{2}$ and $\theta_{k}$, and $\theta_{1}=1-\theta_{k}-\theta_{2}$. Substituting (20) and (21) into (17), we can define the implicit profit function by

$$
\begin{equation*}
\Pi(h ; p) \equiv \pi\left[h, K^{*}(h ; p), D_{2}^{*}(h ; p) ; p\right]=B r^{-\theta_{k}} \theta_{1} h^{1 / \theta_{1}} w_{2}(h)^{-\theta_{2} / \theta_{1}}-w_{1}(h) \tag{22}
\end{equation*}
$$

where $B$ is a positive constant that depends on $\theta_{2}$ and $\theta_{k}$. This function returns the profit the firm can earn at each workweek length if it chooses to employ one type 1 worker. The problem of the firm then boils down to finding an $h$ that maximizes $\Pi(h ; p)$, given prices.

The next step is to characterize the shape of the equilibrium wage schedules. Not surprisingly, individuals' preferences over workweeks determine the shape of the wage schedules. Necessary conditions for a solution to individuals' maximization problems are

$$
\begin{equation*}
\omega v_{i}(h)+v_{i} \geq w_{i}(h) \quad \text { for all } h \in H \tag{23}
\end{equation*}
$$

$$
=w_{i}(h) \quad \text { if } n_{i}(h)>0, \text { and }
$$

$$
\begin{equation*}
v_{i} n_{i}(0)=0 \tag{24}
\end{equation*}
$$

where $\omega$ is the inverse of the marginal utility of consumption, and $v_{i}$ is the multiplier on the constraint that an individual cannot place more than one unit of probability across workweek lengths. The multiplier $v_{i}$ is nonnegative and equals zero if type $i$ people place positive weight on working a workweek of length 0 . That is, vi is zero if type i people are not all working in equilibrium. The conditions in (23) hold with equality if a lottery contract is traded that has a strictly positive probability of working a type i workweek of length $h$.

An interesting feature of this framework is that some workweek lengths are not traded in equilibrium. The equilibrium prices for these workweek lengths are typically not uniquely determined. However, since $\omega$ and $v_{i}$ are determined in equilibrium, the left-hand side of the equations in (23) provide an upper bound for these equilibrium prices. For a given workweek length, this upper bound gives the wage at which people are just indifferent to supplying, or not supplying, that workweek length. These upper bounds can be interpreted as the supply reservation wages of workweeks of different lengths, which are defined as

$$
\begin{align*}
& \mathrm{w}\left(\mathrm{~h} ; \omega_{1}{ }^{*}, v_{1}{ }^{*}\right)=\omega_{1}{ }^{*} \mathrm{v}_{1}(\mathrm{~h})+v_{1}{ }^{*},  \tag{25}\\
& \mathrm{w}\left(\mathrm{~h} ; \omega_{2}{ }^{*}, v_{2}{ }^{*}\right)=\omega_{2}{ }^{*} \mathrm{v}_{2}(\mathrm{~h})+v_{2}{ }^{*}, \quad \text { for all } \mathrm{h} \in \mathrm{H},
\end{align*}
$$

where $\omega_{1}{ }^{*}, \omega_{2}{ }^{*}, v_{1}{ }^{*}$, and $v_{2}{ }^{*}$ are equilibrium values. ${ }^{19} \mathrm{~A}$ useful procedure for selecting

[^9]equilibrium prices for workweeks not traded in equilibrium is to set these prices using the equations in (25).

Supply reservation wages are the wages the firm must pay to attract workers at various workweek lengths. These wage functions can be substituted into the profit functions (17) and (22) to gain insight into the equilibrium work schedules and employment rates that arise in this model under different assumptions for preferences over leisure. This is now done for a particular example.

## A. An Example

In this subsection I present an example and determine the nature of the equilibrium work schedules and employment for different parameter choices. First, let preferences over workweek lengths be given by

$$
\begin{align*}
& \mathrm{V}_{1}(\mathrm{~h})=\gamma_{1} \mathrm{~h}^{\alpha_{1}},  \tag{26}\\
& \mathrm{~V}_{2}(\mathrm{~h})=\gamma_{2} \mathrm{~h}^{\alpha_{2}},
\end{align*}
$$

where $\gamma_{1}>0, \gamma_{2}>0, \alpha_{1} \geq 1$, and $\alpha_{2} \geq 1$. The disutility of a workweek of length h for each type is an increasing, convex function of $h$. The parameters $\alpha_{1}$ and $\alpha_{2}$ can be interpreted as risk aversion parameters that determine how averse people are to randomizing over workweeks of different lengths. Loosely speaking, the larger the value of $\alpha$ the greater is the loss in utility from randomizing.

Next, I characterize the equilibrium workweek lengths and employment rates for various assumptions on the preference parameters. Since we are ultimately interested in equilibria with less than full employment, for the sake of simplicity I restrict my attention to the subset of the parameter space, call it $\Theta$, for which there exists an equilibrium in which there is a strictly positive measure of type 2 people not employed (that is, $n_{2}(0)>0$ ). The propositions given in this subsection apply for the
parameters in the set $\Theta$. Since this example is simply intended to provide insight into the workings of the model, the size of this set is unimportant.

The functional forms of the supply reservation wages are

$$
\begin{align*}
& w\left(h ; \omega_{1}, v_{1}\right)=\omega_{1} \gamma_{1} h^{\alpha_{1}}+v_{1},  \tag{27}\\
& w\left(h ; \omega_{2}, v_{2}\right)=\omega_{2} \gamma_{2} h^{\alpha_{2}}+v_{2},
\end{align*}
$$

where $v_{2}$ is zero in equilibrium for parameters in the set $\Theta$.
The following proposition and corollary guarantee that there is a unique workweek length.

PROPOSITION 3. $\quad \Pi^{\prime}(h ; p)$ equals zero for at most one $h \in(0,1)$.

Proof. This proposition is proved by substituting the supply reservation wages (27) into the profit function (22) and differentiating with respect to $h$. It is then easy to show that, for any $\omega_{1}, \omega_{2}$, and $v_{1}$, there is at most one $h$ that sets this derivative to zero.

COROLLARY 1. There is a unique, strictly positive workweek length $h$ at which all production teams operate.

This corollary follows immediately from proposition 3, the continuity of the profit function $\Pi(\cdot ; p)$, and the zero-profit equilibrium condition.

The next two propositions provide insight into how the equilibrium workweek length and employment vary with the risk aversion parameters.

PROPOSITION 4. If $\alpha_{1}=1$ and $\alpha_{2}=1$, then only production team types with $h=1$ operate in equilibrium.

Proof. This proposition is proved by substituting the supply reservation wages (27) into the profit function (17). The proof is by contradiction. Suppose some aggregate team type with $0<h<1$ earned zero profits. Then increasing $h$ would result in strictly positive profits, since output increases proportionally with h while wage payments go up at most proportionally, and rental payments to capital are unchanged. Since no aggregate team type earns strictly positive profits in equilibrium, this is a contradiction.

The intuition behind this result is straightforward. First, note that a property of the aggregate production team function (2) is that for a given amount of total type 1 hours and type 2 hours, output is greatest when $h$ is set to $1 .{ }^{20}$ With linear preferences over workweek lengths, people care only about their expected hours of work. Since output is increased by setting h higher and using fewer workers, and workers suffer no loss in utility from randomization, the workweek is set to 1 in equilibrium. The proof of this proposition shows that wages cannot be proportional to workweek length if there is to be an equilibrium workweek length less than 1.

PROPOSITION 5. If $\alpha_{i} \geq 1 / \theta_{i}$, then $n_{i}(h)=1$ at the equilibrium $h, i \in\{1,2\}$.

Proof. The strategy of this proof is again to substitute (27) into (22). The proof is by contradiction. Suppose $\alpha_{i} \geq 1 / \theta_{i}$ and $n_{i}(h)<1$. It is then easy to show that the derivative

[^10]of the profit function with respect to $h$ is strictly negative for all positive $h$. This implies strictly negative profits for all strictly positive workweek lengths, and hence no production. But there must be positive production in equilibrium, given the assumption on the preferences over consumption.

This proposition says that if people are sufficiently averse to randomizing over workweek lengths, then no randomization occurs for that type in equilibrium.

To illustrate the possible shapes of the equilibrium profit function $\Pi(\cdot ; p)$, I compute the equilibria of two model economies. The first economy has both $\alpha_{1}$ and $\alpha_{2}$ set to 1 , so that proposition 4 applies. The second economy is identical except that $\alpha_{1}$ is set to 2 , so that proposition 5 applies (in both economies $1 / \theta_{1}=2$ ). The parameter values of these economies are given in table 1, along with the equilibrium employment rates and workweek length for each economy. Figure 1 shows the equilibrium implicit profit functions for the two economies when equilibrium wage rates are set to the supply reservation wages defined in (30).

The example in this subsection illustrates that the crucial elements determining equilibrium workweek length(s) in this framework are the tensions between risk-averse people who may have different preferences over work and a technology in which heterogeneous workers must work in teams and in which the distinction between hours per worker and number of workers is important for productivity. While this specification of preferences is useful as an example, the following proposition shows why it is less useful for applied work.

PROPOSITION 6. Either $n_{1}(0)=0$ or $h=1$ in equilibrium.

Proof. The proof is by contradiction. Suppose that $n_{1}(0)>0$ and $h<1$ in equilibrium. Proposition 3, the continuity of the profit function $\Pi(\cdot ; p)$, and the zero profit equilibrium condition imply that only at the equilibrium $h$ does $\Pi^{\prime}(h ; p)$ equal zero.

This implies that $\Pi(0 ; p)$ is less than zero, which in turn implies that $v_{1}>0$, or equivalently, that $n_{1}(0)=0$, a contradiction.

Recall that in this section I am only considering economies in which some type 2 people are not employed. This proposition states that for such economies either all type 1 workers are employed or the equilibrium workweek length is one. Since ultimately we are likely to be interested in economies with less than full employment of both types and with workweek lengths less than one in equilibrium, I consider a different specification of preferences in the next section.

## 5. A Simple Policy Experiment

As a preliminary step in exploring the potential importance of team production in analyzing labor market policies, I present an illustrative policy experiment that addresses a basic question: Can modeling team production explicitly have important implications for analyzing labor market policies and regulations in which firm-level decisions about the work schedules and employment of heterogeneous workers play a crucial role?

To investigate this question I examine the consequences of an employment tax on high-wage workers in two model economies that are identical except for the specification of the technologies. The first economy uses individual production (teams with one worker). W orkers with different wages produce independently. The second economy uses team production in which workers with different wages must work together.

The experiment is organized as follows: I first select parameter values so that the equilibrium of each economy reproduces a common set of observations that are roughly drawn from U.S. data. I then compare the implications of imposing a per person tax on the employment of high-wage workers for the work schedules, wages,
and employment of both high-wage and low-wage workers in the two economies.
The functional forms for preferences used in both economies are

$$
\begin{align*}
u_{1}(c) & =u_{2}(c)=\log (c), & \text { and } &  \tag{28}\\
v_{i}(h) & =-\gamma_{i}\left[(1-h)^{\sigma}-1\right] / \sigma & & \text { for } \sigma_{i} \neq 0  \tag{29}\\
& =-\gamma_{i} \log (1-h) & & \text { for } \sigma_{i}=0 \quad i \in\{1,2\},
\end{align*}
$$

where $\gamma_{i} \geq 0$ for $i \in\{1,2\}$ and $\sigma<1$. Preferences for type 1 and type 2 people differ only in the parameters $\gamma_{1}$ and $\gamma_{2}$. This specification of $v_{i}(\mathrm{~h})$ guarantees interior solutions for equilibrium workweek lengths, since $v(\mathrm{~h}) \rightarrow \infty$ as $\mathrm{h} \rightarrow 1$.

In both economies the firm solves the following optimization problem:

$$
\begin{align*}
& \max C-r K-\sum_{\mathrm{h}}\left\{W_{1}(\mathrm{~h})+\chi_{1}\right\} N_{1}(\mathrm{~h})-\sum_{\mathrm{h}} W_{2}(\mathrm{~h}) \mathrm{N}_{2}(\mathrm{~h})  \tag{30}\\
& \quad \text { subject to } \\
& \left(\mathrm{C}, \mathrm{~K}, \mathrm{~N}_{1}, \mathrm{~N}_{2}\right) \in \mathrm{Y}, \tag{31}
\end{align*}
$$

where $\chi_{1}$ is the employment tax on type 1 workers, who are parameterized to be the high-wage workers in equilibrium. Tax revenues are thrown into the ocean. The difference between the two economies is the specification of the production set $Y$. For the team production economy this set was defined in section 2. For the individual production economy this set will be defined below.

## A. Team Production Economy

The team production economy is the economy described in sections 2 and 3 , except that there is a continuum of workweek lengths between 0 and 1 and the set J of feasible aggregate production team types is a rectangle in $R^{4}$. For the parameter values used in this experiment, the equilibrium of the team production economy has all teams
operating the same workweek length. This is true even when employment tax rates are different for the two types.
B. Individual Production Economy

The individual production economy is similar to the economy presented in Hornstein and Prescott (1993), but extended to include heterogeneous workers. Production in this economy is done by individuals (teams with one worker), working independently. A type i person working $h$ hours with $k$ units of capital produces

$$
\begin{equation*}
y_{i}=A_{i} h k^{\theta_{k}} \tag{32}
\end{equation*}
$$

units of output. ${ }^{21}$
W hen there is a finite number of possible hours-capital pairs given by the set $J$, where $J \subset R$, the aggregate production possibility set is derived in the manner described in section 2. Let $m$ denote the measure of type $i$ individuals, $i \in\{1,2\}$, working $h_{j}$ hours with $k_{j}$ units of capital, $j \in J$. An aggregate production plan is a pair of measures, $\mathrm{m}^{1}$ and $\mathrm{m}^{2}$. The aggregate production possibility set for the individual production economy is given by
$Y \equiv\left\{\left\{C, K, N_{1}, N_{2}\right\}\right.$ : there exists a production plan $\mathrm{m}^{1}, \mathrm{~m}^{2}$ such that

$$
\begin{array}{ll}
C & \leq \Sigma_{j} m A_{1} h_{j} k_{j}^{\theta_{k}}+\Sigma_{j} m A_{2} h_{j} \\
\Sigma_{j}(m+m) k_{j} & \leq K \\
\Sigma_{\left.j: h_{j}=h\right\}} m & \leq N_{1}(h) \quad \text { all } h \in H
\end{array}
$$

${ }^{21}$ This economy is equivalent to the team production economy where type 1 and type 2 workers are assumed to be perfect substitutes, that is, where output is $y=h$ $K^{\theta_{k}}\left[\left(A_{1} D_{1}\right)^{\rho}+\left(A_{2} D_{2}\right)^{\rho}\right]^{\left(1-\theta_{k} / \rho\right.}$, and $\rho=1$.

$$
\left.\Sigma_{j: \mathrm{hj}}^{\mathrm{h}=\mathrm{h}\}} \mathrm{m} \quad \leq \mathrm{N}_{2}(\mathrm{~h}) \quad \text { all } \mathrm{h} \in \mathrm{H} \quad\right\} .
$$

For the individual production economy used in this section, there is a continuum of feasible workw eek lengths; the set of feasible hours-capital pairs J is a rectangular subset of $R$.

All other aspects of the individual production economy are the same as in the team production economy. A key property of an equilibrium of the individual production economy is given in the following proposition.

PROPOSITION 7. An equilibrium of the individual production economy has type 1 people placing mass only on workweek lengths 0 and $h_{1}$, and type 2 people placing mass only on workweek lengths 0 and $h_{2}$.

Proof. The proof of this proposition is a simple extension of the proof provided in Hornstein and Prescott (1993).

The next step is to choose parameter values for both economies.

## C. Parameter Selection

The parameter values are selected so that the equilibria of the two model economies without taxes are identical. Each economy has nine parameters, chosen so that type 1 people have higher wages and a higher employment rate than type 2 people. The percentage of the population that is type 1 is set to 20 percent. The capital endowments are chosen so that the ratio of the endowments is equal to the ratio of the equilibrium labor incomes, and are normalized so that total capital is 1 . The parameters $\gamma_{1}$ and $\gamma_{2}$ are chosen so that the employment-population ratio of type 1 people is .9, and the total employment-population ratio is .75. The parameter $\sigma$ is
selected so that the hours per worker of both type 1 and type 2 workers is . 40 . A ssuming there are 100 hours in a week that are not allocated to sleep and personal care, this corresponds to a 40 hour workweek. Finally, $\theta_{k}$ is chosen so that capital's share of income is . 36 .

The remaining parameters are ( $\mathrm{A}, \theta_{1}$ ) for the team production economy and $\left(A_{1}, A_{2}\right)$ for the individual production economy. These parameters are selected so that the hourly wage rate of type 1 workers is 2 times the hourly wage rate of type 2 workers, and so that total output is 1 . The parameter values for both economies are given in table 2.

## D. Results

This section reports the consequences of introducing a tax on the employment of high-wage workers in an economy with individual production and in an economy with team production. In each model economy, this tax is chosen to be 1 percent of the equilibrium weekly wage rate of the type 1 workers in that economy without taxes. Table 3 reports how the equilibrium employment, hours, and wages are affected by the employment tax in the two economies.

The first column in table 3 provides equilibrium prices and quantities in the two economies without taxes. Recall that the parameters were chosen so that these equilibria are the same. The second and third columns report the percent change in the equilibrium prices and quantities in the individual and team production economies, respectively, when the employment tax is imposed.

For the individual production economy the tax on type 1 workers has, qualitatively, the expected effect on employment and hours of type 1 workers. That is, the firm hires fewer type 1 workers to work longer hours. The 1 percent tax on type 1 workers results in a decline in their employment rate of 3 percent. Perhaps surprisingly, the hourly wage rate is unchanged. The depressing effect of the tax on type 1 wage rates is offset by the fact that type 1 workers are now relatively more scarce and are working longer hours, both of which suggest higher hourly wage rates. The tax has very little effect on the employment, hours, and wages of type 2 workers.

The consequences of the employment tax on type 1 workers in the team production economy are quite different. While the qualitative effect on employment and hours of type 1 workers is the same, the employment rate of type 1 people falls by only 1.4 percent, while the workweek increases by less than half the increase in the individual production economy. The hourly wage rate of type 1 workers now falls by 0.4 percent. Type 2 people are no longer unaffected by the employment tax on type 1 workers when there is team production. The firm hires 1.0 percent fewer type 2 workers to work the longer workweek. However, the hourly wage rate of the type 2
workers increases by 0.3 percent.
To summarize, the effect of a 1 percent employment tax on type 1 workers is quite different in the model with team production, relative to the model where workers produce independently. In the individual production economy, the decline in employment falls exclusively on type 1 workers, and hourly wage rates are essentially unchanged. In the model with team production, however, the decline in employment is felt by both types, while the hourly wage rate disparity between the two types narrows by 0.7 percent. The hourly wage rate of type 2 workers actually increases in the model with team production.

## 6. Concluding Remarks

The objective of this paper has been to incorporate team production into a tractable general equilibrium framework. The structure of equilibrium work schedules, wages, and employment was explored for a specific example, and an illustrative policy experiment suggested that modeling team production can be important when analyzing labor market policies that affect firm-level decisions about the work schedules and employment of heterogeneous workers.

A potential advantage of this framework for the quantitative analysis of labor market policies is that the technology is specified closer to the establishment level than in models that use an aggregate production function. M odeling the interaction between hours and employment at the establishment level may be crucial when analyzing certain policies. Furthermore, an abundance of data collected at the establishment level and microeconomic studies that use this data are available to guide the specification of the functional forms and to assign parameter values.

As I noted in the introduction, the model presented here captures team production in a simple and stark manner. Furthermore, potentially important considerations such as various fixed costs of employment, for example commuting time
and set-up costs, have been abstracted from. Addressing specific policy questions will generally require extending this framework along such dimensions, which is a task for future work.

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TABLE 1

## PARAMETER VALUES FOR EXAMPLES

Economy 1: $\alpha_{1}=1$
Economy 2: $\alpha_{1}=2$

> | Both Economies |  |
| :---: | :---: |
| $\alpha_{2}=$ | 1.000 |
| $\lambda_{1}=$ | 0.500 |
| $\mathrm{k}_{1}=$ | 1.000 |
| $\mathrm{k}_{2}=$ | 1.000 |
| $\gamma_{1}=$ | 2.308 |
| $\gamma_{2}=$ | 1.852 |
| $\theta_{k}=$ | 0.350 |
| $\theta_{2}=$ | 0.150 |
| $A=$ | 1.000 |

## EQUILIBRIUM WORKWEEKS AND EMPLOYMENT

$$
\text { Economy 1: } \alpha_{1}=1 \quad \text { Economy 2: } \alpha_{1}=2
$$

W orkweek (h) $1.0 \quad 0.583$

Employment R ates

| Type $1\left(n_{1}\right)$ | 0.4 | 1.000 |
| :--- | :--- | :--- |
| Type $2\left(n_{2}\right)$ | 0.2 | 0.343 |

## TABLE 2

PARAMETER VALUES FOR TWO ECONOMIES

| PARAMETER PRODUCTION | TEAM PRODUCTION ECONOMY | INDIVIDUAL PROD. ECONOMY |
| :---: | :---: | :---: |
| $\lambda_{1}$ | 0.2000 | 0.2000 |
| $\mathrm{k}_{1}$ | 1.9355 | 1.9355 |
| $\mathrm{k}_{2}$ | 0.7661 | 0.7661 |
| $\sigma$ | -0.7273 | -0.7273 |
| $\gamma_{1}$ | 1.1495 | 1.1495 |
| $\gamma_{2}$ | 1.4519 | 1.4519 |
| $\theta \mathrm{k}$ | 0.3600 | 0.3600 |
| -- $\quad \theta_{1}$ | 0.2477 | n.a. ${ }^{22}$ |
| A | 4.7665 | n.a. |
| A 1 | n.a. | 4.0810 |
| $A_{2}$ | n.a. | 2.6189 |

${ }^{22}$ not applicable

TABLE 3
EFFECT OF A 1 PERCENT EMPLOYMENT TAX ON TYPE 1 WORKERS

Effect of Employment Tax on Type 1 W orkers

| No | Individual Prod. | Team Prod. <br> Tax |
| :---: | :---: | :---: |
| Economy | Economy |  |

TAXES
$\chi 1$
total tax revenue

WORK SCHEDULES
workweek length

| type 1 | 0.400 | $1.7 \%$ | $0.7 \%$ |
| :--- | :--- | :--- | :--- |
| type 2 | 0.400 | 0.0 | 0.7 |

EMPLOYMENT
employment rates
type 1
0.900
type 2
total
0.713
0.750
total hours per capita 0.300
WAGES per hour
type 1
type 2
3.441
1.720
-3.0
-1.4
0.1
$-0.7$
-1.0
$-1.1$
$-0.3$
$-0.5$

[^11]
[^0]:    ${ }^{8}$ This use of "team" differs from the meaning developed in M arshak (1954) and M arshak and Radner (1972).

[^1]:    ${ }^{9}$ This observation has received considerable attention in recent years in the labor supply literature. See, for example, Altonji and Paxson (1988,1990), K ahn and Lang (1991), and Dickens and Lundberg (1993). In the applied general equilibrium literature, Rogerson (1988) introduced indivisible labor in which people can only choose to work an exogenously given number of hours.

[^2]:    ${ }^{10}$ The technology could be written more generally to include features such as a fixed set-up time for operating and the possibility of decreasing returns in the number of hours a team is operated. One example drawn from the labor demand literature has $y=(h-s)^{\varepsilon} f\left(k, d_{1}, d_{2}\right)$ for $h \geq s \geq 0$, where $s$ is the set-up time per worker and $0<\varepsilon \leq 1$.

[^3]:    ${ }^{11}$ Rosen (1974) works with an economy that has a continuum of differentiated products. M as-Colell (1975) introduced this feature into general equilibrium theory.

[^4]:    ${ }^{12}$ Several decentralizations have been put forth. Hansen (1985) shows that equilibria with lotteries are equivalent to equilibria in which people are paid according to the workweek actually worked as determined by the lottery, but where people have access to actuarially fair insurance. Shell and W right (1993) reinterpret equilibria with lotteries as A rrow-D ebreu equilibria without lotteries when sunspots are introduced. Cole and Prescott (1994) employ an economy with gambles and deterministic exchanges as a decentralization.

[^5]:    ${ }^{13}$ See Hornstein and Prescott (1993) or Cole and Prescott (1994) for a discussion of this restriction.
    ${ }^{14}$ See Stokey and Lucas (1989).

[^6]:    ${ }^{15}$ The consumption possibility set can be made compact by specifying an upper bound on consumption that is greater than the maximum possible output.

[^7]:    ${ }^{16}$ This proposition holds so long as the matrix defined by the coefficients in equations (12) - (15) satisfies a rank condition.

[^8]:    ${ }^{17}$ See Cole and Prescott (1994) and Rios-Rull and Prescott (1991) for a full discussion of equilibrium existence and welfare theorems in similar economies.

[^9]:    ${ }^{19}$ These supply reservation wages correspond to what Lewis (1969) refers to as "employee equalizing wage curves," and what Rosen (1986) terms "worker indifference curves." One can also define demand reservation wages that correspond to "labor demand" curves, and then derive the equilibrium tangency conditions that are central to the definitions of equilibrium used in these papers. The advantage of the current setup is that the prices are linear over the commodities traded and the tools of general equilibrium theory can be applied.

[^10]:    ${ }^{20}$ It is easy to show that the problem of choosing $h, D_{1}$, and $D_{2}$ to maximize $F\left(h, K, D_{1}, D_{2}\right)$ subject to $h * D_{1}=H_{1}$ and $h * D_{2}=H_{2}$ given $K$ is solved by $h=1, D_{1}=H_{1}$, and $D_{2}=H_{2}$. This property need not hold if we assume that output increases less than proportionally with hours, due to factors like worker fatigue or boredom. See footnote 4 for one possible specification.

[^11]:    ${ }^{23}$ Economies have same equilibrium without taxes.

