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**Capital Trading, Stock Trading, and the Inflation Tax on Equity: A Note**

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In “Capital Trading, Stock Trading, and the Inflation Tax on Equity,” Chami, Cosimano, and Fullenkamp (2001) (hereafter, CCF) analyze a cash-in-advance model in which capital goods are explicitly traded. The authors show that there is more responsiveness of consumption and output to changes in the money supply than exists in the standard neoclassical growth models. This note demonstrates that this arises because CCF implicitly imposed an additional equilibrium restriction on the Cooley and Hansen (1989) model. This restriction can be imposed only if the Cooley and Hansen model is subject to real indeterminacy which occurs whenever the risk aversion coefficient (denoted by  $\gamma$  in the Chami et al paper) exceeds 2.

**JEL Classification:** C68, E32, E44

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In “Capital Trading, Stock Trading, and the Inflation Tax on Equity,” Chami, Cosimano and Fullenkamp (2001) (hereafter, CCF) analyze a cash-in-advance model in which capital goods are explicitly traded. In Theorem 1, the paper notes that the equilibrium conditions of the model are consistent with an equality between the price of used capital goods in period  $t$  ( $Q_t$ ) and the price of new capital goods at the end of time  $t-1$  ( $P_{t-1}$ ). This latter price is the price level in time  $t-1$ . Based on this result, the authors show that this equilibrium implies more responsiveness of consumption and output to changes in the money supply than do other neoclassical growth models.

Correspondence with Baier, Carlstrom, and Fuerst, however, has led the six authors of this note to conclude that two related clarifications to the results of CCF should be pointed out. First, there is a close relationship between the CCF model and the seminal work of Cooley and Hansen (1989). In particular, the CCF model arises by imposing an additional equilibrium restriction on the Cooley and Hansen model. Second, this restriction can be imposed only if the Cooley and Hansen model is subject to real indeterminacy which occurs whenever the risk aversion coefficient (denoted by  $\gamma$  in the Chami et al paper) exceeds 2. In the case of  $\gamma > 2$ , the equilibrium in CCF is made possible by the presence of real indeterminacy in the neoclassical growth model with a cash-in-advance constraint. The equilibrium analyzed by CCF is one of many possible equilibria in this model as well as in a broad class of neoclassical growth models with cash-in-advance constraints, exogenous money growth rules, and  $\gamma > 2$ .<sup>1</sup> In this note, we elaborate on the above two clarifications.

The two key behavioral equations in CCF are:

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<sup>1</sup> See Carlstrom and Fuerst (2000).

$$\frac{Q_t}{P_t} = E_t \left[ \frac{mrs_{t+1}}{\Pi_{t+1}} (F_2(\theta_t L_t, K_t) + (1 - \delta) \frac{Q_{t+1}}{P_t}) \right], \quad (1)$$

$$E_t \left[ \frac{mrs_{t+1}}{\Pi_{t+1}} \right] = E_t \left[ mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} (F_2(\theta_{t+1} L_{t+1}, K_{t+1}) + (1 - \delta) \frac{Q_{t+2}}{P_{t+1}}) \right], \quad (2)$$

where the notation is as in CCF, and equations (1) and (2) are equations (12) and (13) in the original paper. CCF refer to (1) as the demand for used capital goods, while (2) is the demand for new capital goods, ie., the investment decision. Substituting (1) into (2) yields<sup>2</sup>

$$E_t \left[ P_t \frac{mrs_{t+1}}{P_{t+1}} \right] = E_t \left[ Q_{t+1} \frac{mrs_{t+1}}{P_{t+1}} \right]. \quad (3)$$

Notice that using (3) we can express (1)-(2) in the following way:

$$\frac{Q_t}{P_t} = E_t \left[ \frac{mrs_{t+1}}{\Pi_{t+1}} (F_2(\theta_t L_t, K_t) + (1 - \delta)) \right] \quad (4)$$

$$E_t \left[ \frac{mrs_{t+1}}{\Pi_{t+1}} \right] = E_t \left[ mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} (F_2(\theta_{t+1} L_{t+1}, K_{t+1}) + (1 - \delta)) \right]. \quad (5)$$

Equation (5) is the capital accumulation equation from the Cooley-Hansen model. The Cooley-Hansen model is defined by (5) and the standard resource constraint. We can simplify equation

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<sup>2</sup> Chami, Cosimano, and Fullenkamp interpret condition (3) as an equilibrium condition that reconciles the used capital decision with the investment decision. When (3) is imposed on the CCF model as shown in this note, this condition leads to an equilibrium that is identical to the CH equilibrium. The equilibrium condition imposed by CCF,  $Q_t = P_{t-1}$ , is an alternative equilibrium, but equally compelling, condition to (3), not an additional equilibrium condition, when  $\gamma > 2$ .

(5) by using the resource constraint and binding cash-in-advance constraint to eliminate consumption and prices from (5), so that we have an expression solely in terms of the capital stock. If we take a log-linear approximation of these equations around the non-stochastic steady-state, we obtain a third order linear difference equation for the capital stock of the form:

$$E_t \begin{pmatrix} k_{t+3} \\ k_{t+2} \\ k_{t+1} \end{pmatrix} = AE_t \begin{pmatrix} k_{t+2} \\ k_{t+1} \\ k_t \end{pmatrix} + Bs_t. \quad (6)$$

where  $s_t$  is the vector of exogenous shocks,  $B$  is the corresponding matrix, and  $E_t$  is the expectations operator. The key issue is whether real behavior is uniquely determined in the Cooley-Hansen model. For there to be a unique equilibrium the matrix  $A$  must have exactly two explosive eigenvalues (see, for example, Benhabib and Farmer (1999) or Farmer (1993)). The key parameter for this issue is  $\gamma$ . When  $\gamma = 2$ , there is one root less than one and two greater than one; that is, there are two explosive roots. Hence, there is a unique equilibrium in this case.<sup>3</sup>

The equilibrium is uniquely given by

$$k_{t+1} = a_1 k_t + b_1 s_t \quad (7)$$

where  $a_1$  is the single eigenvalue within the unit circle. We call this an AR(1) equilibrium as capital depends only on its one lag.

When the coefficient of relative risk aversion,  $\gamma$ , is greater than two there are two roots less than one and one greater than one. As Benhabib and Farmer (1999) point out, when there is only one root outside the unit circle the transversality condition yields only one condition between the three lags of the capital stock. As a result, there are multiple equilibria. First, there are two AR(1) equilibria of the following form:

$$k_{t+1} = a_1 k_t + b_1 s_t \quad (8)$$

$$k_{t+1} = a_2 k_t + b_2 s_t \quad (9)$$

where  $a_1$  and  $a_2$  are the two stable eigenvalues and  $b_1 \neq b_2$  are unique. But there are also AR(2) equilibria in which capital depends upon two lags:

$$k_{t+1} = c_1 k_t + c_2 k_{t-1} + d_1 s_t + e \sigma_t \quad (10)$$

where  $c_1$ ,  $c_2$ , and  $d_1$  are uniquely determined, but  $e$  is entirely free. The variable  $\sigma_t$  is any mean zero, iid random variable. This term can be a “sunspot” variable, but could also be innovations in the exogenous shock processes. To summarize, in the case of  $\gamma > 2$ , there are multiple equilibria in the Cooley-Hansen model and they are given by (8), (9), or (10).<sup>4</sup>

A possible way to uniquely pin down the model when there is indeterminacy is given by CCF’s Theorem 1 where  $Q(t) = P(t-1)$ . Their equilibrium is given by (5), the resource constraint, and an expression arising from this particular choice for the price of used capital. Substituting  $Q_t = P_{t-1}$  into (4), we can express (4) as

$$\left[ \frac{mrs_t}{\Pi_t} \right] = E_t \left[ mrs_t \frac{mrs_{t+1}}{\Pi_{t+1}} (F_2(\theta_t L_t, K_t) + (1 - \delta)) \right]. \quad (11)$$

(See the bottom of page 585 in the CCF paper.) Note that this is of the same form as (5). The CCF model consists of equations (5), (11), and the resource constraint. Hence, the CCF equilibrium is the Cooley-Hansen model *plus* the additional time-t restriction given by (11).

This extra restriction comes from CCF’s choice for the used capital price. This uniquely selects an AR(2) equilibrium of the form:

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<sup>3</sup> Cooley and Hansen (1989) assume  $\gamma = 1$ .

<sup>4</sup>Farmer (1993) demonstrates the existence of multiple equilibria in a cash-in-advance model without physical capital.

$$k_{t+1} = c_1 k_t + c_2 k_{t-1} + d_2 s_t + d_3 s_{t-1}. \quad (12)$$

A few observations are in order: First, the coefficients on lagged capital are the same as in the AR(2) in (10). Second, both the current and the lagged exogenous shocks are in (12). Finally, the extra restriction rules out sunspot equilibria so that there are no sunspot terms in (12).

How does real behavior in CCF compare to Cooley-Hansen? If real behavior is uniquely determined in the Cooley-Hansen model, then CCF's potential equilibrium price:  $Q_t = P_{t-1}$  cannot be an equilibrium. There is no stationary equilibrium that satisfies this restriction. Used capital will be priced by equation (4) but is otherwise irrelevant. But if real behavior is *not* uniquely determined in the Cooley-Hansen model, then there does exist a stationary equilibrium with  $Q_t = P_{t-1}$ . This is a "sticky" asset price equilibrium in that the price of capital does not respond to time- $t$  productivity innovations. CCF demonstrate that in this case the model implies much more responsiveness of consumption and output to changes in the money supply than do other neoclassical growth models because the behavior of the capital stock must do the adjusting. Whenever  $\gamma > 2$  some additional restriction must be placed on the model to make it unique.  $Q_t = P_{t-1}$  is one possible restriction and gives the AR2 equilibrium above. Some argue that the two AR1 equilibria are the more "natural" candidate equilibria. Each of these, however, require a different but equally arbitrary restriction.

In summary, the model studied by CCF is the Cooley-Hansen model plus the additional time- $t$  restriction given by (11). CCF analyze the model when  $\gamma > 2$  so that there is real indeterminacy in the Cooley-Hansen model. Because of this, equation (11) can be imposed as an equilibrium selection device. CCF motivated their selection of this equilibrium based on their modeling of the used capital market. However, if  $\gamma > 2$  this choice is possible whether or not there is a used capital market. Similarly when there is a used-capital market there are many



other possible equilibrium prices for  $Q$ . While the CCF equilibrium implies more responsiveness of consumption and output to changes in the money supply than do other neoclassical growth models when  $\gamma > 2$ , there are of course many other equilibria of the Cooley-Hansen model including the sunspot equilibria given by (10).

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