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A Model of (the Threat of) Counterfeiting

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A simple matching-model of money with the potential for counterfeiting is constructed. In contrast to the existing literature, counterfeiting, if it occurred, would be accompanied by two distortions: costly production of counterfeits and harmful effects on trade. However, application of the Cho-Kreps refinement is shown to imply that there is no equilibrium with counterfeiting. If the cost of producing counterfeits is low enough, then there is no monetary equilibrium. Otherwise, there is a monetary equilibrium without counterfeiting.

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1 Introduction

The small analytical literature on counterfeiting, Green and Weber [2] and Kulti [3], leaves the impression that counterfeiting is not necessarily harmful. In particular, it is not consistent with the reasonable notion that counterfeiting, like other forms of theft, produces two kinds of distortions. First, a counterfeiter devotes productive effort to an unproductive use—namely, producing counterfeit money. Second, those who do not counterfeit take costly actions to protect themselves from the effects of receiving counterfeits. In this paper, we set out a very simple matching-model of money in which counterfeiting would be accompanied by both distortions if it occurred.

We borrow most of the components of the model from existing work. The background environment is that in Shi [4] and Trejos and Wright [5], a setting with divisible and costly-to-produce goods and money holdings restricted to be in the set $\{0, 1\}$. (In contrast, Green and Weber [2] and Kulti [3] assume that both production and money holdings are in the set $\{0, 1\}$.) The informational features are taken from Williamson and Wright [7], in which there is a recognizability problem regarding the quality of goods and none regarding money. Here, we apply their formulation of the recognizability problem to money—to the problem of distinguishing genuine from counterfeit money.[1] And we assume that people, at a cost, can decide to produce counterfeit money. (This, too, differs from Green and Weber [2] and Kulti [3], who assume in most of their analyses that the stock of counterfeit money is exogenous.) Finally, in order to provide a motivation for people to avoid receiving counterfeit money, we assume that counterfeit money lasts just one period; in particular, it immediately disintegrates after it is received in a trade. This assumption builds into the model the fact that most reasonably high-denomination currency ends up at banks after one use where counterfeits tend to be detected. It also prevents counterfeits from alleviating a shortage of money, an implausible role that appears in [2] and [3].

Our model has a surprising implication: There is no equilibrium in which counterfeiting actually occurs. For a set of parameters for which counterfeiting is sufficiently unattractive—essentially, a high enough cost of producing

[1] Velde, Weber, and Wright [6] also apply the Williamson-Wright formulation of the recognizability problem to money—to two commodity monies, monies which throw off real dividends. Because their monies are commodity monies and because they assume that the quantities of both monies are exogenous, their model is very different from our model.

counterfeits and/or a high enough probability that counterfeits are recognized prior to trade—there is a monetary equilibrium in which counterfeiting does not occur. In the complement of that set, there is no monetary equilibrium that satisfies the Cho-Kreps [1] refinement—a refinement that is natural in our setting.

2 The model

As noted above, the background environment is that in Shi [4] and Trejos and Wright [5]. At each date there are $N > 2$ types of perishable goods and N specialization types of people. There is a $[0, 1]$ continuum of each specialization type. A type-n person, $n \in \{1, 2, \ldots, N\}$, consumes only good n and produces only good $n+1$ (modulo N).

Each person maximizes expected discounted utility. Period utility for a type-n person who does not produce counterfeit money is $u(x) - y$ and is $u(x) - y - c$ for a type-n person who does produce counterfeit money, where x is the amount of good n consumed and y is the amount of good $n+1$ produced and $c > 0$. The function u is strictly increasing, twice differentiable, and strictly concave, and $u(0) = 0$, $u'(0) = \infty$ and $u'(\infty) = 0$. The discount factor is $\beta \in (0, 1)$.

People cannot commit to future actions and trading histories are private information. Those two assumptions and the absence of a double-coincidence in any pairwise meeting implied by $N > 2$ imply that trade must involve money. We assume that money is indivisible and that each person's holding at any time is in the set $\{0, 1\}$. There are two kinds of money: genuine money and counterfeit money. The economy is endowed with a fixed stock of genuine money, the amount $m \in (0, 1)$ per specialization type.

The sequence of actions within a period is as follows. The period starts with a fraction m of each specialization type holding genuine money. Those without genuine money then decide whether or not to produce counterfeit money. Let θ denote the fraction of the population who produce counterfeit money so that people are split as follows: the fraction m have a unit of genuine money, the fraction θ have a counterfeit, and the remainder, $1-m-\theta$ have neither money. People are then randomly paired off. The only meetings that matter are *trade meetings*, single-coincidence meetings in which the potential consumer, the buyer, has a unit of money, genuine or counterfeit, and the potential producer, the seller, does not. Following Williamson and

Wright [7], in any trade meeting, the pair receives a signal regarding the "quality" of the money of the buyer: with probability ϕ the signal reveals the type of money held by the buyer—either genuine or counterfeit—and with probability $1-\phi$ the signal is uninformative leaving the seller with the posterior probability $m/(m+\theta)$ that the money is genuine. After the signal is realized, the buyer makes a take-it-or-leave-it lottery offer to the seller. After meetings dissolve, any counterfeit money disintegrates.

We study only allocations that are symmetric across specialization types. For such allocations, our economy has no potentially varying "state:" each date starts with a fraction m of individuals of each specialization type holding genuine money. Therefore, we also restrict our analysis to constant allocations.

3 Equilibrium allocations

No matter how pairwise trade is modeled, our setting does not permit the existence of a separating equilibrium—one in which $\theta > 0$ and in which holders of genuine money distinguish themselves in trade meetings from holders of counterfeit money when the uninformative signal is realized. Suppose, by way of contradiction, that there is such an equilibrium. Then sellers would never produce in exchange for a counterfeit. But separation implies that sellers do produce for genuine money. But, then, a holder of counterfeit money gains by duplicating the strategy of the holder of genuine money and we have a contradiction. Therefore, we study only pooling allocations in which holders of counterfeit money exactly emulate the actions of holders of genuine money when the uninformative signal is realized.

A constant and symmetric allocation consists of three quantities: output in trade meetings when the informative signal is realized, q_I, output in trade meetings when the uninformative signal is realized, q_U, and the fraction of people who produce counterfeits, θ. Let v_0 denote the discounted utility of beginning a date without money, let v_1 denote that of beginning with genuine money, and let $\Delta \equiv \beta(v_1 - v_0)$. Without loss of generality, we describe the buyer take-it-or-leave-it problems for holders of genuine money under the assumption that $\Delta \geq 0$ and under the assumption that there is no lottery over output, just over whether the buyer surrenders money.

Problem 1 *Choose* $(q_j, p_j) \in \mathbb{R}_+ \times [0,1]$ *to maximize* $u(q_j) - p_j\Delta$ *subject to*

$$-q_j + p_j\lambda_j\Delta \geq 0, \tag{1}$$

for $j = I, U$ *with* $\lambda_I = 1$ *and* $\lambda_U = \frac{m}{m+\theta}$.

The objective in this problem is the buyer's gain from trade, $u(q_j) + p_j\beta v_0 + (1-p_j)\beta v_1 - \beta v_1$. Maximizing it is equivalent to maximizing the buyer's pay-off because βv_1 is a constant. Also, because $(q_j, p_j) = (0, 0)$ is feasible in this problem and implies a zero value for the objective, any solution to this problem implies a non negative gain from trade for the buyer. The constraint, (1), is a non negative gain from trade for the seller taking into account that when the uninformative signal is realized, $j = U$, the seller's posterior distribution is that the buyer's money is genuine with probability $\frac{m}{m+\theta}$.

Next, we express v_0 and v_1 in terms of the trades and the choice of whether to counterfeit. We have,

$$(1-\beta)v_0 = \max\{C, \tilde{C}\}, \tag{2}$$

where

$$C = -c + \frac{1-m-\theta}{N}(1-\phi)u(q_U) \tag{3}$$

and

$$\tilde{C} = \frac{m}{N}[\phi(-q_I + p_I\Delta) + (1-\phi)(-q_U + p_U\Delta)] - \frac{\theta}{N}(1-\phi)q_U. \tag{4}$$

And we have

$$(1-\beta)v_1 = \frac{1-m-\theta}{N}\{\phi[u(q_I) - p_I\Delta)] + (1-\phi)[u(q_U) - p_U\Delta]\}. \tag{5}$$

In (3), C is the flow pay-off to counterfeiting and in (4), $\tilde{C}$ is the flow pay-off to not counterfeiting. We can now define an equilibrium.

Definition 1 *An allocation* (q_I, q_U, θ) *is a monetary equilibrium if* $max\{q_I, q_U\} > 0$ *and if there exists* (p_I, p_U) *and* (v_0, v_1) *such that (i)* (q_I, p_I) *and* (q_U, p_U) *solve problem* 1*; (ii) (2)-(5) hold; (iii)* $\max\{C, \tilde{C}\} = \tilde{C}$ *and* $\theta > 0$ *implies* $\max\{C, \tilde{C}\} = C$*; (iv) the Cho-Kreps refinement is satisfied.*

The first condition in (iii) is necessary in order that not everyone counterfeit. If they do, then there are no sellers. As regards condition (iv), the Cho-Kreps refinement for a pooling equilibrium of this model requires that a holder of genuine money not have a profitable deviation that would signal that the money is genuine—a profitable deviation that a holder of counterfeit money would not find it profitable to emulate.

We begin by describing the solution to problem 1.

Lemma 1 *Let* $\hat{q}_j = \lambda_j \Delta$ *and let* q_j^* *be the unique solution to* $u'(q_j^*) = \frac{1}{\lambda_j}$. *The solution to problem* 1 *is* $q_j = \min\{\hat{q}_j, q_j^*\}$ *and (1) at equality.*

Proof. Any solution to problem 1 satisfies (1) at equality. It follows that if the constraint $p_j \leq 1$ is not binding, then the solution is $q_j = q_j^*$. If it is binding, then $\hat{q}_j < q_j^*$ and the solution is $q_j = \hat{q}_j$ and $p_j = 1$. ■

Now we delineate the set of parameters alluded to in the introduction. Let $\hat{q}$ be the unique positive solution to

$$\frac{u(\hat{q})}{\hat{q}} = 1 + \frac{N(1-\beta)}{\beta(1-m)}. \tag{6}$$

(This expression for $\hat{q}$ comes from setting $q_I = q_U = \hat{q}$, $(p_I, p_U) = (1,1)$, and $(v_0, v_1) = (0, \hat{q}/\beta)$ in (5).) Also, let q^* be the unique solution to $u'(q^*) = 1$ and let $\bar{q} = \min\{\hat{q}, q^*\}$. Now let

$$A = \{(c, \phi) : c \geq \frac{1-m}{N}(1-\phi)u(\bar{q})\}. \tag{7}$$

This is a simple set because $\bar{q}$ does not depend on c or ϕ.

Proposition 1 *If* $(c, \phi) \in A$, *then* $\theta = 0$ *and* $q_I = q_U = \bar{q}$ *is a monetary equilibrium.*

Proof. We propose $p_I = p_U = p$. If $\min\{\hat{q}, q^*\} = \hat{q}$, then we propose $p = 1$ and $(v_0, v_1) = (0, \hat{q}/\beta)$. If $\min\{\hat{q}, q^*\} = q^*$, then we propose $v_0 = 0$, $(1-\beta)v_1 = \frac{1-m}{N}[u(q^*) - q^*]$, and p given by (1) at equality. By construction, these proposals satisfy conditions (i) and (ii) of definition 1. As regards condition (iii), the proposal implies $\tilde{C} = 0$ and the definition of A implies that $C \leq 0$. Condition (iv) does not arise. ■

Proposition 2 *If* $(c,\phi) \notin A$*, then there is no monetary equilibrium.*

Proof. Suppose, by way of contradiction, that there is such an equilibrium. Either $\theta = 0$ or $\theta \in (0,1)$. The former is impossible because, as is easily verified, any such equilibrium must be the proposal in proposition 1. And with $(c,\phi) \notin A$, that proposal implies $\tilde{C} = 0$ and $C > 0$, a contradiction. Therefore, the equilibrium must have counterfeiting, which, in turn, implies $q_U > 0$ and $p_U > 0$.

Now we apply the Cho-Kreps refinement. Consider a deviation by a holder of genuine money that satisfies $q < q_U$. Any such offer is viewed by the seller as coming from a holder of genuine money because a counterfeiter cares only about the amount of output obtained. It is easy to see that there exist such deviations that are profitable for a holder of genuine money and that are accepted by the seller. Profitability for a holder of genuine money is

$$u(q) + (1-p)\Delta > u(q_U) + (1-p_U)\Delta, \tag{8}$$

while acceptance by the seller is $-q + p\Delta \geq 0$. Because (q_U, p_U) is part of an equilibrium, it must satisfy $-q_U + p_U \frac{m}{m+\theta}\Delta \geq 0$. Therefore, acceptance by the seller is implied by

$$-q + p\Delta \geq -q_U + p_U \frac{m}{m+\theta}\Delta. \tag{9}$$

Let q' satisfy $u(q') + (1 - p_U \frac{m}{m+\theta})\Delta = u(q_U) + (1-p_U)\Delta$. If $q \in (q', q_U)$ and $p = p_U \frac{m}{m+\theta}$, then (8) and (9) hold. Hence, an equilibrium with $q_U > 0$ does not satisfy the Cho-Kreps refinement. ■

Therefore, as we said at the outset, there is no equilibrium with counterfeiting. If $(c,\phi) \in A$, which should be interpreted as counterfeiting being sufficiently unattractive, then there is a monetary equilibrium in which counterfeiting does not occur. If $(c,\phi) \notin A$, then there is no monetary equilibrium.

4 Concluding remarks

We began by constructing a simple model that we thought would give rise to a region of the parameter space in which counterfeiting occurs. That turned out not to be the case. Our model has many extreme assumptions: the simple nature of the signal; the perishable nature of counterfeit money;

money holdings in the set $\{0,1\}$; the assumption that people are identical (for example, everyone has the same cost of counterfeiting), and the assumption that buyers make take-it-or-leave-it offers. However, it is far from obvious that departures from these assumptions would overturn the non existence we find.

We have given some thought to replacing the last assumption by having buyers make offers with some probability and having sellers make offers with the complementary probability. However, so long as buyers sometimes make such offers, there is no equilibrium with counterfeiting. The argument in the proof of proposition 2 says that output cannot be positive when the buyer makes the offer and the uninformative signal is realized. And lemma 1 says that output is not zero if genuine money is valuable, $\Delta > 0$. Finally, as is well-known, there is no monetary equilibrium in this model if sellers always make the offer.

The non existence of an equilibrium with counterfeiting seems to arise because holders of counterfeit money value output as least as much as do holders of genuine money and value retaining money less than do holders of genuine money.[2] Those features seem reasonable. If we take seriously the non existence of an equilibrium with counterfeiting, then the message is that counterfeiting can be a serious threat even if we do not see it occurring. That is, actions taken to keep the cost of producing counterfeits high and the probability that counterfeits can be recognized high can be worthwhile even if (a significant amount of) counterfeiting is not observed.

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[2] That is, those features, lotteries, and the application of the Cho-Kreps refinement produce the non existence. The lotteries are important. Without them, the Cho-Kreps refinement has no bite because agents cannot make small profitable deviations. Lotteries are not considered in the other models in which there is a recognizability problem concerning money—namely, [2], [3], and [6].

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