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COMPARISON OF UNIVARIATE ARIMA, MULTIVARIATE ARIMA, AND VECTOR AUTOREGRESSION FORECASTING

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Key words: univariate ARIMA, multivariate ARIMA, vector autoregressions, forecasting performance

Abstract

The purposes of this study are two: 1) to compare the forecasting abilities of the three methods: univariate autoregressive integrated moving average (ARIMA), multivariate autoregressive integrated moving average (MARIMA), and vector autoregression (both unconstrained--VAR--and Bayesian--BVAR) and, 2) to study the problem of whether series should be differenced before modeling and subsequent use in forecasting. All of these methods have been shown to provide forecasts that are more accurate than many econometric methods, which require more resources to implement. In terms of whether to difference or not, there does not appear to be a clear consensus. The forecasts from models estimated using level data and differenced data can be quite different. This statement is supported by this study in that the forecast performance of these techniques is shown to be very sensitive to differencing. This is especially true of the VAR forecasts. The results for the other methods were mixed--that is, for some forecast lengths and for some variables, level models provided better forecasts, while for other combinations, differenced models were better. In general, the ARIMA and

MARIMA models provided better forecasts than the VAR or BVAR models when differenced data was used for the MARIMA models, and for both differenced and level data for ARIMA models. Also, VAR models in the differenced data provided better forecasts than BVAR models in either levels or differences.

I. Introduction

The main purpose of this research is to compare forecasts from three popular time series methods of forecasting: ARIMA, MARIMA, and VAR-BVAR. As part of this effort, we examine the problem of whether the series should be differenced before estimating models for forecasting purposes. Although the question of whether to difference or not has been shown to be a major potential problem in univariate ARIMA models (see Dickey, Bell, and Miller [1986] for a discussion of this problem and a list of relevent references in the univariate case), not much research has been carried out for multivariate cases. For the MARIMA models, Tiao and Box (1981) recommend not differencing the data. However, many applications of this method do difference the data and then fit MARIMA models. In VAR-BVAR models, it is not apparent whether the series should be differenced or not. For example, Litterman (1986) suggests an appropriate approximation of the behavior of an economic variable is a random walk around an unknown, deterministic trend. This would seem to imply that the data should not be differenced because generally only undifferenced economic series behave this way. However, Litterman also says that this part of the prior is a likely candidate for modification. In fact, Litterman uses levels, growth rates, log levels, and differences in a seven-variable model.

The data used in this study are the Lydia Pinkham annual data on sales and advertising from 1907 through 1960. Thsee data are given and discussed in Heyse and Wei (1985) and have been used in many other studies: for example: Bhattacharyya (1982); Erickson (1981); Hanssens (1980); Helmer and Johansson (1977); Pack (1979); Pollay (1979); and Schmalensee (1972).

II. Time Series Models

The following is a very brief description of the time series models used in this study. The univariate ARIMA models are discussed in detail in Box and Jenkins (1976), Tiao and Box (1981) provide a more detailed description of the multivariate ARIMA models, and Litterman (1986) discusses the VAR and BVAR models. All of these models are particular versions of the general time series model of order (p,q) given by:

(1) $\underline{\phi}_p(B)\underline{z}_t = \underline{\theta}_q(B)\underline{a}_t + \underline{\theta}_0,$

where

(2) $$\underline{\phi}_p(B) = \underline{I} - \underline{\phi}_1 B - \dots - \underline{\phi}_p B^p,$$
$$\underline{\theta}_q(B) = \underline{I} - \underline{\theta}_1 B - \dots - \underline{\theta}_q B^q,$$

where

B = backshift operator (e.g., $B^s z_{i,t} = z_{i,t-s}$),

$\underline{I}$ = k x k identity matrix,

$\underline{z}$ = vector of k variables in the model,

$\underline{\phi}_j$'s and $\underline{\theta}_j$'s = k x k matrixes of unknown parameters,

$\underline{\theta}_0$ = k x 1 vector of unknown parameters, and

$\underline{a}$ = k x 1 vector of random errors that are identically and independently distributed as $N(0,\Sigma)$.

Thus, it is assumed that the $a_{j,t}$'s at different points in time are independent, but not necessarily that the elements of $\underline{a}_t$ are independent at a given point in time.

The univariate models use only past history of the individual series being modeled. Thus, they do not use any information from other series which may be related to the series being forecast. The MARIMA, VAR, and BVAR models use information from other related series to attempt to obtain better forecasts by using the additional information from these series. These models differ in how they model the relationships among the series. Both VAR and BVAR assume that the relationships can be approximated by using only autoregressive components of the more general autoregressive moving average (ARMA) models. The difference between the VAR and the BVAR models is in the method of estimating the models rather than in their form.

The n-period-ahead forecasts from these models at time t ($\underline{z}_t(n)$) are given by:

(3) $$\underline{z}_t(n) = \underline{\phi}_1[\underline{z}_{t+n-1}] + \ldots + \underline{\phi}_p[\underline{z}_{t+n-p}] + [\underline{a}_{t+n}] - \underline{\theta}_1[\underline{a}_{t+n-1}] - \ldots - \underline{\theta}_q[\underline{a}_{t+n-q}],$$

where, for any value of t,n,m, $[\underline{x}_{t+n-m}]$ implies the conditional expected values of the random variables $\underline{x}_{t+n-m}$ at time t. If n-m is less than or equal to zero, then the conditional expected values are the actual values of the random variables and the error terms. If n-m is greater than zero, then the expected values are the best forecasts available for these random variables and error terms at time t. Because the error terms are uncorrelated with present and past information, the best forecasts of the error terms for n-m greater than zero are their conditional means, which are zero. The forecasts can be generated iteratively with the one-period-ahead forecasts that depend only on known values of the variables and error terms. The longer-length forecasts, in turn, depend on the shorter-length forecasts.

III. Development of Models For Forecasting

Because we wish to test whether models developed using undifferenced or differenced series provided better forecasts, we developed models for all four methods using both level and differenced series. The data set consists of 54 annual observations, which we divided into two periods. The first 40 observations were used to estimate the models, and the last 14 were used to evaluate the forecasting ability of the estimated models.

In the univariate case, there are tests to determine whether the series should be differenced. When we applied the test given by Dickey, Bell, and Miller (1986), the results indicated that both advertising and sales should be differenced. However, it is not clear whether this result carries over to multivariate models.

In the univariate case, we used the model in differenced data given by Helmer and Johansson (1977). For the level series, we used the method of Box and Jenkins (1976) to estimate the models. This method is an iterative one involving: 1) tentatively identify a model by examining autocorrelations of the series, 2) estimate the parameters of this model, and 3) apply diagnostic checks to the residuals. If the residuals do not pass the diagnostic checks, then the tentative model is modified, and steps two and three are repeated. This process continues until a satisfactory model is obtained. These models are given in table 1 for advertising and in table 2 for sales.

For the MARIMA models, we used the model given by Heyse and Wei (1985) for the differenced data and developed a model for the level series by using the method of Tiao and Box (1981). This method is similar to that of Box and Jenkins method for univariate models, except cross-correlations between the series are added and modeled for. The results are also presented in tables 1 and 2.

For the VAR models, we only need to specify a lag length and then estimate the model. To do this, we divided the estimation period into two periods. We estimated the VAR model for lags 1 through 6 over the first 32 observations and then used the resulting models to forecast one-year ahead over the next eight years. We then chose the model that minimized the log determinate of the variance-covariance matrix of the forecasts. The model with the best lag length was then estimated over the first 40 observations. The results for both the level and differenced data are presented in tables 1 and 2.

For the BVAR models, we used two methods of estimation. In the BVAR models there are several parameters that specify the prior distribution used in the estimation process. To specify these parameters, we used the two methods of 1) using the "Minnesota" prior as identified in the RATS program from VAR Econometrics, and, 2) estimating the parameters by a grid search. In both cases, we estimated the models over the first 32 observations and then used the next eight observations to determine the model that minimized the log determinate of the variance-covariance matrix of the forecasts. Each method involved searches over lag length from 1 to 9. The resulting specification was then used in estimating the corresponding model over the first 40 observations. The results are also presented in tables 1 and 2. The results from the grid search estimation are denoted as "optimal" BVAR. Because this method of searching over all parameters in a BVAR is time consuming, a comparison of these two methods is important. If there is essentially no difference in the two methods, or if the "Minnesota" prior gives better results, then we should use the "Minnesota" prior because of time saved in the estimation procedure.

Examining the within-sample standard error of estimate, it appears that the models using levels fit the data better than the models using differenced data for almost all methods and for both advertising and sales. The only exception to this is the MARIMA models for sales. Also, these results indicate that the best-fitting model for advertising is the MARIMA model in levels, while the best-fitting model for sales is the MARIMA model in differences. However, there is no assurance that this result will carry over into forecasting. Consequently, in the next section, we present results from forecasting both advertising and sales using all of the models.

IV. Forecasting Results

The 10 models developed for this study were used to forecast advertising and sales for up to a forecast horizon of eight years over the last 14 years of data. These forecasts were actual forecasts and did not use any information within the forecast horizon. The number of forecast we have for each forecast length, thus, varies. For one-year-ahead forecasts, we have 14 observations; for two years ahead, we have 13 observations, etc. For the purposes of this study, we calculated three measures of forecast accuracy: 1) the root mean square error (RMSE), 2) the mean error (ME), and 3) the mean absolute error (MAE). The results are presented in tables 3 through 8. Tables 9 through 14 present the corresponding ranks for the different methods for the appropriate statistic. We will discuss the RMSE results only. The results for the other statistics are roughly the same.

Advertising

For advertising, we see from tables 3 and 9 that the best method for one-year-ahead forecasting was the VAR in the differenced data followed closely by the MARIMA model for the differenced data. For longer-length forecasts, we observe that the two best forecasts are provided by either the ARIMA or MARIMA models for the differenced data. We also observe that in all cases, the models developed from differenced data out-performs the level models. In some cases, this difference is substantial. This is especially true for the VAR models. We also notice that the BVAR models are no better than fourth best for any forecast horizon or specification. In fact, the VAR model in the differenced data forecasts better than any of the BVAR models for six of the eight horizons considered here. This result is in contrast to other results indicating that BVAR models generally provide better forecasts than VAR models (see, for example, Doan, Litterman, and Sims [1984]). This may be due to the use of only undifferenced data in the other studies. The results of this study from the level data do support the conclusion that BVAR models forecast better than VAR models. Comparing the BVAR models with different priors, we see that the "Minnesota" prior performed better than the "Optimal" prior for level data and worse for differenced data. However, the difference in the two priors for the differenced data are extremely small. Thus, for advertising, it appears one would be better off using the "Minnesota" prior.

For the MARIMA models, the level data model performed much worse than the differenced data, model even though the level model had a better in sample fit. This suggests that one should consider models in the differenced data when estimating multivariate models, even though Tiao and Box (1981) recommend not differencing the data.

Sales

An examination of tables 4 and 10 shows that the results concerning the use of level or differenced sales data are not so clear. There does appear to be a pattern in most of the methods (except for VAR), which suggests that for sales, the differenced models provide better forecast for shorter horizons while level data models provide better forecasts for longer length horizons. For the VAR model, the differenced data models provide much better forecasts at all horizons than the level data models.

Again, we see that the best forecasts are provided by either ARIMA or MARIMA models. Generally, the VAR level model and the BVAR models provide the worse forecasts. The VAR model in differences out-forecasts the BVAR models for every horizon considered here.

Comparing the two priors, we see that for level data models, the "Optimal" prior forecasts better than the "Minnesota" prior for all horizons. For differenced data models, there is essentially no difference in the forecasts. This suggests that the "Optimal" prior would be better.

IV. Summary

In this study, we have compared ARIMA, MARIMA, VAR, and BVAR forecasting for two series that have been studied widely--the Lydia Pinkham annual data on sales and advertising. The results indicate that the ARIMA and MARIMA models provide better forecasts than the VAR and BVAR models when differenced data is used for the MARIMA model and either level or differenced data is used for the ARIMA models. Contrary to popular conception, the VAR model in the differenced data provided better forecasts for both series than the BVAR models for most forecast horizons considered here.

When we compare the forecasts from the two models using the different priors, the results are mixed. For advertising, the "Minnesota" prior is better, while for sales, the "Optimal" prior is better. In both cases, the results are not substantially different. The RMSE ranges from 2 percent to 10 percent larger for the worse case for each series. These results suggest that one would probably do just as well, on average, to use the "Minnesota" prior, rather than spend resources and time to estimate the "Optimal" prior. Of course, since this result is based on only two series, further work is necessary to determine whether this result will be true in general.

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Table 1 Estimated Models for Advertising

	ARIMA		MARIMA		VAR		BVAR - "Minnesota"		BVAR - "Optimal"	
	Levels	Differences	Levels	Differences	Levels	Differences	Levels	Differences	Levels	Differences
Constant	211.171	-------	-------	-------	-45.995	-.121	164.339	14.485	224.447	14.334
Advertising Lag										
1	.794	1.074	.470	.787	.361	.828	.889	1.016	.842	1.015
2	-------	-.481	-------	-.320	-.343	.172	-.041	-.016	-.088	-.015
3	-------	.407	-------	.533	.081	-------	.002	-------	.026	-------
4	-------	-------	-------	-------	.110	-------	.005	-------	.025	-------
5	-------	-------	-------	-------	-------	-------	-.007	-------	-.018	-------
6	-------	-------	-------	-------	-------	-------	-------	-------	-.018	-------
Sales Lag										
1	-------	-------	.285	.568	.535	.406	.002	.001	.011	.005
2	-------	-------	-------	-.568	-.372	-.406	.0002	-.001	.002	-.005
3	-------	-------	-------	-------	.154	-------	.0002	-------	.001	-------
4	-------	-------	-------	-------	.134	-------	.00007	-------	.0005	-------
5	-------	-------	-------	-------	-------	-------	-.00001	-------	.0001	-------
6	-------	-------	-------	-------	-------	-------	-------	-------	-.00005	-------
Moving Average Term Lag										
1	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
2	-------	-------	.376	-------	-------	-------	-------	-------	-------	-------
3	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
4	-.375	-------	-------	-------	-------	-------	-------	-------	-------	-------
SEE	226.632	231.538	181.879	197.904	191.285	243.908	245.448	251.849	234.215	251.577

Notes: All models are expressed in terms of the series levels, whether they were estimated in levels or differences. In the MARIMA models, the moving average terms are all on the error terms of the dependent variable. That is, there are no cross equation error terms.

Table 2 Estimated Models for Sales

	ARIMA		MARIMA		VAR		BVAR - "Minnesota"		BVAR - "Optimal"	
	Levels	Differences	Levels	Differences	Levels	Differences	Levels	Differences	Levels	Differences
Constant	249.293	21.030	-------	-------	318.710	13.903	275.523	27.397	350.831	27.384
Advertising Lag										
1	-------	-------	-.437	-------	-.151	-.013	-.002	.001	-.005	.003
2	-------	-------	-------	-.267	-.362	.013	-.001	-.001	-.002	-.003
3	-------	-------	-------	.267	.056	-------	-.0003	-------	-.001	-------
4	-------	-------	-------	-------	-.109	-------	-.0001	-------	-.0003	-------
5	-------	-------	-------	-------	-------	-------	-.0001	-------	-.0004	-------
6	-------	-------	-------	-------	-------	-------	-------	-------	-.0002	-------
Sales Lag										
1	.877	-------	1.231	1.525	1.256	1.453	.982	1.132	1.051	1.132
2	-------	-------	-------	-.525	-.207	-.453	-.047	-.132	-.094	-.132
3	-------	-------	-------	-------	.0004	-------	-.028	-------	-.046	-------
4	-------	-------	-------	-------	.095	-------	-.018	-------	-.026	-------
5	-------	-------	-------	-------	-------	-------	-.014	-------	-.025	-------
6	-------	-------	-------	-------	-------	-------	-------	-------	-.026	-------
Moving Average Term Lag										
1	.482	-.454	-------	-------	-------	-------	-------	-------	-------	-------
2	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
3	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
4	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
SEE	235.751	240.942	239.105	230.167	238.790	245.700	246.805	251.677	236.119	251.621

Table 3 Comparison of Root Mean Square Forecast Errors - Advertising

	Forecast Horizon in Years (number of observations)							
Method	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	106.09	125.99	152.64	175.93	225.41	244.90	269.37	292.22
Differences	101.14	110.12	125.43	141.88	181.62	190.76	207.75	239.88
Multivariate ARIMA -								
Levels	158.49	234.23	229.97	235.15	260.78	254.73	267.35	275.42
Differences	95.89	136.97	138.37	136.28	160.28	194.21	215.69	223.44
Vector Autoregression -								
Levels	157.95	262.87	278.84	265.61	311.17	364.55	400.97	403.09
Differences	95.11	154.51	163.79	151.88	191.01	236.13	262.06	257.73
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	121.11	163.38	199.81	233.91	278.92	299.70	328.27	358.29
Differences	112.51	140.59	157.87	182.18	242.20	268.95	301.72	335.71
"Optimal" prior								
Levels	126.47	175.98	217.41	252.70	294.14	313.45	340.05	367.47
Differences	112.18	140.46	157.61	181.65	241.58	268.43	301.18	334.92

Table 4 Comparison of Root Mean Square Forecast Errors - Sales

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	120.88	205.04	255.75	298.84	349.13	388.07	437.09	474.07
Differences	117.81	201.26	243.10	290.05	358.53	416.28	490.76	543.46
Multivariate ARIMA -								
Levels	151.41	208.50	207.36	238.45	279.34	284.32	299.44	303.45
Differences	122.51	208.79	216.11	213.17	267.41	318.70	360.73	377.49
Vector Autoregression -								
Levels	179.60	325.94	406.16	451.37	489.68	528.73	557.28	563.07
Differences	129.24	220.29	243.32	271.32	348.91	420.17	490.41	523.91
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	153.35	241.44	303.95	375.64	442.83	490.64	544.24	592.48
Differences	139.71	220.71	267.75	341.83	422.48	483.21	561.60	631.22
"Optimal" prior								
Levels	145.30	230.09	285.10	345.53	409.27	456.39	499.60	535.02
Differences	139.73	220.76	267.76	341.80	422.48	483.23	561.64	631.24

Table 5 Comparison of Mean Forecast Errors - Advertising

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	-54.48	-81.26	-113.29	-142.17	-185.23	-203.10	-237.91	-274.01
Differences	-42.07	-72.36	-97.38	-118.99	-148.04	-158.59	-189.32	-223.66
Multivariate ARIMA -								
Levels	-138.01	-216.03	-215.06	-221.96	-232.07	-229.98	-247.89	-263.42
Differences	-6.54	-19.60	-57.35	-83.50	-103.07	-107.13	-144.27	-174.64
Vector Autoregression -								
Levels	-136.45	-246.31	-266.31	-250.16	-289.00	-343.18	-385.64	-392.09
Differences	-1.79	-10.51	-44.07	-76.02	-104.68	-121.65	-167.77	-191.95
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	-70.35	-118.63	-169.20	-212.95	-253.97	-273.30	-308.46	-347.09
Differences	-45.80	-77.70	-121.86	-162.93	-205.29	-232.18	-278.90	-318.68
"Optimal" prior								
Levels	-82.22	-137.66	-188.08	-230.15	-269.73	-287.04	-318.95	-355.58
Differences	-45.34	-77.10	-121.23	-162.27	-204.53	-231.36	-278.09	-317.75

Table 6 Comparison of Mean Forecast Errors - Sales

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	-58.20	-129.45	-202.11	-268.22	-309.72	-341.88	-399.54	-455.26
Differences	-50.08	-114.00	-191.01	-266.58	-319.85	-370.60	-456.64	-525.84
Multivariate ARIMA -								
Levels	-98.63	-141.73	-162.47	-189.21	-198.69	-211.87	-248.74	-270.57
Differences	-25.38	-53.30	-110.52	-164.55	-189.64	-213.00	-281.82	-328.12
Vector Autoregression -								
Levels	-141.59	-282.99	-376.08	-422.31	-445.11	-471.86	-507.36	-528.78
Differences	-37.33	-81.38	-153.32	-226.41	-275.08	-328.76	-422.07	-481.70
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	-99.87	-182.47	-267.26	-347.36	-403.96	-450.25	-514.13	-576.25
Differences	-78.98	-153.33	-238.12	-322.09	-386.37	-450.96	-542.64	-620.19
"Optimal" prior								
Levels	-77.90	-144.15	-217.59	-288.92	-338.71	-381.18	-441.95	-502.72
Differences	-78.93	-153.27	-238.09	-322.06	-386.33	-450.93	-542.64	-620.18

Table 7 Comparison of Mean Absolute Forecast Errors - Advertising

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	87.98	114.64	132.55	144.71	197.61	218.85	237.91	274.01
Differences	82.69	97.66	103.02	122.34	150.15	167.90	190.32	223.66
Multivariate ARIMA -								
Levels	143.80	216.03	215.06	221.96	232.07	229.98	247.89	263.42
Differences	85.18	120.05	122.59	110.72	122.20	165.93	185.10	177.39
Vector Autoregression -								
Levels	143.03	246.31	266.31	250.16	289.00	343.18	385.64	392.09
Differences	76.47	128.48	151.83	124.98	152.60	201.95	226.99	204.91
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	98.65	146.06	174.90	212.95	253.97	273.30	308.46	347.09
Differences	86.84	122.83	138.81	162.93	208.03	241.73	278.90	318.68
"Optimal" prior								
Levels	104.07	155.60	189.27	230.15	269.73	287.04	318.95	355.58
Differences	86.56	122.54	138.50	162.27	207.55	241.22	278.09	317.75

Table 8 Comparison of Mean Absolute Forecast Errors - Sales

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	90.14	174.71	228.64	268.22	309.72	341.88	399.54	455.26
Differences	90.20	169.90	216.18	266.58	319.85	370.60	456.64	525.84
Multivariate ARIMA -								
Levels	122.43	181.15	195.08	199.48	220.03	235.31	248.74	270.57
Differences	100.92	184.06	182.36	174.22	202.83	248.54	295.90	328.12
Vector Autoregression -								
Levels	160.06	288.46	376.08	422.31	445.11	471.86	507.36	528.78
Differences	106.74	185.52	202.33	226.41	283.50	339.66	422.07	481.70
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	130.42	213.90	267.26	347.36	403.96	450.25	514.13	576.25
Differences	109.55	192.98	246.30	322.09	386.37	450.96	542.64	620.19
"Optimal" prior								
Levels	126.29	206.13	231.42	288.92	346.65	387.30	441.95	502.72
Differences	109.53	193.03	246.25	322.06	386.33	450.93	542.64	620.18

Table 9 Rankings According to Root Mean Square Forecast Errors - Advertising

	Forecast Horizon in Years (number of observations)							
Method	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	4	2	3	4	4	4	5	5
Differences	3	1	1	2	2	1	1	2
Multivariate ARIMA -								
Levels	10	9	9	8	7	5	4	4
Differences	2	3	2	1	1	2	2	1
Vector Autoregression -								
Levels	9	10	10	10	10	10	10	10
Differences	1	6	6	3	3	3	3	3
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	7	7	7	7	8	8	8	8
Differences	6	5	5	6	6	7	7	7
"Optimal" prior								
Levels	8	8	8	9	9	9	9	9
Differences	5	4	4	5	5	6	6	6

Table 10 Rankings According to Root Mean Square Forecast Errors - Sales

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	2	2	5	5	4	3	3	3
Differences	1	1	3	4	5	4	5	6
Multivariate ARIMA -								
Levels	8	3	1	2	2	1	1	1
Differences	3	4	2	1	1	2	2	2
Vector Autoregression -								
Levels	10	10	10	10	10	10	8	7
Differences	4	5	4	3	3	5	4	4
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	9	9	9	9	9	9	7	8
Differences	5	6	6	7	8	7	9	9
"Optimal" prior								
Levels	7	8	8	8	6	6	6	5
Differences	6	7	7	6	7	8	10	10

Table 11 Rankings According to Mean Forecast Errors - Advertising

	Forecast Horizon in Years (number of observations)							
Method	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	6	6	4	4	4	4	4	5
Differences	3	3	3	3	3	3	3	3
Multivariate ARIMA -								
Levels	10	9	9	8	7	5	5	4
Differences	2	2	2	2	1	1	1	1
Vector Autoregression -								
Levels	9	10	10	10	10	10	10	10
Differences	1	1	1	1	2	2	2	2
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	7	7	7	7	8	8	8	8
Differences	5	5	6	6	6	7	7	7
"Optimal" prior								
Levels	8	8	8	9	9	9	9	9
Differences	4	4	5	5	5	6	6	6

Table 12 Rankings According to Mean Forecast Errors - Sales

Method	Forecast Horizon in Years (number of observations)							
	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	4	4	5	5	4	4	3	3
Differences	3	3	4	4	5	5	6	6
Multivariate ARIMA -								
Levels	8	5	3	2	2	1	1	1
Differences	1	1	1	1	1	2	2	2
Vector Autoregression -								
Levels	10	10	10	10	10	10	7	7
Differences	2	2	2	3	3	3	4	4
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	9	9	9	9	9	7	8	8
Differences	7	8	8	8	8	9	9	10
"Optimal" prior								
Levels	5	6	6	6	6	6	5	5
Differences	6	7	7	7	7	8	10	9

Table 13 Rankings According to Mean Absolute Forecast Errors - Advertising

	Forecast Horizon in Years (number of observations)							
Method	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	6	2	3	4	4	4	4	5
Differences	2	1	1	2	2	2	2	3
Multivariate ARIMA -								
Levels	10	9	9	8	7	5	5	4
Differences	3	3	2	1	1	1	1	1
Vector Autoregression -								
Levels	9	10	10	10	10	10	10	10
Differences	1	6	6	3	3	3	3	2
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	7	7	7	7	8	8	8	8
Differences	5	5	5	6	6	7	7	7
"Optimal" prior								
Levels	8	8	8	9	9	9	9	9
Differences	4	4	4	5	5	6	6	6

Table 14 Rankings According to Mean Absolute Forecast Errors - Sales

	Forecast Horizon in Years (number of observations)							
Method	1 (14)	2 (13)	3 (12)	4 (11)	5 (10)	6 (9)	7 (8)	8 (7)
Univariate ARIMA -								
Levels	1	2	5	5	4	4	3	3
Differences	2	1	4	4	5	5	6	6
Multivariate ARIMA -								
Levels	7	3	2	2	2	1	1	1
Differences	3	4	1	1	1	2	2	2
Vector Autoregression -								
Levels	10	10	10	10	10	10	7	7
Differences	4	5	3	3	3	3	4	4
Bayesian Vector Autoregression -								
"Minnesota" prior								
Levels	9	9	9	9	9	7	8	8
Differences	6	6	8	8	8	9	9	10
"Optimal" prior								
Levels	8	8	6	6	6	6	5	5
Differences	5	7	7	7	7	8	10	9