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DYNAMIC OPTIMAL FISCAL AND MONETARY
POLICY IN A BUSINESS CYCLE MODEL WITH
INCOME REDISTRIBUTION

by Kevin J. Lansing

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ABSTRACT

An optimal program of distortionary taxes, money growth, and borrowing to finance a stream of expenditures is computed in a monetary real business cycle model for which distribution issues between the rich and poor play a fundamental role in policy decisions. Specifically, a simple feedback rule links public spending on goods and services to a measure of income inequality, and the government is required to provide poor households with some minimum level of transfers. The stationary equilibrium policy displays positive capital taxation, progressive labor taxes, and moderate (6 percent) inflation. The capital tax and the inflation tax fluctuate over time to absorb budget shocks, while the labor tax remains relatively constant. Model simulations compare favorably in many respects with postwar U.S. time series on tax rates, money growth, and aggregate business cycle variables. The solution method employs the recursive algorithm developed by Kydland and Prescott (1980) to compute optimal policy rules under the assumption of commitment.

1. Introduction

Regardless of one's views on whether government *should* be involved in the business of redistributing income, it seems clear that the complicated U.S. system of taxes and public spending programs has been designed, in large measure, with this objective in mind. Policy debates are often driven by arguments for a more equitable distribution of income, as in this example:

Senate Majority Leader George Mitchell insisted from the beginning that the wealthiest taxpayers pick up more of the tab... Now, the nearly instant availability of the distribution tables casts every offer and counter-offer in terms of fairness between the rich and the poor. Every plan involving a cut in the capital gains tax invariably showed a windfall for the rich.¹

The distribution of wealth and income in the United States is highly skewed, with the top 20 percent of households owning about 80 percent of the wealth and earning about 42 percent of pre-tax income.² In this environment, policymakers and the public have come to view the capital gains tax as being paid primarily by the wealthy. This tax and another capital-type tax, the corporate income tax, are frequently singled out by policymakers as tools for achieving more equity in the U.S. economy. The government has also developed other redistributive tools, including our system of progressive marginal tax rates and a myriad of means-tested assistance programs, commonly known as "welfare."³

In this paper, I formulate a model of dynamic optimal fiscal and monetary policy that incorporates, in a simple way, the government's use of redistributive tools like the capital tax, progressive labor taxation, and means-tested transfers. I then subject the model to the same kind of quantitative comparisons with U.S. data that have been widely used in the real business cycle literature. As a way of approximating the skewed distribution of U.S. wealth and income, capital ownership in the

¹See A. Murray and J. Calmes, "How the Democrats, with Rare Cunning, Won the Budget War," *The Wall Street Journal*, November 15, 1990.

²See McDermed, Clark, and Allen (1989), figures 13.1 and 13.2, and Rosen (1992), table 8.1. The measure of wealth inequality cited here is based on net worth from 1983 household survey data. This measure remained approximately constant from 1962 to 1983.

³The principal means-tested transfer programs used to supplement the earnings of the poor are Aid to Families with Dependent Children (AFDC), Supplemental Security Income (SSI), and the Earned Income Tax Credit (EITC). There are also in-kind transfer programs such as housing assistance, food stamps, job training, and Medicaid. Social insurance programs may be viewed as implicit transfer programs. The three major social insurance programs are Social Security, Medicare, and unemployment insurance. For more details, see *Economic Report of the President 1992*, chapter 4.

model is concentrated in the hands of a single group, labeled "rich" households. The government solves a dynamic version of the Ramsey (1927) optimal tax problem, in which a policymaker chooses a program of distortionary taxes over time to finance a required stream of spending. Monetary policy is incorporated by viewing inflation as an effective tax on real money balances.

A crucial aspect of the model is the manner in which government outlays are determined. In particular, I assume that a simple feedback rule links public spending on goods and services to a measure of income inequality and, further, that the government must provide the poor with some minimum level of transfers. The transfer payments are a proxy for the various means-tested assistance programs in the U.S. economy. However, the infinite-horizon framework abstracts from any life-cycle effects of specific transfer programs, like Social Security. The endogenous policy variables are the tax rate on capital income, tax rates on labor income (for the rich and poor), and the growth rate of the nominal money stock. For simplicity, the steady-state level of government debt is taken to be exogenous. The government's problem is solved using a numerical recursive algorithm based on a method developed by Kydland and Prescott (1980). Specifically, a "pseudo state variable" is defined that permits the use of dynamic programming to compute optimal policy rules under the assumption of commitment.

A primary finding is that equilibrium policy displays positive capital taxation, progressive labor taxes (in the sense that the rich are taxed at a higher marginal rate than the poor), and moderate (6 percent) inflation. In simulations, the capital tax and the inflation tax fluctuate over time to absorb budget shocks, while the labor tax remains relatively constant. As previously identified by Judd (1989) and Chari, Christiano, and Kehoe (1991), the fact that household savings in the form of capital or money balances is inelastic in the short run suggests that state-contingent taxes on these assets can serve as nondistortionary shock absorbers. Budget shocks in the model are caused by changes in the size of the tax base (due to business cycle fluctuations) or by changes in exogenous spending requirements. Predictions for the moments of aggregate economic variables are very close to those of previous monetary real business cycle models. This result is reassuring because it suggests that these models can be extended into new areas without sacrificing a reasonable description of the aggregate economy.

Another finding is that some predictions of partial-equilibrium models that have been used in the past as empirical tests for optimal government behavior are not implied by this general-equilibrium

model. In simulations, the labor tax is negatively correlated with inflation (or money growth) while the capital tax is positively correlated with inflation. Partial-equilibrium models generally do not distinguish between labor and capital taxes and predict a positive correlation between an "income tax" and inflation. This failure to distinguish between factor incomes may help to explain the inconsistent findings of previous empirical studies designed to test the partial-equilibrium result (see Mankiw [1987], Roubini and Sachs [1989], Poterba and Rotemberg [1990], and Roubini [1991]).

Within the infinite-horizon growth framework, models of dynamic optimal fiscal policy have been applied to the study of heterogeneous-agent economies by Judd (1985), Aiyagari and Peled (1991), and Alesina and Rodrik (1991). This paper attempts to go further by bringing in monetary policy and by directly examining the quantitative implications of the model in comparison to U.S. data.⁴ A well-known result that applies to infinite-horizon growth models is that the optimal steady-state tax on capital is zero.⁵ Moreover, Judd (1985) has shown that this result holds regardless of the weights placed on different groups in a social welfare function, even when one group holds the entire stock of physical capital. This seemingly counterintuitive finding obtains because a zero tax on capital leads to higher levels of capital accumulation and hence higher wages, thus benefiting all individuals, not just capital owners. However, variations in the structure of the standard model can overturn the optimality of a zero tax rate on capital, for example, when certain kinds of externalities or constraints are present or when the government faces restrictions on the menu of available policy instruments. Arrow and Kurz (1970), Thompson (1979), Stiglitz (1987), Aiyagari and Peled (1991), and Jones, Manuelli, and Rossi (1992) all provide examples of such cases.

In this paper, I assume that income inequality generates negative externalities that ultimately lead to a drain on productive resources in the form of higher public spending needs. The government's desire

⁴Static models of optimal redistributive taxation can be found in Fair (1971), Mirlees (1971), and Stiglitz (1987). A dynamic model with fixed saving propensities is developed by Pestieau and Possen (1978). The overlapping-generations models of Atkinson and Sandmo (1980) and Stiglitz (1987) examine aspects of intergenerational redistribution. This paper deals with redistribution among rich and poor households of the *same* generation. The assumption of an operative bequest motive among finitely lived households allows the economy to be modeled using an infinite-horizon framework.

⁵The intuition for this result is that the long-run supply elasticity of capital is essentially infinite. Recall that the Ramsey principle of optimal taxation states that taxes should be set in *inverse proportion* to the elasticity of the tax base. The zero tax result is discussed by Arrow and Kurz (1970), pp. 195-203, and has been further elaborated on by Judd (1985) and Chamley (1986).

to combat these externalities motivates a positive tax on capital in the steady state. This simple formulation in which feedback from the economy affects government spending policy introduces a role for the capital tax in reducing income inequality. Support for this idea can be found in the urban development literature, where studies of urban decline often refer to the spillover effects of distributional inequality, such as increased crime, family disintegration, urban population loss, deteriorating neighborhoods, and low-quality schools.⁶ In addition to other harmful consequences, these spillovers contribute to a drop in income and property tax bases in urban areas, forcing cities to rely increasingly on aid from federal and state governments to provide basic public services.⁷ On the empirical side, some recent cross-country studies suggest that government spending and economic growth are both linked to inequality. Easterly and Rebelo (1993) report a statistically significant, positive correlation between income inequality and public spending on education, transportation, and communication, while Persson and Tabellini (1991), Alesina and Rodrik (1991), and Perotti (1992) find that income inequality has a detrimental effect on economic growth. All of these studies are consistent with a model of political equilibrium in which an increase in inequality causes the median voter to support higher levels of government redistribution expenditures and higher taxes on capital.

In an economy with distortionary taxes, the optimal rate of inflation can depend crucially on the specific structure of the model. A common result is that the optimal rate of inflation is negative in steady state, in agreement with the Friedman (1969) optimal money rule. Under this policy, the government reduces the size of the nominal money stock at the rate of time preference, thereby achieving a zero nominal interest rate. However, the presence of externalities or the use of alternative functional forms (for household utility or transaction cost functions) can yield optimal rates of inflation that are positive.⁸

Here, I assume that the government is constrained to provide poor households with some minimum level of transfers. This constraint can be viewed as a way of reflecting that welfare programs represent a "safety net" for the poor that cannot be reduced below some baseline amount. Alternatively,

⁶See, for example, Bateman and Hochman (1971), Bradbury, Downs, and Small (1982), and Mieszkowski and Mills (1993).

⁷From 1950 to 1988, federal and state aid to cities averaged nearly 40 percent of local revenues. See Rosen (1992), table 21.4.

⁸See Chari, Christiano, and Kehoe (1991), Braun (1993), and Croushore (1993) for further discussion.

political factors may render these programs "untouchable" in the eyes of many policymakers. The effect of the constraint is to introduce an externality into the government's decision problem that can motivate the use of a positive inflation tax. Since transfer payments are not taxed but must be financed by distorting taxes, it is efficient for the government to spread the distortionary costs across various tax bases, including consumption. The inflation tax operates as a consumption tax in this model because a subset of consumption goods (known as "cash goods") can only be acquired with previously accumulated cash balances. A steady-state annual inflation rate of 6 percent is obtained when model transfers are calibrated to match the average level of U.S. means-tested transfers (approximately 2 percent of GNP).

The remainder of the paper is organized as follows. Sections 2 and 3 describe the model and the recursive solution method. Section 4 describes the choice of parameter values. Section 5 presents quantitative results from steady-state and dynamic experiments, and section 6 concludes.

2. The Model

The model economy consists of two types of infinitely lived households, identical competitive firms, and the government. The total number of households is normalized to one so that γ and $1-\gamma$ represent the fraction of poor and rich households, respectively, where $0 < \gamma < 1$. The poor's discount factor (β^P) is assumed to be less than that of the rich (β^R). This implies that the poor are more impatient than the rich and thus would rather borrow than save at the steady-state real interest rate.⁹ In equilibrium, ownership of the capital stock is concentrated in the hands of the rich. Although the poor would like to borrow to increase their current consumption, it is assumed that they are prevented from doing so. A rationale for the borrowing constraints is that the rich are unwilling to make loans to the poor because of difficulties in enforcing repayment. The condition $\beta^P < \beta^R$ ensures that the poor will choose not to save for stochastic equilibria of the type examined here, involving sufficiently small fluctuations around the steady state.^{10,11} The assumption regarding discount factors is consistent with

⁹The discount factor β is inversely related to the marginal rate of time preference ρ , according to $\beta=(1+\rho)^{-1}$.

¹⁰In a perfect foresight model, Becker (1980) shows that concentrated ownership of capital results when one household type is more patient than other types. Woodford (1988) applies this result to a stochastic economy. When all households have the *same* discount factor, Becker shows that the steady-state distribution of capital across households is indeterminate, i.e., long-run equilibrium

some recent empirical evidence suggesting that individuals with lower income and lower education tend to be more impatient (see Lawrance [1987, 1991]). Further distinction between households is made by assuming that the labor input of the poor is less productive than that of the rich, based on the idea that the poor have less human capital, perhaps due to less education. Rather than explicitly modeling the accumulation of human capital, I introduce an exogenous labor efficiency parameter ($\hat{\epsilon}$) for the poor. This factor represents the ratio of the poor's productive labor input to their number of hours worked, where $0 < \hat{\epsilon} < 1$. In equilibrium, this factor produces a wage differential between the rich and poor.

2.1 The Household's Problem

Household behavior is described in the context of a "cash-in-advance" economy similar to that of Cooley and Hansen (1992). Household income is either allocated to consumption, held in the form of money, or, in the case of rich households, invested in the formation of productive capital. To ensure that money is held in equilibrium, households are assumed to be subject to a cash-in-advance constraint of the type described by Lucas and Stokey (1983, 1987). A particular type of consumption goods known as cash goods can only be acquired with previously accumulated cash balances. Purchases of the other type, known as credit goods, can be financed out of current income. Household preferences are summarized by the following utility function:

$$E_0 \sum_{t=0}^{\infty} (\beta^i)^t \left\{ \alpha \ln c_{1t}^i + (1-\alpha) \ln c_{2t}^i - A h_t^i \right\}, \quad 0 < \beta^P < \beta^R < 1, \quad 0 < \alpha < 1, \quad i = P, R \quad (1)$$

Superscripts P and R are used throughout the discussion to denote quantities held by poor and rich households, respectively. In (1), c_{1t}^i represents purchases of cash goods and c_{2t}^i is purchases of credit goods. Households are endowed with one unit of time each period and work h_t^i hours ($i=P, R$)

is consistent with *any* wealth distribution. If the discount factor is endogenous (rather than constant), the steady state determines a unique wealth distribution (see Epstein and Hynes [1983]). Appendix A describes how $\beta^P < \beta^R$ yields concentrated capital ownership.

¹¹Borrowing constraints are necessary to ensure a positive level of steady-state consumption for the poor. Otherwise, poor households would borrow up to the present value of their income stream to finance current consumption. Here, no borrowing is allowed. An alternative would be to allow borrowing up to some credit limit that would always be binding.

during period t . The symbol E_t is the expectation operator conditional on information available at time t . The form of the within-period utility function has been chosen for tractability and for comparability with previous business cycle literature. The coefficient of relative risk aversion for consumption is constant and equal to one for this function.

The fact that utility is linear in hours worked reflects "indivisible labor" as described by Rogerson (1988) and Hansen (1985). This means that all variation in economywide hours worked is due to variations in the number of employed workers as opposed to variations in hours per worker. In a decentralized economy, these authors show that the utility function in (1) can be supported by a lottery that randomly assigns workers to employment or unemployment each period, with the firm providing full unemployment insurance. Wage contracts call for households to be paid based on their expected, as opposed to actual, number of hours worked. Real business cycle models with indivisible labor are better able to match some key characteristics of aggregate labor market data. Specifically, U.S. data display a large volatility of hours worked relative to labor productivity and a weakly positive or even slightly negative correlation between hours and productivity.¹²

The weights α and A in the utility functions are assumed to be equal for both the rich and poor. Households maximize the utility function in (1) over consumption and leisure, subject to the following sequence of budget constraints:

Cash-in-advance constraint:

$$P_t c_{1t}^i \leq m_t^i, \quad m_0^i \text{ given}, \quad i = P, R. \quad (2)$$

Budget constraint of poor households:

$$c_{1t}^P + c_{2t}^P + \frac{m_{t+1}^P}{P_t} \leq (1 - \tau_{ht}^P) \hat{e} w_t h_t^P + \frac{m_t^P}{P_t} + TR_t, \quad 0 < \hat{e} < 1. \quad (3)$$

¹²See Hansen (1985), Christiano and Eichenbaum (1992), and Hansen and Wright (1992).

Budget constraint of rich households:

$$c_{1t}^R + c_{2t}^R + x_t + \frac{m_{t+1}^R}{P_t} + b_{t+1} \leq (1-\tau_{ht}^R)w_t h_t^R + (1-\tau_{kt}^R)r_t k_t + \tau_{kt} \delta k_t + \frac{m_t^R}{P_t} + b_t(1+r_{bt}) \quad (4)$$

$$k_{t+1} = (1-\delta)k_t + x_t, \quad 0 < \delta < 1, \quad k_0, b_0 \text{ given.} \quad (5)$$

Equation (2) represents the cash-in-advance constraint faced by all households. A requirement that this constraint hold with equality is imposed in the computation of government policy.¹³ Equations (3) and (4) are the within-period budget constraints of households. Poor households choose not to hold physical capital due to the assumption $\beta^P < \beta^R$. In the above constraints, m_{t+1}^i represents nominal money balances carried into the next period, and P_t is the price level in period t . Poor households are assumed to qualify for lump-sum government transfer payments TR_t . The timing of transfer payments is such that they supplement wage earnings and thus do not enter the cash-in-advance constraint. The term TR_t is intended to summarize, in an approximate way, the many forms of means-tested assistance provided by the government. Although explicit modeling of specific assistance programs is possible (such as the EITC), the basic nature of the results depends not on the form of the transfers, but rather on the assumption that some portion is exogenous and cannot be directly taxed by the government.

The terms x_t and k_t represent the rich household's investment and capital stock, respectively, and r_t is the rental rate on capital. Since the poor have a lower labor efficiency, their equilibrium wage rate ($\hat{e}w_t$) is lower than that of the rich (w_t). The tax rate on labor income (τ_{ht}^i) is allowed to vary between household types. Notice that marginal tax rates are the same as average tax rates here, but rich and poor households face different linear tax schedules. This is an approximation of the nonlinear, progressive U.S. tax system, where marginal tax rates generally exceed average tax rates. Rich households pay taxes on capital income at the rate τ_{kt} , and $\tau_{kt} \delta k_t$ is the depreciation allowance built into the U.S. tax code. In equation (4), b_{t+1} represents real, one-period government bonds carried into period $t+1$ by the rich.

¹³For the constraint to be binding, the condition $\partial U/\partial c_{1t} > \partial U/\partial c_{2t}$ must hold in expected terms. This implies nonsatiation for real money balances. With logarithmic utility, this condition will be satisfied when $E_t [1/(1+\mu_{t+1})] > 1/\beta$, where μ_{t+1} is next period's money growth rate. Since β is generally less than one ($\beta=0.99$ is a typical value for quarterly data), the constraint will bind whenever the expected money growth rate is positive, and even for negative expected growth rates that are sufficiently small. The Friedman (1969) optimal money rule is $\mu=\beta-1$ in steady state, which yields a nominal interest rate of zero.

These bonds can be viewed as being indexed against inflation so as to pay principal and interest of $b_t(1+r_{bt})$ in real terms during period t .¹⁴ Equation (5) is the law of motion for capital, given a constant rate of depreciation δ . Households view tax rates, transfer payments, wages, and interest rates as being determined outside their control. The household decision variables in period t are $c_{1t}^i, c_{2t}^i, h_t^i, m_{t+1}^i, k_{t+1}$, and b_{t+1} , $i = P, R$.

2.2 The Firm's Problem

Output (Y_t) is produced by identical competitive firms using a constant-returns-to-scale technology. Since profits are zero in equilibrium, there is no need to model a market governing the ownership of firms. The production technology is subjected to serially correlated exogenous shocks (z_t) that are revealed to agents at the beginning of period t . These shocks produce equilibrium business cycle fluctuations in the model. The firm's technology can be described as follows:

$$\begin{aligned}
 Y_t &= \exp(z_t) K_t^\theta H_t^{1-\theta}, & 0 < \theta < 1 \\
 K_t &= (1-\gamma) k_t \\
 H_t &= \gamma \hat{e} h_t^P + (1-\gamma) h_t^R, & 0 < \hat{e} < 1 \\
 z_{t+1} &= \rho_z z_t + \varepsilon_{t+1}, & 0 < \rho_z < 1, \quad \varepsilon_{t+1} \sim \text{iid}(0, \sigma_\varepsilon^2), \quad z_0 \text{ given.}
 \end{aligned} \tag{6}$$

In (6), H_t and K_t are the economywide labor and capital inputs.¹⁵ The (per capita) labor input of the poor is $\hat{e} h_t^P$. The firm's problem is static because it simply rents capital and labor services from households each period, with the objective of maximizing profits: $Y_t - r_t K_t - w_t H_t$. The firm's first-order

¹⁴The inflation tax operates only on real money balances in this model. The assumption of indexed debt, though counterfactual, is needed to pin down an equilibrium policy in a model with money, capital, and government bonds. I will come back to this point when I describe the government's decision problem.

¹⁵There is no need to distinguish between variables under the household's control and variables representing per capita quantities, as is necessary when solving directly for a decentralized, competitive equilibrium. As noted by Lucas and Stokey (1983), solution of the government's decision problem yields a set of policies that dictate household equilibrium allocations. These allocations determine the equilibrium prices r_t , w_t , and P_t . Thus, prices are not outside the government's control as they are for households.

conditions yield the following expressions for the rental rate on capital and the real wage:

$$r_t = \exp(z_t) \theta \left[\frac{H_t}{K_t} \right]^{1-\theta} \quad w_t = \exp(z_t) (1-\theta) \left[\frac{K_t}{H_t} \right]^\theta. \quad (7)$$

To facilitate solving for an equilibrium, a transformation of variables is performed to render the household problems stationary. Following Cooley and Hansen (1992), the transformation is defined as

$$\hat{m}_t^i \equiv \frac{m_t^i}{M_t}, \quad i = P, R \quad \hat{P}_t \equiv \frac{P_t}{M_t}. \quad (8)$$

The term M_t is the economywide nominal money stock that evolves according to $M_{t+1} = (1+\mu_t)M_t$, where μ_t is the growth rate observed at time t . The government achieves the desired level of μ_t by injecting new money into the economy through open-market operations. Since households use identical currency, μ_t must be the same for all households, unlike tax rates, which may differ between types. In equilibrium, the economywide money stock is the sum of household money stocks: $M_t = \gamma m_t^P + (1-\gamma)m_t^R$. In transformed variables, the equilibrium condition is $1 = \gamma \hat{m}_t^P + (1-\gamma)\hat{m}_t^R$.

2.3 Household Optimality

As a preliminary step to obtaining the conditions for household optimality, the variable transformations in (8) are applied to equations (1)-(5) and the cash-in-advance constraint is imposed with equality. This procedure yields the following Lagrangians for households:

Poor households:

$$\mathcal{L}^P = E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ \alpha \ln \frac{\hat{m}_t^P}{\hat{P}_t} + (1-\alpha) \ln c_{2t}^P - A h_t^P + \right. \\ \left. \lambda_t^P \left[(1-\tau_{ht}^P) \hat{e} w_t h_t^P + TR_t - \frac{\hat{m}_{t+1}^P (1+\mu_t)}{\hat{P}_t} - c_{2t}^P \right] \right\} \quad (9)$$

Rich households:

$$\begin{aligned} \mathcal{L}^R = E_0 \sum_{t=0}^{\infty} (\beta^R)^t & \left\{ \alpha \ln \frac{\hat{m}_t^R}{\hat{P}_t} + (1-\alpha) \ln c_{2t}^R - A h_t^R + \right. \\ & \left. \lambda_t^R \left[(1-\tau_{ht}^R) w_t h_t^R + (1-\tau_{kt}^R)(r_t - \delta)k_t + k_t + b_t(1+r_{bt}) - (k_{t+1} + b_{t+1}) - \frac{\hat{m}_{t+1}^R(1+\mu_t)}{\hat{P}_t} - c_{2t}^R \right] \right\} \end{aligned} \quad (10)$$

The household first-order conditions with respect to the indicated variables and the associated transversality conditions are

$$\begin{aligned} \hat{m}_{t+1}^i: \quad & \frac{\lambda_t^i(1+\mu_t)}{\hat{P}_t} = \frac{\alpha\beta^i}{\hat{m}_{t+1}^i}, \quad i = P, R \\ c_{2t}^i: \quad & \lambda_t^i = \frac{1-\alpha}{c_{2t}^i}, \quad i = P, R \\ h_t^i: \quad & \lambda_t^i(1-\tau_{ht}^i)\hat{e}^i w_t = A, \quad i = P, R \quad (\hat{e}^R = 1) \\ k_{t+1}: \quad & \lambda_t^R = \beta^R E_t \lambda_{t+1}^R [(1-\tau_{kt+1}^R)(r_{t+1} - \delta) + 1], \\ b_{t+1}: \quad & \lambda_t^R = \beta^R E_t \lambda_{t+1}^R (1+r_{bt+1}) \\ & \lim_{t \rightarrow \infty} E_0 (\beta^R)^t \lambda_t^R k_{t+1} = 0, \quad \lim_{t \rightarrow \infty} E_0 (\beta^R)^t \lambda_t^R b_{t+1} = 0. \end{aligned} \quad (11)$$

2.4 The Government's Problem

The government chooses an optimal program of distortionary taxes, money growth, and borrowing to finance a stream of expenditures and transfers. The problem is a dynamic version of the

classic Ramsey case, involving a Stackelberg game between the government and households.¹⁶ To avoid time-consistency problems, I assume that the government can commit to a set of state-contingent policy rules announced at time zero. Also, to make the problem interesting, lump-sum taxes are ruled out. Otherwise, the government would elect to finance all future expenditures with an initial levy on household assets. With these assumptions, the government's problem can be summarized as follows:

$$\max_{\tau_w, \tau_h, b_t} E_0 \sum_{t=0}^{\infty} (\beta^G)^t \left\{ W(c_{1t}^P, c_{2t}^P, h_t^P, c_{1t}^R, c_{2t}^R, h_t^R) \right\},$$

subject to:

(i) household first-order conditions and budget constraints

(ii) firm profit-maximization conditions

$$(iii) \quad g_t + \gamma TR_t + (1-\gamma)b_t(1+r_{bt}) = \gamma \left[\tau_{ht}^P \hat{e} w_t h_t^P \right] + (1-\gamma) \left[\tau_{ht}^R w_t h_t^R + \tau_{kt}(r_t - \delta) k_t \right]$$

$$+ \frac{\mu_t M_t}{P_t} + (1-\gamma)b_{t+1} \tag{12}$$

$$(iv) \quad M_{t+1} = (1 + \mu_t) M_t, \quad \text{where } M_t = \gamma m_t^P + (1-\gamma)m_t^R$$

$$(v) \quad TR_t \geq \bar{TR} > 0$$

$$(vi) \quad \lim_{t \rightarrow \infty} \frac{(1-\gamma)b_t}{\prod_{i=1}^t (1+r_{b i-1})} = 0.$$

The government employs discount factor β^G in maximizing a sequence of within-period objective functions, $W(\cdot)$. The choice of β^G and $W(\cdot)$ will be discussed shortly. Constraints (i) and (ii) summarize rational maximizing behavior on the part of private agents and constitute "implementability" constraints on the government's choice of policy. Constraint (iii) is the government budget constraint, where the term $\mu_t M_t / P_t$ ($= \mu_t / \hat{P}_t$) represents seigniorage. Constraint (iv) describes the

¹⁶The dynamic Ramsey problem in a representative-agent framework has been studied by numerous authors. A partial list employing general-equilibrium models with either money or capital is Helpman and Sadka (1979), Kydland and Prescott (1980), Turnovsky and Brock (1980), Lucas and Stokey (1983), Chamley (1986), Lucas (1990), Jones, Manuelli, and Rossi (1992), Zhu (1992), Chari, Christiano, and Kehoe (1991), and Braun (1993). Models with both money and capital are analyzed by Drazen (1979) and Chamley (1985a).

evolution of the economywide nominal money stock. Constraint (v) is the assumption that the government must provide poor households with a minimum level of transfer payments, \bar{TR} . In equilibrium, this constraint is always binding such that $TR_t = \bar{TR}$ for all t .¹⁷ Finally, (vi) is a transversality condition that ensures the government budget constraint is satisfied in present-value terms.

Since the model includes heterogeneous households, the question arises as to what form the government objective function should take. In the words of Arrow and Kurz (1970), "no definitive criterion can be given that will withstand all criticism." I suggest, therefore, an analytically tractable version reflecting the basic premise that the government cares about household welfare, where welfare is measured by some concave function of household consumption and leisure. Within this class, the qualitative behavior of the model is robust to the choice of a specific function. To minimize introduction of new parameters, the function $W(\cdot)$ is assumed to be an additively separable combination of household within-period utility functions, $W(\cdot) = \phi U^P(\cdot) + U^R(\cdot)$. This choice implies that the government respects household valuation of utility *within a given period*. The parameter $\phi > 0$ controls how much the policymaker favors one group over the other. For example, the weight assigned to the welfare of a given group may exceed the value implied by the group's relative number in the population.

The government's discount factor β^G need not coincide with household discount factors. The case in which public and private discount factors differ is termed *futurity divergence* by Arrow and Kurz (1970). To support an equilibrium with public debt, however, I assume that the government's discount factor coincides with that of rich households ($\beta^G = \beta^R$). I thus attribute the government with having more patience than the poor, an assumption that seems quite reasonable considering the existence of government-mandated savings programs like Social Security, long-term investments in public infrastructure, and the government's willingness to subsidize activities that build human capital, such as job training and basic education.¹⁸ The choice of $\beta^G = \beta^R$ and the form of $W(\cdot)$ yield a government

¹⁷Removing the constraint essentially provides the government with a nondistortionary tax instrument. In this case, the equilibrium value of TR_t is highly negative, which indicates that the government would like to impose a large lump-sum tax on the poor, allowing other distortionary taxes to be lowered. Lump-sum taxes have been ruled out to focus on a "second-best" equilibrium.

¹⁸If the government is myopic relative to savers ($\beta^G < \beta^R$), an equilibrium with government debt can be restored by introducing an exogenous limit on public borrowing (which would always be binding) or by imposing an exogenous cost of borrowing (which increases with the level of debt). The assumption $\beta^G = \beta^R$ avoids these additional complications, but is not crucial for the basic results.

objective that represents a transformed version of a standard utilitarian social welfare function, where the transformation places more weight on the future utility of the poor.¹⁹

A simple feedback rule is assumed to link government spending on goods and services (g_t) to a measure of income inequality in the economy. The law of motion for g_t is

$$g_t = \bar{g} \exp(v_t) + \eta \left[(1 - \tau_{kt}) r_t k_t + \tau_{kt} \delta k_t + (1 - \tau_{ht}^R) w_t h_t^R - (1 - \tau_{ht}^P) \hat{e} w_t h_t^P - TR_t \right] \quad (13)$$

$$v_{t+1} = \rho_v v_t + \xi_{t+1}, \quad 0 < \rho_v < 1, \quad \xi_{t+1} \sim \text{iid}(0, \sigma_\xi^2), \quad v_0 \text{ given.}$$

In (13), government spending consists of both exogenous and endogenous outlays, where \bar{g} and η are positive constants. Exogenous spending follows a stationary stochastic process subject to serially correlated shocks (v_t) that are revealed to agents at the beginning of period t . Endogenous spending is assumed to depend linearly on the difference in income between households of each type, where income is measured by funds available for consumption after taxes are paid and transfer payments are received.²⁰ The parameter η is the semi-elasticity of government spending with respect to income inequality. The linear form simplifies computations, but is not crucial for any results. For all parameter values examined, endogenous spending is positive. As a further simplification, households derive no direct utility from government spending; it is simply a drain on productive resources.

The vector $\mathcal{P}_t = \{\tau_{ht}, \tau_{kt}, \mu_t\}$ summarizes government policy implemented at time t . The interest rate on government bonds (r_{bt}) is not an independent policy variable in this model. This restriction, combined with the assumption that debt is indexed against inflation, is necessary to pin down a unique policy in equilibrium. As Zhu (1992) and Chari, Christiano, and Kehoe (1991) have shown, allowing

Oudiz and Sachs (1985) examine a structural macroeconomic model with a myopic government. Arrow and Kurz (1970), Atkinson and Sandmo (1980), and Stiglitz (1987) consider the opposite case where government is *less* myopic than households.

¹⁹One version of a social welfare function is $SWF(\cdot) = \gamma \Sigma(\beta^P)^t U_t^P + (1 - \gamma) \Sigma(\beta^R)^t U_t^R$. The government's objective here is $\Sigma(\beta^R)^t (\phi U_t^P + U_t^R)$, which allows for "imperfect altruism." Perfect altruism would imply $\phi = [\gamma / (1 - \gamma)] (\beta^P / \beta^R)^t$. When $\beta^P < \beta^R$, the value of ϕ defined in this way approaches zero as t increases. By restricting $\phi = \text{constant} > 0$, the government's objective represents a transformation of $SWF(\cdot)$.

²⁰Strictly speaking, the funds available for consumption in (13) should also include the after-tax interest payments on government debt held by rich households. For the levels of government debt examined in this model, the additional income is small and is neglected to avoid complication.

the government to choose τ_{kt} and r_{bt} independently results in a fundamental indeterminacy between the equilibrium values of these two variables. To see why, note that the household first-order conditions for k_{t+1} and b_{t+1} in (11) represent an ex ante arbitrage condition on expected returns from bonds and capital. Ex post, after shocks to the economy are revealed, the government is free to alter the combination of τ_{kt} and r_{bt} in many different ways to raise necessary revenue yet still satisfy ex ante arbitrage. A unique policy can be pinned down by imposing restrictions that reduce the degrees of freedom in setting effective bond returns. With indexed bonds, the government is prevented from using state-contingent inflation to manipulate the rate of return. One more degree of freedom can be removed by requiring the arbitrage condition to hold ex post as well as ex ante. The interest rate on government bonds is thus determined by $r_{bt} = (1 - \tau_{kt})(r_t - \delta)$. It must be pointed out, however, that since other restrictions are possible, the model alone cannot pin down a unique prediction for the time-series behavior of τ_{kt} .²¹ Ex post arbitrage imposes "certainty equivalence" on the government's use of debt. Certainty equivalence is also exploited in computing a solution to the model, because the method involves a linear-quadratic approximation of the government's decision problem.

The summation of the household budget constraints and the government budget constraint yields the following resource constraint for the economy. Note that the resource constraint and the government budget constraint are not independent equations. To simplify the formulation, the resource constraint is used in place of the government budget constraint in the recursive version of the problem.

$$\gamma \left[c_{1t}^P + c_{2t}^P \right] + (1 - \gamma) \left[c_{1t}^R + c_{2t}^R + x_t \right] + g_t = Y_t. \quad (14)$$

2.5 Recursive Formulation of the Problem

The government's problem under commitment can be solved using the unique recursive algorithm developed by Kydland and Prescott (1980). Standard recursive methods cannot be used

²¹Other restrictions might be a period-by-period balanced budget constraint (so that government debt is not a state variable) or the assumption that either r_{bt} or τ_{kt} is not state contingent (the equilibrium value is not a function of state variables).

because the problem does not satisfy Bellman's principle of optimality.²² Specifically, households' optimum decision rules, which must be incorporated into the return function $W(\cdot)$, depend not only on current policy, but also on the anticipated sequence of future policies. The value of the return function $W(\cdot)$ at time t is thus dependent on future policy variables τ_{h+j} , τ_{k+j} , and μ_{t+j} for $j>0$. This influence of future policy on current returns destroys the recursivity of the problem. However, the method of Kydland and Prescott allows the problem to be redefined in a way that recovers a recursive structure. The crucial aspect involves defining the lagged shadow prices λ_{t-1}^P and λ_{t-1}^R to be pseudo state variables. Including these prices in the state vector provides a link to the past by which the policymaker at time t takes into account the fact that household decisions in earlier periods depend on current policy by means of expectations. This link to the past is crucial in order to solve the commitment problem using dynamic programming. In a no-commitment regime, the policymaker at time t ignores the effect of current policy on household decisions in earlier periods.²³

To reformulate (12), we first substitute the household first-order conditions in (11) into the transformed household budget constraints, as seen in the Lagrangians. The substitution eliminates τ_h , τ_k , μ_t , c_2^P , and c_2^R and yields the following set of equations:²⁴

$$A h_t^P + \lambda_t^P TR_t - \alpha \beta^P - (1-\alpha) = 0. \quad (15)$$

$$A h_t^R + \frac{\lambda_{t-1}^R}{\beta^R} (k_t + b_t) - \lambda_t^R (k_{t+1} + b_{t+1}) - \alpha \beta^R - (1-\alpha) + u_t^R = 0, \quad E_{t-1} u_t^R = 0. \quad (16)$$

²²Bellman (1957), pp. 81-83. Bellman defines a recursive problem as one in which the optimal decision rules depend only on current-period state variables.

²³In game theoretic terms, the government's optimal strategy under commitment is "memory based," where λ_{t-1}^P and λ_{t-1}^R summarize the history of the game. Under no commitment, there are potentially many equilibrium strategies, including "memoryless" strategies that are functions only of current-period state variables z_t , v_t , k_t , and b_t . Oudiz and Sachs (1985) provide an excellent summary of these equilibrium concepts in the context of a structural macroeconomic model.

²⁴Due to the presence of the expectation operator in the first-order conditions for k_{t+1} and b_{t+1} , the substitution has been accomplished using the expression $E_{t-1} f_t(\cdot) = f_t(\cdot) - u_t$, where $f_t(\cdot)$ is a function of random variables and u_t is the forecast error. The assumption of rational expectations implies $E_{t-1} u_t = 0$.

The household first-order conditions can also be used to eliminate τ_h and τ_k from the expression for income inequality in the feedback rule for government spending. The result is

$$g_t = \bar{g} \exp(v_t) + \eta \left[\frac{\lambda_{t-1}^R}{\beta^R \lambda_t^R} k_t - (1-\delta)k_t + \frac{Ah_t^R}{\lambda_t^R} - \frac{Ah_t^P}{\lambda_t^P} - TR_t \right] + \frac{u_t^G}{\lambda_t^R}, \quad E_{t-1} u_t^G = 0. \quad (17)$$

The first-order condition for \hat{m}_{t+1} can be used to obtain a useful relation between \hat{m}_t^R and \hat{m}_t^P :

$$\hat{m}_t^R = \hat{m}_t^P \frac{\lambda_{t-1}^P}{\lambda_{t-1}^R} \frac{\beta^R}{\beta^P}. \quad (18)$$

Finally, c_{1t}^i and c_{2t}^i ($i=P, R$) are eliminated from both the return function $W(\cdot)$ and the resource constraint (14) using the cash-in-advance constraints and the first-order condition for c_{2t}^i . The vector of state variables for the government's problem is $s_t = \{z_t, v_t, k_t, b_t, \lambda_{t-1}^P, \lambda_{t-1}^R\}$. In the transformed problem, the government's decision variables are $\hat{m}_{t+1}^i, h_t^i, \lambda_t^i, k_{t+1}, b_{t+1}$, $i=P, R$. Using primes ($'$) to denote next-period quantities, the recursive version of the government's problem is shown in (19).

The Bellman equation in (19) summarizes the recursive nature of the problem. The first line of constraints lists the cash-in-advance constraints and the first-order conditions for credit goods. The second line is the relationship between household money stocks from (18) and the transfer payment constraint. The next three lines are the household budget constraints and the resource constraint. The remaining constraints define the production technology and the laws of motion for g, k, z , and v .

The dynamic programming problem defined in (19) applies for all $t > 0$. The problem at $t=0$ must be considered separately, as shown by Kydland and Prescott (1980), Lucas and Stokey (1983), and Chamley (1986). At $t=0$, the stocks of capital, bonds, and money are fixed. Optimal policy thus implies very high values for the initial tax rate on capital and the initial money growth rate, to take full advantage of nondistortionary sources of revenue. I assume that this form of lump-sum taxation is insufficient to finance the entire stream of future expenditures.

$$V(s) = \max_{\substack{\hat{m}^i, k', b' \\ h^i, \lambda^i, i=P,R}} E_{-1} \left\{ W(c_1^P, c_2^P, h^P, c_1^R, c_2^R, h^R) + \beta^G [V(s') | s] \right\},$$

where: $s = \{z, v, k, b, \lambda_{-1}^P, \lambda_{-1}^R\}$

$$W(\cdot) = \phi U^P(c_1^P, c_2^P, h^P) + U^R(c_1^R, c_2^R, h^R)$$

subject to:

$$c_1^i = \frac{\hat{m}^i}{\hat{P}} \quad c_2^i = \frac{1-\alpha}{\lambda^i}, \quad i = P, R$$

$$\hat{m}^R = \hat{m}^P \frac{\lambda_{-1}^R \beta^R}{\lambda_{-1}^P \beta^P}, \quad TR = \bar{TR}$$

$$Ah^P + \lambda^P TR - \alpha \beta^P - (1-\alpha) = 0$$

$$Ah^R + \frac{\lambda_{-1}^R}{\beta^R} (k+b) - \lambda^R (k'+b') - \alpha \beta^R - (1-\alpha) + u^R = 0$$

$$\gamma [c_1^P + c_2^P] + (1-\gamma) [c_1^R + c_2^R + x] + g = Y \quad (19)$$

$$Y = \exp(z_t) [(1-\gamma)k]^\theta [\gamma \hat{e} h^P + (1-\gamma) h^R]^{1-\theta}$$

$$g = \bar{g} \exp(v_t) + \eta \left[\frac{\lambda_{-1}^R}{\beta^R \lambda^R} k + (1-\delta)k + \frac{Ah^R}{\lambda^R} - \frac{Ah^P}{\lambda^P} - TR \right] + \frac{u^G}{\lambda^R}$$

$$k' = (1-\delta)k + x$$

$$z' = \rho_z z + \varepsilon'$$

$$v' = \rho_v v + \xi'$$

The analysis here will focus on policy in stationary stochastic equilibrium, i.e., when t is very large. The linear-quadratic approximation method used to solve (19) is accurate only in the neighborhood of the deterministic steady state. Consequently, I do not solve the $t=0$ problem or compute the transition path to the stationary equilibrium. One complication that arises with this approach is that the steady-state level of government debt cannot be determined solely on the basis of steady-state analysis. Rather, steady-state debt is a function of both the initial level of debt, b_0 , and the entire transition path of taxes

and spending from $t=0$ until the steady state is reached. As an alternative to performing this difficult computation, I simply choose the level of steady-state debt to reflect a debt-to-GNP ratio consistent with U.S. data. I assume that b_0 and the transition path are set such that the government budget constraint is satisfied in present-value terms.²⁵

Kydland and Prescott (1980) prove the existence of a stationary equilibrium in a representative household version of the Ramsey problem. Proving existence and uniqueness of a stationary equilibrium in this model is difficult due to the borrowing constraints imposed on poor households. Instead, I simply assume that some (unspecified) institutional mechanism prohibits the use of time-varying policy rules. Equilibrium is defined as a value function $V(s)$ and an associated set of stationary decision rules that satisfy (19). The decision rules dictate a set of household allocations and prices at time t that can be implemented by means of the government's chosen policy. The government's explicit policy rules for tax rates and money growth can be recovered by substituting the implementable allocations and prices into the household first-order conditions and budget constraints and by imposing $r_{bt} = (1 - \tau_{kt})(r_t - \delta)$.

3. Computation Procedure

The dynamic programming problem in (19) is solved numerically using a variant of the linear-quadratic approximation technique first used by Kydland and Prescott (1982). An approximate version of (19) is obtained by first substituting all nonlinear constraints into the government's objective function $W(\cdot)$ and then forming a quadratic approximation of the resulting expression in terms of the *logarithms* of all variables.²⁶ The solution algorithm exploits the certainty equivalence property of linear-quadratic control problems. The optimal decision rules for the approximated economy can be obtained by solving the deterministic version of the model.²⁷ An initial guess V_0 is made for the optimal value function $V(s)$

²⁵The indeterminacy of steady-state debt is discussed by Chamley (1985b). Auerbach and Kotlikoff (1987) show how the steady-state level of debt can be computed by explicitly modeling the transition path in a life-cycle model with no uncertainty.

²⁶The log-linear version of the Kydland-Prescott method is described in Christiano (1988). A step-by-step guide to linear-quadratic solution algorithms can be found in Hansen and Prescott (1991).

²⁷See Sargent (1987), p. 36. Specifically, the stochastic terms ϵ_t , ξ_t , u_t^R , and u_t^G are set equal to their unconditional means (zero) in the numerical algorithm. With a quadratic objective, the first-order conditions are linear in all variables. This allows the expectation operator in (19) to be passed through the expressions, dropping out any stochastic terms.

in the quadratic version of (19). Sequential candidate value functions V_i are then computed by successively iterating on the Bellman equation until the value function has converged, i.e., until V_i is sufficiently close to V_{i+1} . Once the process has converged, log-linear decision rules that dictate household equilibrium allocations are computed. Log-linear policy rules for τ_{hr} , τ_{kr} , and μ_t can then be computed using the household first-order conditions in (11) and the household budget constraints, log-linearized around the steady state. To improve accuracy during the simulations, the nonlinear versions of the policy rules are used in computing period-by-period values for the policy variables.

4. Calibration of the Model

To explore the quantitative predictions of the model, as many parameters as possible are assigned values in advance based on empirically observed features of postwar U.S. data. Parameter choices are also guided by the desire to obtain steady-state values for key model variables that are consistent with postwar averages in the U.S. economy. For parameters that are difficult to pin down, such as η , ϕ , and \bar{TR} , a range of values is examined. Table 1 summarizes the baseline parameter values and is followed by a brief explanation of how they were selected.

Table 1: Baseline Parameter Set

| Agent | Parameters and Values | | | |
|------------|-------------------------------------------------------------------------|-----------------|---------------------------|-------------------------------------------------|
| Households | $\gamma = 0.80$ | $\alpha = 0.6$ | $A = 2.9$ | $\beta^R = \beta^G = 0.99$ $\beta^P = 0.985$ |
| Firms | $\theta = 0.36$ $\hat{\epsilon} = 0.75$ | $\rho_z = 0.95$ | $\sigma_\epsilon = 0.007$ | $\delta = 0.025$ |
| Government | $\gamma \bar{TR}/Y = 0.02$ $g/Y = 0.22$ $(1 - \gamma) b/Y = 0.25$ | $\rho_v = 0.95$ | $\sigma_\xi = 0.02$ | $\eta = 0.04$ $\phi = 2.7$ |

The relative number of poor households (γ) determines the distribution of wealth and income in the model. Using data from the 1983 *Survey of Consumer Finance*, McDermed, Clark, and Allen (1989) estimate a Lorenz curve that summarizes the highly skewed distribution of wealth in the U.S.

economy. Their results indicate that the richest 20 percent of households own 80 percent of U.S. wealth (based on net worth). Here, poor households have no wealth. The choice $\gamma=0.80$ implies that the richest 20 percent of households in the model own 100 percent of the wealth, a distribution that roughly approximates the empirical Lorenz curve. For the model, it can be shown that the analytical Gini coefficient (based on wealth) is equal to γ . The value $\gamma=0.80$ is very close to empirical estimates of wealth-based Gini coefficients for the U.S. economy. Using data on household net worth, Wolff and Marley (1989) report Gini coefficients of 0.772 for 1962 and 0.788 for 1983.²⁸

The parameter α determines the relative importance of cash versus credit goods in the household utility function. Empirical estimates of this parameter vary, depending on the choice of monetary aggregate and the sample period. Cooley and Hansen (1992) estimate a value of $\alpha=0.84$. Using a somewhat different utility function, Chari, Christiano, and Kehoe (1991) estimate the relative weight on cash goods to be 0.43. The chosen value of 0.6 lies about midway between the two estimates. It turns out that higher values of α induce the government to choose higher rates of inflation in equilibrium.

The parameter A is picked to yield an economywide average number of hours worked close to 0.3. This is consistent with time-use studies, such as Juster and Stafford (1991), which indicate that households spend approximately one-third of their discretionary time in market work.²⁹

The time period in the model is taken to be one quarter. With quarterly time periods, the common discount factor for rich households and the government is set at $\beta^R=\beta^G=0.99$. This value implies an *annual* rate of time preference equal to 4 percent. The discount factor for poor households is set at $\beta^P=0.985$, which implies an annual time preference rate of 6.2 percent. Engen (1992) estimates annual rates of time preference in the range of 4 to 7.9 percent, while Lawrance (1991) estimates rates in the range of 0 to 19 percent. In Lawrance's study (tables 3 and 5), time preference rates of households with below-median incomes are 2 to 5 percentage points *higher* than those with above-median incomes.

²⁸The Gini coefficient is a measure of inequality that ranges between zero and one. A value of zero implies no inequality among households. A value of one implies all wealth (or income) accrues to a single household. The coefficient can be computed by taking twice the area between the Lorenz curve and the 45 degree diagonal.

²⁹In the model, poor households spend a larger fraction of their time working than do rich households, $h^P > h^R$. This is because the rich derive a substantial portion of their income from capital and earn a higher hourly wage.

Households without college educations have time preference rates about 2 percentage points higher than those with college educations. The chosen values for β^P and β^R imply time preference rates for the poor that are about 2 percentage points higher than rates for the rich.

The share of output that represents payments to capital (θ) is set at 0.36, about midway in the range of 0.25 to 0.43 estimated by Christiano (1988). The quarterly depreciation rate of $\delta=0.025$ is commonly used and, together with β^R and θ , yields a realistic steady-state ratio of economywide capital to output of 8.5 and a ratio of total investment to output of 0.21. The process governing technology shocks has been estimated by Prescott (1986). The parameters governing the shocks, $\rho_z=0.95$ and $\sigma_\epsilon=0.007$, represent values commonly used in real business cycle models.

Empirical estimates of the labor efficiency parameter (\hat{e}) as a function of savings behavior or wealth are not available. Estimates are available, however, as a function of age and education. Using panel data on labor earnings, Engen (1992) estimates \hat{e} as a quadratic function of age over an individual's lifetime for various education levels. Three-fourths of the sample consumers have no college education. The ratio of the average lifetime \hat{e} for individuals with no college education to those with a college education is about 0.75. If the non-saving, poor households in this model are viewed as representing individuals with no college education, then the empirical evidence would suggest a value of $\hat{e}=0.75$. The values of \hat{e} and γ affect the skewness of the *income* distribution in the model. As an additional calibration source, the distribution of income in the model can be compared to the U.S. economy. Rich households in the model earn 42 percent of total income (before taxes and transfers). This figure coincides with the average share earned by the top fifth of U.S. households from 1947 to 1989. The model's income-based Gini coefficient is 0.22, a value somewhat lower than the average value of 0.37 for the postwar U.S. economy.³⁰

The semi-elasticity parameter η controls the degree to which government spending responds to income inequality. It turns out that the value of η (together with β^R and γ) determines the steady-state level of τ_k . Given the values for β^R and γ described above, η is set to yield $\tau_k=0.41$. This tax rate is about midway in the range of estimates for the average marginal tax rate on capital in the U.S. economy.

³⁰See Rosen (1992), table 8.1, and *Economic Report of the President, 1992*, chapter 4, chart 4-4.

With $\eta=0.04$, the steady-state level of endogenous government spending is equal to 4.5 percent of GNP. To put this number in perspective, federal and state aid to local governments averaged 3.6 percent of GNP from 1950 to 1988 (see Rosen [1992], table 21.4). A range of values for η is also investigated.

The law of motion for exogenous spending, $\bar{g} \exp(v_t)$, is designed to mimic the quarterly time series of government purchases of goods and services in the U.S. economy. Data on *total* government purchases were used in the estimation because it is not possible to isolate and exclude that portion driven by income inequality. Exogenous spending accounts for about 80 percent of g_t in the model, however. The value of \bar{g} is set to yield a steady-state ratio of total government purchases to GNP of 0.22, the postwar U.S. average. The parameters ρ , and σ_ξ govern the behavior of the exogenous spending shocks.³¹

The value of \bar{TR} is set to approximate the average ratio of transfer payments to GNP in the U.S. economy. This ratio varies, depending on the type of payments included in the definition. Means-tested transfer payments (including in-kind transfers) increased from 1.2 percent of GNP in 1965 to 3.6 percent in 1988. If social insurance programs (Social Security, Medicare, and unemployment insurance) are included in the definition, the average level of transfer payments from 1950 to 1990 increases to more than 6 percent of GNP. It turns out that the value of \bar{TR} significantly affects the government's equilibrium choice of money growth. Therefore, the steady-state ratio of transfer payments to GNP ($=\gamma\bar{TR}/Y$) is set at 0.02, and a range of values is investigated. The steady-state ratio of government debt to GNP is set at 0.25. This value is at the lower end of the range of net federal debt as a share of GNP since 1950. The basic results are not significantly affected by the level of steady-state debt.³²

The parameter ϕ controls how much the government favors one group relative to the other and thus significantly affects the progressivity of equilibrium labor taxes. I choose ϕ such that the revenue-weighted average of τ_r across all households is close to estimates for the U.S. economy. The baseline

³¹The law of motion for exogenous government spending is equivalent to the following AR(1) specification: $\ln g_t = (1-\rho)\ln \bar{g} + \rho \ln g_{t-1} + \xi_t$. Using this form, Christiano and Eichenbaum (1992) estimate $\rho=0.96$ and $\sigma_\xi=0.02$.

³²Data on government purchases and total transfer payments are from Citibase. Data on net federal debt to GNP are from *Federal Debt and Interest Costs*, Congressional Budget Office (1993). Data on means-tested transfers are from Rosen (1992) and *Economic Report of the President, 1992*, chapter 4.

value is set at $\phi=2.7$, and a range of values is investigated.³³

5. Quantitative Properties of the Model

5.1 Steady-State Experiments

Figures 1-4 show the effect on steady-state policy of varying four key parameters in the model, namely, η , ϕ , $\bar{T}R$, and \bar{g} . In each figure, only a single parameter is varied, with remaining parameters set at the baseline values in table 1.

Figure 1 shows the effect of varying η , which controls the sensitivity of government spending to income inequality. An analytical expression for the optimal steady-state tax on capital as a function of η can be derived and is shown below.

$$\tau_k = 1 - \frac{\rho}{\rho + \eta \frac{\rho + \delta}{1 - \gamma}}, \quad \text{where } \rho = \frac{1}{\beta^R} - 1 \quad (\beta^R = \beta^G). \quad (20)$$

Equation (20) is derived by combining the government's first-order condition for k_{t+1} with the corresponding household first-order condition in (11) and making use of the assumption $\beta^R = \beta^G$. When $\eta=0$, the result is $\tau_k=0$. Notice that the steady-state tax on capital is not affected by ϕ , the weight placed on the poor's welfare in the government objective function. These results agree with those proved in Judd (1985) and Chamley (1986) in models with no externalities. From (20), we see that $\partial\tau_k/\partial\eta > 0$ and $\partial\tau_k/\partial\gamma > 0$. Higher levels of capital accumulation accentuate income inequality. This effect imposes a negative externality on the economy (as determined by η) in the form of higher public spending because the additional spending must be financed by distortionary taxation. Positive values of τ_k force rich households to help pay for this externality. An increase in η also tends to reinforce the progressivity of labor taxes. As the number of poor households (γ) increases, the income distribution becomes more skewed. This increase in inequality causes more spending, calling for higher levels of τ_k . Figure 1 shows that the amount of endogenous spending necessary to induce high levels of τ_k is relatively small,

³³With $\phi=2.7$, the government places less weight on the within-period utility of the poor than is implied by their relative number. With $\gamma=0.80$ and $1-\gamma=0.20$, there are four times as many poor households as rich. Here, the government places only 2.7 times as much weight on the poor's within-period utility. This behavior might be justified either as a way of compensating for $\beta^G > \beta^P$ or as a reflection of lower voting rates among the poor. The model abstracts from an explicit description of political equilibrium, however.

about 4 to 5 percent of GNP.

Figure 2 shows the effect of varying the political weighting factor ϕ . When $\phi=0$, the government's optimal policy calls for highly *regressive* labor taxes. This is because the government views the poor's labor supply as completely inelastic and thus it imposes a very high tax on this activity, in accordance with Ramsey's principle of optimal taxation. As ϕ increases, labor taxes become more progressive. Due to the diminishing marginal utility property of $U^i(\cdot)$, the government perceives more benefits from a dollar in the hands of the poor than a dollar in the hands of the rich.

Figure 3 shows that the quarterly money growth rate (which equals the quarterly inflation rate in steady state) increases rapidly with the level of required transfer payments \bar{TR} . In a standard cash-in-advance model with utility functions of the form used here and no externalities, optimal money growth adheres to the Friedman rule.³⁴ In this model, transfer payments represent a negative externality for the government because they are not taxed but must be financed by distortionary taxation. This drives a wedge between the government's marginal utility of consumption and that of households. Moreover, transfer payments induce the poor to work less and cause their labor supply to become more elastic, thus increasing the distortionary costs of labor taxation. To spread out distortionary costs across tax bases, the government levies a tax on consumption in the form of inflation. In a representative household version of the model, with $\gamma=1$ and $\beta^G=\beta^P=\beta$, it is possible to derive the following steady-state expression for optimal money growth:

$$\mu = \beta - 1 + \frac{\Lambda_1 \beta \bar{TR}}{1-\alpha}, \quad \text{where } \Lambda_1 = \frac{\partial W(\cdot)}{\partial c_2} > 0. \quad (21)$$

From (21), when $\bar{TR}=0$ the result is $\mu=\beta-1$ (the Friedman rule). The term $\Lambda_1 > 0$ is the Lagrange multiplier on the household budget constraint in the government's first-order conditions. This represents the perceived benefit to the government of increasing private consumption by one unit. When $\bar{TR} > 0$, optimal monetary policy calls for a positive nominal interest rate in steady state. The government's inability to tax transfers directly motivates the imposition of a tax through the back door, by raising the

³⁴Recall that the Friedman rule in steady state is $\mu=\beta-1$. For further discussion, see the references cited in footnote 8.

nominal price of consumption.³⁵ Equation (21) illustrates the well-known fact that standard optimality results may not go through in the presence of externalities. At the baseline level of transfers, quarterly money growth is $\mu=0.014$, implying an optimal inflation rate of about 6 percent per year.

Figure 4 shows the effect of increasing the steady-state ratio of government purchases to GNP. The ratio is varied by increasing \bar{g} , which controls the level of *exogenous* purchases. As g/Y increases, tax rates on labor increase in a linear fashion. It is efficient for the government to finance long-run (steady-state) increases in g with labor taxes because the long-run elasticity of labor supply is less than the long-run elasticities of capital or money balances. As labor tax rates approach 0.60, the money growth rate accelerates dramatically. At this point, maximum revenue is being collected from labor taxes. As required spending continues to go up, the government is forced to rely more heavily on seigniorage. Revenues from seigniorage are limited by households' willingness to hold money balances, as measured by the parameter α . From (21), higher values of α result in higher money growth rates.

As a final steady-state experiment, table 2 compares revenues collected from various sources in the model and in the postwar U.S. economy. Model results are for the baseline parameters, and all revenues are normalized by GNP. The labor tax is the largest source of revenue. The capital tax provides significantly less revenue than the labor tax, even though the tax rate on capital is higher in the model. This is due to the depreciation allowance. Finally, seigniorage is the smallest source of revenue. The relative sizes of revenue compare remarkably well with the U.S. averages. However, revenue sources in the data do not always fit neatly into one of the three categories.

Table 2: Revenue/GNP from Different Sources

| Source of Revenue | Model | U.S. Economy ^a |
|--------------------|--------|---------------------------|
| Labor Income Tax | 0.177 | 0.159 |
| Capital Income Tax | 0.062 | 0.067 |
| Seigniorage | 0.0046 | 0.0035 |

^aTax revenues are average values from various issues of *Revenue Statistics of OECD Member Countries, 1965-1990*, table 61. Labor tax revenue is defined to include federal and state individual income taxes and Social Security taxes. Capital tax revenue is defined to include federal and state corporate taxes, capital gains taxes, and property taxes. Seigniorage is from Neumann (1992) for 1951-90, defined as $(M_t - M_{t-1})/P_t$, where M_t is the monetary base.

³⁵This interpretation is based on a discussion of transfers in Jones, Manuelli, and Rossi (1992), p. 36.

5.2 Dynamic Experiments

5.2.1 Optimal Policy Rules

The solution to the approximate version of (19) yields the following set of log-linear optimal policy rules, which are valid in the neighborhood of the deterministic steady state.

Table 3: Optimal Policy Rules

| | Constant | z_t | v_t | $\ln(k_t)$ | $\ln(b_t)$ | $\ln(\lambda_{t-1}^P)$ | $\ln(\lambda_{t-1}^R)$ |
|-----------------|----------|--------|--------|------------|------------|------------------------|------------------------|
| $\tau_{ht}^P =$ | -1.716 | -0.675 | -0.083 | 0.636 | 0.015 | 0.000 | 0.470 |
| $\tau_{ht}^R =$ | -1.570 | -0.532 | -0.083 | 0.596 | 0.015 | 0.000 | 0.490 |
| $\tau_{kt} =$ | 2.715 | -3.218 | 0.864 | -0.346 | 1.135 | 0.000 | -1.314 |
| $\mu_t =$ | 0.760 | -0.214 | 0.056 | -0.165 | -0.002 | 0.000 | -0.230 |

Shocks to the government budget are caused by unexpected changes in the size of the tax base or by unexpected increases in exogenous spending requirements. The government's optimal response to these shocks can be seen by examining the coefficients on state variables z_t and v_t . For example, a positive technology shock causes large *decreases* in τ_{ht} and μ_t (in proportion to their steady-state values) relative to τ_{ht} . A positive z_t causes GNP and household incomes (the tax base) to rise, allowing revenue requirements to be met with lower taxes. In contrast, a positive expenditure shock (v_t) calls for an *increase* in τ_{kt} and μ_t to collect additional required revenue. Absorbing shocks in this way is efficient because capital and money balances are completely inelastic within a given period. Judd (1989) and Chari, Christiano, and Kehoe (1991) also obtain shock-absorbing behavior in related models. Notice that the policy rule for μ_t reflects the notion of countercyclical monetary policy, in that money growth moves opposite to output fluctuations. However, the neoclassical framework precludes any role for "stabilization" in the sense of preventing large swings in unemployment over the business cycle.

The shock-absorbing features of τ_{kt} and μ_t allow the government to maintain relatively stable tax rates on labor, reminiscent of the tax-rate-smoothing hypothesis of Barro (1979, 1986). This hypothesis has been the subject of numerous empirical studies designed to test whether tax rates or inflation follows a random walk (or martingale).³⁶ In this model, however, the optimal policy rules show

³⁶See, for example, Sahasakul (1986), Mankiw (1987), and Bizer and Durlauf (1990).

that there is no theoretical prediction that policy variables follow a random walk.³⁷

As a direct test of the model, it would be desirable to compare the policy rules in table 3 with empirical versions estimated with U.S. data. An estimation problem exists, however, because shadow prices λ_{t-1}^p and λ_{t-1}^r are unobservable. Kydland and Prescott (1980) point out that the ratio $\lambda_{t-1}^p/\lambda_t$ could conceivably be estimated from the household's first-order conditions, but this still allows the shadow prices to be scaled in an arbitrary way. Empirical testing of key characteristics of the optimal policy rules is an area for future research.³⁸

5.2.2 Policy Simulations

Figures 5-10 plot simulated policy from the model together with U.S. data on marginal tax rates and money growth. Tables 4 and 5 provide a quantitative comparison of the series. In these tables, the inflation tax rate, defined as $\pi_t = (P_t - P_{t-1})/P_t$, has also been included. Although μ_t is the instrument of monetary policy directly under the government's control, π_t has the advantage of lying between zero and one, analogous to the other tax rates τ_{kt} and τ_{lt} . The two measures of monetary policy are related by $\pi_t = 1 - \hat{P}_{t-1} / [\hat{P}_t (1 + \mu_{t-1})]$, where \hat{P}_t is defined in (8).

The model does reasonably well in capturing the standard deviations and serial correlations of the policy variables (table 4), but is less successful regarding the contemporaneous correlations (tables 5a and 5b). A basic prediction is that the capital tax and the inflation tax should both be more volatile than the labor tax, a feature generally confirmed by the data. The capital tax series estimated by Jorgenson and Sullivan (1981), shown in column 3, has a much higher standard deviation than the series estimated by Joines (1981), shown in column 4. The values are 16.38 percent and 5.09 percent, respectively. The Jorgenson and Sullivan series is an estimate of the effective corporate tax rate, while the Joines series also includes property taxes and taxes paid by individuals on capital gains and dividends. Neither series takes into account the imputed subsidy on investment in residential housing.

³⁷This point was originally made by Chari, Christiano, and Kehoe (1991).

³⁸The coefficients on λ_{t-1}^p are equal to zero in table 2 because poor households do not save. In fact, since λ_{t-1}^p is directly related to λ_{t-1}^r by (18), λ_{t-1}^p could have been eliminated as a state variable.

Other estimates of U.S. tax rates on capital can be found in Auerbach and Poterba (1988), Fullerton and Karayannis (1987), King and Fullerton (1984), Jorgenson and Yun (1989), and Judd (1989).

Table 4: Simulated Policy versus U.S. Economy (All Variables Detrended)

| | Model ^a | U.S. Economy ^b | U.S. Economy ^c | |
|-------------|--------------------|---------------------------|---------------------------|-------|
| τ_{hr} | Mean | 0.278 | 0.272 | 0.222 |
| | Std. Dev. (%) | 0.99 | 5.65 | 4.44 |
| | corr (-1) | 0.49 | 0.43 | 0.38 |
| | corr (-2) | 0.01 | -0.19 | -0.16 |
| | corr (-3) | -0.22 | -0.67 | -0.59 |
| τ_{kr} | Mean | 0.412 | 0.299 | 0.540 |
| | Std. Dev. (%) | 8.41 | 16.38 | 5.09 |
| | corr (-1) | 0.43 | 0.51 | 0.19 |
| | corr (-2) | -0.04 | -0.04 | -0.37 |
| | corr (-3) | -0.21 | -0.24 | -0.37 |
| μ_t | Mean | 0.061 | 0.048 | 0.050 |
| | Std. Dev. (%) | 22.50 | 50.50 | 29.78 |
| | corr (-1) | 0.48 | 0.03 | 0.13 |
| | corr (-2) | -0.01 | -0.30 | -0.21 |
| | corr (-3) | -0.22 | -0.18 | -0.24 |
| π_t | Mean | 0.056 | 0.040 | 0.041 |
| | Std. Dev. (%) | 28.01 | 49.75 | 36.28 |
| | corr (-1) | 0.38 | 0.33 | 0.15 |
| | corr (-2) | -0.05 | -0.38 | -0.11 |
| | corr (-3) | -0.21 | -0.36 | -0.23 |

^aModel statistics are means over 100 simulations, each 124 quarters long. During each simulation, annualized series were constructed using revenue-weighted averages to compute tax rates and end-of-year money stocks and prices to compute μ_t and π_t . The annualized series were then detrended using the Hodrick-Prescott filter with a smoothing parameter of 100.

^bHere, τ_{hr} is from Barro and Sahasakul (1986) for 1947-83, τ_{kr} is from Jorgenson and Sullivan (1981, table 11) for 1947-80, μ_t is based on the M1 series constructed by Rasche (1987) for 1947-89, and π_t is based on the CPI (all items) from Citibase for 1947-89. Data for μ_t and π_t were annualized as in the model, and all variables were detrended using the Hodrick-Prescott filter.

^cHere, τ_{hr} and τ_{kr} are from Joines (1981, tables 2 and 10) for 1947-75, where τ_{hr} is "MTRL4" and τ_{kr} is "MTRK." Data for μ_t are from the monetary base series in Citibase for 1947-89, and π_t is based on the GNP deflator for 1947-89, also from Citibase.

Table 5a: Contemporaneous Correlation in Model^a

| | τ_{hr} | τ_{kr} | μ_r | π_r | Y_r | g_r |
|-------------|-------------|-------------|---------|---------|-------|-------|
| τ_{hr} | 1.00 | -0.53 | -0.47 | -0.59 | 0.91 | 0.68 |
| τ_{kr} | | 1.00 | 0.83 | 0.90 | -0.66 | 0.17 |
| μ_r | | | 1.00 | 0.94 | -0.60 | 0.18 |
| π_r | | | | 1.00 | -0.68 | 0.07 |

^aModel statistics are means over 100 simulations, where all series have been annualized and detrended as in table 4.

Table 5b: Contemporaneous Correlation in U.S. Economy^a

| | τ_{hr} | τ_{kr} | μ_r | π_r | Y_r | g_r |
|-------------|-------------|-------------|---------|---------|--------|--------|
| τ_{hr} | 1.00 | 0.42 | 0.19 | 0.39 | 0.39 * | 0.51 * |
| τ_{kr} | 1.00 | 0.71 | 0.32 | 0.27 | 0.16 * | 0.28 * |
| μ_r | | 1.00 | -0.20 | 0.41 * | 0.12 | 0.31 * |
| π_r | | 1.00 | 0.48 * | 0.43 * | 0.45 | 0.33 * |
| Y_r | | | 1.00 | 0.00 | 0.27 | 0.23 * |
| g_r | | | 1.00 | 0.12 * | 0.40 | 0.43 * |
| π_r | | | | 1.00 | 0.31 | -0.02 |
| Y_r | | | | 1.00 | 0.12 | -0.21 |

* An asterisk indicates that the correlation coefficient has the same sign as in the model. The top and bottom numbers in each cell represent correlations using the U.S. variables described in footnotes a and b, respectively, of table 4. The U.S. series were each annualized and detrended over periods for which a full set of variables was available. For the top numbers, this period was 1947-80. For the bottom numbers, the period was 1947-75.

Also from table 4, we see that the tax rate on labor in the model has a much lower standard deviation than either U.S. series (0.99 percent versus 5.65 or 4.44 percent). Money growth and the inflation tax both display very high standard deviations (more than 20 percent). Comparisons with the data are slightly more favorable for the monetary base series (as opposed to M1) and the GNP deflator series (as opposed to the CPI index). In a related model, Chari, Christiano, and Kehoe (1991) report a much higher standard deviation for simulated money growth than the value shown here. However, their model includes *nominal* government debt, and the inflation tax is the only available shock absorber.

The correlation coefficients in the model match the signs in U.S. data for about half the cases in table 5. The model generally predicts strong correlations among the variables, while many U.S. correlations (which are based on only 29 to 34 observations) are quite weak and can even vary in sign,

depending on the source. All variables have been detrended in table 5 because the U.S. labor tax and money growth rate both display upward trends, while the U.S. capital tax displays a slight downward trend (see figures 5, 7, and 9). These trends have no counterpart in the model because the ratio of government outlays to GNP is stationary. In U.S. data, the ratio of outlays to GNP has risen over time, mainly due to the rapid growth in transfer payments. From figures 3 and 4, the model predicts that higher steady-state outlays relative to GNP should be accompanied by increases in the labor tax and the money growth rate.³⁹

Another basic prediction of the model is that the labor tax should be *negatively* correlated with inflation (and money growth), while the correlation between the capital tax and inflation should be *positive*. Partial-equilibrium models generally do not distinguish between labor and capital taxes and predict a positive correlation between a single "income tax" and inflation. This failure to distinguish between factor incomes may help to explain the conflicting findings of previous U.S. and cross-country empirical studies designed to test for the partial-equilibrium result (see Mankiw [1987], Roubini and Sachs [1989], Poterba and Rotemberg [1990] and Roubini [1991]).

As a final check of the model's dynamic behavior, tables 6 and 7 summarize predictions for key business cycle statistics. Table 6 shows the corresponding statistics from Cooley and Hansen (1989), who study a cash-in-advance model with no distortionary taxes and *exogenous* stochastic money growth. The model statistics are virtually identical to the Cooley-Hansen results. Table 7 displays the model predictions for two labor market statistics that have received particular attention in recent real business cycle literature, namely 1) the volatility of hours worked relative to labor productivity, $\sigma_H/\sigma_{Y/H}$, and 2) the contemporaneous correlation between hours and productivity, $\text{corr}(H, Y/H)$. The model results are comparable to those obtained by Christiano and Eichenbaum (1992) and Hansen and Wright (1992) in models without money or distorting taxes. These results are encouraging because they suggest that monetary real business cycle models can be extended into new areas, such as policy analysis or perhaps even forecasting, without sacrificing a reasonable description of the aggregate economy.

³⁹In the United States, the upward trend in τ_{lr} is possibly linked to the trend in μ , by the phenomenon of "bracket creep," which existed before tax schedules were indexed for inflation in 1985. Regarding the capital tax τ_{kr} , Auerbach and Poterba (1988) argue that the downward trend is due to increasingly generous investment tax credits and accelerated depreciation schedules.

Table 6: Business Cycle Statistics for Models and U.S. Economy

| Series | U.S. Economy ^a | Standard Deviation in Percent | |
|---------------|---------------------------|-------------------------------|----------------------------|
| | | Model ^b | Cooley-Hansen ^c |
| Output | 1.74 | 1.74 | 1.73 |
| Consumption | 0.81 | 0.61 | 0.62 |
| Investment | 8.45 | 5.79 | 5.69 |
| Capital Stock | 0.38 | 0.49 | 0.48 |
| Hours Worked | 1.41 | 1.24 | 1.33 |
| Productivity | 0.89 | 0.64 | 0.50 |
| Price Level | (CPI) 1.59 (GNP) 0.98 | 1.35 | 1.70 |

| Series | U.S. Economy | Contemporaneous Correlation with Output | |
|---------------|----------------------------|-----------------------------------------|---------------|
| | | Model | Cooley-Hansen |
| Consumption | 0.65 | 0.71 | 0.72 |
| Investment | 0.91 | 0.97 | 0.97 |
| Capital Stock | 0.28 | 0.03 | 0.06 |
| Hours Worked | 0.86 | 0.96 | 0.98 |
| Productivity | 0.59 | 0.84 | 0.87 |
| Price Level | (CPI) -0.48 (GNP) -0.53 | -0.27 | -0.27 |

^aThe U.S. statistics are from table 1 of Cooley and Hansen (1989) for the period 1955:IIIQ to 1984:IQ (115 quarters).

^bModel statistics are mean values over 100 simulations, each 115 quarters in length. All variables were logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. The value $\sigma_\varepsilon=0.0077$ was used for the technology shock to achieve a standard deviation of output equal to 1.74. Productivity is defined as output/hours.

^cStatistics are from Cooley and Hansen (1989), table 1, with quarterly money growth of 0.015 and $\sigma_\varepsilon=0.00721$.

Table 7: Comparison of Labor Market Statistics

| Statistic | U.S. Economy ^a | Model ^b | Christiano-Eichenbaum ^c | Hansen-Wright ^d |
|-------------------------|---------------------------|--------------------|------------------------------------|----------------------------|
| $\sigma_H/\sigma_{Y/H}$ | 1.37 2.15 | 1.95 | 1.44 | 1.92 |
| $\text{corr}(H, Y/H)$ | 0.07 -0.14 | 0.64 | 0.58 | 0.49 |

^aThe U.S. statistics are from Hansen and Wright (1992), table 2, for the period 1947:IQ to 1991:IIIQ (179 quarters). The top and bottom numbers refer to the household and establishment surveys, respectively.

^bModel statistics are means over 100 simulations, each 179 quarters in length, with $\sigma_\varepsilon=0.007$.

^cChristiano and Eichenbaum (1992), table 4, with government consumption, indivisible labor, and $\sigma_\varepsilon=0.012$.

^dHansen and Wright (1992), table 3, with home production and $\sigma_\varepsilon=0.007$.

6. Concluding Remarks

The goal of any quantitative model of the economy should be to capture the basic incentives and interactions among agents that govern the process of interest. In the case of government policy, it is clear that real-world policymakers are fundamentally concerned with distribution issues. Monetary real business cycle models have been reasonably successful in describing the behavior of aggregate fluctuations. This paper uses such a framework as the starting point for endogenizing the choice of fiscal and monetary policy over time in a model with the following characteristics: 1) the distribution of wealth and income among households is highly skewed, 2) income inequality affects tax and spending policies, and 3) the government must provide transfers to the poor.

I subjected the model to comparisons with postwar U.S. data on tax rates, money growth, and inflation, and obtained varying degrees of success in capturing observed behavior of the various time series. Comparisons with the data are difficult, however, because estimates of average marginal tax rates are available only at annual frequency and consist of a small number of observations. A noteworthy result is that the model predicts distinctly different behavior for the labor tax and the capital tax regarding the optimal interaction with inflation, thereby pointing out the importance of distinguishing between these taxes in empirical tests for optimal government behavior. Finally, the model was shown to deliver business cycle statistics very close to models in which government policy is treated as an exogenous state variable.

The methodology of this paper can be used to perform quantitative studies in other important policy areas, such as characterizing the optimal behavior of public investment over the business cycle or quantifying the effects of international policy coordination on aggregate fluctuations. Regarding monetary policy, a more complete description of the banking sector (which captures the liquidity effect of a money shock) would be desirable. It would also be interesting to perform the policy simulations done here in the context of an overlapping-generations framework (see Ríos-Rull [1992]) to allow for age heterogeneity as well as for rich and poor households. For example, such a model would allow consideration of optimal Social Security policy (see İmrohoroğlu, İmrohoroğlu, and Joines [1992]).

APPENDIX A

Equilibrium with Different Discount Factors

This appendix briefly explains how the assumption of different discount factors, $\beta^P < \beta^R$, results in concentrated ownership of capital in the hands of the more patient, rich households. See Becker (1980) for a formal treatment of equilibrium when households have different discount factors. The argument here is based on Woodford (1988). If we assume that all households face the same after-tax interest rate, the following condition must hold if poor households would rather borrow than save:

$$\lambda_t^P > \beta^P E_t \lambda_{t+1}^P [(1 - \tau_{k,t+1})(r_{t+1} - \delta) + 1]. \quad (\text{A.1})$$

The condition for rich households to save is

$$\lambda_t^R = \beta^R E_t \lambda_{t+1}^R [(1 - \tau_{k,t+1})(r_{t+1} - \delta) + 1]. \quad (\text{A.2})$$

In steady state, these two conditions become

$$\begin{aligned} 1 &> \beta^P [(1 - \tau_k)(r - \delta) + 1], \\ 1 &= \beta^R [(1 - \tau_k)(r - \delta) + 1]. \end{aligned} \quad (\text{A.3})$$

Combining the expressions in A.3 yields the condition $\beta^P < \beta^R$ for concentrated ownership of capital. It should be pointed out, however, that equations A.1 and A.2 can be satisfied even if $\beta^P = \beta^R$. An example is when the poor face a lower after-tax interest rate than the rich. This situation might arise if the poor incur some type of transaction cost for investing small amounts that effectively reduces their rate of return. Judd (1985) uses an argument similar to this to justify concentrated ownership of capital when all households have the same discount factor.

FIG 1: TAX RATES vs FEEDBACK EXTERNALITY

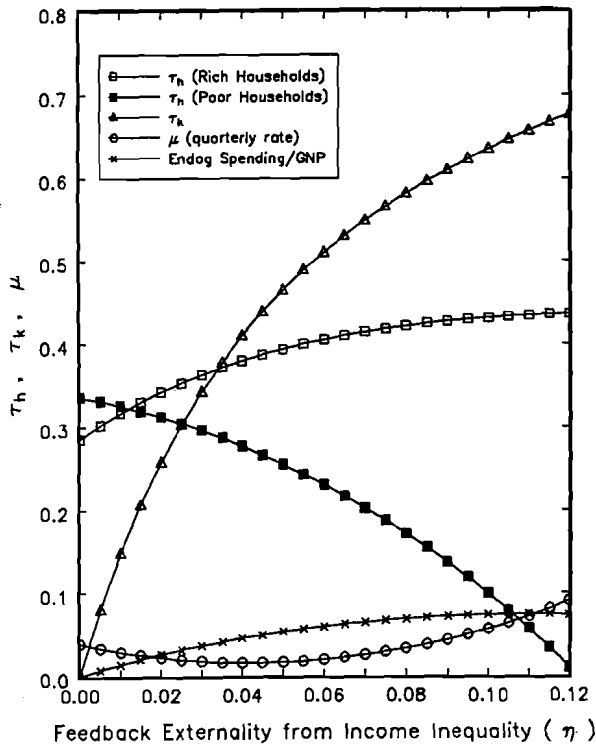


FIG 2: TAX RATES vs WEIGHT ON U(P)

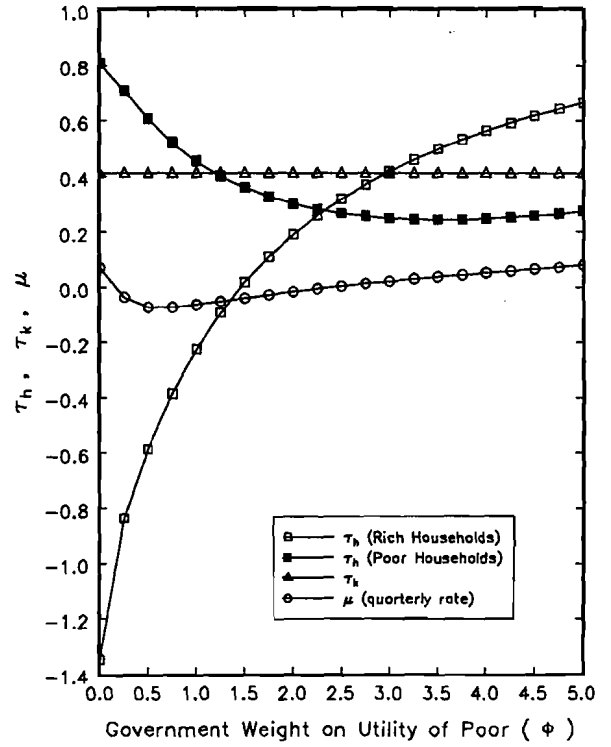


FIG 3: TAX RATES vs TRANSFERS/GNP

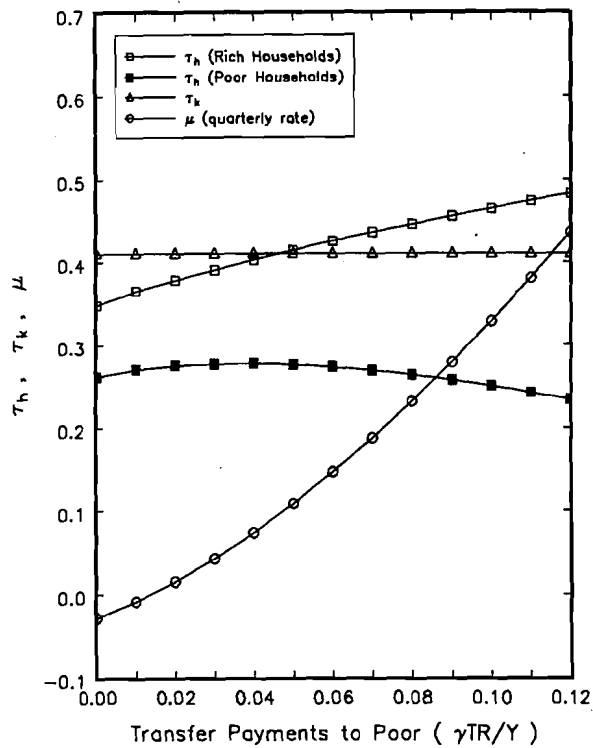
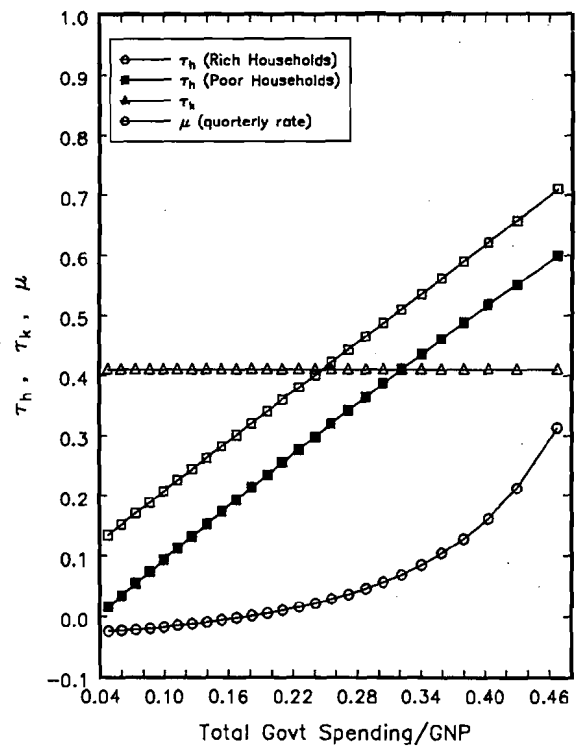


FIG 4: TAX RATES vs GOVT SPENDING/GNP



Source: Author's calculations.

FIG 5: U.S. TIME SERIES FOR LABOR TAX (Barro & Sahasakul-1988)

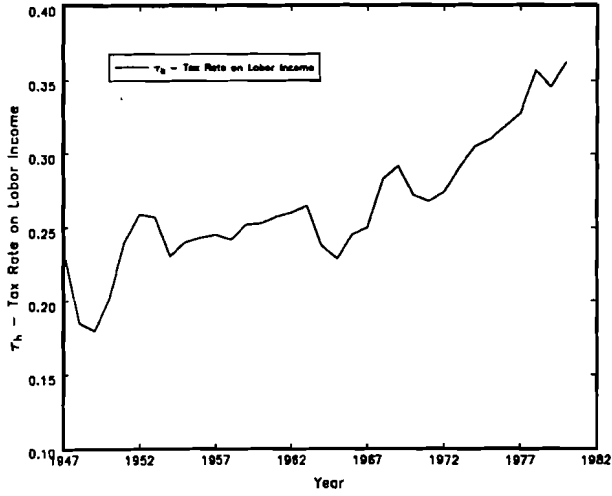


FIG 6: SIMULATED TIME SERIES FOR LABOR TAX

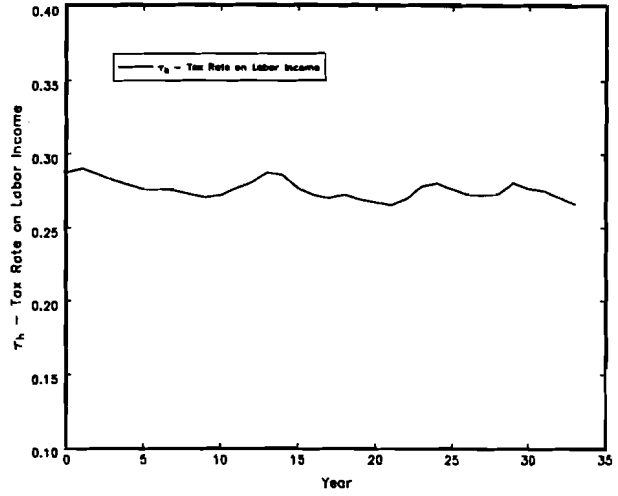


FIG 7: U.S. TIME SERIES FOR CAPITAL TAX (Jorgenson & Sullivan-1981)

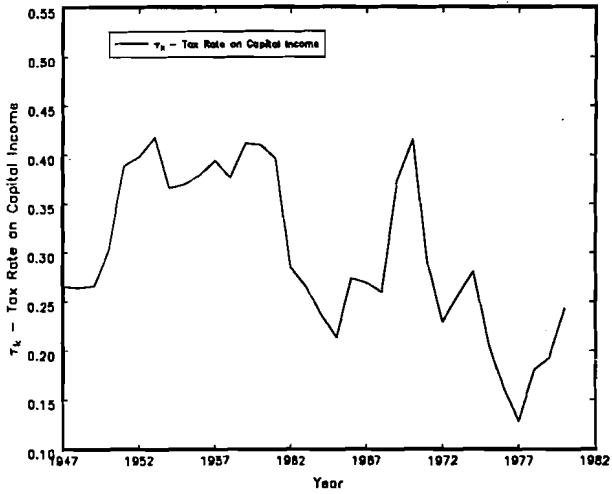


FIG 8: SIMULATED TIME SERIES FOR CAPITAL TAX

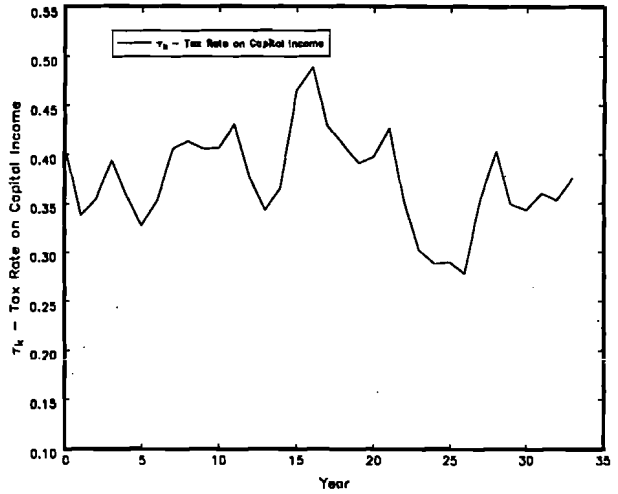


FIG 9: U.S. TIME SERIES FOR MONEY GROWTH (M1 data)

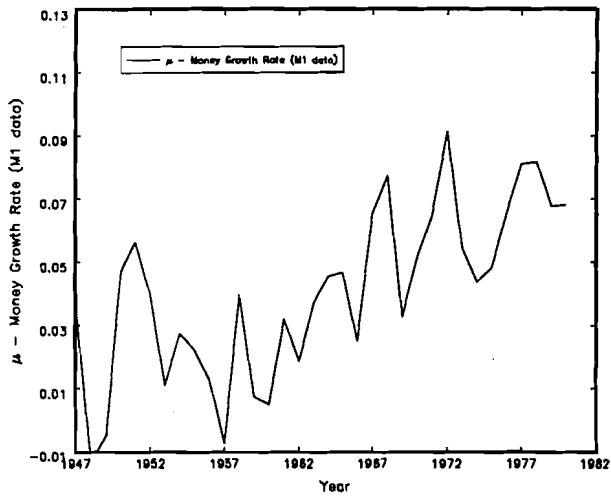
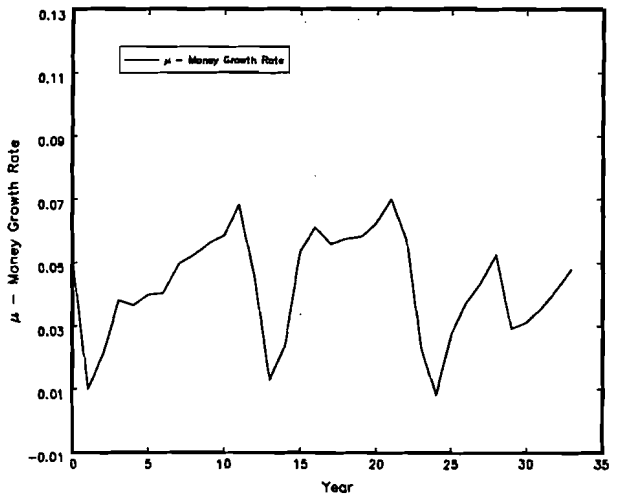


FIG 10: SIMULATED TIME SERIES FOR MONEY GROWTH



Source: Author's calculations.

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