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We revisit classic questions concerning the effects of money on investment in a new framework: a two-sector model where some trade occurs in centralized and some in decentralized markets, as in recent monetary theory, but extended to include capital. This allows us to incorporate novel elements from the microfoundations literature on trading with frictions, including stochastic exchange opportunities, alternative pricing mechanisms, etc. We calibrate models with bargaining and with price taking in the decentralized market. With bargaining, inflation has little impact on investment, but a sizable impact on welfare: going from 10% inflation to the Friedman rule e.g. barely affects capital, but is worth 3% of consumption. With price taking, this policy increases capital between 3% and 5%, and is worth 1.5% of consumption across steady states or 1% with transition. Fiscal distortions are also big. So is the impact of holdup problems from bargaining, even if the decentralized market accounts for only 5% of output. Many of these numbers are quite different from previous studies. Our two-sector specification is a key to the results.

Key words: bargaining, price taking, centralized markets, decentralized markets, capital, inflation, investment, welfare.

JEL code: C78, E44

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1 Introduction

This paper studies the effects of monetary policy on capital formation in the long run. By this we do not mean that we focus only on steady states, since dynamic transitions are central in the analysis, but that we focus on fully-anticipated inflation and abstract from signal-extraction problems, nominal rigidities, and other complications that we think are more likely to more important only in the shorter run. This long-run relation between money and capital is one of the classic issues in macroeconomics, going back at least to Tobin (1965) and Sidrauski (1967a, 1967b), continuing through Stockman (1981), Cooley and Hansen (1989, 1991), Gomme (1993), Ireland (1994) and many others. All of these papers adopt a reduced-form approach, in the sense that they put real balances directly into the utility function, or impose cash-in-advance constraints, e.g., in an attempt to implicitly capture the role of money in the exchange process, but in other respects use more or less frictionless models.

While this has some advantages, an alternative approach has developed over the years that attempts to be relatively explicit about the frictions that make a medium of exchange essential in the first place, and in this effort, the papers have introduced a variety of new elements into monetary economics, including detailed descriptions of specialization and information, bilateral matching and trading, alternative pricing mechanisms such as bargaining, explicit multi-sector models, and so on. See e.g. Kiyotaki and Wright (1989, 1993), Aiyagari and Wallace (1991), Shi (1995), Trejos and Wright (1995), Kocherlakota (1998), Wallace (2001), and some other work that we discuss in more detail below. Our goal is to revisit the relation between money and capital through the lens of this literature on the microfoundations of trading with frictions, to see if the above-mentioned elements matter for the issue, they way they have been shown to matter for some other issues.[1]

[1] See Lagos and Wright (2005), Molico (2006), Craig and Rocheteau (2006), Aruoba, Rocheteau and Waller (2007), and Ennis (2007) for recent examples where incorportaing frictions like search, bargaining, or priviate information makes a difference not only theoretically but also quantitatively for issues in monetary economics.

Our starting point is the two-sector framework developed in Lagos and Wright (2005), where some economic activity takes place in centralized markets and some takes place in decentralized markets. This is useful because, in addition to potentially providing a role for media of exchange, decentralized markets allow us to introduce ingredients like stochastic exchange opportunities and pricing mechanisms such as bargaining, while centralized markets allow us to incorporate capital exactly as in standard growth theory. This contrasts sharply with earlier attempts to study money and capital in models with frictions, including Shi (1999), Shi and Wang (2006), and Menner (2006), who build on Shi (1997), and Molico and Zhang (2005), who build on Molico (2006). While that is certainly admirable work, those models have *only* decentralized markets. We find it much easier to connect with mainstream macro, and to incorporate other potentially relevant ingredients like fiscal policy, in addition to capital, when there is also some centralized trade.[2]

It is, however, not trivial to put capital into the Lagos-Wright model, in the sense that when one does so in an obvious way one is lead to undesirable implications: the version in Aruoba and Wright (2003) e.g. displays a strong *dichotomy*, which means that one can solve independently for allocations in the centralized and decentralized markets. Among the undesirable implications, in that model, monetary policy has no impact on investment, employment or consumption in the centralized market; indeed, based on these results, one might say that money and growth theory have not been integrated at all (Howitt 2003; Waller 2003). The model here does not dichotomize, in general, because of explicit feedback across sectors. In our baseline model, this is because capital used in decentralized market

[2] Reducing the gap between models of decentralized trade and mainstream macro has been a challenge for a while. As Azariadis (1993) put it, "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1982) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." As Kiyotaki and Moore (2001) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets." For us, combining centralized and decentralized markets is not just a device to reduce complexity, as it was in Lagos and Wright (2005), but a way to bring back some competitive markets and make models with trading frictions closer to standard macro and growth theory.

production is itself produced in the centralized market. This allows potentially rich feedback from money to investment and other centralized market variables, and the issues then become quantitative. So we calibrate the model to study numerically the effects of monetary and fiscal policy on capital formation, welfare, and other variables.

A finding that is perhaps surprising – or, at least, that is not seen in previous analyses – is that the results depend critically on what one assumes about price formation in the decentralized sector. If we assume generalized Nash bargaining between buyers and sellers in this sector, inflation has almost no impact on capital formation, due to a double holdup problem affecting both investment and money demand. Intuitively, if sellers have too little bargaining power their return in this market is to small to matter for aggregate investment; and if they have too much bargaining power the size of the sector (or, the value of money) becomes too small to matter for any aggregate variable. In fact our numerical results indicate that for any bargaining power the impact of inflation on investment is quite small. However, inflation can still have a sizable impact on welfare, because it reduces decentralized market consumption, which is already low due to the holdup problems. To put a number on it, going from 10% inflation to the Friedman rule and making up any revenue loss by lump sum taxation is worth about 3% of total consumption.

If we switch to a pricing mechanism that avoids the holdup problems, which in our case could be either price taking or price posting, we find something completely different. Now going from 10% inflation to the Friedman rule can increase the long-run capital stock between 3% and 5%. This is a lot, and shows the relation between money and capital hinges critically on how the terms of trade are determined in decentralized trade. Inflation is also costly in terms of welfare in this version of the model, although not as much, and for different reasons: in addition to reducing investment by a lot, compared to the bargaining version, inflation also reduces decentralized market consumption by a lot, but this is less painful, since for any given inflation rate consumption is not so low without holdup problems. The relevant welfare number is now around 1.5% of total consumption across steady states, and around

1% when we take into account the transition because of course agents have to work and save to accumulate the additional capital (transitions are not important with bargaining because the capital stock does not change much).

Some of these results are quite different from previous findings.[3] We also find sizable effects from fiscal policy, and on balance, if we have to make up revenue with distortionary instead of lump sum taxes, reducing inflation may or may not be a good idea, depending on details. Thus, given existing tax rates, the Friedman rule is not necessarily the best policy, even though it would be optimal without distortionary fiscal policy. We also show the holdup problems in the demand for money and capital can be quantitatively important, even though we only have bargaining in the decentralized market and we find this accounts for only around 5% of aggregate output. These results are reasonably robust to several variations on the specification. For example, they do not depend much on the length of a period – a year, a month or whatever – mainly because we allow stochastic trading opportunities. This means that velocity in our model is not constrained by period length, as in typical cash-in-advance models: when we reduce period length, the probability of needing the money in the decentralized market can fall, and so velocity need not go up. Hence, unlike a typical cash-in-advance model, our results are robust at least on this dimension.

Our overall conclusion is that bringing in elements from modern monetary economics can make a difference, not only in theory, but for policy-relevant quantitative issues. The two-sector structure in particular is key. Intuitively, inflation is a tax on cash-intensive

[3]Cooley and Hansen (1989,1991) find much smaller effects on capital and welfare in cash-in-advance models – e.g. steady state welfare numbers substantially below 1%. Gomme (1993) gets even smaller effects in his endogenous growth version. Ireland (1994) gets welfare numbers around 0.67% (although he finds reverse causation from growth to the monetary system can be substantial). Lucas (2000) has no capital, or even a model, but using the approach in Bailey (1956) he gets welfare numbers below 1%; earlier efforts at this approach by Lucas (1981) and Fischer (1981) e.g. get 0.3% to 0.45%. Imorhoroglu and Prescott (1991) also get less than 1%. A few papers find somewhat larger effects, such as Dotsey and King (1996), in part because even though inflation does not affect total capital very much it affects the amount of resources used in intermediation (see also Freeman and Kydland 2000 or Aiyagari, Braun and Eckstein 1998); we abstract from these endogenous intermediation effects here. In a search model, Molico and Zhang (2005) actually get steady state capital and welfare to increase with inflation over the range we consider, due to distributional effects like those in Molico (2006), while the other search models in fn. 1 find a big cost of inflation, sometimes 4% or more, but those models have no capital.

consumption goods, and if these goods are produced in a sector that uses capital produced elsewhere in the economy, then monetary policy can have important effects on the entire economy even if the cash-intensive sector is small. For this to work, it is important not only to have both "cash goods" and "credit goods" but for these goods to be produced in different sectors where capital plays different roles – which occurs here because capital is an input in the decentralized market and an input plus an output in the centralized market.[4] Given this, we actually do not want in this paper to make too much of "microfounded versus reduced-form" monetary economics; for the questions at hand, the interesting aspects are the impact of stochastic trading opportunities, bargaining, and the two-sector structure, not methodological issues per se.

Related to that point, we do not try to provide a detailed analysis of why claims to capital (or other inside money) might not displace outside money as medium of exchange. We do offer a story to this effect that one may find useful, the way many people seem to find e.g. Lucas' (1980) story of the "worker-shopper pair" useful for cash-in-advance models. But the "worker-shopper" parable actually rationalizes only the use of *some* medium of exchange, not whether it should be inside or outside money. So we endeavor to go a little further, appealing to long-standing ideas about portability and recognizability as a way to motivate why currency may have a role when capital is a factor. But to be clear, this is not intended to be our main contribution. While it might be nice to have a compelling and quantitatively relevant explanation for the coexistence of money and higher-return assets, independent of that, it seems worth asking how elements from the literature on trading with frictions affect answers to important questions.[5]

The rest of the paper is organized as follows. In Section 2 we describe the model. In

[4] For comparison, we also consider some alternative scenarios, where e.g. new capital is produced and traded in the decentralized market sector, as in Shi (1999). Our basic message is fairly robust, as long as there are multiple sectors and there is feedback via capital across sectors.

[5] In some other search models, money and capital can both play a role, but only if they have the same rate of return in equilibrium (e.g. Lagos and Rocheteau 2006; Geromichalos, Licari, and Suarez-Lledo 2007). There is also a literature using overlapping generations models of money to study capital and growth (e.g. Schreft and Smith 1997, 1988), but the empahsis is very different.

Section 3 we discuss calibration. In Section 4 we present quantitative results. In Section 5 we conclude. The Appendix contains details of the analysis and alternative specifications.

2 The Basic Model

2.1 General Assumptions

A $[0,1]$ continuum of agents live forever in discrete time. In order to integrate elements of both mainstream macro and search theory, we adopt the sectoral structure in Lagos and Wright (2005), hereafter LW, where each period the agents engage in two types of economic activity. Some of this activity takes place in a frictionless centralized market, referred to as the CM, and some takes place in a decentralized market, referred to as the DM, with two main frictions: a *double coincidence problem* and *anonymity.* These frictions combine to make some medium of exchange essential in the DM. As this is not the place to go into all of the details, for formal discussions of essentiality and the role of anonymity we refer readers to Kocherlakota (1998), Wallace (2001), and Aliprantis, Camera and Puzzelo (2007).[6]

Given a medium of exchange is essential, one issue in monetary theory is to determine endogenously which objects serve this function (Kiyotaki and Wright 1989). In order to focus on other questions, however, other papers avoid this by assuming there is a unique storable asset that qualifies for the role. Since we obviously cannot assume a unique storable asset in a paper called "Money and Capital" we need to say a few words about the issue. As we said, what we have to offer is a story along the lines of the "worker-shopper" story in Lucas' (1980), extended slightly based on ideas about the origins of monetary exchange going back at least to Menger (1892) involving *portability* and *recognizability.* First, in terms of portability, we assume that in the DM agents have their capital physically fixed in place

[6]Lest there is confusion, note that Aliprantis et al. (2006) construct an environment with both CM and DM meetings where money is *not* essential. But this environment differs from LW in an important way having to do with their aggregate production function, which allows a single individual to shut down the economy by withdrawing his labor services, and hence allows the use of trigger strategies. This cannot happen in LW, so the result says nothing about essentiality in LW. In any case, the point is moot: Aliprantis et al. (2007) show money *is* essential, even with their production function, in a variant of the environment where some meetings are centralized only in the sense that there are enough agents for the law of large numbers, not in the sense that everyone is in the same place at the same time.

at production sites. Thus, when you want to buy something from someone, you must visit his location, and since you cannot bring your capital it cannot be used in payment. Since cash is portable, it can. This appeal to spatial separation is very much in the spirit of the "worker-shopper" idea. But one really ought to go beyond this, and ask why claims to capital (or perhaps other claims) cannot overcome such spatial separation.

Although this is a tough question, to which we do not have a definitive answer, one approach is to invoke recognizability. A blunt version of this is to assume agents in the DM can costlessly counterfeit claims, other than currency, perhaps because the monetary authority has a monopoly on the technology for producing hard-to-counterfeit notes. Given this, a DM seller would never accept a claim to capital from an anonymous buyer, any more than a personal IOU, and money may be needed as medium of exchange even if capital is a storable factor of production. As an aside, later we suggest that the money need not necessarily be in your pocket, and can for instance be in your bank, if it also has access to the technology for producing hard-to-counterfeit objects. As long as banks must hold reserves – i.e. as long as currency is held by *someone* – the theory of inflation as a tax on currency or currency-backed objects should apply, as long as capital-backed assets do not drive these objects out, as can be the case when recognizability is an issue.[7]

While more work needs to be done on the coexistence of money and other assets, in general, we now continue with the task of this paper: analyzing capital-theoretic issues in models incorporating elements from the literature on trading with frictions. As in the neoclassical growth model, in the CM there is a general good that can be used for consumption or investment, produced using labor H and capital K hired by firms in competitive markets. Profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r the rental rate, and w the real wage; constant returns implies equilibrium profits are 0. In

[7]Monetary economics following the approach discussed in Wallace (1996, 2001) e.g. involves describing as part of the model a physical environment where money has an interesting role – i.e. it helps overcome certain frictions – and this artificial economy should be internally consistent if not literally realistic. We think recognizability is an interesting friction, and plausibly a realistic one, that should be considered seriously when it comes to understanding the role of money and other assets, but pursuing this rigorously is not the subject of this paper. See Lester, Postlewaite and Wright (2006) for a stab in that direction.

the DM these firms do not operate, but an agent's own effort e and capital k can be used with technology $f(e, k)$ to produce a different good. Notice k appears as an input the DM, even if it cannot be used as a means of payment, because when you go to a seller's location he has access to his capital even if you do not. This is important, because the fact that capital produced in the CM is productive in the DM is what breaks the above-mentioned dichotomy and allows money to have interesting effects on the CM.

We generate our double-coincidence problem as follows. With probability σ an agent in the DM discovers he is a *buyer*, which means he wants to consume but cannot produce, in which case he visits the location of someone chosen randomly from the set of agents that can produce. With probability σ an agent is a *seller*, which means he can produce but does not want to consume, in which case he waits at his location for someone to visit him. And with probability $1 - 2\sigma$ he is a *nontrader*, in which case we can either interpret him as neither producing nor consuming, or as producing for his own consumption (it does not matter for anything we do). In most contexts this taste and technology shock specification would be equivalent to a bilateral matching specification, where there is a probability σ of wanting to consume something produced by a random partner. We use taste and technology shocks, instead of matching, because we think it fits well with assumptions about spatial separation with buyers visiting sellers' locations, but not too much really hinges on this.

Instantaneous utility for everyone in the CM is $U(x) - Ah$, where x is consumption and h labor hours. Linearity in h reduces the complexity of the model (see Chiu and Molico 2007 for what happens without this assumption in a related model). Alternatively, Rocheteau et al. (2006) show how to get exactly the same results with general utility by assuming indivisible labor and lotteries, à la Rogerson (1988). In the DM, with probability σ you are a consumer and have utility $u(q)$, and with probability σ you are a producer and have disutility $-\ell(e)$, where q is consumption and e labor effort. If we solve $q = f(e, k)$ for $e = \xi(q, k)$, then $c(q, k) \equiv \ell[\xi(q, k)]$ is the utility cost of producing q, given k.[8] Since it is

[8]In Appendix B.1 we show $c_q > 0$, $c_k < 0$, $c_{qq} > 0$, and $c_{kk} > 0$, under the usual monotonicity and convexity assumptions on f and ℓ, and $c_{qk} < 0$ if $f_k f_{ee} < f_e f_{ek}$, which holds under the additional assumption

hard to disentangle technology and preferences at this level, we normalize $\ell(e) = e$, which is basically choosing units for effort, and is of little consequence here since we do not regard e as observable. Otherwise preferences have the usual properties. Agents discount between the CM and DM at rate β but not between the DM and CM.

Government sets the money supply so that $M_{+1} = (1+\tau)M$, where $+1$ denotes next period. We use τ as our monetary policy instrument, but as long as the Fisher equation holds, which is reasonable for long-run analysis, it is equivalent to target the inflation rate or the nominal interest rate.[9] Government also consumes G, levies a lump-sum tax T, a labor income tax t_h, a capital income tax t_k, and a sales tax t_x in the CM. We omit sales tax in the DM to streamline the presentation, and because it matters little for quantitative results. Letting δ be the depreciation rate of capital, which is tax deductible, and p the CM price level, the usual government budget constraint is

$$G = T + t_h wH + (r - \delta) t_k K + t_x X + \tau M/p.$$

If $W(m,k)$ and $V(m,k)$ are the value functions in the CM and DM, then the DM problem for the representative agent is

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1},k_{+1}} \{U(x) - Ah + \beta V_{+1}(m_{+1}, k_{+1})\} \qquad (1) \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1 + (r-\delta)(1-t_k)]\,k - k_{+1} - T + \frac{m - m_{+1}}{p}. \end{aligned}$$

Eliminating h using the budget and taking FOC, assuming interiority, we get

$$\begin{aligned} x &: U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\ m_{+1} &: \frac{A}{pw(1-t_h)} = \beta V_{+1,m}(m_{+1}, k_{+1}) \qquad (2) \\ k_{+1} &: \frac{A}{w(1-t_h)} = \beta V_{+1,k}(m_{+1}, k_{+1}). \end{aligned}$$

that k is a normal input.

[9] The Fisher equation can be interpreted as a no-arbitrage condition if we use standard methods to price nominal claims between one CM and the next CM. If one goes that route, one has to assume these claims cannot be used as a means of payment in the DM; this is the same as worrying about claims to capital in the DM, and again recognizability can be brought to bear. Alternatively, even if no such claims exist, the Fisher equation can be interpreted merely as a piece of notation defining in terms of the inflation and real interest rate a variable i that we call the nominal rate. We often use i in the discussion, but nothing hinges on this; one can recast all the theory and quantitative work in terms of inflation or money growth if one prefers.

We immediately have two results. First, W is linear in (m,k),

$$W_m(m,k) = \frac{A}{pw(1-t_h)} \tag{3}$$

$$W_k(m,k) = \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)}; \tag{4}$$

and second, since (m,k) does not appear in (2), for any distribution of (m,k) across agents entering the CM the distribution of (m_{+1},k_{+1}) is degenerate.[10]

Moving to the DM, we have

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k), \tag{5}$$

where the values to being a buyer and a seller are

$$V^b(m,k) = u(q_b) + W(m-d_b,k) \tag{6}$$

$$V^s(m,k) = -c(q_s,k) + W(m+d_s,k), \tag{7}$$

with q_b and d_b (q_s and d_s) denoting goods and money exchanged when buying (selling). Using (3) we have

$$V(m,k) = W(m,k) + \sigma\left[u(q_b) - \frac{d_b A}{pw(1-t_h)}\right] + \sigma\left[\frac{d_s A}{pw(1-t_h)} - c(q_s,k)\right]. \tag{8}$$

This yields the envelope conditions

$$\begin{aligned} V_m(m,k) &= \frac{A}{pw(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ &\quad + \sigma\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} V_k(m,k) &= \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial k}\right] \\ &\quad + \sigma\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right]. \end{aligned} \tag{10}$$

[10] This is the simplification that comes from quasi-linear utility (or, alternatively, indivisible labor). Note that it does *not* mean two people with *very* different (m,k) will look identical after one period – for interiority and hence for the result to go through, (m,k) cannot be too disperse, but if agents start with similar (m,k) then they stay similar. Also, we are simply assuming the concavity of V and interiority here; one can adapt the discussion in LW to guarantee this is valid, but we do not bother, since we check it directly in the quantitative analysis.

Once we specify how q and d are determined, in equilibrium, we can substitute for their derivatives in (9) and (10). But first, as a benchmark, consider the planner's problem *without* anonymity, so that money is *not* essential:

$$\begin{aligned} J(K) &= \max_{q,X,H,K_{+1}} \{U(X) - AH + \sigma [u(q) - c(q,K)] + \beta J_{+1}(K_{+1})\} \\ \text{s.t. } X &= F(K,H) + (1-\delta)K - K_{+1} - G \end{aligned} \tag{11}$$

Eliminating X, and again assuming interiority, we have the FOC

$$\begin{aligned} q &: u'(q) = c_q(q,K) \\ H &: A = U'(X)F_H(K,H) \\ K_{+1} &: U'(X) = \beta J'_{+1}(K_{+1}). \end{aligned} \tag{12}$$

The envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$ implies

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1},H_{+1}) + 1 - \delta] - \sigma\beta c_k(q_{+1},K_{+1}). \tag{13}$$

From the first condition in (12), $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1},H,X) satisfy the Euler equation (13), the second FOC in (12), and the constraint in (11). This characterizes the first best.[11]

Note the presence of the term $-\beta\sigma c_k(q_{+1},K_{+1}) > 0$ in (13), which reflects the fact that investment affects DM as well as CM productivity because K is used in both sectors. If K did not appear in $c(q)$ the system would dichotomize: we could first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions independently for (K_{+1},H,X). The fact that K is used in the DM and produced in the CM breaks this dichotomy: in general

[11] To be precise, assume the environment is stationary. Then the methods in Stokey and Lucas (1989) tell us the solution is fully characterized by the FOC and envelope condition, or, equivalently, we can replace the FOC for K_{+1} and envelope condition by the Euler equation and standard transversality condition. This same comment applies when we define equilibrium, but we will not dwell on it, below. Also, one can use standard methods to check when there is a unique steady state and the planner's solution converges to it under the typical kind of assumptions, since once we substitute $q = q^*(K)$, except for notation, this is a standard optimal growth problem. Things are more complicated for equilibria because we allow for fiscal, monetary, and bargaining distortions. In Appendix B.4 we show analytically that for the price-taking version there exists a unique steady state for the functional forms we use in the calibration; in the bargaining version we rely on numerical results.

one must solve simultaneously for the paths for all the variables (q, K_{+1}, H, X). Note also that, although here we assume it is the same K used in both sectors, Appendix A.1 presents a version with two distinct capital goods. Also, Appendix A.2 presents a version where K is used only in the CM but produced traded in the DM (which is reminiscent of Stockman 1981). As discussed in Section 4.3, these variations do not affect the main results too much, since in each case there is feedback across sectors via the capital stock.

2.2 Bargaining

Suppose now that each agent with a desire to consume in the DM visits one who can produce. As suggested above, when physical capital is immobile, claims other than cash are counterfeitable, and buyers are anonymous, trade requires money. If the buyer's and seller's states are (m_b, k_b) and (m_s, k_s), we assume for now that the terms of trade (q, d) solve the generalized Nash problem, with bargaining power for the buyer given by θ, and threat points given by continuation values. Since the buyer's payoff from trade is $u(q) + W(m_b - d, k_b)$ and his threat point is $W(m_b, k_b)$, (3) implies his surplus is $u(q) - Ad/pw\,(1 - t_h)$. Similarly, the seller's surplus is $Ad/pw\,(1 - t_h) - c(q, k_s)$. Hence the bargaining solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw\,(1-t_h)}\right]^{\theta} \left[\frac{Ad}{pw\,(1-t_h)} - c(q, k_s)\right]^{1-\theta} \text{ s.t. } d \leq m_b.$$

As in LW, one can show that in any equilibrium $d = m_b$, and this implies $q \leq q^*(k_s)$ where $q^*(k_s)$ is the solution to $u'(q) = c_q(q, k_s)$. Inserting $d = m_b$ and taking the FOC with respect to q, we get

$$\frac{m_b}{p} = \frac{g(q, k_s)w\,(1 - t_h)}{A}, \tag{14}$$

where

$$g(q, k_s) \equiv \frac{\theta c(q, k_s)u'(q) + (1 - \theta)u(q)c_q(q, k_s)}{\theta u'(q) + (1 - \theta)c_q(q, k_s)}. \tag{15}$$

Writing $q = q(m_b, k_s)$, where $q(\cdot)$ is given by solving (14) for q as a function of (m_b, k_s), we can now compute the relevant derivatives in (9) and (10) as $\partial d/\partial m_b = 1$, $\partial q/\partial m_b =$

$A/pw\,(1-t_h)\,g_q > 0$ and $\partial q/\partial k_s = -g_k/g_q > 0$, where

$$
\begin{aligned}
g_q &= \frac{u'c_q[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)[(u'c_{qq} - c_q u'')}{[\theta u' + (1-\theta)c_q]^2} > 0 \\
g_k &= \frac{\theta u' c_k\,[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)u'c_{qk}}{[\theta u' + (1-\theta)c_q]^2} < 0.
\end{aligned}
$$

Inserting these results and imposing $(m,k) = (M,K)$, (9) and (10) reduce to

$$
\begin{aligned}
V_m(M,K) &= \frac{(1-\sigma)A}{pw\,(1-t_h)} + \frac{\sigma A u'(q)}{pw\,(1-t_h)\,g_q(q,K)} && (16) \\
V_k(M,K) &= \frac{A + A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)} - \sigma\gamma(q,K), && (17)
\end{aligned}
$$

where it is understood that $q = q(M,K)$, and

$$
\gamma(q,K) \equiv c_k + c_q \frac{\partial q}{\partial K} = \frac{c_k(q,K)g_q(q,K) - c_q\,(q,K)\,g_k(q,K)}{g_q(q,K)} < 0. \qquad (18)
$$

Substituting (16) and (17), as well as prices $p = AM/w\,(1-t_h)\,g(q,K)$, $r = F_K(K,H)$, and $w = F_H(K,H)$, into the FOC for m_{+1} and k_{+1}, we get the equilibrium conditions

$$
\begin{aligned}
\frac{g(q,K)}{M} &= \frac{\beta g(q_{+1},K_{+1})}{M_{+1}} \left[1 - \sigma + \sigma \frac{u'(q_{+1})}{g_q(q_{+1},K_{+1})}\right] && (19) \\
U'(X) &= \beta U'(X_{+1}) \left\{1 + [F_K(K_{+1},H_{+1}) - \delta]\,(1-t_k)\right\} && (20) \\
&\quad -\sigma\beta\,(1+t_x)\,\gamma(q_{+1},K_{+1}).
\end{aligned}
$$

Two other conditions come from the FOC for X and the resource constraint,

$$
\begin{aligned}
U'(X) &= \frac{A\,(1+t_x)}{(1-t_h)\,F_H(K,H)} && (21) \\
X + G &= F(K,H) + (1-\delta)K - K_{+1}. && (22)
\end{aligned}
$$

An *equilibrium with bargaining* is defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (19)-(22), given policy and the initial condition K_0. A monetary equilibrium satisfies $q > 0$ at every date. A nonmonetary equilibrium satisfies $q = 0$ at all dates, while (K_{+1}, H, X) solves (20)-(22) with $\gamma = 0$, which is exactly the equilibrium for a standard nonmonetary model with these preferences (e.g. Hansen 1985). Although we are interested in dynamics, as a reference point, when $M_{+1} = (1+\tau)\,M$ for fixed τ we can define a *steady* *state* as a constant solution (q, K, H, X) to (19)-(22). In steady state inflation is τ and the nominal interest rate is given by the Fisher equation $i = (1+\rho)(1+\tau)-1$, where $\rho = 1/\beta - 1$ is the real rate. Then (19)-(20) simplify to

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q, K)} - 1 \tag{23}$$

$$\rho = [F_K(K, H) - \delta](1 - t_k) - \sigma(1 + t_x)\frac{\gamma(q, K)}{U'(X)}. \tag{24}$$

If capital is not used in the DM, then $c(q, K) = c(q)$ and $\gamma(q, K) = 0$. This version dichotomizes, and since M appears in (19) but not (20)-(22), monetary policy affects q but not (K_{+1}, H, X). Equilibrium does not dichotomize when K enters $c(q, K)$; as in the planner's problem, this implies we cannot solve independently for q and the CM variables. Notice however that $\theta = 1$ implies $\gamma(q, K) = 0$ even if K enters $c(q, K)$. In this case, (20)-(22) can be solved for (K_{+1}, H, X), then (19) determines q. So if $\theta = 1$ money still does not influence CM variables, even though anything that affects the CM (e.g. taxes) influences q. Intuitively, when $\theta = 1$ sellers do not get any of the surplus from DM trade, and so investment decisions are based solely on returns to K that accrue in the CM. This is an extreme version of a holdup problem in the demand for capital. More generally, for any $\theta > 0$, sellers do not get the full return on their capital from DM trade, and hence compared to the efficient outcome they underinvest.

This capital holdup problem is not present in standard models, and constitutes a distortion over and above the inefficiencies that arise from $t_h, t_k, t_x > 0$, from $i > 0$, and from the holdup problem in money demand emphasized in Lagos and Wright (2005).[12] If we run the Friedman Rule by setting $i = 0$ and levy only lump-sum taxes, we are left with the two holdup problems. In many models all such problems can be resolved simultaneously if one simply sets the bargaining power parameter θ correctly (Hosios 1990). In the present model

[12] Generally, capital holdup problems have been perhaps neglected in macro, although as Caballero and Hamour (1998) say: "From a macroeconomic perspective, the prevalence of unprotected specific rents makes it a potentially central factor in determining the functioning of the aggregate economy." Caballero (1999) further says "the quintessential problem of investment is that is almost always sunk .. opening a vulnerable flank," and the problem is more serious "when the exposed flanks are largely controlled by economic agents with the will and freedom to behave opportunistically." This is exactly what happens here.

this is impossible: $\theta = 1$ resolves the problem in the demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this is the worst case for money. Under bargaining there is no θ that can eliminate the double holdup problem, and as we shall see, this has implications for both the empirical performance of bargaining models and their policy implications.

2.3 Price Taking

While our two holdup problems cannot simultaneously be solved by bargaining, other solution concepts work better. It is by now well known that *competitive search equilibrium* with price posting resolves, multiple holdup problems, as does *competitive equilibrium* with Walrasian price taking – aat least in our model, if not in all models.[13] Since it is easier to present, relative to posting, we present price taking in the DM. One can interpret this as a monetary version of the labor-market model in Lucas and Prescott (1974), where large numbers of buyers visit 'islands' with large numbers of sellers, and on each 'island' prices are taken as given. This fits well with our assumptions about locations and taste-technology shocks. Thus, to pursue the analogy with maro-labor, the difference between our price-taking and bargaining models is essentially that the former assumes buyers and sellers meet in large groups, as in Lucas and Prescott (1974), while the latter assumes they meet bilaterally, as in Mortensen and Pissarides (1994).

With price taking, the DM value function has the same form as (5), but now

$$V^s(m,k) = \max_q \{-c(q,k) + W(m + \tilde{p}q, k)\} \tag{25}$$

$$V^b(m,k) = \max_q \{u(q) + W(m - \tilde{p}q, k)\} \text{ s.t. } \tilde{p}q \le m \tag{26}$$

where $\tilde{p}$ is the DM price level (which generally differs from the CM price level p). Market clearing implies buyers and sellers choose the same q. As with bargaining, buyers spend all

[13] Shimer (1995), Moen (1997) and Acemoglu and Shimer (1999) discuss competitive search equilibrium in the context of labor markets; Rocheteau and Wright (2005) extend this to monetary economies, and also show that competitive equilibrium works well without search externalities but not as well as competitive search equilibrium with search externalities.

their money, so $q = M/\tilde{p}$. The FOC from (25) is $c_q(q,k) = \tilde{p}W_m = \tilde{p}A/pw\,(1-t_h)$. Inserting $\tilde{p} = M/q$, we get the analog to (14) from the bargaining model

$$\frac{M}{p} = \frac{qc_q(q,k)w\,(1-t_h)}{A}. \tag{27}$$

Then the analogs to (16) and (17) are

$$\begin{aligned} V_m(m,k) &= \frac{(1-\sigma)A}{pw\,(1-t_h)} + \frac{\sigma u'(q)}{\tilde{p}} \\ V_k(m,k) &= \frac{A + A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)} - \sigma c_k(q,k). \end{aligned}$$

Inserting these into (2) yields the analogs to equilibrium conditions (19) and (20)

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] \tag{28}$$

$$\begin{aligned} U'(X) = {} & \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)\right\} \\ & -\sigma\beta\,(1+t_x)\,c_k(q_{+1},K_{+1}). \end{aligned} \tag{29}$$

The other equilibrium conditions (21)-(22) are the same as in the previous model. An *equilibrium with price taking* is given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying these conditions, given policy and K_0. The difference between bargaining and price taking is the difference between (19)-(20) and (28)-(29). Notice the equilibrium condition for q in this model looks like the one from the bargaining model when $\theta = 1$, and the equilibrium condition for K looks like the one from the bargaining model when $\theta = 0$, suggesting that price taking avoids both holdup problems.

We now verify this. First set $t_k = t_h = t_x = 0$. Then under price taking the equilibrium conditions for (K_{+1}, H, X) are the same as those for the planner problem in Section 2.1. Hence the equilibrium coincides with the first best iff $u'(q) = c_q(q,K)$. From (28), this is equivalent to

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}.$$

Using (27) this reduces to $1/pw = \beta/p_{+1}w_{+1}$. Since $w = A/U'(X)$, it further reduces to $p/p_{+1} = U'(x)/\beta U'(X_{+1})$. Since in any equilibrium the slope of the indifference curve

$U'(x)/\beta U'(X_{+1})$ equals the slope of the budget line $1+\rho$, with ρ equal to the real interest rate, the relation in question finally reduces to $p_{+1}/p = 1/(1+\rho)$. Using the Fisher equation, this holds and hence $q = q^*(K)$ solves (28) iff we set the nominal rate to $i = 0$. We therefore conclude that when $i = 0$ and we use only lump-sum taxes, under price taking, we get the first best allocation.

2.4 A Short Digression on Banking

At first blush it might seem the relevant notion of money in the model should be *coins and currency*, or $M0$. This is one interpretation, but we want to suggest that it is not the only possible one. Without going into too much detail, we mention that one can introduce *banks* into the framework following e.g. the approach in Berentsen, Camera and Waller (2006) or the approach in He, Huang and Wright (2005, 2006). Berentsen et al. assume that after trading stops in the CM, so that agents have decided on their $\hat{m}$, it is revealed which ones will want to consume and which ones will be able to produce while banks are still open but before agents go to the DM.[14] As the sellers have no use for money, they deposit it into the banks, who then lend it to buyers. One can imagine them lending out the same physical currency, or as keeping it in the vault and issuing bank-backed securities that can be used in payments, at least as long as these securities are also not easily counterfeitable. This changes some details, and it may be worth studying the differences quantitatively in future work, but basically that model with explicit banks is not so different qualitatively from what we have here.

He et al. study a similar environment, but also assume that cash can be *stolen* while bank-backed securities cannot (think of traveler's checks or debit cards protected by a PIN; theft and counterfeiting are obviously related issues). Assuming fractional reserves, one can derive the money multiplier, albeit in a simplified version of that model, with $M1$ determined endogenously as a function of $M0$ and the legal reserve ratio. If the resource cost of banking goes to zero, in that model, cash may actually stop circulating, to be replaced in all DM

[14] Chiu and Meh (2007) do something similar.

transactions by bank-backed securities. Otherwise, that model is also fairly close to what we have here. Now, we would certainly agree that more work needs to be done to seriously address many interesting issues related to financial intermediation in these kinds of models, and we are there yet; we digress on the topic here only because we do not necessarily want to take M literally as coins and currency in the following exercises. That is, we want to use $M1$ for some of the quantitative work, mainly to facilitate comparison with previous studies, although we will also present results for other measures including $M0$.[15]

3 Quantitative Analysis

3.1 Preliminaries

We first need to do some accounting. The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1 - t_h)\, g\,(q, K)\, F_H(K, H)} \tag{30}$$

in the bargaining version of the model by (14), and

$$p = \frac{AM}{(1 - t_h)\, qc_q\,(q, K)\, F_H(K, H)} \tag{31}$$

in the price-taking version by (27). Nominal output is $pF\,(K, H)$ in the CM, and σM in the DM (since each of the σ buyers spends M dollars). Using p as the deflator, real output in the CM is $Y_C = F\,(K, H)$ and in the DM is $Y_D = \sigma M/p$. Hence total real output is $Y = Y_C + Y_D$.

Define the *share* of output in the DM, $s_D = Y_D/Y$, which we do not calibrate, but compute indirectly from other variables. To see how this works, note that velocity is $v = pY/M = \sigma Y/Y_D$, since $Y_D = \sigma M/p$. Hence, if we know σ and v we know $s_D = \sigma/v$. The maximum σ can be is 1/2, and given $M1$ velocity is around 5, e.g., s_D is bounded above

[15] As Lucas (2000) discusses, there is often a disconnect between monetary theory, where it looks like $M0$ is the relevant variable, and quantitative work, where people use $M1$. The papers mentioned in this digression are in part motivated as a way to address this issue.

by 10%. With our calibrated value of σ, s_D is actually closer to 5%. There are two points we want to emphasize. First, to discuss the relative size of the two sectors, one does not have to take a stand on which goods and services are traded in the DM and CM (although this certainly might be an interesting avenue for future work). Second, the results presented below cannot be said to depend on having an unreasonably large amount of decentralized trade – around 95% of economic activity here looks just like what ones see in standard frictionless models.

We will also discuss the *markup* μ, defined by equating $1+\mu$ to the ratio of price to marginal cost. The markup in the CM market is 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, however, the markup in the DM is derived as follows. Marginal cost in terms of utility is $c_q(q,K)$. With our utility function, a dollar is worth $A/p(1-t_h)w$ utils, so marginal cost in dollars is $c_q(q,K)p(1-t_h)w/A$. Since the DM price is $\tilde{p}=M/q$, the DM markup μ_D is given by

$$1+\mu_D=\frac{M/q}{c_q(q,K)p(1-t_h)w/A}=\frac{g(q,K)}{qc_q(q,K)},$$

after eliminating M using (30). The aggregate markup is then $\mu=s_D\mu_D$.

Finally, we will also discuss certain elasticities, including the interest elasticity of money demand $\xi=\frac{\partial M/p}{\partial i}\frac{i}{M/p}$. As mentioned above, we could do everything in terms of the elasticity with respect to the inflation rate, and the results will be the same to the extent that the Fisher equation holds in the data. Indeed, we do use the inflation rate when we discuss the elasticity of investment, but for money demand we use the interest rate mainly because this is what most others do. In any event, consider ξ under bargaining (price taking is similar). Using (30) and differentiating, we get

$$\xi=\left(g_q\frac{\partial q}{\partial i}+g_k\frac{\partial K}{\partial i}\right)\frac{i}{g}+\left(F_{HH}\frac{\partial H}{\partial i}+F_{HK}\frac{\partial K}{\partial i}\right)\frac{i}{F_H}. \tag{32}$$

It is now a matter of substituting $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$ (see Appendix B.2) to yield ξ as a function of the allocation and parameters.

3.2 Calibration

Consider the following functional forms for preferences and technology:

$$\text{CM:} \qquad U(x) = B\frac{x^{1-\varepsilon}-1}{1-\varepsilon} \text{ and } F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$\text{DM:} \qquad u(q) = C\frac{(q+b)^{1-\eta}-b^{1-\eta}}{1-\eta} \text{ and } c(q,k) = q^{\psi}k^{1-\psi}$$

The cost function $c(\cdot)$ comes from $\ell(e) = e$ and $q = e^{\chi}k^{1-\chi}$, with $\psi = 1/\chi$; if $\psi = \chi = 1$ then k does not appear in $c(\cdot)$ and the model dichotomizes. The parameter b is introduced so that $u(0) = 0$, which is helpful for technical reasons (it keeps the threat point in bargaining well defined for all η). This means relative risk aversion is not constant, but if $b \approx 0$ then relative risk aversion is approximately constant at $\eta q/(q+b) \approx \eta$. We set $b = 0.0001$ and $\varepsilon = \eta = 1$ as a benchmark to facilitate comparison with the literature, but we show that the results are fairly robust to these choices in Section 4.3. We normalize $C = 1$ with no loss in generality.[16]

To describe our calibration strategy for the remaining parameters, we start with a heuristic description, then provide details. The first point we want to establish is that our approach is a natural extension of standard methods. To pick one typical application, Christiano and Eichenbaum (1992) e.g. study a one-sector growth model parameterized by

$$U = \log(x) + A(1-h) \text{ and } Y = K^{\alpha}h^{1-\alpha}$$

(for their indivisible-labor version; for the divisible-labor version replace $1-h$ by it's logarithm). We call this *the standard model.* There are four parameters that they calibrate, one at a time, as follows. First, set the discount factor $\beta = 1/(1+\rho)$ where ρ is some

[16] We are not over-normalizing here. Consider the following argument for the case of $b = 0$ (i.e. for log utility). Before normalizing, the utility function is $\mathcal{U} = B\log(x) - Ah + C\log(q) - De$. Write the DM production function as $q = Ze^{\chi}k^{1-\chi}$. Then making a change of variable $\tilde{q} = qD/Z^{1/\chi}$ we have $\mathcal{U} = B\log(x) - Ah + C\log(\tilde{q}Z^{1/\chi}/D) - \tilde{q}k^{(\chi-1)/\chi}$ which is basically what we use. Obviously we can normalize $C = 1$. With log utility, the choice of constants Z and D has no effect on individual decisions (it affects only the units of DM output, but this will be neutralized by the relative price), so we can normalize these how we wish. Now this argument is literally true only for log utility, and we have b small but positive. However, for the values of b we used, in practice, we varied D from 1 to 100 for example without affecting the numerical results very much at all.

observed average interest rate. Then set the depreciation rate $\delta = I/K$ to match the observed investment-capital ratio. Then set α to match either labor's share of income LS or the capital-output ratio K/Y, since these yield the same result, given there are no distorting taxes in the model (see below). Finally, set A to match observed average hours worked h.

This standard method can be adapted to many scenarios. For example, Greenwood et al. (1995) calibrate a two-sector model with home production as follows. Consider

$$U = \log(x) + A(1 - h_m - h_n),\ Y_m = K_m^{\alpha_m} h_m^{1-\alpha_m} \text{ and } Y_n = K_n^{\alpha_n} h_n^{1-\alpha_n},$$

where $x = [Dx_m^{\varepsilon} + (1 - D)x_n^{\varepsilon}]^{1/\varepsilon}$, and x_m, h_m and k_m are consumption, hours and capital in the market while x_n, h_n and k_n are consumption, hours and capital in the nonmarket (home) sector. After setting $\beta = 1/(1 + \rho)$, as above, the two-sector version of the standard method is this: set δ_m and δ_n to match I_m/K_m and I_n/K_n; set α_m and α_n to match K_m/Y_m and K_n/Y_m; and set A and D to match h_m and h_n. We are left with ε, which is hard to pin down based on steady state observations, and is therefore typically set based on estimates of relevant elasticities.

Since we also have a two-sector model, we use a variant of the home-production calibration method, although it differs slightly because we do not want to (we do not have to) take a stand on what the CM and DM correspond to in the data. Thus, first set β, δ and A as above. Then set α and ψ to match *both* K/Y and LS. As we said, in the standard one-sector model, without taxes, it does not matter if one calibrates α to LS or K/Y, but with proportional taxes on capital income, calibrating α to LS yields a value for K/Y that is too low (Greenwood et al. 1995; Gomme and Rupert 2005). The idea here is to set α to match LS, then try to use ψ to match K/Y, since DM production provides an extra kick to the return on K. Given this, we set the utility parameter B and probability σ to match some money demand observations discussed below, which is the analog of picking ε in home production framework, and must be done in any calibrated monetary model.

This completes the heuristic description. We now go into detail. Our benchmark model is annual, to facilitate comparison with the literature, but we also discuss quarterly and monthly results below. Then we pin down $\beta = 1/(1+\rho)$ by $\rho = 0.025$, which is the annual, after-tax, real interest rate in the 1951-2004 U.S. data, based on an average pre-tax nominal rate on Aaa-rated corporate bonds of 7.2%, an inflation rate from the GDP deflator of 3.6%, and a tax on bond returns of 30% from the NBER TAXSIM model.[17] We more or less directly observe other taxes as well; we use $t_h = 0.242$ and $t_k = 0.548$, the average effective marginal rates in McGrattan et al. (1997); Gomme, Ravikumar and Rupert (2005) and Gomme and Rupert (2005) have similar numbers. We compute $t_x = 0.069$ directly as the average of excise plus sales tax revenue divided by consumption. We also observe $G/Y = 0.25$. We set $\delta = I/K = 0.070$, using residential and nonresidential structures plus producer equipment and software for K. We set $\alpha = 0.288$ to get $LS = 0.712$, which we get using the method in Prescott (1986).

This pins down all but five parameters, A and B from preferences, the cost parameter ψ, the probability σ, and in the bargaining model θ; see Table 1 (all tables are at the end of the paper). These parameters are determined simultaneously to match the following targets. First, hours worked as a fraction of discretionary time, $H = 1/3$ (Juster and Stafford 1991). Second, average velocity, $v = 5.29$ which we measure directly using $M1$. Third, $K/Y = 2.32$ when we measure K as defined above. Fourth, a money demand elasticity of $\xi = -0.226$, estimated as discussed in Appendix B.3. Fifth, in the bargaining model, the markup $\mu = 0.10$ (Basu and Fernald 1997). We choose the parameters simultaneously to minimize the distance between the targets in the data and model. Since we can have more targets than parameters when we minimize deviations, sometimes we add the long-run elasticity of investment with respect to inflation as a target, which we estimate as $\zeta = -0.023$ (on quarterly data) using same method used for money demand.[18]

[17] At the risk of being redundant, given what was said above about the Fisher equation, we mention the following. Although we did not explicitly incorporate them in the discussion of equilibrium, obviously we can price bonds that trade between meetings of the CM in the standard way, and use observations on interest rates to pin down β. But, as suggested above regarding assets other than money, in general, we assume these bonds do not circulate in the DM. Again, one could guarantee this with the right assumptions about counterfeiting and recognizability.

[18] Adding ζ as a target is something like adding the empirical labor supply elasticity in calibrating a standard business cycle model, which one may or may not like; in any case we report results with and

3.3 Decision Rules

We first scale all nominal variables by M, so that $\hat{m} = m/M$, $\hat{p} = p/M$ etc. Then the individual state becomes $(\hat{m}, k, K)$, where in equilibrium $\hat{m} = 1$ and $k = K$. Although parts of the above presentation were more general, we are now interested in recursive equilibrium, given by time-invariant decision rules and value functions $[q(K), K_{+1}(K), H(K), X(K)]$ and $[W(K), V(K)]$ solving the relevant equations – e.g. (19)-(22), (1) and (8) in the bargaining model. We solve these equations numerically using a nonlinear global approximation, which is important for accurate welfare computations if we move far from steady state.[19] Figure 1 plots the decision rules and value function for two preferred parameterizations (Models 3 and 5 as described in the next section) for four scenarios: the planner's problem; monetary equilibrium at the Friedman rule; monetary equilibrium at 10% inflation; and nonmonetary equilibrium. We discuss the economic content of these pictures below.

4 Results

4.1 Model 'Fit'

The basic calibration results are in Table 2. One column lists the relevant moments in the data, while the others list the moments from five specifications of the model. Model 1 uses bargaining in the DM with bargaining power $\theta = 1$, giving up on the markup μ as a target; it is presented mainly as a benchmark since we already proved that $\theta = 1$ implies money cannot affect the CM variables at all. Models 2 and 3 use bargaining with θ calibrated along with the other parameters; the difference between Models 2 and 3 is that the latter adds the investment elasticity ζ as a target while the former does not. Models 4 and 5 use price taking in the DM, so there is no θ, calibrating the rest of parameters to match the targets other than the markup; the difference between Models 4 and 5 is again that the latter adds

without targeting ζ. Although the estimated value of -0.023 may appear small, it is statistically significant and economically relevant: raising inflation from our benchmark value to 7% e.g. is predicted based on the regression to reduce investment by around 2.3%, which is nothing to scoff at.

[19] Specifically, we use the Weighted Residual Method with Chebyshev Polynomials and Orthogonal Collocation. See Judd (1992) for details, and Aruoba et al. (2006) for a recent comparison of solution methods.

ζ as a target while the former does not.

We do well matching the targets with two exceptions. First, we match the markup μ only if we assume bargaining and calibrate θ, rather than fixing it at 1 or assuming price taking, for obvious reasons. Second, we do a good job matching K/Y and ζ only in the price-taking model, for reasons that we now explain. Intuitively, our calibration sets the CM technology parameter α to match LS and then tries to hit K/Y using the DM technology parameter ψ (although we think this way of looking at things is instructive, it is meant only to be suggestive, since in fact we pick all of the parameters simultaneously). When $\psi = 1$, K is not used in the DM, and K/Y is too low, as in the standard model once taxes on capital income are introduced. As we increase ψ above 1 the return on K from its use in the DM increases and hence so does K/Y. But, in practice, with bargaining, this effect is tiny because the holdup problem eats up most of the DM return on K.

Of course, the size of this effect depends on bargaining power, but even if we pick θ to maximize K/Y we cannot get it big enough. This is due to the *double* holdup problem: if θ is big then buyers have all the bargaining power, which makes q big, other things being equal, but gives little return from DM trade to sellers; and if θ is small then sellers have all the bargaining power, which gives them a big share of the return, but only on a very small q. There is no way around it with bargaining. With price taking, however, the holdup problems vanish, and we can pick ψ to match K/Y exactly. The same intuition about how holdup problems affect the level of K/Y also explains how they affect the elasticity ζ: with bargaining, any extra return on K from DM production will not increase aggregate K much, since the DM return is a small fraction of the overall return to investment. Again, this is not a problem with price taking, and we can hit ζ perfectly.

Earlier we alluded to the fact that the DM share s_D is around 5%, as shown in Table 2. We think this is reasonable, in the sense that we would be uncomfortable if s_D were too big, since then we would be very far from the standard growth model. Because s_D is small, however, we need a markup in the DM of around 200% to match the economy-wide μ. This may seem high, but there several points to be made. First, although 10% is a reasonable aggregate target, actual markups vary a lot in micro data.[20] Second, if we relax our extreme assumption of a perfectly competitive CM we could get away with a much smaller DM markup. Third, it turns out that we can actually recalibrate μ_D to 1.0, 0.4 or even 0.1 rather than 2.0, and the key results will not change very much – which may be surprising, but it will be explained in Section 4.3. Given this, we keep the aggregate target of $\mu = 0.1$, with an implied μ_D of around 2.0, in the baseline model.

Finally, one might ask how we match the empirical money demand curve, which is often taken to be a measure of 'fit' in monetary calibration exercises (see e.g. Lucas 2000, Lagos and Wright 2005, or Craig and Rocheteau 2006). Comparing plots of i versus M/pY from the data and from our model, we see something similar to what sees with other models in the literature. In particular, as with other approaches, it is not easy to match both the observations with very low i and high M/pY from the first decade and those with low M/pY from the last decade in the sample – which is no surprise given changes over time in transactions technologies. Although the 'fit' is about as good on this dimention as in other models, we actually do not put much weight on looking at plots of i versus M/pY, since this specification for money demand assumes a unit income elasticity which is rejected by the data in the regression results reported in Appendix B.3.

4.2 Experiments

Here we consider experiments where, starting in a steady state, we make a once-and-for-all change in the growth rate of money τ, and track the economy over time. Since inflation in steady state equals τ, we abuse language slightly and describe our experiments as a change in inflation, but note that inflation actually does not jump to the new steady state level in the short run (i.e. it may not equal τ during the transition). Table 3 contains results for

[20] As Faig and Jerez (2005) report, data at http://www.census.gov/svsd/www/artstbl.html indicate the following: at the low end, warehouse clubs, superstores, car dealers and gas stations have μ between 17% and 20%; and at the high end, specialty food, clothing, footwear and furniture stores are over 40%. None of these are as high as 200%, although the sample does not include convience stores, magazine stands, and so on, where markups may be considerably higher.

each of the five models when we perform a common experiment and change $\tau^1 = 0.1$ to the Friedman rule, which is $\tau^2 = -0.0239$ in the baseline calibration. For now, we make up any change in government revenue with the lump-sum tax T, and consider other fiscal options below. Table 3 presents ratios of equilibrium values of several variables at the two inflation rates.[21]

The first thing to note is that q^1/q^2 is considerably less than 1, varying between 0.67 and 0.87, depending on the model. Again, the idea is that inflation is a tax on DM activity, and these results show that this tax is quantitatively very important. In Model 1 this is the only effect, since $\theta = 1$ implies monetary policy has no impact on the CM. In Models 2 and 3, in theory monetary policy does affect the CM, but the impact is tiny as one should expect from the discussion in Section 4.1. Based on this, Models 1-3 all predict that going to the Friedman rule increases aggregate output Y by 2%, almost all due to the change in q. In Models 4 and 5 the effects are different. First, q actually changes by more; and second, now K changes, and by quite a lot – either 3% or 5%, according to Model 4 or 5. This makes CM consumption X change by about 1%, and the net impact on Y is now 3%.

Before discussing the intuition for these results, consider welfare. As is standard, we solve for Δ such that agents are indifferent between reducing τ and increasing total consumption (X and q) by a factor Δ. We report the answer comparing across steady states – jumping instantly from τ^1 and K^1 to τ^2 and K^2 – as well as the cost of the transition from K^1 to K^2 and the net gain to changing τ starting at K^1. This net gain is the true benefit of the policy change, although the steady state comparison is still interesting, as it tells us how much an agent facing τ^1 and K^1 would pay to trade places with someone facing τ^2 and K^2. In Model 1 there is no transition since τ does not affect K, and in Models 2 and 3 we expect it to be unimportant, since τ does not affect K very much, but in Models 4 and 5 the transition could be significant. We also report the net gain to reducing τ to 0, instead of all the way to Friedman rule, to check how much of the gain comes from eliminating inflation and how

[21] When a *1* appears in italics, the true number is not exactly unity but shows up this way due to rounding, to distinguish effects that are theoretically 0 from those that not exactly 0 but numerically very small.

much comes from deflation (most comes from the former).

In Model 1, with $\theta = 1$, going from 10% inflation to the Friedman rule is worth around 3/4 of 1% of consumption, commensurate with many previous findings (recall fn. 3). In Models 2 and 3, with $\theta \approx 3/4$, this policy is worth over 3% of consumption. Intuitively, at $\theta \approx 3/4$ the money holdup problem makes q very low, and so any additional reduction is very costly. In Models 4 and 5 the steady state gain is about half that in Models 2 and 3, since the economy is closer to the first best with price taking than it is with bargaining. In Models 4 and 5 inflation has a sizable impact on K and X, but since much of the gain accrues only in the long run and agents must work more and/or consume less during the transition, the net gain is closer to 1%. Figure 2 shows the transitions for Models 3 and 5. In Model 5, e.g., in the short run H increases by around 1% and X falls slightly before settling down to their new steady state levels, q jumps on impact by around 50% and quickly settles at the new steady state.

Table 4 compares the Friedman rule and first best allocations. The differences are big, mainly due to taxation (McGrattan et al. 1997 find similar results in standard nonmonetary models with taxes). We also report the gain to moving from the Friedman rule to the first best after setting $t_h = t_k = t_x = 0$ and recalibrating other parameters. In Models 4 and 5, the gain in this case is 0 because as we showed the Friedman rule implements the first best. In Model 1, with a capital holdup but no money holdup problem, the steady state gain is around 4%, although much is lost in transition. In Models 2 and 3, with both holdup problems, it is around 5% and 15%. These calculations provide measures of the impact of holdup problems: based on the steady state comparisons, e.g., one could say 4% of consumption is the cost of capital holdup and an additional $1\% - 11\%$ is the cost of money holdup. Although there is no single 'correct' way to decompose these effects, this suggests holdup problems may be quantitatively important, even though we have bargaining only in the DM and s_D is only around 5%.[22]

[22] The calibrated parameters differ across the columns in Table 4. Suppose we instead fix the parameters as in Model 3, and consider three cases: (i) $\theta = 1$; (ii) θ calibrated; and (iii) price taking. With taxes, going

Table 5 reports the actual allocations, not just the ratios of the allocations, at different τ, to facilitate comparisons across models. Notice e.g. that q is considerably lower in Models 2 and 3 than in other models, showing the impact of the money holdup problem. Also, comparing Models 4 and 5, notice how the latter has a considerably bigger K/Y ratio for moderate inflation rates, although K/Y is basically the same at the first best; in other words, K is much more sensitive to inflation in Model 5. The table also reports the allocation in the nonmonetary equilibrium, which can be considered the limit as inflation goes to ∞. Although we can of course compute the cost of very high inflation – e.g., going from 100% inflation to the Friedman rule is worth around 14% in Model 3 and 8% in Model 5 – one should take these calculations cautiously, since agents may well devise other ways to trade in the DM at very high inflation (like using foreign currency) and since our numerical results are sensitive to parameter choices at very high inflation.

At the risk of redundancy, we also discuss results using the decision rules. In Figure 1, for Model 5 we see that as we lower τ the decision rule for K_{+1} shifts up, and steady state K increases, although it is still far from the first best even at the Friedman rule (the symbols on each curve show the location of the steady state, but the first best steady state $K = 2.23$ is off the chart in this case). Also, the decision rule for q shifts up, increasing q in the short run and more in the longer run as we move along the decision rule for q with the growth in K. The latter effect is important here, since K grows a lot. In Model 3 the decision rule for K_{+1} and hence steady state K change little. The decision rule for q shifts, giving a short-run effect, but there is little additional long-run effect. Still, inflation is very costly in Model 3 because the decision rule for q at the Friedman rule is quite far from the decision rule at the first best, so any change in q matters a lot, while in Model 5 the decision rules for q at the

from the FR to FB in these three scenarios is worth, in terms of steady state (net) comparisons: (i) 28.56 (15.85); (ii) 39.89 (26.12); and (iii) 10.30 (5.47). With taxes set to 0 we get: (i) 5.55 (1.42); (ii) 15.29 (10.81); and (iii) 0 (0). Looking at the results without taxes, one could say the cost of capital holdup in terms of steady state is 5.55, or 1.42 with transition, and the cost of money holdup is 9.74, or 9.39 with transition. With taxes the cost of capital and money holdup including transitions are almost identical, 10.38 and 10.27. Again, there is no single 'correct' way to measure these effects, but all of this indicates that holdup problems can be important.

Friedman rule and first best are virtually coincident.

One can also consider lowering τ and making up the revenue shortfall with proportional rather than lump sum taxes. Cooley and Hansen (1991) e.g. find that if proportional taxes must be used then eliminating inflation is *not* beneficial. Table 6 reports our results for the case where we make up the revenue with lump-sum taxes, reproducing Table 3, and with labor or consumption taxes.[23] Going to the Friedman rule and making up revenue with labor taxes requires raising t_h from 25% to around 30%. This reduces Y around 3% in Models 1-3, 2% in Model 4 and 1% in Model 5. The net welfare gain is positive with bargaining and θ calibrated, but negative with price-taking. A similar discussion applies to t_x. In general, these results are quite sensitive, but we think it is interesting that when existing tax rates are given, going to the Friedman rule may not be the best policy. More work needs to be done on this issue, as we discuss further in the Conclusion.

4.3 Robustness

We redid all the calculations for many alternative specifications, but in the interest of space, in Table 7 we report the results in terms of one statistic: the net welfare gain of going from 10% inflation to the Friedman rule. The first row is the benchmark model. The first robustness check involves shutting down the distorting taxes, both for the case where other parameters are kept at benchmark values, and when they are recalibrated. Most of the results are similar to the benchmark calibration, except for Model 5 and, to a lesser extent, Model 4, where the cost of inflation is somewhat lower without distorting taxes. This is because the Friedman rule achieves the first best in price-taking models without distortionary taxes, and hence the cost of moderate inflation is low by the envelope theorem. We do not think this is a problem: it is no surprise that the results can depend on what one assumes about taxation, and since taxes are a fact of life, we trust the benchmark calibration.

[23] We could not solve the case where we try to make up the shortfall with capital taxation, since increasing t_k lowered K by so much that sufficient revenue was not forthcoming (as in Cooley and Hansen 1991). Also, for these experiments, we allow the government to issue a bond paying interest equal to the discount rate so that we do not have to adjust taxes each period during the transition.

We report the effects of varying utility parameters b, ε and η. One can look at the numbers for oneself, but we conclude the results are not overly sensitive. One can also vary β, δ etc. over reasonable ranges without affecting things too much (not reported). We also show that changing our target for the markup does not matter much: e.g. lowering μ_D to 40%, which makes the aggregate μ closer to 2% instead of 10%, only reduces the welfare cost from 3.08% to 2.77% in Model 2 and from 3.43% to 2.96% in Model 3. This may be surprising, but it can be understood as follows. First, note that when $\theta = 1$ the markup is actually negative in Table 2, because take-it-or-leave-it offers by buyers means $p = AC$ and $AC < MC$. Thus, just to get $\mu > 0$ we need θ significantly below 1: e.g. $\mu_D = 0.01$ requires $\theta \approx 0.927$, which implies the money holdup problem is already important enough to generate a sizable welfare cost of around 2.76%. Since it is not so clear exactly what the right target for the markup should be, or what is the best way to get this out of a model, it is good that the results do hinge on μ_D.

The table also shows that the results are not too sensitive to using different time periods for the calibration, although more work could be done to investigate this in more detail. What is clear is that the results are not at all sensitive to assuming a different length for a period – a quarterly or monthly instead of an annual model delivers very similar predictions. This is easy to understand: to go from an annual to a quarterly or monthly model, we simply adjust inflation, velocity, interest rates, K/Y and I/K by the relevant factor. The calibrated σ declines, because a shorter period reduces the probability of consuming in any given DM, but the welfare conclusions do not change. We find this important because changing frequency typically *does* change the results, including welfare implications, in many models, including the typical cash-in-advance model where agents generally spend all their money every period. This is not the case when $\sigma \in (0, 1)$, which means that agents in the DM do not all spend all their money.[24]

[24] One might say that this model has a 'precautionary demand' for money. We do not contend that one *couldn't* somehow introduce randomness and a 'precautionary demand' into cash-in-advance models; we are rather suggesting that one *should*.

What does matter is the empirical measure of money, $M0$, $M1$, $M2$ or $M3$. One reason is these alternative measures imply different values for average velocity, and given our calibration method, this changes the size of the DM and thus the cost of inflation (although this is not the whole story, since the different measures also lead to different estimates of the elasticity ξ, which explains e.g. how $M3$ can yield a lower welfare cost than $M2$). One intuition comes from the traditional method of computing the cost of inflation as the area under the money demand curve: using a narrower definition of M shifts the curve down and reduces the estimated cost. This is not meant as an endorsement of the traditional method, however, since our different models all generate very similar money demand curves, yet imply quite different welfare numbers. In any case, while it is perhaps unfortunate that results depend on the definition of M, at least we understand why it is so, and it is bound to be true for *any* theory of money.

One can go beyond these issues and consider robustness with respect to larger modeling choices. We mentioned earlier a version of the model with two capital stocks, K_C and K_D (see Appendix A.1). Tables 8 and 9 report results for this model with bargaining and with price taking, called Models 6 and 7. These two-capital analogs of Models 3 and 5 do about as well as in matching the targets. In Models 6 and 7, q actually increases by more than in the baseline models when we reduce τ, which tends to make inflation more costly. However, there are also other effects, and hence the net cost of inflation is actually lower in Model 7 than 5. These other effects occur because in Model 5, e.g., the same K is used to produce q, X and K, but in Model 7 q is produced with K_D while X and K_D are produced with K_C. Despite this detail, the overall picture from a two-capital-stock analysis is similar to the base case.

Tables 8 and 9 also report results from another extension (see Appendix A.2), where K is used as an input in the CM only but is produced and traded in the DM, as in Shi (1999), which means one needs cash to buy new capital, as in Stockman (1982). The bargaining and price-taking versions are called Models 8 and 9. Now inflation taxes capital accumulation directly, and not only indirectly via q. We can see that this yields a sizable effect of τ on K under bargaining, as well as under price taking. Overall, the results are not so different from the base case, however, even if the welfare cost estimates are affected somewhat. It may be worth studying these alternative models in more detail in the future, although to do so one might want to rethink the strategy for calibration. We presented them here mainly to show that the basic ideas carry over to alternative formulations, and that many of the numerical results do not hinge too critically on some of the details.

4.4 Summary of Results

Here is what we think we learn from all of this:

- One can integrate elements of models with explicit trading frictions and standard growth theory in a way that generates interesting effects of money – i.e., there is no dichotomy.

- One can use textbook methods to calibrate the model, even though it contains some novel parameters.

- To do so one does not need to take a stand on which goods are traded in the DM and CM, even if one wants to estimate the relative size of the two sectors; our numbers imply the DM accounts for around 5% of total output.

- We do a good job matching most of the targets, although (obviously) with price taking we cannot match the markup, and (less obviously) with bargaining we cannot match K/Y, or the elasticity of K with respect to i, due to holdup problems.

- Inflation is a tax on DM consumption, and the effects are big.

- Qualitatively, given K is useful for producing q, inflation reduces investment; quantitatively, this effect is tiny under bargaining, and big (3 to 5%) under price taking.

- Under price taking, reducing inflation from 10% to the Friedman rule is worth 1.5% across steady states, and 1% taking into account the transition; it is worth over 3% under bargaining.

- With either bargaining or price taking much of the gain is achieved by reducing inflation to 0 rather than going all the way to the Friedman rule.

- The costs of fiscal distortions are quite big, and hence it may or may not be desirable to replace inflation with proportional taxes, depending somewhat delicately on parameter values and other details.

- The welfare cost of holdup problems can be big even if the DM is fairly small.

- Most of these results are fairly robust to modeling choices and parameter values, although the empirical measure of M does make a difference.

- Many of these results differ from findings in the literature.

- Key elements in the analysis include: allowing for bargaining instead of (i.e. in addition to) price taking; having a demand for money based on preference shocks; and using an explicit two-sector structure.

5 Conclusion

This paper has developed an explicit multi-sector model with trading frictions in some markets, as in some recent monetary theory, but extended to include capital, as in the neoclassical growth model. There is feedback across sectors because capital produced in the centralized market is used as an input in the decentralized market. This means that monetary policy, which we model in terms of a choice of the (fully-anticipated) inflation rate, and which has the direct effect of a tax on decentralized market activity, can in principle have interesting effects on investment and other centralized market variables. We think that these results constitute some progress on the challenge of integrating of models with trading frictions and mainstream macro, that they extend each of those paradigms individually, and that they potentially open up doors to much additional work. The model is not difficult to calibrate, and we have already summarized the main quantitative findings. Here we discuss some possible directions for future research.

It might be interesting to take more seriously the details of commercial or central banking in this type of model. We do not necessarily expect this to affect the overall message too much – as long as monetary policy helps determine inflation, and inflation is a tax on either currency or currency-backed objects that serve as a medium of exchange, the basic economic intuition would seem to apply – although this needs to be checked. But the quantitative results could change once the amount of resourced used in financial intermediation becomes endogenous, as in some of the papers mentioned in footnotes in the Introduction. This may also help to sort out which measures of money is appropriate, by allowing different measures ($M0$, $M1$, etc.) to be determined endogenously in the model. Again, some papers mentioned already make some progress on this, but there is more to be done. More generally, some financial institutions develop in part in response to trading frictions like those in the model – money being only one example – and so this may provide a natural framework to think about these other institutions.

It may be interesting to consider more general preferences, which allows one to break the dichotomy even without feedback via capital across sectors. Rochetaeu, Rupert and Wright (2007) e.g. consider a utility function is still (effectively) quasi-linear but allows non-separabilities between CM and DM consumption. Thus, anything that affects q, including money, affects x depending on whether these goods are complements or substitutes. That model has no capital and has not been quantified – and in fact it is not straightforward how one would calibrate it – but in principle in could be done. The approach in this project was to explore feedback through the technology side, again by having capital produced in one sector and used in the other, and for this we came up with a simple calibration strategy. But it might be interesting to build models with feedback across sectors coming from both preferences and technology. Also, as suggested earlier, it would surely be interesting to study optimal policy, to see e.g. what the best monetary policy is when we do not restrict attention to empirically given proportional tax rates. This is somewhat of an involved exercise, however, and beyond the scope of the current project; see Aruoba and Chugh (2006).

One could also pursue alternative approaches to picking parameters; here we concentrated on a simple calibration procedure. We then used the theory to measure the predicted effects of monetary and fiscal policy; one could go beyond this and compare these predictions with time-series or cross-country data, but again that is beyond the scope of the paper. One could also add shocks to study the quantitative business-cycle properties of the model and compare those with the data; here we focused only on the long-run implications for money and capital. Finally, a big outstanding issue not only for this type of model, but for all of monetary economics, is to delve deeper into the coexistence of money and other assets. We discussed some ideas in this regard, but a rigorous analysis of that difficult topic was not the object of the exercise. We hope people agree that it is worth exploring issues like the effect of money on capital in models that include ingredients from theoretical monetary economics, even if one does not have a definitive formal explanation for coexistence. We believe the approach in this paper can provide interesting quantitative insights, while formal monetary theorists continue to pursue the quest for deeper and better theories.

A Appendix: Alternative Specifications

We sketch two alternative models mentioned in the text. First, suppose there are two distinct capital goods: k_C is used in the CM, and k_D in the DM. Both are produced in the CM, and neither can be used for payment in the DM. They depreciate at rates δ_C and δ_D. For simplicity, there is no tax on k_D, and we present only the bargaining version (price taking and the planner's problem are similar). The CM problem is

$$\begin{aligned} W(m, k_C, k_D) &= \max_{x,h,m_{+1},k_{C+1},k_{D+1}} \{U(x) - Ah + \beta V(m_{+1}, k_{C+1}, k_{D+1})\} \\ \text{s.t. } (1+t_x)\, x &= w\,(1-t_h)\, h + [1 + (r-\delta_C)(1-t_k)]\, k_C - k_{C+1} - T + \frac{m - m_{+1}}{p} \\ &\quad + (1-\delta_D)\, k_D - k_{D+1}. \end{aligned}$$

Eliminating h using the budget constraint, FOC are

$$\begin{aligned} x &: U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \frac{A\,(1+t_x)}{pw\,(1-t_h)} = \beta V_m(m_{+1}, k_{C+1}, k_{D+1}) \\ k_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_k(m_{+1}, k_{C+1}, k_{D+1}) \\ z_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_z(m_{+1}, k_{C+1}, k_{D+1}). \end{aligned}$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way. The usual logic implies the distribution of (m, k_C, k_D) is degenerate for agents leaving the CM.

The DM is as before, except we replace k with k_D. The value function and envelope conditions are derived in the obvious way. This leads to

$$\frac{g(q, K_D)}{M} = \frac{\beta g(q_{+1}, K_{D+1})}{M_{+1}} \left[1 - \sigma + \sigma \frac{u'(q_{+1})}{g_q(q_{+1}, K_{D+1})}\right] \tag{33}$$

$$U'(X) = \beta U'(X_{+1}) \{1 + [F_K(K_{C+1}, H_{+1}) - \delta_C](1-t_k)\} \tag{34}$$

$$U'(X) = \beta U'(X_{+1}) \left[1 - \delta_D - \frac{(1+t_x)\,\sigma\gamma(q_{+1}, K_{D+1})}{U'(x_{+1})}\right] \tag{35}$$

$$U'(X) = \frac{A\,(1+t_x)}{F_H(K_C, H)\,(1-t_h)} \tag{36}$$

$$X + G = F(K_C, H) + (1-\delta_C)\, K_C - K_{C+1} + (1-\delta_D)\, K_D - K_{D+1}. \tag{37}$$

An *equilibrium* is now given by paths for $(q, K_{C+1}, K_{D+1}, H, X)$ satisfying (33)-(37). This model does not dichotomize, in general, since K_D is used in the DM and produced in the CM.

For the next alternative model, we revert to one capital good, but suppose new k is acquired in the DM. As in Shi (1999), agents do not consume DM output q, but use it as an intermediate input that is transformed one-for-one into k, which is an input to CM production. Each period a fraction σ of agents in the DM can produce the intermediate input, and a fraction σ can transform it into capital. Although agents cannot acquire new capital in the CM, they are allowed to trade used capital. We again present only the bargaining version.

Let k be the amount of capital held by an agent entering the CM and k'_{+1} the amount of capital taken out, and hence into the next DM. We show how to construct equilibrium where the distribution of (m, k') coming out of the CM is degenerate, even though the distribution going in is not. To begin, the CM problem is

$$
\begin{aligned}
W(m,k) &= \max_{x,h,m_{+1},k'_{+1}} U(x) - Ah + \beta V_{+1}(m_{+1}, k'_{+1}) \\
\text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [r-(r-\delta)\,t_k]k + (1-\delta)\,\phi k - \phi k'_{+1} - T + \frac{m-m_{+1}}{p}
\end{aligned}
$$

where ϕ is the goods price of used capital in terms of x. The FOC are:

$$
\begin{aligned}
x &: \; U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\
m_{+1} &: \; \frac{A}{pw\,(1-t_h)} = \beta V_{+1,m}(m_{+1}, k'_{+1}) \\
k'_{+1} &: \; \frac{A\phi}{w\,(1-t_h)} = \beta V_{+1,k}(m_{+1}, k'_{+1})
\end{aligned}
\tag{38}
$$

The envelope conditions are obtained in the obvious way.

Buyers in the DM spends all their money, and bring $k = k' + q$ to the CM. The bargaining solution now implies that q solves $m_b/p = g(q, r, w, \phi)$ where

$$
g(q,r,w,\phi) \equiv \frac{(1-t_h)\,w\,[\theta c\,(q) + (1-\theta)\,c'\,(q)\,q]\,[r-(r-\delta)\,t_k + (1-\delta)\,\phi]}{\theta A[r-(r-\delta)\,t_k + (1-\delta)\,\phi] + (1-\theta)\,(1-t_h)\,wc'\,(q)}.
$$

Then we have

$$
V(m,k') = W(m,k') + \sigma\left\{\tfrac{A[r-(r-\delta)t_k+(1-\delta)\phi]q(m)}{w(1-t_h)} - \tfrac{Am}{pw(1-t_h)}\right\} + \sigma E\left\{\tfrac{A\tilde{m}}{pw(1-t_h)} - c\,[q\,(\tilde{m})]\right\},
$$

where the expectation is with respect to the money holdings $\tilde{m}$ of agents and we assume you visit one at random (we will establish, but have not yet established, that $\tilde{m} = M$ is degenerate). Then

$$
\begin{aligned}
V_m(m,k') &= \frac{(1-\sigma)A}{pw(1-t_h)} + \frac{\sigma[r-(r-\delta)t_k+(1-\delta)\phi]}{pw(1-t_h)g_q(q,r,w,\phi)} \\
V_k(m,k') &= \frac{A[r-(r-\delta)t_k+(1-\delta)\phi]}{(1-t_h)w}.
\end{aligned}
$$

Since V_m is independent of k', the FOC for m_{+1} in (38) implies m_{+1} is independent of k'_{+1} and hence degenerate.

The analog to (19) is

$$
\frac{\hat{g}(q,K,H,\phi)}{F_H(K,H)M} = \frac{\beta\hat{g}\left(q_{+1},K_{+1},H_{+1},\phi_{+1}\right)}{F_H(K_{+1},H_{+1})M_{+1}}\left[1-\sigma+\sigma\Xi(q_{+1},K_{+1},H_{+1},\phi_{+1})\right] \tag{39}
$$

where

$$
\begin{aligned}
\hat{g}(q,K,H,\phi) &\equiv g[q,F_K(K,H),F_H(K,H),\phi] \\
\Xi(q,K,H,\phi) &\equiv \frac{F_K(K,H)(1-t_k)+\delta t_k+(1-\delta)\phi}{\hat{g}(q,K,H,\phi)}.
\end{aligned}
$$

The FOC for k'_{+1} is

$$
\frac{\phi}{F_H(K,H)} = \frac{\beta\left[F_K(K_{+1},H_{+1})(1-t_k)+\delta t_k+(1-\delta)\phi_{+1}\right]}{F_H(K_{+1},H_{+1})}, \tag{40}
$$

which is an arbitrage condition that implies the demand for k'_{+1} is indeterminate. Hence we can set $k'_{+1}=(1-\delta)K$ for all agents, so (m_{+1},k'_{+1}) is degenerate. The other conditions are

$$
\begin{aligned}
K_{+1} &= (1-\delta)K+\sigma q_{+1} && (41) \\
U'(X) &= \frac{A(1+t_x)}{(1-t_h)F_K(K,H)} && (42) \\
X+G &= F(K,H) && (43)
\end{aligned}
$$

An *equilibrium* is now given by paths for (q,ϕ,K_{+1},H,X) satisfying (39)-(43). Again, it obviously does not dichotomize, in general.

B Appendix: Details

B.1 The Cost Function

Here we verify the properties of the DM cost function $c(q,k)$, derived from a production function $q=f(k,e)$ that is strictly increasing and concave, and a disutility of effort $\ell(e)$ that is strictly increasing and convex. By definition, saying k is a normal input means that, in the problem $\min\{we+rk\}$ s.t. $f(k,e)\geq q$, the solution satisfies $\partial k/\partial q = f_e f_{ek} - f_k f_{ee} > 0$. To proceed, rewrite $q=f(k,e)$ as $e=\xi(q,k)$ Then $\partial e/\partial q = \xi_q = 1/f_e > 0$ and $\partial e/\partial k = \xi_k = -f_k/f_e < 0$. Also $\xi_{qq} = -f_{ee}/f_e^3 > 0$, $\xi_{kk} = -\left(f_e^2 f_{kk} - 2f_e f_k f_{ke} + f_k^2 f_{ee}\right)/f_e^3 > 0$, and $\xi_{kq} = -\left(f_{ek}f_e - f_{ee}f_k\right)/f_e^3$. Hence, $c_q = \ell'/f_e > 0$, $c_k = -\ell' f_k/f_e < 0$, $c_{qq} = \left[\ell''\ell'^2 f_e - \ell' f_{ee}\right]/f_e^3 > 0$, $c_{kk} = -\left[\ell'\left(f_e f_{kk} - 2f_e f_k f_{ke} + f_k^2 f_{ee}\right) - f_e f_k^2 \ell''\right]/f_e^3 > 0$ and $c_{qk} = -\left[\ell'' f_e f_k - \ell'\left(f_k f_{ee} - f_e f_{ek}\right)\right]/f_e^3$. These results establish that c is increasing and convex in q and decreasing and convex in k, and that $c_{qk} < 0$ if k is a normal input, as claimed.

B.2 Money Demand Elasticity

The interest elasticity of money demand is $\xi = \frac{\partial(M/P)}{\partial i}\frac{i}{M/P}$. To compute this in the bargaining model (price taking is similar) we need to determine $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$ and substitute them into (32). Eliminating X, we can write the steady state conditions as 3 equations in (q,K,H):

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q,K)} - 1$$
$$\rho = \left[F_K(K,H)-\delta\right](1-t_k) - \frac{\sigma\left(1+t_x\right)\gamma(q,K)}{U'\left[F(K,H)-\delta K - G\right]}$$
$$U'\left[F(K,H)-\delta K-G\right]F_H(K,H) = \frac{A\left(1+t_x\right)}{\left(1-t_h\right)}$$

We take the total derivative of this system to obtain

$$B\begin{bmatrix} dq \\ dK \\ qH \end{bmatrix} = \begin{bmatrix} di \\ 0 \\ 0 \end{bmatrix}$$

where

$$B = \begin{bmatrix} \frac{\sigma(g_q u'' - u' g_{qq})}{g_q^2} & -\frac{\sigma u' g_{qk}}{g_q^2} & 0 \\ -\frac{\sigma(1+t_x)\gamma_q U'}{U'^2} & \Theta & \frac{(1-t_k)U'^2 F_{KH} + \sigma(1+t_x)\gamma U'' F_H}{U'^2} \\ 0 & \left(F_K-\delta\right)F_H U'' + F_{KH}U' & F_H^2 U'' + F_{HH}U' \end{bmatrix}$$

and $\Theta = (1-t_k)F_{KK} - \frac{\sigma(1+t_x)}{(U')^2}\left[\gamma_k U' - \left(F_K-\delta\right)\gamma U''\right]$. We can now compute the partials as

$$\frac{\partial q}{\partial i} = B_{11}^{-1}\ \frac{\partial K}{\partial i} = B_{21}^{-1}\ \frac{\partial H}{\partial i} = B_{31}^{-1}$$

where B_{ij}^{-1} refers to the (i,j) element of B^{-1}.

B.3 Money Demand Estimation

Here we clarify how we get our empirical elasticity of money demand with respect to the nominal rate, ξ. Following a common specification in the literature (e.g. Goldfeld and Sichel 1990), we write the log of real money ($\tilde{m}_t$) as a linear function of log nominal interest ($\tilde{\imath}_t$) and log real output ($\tilde{y}_t$), allowing for first-order autocorrelation in the residuals. We estimated this using levels and first differences, but since the relevant results are statistically identical we report only the latter:

$$\Delta\tilde{m}_t = \beta_y\Delta\tilde{y}_t + \beta_i\Delta\tilde{\imath}_t - \rho\beta_y\Delta\tilde{y}_{t-1} - \rho\beta_i\Delta\tilde{\imath}_{t-1} + \rho\Delta\tilde{m}_{t-1} + \nu_t$$

$$\beta_y = 0.369 \ \ (0.124)\,, \ \ \beta_i = -0.226 \ \ (0.045)\,, \ \rho = 0.347 \ \ (0.131)\,, \ R^2 = 0.423$$

Here ρ is the AR(1) coefficient for the residuals in the original equation in levels and the numbers in parentheses are standard errors. The long-run interest elasticity is $\xi = -0.226$, with a relatively small standard error of 0.05.

B.4 Existence and Uniqueness

Here we show that for the functional forms we use in the calibrated model, under pricing taking, a steady state exists and under certain conditions is unique. With the functional forms in question, (28), (29), (21) and (22) can be written:

$$\frac{K^{1-\psi}}{q^{-\psi}} = \frac{\beta}{1+\pi}\left[(1-\sigma)\frac{K_{+1}^{1-\psi}}{q_{+1}^{-\psi}} + \sigma\psi(q_{+1}+b)^{-\eta}q_{+1}\right] \tag{44}$$

$$\frac{X_{+1}^{\varepsilon}}{X^{\varepsilon}} = \beta(1-t_k)\left[\alpha\left(\frac{K_{+1}}{H_{+1}}\right)^{\alpha-1} + 1 - \delta\right] - \frac{\sigma\beta(1+\pi_x)(1-\psi)}{B}\frac{X_{+1}^{\varepsilon}K_{+1}^{-\psi}}{q_{+1}^{-\psi}} \tag{45}$$

$$X = \left[\frac{B(1-\alpha)(1-t_h)}{A(1+t_x)}\frac{K^{\alpha}}{H^{\alpha}}\right]^{1/\varepsilon} \tag{46}$$

$$X = K^{\alpha}H^{1-\alpha} + (1-\delta)K - K_{+1} - G \tag{47}$$

Let $\Bbbk = K/H$, and combine (47) and (46) to get

$$\frac{\Bbbk}{K}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}\Bbbk^{\alpha}\right]^{1/\varepsilon} = \Bbbk^{\alpha} + (1-\delta)\Bbbk + \frac{H_{+1}}{H}\Bbbk_{+1} - \frac{G}{K}\Bbbk.$$

Hence, in steady state,

$$K = \frac{\Bbbk^{1-\alpha}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\Bbbk^{\alpha}\right]^{1/\varepsilon}}{1-(\delta+\frac{G}{K})\Bbbk^{1-\alpha}}. \tag{48}$$

Given $b \approx 0$, (44)-(46) reduce to:

$$q = \left[\frac{\sigma}{\psi(i+\sigma)}\right]^{\frac{1}{\psi+\eta-1}} K^{\frac{\psi-1}{\psi+\eta-1}} \tag{49}$$

$$X = \left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} \tag{50}$$

$$1 = \beta\left[1+(\alpha\mathbb{k}^{\alpha-1}-\delta)(1-t_k)\right] + \frac{(\psi-1)\sigma\beta(1-\alpha)(1-t_h)}{A}\left[\frac{\sigma}{\psi(i+\sigma)}\right]^{\frac{\psi}{\psi+\eta-1}} \mathbb{k}^{\frac{\alpha(\psi+\eta-1)-(1-\alpha)\psi\eta}{\psi+\eta-1}} \left\{\frac{1-(\delta+G/K)\mathbb{k}^{1-\alpha}}{\left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}\right]^{1/\varepsilon}}\right\}^{\frac{\psi\eta}{\psi+\eta-1}} \tag{51}$$

Notice (51) is one equation in $\mathbb{k}$. The RHS approaches ∞ as $\mathbb{k} \to 0$ and approaches a value less than 1 as $\mathbb{k} \to (\delta + G/K)^{1/(\alpha-1)}$. Hence it has a solution. The solution is unique if we assume $\alpha(\psi+\eta-1) < (1-\alpha)\,\psi\eta$, since then the RHS is strictly decreasing. Given $\mathbb{k}$, (48) yields K, (49) yields q, (50) yields X, and $H = \mathbb{k}/K$. So we have existence, and uniqueness under a simple restriction.

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Table 1 - Benchmark Calibration

(a) 'Simple' Parameters

Parameters	b	$\varepsilon = \eta$	β	t_h	t_k	t_x	G/Y	δ	α
Targets	0.0001	1	0.976	0.242	0.548	0.069	0.25	0.070	0.288

(b) Remaining Parameters

Parameters	A	B	ψ	σ	θ
Targets	H	v	K/Y	$-\xi$	μ
Target Values	0.33	5.29	2.32	0.23	0.10

Table 2 - Calibration Results

	Data	Model 1 $\theta = 1$	Model 2 calibrate θ	Model 3 calibrate θ calibrate ζ	Model 4 price taking	Model 5 price taking calibrate ζ
Calibrated Parameters						
σ		0.21	0.22	0.24	0.22	0.22
B		1.47	0.69	1.09	2.40	2.38
ψ		1.65	3.51	2.08	1.15	1.30
A		3.90	1.83	2.89	6.45	6.46
θ		–	0.72	0.75	–	–
Calibration Targets						
μ	10.00	-1.56 (*)	10.00	9.78	0.00 (*)	0.00 (*)
K/Y	2.32	2.23	2.23	2.22	2.32	2.39
H	0.33	0.33	0.33	0.33	0.33	0.33
v	5.29	5.30	5.30	4.92	5.29	5.28
ξ	-0.23	-0.23	-0.23	-0.22	-0.23	-0.23
ζ	-0.023	0.000 (*)	-0.001(*)	-0.001	-0.013 (*)	-0.023
Miscellaneous						
s_D		3.96	4.10	4.76	4.06	4.12
μ_D		-39.3	244.2	205.3	0.00	0.00
Sq. Error		0.0016	0.0015	0.9210	0.0000	0.0010

Note: The calibration targets marked with (*) are not targeted in the corresponding model and is not included in the computation of the squared error.

Table 3 - $\tau = 0.1$ vs. Friedman rule

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.75	0.87	0.81	0.67	0.69
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95
H^1/H^2	1.00	*1.00*	*1.00*	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	*1.00*	*1.00*	0.99
Y_C^1/Y_C^2	1.00	*1.00*	*1.00*	0.99	0.98
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97
Welfare					
ss gain	0.73	3.09	3.46	1.30	1.69
transition	0.00	−0.02	−0.02	−0.28	−0.50
net gain	0.73	3.08	3.43	1.02	1.19
net gain to 0	0.69	2.35	2.64	0.88	0.99

Table 4 - Friedman rule vs. first best

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.72	0.20	0.25	0.92	0.87
K^1/K^2	0.43	0.28	0.37	0.53	0.55
H^1/H^2	0.74	0.68	0.72	0.75	0.75
X^1/X^2	0.61	0.55	0.59	0.64	0.65
Y_C^1/Y_C^2	0.63	0.53	0.59	0.68	0.69
Y^1/Y^2	0.61	0.48	0.56	0.68	0.69
Welfare					
ss gain	22.35	60.33	39.84	15.70	15.22
transition	−10.07	−21.18	−13.77	−7.22	−6.95
net gain	12.28	39.15	26.07	8.48	8.26
Welfare with no Taxes					
ss gain	4.23	5.12	16.10	0.00	0.00
transition	−3.36	0.00	−4.59	0.00	0.00
net gain	0.87	5.12	11.51	0.00	0.00

Table 5 - Allocations

	Model 1	Model 2	Model 3	Model 4	Model 5
First Best					
q	1.06	1.83	1.23	0.98	0.98
Y_C	0.74	0.88	0.78	0.70	0.70
Y	0.80	1.03	0.88	0.73	0.74
K	2.51	3.83	2.93	2.17	2.23
H	0.45	0.48	0.46	0.44	0.44
X	0.44	0.49	0.46	0.42	0.42
K/Y	3.14	3.72	3.32	2.96	3.02
Equilibrium at Friedman rule					
q	0.76	0.36	0.30	0.90	0.86
Y_C	0.46	0.46	0.47	0.47	0.48
Y	0.49	0.49	0.49	0.50	0.51
K	1.08	1.08	1.08	1.16	1.23
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.27	0.27	0.27	0.27
K/Y	2.20	2.20	2.19	2.32	2.41
Equilibrium at $\tau = 0$					
q	0.71	0.35	0.29	0.82	0.79
Y_C	0.46	0.46	0.47	0.47	0.48
Y	0.49	0.49	0.49	0.50	0.50
K	1.08	1.08	1.08	1.15	1.21
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.27	0.27	0.27	0.27
K/Y	2.21	2.22	2.20	2.32	2.40
Equilibrium at $\tau = 0.1$					
q	0.57	0.32	0.24	0.60	0.59
Y_C	0.46	0.46	0.46	0.47	0.48
Y	0.48	0.48	0.48	0.49	0.49
K	1.08	1.08	1.08	1.13	1.17
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.27	0.27	0.27	0.27
K/Y	2.24	2.25	2.23	2.32	2.38
Nonmonetary Equilibrium					
q	0.00	0.00	0.00	0.00	0.00
Y_C	0.46	0.46	0.46	0.46	0.46
Y	0.46	0.46	0.46	0.46	0.46
K	1.08	1.07	1.08	1.07	1.07
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.27	0.27	0.26	0.26
K/Y	2.32	2.32	2.32	2.32	2.32

Table 6 - $\tau = 0.1$ vs Friedman rule and...

	Model 1	Model 2	Model 3	Model 4	Model 5
Making up Revenue by T					
q^1/q^2	0.75	0.87	0.81	0.67	0.69
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95
H^1/H^2	1.00	1.00	1.00	*1.00*	*1.00*
X^1/X^2	1.00	1.00	*1.00*	0.99	0.99
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97
T^1/Y^1	-3.43	-3.37	-3.39	-2.97	-2.63
T^2/Y^2	-1.28	-1.22	-1.08	-0.89	-0.58
ss gain	0.73	3.09	3.46	1.30	1.69
transition	0.00	-0.02	-0.3	-0.29	-0.51
net gain	0.73	3.08	3.43	1.02	1.19
Making up Revenue by t_h					
q^1/q^2	0.77	0.90	0.83	0.67	0.70
K^1/K^2	1.05	1.05	1.06	1.02	1.00
H^1/H^2	1.05	1.05	1.06	1.05	1.04
X^1/X^2	1.08	1.08	1.08	1.06	1.05
Y^1/Y^2	1.03	1.03	1.03	1.02	1.01
New t_h	0.296	0.296	0.301	0.293	0.291
ss gain	-2.19	-0.12	0.13	-1.40	-0.93
transition	0.48	0.52	0.53	0.19	-0.02
net gain	-1.71	0.39	0.66	-1.20	-0.94
Making up Revenue by t_x					
q^1/q^2	0.76	0.90	0.82	0.67	0.70
K^1/K^2	1.04	1.04	1.04	1.01	0.98
H^1/H^2	1.04	1.04	1.04	1.04	1.03
X^1/X^2	1.06	1.06	1.06	1.05	1.04
Y^1/Y^2	1.02	1.02	1.02	1.01	*1.00*
New t_x	0.130	0.130	0.136	0.127	0.126
ss gain	-1.45	0.69	0.99	-0.67	-0.20
transition	0.37	0.39	0.39	0.05	-0.19
net gain	-1.09	1.07	1.38	-0.63	-0.38

Table 7 - Robustness

	Model 1	Model 2	Model 3	Model 4	Model 5
Benchmark	0.73	3.08	3.43	1.02	1.19
Only Lump-sum Tax					
Recalibrated	0.82	3.63	3.83	0.89	0.53
Not	0.73	3.06	3.41	0.67	0.60
Utility Parameters ε and η					
$\varepsilon = 2$, $\eta = 1$	0.75	3.34	3.46	0.92	1.05
$\varepsilon = 5$, $\eta = 1$	0.78	3.49	3.60	0.87	0.96
$\varepsilon = 1$, $\eta = 1/2$	0.74	5.45	7.31	1.33	1.25
$\varepsilon = 2$, $\eta = 1/2$	0.54	5.51	7.61	1.15	1.13
$\varepsilon = 5$, $\eta = 1/2$	0.57	5.94	8.38	1.06	1.05
$\varepsilon = 1$, $\eta = 2$	0.75	2.33	6.38	0.81	1.06
$\varepsilon = 2$, $\eta = 2$	0.75	2.18	6.66	0.77	0.91
$\varepsilon = 5$, $\eta = 2$	0.76	2.25	6.99	0.74	0.81
Utility Parameter b					
$b = 0.00001$	0.73	3.26	3.36	1.02	1.19
$b = 0.0001$	0.73	3.08	3.43	1.02	1.19
$b = 0.001$	0.73	3.37	3.60	1.02	1.19
$b = 0.01$	0.73	3.56	4.20	1.02	1.12
$b = 0.1$	0.74	3.92	3.92	1.07	1.21
Markup Target					
$\mu_D = 10\%$	–	2.48	2.80	–	–
$\mu_D = 40\%$	–	2.77	2.96	–	–
$\mu_D = 100\%$	–	2.81	3.16	–	–
Measures of Money					
$M0$	0.04	0.37	0.37	0.05	0.05
$M1$	0.73	3.08	3.43	1.02	1.19
$M2$	2.06	7.22	9.84	2.67	2.39
$M3$	1.42	5.92	7.31	1.94	1.62
Frequency					
Quarterly	0.73	3.05	3.28	0.95	1.16
Monthly	0.72	3.03	3.23	0.95	1.18
Period					
1961-2004	0.64	3.06	2.88	1.22	0.68
1951-1998	0.74	3.45	3.62	0.78	1.21
1986-2004	0.84	2.87	3.14	1.61	0.96

Table 8 - More Robustness : Calibration Results

	Data	Model 6	Model 7	Model 8	Model 9
Calibrated Parameters					
σ		0.24	0.21	0.24	0.19
B		0.96	2.35	1.01	0.13
ψ		2.41	1.96	3.62	7.70
A		2.55	6.41	2.09	0.28
G		0.12	0.12	0.13	0.15
θ		0.61	–	0.20	–
Calibration Targets					
μ	10.00	9.86	0.00 (*)	10.02	0.00 (*)
K/Y	2.32	2.21	2.43	2.38	2.69
G/Y	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33
v	5.29	5.08	5.28	5.51	1.53
ξ	−0.23	−0.22	−0.23	−0.08	−0.20
ζ	−0.023	−0.001	−0.023	−0.025	−0.025
Miscellaneous					
s_D		4.67	3.98	4.42	12.46
μ_D		211.36	0.00	226.76	0.00
Sq. Error		0.9456	0.0024	0.4444	0.5516

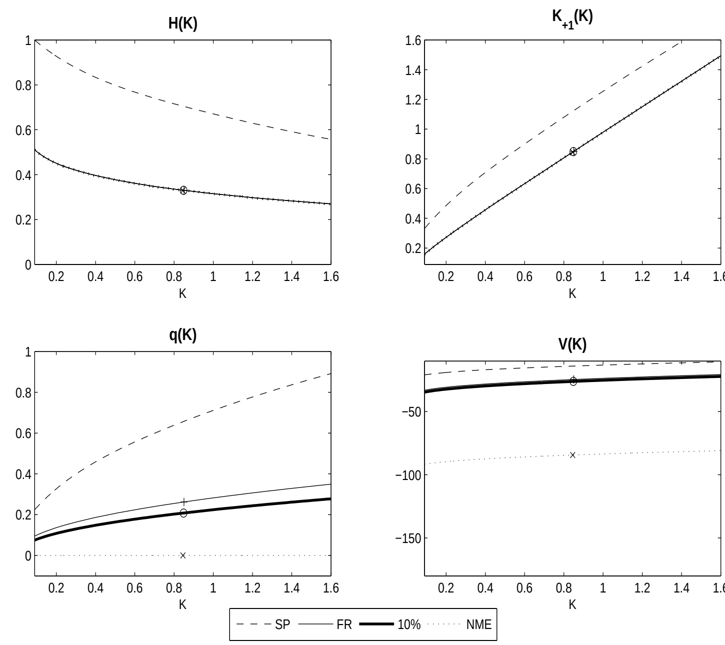
Note: The calibration targets marked with (*) are not targeted in the corresponding model and not included in the computation of the squared error.

Table 9 - More Robustness : $\tau = 0.1$ vs. Friedman rule

	Model 6	Model 7	Model 8	Model 9
Allocation				
q^1/q^2	0.67	0.62	0.80	0.93
K^1/K^2	1.00	0.99	0.80	0.93
Z^1/Z^2	0.68	0.62	–	–
ϕ^1/ϕ^2	–	–	1.10	1.03
H^1/H^2	1.00	0.99	1.02	1.01
X^1/X^2	1.00	*1.00*	0.93	0.98
Y_C^1/Y_C^2	1.00	0.99	0.95	0.99
Y^1/Y^2	0.98	0.97	0.92	0.92
Welfare				
ss gain	7.77	1.59	7.61	1.41
transition	−0.12	−0.49	−1.17	−0.38
net gain	7.65	1.10	6.44	1.03
net gain to 0	5.82	0.96	4.68	0.83

Figure 1 - Decision Rules and Value Functions

(a) Model 4



(b) Model 5

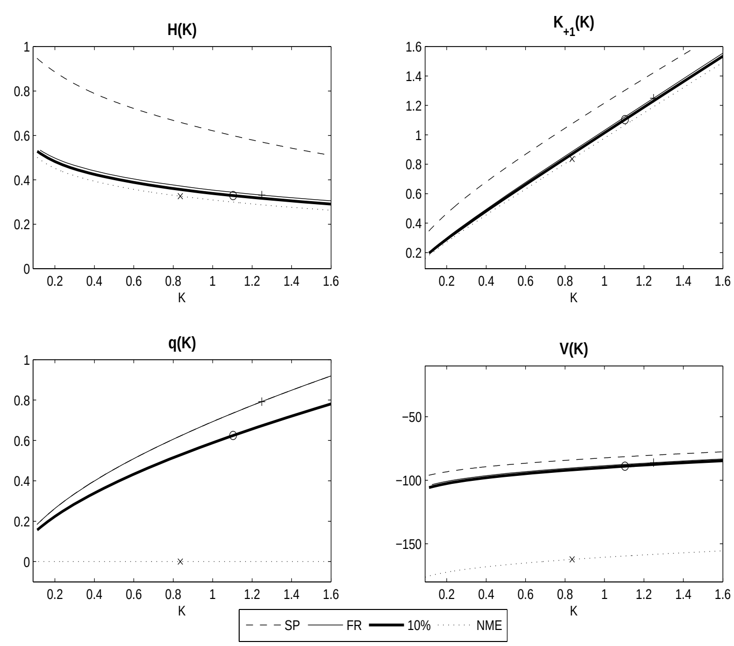
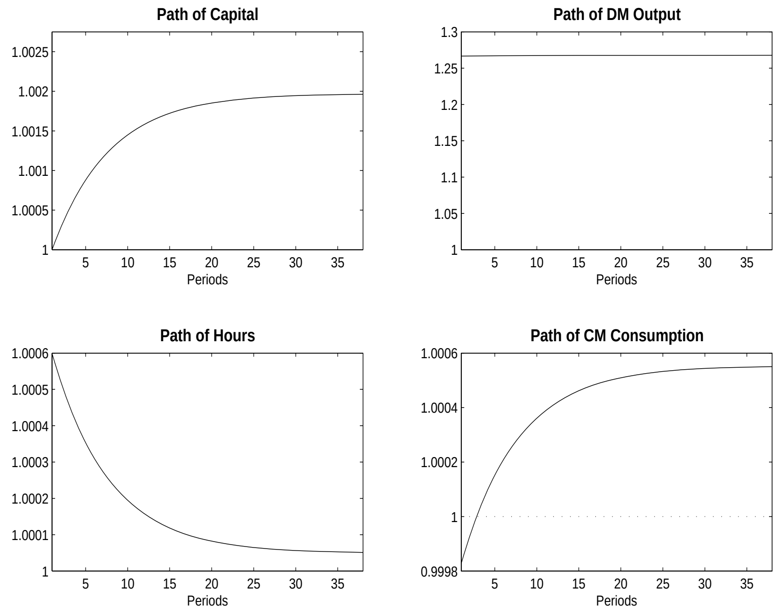


Figure 3 -10% to FR: Transitions

(a) Model 4



(b) Model 5