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Adaptive Learning, Endogenous Inattention, and Changes in Monetary Policy

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This paper develops an adaptive learning formulation of an extension to the Ball, Mankiw, and Reis (2005) sticky information model that incorporates endogenous inattention. We show that, following an exogenous increase in the policymaker's preferences for price vs. output stability, the learning process can converge to a new equilibrium in which both output and price volatility are lower.

JEL Classifications: E52; E31; D83; D84

Key Words: expectations, optimal monetary policy, bounded rationality, economic stability, adaptive learning.

William A. Branch is at the University of California at Irvine, John B. Carlson is at the Federal Reserve Bank of Cleveland and can be reached at john.b.carlson@clev.frb.org, George W. Evans is at the University of Oregon, and Bruce McGough is at Oregon State University. The authors are indebted to Larry Ball, Bruce Preston, and Ricardo Reis for their detailed comments. Financial support from the National Science Foundation Grant is gratefully acknowledged.

1 Introduction

Recently, a number of researchers have conjectured that the decline in U.S. economic volatility in the post 1984 period – the "Great Moderation" – was the result of improved monetary policy, e.g. Orphanides (2002), Bernanke (2004), Clarida, Gali and Gertler (2000). These papers suggest that a change in the stance of monetary policy towards greater price stability helped induce the lower volatility in output and prices. In the monetary policy literature there is not a consensus on the channels through which policy would be able to achieve an overall improvement in stability. Thus an open question is how a monetary authority, also ignorant of the precise monetary policy transmission mechanism, can induce stability.

This paper extends Branch, Carlson, Evans and McGough (2006) (BCEM) by incorporating real-time adaptive learning by both the policy authority and private sector agents. BCEM is, itself, an extension of the Ball, Mankiw and Reis (2005) (BMR) model of sticky information since it endogenizes the rate of information acquisition. In BCEM we studied output and price volatility as a function of the policymaker's preference parameter ω, and we showed that it is possible for the resulting policy "frontier" to be upward sloping if the rate of information acquisition λ, i.e. the rate of inattention $1-\lambda$, is sufficiently responsive. The parameter ω measures the weight in the policymaker's loss function placed on price variance relative to output variance and we refer to a high value of ω as a low degree of activism since in effect it corresponds to a reduced desire to smooth output. The key is that by switching emphasis to greater price stability, the policymakers can induce an endogenous response in λ that lowers overall economic volatility – thereby avoiding the usual trade-off between price and output stability.

This paper elaborates on the mechanics of this process by developing a natural setting in which policymakers, as well as private agents, are neither naive nor fully informed rational, but instead are boundedly rational in the spirit of Marcet and Sargent (1989), Sargent (1999) and Evans and Honkapohja (2001). We go on to study whether in an adaptive learning setting the simultaneous decline in economic volatility remains a possibility. If so, the results presented here would provide additional theoretical support for a monetary policy channel of the Great Moderation.

We consider a system initially in equilibrium and look, numerically, at the impact of an exogenous increase in ω, i.e. a permanent decrease in policy activism. The numerical results track the BCEM theoretical results showing that a simultaneous decline in price and output volatility is possible, but with one significant difference. Initially, when the new policy rule is implemented, output volatility rises in line with the "standard" view of a trade-off, reflecting the transitional period in which λ adapts over time to its new lower equilibrium level. However, in the long-run, output as well as price volatility decline permanently. Our adaptive learning version of the model provides results that are more hopeful than those of Sargent (1999) in the sense that with appropriate policy a permanent decrease in volatility is possible.

2 BMR Model in Reduced Form

BMR develop a simple model, along the lines of Woodford (2003), with monopolistic competition and optimal monetary policy. Their novelty is the information structure: agents update their information with exogenous probability $0 < \lambda < 1$ each period, and each agent sets a price path optimally every period, subject to their information constraint.[1] The equilibrium for the model is represented by two reduced-form equations,

$$\begin{aligned} p_t &= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} \left(p_t + \alpha y_t + u_t \right) && (1) \\ y_t &= \hat{m}_t - p_t, && (2) \end{aligned}$$

where p_t is the price-level, y_t is aggregate output, and $u_t = \rho u_{t-1} + \varepsilon_t$ are mark-up shocks that follow a stationary AR(1) process.[2] Equation (1) is a Phillips curve and represents the aggregate supply relationship of the economy. Equation (2) is a quantity-equation theory of aggregate demand and is derived from a cash-in-advance constraint. The variable $\hat{m}_t$ is the policy instrument set at time $t-1$ according to a rule that maximizes a second order welfare approximation. BMR show that the equilibrium has an $MA(\infty)$ representation of the form

$$p_t = \sum_{j=0}^{\infty} \phi_j \varepsilon_{t-j} \text{ and } y_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j},$$

for appropriately defined ϕ_j, φ_j. The key insight of BCEM is that, when λ is endogenous, ϕ, φ depend on λ and vice-versa.

In BCEM agents are assumed to choose their information acquisition rate λ, given aggregate $\bar{\lambda}$, according to the best-response function

$$T(\bar{\lambda}) = \arg \min_{0 \leq \lambda \leq 1} \left(E \left(\hat{p}_t(\lambda) - p_t^*(\bar{\lambda}) \right)^2) + C\lambda^2 \right) \tag{3}$$

where $C\lambda^2$, $C > 0$, is the cost to updating and utilizing information at rate $0 \leq \lambda \leq 1$. Here $p_t^*(\bar{\lambda})$ is the optimal (full information) price at t and $\hat{p}_t(\lambda)$ is the (agent-specific stochastic process for the) price set at t by a firm with information acquisition rate λ. Like BMR, we interpret broadly the costs of information updating, to include not just the cost of obtaining but also the cost of processing the information. An *Endogenous Inattention* equilibrium is defined by the fixed point $\lambda^* = T(\lambda^*)$. This a symmetric Nash equilibrium, between the continuum of private agents and also the policymaker. In this "game" private agents choose λ and the central bank chooses its policy.

In BCEM we considered the following policy experiment. If ω, the relative weight on cross-sectional price variance in the central bank's loss function, exogenously increases then, for fixed λ, price variance decreases and output variance increases. However, with endogenous

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[1] Reis (2006) provides further microfoundations to this approach.

[2] All variables are in log deviations form. BMR and BCEM also include demand shocks. We omit these shocks to ease exposition.

λ, higher ω implies that λ^* will decrease, reducing both price and output volatility. The overall impact on output variance depends on the relative strengths of these effects, and BCEM show that a simultaneous reduction in both price and output volatility is indeed possible. The intuition for this result is clear: policy that responds more aggressively to price volatility will induce an endogenous response in λ that will also lower output volatility.

3 Adaptive Learning and Changes in Monetary Policy

The result of the policy experiment discussed above depends on the timing of the "game" between policymakers and private agents. The structure of the model assumes a simultaneous move game with ω parameterizing the preferences of the government. This timing assumption results in a prisoner's dilemma and the economy can be trapped in an inefficient outcome, as in Kydland and Prescott (1977) and Rogoff (1985). A decline in both price and output volatility can obtain in the Nash equilibrium of this game, given an exogenous increase in preferences ω. As an alternative, we could specify the structure as a Stackelberg game with the government as the large player who moves first. In such a setting, policymakers could announce a policy consistent with preferences less activist than their own, and thereby choose their preferred point on the frontier.

Our own view is that it is more plausible to extend the bounded rationality viewpoint to policymakers, as well as private agents, and to think in terms of an evolution and improvement over time in the exercise of monetary policy.[3] This view is in line with the hypothesis of Bernanke (2004) and seems implicit in the discussions of monetary policy in both Svensson (2003) and McCallum (2000). In this paper we will therefore assume that policymakers over time arrive at the view that decreased policy activism can improve economic performance, without them necessarily understanding all of the relevant mechanisms.

We model the expectations of both policymakers and private agents using the adaptive learning approach described in Evans and Honkapohja (2001). Least squares learning allows policymakers and private agents to learn how to make optimal forecasts, given their information sets, without knowing structural parameters, and also allows them to appropriately track structural change. This makes policy and λ time-dependent. A natural question is: to what type of equilibrium will this adaptive version of the economy converge (if any)? If, after removing the strategic interaction of the model, the economy converges to the Nash equilibrium/endogenous inattention outcome, then this provides additional support for the BMR/BCEM model. This approach also allows us to consider the output-price volatility trade-off in terms of stability under adaptation. If the relevant equilibria are stable then an exogenous change in policymaker preferences could cause the economy to move to a lower point on the upward sloping section of the policy frontier, thus resulting in reduced volatility in both prices and output.

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[3] In his comments on Orphanides and Williams (2005), Evans (2005, pp. 241-2) stresses the advantages of models with "cognitive consistency" between private agents, policymakers and economists.

3.1 Real-time Learning Version of the Model

Policymakers choose their policy instrument in order to satisfy the first-order condition $E_{t-1}y_t = -\alpha\omega E_{t-1}p_t$ given their forecast for the price-level.[4] This implies a rule

$$\hat{m}_t = (1-\kappa)E_{t-1}p_t, \tag{4}$$

where $\kappa = \alpha\omega$, for setting the policy instrument $\hat{m}_t$, at time $t-1$. Such a rule is close to the one studied, for example, by Taylor (1980), who refers to $1-\kappa$ as the "degree of accommodation" (to price shocks).[5] In the numerical simulations below, it is convenient to report the effects of the policy shift in terms of an increase in ω, but the policy change can equivalently be interpreted simply in terms of a reduction in activism or in the degree of accommodation. Implementation of this rule still requires forecasts of prices. Since we do not want to assume full knowledge of the structure by policymakers we replace $E_{t-1}p_t$ by an econometric forecast $\hat{E}_{t-1}p_t$ based on a reduced form time-series model.[6] In equilibrium, the price process is $MA(\infty)$ and it is natural to assume that policymakers approximate this process using an ARMA(r,q) specification. In addition we assume that the exogenous shocks ε_t are observable at t, so that policymakers can use recursive least squares (RLS) to update the estimates of their ARMA model's parameters.[7] Policymakers thus set $\hat{m}_t$ according to (4) with $E_{t-1}p_t$ replaced by $\hat{E}_{t-1}p_t$.

In BMR, firms are price setters and would prefer to set price to

$$p_t^* = p_t + \alpha y_t + u_t$$

each period. However, there is a cost to processing new information. We assume that firms do not know the full economic structure and are thus unable to form fully rational expectations or to compute the optimal λ, given their costs. Instead, firms hire consultants to provide real-time estimates of both optimal price forecasts and of λ_t, given the costs to the firm of updating prices at frequency λ. We think this set-up is a reasonable stylized description of actual agent behavior.[8]

Consultants act as information gatherers, providing to firms forecasts of future optimal prices as well as the optimal rate of information processing.[9] Consultants, like the policy-

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[4] In BMR the loss function is $Var(y_t) + \omega E(Var_i(p_{it} - p_t))$ and is minimized subject to (1).

[5] The F.O.C. is a "specific targeting rule" of the type advocated and discussed in detail by Svensson (2003). As stressed by Svensson (2003), one of the advantages of this type of rule is that its specification does not require knowledge of the full structure of the economy. We note, however, that Svensson does not advocate money supply rules.

[6] Other implementations of bounded rationality are possible in which policymakers make use of their knowledge of the structure. For a discussion of optimal monetary policy with structural parameter learning see Evans and Honkapohja (2003). The key qualitative results of the current paper are unlikely to depend on the detailed implementation of learning.

[7] See Evans and Honkapohja (2001) for a detailed discussion of least-squares learning in dynamic macroeconomics.

[8] For example, Carroll (2003) provides evidence that consumer expectations follow a distributed lag of professional forecasters.

[9] The notion of a consultant is a descriptive device designed to remove the explicit strategic interaction between agents. Some of the roles of consultant could be served by newspapers, business publications, Central Bank forecasts or the forecasting community more generally.

makers, are assumed not to know the full structure of the economy and to forecast using a reduced form ARMA model. Each period consultants forecast the value of p_t^* using an ARMA(r, q) specification, with ε_t observable.[10] As before, the ARMA(r, q) can be estimated using RLS. Consultants are willing to provide $\hat{E}_t p_{t+k}^*$, for $k = 0, 1, 2, \ldots$, either free of charge or for a fixed fee willingly paid by all firms; however, the consultants are aware that firms incur a cost of information processing. The consultants therefore also provide to firms an estimate of the optimal rate of information accrual, λ_t, by solving the firm's optimization problem (3) using an estimate of the mean-square forecast error in (3), based on their estimated ARMA(r, q) process for p_t^*.

It is worth emphasizing that although the consultants know the value of λ_t and have memory of the conditional forecasts $\hat{E}_{t-j} p_t^*$, the consultants do not know the full structural equations and so do not know how this translates into actual prices and, hence, actual optimal prices. This learning set-up is constructed specifically so that none of the agents know how λ_t affects the actual dynamics. Convergence to a Nash Equilibrium then provides additional theoretical support for the endogenous inattention equilibrium concept.

The following system, written in recursive causal ordering, describes the evolution of the economy under adaptive learning (and summarizes the preceding discussion):

$$\hat{E}_{t-1} p_t = \{\mathrm{ARMA(r, q)\ \ Policy\ \ Maker\ \ Forecast}\}$$

$$\hat{m}_t = (1 - \kappa)\hat{E}_{t-1} p_t, \text{ where } \kappa = \alpha\omega.$$

$$\hat{E}_t p_{t+k}^*,\ k = 0, 1, \ldots = \{\mathrm{ARMA(r, q)\ \ Consultant\ \ Forecast}\}$$

$$\lambda_t = \{\mathrm{Consultant\ \ Computed}\}$$

$$p_t = \sum_{j=0}^{\infty} \lambda_{t-j} \prod_{i=0}^{j-1} (1 - \lambda_{t-i}) \hat{E}_{t-j} p_t^*$$

$$p_t^* = \alpha \hat{m}_t + (1 - \alpha) p_t + u_t,$$

where the last equation is obtained using the AD relation and the definition of p_t^*.

We now address two questions: First, will this economy converge to the equilibrium associated with a stable Nash equilibrium of the non-adaptive model? Second, suppose that ω increases exogenously. Will the economy converge to a new, more "moderate" Nash equilibrium?

3.2 Numerical Results

We first start with what BCEM term the benchmark case.[11] We set $\alpha = .1, \sigma_\varepsilon^2 = .1, C = 5, \rho = .85$. We set the ARMA parameters to $r = 1, q = 5$ as these provide a good approx-

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[10] We could instead assume that policymakers forecast with an ARMA(r', q') with (r', q') possibly different from (r, q). However, this would not change the results below. Similarly, we could instead have the consultants forecast p_t, y_t and u_t separately, and then combine them to construct the forecast of p_t^*. The impact on our results of this alternative set-up would be minimal.

[11] These parameter values are chosen for illustrative purposes and are not calibrated in any serious sense.

imation to the actual stochastic process. This parameterization yields an upward sloping policy frontier. Figure 1 illustrates the results from a typical simulation when $\omega = 15$.

INSERT FIGURE 1 HERE

As indicated by Figure 1, λ_t converges to its Nash equilibrium value, marked by the horizontal line in the top panel. In the bottom two panels, the time t estimates of the unconditional variances of price and output are plotted. These estimates were obtained using a moving average with window length 500; thus the horizontal scales in these figures do not include the transient period. The horizontal lines in these panels correspond to the theoretical variances of output and price at the associated Nash equilibrium.

The results of Figure 1 strongly suggest that the Nash outcome is stable under our adaptive model. The intuition for this stability is as follows. For fixed λ, the ARMA models are approximations to the true MA(∞) equilibrium price process. Since the true process depends on the underparameterized ARMA models – through policy and λ_t – the equilibrium here is similar to the Restricted Perceptions Equilibrium (RPE) defined in Evans and Honkapohja (2001). Moreover, the RPE in models with an expectational structure similar to the one presented here are stable under adaptive learning. Furthermore, for a fixed price process we restrict attention to Nash equilibria that are stable fixed points of our T-map in the sense that $T'(\lambda^*) < 1$. Thus, it is not surprising (though not obvious) that these two stable mechanisms imply convergence.

We now turn to examining the simultaneous decline in output and price volatility in real-time. The conjecture is that as policy becomes less activist, there is a tendency for price variance to decrease, resulting in an eventual decrease in equilibrium attentiveness, λ, that induces lower output variance. We thus now assume that during the simulation there is an exogenous increase in ω from $\omega = 15$ to $\omega = 30$. The increase in ω could be due to a shift in policy stance accompanying the appointment of a conservative central banker, and could be thought of either as exogenous or as a response by the government to a series of adverse price shocks.

Figure 2 illustrates the results from a typical simulation. Initially, (after a transient period of length 600), the economy is near the equilibrium corresponding to $\omega = 15$. At time $t = 800$, ω increases abruptly from 15 to 30. Figure 2 demonstrates that, prior to the change in policy, the real-time learning dynamics are near their Nash equilibrium values. Immediately following the policy change, price volatility plummets as predicted, but output volatility rises. This reflects the fact that though λ_t is falling from its pre-shock level, it has not yet reached its new equilibrium level; thus, temporarily, the usual trade-off exists. As λ_t gets close to its new equilibrium level, however, both volatility time-series converge to levels lower than those of the pre-shock equilibrium, and the economy exhibits a "Great Moderation."

INSERT FIGURE 2 HERE

We emphasize that we have not attempted to calibrate our model to provide a description of the actual historical experience. Any serious exercise along these lines would require significantly more elaborate detail for both the aggregate demand and the aggregate supply sides of the model. Nonetheless, the finding that a Great Moderation is possible in the model, with a boundedly rational policymaker and adaptive learning by all agents, is significant. These results suggest that a possible application of the theoretical approach taken in this paper would be an empirical examination of the Great Moderation, allowing for policy change and an endogenous response in the attentiveness of economic agents. Such an undertaking is beyond the scope of the current paper.

4 Conclusion

This paper formulates an adaptive learning version of the endogenous inattention model of Branch, Carlson, Evans and McGough (2006). In this formulation policymakers set policy in order to satisfy their first order optimality condition but replace expectations based on the structural model with recursively updated ARMA forecasts. Similarly, private sector agents use an ARMA model to forecast their optimal price, given their rate of information acquisition. It was shown that this economy converges to the Endogenous Inattention equilibrium of Branch, Carlson, Evans and McGough (2006). Moreover, with a change in policymaker preferences, the central bank and the private-sector can eventually learn a new equilibrium with both lower price and output variance. These results provide additional support for the theoretical results in Branch, Carlson, Evans and McGough (2006), Ball, Mankiw and Reis (2005) and others, since they show that the interaction of optimal policy and limited attention on the part of private sector agents has important practical policy implications that are not sensitive to the particular timing protocol of these models.

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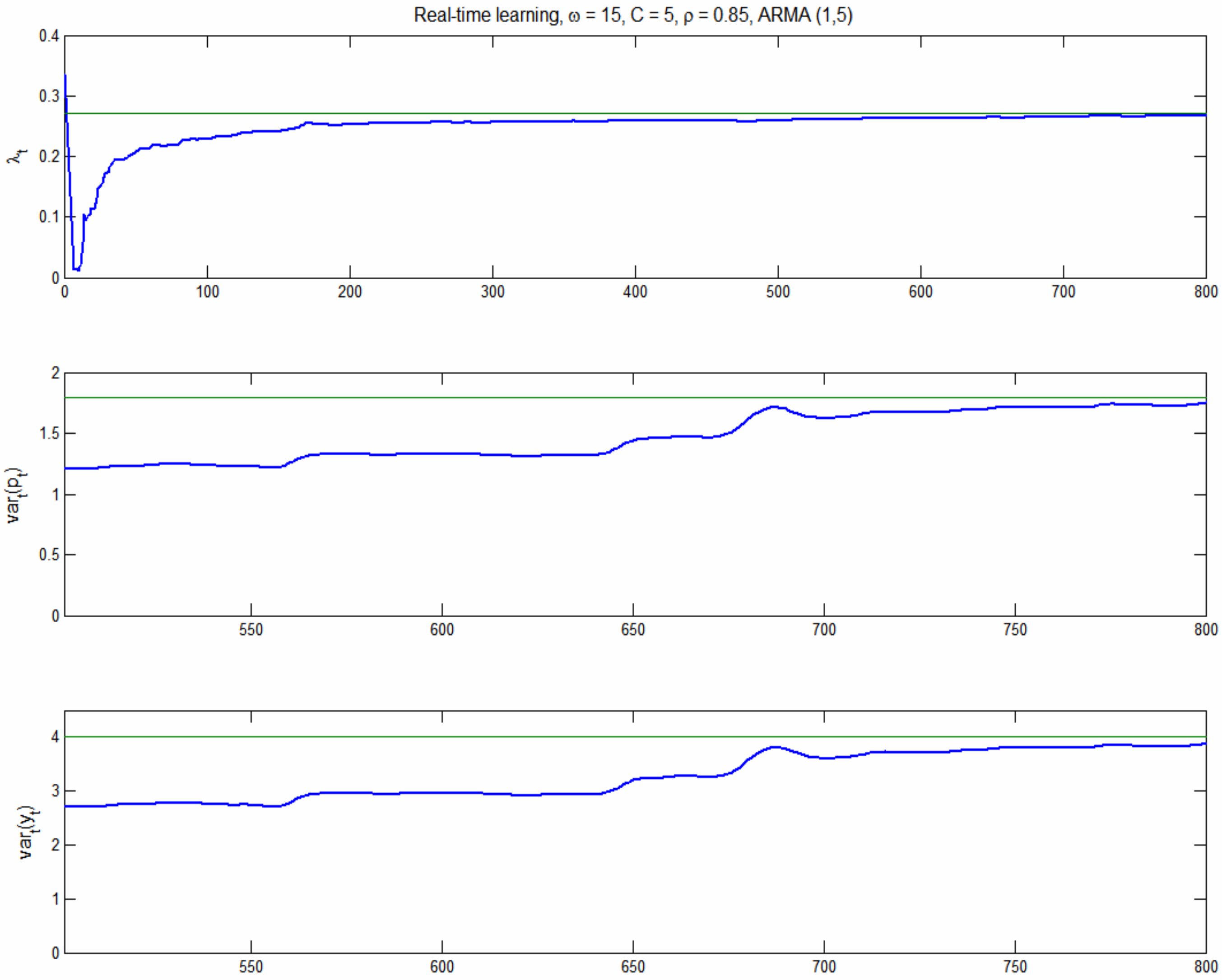

Figure 1. Stability of Endogenous Inattention under Adaptive Learning.

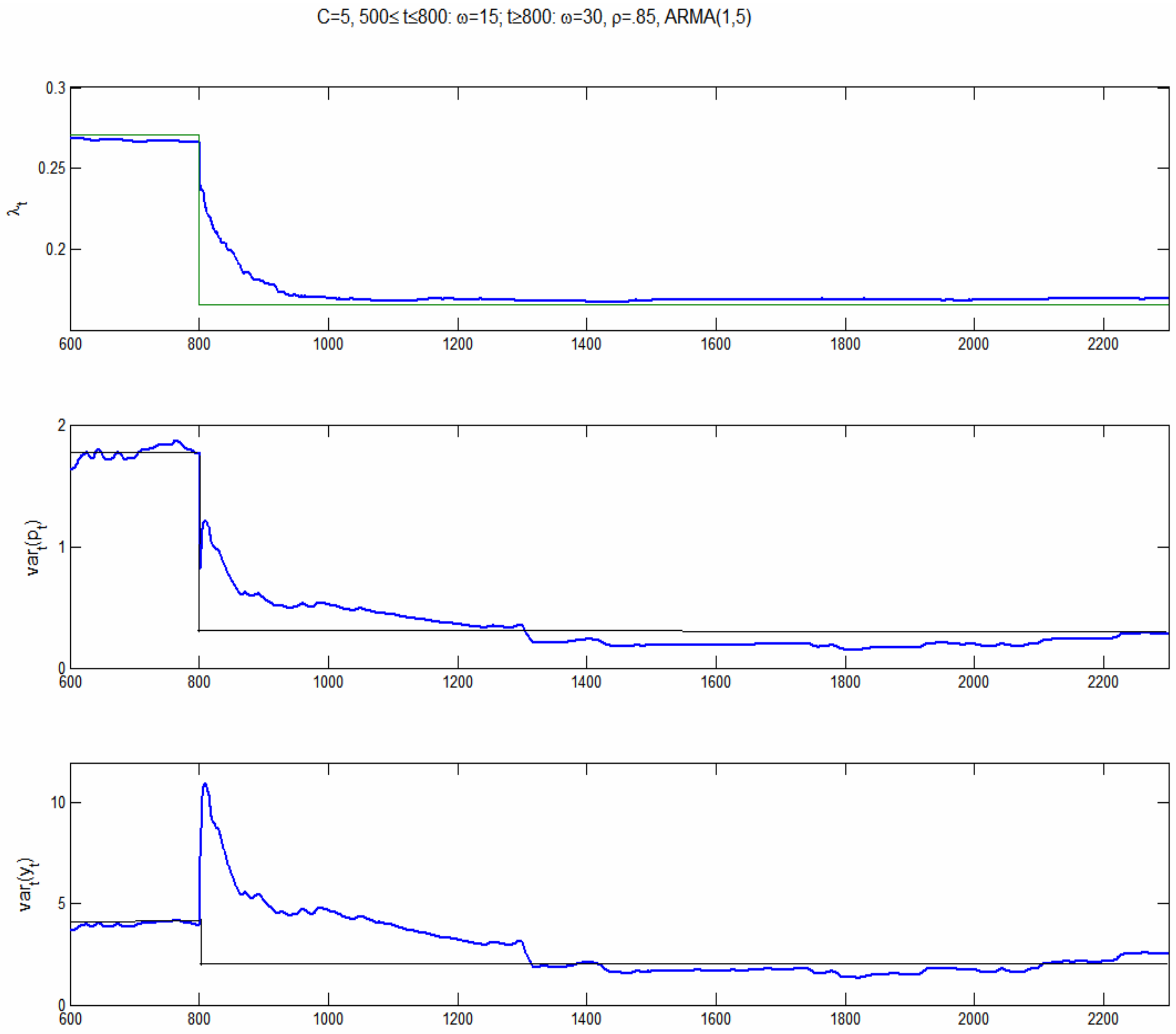

Figure 2. Changes in policy preferences in real-time. At time 800 ω increases to 30.