working paper 0119

A Strategic Approach to Hedging and Contracting

by David Downie
and Ed Nosal

FEDERAL RESERVE BANK OF CLEVELAND

Working Paper 01-19 October 2001

A Strategic Approach to Hedging and Contracting

by David Downie and Ed Nosal

This paper provides a new rationale for hedging that is based, in part, on non-competitive behavior in product markets. We identify a set of conditions which imply that a firm may want to hedge. Empirically, these conditions are not inconsistent with what is observed in the market place. The conditions are: (i) firms have some market power in their product market, (ii) firms have limited liability, and (iii) firms can contract to sell their output at a specified price before all factors which can affect their profitability are known. For some parameter specifications, however, our model predicts that firms will not want to hedge. This result is important as our hedging results since, in practice, although a large fraction of firms do hedge their cash flows, a substantial number of firms do not.

JEL Codes: G30, L13, D43.
Key Words: product market competition, hedging, default

David Downie is at the Royal Bank of Canada. Ed Nosal is at the Federal Reserve Bank of Cleveland. The authors thank David Andolfatto, Lutz Busch, and Ken Vetzel for their comments on earlier versions.

David Downie may be reached at david.downie@royalbank.com.
Ed Nosal may be reached at Ed.Nosal@clev.frb.org.

1 Introduction

Risk management has become an integral part of the financial management of companies. According to a survey of large US non-financial firms (Smithson [1996]), 65% of the responding firms have used derivatives to manage risk. A similar Canadian study (reported in Smithson [1996]) found that 80% of large firms used derivatives to manage risk. Why do these firms hedge?

In order to provide a rationale for the hedging behavior of firms, existing theories have relied upon the existence of taxes (Smith and Stulz (1985) and Graham and Smith (1996)), asymmetric information, (DeMarzo and Duffie (1991), Ljungqvist (1994), Breeden and Viswanathan (1996) and Degeorge, Moselle and Zeckhauser (1996)), risk-aversion (Stulz (1990) and DeMarzo and Duffie (1995)) and costly external capital (Froot, Scharfstein and Stein (1994)). All these papers take the firm as the basic 'unit of analysis'. That is, cash flows under alternative hedging scenarios are exogenously specified and the firm's problem is to choose that hedging strategy which maximizes its expected payoff. Following the seminal contributions of Brander and Lewis (1986) and Maksimovic (1986)—which point out that there is an intimate relationship between product market competition and a firm's choice of capital structure—we provide a new explanation for hedging that is based on non-competitive product market competition.[1] The general flavour of the Brander and Lewis (1986) and Maksimovic (1986) results carries over to a model where the firm's financial decision is not one of choosing an appropriate debt/equity mix but instead deals with the amount of futures contracts it should buy or sell. In particular, the firm may be able to act more aggressively in the product market and, as a result, may be able to attain a higher payoff when it hedges it cash

[1] Maksimovic (1995) provides a nice summary of the financial structure and product market competition literature. Our paper is somewhat related to papers by Allaz (1992) and Allaz and Villa (1993). These papers examine pricing and output policies of imperfectly competitive firms that can buy and sell forward contracts on their output. Firms, however, do not buy or sell futures contracts.

flows. Hence, just as debt can be viewed as a device that commits the firm to act more aggressively in the product market in the models of Brander and Lewis (1986) and Maksimovic (1986), futures contracts can be viewed as providing the firm with the same kind of commitment in our model.

The intuition that underlies our model of corporate hedging is as follows. Firms compete in a non-cooperate manner in the product market. A firm may be able to achieve a 'first-mover advantage' over rival firms and does so by selling its output contractually at a predetermined (delivery) price, instead of waiting to compete directly in the spot market. If the firm does sell its output contractually at a predetermined price, then it subjects itself to default risk since it agrees to sell its output at a fixed price before all factors affecting its productivity are known. For example, if *ex post* input prices turn out to be 'high', the firm may default on its contractual obligations to deliver output owing to insufficient resources. Consumers are, of course, rational. The fact that the firm may default in some states of the world will be, *ex ante*, impounded into the delivery price. If the firm could somehow commit to delivery in all states of the world, consumers would bid up the delivery price. The firm would *like* to commit to delivery if the expected profit associated with delivery in all states is greater than the expected profit associated with default in some states. In such circumstances, the firm *can* commit to delivery in all states of the world by purchasing futures contracts whose underlying asset is sufficiently correlated with its input prices. The futures contracts will 'pay off' precisely at the time when the firm's resources are strained. Hence, futures contracts have value because they prevent the firm from defaulting on a contractual obligation when 'not defaulting' is (*ex ante*) important.

It is not the case, however, that firms who sell their output contractually will want to hedge their cash flows in order to prevent default. (Surprisingly, it may not even be the case that a firm would want to exercise its first mover advantage!) For some firms, hedging cash flows to prevent default on contractual obligations may actually *lower* expected payoffs. These firms will not use futures contracts. This is an important result because although, in practice, a large proportion of firms hedge (65% in the U.S.), a large number of firms do not. Any theory that attempts to provide a rationale for corporate hedging must at the same time be consistent with the fact that not all firms want to hedge. Our theory is consistent with both the strict preference of hedging and strict preference with not hedging.

Below, we provide a model where a firm may hedge its input prices. This is but one example why a firm might want to hedge risk. We could have, alternatively, formulated our model in an international context. For example, a domestic firm may agree to sell its output abroad at a fixed delivery price denominated in a foreign currency. At the time when the firm is to produce and ship the goods abroad, the exchange rate may move against the firm, implying that the firm will default on its delivery contract. The firm can, however, avoid default by entering into a foreign exchange futures contract. If the firm's expected payoff is higher if it does not default, compared to its expected payoff if it does default in some states, then the firm will, in fact, hedge its foreign exchange exposure.

The paper is organized as follows. A model where two firms compete in the product market is presented in the next section. Section 3 describes the equilibrium outcomes when a Stackelberg market structure is assumed, i.e., one firm competes by forward selling delivery contracts and the other firm's production decision is made after the first firm delivers on its contractual obligations. Section 4 describes the equilibrium outcomes when an *ex post* Cournot market structure is assumed, i.e., both firms wait until the state of the world is revealed and compete à la Cournot.

Section 5 characterizes the ‘equilibrium market structure’, i.e., a firm can choose to be a leader and competes contractually, or can choose to compete simultaneously in the *ex post* spot market. Section 6 concludes the paper.

2 The Model

Two firms compete for a given market demand. The firms are endowed with identical constant returns to scale production technologies, are risk neutral and have limited liability. Firms have no outside wealth.

One can interpret the firms as playing a game over two dates, date 1 and date 2. At date 1 the unit costs of production are unknown but, between dates 1 and 2, these costs are revealed. We assume, for simplicity, that there are only two states of the world: a low cost state of the world, l, which occurs with probability θ and a high cost state of the world, h, which occurs with probability $1-\theta$. We denote the unit cost of production as ω_s, $s \in \{h, l\}$, where $\omega_h > \omega_l$, and the expected unit cost of production as $\bar{\omega} = \theta\omega_l + (1-\theta)\omega_h$.

Let x_i represent the output that is supplied to the market by firm $i \in \{1, 2\}$ at date 2. Market demand is represented by the linear inverse demand curve

$$p = a - x, \quad a > 0,$$

where p represents the price of the good and x is market demand which equals market supply, $x_1 + x_2$.

Firm 1 can choose *when* to sell its output to consumers. Firm 1 can either,

1. sell its output by writing contracts with consumers at date 1 or

2. sell its output at date 2—after the state of the world is revealed—in the 'spot market'.

If firm 1 sells its output contractually at date 1, then each contract promises to deliver a one unit of output at date 2 at a prespecified unit price, f. We will refer to such contracts as *forward* contracts. Because the unit production costs are unknown at the time when a forward contract is written, the date 2 payoff associated with a forward contract is uncertain. Denote the total number of forward contracts written by firm 1 by X_1. It is rather important to emphasize that, owing to limited liability, firm 1 may end up defaulting on its forward contract obligations at date 2. This could happen if, for example, the forward contract price, f, is less than the unit cost of production.

Firm 2 makes its production decision at date 2 and sells its output in the spot market.[2]

Firms may have an incentive to *hedge* their input prices. Although the purpose of this paper is to understand why firms hedge, one can conjecture that firm 1 may wish to hedge in order to avoid the possibility of defaulting on its forward contracts or that either firm 1 or 2 may choose to hedge for strategic reasons. We suppose that at date 1 there exists a futures market that trades futures contracts on the unit cost of production. As is convention, the futures price, F_ω, is set so that the value of the futures contract at inception is zero. The payoff to a futures contract is realized at date 2, where the payoff is a function of the difference between the futures price, F_ω, and the date 2 unit cost of production, ω_s. If the firm buys—or is 'long'—in a futures contract the date 2 payoff is $\omega_s - F_\omega$ in state $s \in \{h, l\}$.[3] A firm is said to hedge its input costs if the payoff to its futures

[2]The results of this paper will not be altered if we assume that firm 2 has the same choices, in terms of *when* to compete, as firm 1. We show this in Section 5. Hence, the assumption that firm 2 can only compete in the spot market should be viewed as a simplifying one.

[3]Although a futures contract pays off in dollars, one can conceptually think about a long futures position, i.e., buying a futures contract, as agreeing to purchase the input at date 2 for F_ω. The buyer ends up paying F_ω for an object whose value is ω_s: the buyer's payoff is, therefore, $\omega_s - F_\omega$.

contract position is positive when input prices are high and negative when input prices are low. Hence, a firm hedges by taking a long position in futures contracts.

Because firms have limited liability, the lowest payoff that they can receive in any state is zero. More formally, if $X_1 > 0$, then firm 1's payoff in state $s \in \{h, l\}$ is given by

$$\Pi_1^s(X_1) = \max\{\pi_f^s, 0\},$$

where

$$\pi_f^s = (f - \omega_s)X_1 + (\omega_s - F_\omega)n_1. \tag{1}$$

The variable n_1 in equation (1) represents firm 1's position in the futures market. $n_1 > 0$ indicates that firm 1 has a long position of n_1 futures contracts and $n_1 < 0$ indicates that it has a net short position of $|n_1|$ futures contracts. Note that if $\pi_f^s < 0$, then firm 1 does not have sufficient resources to honor all of its contractual obligations in state s. In this case, firm 1 defaults and receives a payoff of zero.

Firm 2's state s payoff and firm 1's state s payoff in the event that it chooses to compete in the spot market, i.e., $X_1 = 0$, is given by

$$\Pi_i^s = \max\{\pi_i^s, 0\},$$

where

$$\pi_i^s = (p_s - \omega_s)x_i^s + (\omega_s - F_\omega)n_i, \quad i \in \{1, 2\}, \tag{2}$$

where p_s represents the spot price in state s. If there does not exist a level of spot market production $x_i^s > 0$ such that firm $i \in \{1, 2\}$ has sufficient resources ($p_s x_i^s$) to honor its obligations ($\omega_s x_i^s + (F_\omega - \omega_s)n_i$) in state s, then it defaults on all of its contractual obligations in state s and receives a payoff of zero.[4]

The futures market is perfectly competitive, i.e., no single trader can influence the futures price. Financial market participants are assumed to be risk-neutral. A futures exchange initially acts as an intermediary, matching long and short positions that are requested by financial market participants. After parties are 'matched', the futures exchange guarantees performance on all contracts, i.e., financial market participants view that their contract is with the futures exchange. As guarantor the exchange may limit the number of contracts that a firm buys or sells. In particular, the exchange will buy and sell contracts from a firm as long as the (equilibrium) expected payoff to the exchange for the transactions is greater than or equal to zero, the assumed competitive reservation value. A firm will default on its futures contracts if it does not have sufficient *ex post* resources to pay off the contract in some state of the world.[5]

For simplicity, it is assumed that the discount rate between dates 1 and 2 is zero. An implication

[4]The simplest way to think about what happens when $\pi_i^s < 0$, is that firm i 'disappears' and receives a zero payoff and all contracts written by firm i become null and void. We could have, alternatively, closed the model by having the productive and financial assets of the firm auctioned off, where the proceeds of the auction are distributed to individuals who hold claims on the defaulting firm. The new owner of the firm, i.e., the person who purchased the productive assets of the firm, now competes at date 2 in the spot market. However, our results pertaining to the hedging behavior of the firms are, qualitatively speaking, insensitive to the precise specification of the market and ownership structure *in the event of a default*. The intuition for this invariance is that, independent of how things are resolved after a default, the owner of the defaulting firm is out of the market and receives a zero payoff. The fact that the market continues and other agents are receiving possibly positive payoffs is irrelevant to the defaulting firm. It is for this reason that we close the model in the (analytically) simplest way.

[5]In practice, a futures exchange requires parties to post margin accounts so that it (the exchange) can credibly guarantee performance on all contracts. If a party is unable to post a sufficient margin, then the party will be unable to buy or sell the amount of contracts that it 'desires', i.e., the party will be quantity constrained. Since we assume that the firm does not have any outside wealth the firm will be unable to post a margin. The exchange, therefore, guarantees performance by limiting the number of contracts that it will buy or sell from a firm. In Section 5 we discuss the implications of requiring firms to post margin accounts.

of the zero discount rate assumption (along with the assumption that financial market participants are risk-neutral) is that $F_\omega = \bar{\omega}$, i.e., the futures price equals the expected unit cost.

The timing of events for the our model is as follows. At date 1:

- Firms simultaneously offer to take positions N_1 and N_2 with the futures exchange.
- The futures exchange accepts $0 \leq n_i \leq N_i$ from firm $i \in \{1, 2\}$ if $N_i \geq 0$ and $0 \geq n_i \geq N_i$ from firm i if $N_i \leq 0$.
- Firm 1 offers output contracts $X_1 \geq 0$ for delivery at date 2.
- The representative consumer purchases all of X_1; contract price f is established.[6]

This ends date 1. Before date 2 begins, the state of the world $s \in \{h, l\}$ is revealed. At date 2:

- If $X_1 > 0$, firm 1 produces output $x_1^s \leq X_1$ in state s and delivers it to the representative consumer. Firm 2 then chooses output level x_2^s to supply in the spot market in state s. All futures contracts are settled.
- If $X_1 = 0$, firm 1 and 2 simultaneously choose output levels x_1^s and x_2^s, respectively, to supply to the spot market in state s. All futures contracts are settled.

Note that when $X_1 > 0$ and firm 1 delivers $x_1^s \leq X_1$ contracts at date 2, then firm 2 effectively faces the (residual) inverse demand curve

$$p_s = a' - x_2^s$$

[6]One could model the representative consumer as behaving 'strategically', i.e., the representative consumer can purchase any amount of output contracts *less than* or equal to X_i from firm $i \in \{1, 2\}$. The idea here is that representative consumer will purchase that amount of output contracts which minimizes the expected product price. (It is not necessarily the case that purchasing all of X_1 minimizes the expected price of the good.) Modeling the representative consumer as a strategic agent does not qualitatively alter the main results of this paper, i.e., the reasons for why a firm may want to hedge remains valid if we allow consumers to act strategically.

at date 2, where $a' = a - x_1^s$. We shall assume that the input price in the high cost state is not 'too' high in that $a - \bar{\omega} > 2\omega_h$.[7]

In terms of the information structure, we assume that all market participants can observe the actions taken by all players and can observe all market outcomes, i.e., information is complete. There is, however, imperfect information between firms 1 and 2 at date 1 when firms make their futures contract decisions and at date 2 when firms make their output decisions in the event that $X_1 = 0$.

The equilibrium concept that will be used is that of a subgame perfect Nash equilibrium (SPE). A SPE requires that candidate equilibrium strategies are Nash at each and every subgame. Very loosely speaking, in the description of the timing of the game above, each 'bullet' represents a subgame.

We will proceed by first assuming that firm 1 can only compete by selling forward contracts. This situation will be referred to as a 'Stackelberg market structure' since firm 1 gets to choose its output level before firm 2 does. We then assume that firm 1 can only compete by producing and selling in the spot market. This situation will be referred to as an '*ex post* Cournot market structure' since firms compete simultaneously after the state of the world is revealed. Firm 1's equilibrium behavior can be determined by simply comparing the expected profits that it generates under the Stackelberg market structure with the expected profits generated under the *ex post* Cournot market structure.

[7]The intuition behind this inequality will be explained at the time that it is introduced into the analysis, see footnote 9.

3 Analysis of a 'Stackelberg Market Structure'

As is standard, the model can be solved in two stages: the first stage characterizes the equilibrium behavior of firm 2; the second stage characterizes the equilibrium behavior of firm 1 and identifies the equilibrium to the game. Before we begin the stage 1 analysis we can present a preliminary result that deals with firms' futures positions.

If firm $i \in \{1, 2\}$ defaults in one state of the world, then, in equilibrium, its futures contract position must be zero, i.e., $n_i = 0$.

This result reflects that fact if firm i defaults in one state of the world, then either the futures exchange or firm i will have entered into a contractual arrangement that has a strictly negative expected payoff. That is, in the state of the world where firm i defaults, the payoff associated with the futures contract is zero to both parties; in the state of the world where firm i does not default, firm i must either make or receive a positive payoff from its futures position. The party who must make the positive payment in the non-defaulting state can make itself better off by not entering into the contract in the first place. Note that this result is independent of the assumed structure of the product market.

3.1 Stage 1: Equilibrium Behavior for Firm 2

Suppose that at date 1, firm 1 has a futures position n_1, firm 2 has a futures position n_2, and at date 2 firm 1's output level is x_1^s in state s. At date 2, firm 2 will select a level of output, x_2^s, that maximizes

$$\pi_2^s = [a - (x_1^s + x_2^s)]x_2^s - \omega_s x_2^s + (\omega_s - F_\omega)n_2, \quad s \in \{h, l\}.$$

If π_2^s is negative for all values of $x_2^s \geq 0$, then firm 2 defaults in state s and its payoff will be zero. If π_2^s is non-negative for some values of $x_2^s \geq 0$, then firm 2's SPE quantity choice is the maximizing value of x_2^s, i.e., firm 2's best response function is given by

$$x_2^s = \begin{cases} (a - x_1^s - \omega_s)/2 & \text{if} a - x_1^s - \omega_s > 0 \\ 0 & \text{if } a - x_1^s - \omega_s \leq 0 \end{cases}, \quad s \in \{h, l\}. \tag{3}$$

In any equilibrium firm 2 never defaults. To see this suppose, to the contrary, that there is an equilibrium where firm 2 defaults. Result 1 implies that firm 2's futures position must be zero, i.e., $n_2 = 0$, meaning that firm 2's only source of payoff comes from production. But firm 2's best response function, (3), implies that it will only produce a positive level of output if its payoff is greater than zero. Hence, firm 2 does not default, a contradiction.

Since firm 2's best response function, (3), does not depend upon its own futures contract position, n_2, in equilibrium, firm 2's futures position does not *directly* affect its behavior in the output market. In determining its own futures and output contract positions, firm 1 will use the best response function (3) to predict the behavior of firm 2. Since, in equilibrium, firm 2's best response function does not depend upon its own futures contract position, *firm 1's* choice of output and futures contracts will also be independent of firm 2's futures position. Hence, in equilibrium, firm 2's futures contract position can not *indirectly*—i.e., via firm 1—affect its own production decision. Finally, since in any equilibrium firm 2 does not default, any (equilibrium) futures contract position it takes has a zero expected value. All these observations imply,

In equilibrium, firm 2's futures market position can not affect its expected payoff: There does not exist an economic rationale for firm 2 to hedge.

At one level, this result may appear to be somewhat counterintuitive. In particular, if one interprets a futures contract as being a vehicle for altering *ex post* unit costs of inputs, then, because best response functions depend upon unit costs, one should expect that equilibrium quantities would be affected by the purchase or sale of futures contracts. This intuition is, however, misguided because a futures contract just provides the firm with a state contingent "cash" payoff and does not *directly* affect the real resource costs of the inputs. Since firm 2 makes its production decision *after* input costs are known, it will base its output decision on the actual resource costs that prevail at the time the production decision is made. As a result, the buying or selling futures contracts prior to the resolution of uncertainty does not confer any (*ex post*) strategic advantage the firm 2. Henceforth, we shall assume through out that if futures contracts do not affect firm i's behavior and payoff, where $i = 1, 2$, then firm i will take a zero position in the futures market, i.e., $n_i = 0$.

Generally speaking, the above result implies that the existence of limited liability and non-competitive behavior can not *by themselves* rationalize a firm's use of futures contracts. This is an interesting observation since the assumption of non-competitive behavior in the output market is a departure from the Modigliani-Miller world of perfect markets: hence the relaxation of some 'perfect markets' assumption may, in equilibrium, still lead to 'perfect markets' outcomes.

3.2 Stage 2: Equilibrium Behavior for Firm 1

Firm 1 formulates its futures and forward positions knowing that firm 2 will behave according to (3) in the output market. We will consider firm 1's optimal choice of forward contracts, X_1, assuming first that it never defaults and then assuming that it defaults in state h.

3.2.1 No default outcomes

Suppose that firm 1 delivers output $x_1^s = X_1$ in $s \in \{1, 2\}$, i.e., it does not default. Given the behavior by firm 2, equation (3), the forward price for delivery of output contract, f, will be[8]

$$f = \frac{a - X_1 + \bar{\omega}}{2}.$$

Hence, firm 1's expected profit is,[9]

$$(\frac{a - X_1 + \bar{\omega}}{2} - \bar{\omega})X_1. \tag{4}$$

Given that firm 1 does not default, the optimal number of forward contracts that it offers, X_1, will maximize the expected profit function (4). Denote this number of forward contracts as X_N^1, ('N' for no-default), where

$$X_1^N \equiv \frac{a - \bar{\omega}}{2}. \tag{5}$$

[8]In state s total output supplied is $(a - X_1 - \omega_s)/2 + X_1$. The expected price of output (which is the forward price f) is

$$\begin{aligned} & a - (\theta \frac{a + X_1 - \omega_l}{2} + (1 - \theta)\frac{a + X_1 - \omega_h}{2}) \\ = & \frac{a - X^1 + \bar{\omega}}{2} \end{aligned}$$

[9]We have assumed that $a + \bar{\omega} > 2\omega_h$. This implies that if firm 1 chooses the value of X^1 that maximizes (4) *and* does not default, then firm 2 will supply strictly positive levels of output in both states of the world. In this situation, the expected spot price will be $\frac{a - X^1 + \bar{\omega}}{2}$. One can interpret $a + \bar{\omega} > 2\omega_h$ as assuming that the unit cost in the high cost state is not 'too high' in the sense that if firm 1 chooses that level of output contracts which maximizes its expected payoff (assuming that it does not default), then there will still be a strictly positive residual demand for firm 2 in state h.

If firm 1 sells X_1^N forward contracts and does not default, then, given that firm 2 behaves optimally, firm 1's expected profit, $E(\Pi_1(X_1^N))$, is given by

$$E(\Pi_1(X_1^N)) \equiv \frac{(a-\bar{\omega})^2}{8}.$$

Note that the expressions for output and profit both the 'leader' and 'follower' correspond to the 'standard' Stackelberg formulae for the leader's and follower's output and profit.

3.2.2 Default outcomes

Suppose now that firm 1 defaults in state h. Hence, it must be the case that $n_1 = 0$. At date 1 the representative consumer understands that firm 1 will default in state h: accordingly, he will price the forward contracts consistent with delivery only in state l. If X_1 forward contracts are purchased and firm 1 defaults on delivery in state h, then the forward price, f, will be

$$f = p_l = \frac{a - X_1 + \omega_l}{2}.$$

The firm 1's expected profit is

$$\theta(\frac{a - X_1 + \omega_l}{2} - \omega_l)X_1. \tag{6}$$

Let X_1^{D*}, ('D' for default), be that quantity of forward contracts that maximizes the profit function (6), i.e.,

$$X_1^{D*} \equiv \frac{a - \omega_l}{2}.$$

Note that $X_1^{D^*} > X_1^N$. If firm 1 sells $X_1^{D^*}$ forward contracts and defaults in state h, then its expected profit, $E(\Pi_1(X_1^{D^*}))$, will be

$$E(\Pi_1(X_1^{D^*})) \equiv \theta \frac{(a-\omega_l)^2}{8}.$$

It is important to note that if firm 1 sells $X_1^{D^*}$ forward contracts it does not imply that it will default in state h. For example, if $f > \omega_h$ and $n_1 = 0$, then the firm 1 will not default in either state of the world. Define $X_1^{D^{crit}} = a + \bar{\omega} - 2\omega_h$ as that critical level of forward contracts such that if firm 1 offers more than $X_1^{D^{crit}}$ forward contracts and if $n_1 = 0$, then it will default in state h.[10] Assuming that firm 1 defaults in state h, the expected payoff to firm 1 associated with offering $X_1^{D^{crit}}$ forward contracts, $E(\Pi_1(X_1^{D^{crit}})$, is

$$E(\Pi^1(X_1^{D^{crit}})) = \theta\omega_h(a + \bar{\omega} - 2\omega_h)$$

It will be convenient to define $X_1^D = \max\{X_1^{D^*}, X_1^{D^{crit}}\}$ and the expected payoff to firm 1 associated with offering X_1^D output contracts as $E(\Pi_1(X_1^D))$.

3.3 Equilibrium

The SPE outcomes for the Stackelberg market structure game are determined by simply comparing the magnitudes of $E(\Pi_1(X_1^N))$ and $E(\Pi_1(X_1^D))$.

[10]This critical level of output contracts is determined by the equality of the output contract delivery price, f, with the unit cost in the high state of the world, ω_h, i.e.,

$$\omega_h = \frac{a - X_1 + \bar{\omega}}{2}.$$

Earlier, we have assumed that $a + \bar{\omega} > 2\omega_h$. In the context of the above equation, the assumed inequality implies that there exists a value of forward contract, X_1, such that firm 1 defaults in state h and not in state l.

Forward contracting without default If $E(\Pi_1(X_1^N)) \geq E(\Pi_1(X_1^D))$, then there will exist an SPE in which firm 1 offers X_1^N forward contracts. In addition, if $(f - \omega_h) < 0$, then an SPE that has firm 1 offering X_1^N forward contracts can *only* be supported if firm 1 purchases futures contracts. That is, in this situation if firm 1 *does not* hedge its input prices, then its profit will be strictly less than $E(\Pi_1(X_1^N))$: *hence, hedging is valuable to firm 1*.[11] If, however, $(f - \omega_h) > 0$, then the unique SPE will be characterized by firm 1 offering X_1^N forward contracts. But hedging is not required since the state contingent revenues will always exceed the state contingent total costs.[12]

Forward contracting with default If $E(\Pi_1(X_1^D)) > E(\Pi_1(X_1^N))$, then the unique SPE will be characterized by firm 1 offering X_1^D forward contracts. Since, in this equilibrium, firm 1 defaults in the high cost state of the world, it does not purchase or sell any futures contracts.[13]

We have describe the various equilibria that can arise in the Stackelberg market structure game. We have not, however, addressed the issue of existence of equilibrium. We defer this discussion to Section 5, when we characterize the equilibrium to the overall game.

---

[11]It can be shown that if $E(\Pi_1(X_1^N)) \geq E(\Pi_1(X_1^D))$ and $X_1^N > X_1^{D^{crit}}$, then $X_1 = X_1^N$, $N_1 = n_1 > 0$, $N_2 = 0$ and $x_2^s = (a - X_1^N - \omega_s)/2$ can be supported as a subgame perfect Nash equilibrium, where

$$\frac{(a-\bar{\omega})(a+3\bar{\omega}-4\omega_h)}{8(\omega_h-\bar{\omega})} < N_1 < \frac{(a-\bar{\omega})(a+3\bar{\omega}-4\omega_l)}{8(\bar{\omega}-\omega_h)}.$$

[12]It can be shown that $X_1 = X_1^N$, $N_1 = N_2 = 0$, and $x_2^s = (a - X^N - \omega_s)/2$ can be supported as a subgame perfe1ct Nash equilibrium when $E(\Pi_1(X_1^N)) \geq E(\Pi_1(X_1^D))$ and $X_1^N < X_1^{D^{crit}}$.

[13]It can be shown that if $E(\Pi_1(X_1^D)) \geq E(\Pi_1(X_1^N))$, then $X_1 = X_1^D$, $N_1 = N_2 = 0$, $x_2^l = (a - X_1^N - \omega_l)/2$ and $x_2^h = (a - \omega_h)/2$ can be supported as a subgame perfect Nash equilibrium.

4 Analysis of an ‘Ex post Cournot Market Structure’

In this section we assume that firm 1 does not offer any forward contracts, i.e., $X_1 = 0$, and instead competes with firm 2 in the date 2 spot market. Without loss of generality, we will assume that neither firm buys or sells futures contracts.[14] The game boils down to a very simple and familiar structure: Both firms observe the state of the world and then, at date 2, simultaneously choose their outputs.

In state s, both firms will produce the ‘standard’ Cournot levels of output,

$$x_1^s = x_2^s = \frac{(a - \omega_s)}{3}$$

and profit in state s will be

$$\pi_1^s = \pi_2^s = \frac{(a - \omega_s)^2}{9}.$$

The level of expected profit for both firms, denoted as $E\Pi^C$, is simply

$$\theta \frac{(a - \omega_l)^2}{9} + (1 - \theta) \frac{(a - \omega_h)^2}{9}.$$

5 Equilibrium

We now allow firm 1 to choose whether to compete by writing forward contracts with consumers at date 1 or to compete in the date 2 spot market.

[14]Recall that in the Stackelberg market structure, there is no strategic value in purchasing futures contracts for the firm that competes in the *ex post* spot market. This result also applies to the situation where both firms compete in the *ex post* spot market.

If, in equilibrium, firm 1 chooses to compete by writing forward contracts, i.e., firm 1 chooses $X_1 > 0$ at date 1, then the equilibrium outcomes are those described in Section 3; if, in equilibrium, firm 1 chooses to compete in the *ex post* spot market, i.e., firm 1 chooses $X_1 = 0$ at date 1, then the equilibrium outcomes are those described in Section 4. The equilibrium that prevails can be determined by simply comparing the various expected profits that firm 1 can generate. Depending upon model parameters, it is possible to have equilibria where: (1) both firms compete *ex post*; (2) firm 1 competes in forward contracts and defaults in the high cost state; (3) firm 1 purchases futures contracts, competes in forward contracts and does not default and; (4) firm 1 does not purchase futures contracts, competes in forward contracts and does not default.

Spot Market competition If $E\Pi^C > \max\{E(\Pi_1(X_1^N)), E(\Pi_1(X_1^{D^*}))\}$, then the unique SPE to the game has firm 1 (and firm 2) competing in the date 2 spot market. There can not exist an equilibrium where firm 1 offers forward contracts because firm 1 can increase its expected payoff by simply offering no forward contracts at date 1, $X_1 = 0$, and competing in the spot market. The strategies along the equilibrium path are: $X_1 = 0$, $N_1 = N_2 = 0$ and $x_i^s = (a - \omega_s)/3$ for $s = 1, 2$ and $i = 1, 2$.

This equilibrium can exist for certain model parameters. Note that the condition $E\Pi^C > E(\Pi_1(X_1^N))$, can be rewritten as

$$\theta(1-\theta)(\omega_h - \omega_l)^2 > \frac{(a-\bar{\omega})^2}{8} \tag{7}$$

and, assuming that $X_1^{D^*} > X_1^{crit}$, condition $E\Pi^C > E(\Pi_1(X_1^D))$ can be rewritten as

$$(1-\theta)(a-\omega_h)^2 > \theta\frac{(a-\omega_l)^2}{8} \tag{8}$$

Both of the above inequalities—as well as $X_1^{D^*} > X_1^{crit}$—are satisfied, and hence the equilibrium exists, for the following parameters: $\theta = .5$, $\omega_h = 1.5$, $\omega_l = 0$ and $a = 2$.

Intuitively, when would such an equilibrium prevail? Condition (7) can hold if $\omega_h - \omega_l$ and $\theta(1-\theta)$ are, relatively speaking, 'large'. Note that the value of $\theta(1-\theta)$ is maximized at $\theta = .5$. Hence, if there is a lot of uncertainty in terms of which state will prevail and the state contingent unit input costs are substantially different from one another, then firm 1 may prefer to compete *ex post*. It is true that in this equilibrium firm 1 gives up its 'first-mover' advantage but by giving up this first-mover advantage, firm 1 is able to make better (more profitable) *ex post* production decisions by waiting to observe the state of the world. If firm 1 knew for sure what state would prevail at date 2, it would sell $(a-\omega_s)/2$ forward contracts in state s. However, since it does not know what state will prevail, if it does choose to sell forward contracts, it will sell $(a-\bar{\omega})/2$, i.e., the 'average' of the optimal state contingent levels. But if $\theta = .5$ and the difference between ω_h and ω_l is 'large', then the difference between the 'optimal' amount of forward contracts to sell, $(a-\omega_s)/2$ in state s, and the actual amount sold, $(a-\bar{\omega})/2$, will also be large. It, therefore, may be more profitable to forsake the first-mover advantage to be able to produce output on a state contingent basis. Condition (8) can hold if $(a-\omega_h)$ is, relatively speaking, 'large'. A 'large' $(a-\omega_h)$ means that it is profitable to produce in the high cost state. If it was not too profitable to produce in the high cost state, then firm 1 would prefer to essentially ignore the high cost state—that is, it would default in this state—and offer $(a-\omega_l)/2$ forward contracts at date 1.

Forward contracting with default If $E(\Pi_1(X_1^D)) > \max\{E\Pi^C, E(\Pi_1(X_1^N))\}$, then the unique equilibrium outcome has firm 1 competing contractually and defaulting in the high cost state. In this equilibrium neither firm uses the futures contracts. The strategies along the equilibrium path are: $X_1 = X_1^D$, $N_1 = N_2 = 0$ and $x_2^s = (a - x_1^s - \omega_s)/2$ for $s = 1, 2$ and $i = 1, 2$.

This equilibrium can exist for certain model parameters. Assuming that $X_1^{D*} > X_1^{crit}$, condition $E\Pi_1(X_1^D)) > E(\Pi_1(X_1^N))$, can be rewritten as

$$\theta(a - \omega_l)^2 > (a - \bar{\omega})^2 \tag{9}$$

and condition $E(\Pi_1(X_1^D)) > E\Pi^C$ can be rewritten as

$$\theta\frac{(a - \omega_l)^2}{8} > (1 - \theta)(a - \omega_h)^2 \tag{10}$$

Both of these inequalities—as well as $X_1^{D*} > X_1^{crit}$—will be satisfied, and hence the equilibrium exists, for, $\theta = .8$, $\omega_h = .6$, $\omega_l = 0$ and $a = 1$.

A forward contracting equilibrium with default can occur if θ is relatively large, see conditions (9) and (10). A large θ means that the high cost state's contribution to *expected* profit will be small. As well, for such an equilibrium to exist, it will also be required that $(a - \omega_h)$ is, relatively speaking, small, see condition (10). A small $(a - \omega_h)$ implies that *actual* profit in the high cost state will be low. If both of these conditions hold, then it will be optimal to essentially ignore (i.e., default in) the high cost state. Hence, firm 1 will sell that amount of forward contracts which maximizes profit in the low cost state, $(a - \omega_l)/2$.

Forward contracting without default: If $E(\Pi_1(X_1^N) > \max\{E\Pi^C, E(\Pi_1(X_1^D))\}$, then the unique equilibrium has firm 1 offering X_1^N forward contracts at date 1. If, in addition $X_1^N > X_1^{crit}$, then this equilibrium can only be supported if firm 1 hedges its input prices. Assuming that $X_1^{D*} > X_1^{crit}$, condition $E(\Pi_1(X_1^N)) > E(\Pi_1(X_1^D))$, can be rewritten as

$$(a - \bar{\omega})^2 > \theta(a - \omega_l)^2 \tag{11}$$

and condition $E(\Pi_1(X_1^N)) > E\Pi^C$ can be rewritten as

$$\frac{(a - \bar{\omega})^2}{8} > (1 - \theta)(\omega_h - \omega_l). \tag{12}$$

Both of these inequalities—as well as $X_1^{D*} > X_1^{crit}$—are satisfied for $\theta = .7$, $\omega_h = 1$, $\omega_l = 0$ and $a = 3$. Note also that for these parameters values $X_1^N > X_1^{crit}$, which implies that firm 1 *must* hedge in this equilibrium in order to achieve the payoff of $E(\Pi_1(X_1^N))$.[15]

A forward contracting equilibrium without default can occur if the probability of the good state occurring is 'not too large', see condition (11). Otherwise it would be optimal to sell more forward contracts and default in the high cost state. As well, the difference between state contingent input costs can not be too great, see condition (12). The importance of this condition is that while forward contracting implies that firm 1 produces 'too little' in the low cost state and 'too much' in the high cost state, the difference between what the leader would ideally like to sell and what it actually sells is not that great. Here, the first-mover advantage outweighs the benefit of being able to produce on a state contingent basis.

[15]The above inequalities are also satisfied for the parameters $\theta = .7$, $\omega_h = 1$, $\omega_l = .5$ and $a = 3.5$. However, for these parameters $X_1^N < X_1^{crit}$, meaning that firm 1 does not have to hedge in this equilibrium.

Discussion

General Characterization of Equilibria Generally speaking, there are four possible equilibria that may arise: (a) firm 1 sells non-defaulting forward contracts but *must* purchase futures contracts to ensure that it does not default on its forward contracts; (b) firm 1 sells non-defaulting forward contracts and does not need to purchase futures contracts; (c) firm 1 sells forward contracts and defaults in the high cost state; and (d) firm 1 does not sell any forward contracts but, instead, competes in the *ex post* spot market. In case (a) firm 1 has a strict incentive to purchase futures contracts because it can achieve a level of expected profits that is unattainable in the absence of purchasing futures contracts. In cases (b)-(d), firm 1 and firm 2 have no strict incentive to purchase futures contracts because, in these cases, purchasing futures contracts does not add any value to the firm. (In fact, for case (c) if firm 1 purchases sufficient number of futures contracts so it does not default, the value of the firm will actually fall.) Hence our model predicts that some firms will hedge their cash flows and others will not. This result is consistent with the observation (Smithson [1996]) that while a large fraction (65%) of large US non-financial firms used derivatives to manage risk, there also exists a large number of firms that do not.

Relax Restrictions on Firm 2 We have structured the model so that firm 1 has a choice between selling its output contractually or selling it in the *ex post* spot market. But we have restricted firm 2 to sell its output only in the *ex post* spot market. To what extent do our results and insights depend upon this restriction? It turns out that if firm 2 is given the same choice as firm 1, then the only equilibrium allocations that exist are (qualitatively speaking) those that are described above. Specifically, the only equilibria that can exist are characterized by one of the following:

1. one firm sells non-defaulting forward contracts and must purchase futures contracts to ensure that it does not default while the other firm produces for the spot market,

2. one firm sells non-defaulting forward contracts that does not need to purchase futures contracts while the other firm produces for the spot market,

3. one firm sells forward contracts that it defaults on in the high cost state while the other firm produces for the spot market, or

4. both firms produce for the spot market.

Perhaps, surprisingly, there does *not* exist an equilibrium where both firms compete in forward contracts. To understand this, suppose that there is an equilibrium where both firms sell forward contracts at date 1 and, in this equilibrium, neither firm defaults on it forward contract obligation[16]: firm 1 sells X_1 forward contracts and firm 2 sell X_2 forward contracts. Suppose that firm 2 defects from proposed play by selling zero forward contracts at date 1: this defection implies that firm 2 will produce for the spot market. Firm 2 will maximize its *ex post* payoff by producing $x_2^s = (a - X_1 - \omega_s)/2$ units of output in state s. Note that it will never be the case that $x_2^h = x_2^l$. Given that firm 1 produces X_1, if $x_2^s \neq X_2$ for both states of the world, then firm 2 will be able to increase its profit in both states of the world by defecting from equilibrium play; if $x_2^s = X_2$ for one state of the world, say h, then $x_2^l \neq X_1$ and, hence, by defecting from equilibrium play, firm 2 can increase its profit in state l while maintaining the same level of profit in state h. Therefore, firm 2 will always defect from proposed equilibrium play. The intuition behind this result is straightforward: by defecting from proposed play firm 2 has the flexibility of supplying the *ex post* profit maximizing

[16]The no-default assumption is made simply for illustrative purposes. If either one or both firms do default, then the logic of the argument that follows still applies.

level of output, given that firm 1 always produces X_1. Since the *ex post* profit maximizing level of output for firm 2, given that firm 1 produces X_1, is not constant, defecting from proposed equilibrium play will unambiguously increase firm 2 expected payoff. Hence, one should interpret the model restriction that firm 2 can only produce for the spot market as a simplifying assumption.

Margin Accounts In practice, participants in futures markets must post or deposit margin accounts with the futures exchange. Intuitively, the size of an individual's margin account equals the loss that the individual's futures position can sustain if prices move against it. Hence, a margin account protects the futures exchange from default.[17] We have assumed that firms do not have any outside wealth and, as a result, can not post a margin. Consider the equilibrium where it is optimal for firm 1 to sell X_N^1 forward contracts but to do so it *must* purchase futures contracts (because $X_1^N > X_1^{D^{crit}}$). Suppose now that firm 1 has precisely that amount of outside wealth to cover the loss that its futures position would incur if input prices turned out to be high. It might appear that the purchase of futures contracts is redundant since the firm is now able to commit to producing X_1^N in the high cost state by 'pledging' its outside wealth. Let's slightly generalize our model and suppose that the economy is repeated twice, where the states of the world—which will be revealed between dates 1 and 2 and dates 3 and 4—are independently distributed. In this environment, futures contracts will continue to have value to firm 1. To see this, suppose that firm 1 does not purchase futures contracts at date 1—it commits to producing X_1^N in the high cost state through its outside wealth—and the state of the world (at date 2) turns out to be the high cost state. At date 3, firm 1 will have no outside wealth since it was needed to produce output at date 2. Now, firm

[17]If an individual attempts to 'default' on its position, the futures exchange will immediately reverse the position, i.e., liquidates the position, and any losses that the exchange encurs is taken out of the margin account.

1 will be unable to commit to producing X_1^N at date 4: it has no outside wealth and can not post the required margin to purchase futures contracts. Suppose instead that firm 1 purchases futures contracts at date 1, (its outside wealth is placed in a margin account). If, at date 2, the state of the world turns out to be the high cost state, firm 1's margin account will be credited and firm 1 will be able to produce X_1^N from the payoff of its futures contract. Most importantly, firm 1's outside wealth will remain in tact. Hence, firm 1 will be able to commit to producing X_1^N at date 4. If, at date 2, the state of the world turns out to be the low cost state, firm 1's outside wealth, which was deposited in a margin account, will be used to pay off its futures contract losses. However, the profit that firm 1 makes from its production will strictly exceed its (initial) outside wealth. Hence, firm 1 will have sufficient resources at date 3 to guarantee a production level of X_1^N at date 4. In summary, a firm that has sufficient internal resources to guarantee production in either state of the world may have a strict incentive to purchase futures contracts when margin accounts must be posted.

6 Conclusions

Suppose that a limitedly liable firm with market power sells its output before all factors that can affect its profitability are known. By hedging its cash flows, the firm may be able to increase the set of allocations from which it can choose. In a way, hedging allows the firm to commit to delivering levels of output it otherwise could not do. In some circumstances, this commitment turns out to be valuable. In particular, if one of the 'commitment' allocations can generate a higher expected payoff than all of the unhedged allocations, then the firm will find it strictly optimal to hedge. In other circumstances, this commitment has no value. In fact, it may turn out that all of the ‘commitment’ allocations result in strictly lower payoffs compared to some unhedged allocations. In these situations, the firm strictly prefers not to hedge. Hence, we have provided a theory that is consistent with the stylized fact that while some firms may find it beneficial to hedge their cash flows, others do not.

References

Allaz, B., 1992, ‘Oligopoly, uncertainty and strategic forward contracts’, *International Journal of Industrial Organization*, pp. 297-308.

Allaz, B. and J.-L. Vila, 1993, ‘Cournot competition, forward markets and efficiency’, *Journal of Economic Theory*, pp. 1-16.

Brander, J. and T. Lewis, 1986, ‘Oligopoly and financial structure: the limited liability effect’, *American Economic Review*, 956-970.

Breeden, D. and S. Viswanathan, 1996, ‘Why do firms hedge? An asymmetric information model’, Fuqua School of Business Working Paper.

DeMarzo, P. and D. Duffie, 1991, ‘Corporate financial hedging with proprietary information’, *Journal of Economic Theory*, 261-286.

DeMarzo, P. and D. Duffie, 1995, ‘Corporate incentives for hedging and hedge accounting’, *Review of Financial Studies*, pp. 734-771.

Degeorge, F., B. Moselle and R. Zeckhauser, 1996, ‘Hedging and gambling: Corporate risk choice when informing the market, CEPR Discussion Paper No. 1520.

Froot, K., D. Scharfstein and J. Stein, 1993, ‘Risk management: Coordinating corporate investment and financing policies’, *The Journal of Finance*, pp. 1629-1658.

Graham, J. and C. Smith, 1996, ‘Tax incentives to hedge’, Working paper.

Ljungqvist, Lars, 1994, ‘Asymmetric information: A rationale for corporate speculation, *Journal of Financial Intermediation*, pp. 188-203.

Maksimovic, V., 1986, ‘Optimal capital structure in oligopolies’, Ph.D. Harvard University, unpublished.

Maksimovic, V., 1995, ‘Financial structure and product market competition’, in R. Jarrow, V. Maksimovic, W. Ziemba eds., *Handbooks in operations research and management science, Finance*, Volume 9, North Holland, Amsterdam, pp.887-920.

Smith, C., 1995, ‘Corporate risk management: theory and practice, *Journal of Derivatives*, pp. 21-30.

Smith, C. and R. Stultz, 1985, ‘The determinants of firm’s hedging policies’, *Journal of Financial and Quantitative Analysis*, pp. 391-405.

Smithson, C., 1995, *Managing financial risk*, Chase, NY, NY.

Stulz, R., 1984, Optimal hedging policies, *Journal of Financial and Quantitative Analysis*, pp. 127-140.

Stulz, R., 1990, ‘Managerial discretion and optimal financing policies’, *Journal of Financial Economics*, pp. 3-27.