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IMPLICIT CONTRACTS, ON-THE-JOB SEARCH
AND INVOLUNTARY UNEMPLOYMENT

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Abstract

This paper extends the implicit contracts framework to allow for on-the-job search. It is shown that involuntary unemployment can arise in such a framework without placing any a priori restrictions on either wages or severance payments. The model also implies that firms will practice a two-tier system of adjusting their labor force. In the first stage, workers who receive outside job offers leave the firm. The second stage consists of firms hiring additional workers during good states of nature, and laying off workers during bad states of nature. Furthermore, during "bad enough" states of nature, firms will offer a severance payment or bonus for those who want to voluntarily leave, and then lay off workers without offering a large enough severance payment to compensate them for being unemployed.

INTRODUCTION

Traditional models of equilibrium unemployment have failed to explain why some unemployment might be involuntary. For example, sequential search models, such as Lucas and Prescott's (1974) paper, imply that workers will become unemployed when their expected present discounted value of future utility is greater when they are unemployed than employed. Another objection to the search model's explanation of unemployment is the assumption that unemployed search is more productive than employed search. This assumption has been frequently questioned and recent evidence suggests the opposite might be true.¹

The purpose of this paper is to provide a consistent story of involuntary unemployment without placing any a priori restrictions on either wages or severance payments. While the existence of involuntary unemployment is by no means universally accepted, most economists accept it as a stylized fact of the labor market.² It therefore warrants an explanation within the traditional framework of equilibrium economics. This paper imbeds a simple model of on-the-job search in an implicit contracts framework. Implicit contracts provided one of the first equilibrium attempts to explain involuntary unemployment. In Azariadis' seminal work, involuntary unemployment results from three assumptions: 1) Workers are risk averse while firms are risk neutral. 2) Working is a 0 or 1 decision, that is, hours worked per worker is not a choice variable. 3) Firms cannot make severance payments to unemployed workers. Given these assumptions, involuntary unemployment results. Ex ante the optimal contract calls for workers to become unemployed during certain states of nature and to consume the value of their leisure, thus truncating bad

states of nature. Since workers are risk averse, however, and desire a constant consumption stream, it is not optimal to lower the wages of employed workers in order to induce them to leave. Similarly, by assumption, firms cannot make severance payments in order to induce workers to voluntarily leave.

Another characteristic of Azariadis' model is that there is overemployment. That is, even though there is involuntary unemployment in the sense that laid-off workers are worse off than their employed counterparts, there is over-employment because there is more employment and less unemployment than would occur in a pure Walrasian market. Workers remain employed even though their marginal productivity of labor is less than their reservation wage. Both involuntary unemployment and overemployment result from the assumption that firms cannot make severance payments to laid-off workers. This inability to pay severance payments implies that firms will partially insure workers against the risk of being laid-off by remaining employed longer than they would in a pure Walrasian market. Once severance payments are allowed, unemployment becomes purely voluntary and there is production efficiency.

The goal of this paper is to integrate a simple model of on-the-job search in an implicit contracts framework. This paper investigates the conditions under which involuntary unemployment will occur without placing any a priori restrictions on severance payments. Like Azariadis' model, an explanation of involuntary unemployment will necessitate their seeing overemployment. This is in contrast to **Grossman** and Hart who attempted to explain underemployment. That is, **Grossman** and Hart attempt to explain the ex post regret on the part of firms in the sense that they are laying off workers who ex post they would

want to remain employed. However, all unemployment was voluntary. A recent paper by Oswald provides the first attempt to explain both involuntary unemployment and underemployment, but to do so he exogenously assumed that severance payments were zero. On the other hand, this paper attempts to explain involuntary unemployment, i.e. the ex post regret of workers in the sense that ex post they would rather remain employed with the firm given the prevailing wage rate.

In order to explain involuntary unemployment, it is promising to follow the lines of Kahn (1985). He showed that complete insurance is not possible (or that wages will not be independent of the state of the world) when a firm cannot monitor a worker's alternative wage offer. Arvan (1986) extended Kahn's analysis and suggested this might explain why involuntary layoffs occur. In Arvan's model, firms cannot insure against layoffs because of the need to promote on-the-job search. However, Arvan implicitly constrains the severance payment to laid-off workers to equal the severance payment offered those who voluntarily quit their jobs. It is this assumption that is crucial to explaining involuntary unemployment in his model.

This paper is similar to both Kahn's and Arvan's in that it integrates the original implicit contract model with a simple model of on-the-job search. The structure of the model differs from theirs by assuming that on-the-job search may or may not result in a job offer, and by assuming that searching does not affect the resulting wage offer. If a worker receives a job offer, the present paper assumes the offer is exogenously given. These assumptions are not necessary and are meant to simplify the analysis. To explain involuntary unemployment, no a priori restrictions will be placed on the structure of severance payments. The restrictions placed on severance

payments result from the incentive compatibility constraints. However, the last section does assume that severance payments must be non-negative.

The structure of the paper is as follows: Section one considers the symmetric information case when a firm can observe both a worker's search intensity and whether or not the worker receives a job offer. The optimal contract in this case implies complete insurance. Section two drops the assumption that a firm can observe a worker's search intensity, but assumes the firm can observe which workers receive job offers by making severance payments conditional on the worker accepting an offer. The section shows that the inability of a firm to observe a worker's search efforts is not sufficient to explain involuntary unemployment. However, the model results in incomplete risk-sharing because firms trade off their desire to provide incentives for on-the-job search and to insure workers against future wage changes. The optimal contract is also characterized by production efficiency for laid-off workers. However, workers who receive job offers are shown to leave more often than would occur in a Walrasian world, implying that identical workers (in terms of productivity) will leave the firm and commence working for the firm simultaneously. The third section investigates the conditions necessary to explain involuntary unemployment. It shows that when firms cannot observe both a worker's search efforts and whether or not a worker receives a job offer, the incentive compatible contract implies that laid-off workers will be better off than their employed counterparts. However, this result assumes that firms can tax departing workers. If this assumption is dropped, the optimal contract results in involuntary unemployment. This occurs in order to provide the proper incentive in "bad enough" states of nature for job finders to truthfully reveal that they received an offer. The section also discusses

how the preceding analysis would change if a worker could save or borrow for himself rather than firms also acting as a bank for workers. The last section concludes and discusses possible extensions for future research.

II. THE MODEL WITH SYMMETRIC INFORMATION

Consider an economy that lasts for two periods indexed by $t = 1, 2$. Labor is hired in the first period where production takes place according to a deterministic production function, $f(N)$. Production in the second period is subject to a random shock, θ , where the range of θ is the closed interval $[0, \theta']$, with distribution functions $g(\theta)$ and $G(\theta)$, respectively. The model may be interpreted in a sectoral shifts framework. Workers search for alternate work in the first period in case the demand for the industry's output falls substantially in the second period or, alternatively, if there is a bad shock to production in the second period.

In the first period, workers choose their search effort, λ , where λ represents the probability that a worker will receive a job offer. A worker's search intensity is chosen in the first period before the realization of the random shock to firm production is realized. Searching is assumed not to affect the productivity of a worker. For simplicity, it is also assumed that searching does not require any monetary cost, but requires instead a "psychic" cost $c(\lambda)$, which is assumed not to affect a worker's marginal utility of income. The assumption that search effort enters separably in the worker's utility function is not crucial; it is meant to aid comparison with other implicit contract models. If a worker receives a job offer, it is assumed to be for an exogenous amount, w' . Workers are assumed not to be able to affect

this wage offer through searching. It is temporarily assumed that the firm cannot hire additional labor in the second period. This assumption will later be dropped so that additional labor can be hired in period two at the market wage rate, w' .

Firms compete for workers in the first period by offering an employment contract. Competition among firms for workers implies that the equilibrium contract will be chosen to maximize the expected utility of a representative worker subject to a zero expected-profit condition for the firm.

Contracts consist of wages, severance payments, lay-off probabilities and search intensity. That is, a contract consists of $\{w_1, w_2(\theta), l(\theta), q(\theta), s_l(\theta), s_q(\theta), A\}$, where w_1 is the first period wage; $w_2(\theta)$ is the second period wage chosen after the realization of θ ; $l(\theta)$ and $q(\theta)$ are the respective separation probabilities in the second period for workers who did not receive a job offer and did receive a job offer after the end of the first period; $s_l(\theta)$ and $s_q(\theta)$ are the severance payments (or taxes) given to (or applied to) workers who did not receive job offers and workers who did receive job offers respectively. For the full information case considered below, one can think of the firm as also choosing the search intensity of workers, A .

Assuming that workers cannot save or dissave (this assumption will be dropped later), so that their income in every period is identical to their wage in that period, the expected utility of a representative worker equals:³

$$EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), l(\theta), \lambda)$$

$$= U(w_1) - c(\lambda) + \int \{\lambda(1-q(\theta))U(w_2(\theta)) + \lambda q(\theta)U(w'+s_q(\theta))\}g(\theta)d\theta$$

$$+ \int \{ (1-\lambda)(1-l(\theta))U(w_2(\theta)) + (1-\lambda)l(\theta)U(B+s_1(\theta)) \} g(\theta) d\theta,$$

where B is the reservation wage of a worker, or the income equivalent of a worker consuming his endowed labor. It is also assumed that $U'(\cdot) < 0$, -- or equivalently, that workers are risk averse. The intuition behind the above equation is as follows: $\lambda(1-q(\theta))$ is the probability that a worker receives a job offer, but remain employed at the firm earning w_1 ; $\lambda q(\theta)$ is the probability that a worker receives a job offer and accepts it, in which case the worker earns w_1 plus the severance payment s_1 ; $(1-\lambda)(1-l(\theta))$ is the probability that a worker does not find other employment and is not laid off, in which case he earns w_2 ; $(1-\lambda)l(\theta)$ is the probability that the worker does not receive an offer and is subsequently laid off, in which case he earns the value of his leisure, B , and the severance payment $s_1(\theta)$. Assuming the firm is risk neutral, it has preferences given by

$$\begin{aligned} & E\Pi(w_1, w_2(\theta), s_q(\theta), s_1(\theta), q(\theta), l(\theta), \lambda, N) \\ = & f(N) - w_1 N + \int \{ \theta f([\lambda(1-q(\theta)) + (1-\lambda)(1-l(\theta))]N) \\ & - [(1-q(\theta))\lambda + (1-\lambda)(1-l(\theta))]w_2(\theta)N - \lambda q(\theta)s_q(\theta)N \\ & - (1-\lambda)l(\theta)s_1(\theta)N \} \lambda(\theta) d\theta \end{aligned}$$

The optimal employment contract is for the firm or the planner to choose $\{w_1, w_2(\theta), s_q(\theta), s_1(\theta), q(\theta), l(\theta), A, N\}$ to solve:

$$\max EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda)$$

s. t.

$$E\Pi(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda, N) \geq 0$$

The first-order conditions for this problem are:

1)

i) $U'(w_1) = \gamma_1$

ii) $U'(w_2(\theta)) = \gamma_1$

iii) $U'(w' + s_q(\theta)) = \gamma_1$

iv) $U'(B + s_l(\theta)) = \gamma_1$

va) $\theta f'([1 - \lambda q(\theta)]N) = w'$ when $\theta > \theta'$

vb) $\theta f'([(1 - l(\theta))(1 - \lambda)]N) = B$ when $\theta < \theta'$

where $\theta < \theta' \Rightarrow q(\theta) = 1, 0 < l(\theta) < 1$ and $\theta > \theta' \Rightarrow l(\theta) = 0, 0 < q(\theta) < 1$.

vi) $f'(N) = w_1$

vii) $c'(\lambda) = \gamma_1(w' - B)G(\theta')$

where $1(\theta')=0$ and $q(\theta')=1$ and γ are the Lagrangian associated with the expected profit constraint and $\gamma_1 = N\gamma$.

The solution to this problem is straightforward. Since there are no informational asymmetries, the optimal contract involves both perfect risk sharing according to Borch's rule and production efficiency. Workers are guaranteed the same income during all states of the world, independent of both the state of nature and whether or not a worker receives a job offer. Workers successful in their job search subsidize those who were unsuccessful. Production efficiency implies that the first workers to leave are those with the best outside opportunities, i.e. the workers who receive offers. After all the workers who have found jobs leave, firms must adjust the labor force by laying off workers. Firms lay off workers until the marginal productivity is equal to the reservation wage of the marginal worker. Workers are assumed to have non-market opportunities that give the agent an income equivalent of B . Firms then subsidize workers who are laid off by giving them a severance payment so that the worker is indifferent between staying with the firm or leaving the firm.

Firms also force workers to supply the optimum amount of search intensity given by $viii$). One can think of wages being set equal to zero when workers supply less than the required amount of search effort. The marginal cost of searching is equal to the marginal benefit of searching. The marginal benefit of searching is the difference between what the worker will earn in an alternate job, w' , and what he would produce in the current job, $\theta f'(\cdot)$. In good states of nature ($\theta > \theta'$), this difference is zero from production efficiency, while in bad states of nature the difference is $w'-B$. The

marginal benefit of searching is thus the probability that workers are laid off, $G(\theta')$, multiplied by the value in utils of earning w' versus B . Since the marginal cost of searching is in units of utils this quantity is multiplied by a worker's marginal utility of income.

Not surprisingly, the optimal contract with full information implies complete insurance and, hence, with asymmetric information it would not provide workers with any incentive to search. The next section considers the optimal contract when a firm cannot monitor a worker's search intensity and can choose the separation probabilities of workers, that is, the firm can tax departing workers. Both of these assumptions are maintained until section 3.

II. IMPERFECT MONITORING

In this section, it is assumed that a worker's search intensity is private information to the worker. However, firms can observe which workers receive jobs in the second period in the following manner: severance payments can be made conditional on the worker accepting a job offer. With asymmetric information, firms choose the optimal contract on the assumption that workers will then choose λ to maximize their utility given this contract. That is, given a contract $\{w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), N\}$ workers will choose their desired search intensity, λ^* such that:

$$\lambda^* = \operatorname{argmax}_{\lambda \in [0,1]} EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda)$$

Replacing the above condition with the first-order condition for an agent's search effort yields the reaction function for workers. It gives how agents will choose λ in response to the employment contract. This incentive compatibility constraint is appended to the planner's (or the firm's) problem in the previous section, so that the optimal contract is to choose $\{w_1(\theta), w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda, N\}$ in order to solve the following problem:

$$\max EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda)$$

s.t.

$$E\Pi(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda, N) \geq 0$$

s.t.

$$\int \{1-q(\theta)\}U(w_2(\theta)) + q(\theta)U(w'+s_q(\theta))\}g(\theta)d\theta - \int \{(1-l(\theta))U(w_2(\theta)) + l(\theta)U(B+s_l(\theta))\}g(\theta)d\theta + c'(\lambda) = 0$$

The first-order conditions for this problem are

1)

$$i) \quad U'(w_1) = \gamma_1$$

$$ii) \quad U'(w_2(\theta)) = \frac{\gamma_1[\lambda(\theta)(1-q(\theta))+(1-\lambda)(1-l(\theta))]}{[(\lambda+\gamma_2)(1-q(\theta))+(\lambda-\gamma_2)(1-l(\theta))]}$$

$$iii) \quad U'(w'+s_q(\theta)) = \frac{\lambda\gamma_1}{(\lambda+\gamma_2)}$$

$$\text{iv)} \quad U'(B+s_1(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2)}$$

$$\begin{aligned} \text{va)} \quad & \theta f'([1-\lambda q(\theta)]N) - w' \\ & = \{U(w'+s_q) - U(w_2) - U'(w'+s_q)[(w'+s_q) - w_2]\} / U'(w'+s_q) \quad \text{when } \theta > \theta' \end{aligned}$$

$$\begin{aligned} \text{vb)} \quad & \theta f'([(1-l(\theta))(1-\lambda)]N) = B \quad \text{when } \theta < \theta' \\ & \text{where } l(\theta') = 0 \text{ and } q(\theta') = 1. \end{aligned}$$

$$\text{vi)} \quad f'(N) = w,$$

$$\text{vii)} \quad \gamma_2 c''(\lambda) = \gamma_1 \int q(\theta) \{w_2(\theta) - (\theta f'(\cdot) + s_q(\theta))\} g(\theta) d\theta$$

Using va) and vb) to simplify ii) yields:

$$\text{ia)} \quad U'(w_2(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2)} \quad \text{when } \theta < \theta'.$$

$$\text{ib)} \quad U'(w_2(\theta)) = \frac{\gamma_1(1-\lambda q(\theta))}{[1-(\lambda+\gamma_2)q(\theta)]} \quad \text{when } \theta > \theta'$$

In bad states of nature when lay offs occur, $\theta < \theta'$, we have complete insurance for laid-off workers, i.e. $B + s_1 = w_2$. Workers who receive job offers are subsidized and earn more than those who do not find other employment, i.e. $w' + s_q > w_2$. The sign of s_q depends on the magnitude of the outside wage offer w' . If w' is small, it will always be a subsidy, while if w' is large it may be a tax. In good states of the world where $\theta > \theta'$, no workers are laid off, and the workers receiving job offers earn more than those who did not find alternate employment. However, this differential gets smaller with better states of nature. This implies the paradoxical result

that the marginal productivity of labor decreases with better states of nature when $\theta > \theta'$. In the limit, when $q(\theta) = 0$, and thus when no workers leave the firm, workers will earn equal wages in both the first period and the second period. It should be noted that this solution implicitly assumes that firms have the power to either subsidize or tax workers who leave. That is, even though workers who find alternate employment might earn more at their new jobs, $w' > w_2$, firms are assumed to be able to tax them (or sabotage their future job prospects) to prevent them from leaving, thus regulating the number of workers who leave the firm.

Since workers respond optimally to changes in the contract offered to them, equation vii) states that a worker's search intensity will be chosen so that the change in the marginal cost to workers from increasing their search effort is equal to the marginal benefit (expressed in units of utility) to the firm resulting from workers increasing their search effort. The marginal benefit from increasing a worker's search intensity is the difference between what the worker is paid, w_2 , and the sum of what he produces, $\theta f'(\cdot)$, and the severance payment given to departing workers, $s_q(\theta)$. This implies the familiar result: that the optimal contract will specify less search effort than the full information contract when $\gamma_2 > 0$. That is, λ is chosen such that the marginal benefit to increasing search effort is strictly positive. The proof that γ_2 is strictly positive follows from the first-order conditions. If $\gamma_2 < 0$ then workers would not supply any search effort. A sufficient condition for an interior solution to occur is that $c'(0) = 0$, $c'(1) = \infty$ and $w' > B$, i.e. it is costless to exert a little search effort and there is a positive benefit to searching, while the marginal cost of searching, so that a job offer is certain, is sufficiently costly so the probability a worker will receive an offer is less than one.

Notice, there is production efficiency when firms lay off workers. This is not surprising since there is complete insurance for laid-off workers. When the marginal worker to leave, however, is a worker who has received a job offer, i.e. when no workers are laid off, there is underemployment. Workers who find jobs leave more often than in a Walrasian market. This is symmetric with the result in Azariadis' model that there will be overemployment when there is involuntary unemployment. The intuition behind the present result is that on-the-margin firms find it optimal to provide additional incentive for on-the-job search by allowing workers to earn more after they find another job, and also by allowing them to leave more often than they would in the full information case. From v_a , the amount production differs from that which would occur in Walrasian market is dependent on the curvature of the utility function, or how risk averse workers are. The more risk averse are workers, the greater the need to insure a worker's income. Because this results in less search effort, there is a greater need to provide incentives for on-the-job search by allowing them to leave more often than in a world with symmetric information.

Given that $\theta f'(\cdot) > w'$, there is an incentive for workers who receive job offers to recontract with the firm. This is not possible however, given the assumption that firms can only observe which workers received job offers after the offers were accepted. In addition, there is the implicit assumption that firms cannot hire these workers back after the offer has been accepted. This is meant to imply that offers cannot be costlessly observed. If the firm could costlessly observe a worker's offer, there would always be production efficiency because firms could bribe workers who find jobs to continue employment by offering them a higher wage rate, w' . If the marginal productivity of labor is greater than w' , then the firm has an incentive to induce a worker who received an offer to stay since they can produce more at their present job than they can at an alternative job.

Underemployment also results when $\theta > \theta'$ because firms, by assumption, cannot hire workers in the second period at the market wage rate, w' . If additional labor can be hired, then an interesting result occurs. Workers will simultaneously leave the firm and accept employment with the firm. Since the marginal productivity of labor is greater than w' , the firm will have an incentive to hire additional workers at w' . Although ex post this seems wasteful (although no mobility costs are built into the model), ex ante such behavior is necessary in order to provide workers with the proper incentives to search in the first period.

To formalize this, assume that the firm can hire $n(\theta)$ workers in the second period at a market wage rate of w' . The optimal contract is then to choose $\{w_1(\theta), w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta); \lambda, n(\theta), N\}$ in order to solve the following problem:

$$\max EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda)$$

s.t.

$$E\Pi(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), X, N) > 0$$

s.t.

$$\int \{1 - q(\theta)\} U(w_2(\theta) + q(\theta)U(w' + s_q(\theta))) g(\theta) d\theta - \\ \int \{(1 - l(\theta))U(w_2(\theta) + l(\theta)U(B + s_l(\theta)))\} g(\theta) d\theta + c'(\lambda) = 0$$

s.t.

$$n(\theta) > 0$$

The first-order conditions for this problem are

1)

$$i) \quad U'(w_1) = \gamma_1$$

$$ii) \quad U'(w_2(\theta)) = \frac{\gamma_1[\lambda(1-q(\theta))+(1-\lambda)(1-l(\theta))]}{[(\lambda+\gamma_2)(1-q(\theta))+(\lambda-\gamma_2)(1-l(\theta))]}$$

$$iii) \quad U'(w'+s_q(\theta)) = \frac{\lambda\gamma_1}{(\lambda+\gamma_2)}$$

$$iv) \quad U'(B+s_l(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2)}$$

$$v) \quad q(\theta) = 1 \quad \text{if} \quad \theta f'([1-\lambda q(\theta)]N) - w' <$$

$$vi) \quad \begin{cases} \{U(w'+s_q) - U(w_2) - U'(w'+s_q)[(w'+s_q) - w_2]\}/U'(w'+s_q) & \text{for } \theta > \theta' \\ \theta f'([1-\lambda q(\theta)]N + n(\theta)) = w' & \text{for } \theta > \theta' \end{cases}$$

$$vii) \quad \theta f'([(1-l(\theta))(1-\lambda)]N) = \theta \quad \text{for } \theta < \theta', \text{ where } l(\theta') = 0 \text{ and } q(\theta') = 1.$$

$$viii) \quad f'(N) = w_1$$

$$ix) \quad \gamma_2 c''(\lambda) = \gamma_1 \int q(\theta) \{w_2(\theta) - (\theta f'(\cdot) + s_q(\theta))\} g(\theta) d\theta$$

Using v) and vi) to simplify ii) yields:

$$iia) \quad U'(w_2(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2)}$$

The results of this exercise are as follows: Workers who stay with the firm earn a wage rate, w_2 , which is independent of the state of the world.

Workers who receive job offers receive a severance payment from the firms and will always accept outside job offers. When workers are laid off by the firm, $\theta < \theta'$, complete severance payments will be offered to them, thus there will be neither under nor overemployment, the marginal productivity of labor will be equal to B . No additional workers will be hired in these states of nature. When $\theta > \theta'$; however, so that no workers are being laid off, the firm will hire additional workers at a wage of w' until production efficiency prevails.

This contract implies a two-tier system for adjusting a firm's work force. Firms first offer a severance payment to workers who wish to voluntarily leave the firm. Every worker who has found another job will then accept this offer. In more complex models, one can think of the severance payment offered to departing workers as also consisting of possible early retirement benefits, etc.. After workers accept this offer, the firm then adjusts the labor force by laying off workers or hiring new workers until it reaches the desired level of employment. This sort of two-tier system does seem to have its counterpart in the world. The implication that workers, will be induced to quit while the firm hires new workers also seems to occur. Although the current analysis indicates that those who find jobs will always leave the firm, this result is because there are no adjustment costs incurred when hiring new workers. If there were adjustment costs (or firm specific human capital), not all of the workers who found jobs would leave the firm.

It should be noted that since every worker who receives an outside job offer is allowed to accept the offer, the assumption that firms have the power to tax workers who leave is no longer necessary. Condition **iii)** assumes that the severance payment to workers who receive job offers might be negative. Dropping this assumption, however, would not change the nature of the results.

The condition for the optimum search intensity can be determined by substituting ii), iv) and v) into ix):

$$\begin{aligned} \gamma_2 c''(\lambda) &= \gamma_1 \int l(\theta) \{s_1(\theta) - s_q(\theta)\} g(\theta) d\theta \\ &- \gamma_1 \theta' \int \{w' + s_q - w_2\} g(\theta) d\theta \end{aligned}$$

Since γ_2 , $c''(\lambda)$, $\gamma_1 > 0$ the optimal contract implies that $s_1(\theta) > s_q(\theta)$. The intuition behind this result is straightforward. Consider the optimal contract when workers are risk neutral. In this case, production efficiency results and workers are paid the value of their marginal productivity in every state of the world. Assuming risk neutrality, workers would earn B , when $\theta < \theta'$, and would earn w' when $\theta > \theta'$. The first-period wage would be chosen so that firms earn zero expected profits. As workers become risk averse, firms trade off the incentives of providing on-the-job search for insuring workers against wage changes. First-period wages would be reduced in order to reduce the dispersion in second-period earnings; that is, $s_1(\theta) > s_q(\theta)$. Otherwise, it would have been preferable to keep the contract that resulted when workers were risk neutral as it also provided the proper incentives for on-the-job search.

The above contract must be modified when the assumption that firms can observe which workers receive job offers is dropped, since the above contract will not be incentive compatible. This is because the severance payment offered to workers who find alternate employment is less than the one offered to workers who are laid off (which just compensates a worker for being unemployed); workers who did not receive job offers will never want to pretend

that they did receive a job offer. The opposite is not true, however, when a large fraction of the labor force is being laid off. Workers who found other jobs will wish to pretend they did not receive a job offer so they can be laid off and thereby collect the larger severance payment offered laid-off workers. The next section considers the optimal contract when the firm cannot observe a worker's search intensity, or whether or not a worker receives a job offer.

III. IMPERFECT MONITORING OF SEARCH EFFORTS

The assumption that firms can hire additional labor in the second period will be maintained in this section, although this assumption is not necessary for the following results. If firms cannot monitor who receives job offers, then the optimal contract in the previous section is not incentive compatible. The incentive-compatible contract will be characterized by either involuntary unemployment or the opposite: involuntary employment, where unemployed workers are better off than their employed counterparts. The condition under which the first occurs, is if the firm cannot tax departing workers. The second result occurs if the firm has the power to tax departing workers, and thus the power to choose the separation rates for workers.

To solve for the optimal contract, when firms cannot observe which workers receive job offers, the following incentive compatibility constraint must be placed on the problem:

$$\begin{aligned} & (1-q(\theta)U(w_2) + q(\theta)U(w'+s_q(\theta))) > \\ & l(\theta)U(w'+s_l(\theta)) + (1-l(\theta))U(w_2(\theta)). \end{aligned}$$

The left hand of the above equation is the expected utility of a worker **if** he admits he received a job offer, while the right-hand side is the expected utility of a worker **if** he does not admit he received a job offer. In this case, when he is not laid off he earns $w_2 < w'$, since the firm can restrict his mobility. The optimal contract with this restriction will choose $\{w_1(\theta), w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda, N, n(\theta)\}$ to solve:

$$\max EV(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), \lambda)$$

s. t.

$$\Pi(w_1, w_2(\theta), s_q(\theta), s_l(\theta), q(\theta), l(\theta), A, N) > 0$$

s. t.

$$\int \{1-q(\theta)\}U(w_2(\theta) + q(\theta)U(w'+s_q(\theta)))g(\theta)d\theta - \int \{(1-l(\theta))U(w_2(\theta) - l(\theta)U(B+s_l(\theta)))\}g(\theta)d\theta - c'(\lambda) = 0$$

s. t.

$$q(\theta)U(w'+s_q(\theta)) \geq l(\theta)U(w'+s_l(\theta)) + (1-l(\theta))U(w_2(\theta))$$

s. t.

$$n(\theta) \geq 0$$

The first-order conditions for this problem are:

1)

$$i) \quad U'(w_1) = \gamma_1$$

$$ii) \quad U'(w_2(\theta)) = \frac{\gamma_1 [\lambda(1-q(\theta)) + (1-\lambda)(1-l(\theta))]}{[(\lambda+\gamma_2)(1-q(\theta)) + (1-\lambda-\gamma_2-\gamma_3(\theta))(1-l(\theta))]}$$

$$iii) \quad U'(w'+s_q(\theta)) = \frac{\lambda\gamma_1}{(\lambda+\gamma_2+\gamma_3)}$$

$$\text{iv)} \quad U'(B+s_1(\theta)) = \frac{\gamma_1(1-\lambda) + \gamma_3(\theta)U'(w'+s_1(\theta))}{(1-\lambda-\gamma_2-\gamma_3(\theta))}$$

$$\text{va)} \quad q(\theta) = 1 \quad \text{if} \quad \theta f'([1-\lambda q(\theta)]N) - w' < \{U(w'+s_q) - U(w_2) - U'(w'+s_q)[(w'+s_q) - w_2]\}/U'(w'+s_q)$$

$$\text{vb)} \quad \theta f'([1-\lambda q(\theta)]N + n(\theta)) = w'$$

$$\text{vi)} \quad U'(w_2(\theta))\{\theta f'([(1-l(\theta)(1-\lambda)]N) - B = [U(B+s_1(\theta)) - U(w_2(\theta))] - U'(w'+s_q(\theta))[B+s_1(\theta)-w_2(\theta)] + \gamma_3(\theta)/(1-\lambda-\gamma_1-\gamma_2)[U(B+s_1(\theta)) - U(w'+s_1(\theta))]\}$$

where $l(\theta') = 0$, and $q(\theta') = 1$.

$$\text{vii)} \quad f'(N) = w_1$$

$$\text{viii)} \quad \gamma_2 c''(\lambda) = \gamma_1 \int q\{w_2(\theta) - (\theta f'(\cdot) + s_q(\theta))\}g(\theta)d\theta$$

Using va) and vb) to simplify ii) yields:

$$\text{ia)} \quad U'(w_2(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2-\gamma_3(\theta))}$$

The solution to this problem is identical to that given in the previous section except for the inclusion of the costate variable, $\gamma_3(\theta)$, which becomes binding in "bad enough" states of nature. It can be shown that $\gamma_3(\theta) > 0$, implying that in states of nature where a large fraction of a given cohort of workers is being laid off, the severance payment offered to departing workers increases, while the wage offered to the job stayers and the severance payment given to laid-off workers decreases. However, this is not sufficient to explain involuntary unemployment. In fact, laid-off workers are

better off than their employed counterparts.⁴ This occurs because firms can tax departing workers. If a worker receives a job offer and pretends that he did not find other employment, then the worker will earn $w_2(\theta)$ when he is not laid off. To see the importance of this assumption, consider the case when firms cannot tax departing workers. In this case, the worker can quit and earn w' instead of staying and earning $w_2(\theta)$. The effect of the incentive compatibility constraint will be to reduce $s_1(\theta)$ and increase $s_q(\theta)$. The severance payment to laid-off workers will then be less than necessary to compensate them for being laid off.

To modify the problem so that firms cannot choose the separation rates of workers, the incentive compatibility given earlier must be modified:

$$(1-q(\theta)U(w_2) + q(\theta)U(w'+s_q(\theta))) \geq \\ 1(\theta)U(w'+s_1(\theta)) + (1-1(\theta))U(w').$$

The optimal contract will then be to choose $\{w, (\theta), w_2(\theta), s_q(\theta), s_1(\theta), \gamma_1(\theta), \gamma_2(\theta), 1(\theta), X, n(\theta), N\}$ to solve:

$$\max EV(w_1, w_2(\theta), s_q(\theta), s_1(\theta), q(\theta), 1(\theta), \lambda)$$

s.t.

$$E\Pi(w_1, w_2(\theta), s_q(\theta), s_1(\theta), q(\theta), 1(\theta), \lambda, N) \geq 0$$

s.t.

$$\int \{(1-q(\theta))U(w_2(\theta)) + q(\theta)U(w'+s_q(\theta))\}g(\theta)d\theta - \\ \int \{(1-1(\theta))U(w_2(\theta)) + 1(\theta)U(w'+s_1(\theta))\}g(\theta)d\theta + c'(\lambda) = 0$$

s.t.

$$q(\theta)U(w'+s_q(\theta)) > 1(\theta)U(w'+s_1(\theta)) + (1-1(\theta))U(w')$$

s.t.

$$n(\theta) \geq 0, s_1(\theta) \geq 0, s_q(\theta) \geq 0.$$

To solve this problem, I will assume that w' is large enough so that states of the world will exist such that $w_2 < w'$. Otherwise, the results will be identical to that given above and thus will not explain involuntary unemployment. The first-order conditions for this problem are:

1)

$$i) \quad U'(w_1) = \gamma_1$$

$$ii) \quad U'(w_2(\theta)) = \frac{\gamma_1 [\lambda(1-q(\theta)) + (1-\lambda)(1-l(\theta))]}{[(\lambda+\gamma_2)(1-q(\theta)) + (1-\lambda-\gamma_2)(1-l(\theta))]}$$

$$iii) \quad U'(w'+s_q(\theta)) = \frac{\lambda\gamma_1}{(\lambda+\gamma_2+\gamma_3(\theta))}$$

$$iv) \quad U'(B+s_1(\theta)) = \frac{\gamma_1(1-\lambda) + \gamma_3(\theta)U'(w'+s_1(\theta))}{(1-\lambda-\gamma_2)}$$

$$v) \quad q(\theta) = 1 \text{ if } \theta f'([1-\lambda q(\theta)]N + n(\theta)) - w' < \\ \{U(w'+s_q) - U(w_2) - U'(w'+s_q)[(w'+s_q) - w_2] \text{ if } \theta > \theta'.$$

$$vi) \quad \theta f'([1-\lambda q(\theta)]N + n(\theta)) = w'$$

$$vii) \quad U'(w_2(\theta))\{\theta f'([(1-l(\theta))(1-\lambda)]N) - B\} \\ = U'(w_2(\theta))[w_2(\theta) - (B+s_1(\theta))] - [U(w_2(\theta)) - U(B+s_1(\theta))] \\ + U'(w_2(\theta)) - U'(B+s_1(\theta)) \text{ where } l(\theta') = 0 \text{ and } q(\theta') = 1.$$

$$viii) \quad f'(N) = w_1$$

$$ix) \quad \gamma_2 c''(\lambda) = \gamma_1 \int q(\theta) \{w_2(\theta) - (\theta f'(\cdot) + s_q(\theta))\} g(\theta) d\theta \\ + \gamma_1 \int l(\theta) \{\theta f'(\cdot) - (w_2(\theta) - s_1(\theta))\} g(\theta) d\theta$$

Using v) and vi) to simplify ii) yields:

$$\text{ia)} \quad U'(w_2(\theta)) = \frac{\gamma_1(1-\lambda)}{(1-\lambda-\gamma_2-\gamma_3(\theta))} \quad \text{for all } \theta.$$

When no workers are laid off, the model produces a result similar to that in the previous section; all the workers who receive job offers will be permitted to leave. When workers are laid off, however, there is production efficiency except in certain bad states of nature where there is overemployment. This occurs because comparing ia) and iv) indicates that when a large fraction of the labor force is being laid off, there will be involuntary unemployment. The reason is that in order to get the job finders to truthfully reveal that they have found jobs, the severance payment to laid-off workers needs to be constrained. This differs from the result given earlier in this section, because when firms cannot tax workers only severance payments need to be constrained, rather than both severance payments and second-period wages. Job finders will then truthfully reveal when they receive offers.

The condition for involuntary unemployment to exist seems particularly strong, since it requires a large fraction of the firm's labor force to be laid off. However, the condition does not seem unreasonable if the condition is reinterpreted as a plant closing, or where a large fraction of a given cohort of workers is laid off. The latter might arise in more complex models with firm-specific human capital which have a lay-off rule based on seniority.

The model predicts that severance payments to both quits and lay offs will be state independent except during downturns. During severe downturns, the

severance payment or bonus offered in the first phase of the labor force adjustment will actually increase. This is so that workers who find jobs will truthfully reveal their job offers. In addition, during these downturns the severance payments to laid-off workers will decrease so that they are involuntarily laid off.

Because of the complexity of notation, I have assumed that $s_q > 0$, or that the non-negativity constraint on the severance payment offered to those who find outside offers is not binding. Allowing this constraint to be binding does not affect the results.

A criticism of the current model is that while it extends Azariadis' model, it still predicts that severance payments should be paid to laid-off workers. This is in light of recent evidence by Oswald, that relatively few industries actually offer some form of severance payments, although over 50 percent of manufacturing industries do offer such payments. If this model is extended, however, so that workers can save, instead of all savings and dissavings being provided by the firm, the present model will predict that severance payments might not be as prevalent as earlier predicted.

Once savings are permitted one would not necessarily predict firms to save for workers. If savings are permitted before the realization of the shock to firm production in the second period, then workers would save in the first period an amount equal to the severance payment offered to quits in the previous analysis. That is, savings would be $s_q(\theta)$ for $\theta > \theta'$ where θ' is defined to be the cutoff at which s_q increases and s_l decreases. The result would be severance payments offered only to workers who want to voluntarily leave the firm during bad shocks to industry demand. Casual empiricism suggests that indeed during downturns firms do offer bonuses to

those who want to voluntarily leave the firm. However, this prediction could be verified by future empirical work. The model with savings also decreases the amount of severance payments that is offered to laid-off workers. This might help explain the seeming lack of severance payments to workers in most industries. Of course, allowing workers to save will change the first order conditions, since there would be another constraint placed on the problem. However, this constraint would be independent of θ and thus would not affect the qualitative results of the paper.

IV. CONCLUSIONS

The goal of this paper was to investigate the conditions under which involuntary unemployment can result in a standard implicit contracts model that includes the possibility of on-the-job search. The results were encouraging; it was shown that under certain conditions involuntary unemployment can exist. The conditions necessary to achieve this result were: 1) Firms cannot observe a worker's search intensity, 2) Firms cannot monitor which workers receive job offers, and 3) Firms cannot tax departing workers. The question of whether or not the conditions necessary to explain involuntary unemployment occur often enough to explain the "observed" involuntary unemployment cannot be answered. The paper also showed that firms will have a two-tier procedure for adjusting its labor force to current economic conditions. In the first round, a model with savings implies that workers with outside offers would leave without the inducement of severance payments; in the second round, the firm adjusts its labor force by either laying off

additional workers or hiring new workers. The model implies that workers will leave the firm and new workers will be hired by the firm, although all workers are assumed to be equally productive. This occurs because, ex ante, firms have to offer contracts, which gives workers the necessary incentives to engage in on-the-job search, which also implies subsidizing them when they leave the firm. However, ex post, since firms cannot observe who receives job offers, they will find **it** profitable to hire new workers to replace those who quit.

One frequent criticism of the above analysis is that **it** implies that firms are subsidizing workers to engage in more on-the-job search. Ex ante contracts will be chosen so that workers will find **it** optimal to engage in such search activity, however, ex post **it** would not be surprising to think that firms are in some sense antagonistic to such activity. Firms will, of course, wish that none of their workers are successful in their job search. Similarly, another way of thinking about the problem is that firms sign contracts that reduce worker mobility in order to partially insure workers against income changes.

This paper showed why complete insurance to laid-off workers would not be optimal, given the incentive compatibility constraints. Additional empirical work is necessary to answer the question of whether the amount of severance payments predicted by models, such as the present one, occurs in the world. One reason the amount of severance payments offered by firms might not be that extensive, is because of state-mandated unemployment benefits. Theory suggests that the two are substitutes; thus, increases in state-provided unemployment insurance should decrease private severance payment programs. Future empirical work can be conducted to see **if** privately financed unemployment benefits decrease with increases in state-provided unemployment insurance.

Endnotes

1) For example, Blau (1986) finds that for less effort, employed searchers receive more job offers than unemployed job searchers. However, due to unobserved differences between employed and unemployed searchers, his data remains purely suggestive.

2) For example, Lucas has argued: "Involuntary unemployment is not a fact or a phenomenon which is the task of theorists to explain. **It** is, on the contrary, a theoretical construct which Keynes introduced in the hopes **it** would be helpful in discovering an explanation for a genuine phenomenon: large-scale fluctuations in total employment."

3) The problem is actually the social planner's problem. The "dual" problem where the firm maximizes its profits subject to an individual rationality constraint for the worker, does not affect the results.

4) To see this, compare *ii*) with *iii*). Since $U'(w_1 + s_1(\theta)) < U'(B + s_1(\theta))$ then $U'(w_2) > U'(B + s_1(\theta))$ or that $B + s_1(\theta) > w_2(\theta)$.

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