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by Ben R. Craig and Guillaume Rocheteau

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This paper investigates the welfare effects of inflation in economies with search frictions and menu costs. We first analyze an economy where there is no transaction demand for money balances: Money is a mere unit of account. We determine a condition under which price stability is optimal and a condition under which positive inflation is desirable. We relate these conditions to a standard efficiency condition for search economies. Second, we consider a related economy in which there is a transaction role for money. In the absence of menu costs, the Friedman rule is optimal. In the presence of menu costs, the optimal inflation rate is negative for all our numerical examples. A deviation from the Friedman rule can be optimal depending on the extent of the search externalities.

JEL Codes: E31, E40, E50.
Keywords: search, money, inflation, menu cost.

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1 Introduction

This paper investigates the welfare effects of long-run inflation in economies with search and nominal frictions. We disentangle different inefficiencies associated with inflation by considering two types of economies: Economies where there is no transaction demand for money balances, called cashless economies, and monetary economies where there is a role for monetary exchange.[1] First, we determine a condition under which price stability is optimal in cashless economies and a condition under which positive inflation is desirable. We can relate these conditions to a standard efficiency condition for search economies derived by Hosios (1990). Second, we show that the result according to which the optimal inflation rate is nonnegative no longer holds when one introduces monetary exchange. For all our numerical examples, the optimal inflation rate is negative for monetary economies. We also establish that whether the Friedman rule is (sub)optimal or not depends on the extent of search externalities. Finally, from a methodological perspective, our paper is a step towards introducing nominal rigidities, which many view as important for the study of monetary policy, into a search-theoretic model that treats money in an internally consistent way.[2]

Building on the literature pioneered by Kiyotaki and Wright (1991, 1993) and extended by Lagos and Wright (2005), we adopt a model where agents trade in bilateral meetings and where means of payment are needed to mitigate a double-coincidence-of-wants problem. We also endogenize the frequency of trades through a free-entry condition so that inflation affects both the quantities traded in individual matches (i.e., the intensive margin) and the number of matches (i.e., the extensive margin). The introduction of nominal rigidities is based on the model of Sheshinki and Weiss (1977). Sellers who incur a fixed cost to change their prices adjust prices only infrequently by following an endogenous (S, s) rule.[3]

[1] The term "cashless economy" has been inspired by the terminology of Woodford (2003). He considers an economy in which no monetary assets are needed to facilitate transactions.

[2] For recent empirical evidence suggesting that price adjustments are infrequent, see Levy, Bergen, Dutta, and Venable (1997) and Bils and Klenow (2004).

[3] The macroeconomic literature on state-dependent price adjustments was pioneered by Caplin and Spulber (1987) and Caplin and Leahy (1991), and in search-theoretic environments, by Benabou (1988, 1992), Diamond (1993), and Diamond and Felli (1992). Recent contributions include Dotsey, King, and Wolman (1999) and

The result according to which positive long-run inflation can be socially desirable in cashless economies with search frictions was first pointed out through numerical examples by Benabou (1992) and Diamond (1993). One of our contributions is to derive a condition under which price stability is optimal in a tractable framework that generalizes Diamond (1993) by introducing an intensive margin and giving buyers some market power. This condition states that inflation (or deflation) can be good for society when sellers have too much market power, or equivalently, when the congestion externality in the goods market is too severe. In the presence of sticky prices, a deviation from price stability mitigates this inefficiency by reducing sellers' incentives to enter the market. Also, while many papers restrict themselves to non-negative inflation rates, we allow for deflation and show that the optimal inflation rate is always non-negative.

The introduction of a transaction role for money into the previous environment brings two additional insights. First, the result obtained in cashless economies according to which the optimal inflation is non-negative is not robust to the introduction of an inflation tax. In the absence of menu costs, the Friedman rule is optimal, and in the presence of menu costs, the optimal inflation rate is negative. Second, the presence of nominal rigidities matters for the (sub)optimality of the Friedman rule. Depending on the extent of the search externalities, it is sometimes optimal to keep inflation above the level prescribed by the Friedman rule .[4] Also, we illustrate how the presence of nominal frictions enhances the welfare gains associated with the optimal deflation and eliminates a real indeterminacy at the Friedman rule.

The paper is organized as follows. Section 2 describes the environment. Section 3 solves a model in which there is no money but prices are posted in terms of a unit of account. Section 4 investigates a monetary model with flexible prices, and then with sticky prices. All proofs of lemmas and propositions are relegated to the appendix.

Golosov and Lucas (2003).

[4] These results are not inconsistent with findings in new Keynesian models based on monopolistic competition and time-dependent pricing. In those models, the Friedman rule is optimal if prices are flexible despite the presence of imperfect competition. With sticky prices, deflation is still optimal but the deflation rate can be lower than the one at the Friedman rule. For details and references, see Khan, King, and Wolman (2002).

2 The environment

Time is continuous and goes on forever. Some trades take place in a centralized market, and others in a decentralized market with bilateral random matching. There are two types of perishable goods, a *special good*, produced in different varieties, and a *general good*. Whereas the general good is produced and traded in the centralized market, special goods are traded in the decentralized market.[5]

The economy is populated with a continuum of infinitely-lived agents divided into two categories, called *buyers* and *sellers* to reflect their trading behaviors in the decentralized market. The measure of buyers is normalized to one. The measure of sellers, denoted n, will be endogenous. Buyers differ from sellers both in the goods they produce and in their consumption preferences. Whereas both types of agents consume and produce the general good, buyers, unlike sellers, also want to consume some special goods, and sellers, unlike buyers, produce special goods.

	Buyers	**Sellers**
General good	consume produce	consume produce
Special goods	consume	produce

Agents' trading behaviors

The utility of consuming q units of special goods is $u'q$ with $u' > 0$.[6] The disutility of producing special goods is $c(q)$ with $c(0) = c'(0) = 0$, $c'(q) > 0$ and $c''(q) > 0$ for $q > 0$, and $c(q) = u'q$ for some $q > 0$. The instantaneous utility function of buyers and sellers in the centralized market is simply x, where x is the net consumption flow of general goods.[7] Given this specification, producing the general good for oneself is worthless. Buyers and sellers

[5] The assumption that trades take place in both centralized and decentralized markets was introduced by Lagos and Wright (2005). The environment described in this paper is closer to Rocheteau and Wright (2005).

[6] The linearity of the utility function in terms of special goods, also used by Benabou (1988), will simplify greatly sellers' pricing strategy.

[7] The linear specification for the utility functions for centralized market goods is a key assumption to obtain a tractable model in which the distribution of money holdings is easy to handle. See Lagos and Wright (2005).

discount future utility at the same rate, $\rho > 0$.

Unmatched agents trade in the centralized market. They are thrown into a bilateral match, i.e., in the decentralized market, according to a stochastic Poisson process. When matched, agents do not have access to the centralized market.[8] Matched agents choose whether or not to trade, split apart immediately after the trade has occurred, and return to the centralized market. Since an agent does not have the ability to produce general goods while in the decentralized market, he can only transfer the special good he produces or the assets he holds at the time he is matched.[9]

Regarding agents' abilities to use credit, we will consider two polar cases. We will first consider an economy in which buyers are able to commit to repay their debts and therefore can use IOUs to trade in the decentralized market. We will call this economy a *cashless economy*.[10] We will then consider a *monetary economy* where buyers are unable to commit to repay their debt and need to use money in order to trade in the decentralized market.

The trading opportunities in the decentralized market are described by a standard random-matching technology. The instantaneous matching probability of a buyer is $\alpha(n)$, whereas the instantaneous matching probability of a seller is $\alpha(n)/n$. Furthermore, $\alpha' > 0$ and $\alpha'' < 0$, $\alpha(0) = 0$, $\alpha'(0) = \infty$, $\alpha'(\infty) = 0$ and $\lim_{n\to\infty} \alpha(n) = \infty$. We denote $\eta(n) = \alpha'(n)n/\alpha(n)$ the elasticity of the matching function. As we also want to endogenize n, we assume that sellers who participate in the decentralized market incur a flow cost, $k > 0$, to search for buyers and to advertise their products.[11]

There exists a good called money that is intrinsically useless but that serves as a unit of account. The monetary price of the general good is $w(t)$. It will be exogenous in the cashless economy, and will be determined by a market-clearing condition in the monetary economy. In

[8] For a somewhat related formalization where centralized and decentralized markets open concurrently, see Williamson (2004).

[9] One could assume instead that even though general goods can be produced in bilateral matches, the seller does not wish to consume the general good produced by the buyer in the match.

[10] Related cashless economies with state-dependent pricing are studied in Caplin and Spulber (1987), Benabou (1988, 1992), Diamond (1993), Golosov and Lucas (2003), among others.

[11] The assumption of free-entry is standard in the search literature to endogenize the number of trades. See, among others, Pissarides (2000), Diamond (1993) and Rocheteau and Wright (2005).

the decentralized market, we adopt the following pricing protocol. Unmatched sellers post a monetary price. They can change their posted prices at any time at the cost γ in terms of utility. When a match occurs, the transaction price is chosen as follows. With probability $1-\theta$, every unit that is produced is sold at the seller's posted price. The quantity traded is the minimum of the buyer's demand and the seller's supply at this price. With probability $\theta \in (0,1)$, however, the buyer makes a take-it-or-leave-it offer. If this offer is rejected by the seller, no trade takes place. One can think of this as price posting with imperfect commitment.[12] While this pricing procedure is very stylized, it is realistic in that transaction prices often differ from posted prices. There are two additional reasons to give buyers some market power by allowing them to make offers in some matches. First, without this assumption there would be no monetary equilibrium in the economy with flexible prices.[13] Second, this assumption will allow us to derive a simple condition on θ and $\eta(n)$ under which a deviation from price stability is optimal in the cashless economy.

3 Cashless economies

In this section, we describe an economy in which there are no monetary frictions and agents do not hold nominal assets; they use credit arrangements to trade in the decentralized market.[14] Buyers commit to repay their debt in the general goods market straight after a trade has occurred. Sellers post a monetary price at which they commit to sell their output. The monetary price of general goods, $w(t)$, is exogenous and is growing at rate $\pi \geq -\rho$. In the following, we will refer to the real price p as the nominal price posted by sellers divided by the price of general

[12] For instance, a seller instructs a sales clerk to sell his output at the posted price. The commitment technology is imperfect in the sense that the sales clerk is not always in the shop, let's say, because it is too costly to have an employee full time. This is related to the assumption of costly price commitment introduced by Bester (1994). One can also view this pricing mechanism as a bargaining game with nominal rigidities.

[13] In order to allow for the existence of a monetary equilibrium in an economy with price posting, one can introduce heterogeneity across buyers. See Curtis and Wright (2004) and Ennis (2004).

[14] Our model is related to the one in Diamond (1993) but it differs from it in several dimensions. First, buyers can appropriate the surplus of a match in a fraction θ of the meetings, whereas in Diamond's model, θ is assumed to be 0. Second, the quantity traded in each match is endogenous, whereas in Diamond, it is set exogenously at 1. Third, buyers can trade repeatedly in the search market whereas buyers only trade once in Diamond's environment. As we will show, none of these assumptions is crucial to Diamond's results.

goods, w. Note that p decreases at rate π as long as the monetary price remains unchanged.

Consider a buyer. The Poisson arrival rate of a match in the decentralized market is $\alpha(n)$. With probability θ, the buyer makes a take-it-or-leave-it offer (q_b, d_b), where q_b is the quantity of special goods produced by the seller, and d_b is the quantity of general goods that the buyer commits to deliver (the subscript b reflects the assumption that the offer is made by a buyer). With probability $1-\theta$, the buyer trades at the posted price: He consumes q_s in exchange for $d_s = pq_s$ units of general goods. The quantity q_s, which is the minimum of the buyer's demand and the seller's supply at the posted price, is a function of p. The value function of a buyer, W^b, satisfies the following flow Bellman equation

$$\rho W^b = \alpha(n) \left\{ \theta \left[u(q_b) - d_b \right] + (1-\theta) \int \left[u' q_s(p) - p q_s(p) \right] dH(p) \right\}, \tag{1}$$

where $H(p)$ is the distribution of real prices across sellers.

Consider next a seller. As shown by Sheshinsky and Weiss (1977), in the presence of menu costs sellers change their price according to an (s, S) rule. If $\pi > 0$, the real price of the seller falls until it reaches the *trigger point* s and is then readjusted to the *target point* S. Conversely, if $\pi < 0$ the real price increases steadily from s to S. The length of the period of time between two price adjustments is denoted τ. Suppose the seller has not adjusted his price for a period of time of length $h \in (0, \tau)$. His posted price expressed in terms of the general good is $p(h) = Se^{-\pi h}$ if $\pi > 0$, and $p(h) = se^{-\pi h}$ if $\pi < 0$. The seller's expected utility, $W^s(h)$, obeys the following Bellman equation (see the Appendix)

$$\rho W^s(h) = -k + \frac{\alpha(n)}{n} G\left[p(h)\right] + \frac{\partial W^s(h)}{\partial h}, \tag{2}$$

where $G(p)$, the seller's expected trade surplus, satisfies

$$G(p) = (1-\theta) \left\{ q_s(p)p - c\left[q_s(p)\right] \right\} + \theta \left\{ d_b - c(q_b) \right\}. \tag{3}$$

Equation (2) has the following interpretation. The seller incurs the cost k to participate in the market. A match occurs with instantaneous probability $\alpha(n)/n$, in which case the seller's expected surplus is $G(p)$. The last term on the right-hand side of (2) reflects the fact that the seller's value function is not constant over the (S, s) cycle.

At the end of the (S, s) cycle, i.e., when $h = \tau$, the seller readjusts his price and starts a new cycle so that $W^s(\tau) = W^s(0) - \gamma$. Furthermore, the free entry of sellers implies $W^s(0) - \gamma = 0$, and $W^s(\tau) = 0$. This simply means that a seller who readjusts his price is in the same position as a new entrant. Using this terminal condition, equation (2) can be written in integral form as

$$W^s(h) = \int_h^\tau e^{-\rho(t-h)} \left[-k + \frac{\alpha(n)}{n} G\left[p(t)\right] \right] dt. \tag{4}$$

Finally, $W^s(0) = \gamma$ yields

$$\int_0^\tau e^{-\rho t} \left[-k + \frac{\alpha(n)}{n} G\left[p(t)\right] \right] dt = \gamma. \tag{5}$$

According to (5), the expected discounted utility of a seller over the (S, s) cycle has to be equal to the cost of setting a new price.

3.1 Equilibrium

To characterize equilibrium, we need to specify how terms of trade are formed in the decentralized market. Consider a match between a buyer and a seller whose posted price is p. As previously stated, the transaction price differs from the posted price with probability θ. In this case, the buyer makes a take-it-or-leave-it offer (q_b, d_b) in order to maximize his utility $u(q_b) - d_b$, subject to the sellers' participation constraint, $-c(q_b) + d_b \geq 0$. The solution is then $q_b = q^*$ and $d_b = c(q^*)$, where q^* solves $u' = c'(q^*)$.

With probability $1 - \theta$, agents trade at the posted price. However, the seller can choose not to serve all the buyer's demand at that price. The buyer's demand corresponds to the value of q that maximizes his surplus $u'q - pq$. It is unbounded if $u' > p$ and it is 0 if $u' < p$.[15] We call u' the buyer's reservation price. Also, sellers produce no more than the quantity q that maximizes $qp - c(q)$. Therefore, $q_s(p)$ is given by

$$q_s(p) = \begin{cases} c'^{-1}(p) & \text{if} \quad p \leq u' \\ 0 & \text{otherwise} \end{cases} . \tag{6}$$

[15] In the knife-edge case, where $u' = p$, the buyer is indifferent between trading or not. To guarantee that the seller's pricing problem has a solution, we assume that the buyer's demand is at least equal to the quantity q^* that the seller is willing to produce at this price.

The optimal pricing policy of sellers is such that S and τ maximize sellers' expected utility as given by the left-hand side of (5). It is characterized in the following Lemma.

Lemma 1 *(i) If $\gamma = 0$ then sellers set a real price equal to u'. (ii) Assume $\gamma > 0$. The sellers' optimal pricing policy (S,τ) satisfies $S = u'$ and*

$$\alpha(n)G(u'e^{-|\pi|\tau}) = \begin{cases} nk, & \text{if} \quad \pi > 0 \\ n(\rho\gamma + k), & \text{if} \quad \pi < 0 \end{cases}, \tag{7}$$

and $\tau = \infty$ if $\pi = 0$.

For all inflation rates, sellers target the buyers' reservation price u'. They do not let their price go beyond this target. This result is intuitive since a seller's expected sales fall to 0 if his price is above u'. In the presence of inflation, the opportunity cost of delaying the price adjustment is $\rho W^s(\tau) = 0$ (in flow terms), whereas the instantaneous benefit is $\frac{\alpha(n)}{n}G[p(\tau)] - k$. Therefore, the seller adjusts his price when his instantaneous utility falls to 0.[16] In the case of deflation, the seller readjusts his price when it reaches the buyer's reservation price, u', irrespective of his choice for s. The opportunity cost for the seller of delaying the price adjustment by setting a price smaller than s is equal to $\rho W^s(0) = \rho\gamma$. From (7) there is a symmetry between inflation and deflation only when $\rho \to 0$.

All through the paper, we focus on time-invariant cross-sectional distributions of real prices by assuming that real prices are log-uniformly distributed over $[s, S]$. As shown by Caplin and Spulberg (1987) and Benabou (1988), the log-uniform distribution is the only one that is time invariant and consistent with the (s, S) rule.[17] Equivalently, the length of the period of time during which a seller's price has been kept unchanged is uniformly distributed on $[0, \tau]$.

Definition 1 *An equilibrium is a pair (n, τ) that satisfies (5) and (7) if $\pi \neq 0$ or (5) and $\tau = \infty$ if $\pi = 0$.*

[16] One may wonder why the menu cost γ does not appear in Equation (7) when $\pi > 0$. The reason is that the continuation value of a seller who readjusts his price is $-\gamma + W^s(0)$. From the free-entry condition, this term is 0. Note, however, that the menu cost γ appears in the free-entry condition.

[17] Under a steady inflation, a log-uniform distribution of prices is equivalent to a uniform staggered timing (Rotemberg, 1983).

We illustrate the determination of equilibrium in Figure 1. The free-entry curve corresponds to (5), whereas the pricing curve corresponds to (7). The pricing-curve slopes downward since sellers need to readjust their prices more frequently when the market is congested. The pricing-curve intersects the free-entry-curve when the latter reaches a maximum: The number of sellers is highest when the frequency of price adjustment is chosen optimally. As π increases, the free-entry-curve shifts downward (see dotted curve), n decreases, and $\pi\tau$ increases. Inflation drives sellers out of the market and raises (real) price dispersion.[18] These results are summarized in the following proposition.

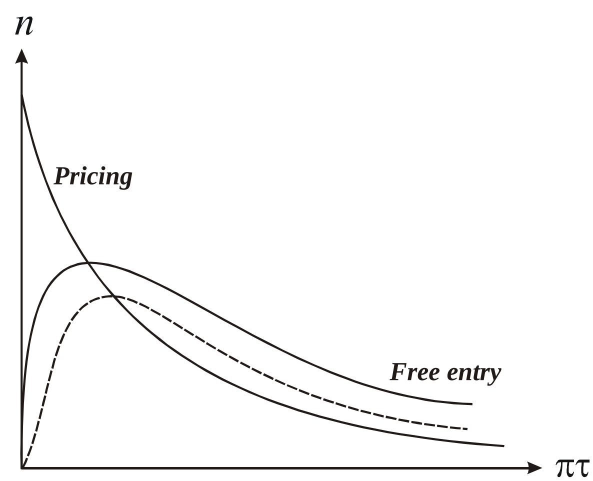


Figure 1: Equilibrium.

Proposition 1 *If $\theta < 1$, equilibrium exists and is unique. If $\gamma = 0$, n is independent of π. If $\gamma > 0$, $\partial\pi\tau/\partial\pi > 0$, and $\partial n/\partial\pi < 0$.*

We measure society's welfare as the expected surplus of buyers per unit of time.

$$\mathcal{W}^b = \alpha(n)\left\{\theta\left[u'q^* - c(q^*)\right] + (1-\theta)\int_0^{|\pi|\tau} u'q_s(u'e^{-h})\left(1-e^{-h}\right)\frac{dh}{|\pi|\,\tau}\right\}. \tag{8}$$

This welfare measure is legitimate since the free-entry condition drives sellers' expected utility to 0. Denote n_0 the measure of sellers when $\pi = 0$.

[18]For a search-theoretic model with endogenous price dispersion and flexible prices, see Head and Kumar (2003) and Head, Kumar and Lapham (2004).

Proposition 2 *If $\gamma = 0$, equilibrium is efficient iff $1 - \theta = \eta(n_0)$.*

The condition in Proposition 2 is similar to the one derived by Hosios (1990) for an efficient allocation in the presence of congestion externalities. It states that the measure of sellers is socially efficient if the fraction of the matches where sellers appropriate the whole surplus of a match coincides with sellers' contribution to the matching process as measured by the elasticity of the matching function. If $1 - \theta > \alpha'(n)n/\alpha(n)$, n is too high. Although it may not be well known, this is the main inefficiency in the Diamond (1993) economy.

Society's welfare can be expressed as a function of the price dispersion that prevails in equilibrium, $|\pi|\,\tau$,

$$\mathcal{W}^b\left(|\pi|\,\tau\right) = \alpha\left[n(|\pi|\,\tau)\right]\left\{\theta\left[u'q^* - c(q^*)\right] + (1-\theta)\int_0^{|\pi|\tau} u'q_s(u'e^{-h})\left(1 - e^{-h}\right)\frac{dh}{|\pi|\,\tau}\right\},$$

where $n(\cdot)$ is implicitly defined by (7).

Proposition 3 $\left.\frac{d\mathcal{W}^b}{d|\pi|\tau}\right|_{|\pi|\tau=0} > 0$ *iff $\theta < \left[1 - \eta(n_0)\right] / \left[1 + \eta(n_0)\right]$.*

Proposition 3 indicates under which circumstances price dispersion is desirable taking into account the effect of price dispersion on the number of sellers. Price dispersion has two opposite effects on buyers' welfare. It raises the ability of buyers to extract a higher surplus, but it also reduces the number of sellers and therefore the frequency of trades. If θ is low, the first effect dominates and price dispersion raises buyers' welfare. In Diamond's (1993) economy, $\theta = 0$ so the condition in Proposition 3 is satisfied. As a corollary, Proposition 3 indicates under which circumstances a deviation from price stability is optimal since we know from Proposition 1 that $\partial\,|\pi\tau|\,/\partial\,|\pi| > 0$. Assuming γ is close to 0 so that $\lim_{\pi\downarrow 0}\pi\tau \simeq 0$, then a deviation from price stability is optimal if $\theta < \left[1 - \eta(n_0)\right] / \left[1 + \eta(n_0)\right]$. However, we cannot infer from Proposition 3 whether it is better to generate price dispersion through inflation or deflation.

Proposition 4 *Assume γ is positive but close to 0. The optimal inflation rate is non-negative.*

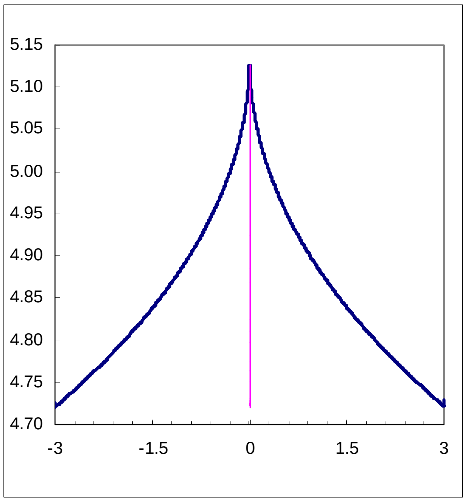
The intuition for this result goes as follows.[19] If buyers could choose the inflation rate, they would face a trade-off between the larger share in the gains from trade that is associated with higher price dispersion and the lower frequency of trade that is associated with the smaller number of sellers in the market. For a given price dispersion $|\pi\tau|$, inflation hurts sellers less than deflation does. Indeed, if $\pi > 0$, sellers set their prices to S and get high profits at the beginning of the cycle, whereas if $\pi < 0$, they set their prices to s and get low profits first. Therefore, it is optimal to reduce sellers' market power by running a positive inflation instead of a deflation.

3.2 Numerical examples

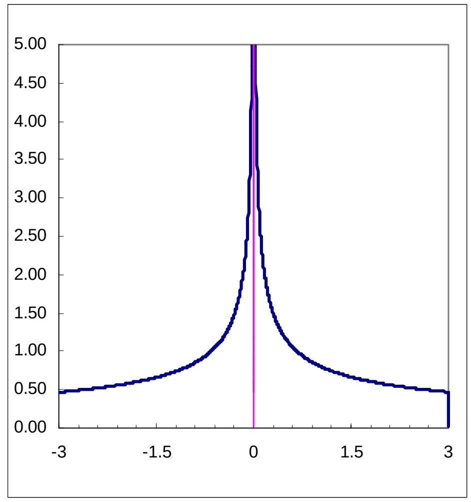
We illustrate Propositions 3 and 4 through numerical examples. A unit of time corresponds to a year, and agents' rate of time preference is 3%. As a normalization, we take $u' = 1$. The production cost in the decentralized market is $c(q) = q^\delta/\delta$, with $\delta = 1.5$, and thus $q^* = 1$. The matching function is $\alpha(n) = n^\eta$, with $\eta = 0.5$, so that buyers and sellers contribute equally to the formation of matches. Sellers' participation cost is $k = 0.1$, which is 20% of the total value of a match, and the cost to adjust prices is $\gamma = 0.001$, which is 1% of the sellers' participation cost.[20] We take as our benchmark $\theta = 0.32$.

[19] This result explains the numerical example provided by Diamond and Felli (1992).

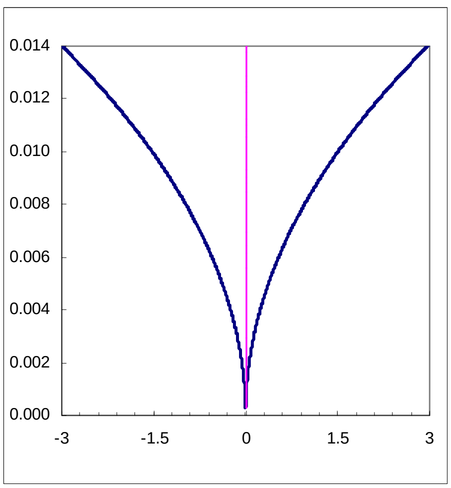
[20] Given our choice for γ, prices are adjusted every 6 months when the inflation rate is about 2%, which broadly accords with the evidence by Bils and Klenow (2002).



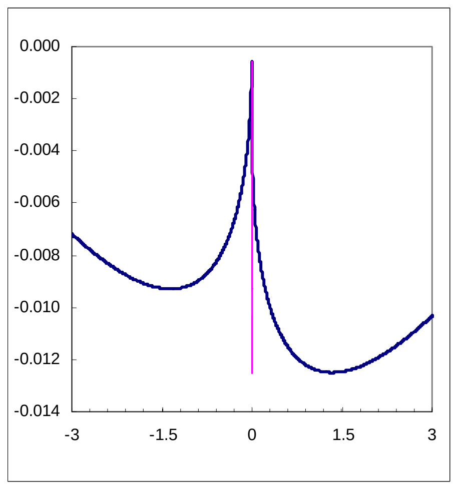
n as a function of π (%).



τ as a function of π.



$|\pi|\,\tau$ as a function of π.

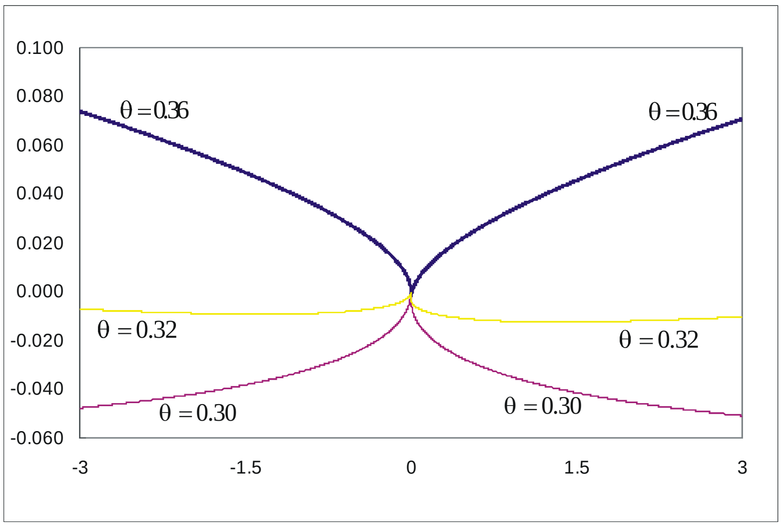


Δ (%) as a function of π.

Our numerical examples show that inflation and deflation have almost symmetric effects on n and τ. A deviation from price stability reduces the number of sellers and raises price dispersion (See Proposition 1). We use $\mathcal{W}^b$ as a measure of welfare, and we determine the welfare cost of inflation, Δ, as the fraction of total consumption that buyers would be willing to sacrifice to be in an economy with price stability. Consider an economy with $\pi = 0$, where buyers' consumption is reduced by a factor $1 - \Delta$. Buyers' welfare is

$$\mathcal{W}_0^b(\Delta) = \alpha(n_0)\left\{u'q^*(1-\Delta) - \left[\theta c(q^*) + (1-\theta)u'q^*\right]\right\},$$

where n_0 is the measure of sellers in equilibrium. Let $\mathcal{W}_\pi^b$ be buyers' welfare in an economy with π inflation. The cost of inflation is measured by the Δ that solves $\mathcal{W}_\pi^b = \mathcal{W}_0^b(\Delta)$. For $\theta = 0.32$ and $\pi > 0$, the welfare cost of inflation, Δ, is a U-shaped function of π. As shown in Proposition 4, the optimal inflation rate is positive. Finally, we plot the welfare cost of inflation for $\theta = 0.3$ and $\theta = 0.36$. In accordance with Proposition 3, in this economy inflation raises welfare whenever $\theta < 1/3$. Also the welfare gain associated with the optimal inflation increases as θ decreases.



Δ as a function of π (%).

4 Monetary economies

We now introduce a transaction role for money by assuming that buyers cannot commit to repay their debts. In the absence of credit arrangements, trades in bilateral matches need to be quid pro quo, and this requires buyers hold money balances. The price of general goods in terms of money is now endogenous. We will determine the optimal monetary policy in the absence of menu costs, and we will investigate how the presence of nominal frictions affects policy.

The quantity of fiat money per buyer is $M(t) > 0$. The growth rate of the money supply is constant over time and equal to $\pi \geq -\rho$; that is, $\dot{M} = \pi M$. Money is injected (withdrawn if $\pi < 0$) by lump-sum transfers (taxes). For simplicity, transfers go only to buyers. We will restrict our attention to steady-state equilibria in which the real value of money M/w is constant over time, i.e., $\dot{w} = \pi w$.

Let $W^b(z)$ be the value of an unmatched buyer holding z units of real money (expressed in terms of the general good). The stochastic time for a buyer to find a seller, denoted T_b, is characterized by an exponential distribution with mean $1/\alpha$. The value function $W^b(z)$ satisfies

$$W^b(z_0) = \max_{\{x(t),z(t)\}} \mathbb{E}\left[\int_0^{T_b} e^{-\rho t} x(t) dt + e^{-\rho T_b} V^b\left[z(T_b)\right]\right] \tag{9}$$

$$\text{s.t.} \quad x + \dot{z} = L - \pi z, \tag{10}$$

$$z(0) = z_0, \tag{11}$$

where $x(t)$ is the net consumption flow of general goods at time t, where $V^b(z)$ is the value function of a matched buyer holding z units of real money, and where the trajectory $\{x(t), z(t)\}$ is contingent on $t < T_b$.[21] The first term on the right-hand side of (9) is the utility of consumption minus the disutility of production over the time interval $[0, T_b]$. The second term is the present value of being matched at time T^b with $z(T_b)$ units of real money. Equation (10) is a budget identity. The term L on the right-hand side is a lump-sum transfer expressed in terms

[21] Implicitly, we allow for jumps in the state variables. For a presentation of optimal control problems with jumps in state variables, see Seierstad and Sydsaeter (1987, chapter 3).

of the general good, and the last term reflects the fact that real balances depreciate at rate π.[22] The initial condition for real balances is given by (11).

From the assumption that T_b is exponentially distributed, (9) can be rewritten

$$W^b(z_0) = \max_{\{x(t),z(t)\}} \int_0^{\infty} e^{-[\rho+\alpha(n)]t} \left[x(t) + \alpha(n)V^b\left[z(t)\right]\right] dt, \tag{12}$$

subject to (10) and (11).[23] Interestingly, (12) is analogous to a deterministic optimal control problem in which the effective discount rate is $\rho + \alpha(n)$ and the instantaneous utility is $x + \alpha(n)V^b(z)$.

Lemma 2 *Assume $V^b(z)$ is strictly concave. Buyers adjust their real balances instantly to $\hat{z}$ that satisfies*

$$V_z^b(z) = 1 + \frac{\rho + \pi}{\alpha(n)}. \tag{13}$$

The left-hand side of (13) is the benefit for the buyer of an additional unit of real balances, whereas the right-hand side is the marginal cost of real balances. This cost, measured in terms of the general good, is the sum of the forgone unit of the general good and the cost of holding real balances, as measured by the sum of the discount rate and the inflation rate, over a period of time of length $1/\alpha(n)$. Assuming $V^b(z)$ is strictly concave (it will be), the solution to (13) is unique and the steady-state distribution of real balances across buyers is degenerate at $z = \hat{z}$. Given that the buyer adjusts his real balances to $\hat{z}$ instantly, we have $W^b(z) = -(\hat{z}-z)+W^b(\hat{z})$, and in particular, $W^b(z) = z + W^b(0)$.

The value function of a matched buyer satisfies

$$\begin{aligned} V^b(z) &= \theta\left\{u'q_b(z) + W^b\left[z - d_b(z)\right]\right\} \\ &+(1-\theta)\int\left\{u'q_s(z,p) + W^b\left[z - pq_s(z,p)\right]\right\} dH(p), \end{aligned} \tag{14}$$

where q_s depends on both the buyer's real balances and the real price p posted by the seller. With probability θ, the buyer has the ability to make an offer (q_b, d_b). He enjoys the utility of

[22] Let m be the buyer's nominal balances. Then, $z = m/w$ and $\dot{z} = -(\dot{w}/w)(m/w) + \dot{m}/w$. To obtain (10), use the fact that $\dot{w}/w = \pi$ and $\dot{m}/w = -x + L$.

[23] See the Appendix for a derivation of (12).

consumption $u'q_b$ and becomes unmatched with $z - d_b$ units of real balances. With probability $1-\theta$, the buyer trades at the posted price p and consumes q_s units of goods for pq_s units of real balances, where q_s is determined as before. Using the linearity of $W^b(z)$, (14) can be rewritten

$$V^b(z) = \theta \left\{ u'q_b(z) - d_b(z) \right\}$$
$$+(1-\theta) \int \left(u' - p \right) q_s(z,p) dH(p) + z + W^b(0). \tag{15}$$

The seller's value function obeys the Bellman equation (2), where G is now a function of $F(z)$, the distribution of real balances across buyers,

$$G(p) = \int (1-\theta) \left\{ q_s(z,p)p - c\left[q_s(z,p)\right] \right\} + \theta \left\{ d_b(z) - c[q_b(z)] \right\} dF(z). \tag{16}$$

Let us turn to the determination of prices. In the fraction θ of the matches where the buyer has the ability to make a take-it-or-leave-it offer, he proposes (q_b, d_b) that satisfies

$$\max_{(q_b,d_b)} \left[u'q_b - d_b \right] \quad \text{s.t.} \ -c(q_b) + d_b = 0 \quad \text{and} \quad d_b \leq z. \tag{17}$$

The main difference with respect to the previous section is that the transfer d_b is constrained by the buyer's (monetary) wealth. The solution to (17) is

$$q_b = \begin{cases} q^* & \text{if} \quad z \geq z^* \equiv c(q^*) \\ c^{-1}(z) & \text{otherwise} \end{cases}. \tag{18}$$

According to (18), if $z > z^*$, then $d_b \leq z$ is not binding and agents trade the efficient quantity q^*. If $z < z^*$, then the constraint binds and the buyer spends all his real balances to buy less than q^*. In the fraction $1-\theta$ of the matches where agents trade at the posted price, the buyer demands z/p if $u' \geq p$ and 0 otherwise, and sellers are willing to produce up to $\bar{q}$ such that $c'(\bar{q}) = p$. Therefore,

$$q_s(z,p) = \begin{cases} \min\left(c'^{-1}(p), \frac{z}{p}\right), & \forall p \leq u', \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

4.1 Flexible prices

The case where prices can be adjusted at no cost ($\gamma = 0$) will allow us to contrast the monetary economy with the economy described in the previous section.[24] In particular, we will show that even though inflation drives sellers out of the market in both versions of the model, this effect is not welfare-enhancing in the monetary economy, as long as prices are flexible.

When prices can be adjusted at no cost, the sellers' optimal pricing strategy is $p = u'$ for any distribution of real balances $F(z)$. The reasoning is similar to the one in Lemma 1. If buyers choose $p > u'$, they make no trade, and sellers have no incentive to choose a price lower than u' since for all $p \leq u'$ buyers spend all their real balances. This pricing strategy allows sellers to extract all the surplus of a match whenever agents trade at the posted price. From (15), the Bellman equation for the value function of a buyer can be simplified to

$$V^b(z) = \theta \left\{ u' q_b(z) - c\left[q_b(z)\right] \right\} + z + W^b(0).$$

Differentiate $V^b(z)$ to get

$$V_z^b(z) = \begin{cases} \theta \left\{ \frac{u'}{c'[q_b(z)]} - 1 \right\} + 1 & \text{if} \quad z < z^*, \\ 1 \quad \text{otherwise.} \end{cases} \tag{20}$$

Equation (20) can be interpreted as follows. Consider a match where the buyer makes a take-it-or-leave-it offer. If the buyer brings one additional unit of real balances, he increases his consumption by $1/c'(q_b)$, which is worth $u'/c'(q_b)$ in terms of utility. Consider next a match where the buyer trades at the posted price. The buyer can increase his consumption by $1/u'$, which is worth 1 in terms of utility. From (20), $V_{zz}^b < 0$ for all $z < z^*$. From Lemma 2, there is a unique solution $\hat{z} < z^*$ to (13), and the distribution of real balances across buyers is degenerate.[25] Furthermore, (13) and (20) yield:

$$\frac{\rho + \pi}{\alpha(n)\theta} = \frac{u'}{c'\left[q_b(z)\right]} - 1. \tag{21}$$

[24] This model is related to the search equilibrium described in Rocheteau and Wright (2005).

[25] To check that $z < z^*$, note first that $V_z^b(z)$ is strictly decreasing for all $z < z^*$. Furthermore, $\rho + \alpha(n) + \pi > \alpha(n) V_z^b(z)$ for all $z \geq z^*$ and for all $\pi > -\rho$.

Free entry of sellers implies $W^s(0) = 0$, which gives from (2)

$$\frac{\alpha(n)}{n}(1-\theta)\left\{u'q_s(z,u') - c\left[q_s(z,u')\right]\right\} = k. \tag{22}$$

Definition 2 *A monetary equilibrium with flexible prices is a list* (z, n) *that satisfies (21) and (22).*

Assume $\pi > -\rho$ (We will treat the case $\pi = -\rho$ separately). We cannot rule out multiplicity of steady-state equilibria. When such multiplicity arises, we focus on the equilibrium with the highest values for q and n as it converges to an equilibrium at the Friedman rule.

Proposition 5 *If* $\theta = 0$ *or* $\theta = 1$, *there is no monetary equilibrium. If* $\theta \in (0, 1)$, *there exists* $\bar{\pi} > -\rho$ *such that a monetary equilibrium exists for all* $\pi \in (-\rho, \bar{\pi})$. *At the equilibrium with the highest* z, $\partial z/\partial\pi < 0$ *and* $\partial n/\partial\pi < 0$.

If $\theta = 0$, there is no demand for real balances and hence no monetary equilibrium. Similarly, if $\theta = 1$, there is no active equilibrium because sellers have no incentive to enter the market. So, for a monetary equilibrium, we need $\theta \in (0, 1)$. In contrast to the model in the previous section, inflation is no longer neutral when $\gamma = 0$. An increase in π reduces the quantities traded in bilateral matches as well as the measure of sellers in the market. Therefore, one may conjecture that the result for cahsless economies, that inflation is welfare improving when n is too high, carries over to monetary economies. The following proposition shows that this conjecture is wrong.

Proposition 6 *Welfare is decreasing with* π, *and the optimal monetary policy is the Friedman rule. Furthermore,* $\lim_{\pi\downarrow-\rho} q_b = q^*$ *and* $\lim_{\pi\downarrow-\rho} q_s < q^*$.

According to Proposition 6, the monetary equilibrium is inefficient since the quantities traded in matches where agents trade at the posted price are always inefficiently low, even at the limit when π approaches $-\rho$. This monopolistic competition inefficiency is related to the fact that sellers do not internalize the effects of their pricing decisions on buyers' real balances.[26] Even if the quantities traded in bilateral matches were efficient, the entry of sellers would generically be inefficient because of the presence of search externalities.

Let us turn to the case $\pi = -\rho$. Denote n^* the value of n that satisfies (22) when $q_s = q^*$.

Proposition 7 *If $\pi = -\rho$ then any (z, n) such that $z \in [c(q^*), u'q^*]$ and n satisfies (22) is an equilibrium. Equilibria are strictly positively Pareto-ranked according to z. There is an equilibrium that generates the first-best allocation iff $\theta = 1 - \eta(n^*)$.*

From the previous proposition, there is a real indeterminacy at the Friedman rule. There exists a continuum of equilibria and these equilibria are Pareto-ranked. Intuitively, when $\pi + \rho = 0$ there is no cost of holding real balances so that buyers are indifferent between any level of real balances above $c(q^*)$. However, an increase in real balances above $c(q^*)$ allows sellers to extract a higher surplus in matches where buyers trade at the posted price. If one selects the equilibrium by taking the limit $\pi \to -\rho$, this equilibrium corresponds to the one with the lowest welfare, i.e., the one with the lowest real balances.

4.2 Sticky prices

Assume now that sellers must incur a fixed cost to change prices. As in the previous section, we focus on steady-state equilibria in which the distribution of (real) prices, $H(p)$, is time-invariant and price adjustments are uniformly staggered. The seller's value function obeys a flow Bellman equation analogous to condition (2), but where G is given by (16). The pricing policy of sellers is still given by Lemma 1.

We describe the buyer's choice of real balances in the case where the growth rate of the money supply is positive ($\pi > 0$). If a buyer meets a seller at random, the price posted by the seller is $p(h) = u'e^{-\pi h}$, where h is uniformly distributed over $[0, \tau]$. The quantity q_s satisfies

$$q_s(z, u'e^{-\pi h}) = \begin{cases} z/u'e^{-\pi h} & \text{if } h \leq \tilde{h}, \\ c'^{-1}\left(u'e^{-\pi h}\right) & \text{if } h > \tilde{h}, \end{cases}$$

[26] This inefficiency should be distinguished from the bargaining inefficiency based on the nonmonotonicity of the generalized Nash solution in Lagos and Wright (2005).

where $\tilde{h}$ is the value of h such that $c'(q_s) = p(\tilde{h})$; i.e., $ze^{\pi\tilde{h}} = u'c'^{-1}\left(u'e^{-\pi\tilde{h}}\right)$. The seller serves all the buyer's demand if $p(h) \geq p(\tilde{h})$; otherwise, the buyer is rationed. From (15) and (19), the expected utility of a matched buyer can be rewritten as[27]

$$V^b(z) = \theta\left\{u'q_b(z) - c\left[q_b(z)\right]\right\} + (1-\theta)z\int_0^{\min\left[\tilde{h}(z),\tau\right]} \tau^{-1}\left(e^{\pi h} - 1\right)dh$$

$$+(1-\theta)u'\int_{\min\left[\tilde{h}(z),\tau\right]}^{\tau} \tau^{-1}c'^{-1}\left(u'e^{-\pi h}\right)\left(1 - e^{-\pi h}\right)dh + z + W^b(0). \tag{23}$$

Let us interpret the second term on the right-hand side of (23). If $h < \tilde{h}$, the seller satisfies all the buyer's demand and the buyer's surplus is equal to $(u'-p)q_s = z\left(\frac{u'-p}{p}\right) = z\left(e^{\pi h} - 1\right) > 0$. Therefore, the buyer can extract a positive surplus even when trading at the posted price. The third term on the right-hand side has a similar interpretation.

Differentiate $V^b(z)$ to get

$$V_z^b(z) = \theta\left[\frac{u'}{c'\left[q_b(z)\right]} - 1\right]^+ + (1-\theta)\int_0^{\min\left[\tilde{h}(z),\tau\right]} \tau^{-1}\left(e^{\pi h} - 1\right)dh + 1, \tag{24}$$

where $[x]^+ = \max(x, 0)$. If the buyer can make a take-it-or-leave-it-offer, one additional unit of real balances allows him to raise his utility by $\frac{u'}{c'[q_b(z)]}$. If the buyer trades at the posted price, $p(h) = u'e^{-\pi h}$, and assuming that the seller satisfies all his demand, an additional unit of real balances allows him to buy $e^{\pi h}/u'$ units of special goods which is worth $e^{\pi h}$ in terms of utility.

From (24), $V^b(z)$ is concave, and strictly concave for all $z < z^*$ and for all z such that $\tilde{h}(z) < \tau$. From (13) and (24),

$$\frac{\rho + \pi}{\alpha(n)} = \theta\left[\frac{u'}{c'\left(q_b\right)} - 1\right]^+ + (1-\theta)\int_0^{\min\left[\tilde{h}(z),\tau\right]} \tau^{-1}\left(e^{\pi h} - 1\right)dh. \tag{25}$$

Inflation has two opposite effects on the value of money. It raises the opportunity cost of holding cash, the left-hand side of (25), and it transfers some market power to the buyer, the second term on right-hand side of (25). The second effect is absent from the model with flexible prices.

[27] To obtain (23), we use the fact that for all $h < \tilde{h}$, $(u' - p\phi)\,q_s = ze^{\pi h} - z$, whereas for all $h > \tilde{h}$, $(u' - p\phi)\,q_s = u'\left(1 - e^{-\pi h}\right)c'^{-1}(u'e^{-\pi h})$.

The buyer's choice of real balances in the case of deflation ($\pi < 0$) is derived using similar reasoning,

$$\frac{\rho + \pi}{\alpha(n)} = \theta \left[\frac{u'}{c'(q_b)} - 1 \right]^+ + (1 - \theta) \int_0^{\min\left[\tilde{h}(z), \tau\right]} \tau^{-1} \left[e^{-\pi l} - 1 \right] dl, \tag{26}$$

where $\tilde{h}(z)$ satisfies $ze^{-\pi \tilde{h}} = u'c'^{-1}(u'e^{\pi \tilde{h}})$.

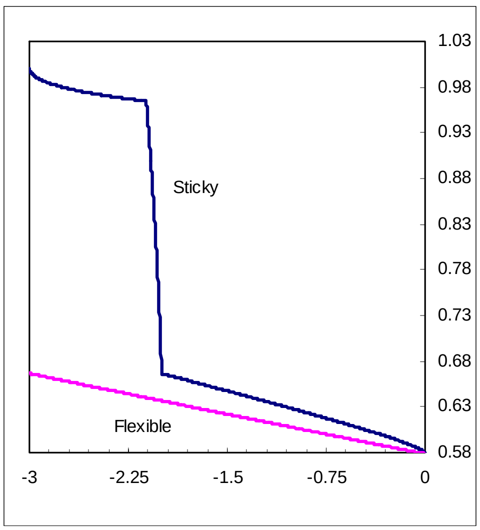
Definition 3 *A steady-state monetary equilibrium with menu cost is a list* (z, n, τ) *that satisfies (5), (7) and, if* $\pi > 0$, *(25), or if* $\pi < 0$, *(26).*

At the Friedman rule, $\rho + \pi = 0$ and, from (26), $q_b = q^*$, and $\tilde{h}(z) = 0$. Therefore, the quantities produced and consumed in matches where agents trade at the posted price obey $c'(q_s) = u'e^{-|\pi|h}$ for all $h \in [0, \tau]$. In particular, $z = u'q^*$ and $q_s(0) = q^*$. The economy is then analogous to the cashless economy studied in the previous section.

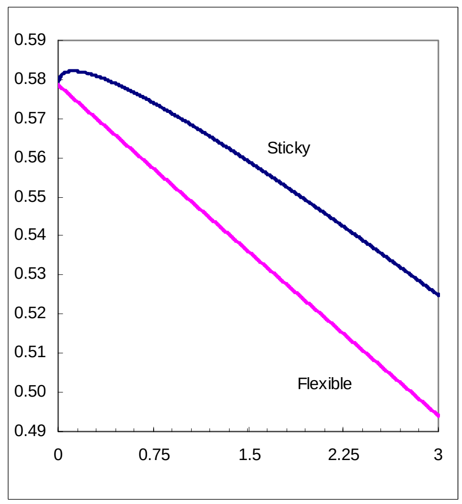
4.3 Numerical examples

The model with endogenous real balances and state-dependent pricing is hard to study analytically. Therefore, we conduct our analysis through numerical examples. While we report results for only one set of parameter values, the one used in the previous section, our reported results are fairly typical of all the examples we have considered. The panels on the left (right) plot the endogenous variables for negative (positive) inflation rates. The lines labelled "Flexible" indicate the values for z and n in the absence of menu costs.

In presence of menu costs and positive inflation (right panel), z is a hump-shaped function of π. As outlined in (25), inflation raises the cost of holding real balances, but it also allows buyers to extract a larger share of the gains from trade. For very low but positive inflation rates, the second effect dominates, whereas above a certain threshold, the first effect takes over. Note also that real balances are higher in the presence of menu costs since the second effect of inflation is absent from the model with flexible prices. Consider next the case of negative money growth rates (left panel). An increase in the deflation rate reduces the cost of holding real balances and it raises buyers' average share in the gains from trade. As a consequence, z increases. The relationship between z and π exhibits an inflection point when z is in the neighborhood of $z^* = 2/3$. The intuition for this is as follows. At $z = z^*$, $\tau < \tilde{h}$ so that buyers are never rationed. When the inflation rate falls below the value that generates z^*, buyers increase their real balances until they get rationed in some matches: z increases to the value that satisfies $\tilde{h}(z) = \tau$. It is also interesting to notice that the presence of nominal frictions eliminates the real indeterminacy of the flexible price economy at the Friedman rule. As π tends to $-\rho$, z approaches $u'q^*$, the level of real balances that maximizes the seller's surplus in the flexible price economy.

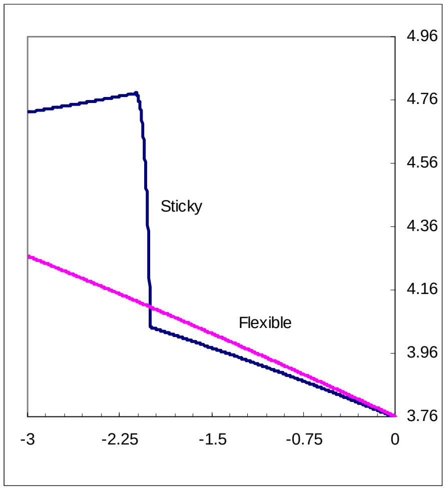


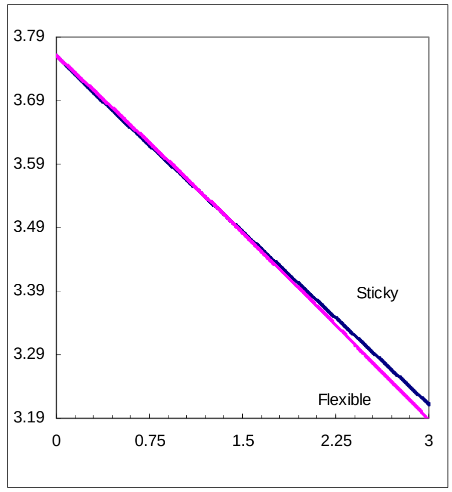
z as a function of π (%).



z as a function of π (%).

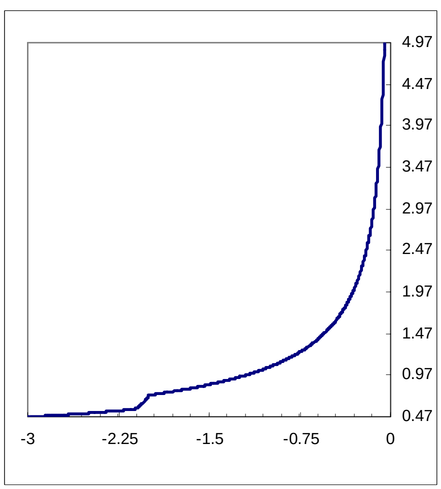
Let us turn to the effects of positive inflation on the number of sellers. On the one hand, inflation reduces sellers' incentives to enter the market since they have to readjust prices more frequently. On the other hand, for low inflation rates, sellers benefit from meeting buyers with higher real balances. For our parameter values, the first effect dominates and n decreases with π. Deflation has two effects on the measure of sellers. Since buyers carry more real balances, sellers have higher incentives to enter the market. But deflation also implies that sellers need to readjust their prices more often. For low deflation rates, the first effect dominates while when π gets closer to the Friedman rule, the second effect dominates. The inflection point for real balances in the neighborhood of $z = z^*$ is associated with a sharp increase in n.

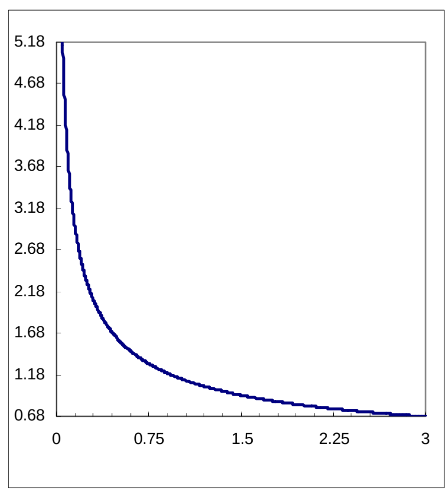




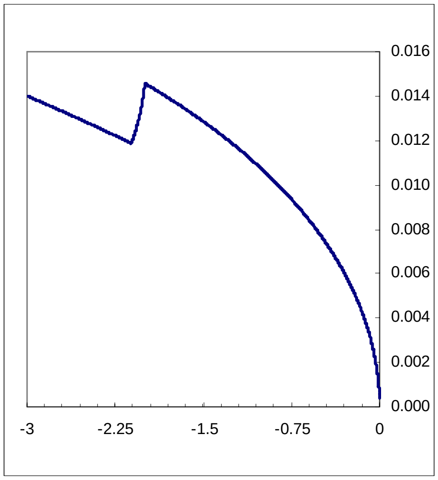
n as a function of π (%) n as a function of π (%)

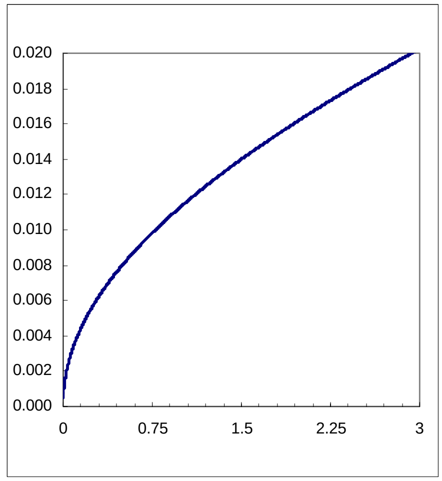
The effects of inflation on the length of the (S, s) cycle and price dispersion are in accordance with those obtained in cashless economies. As inflation increases, sellers need to readjust their prices more often but price dispersion increases.





τ as a function of π (deflation) τ as a function of π (inflation)

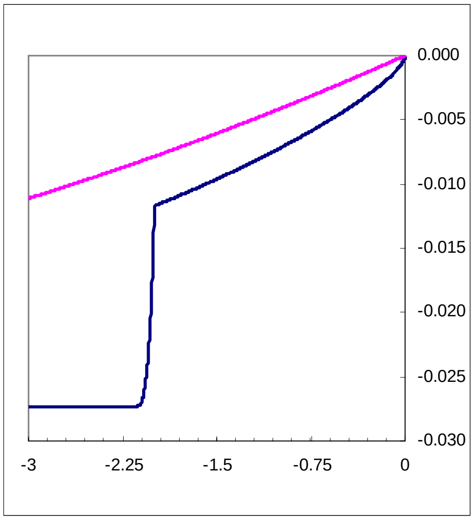


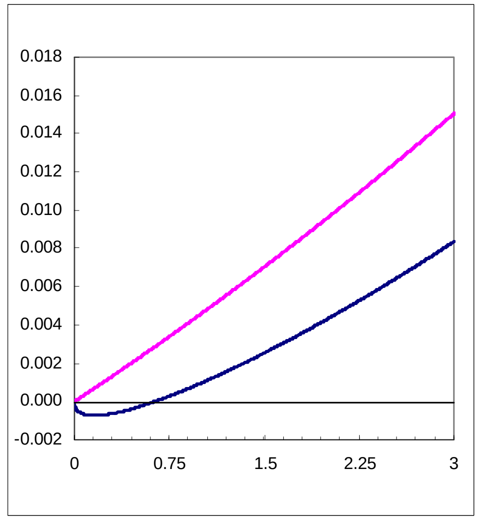


$|\pi|\,\tau$ as a function of π (%)

$\pi\tau$ as a function of π (%).

For positive money growth rates, the welfare cost of inflation is a U-shaped function of π. A small increase in inflation above price stability is welfare-improving because inflation raises buyers' gains from trade and therefore their incentives to invest in real balances. Note however that the welfare gains of positive inflation are tiny, less than 0.1% of consumption. A stronger effect occurs when one reduces π below 0 since lowering the inflation rate reduces the cost of holding real balances. In particular, the sharp increase of real balances in the neighborhood of $z = z^*$ has a large positive effect on welfare. For our example, the welfare gain of reducing π from 0 to the Friedman rule is about 2.5% of consumption. So, deflationary policies dominate inflationary ones.



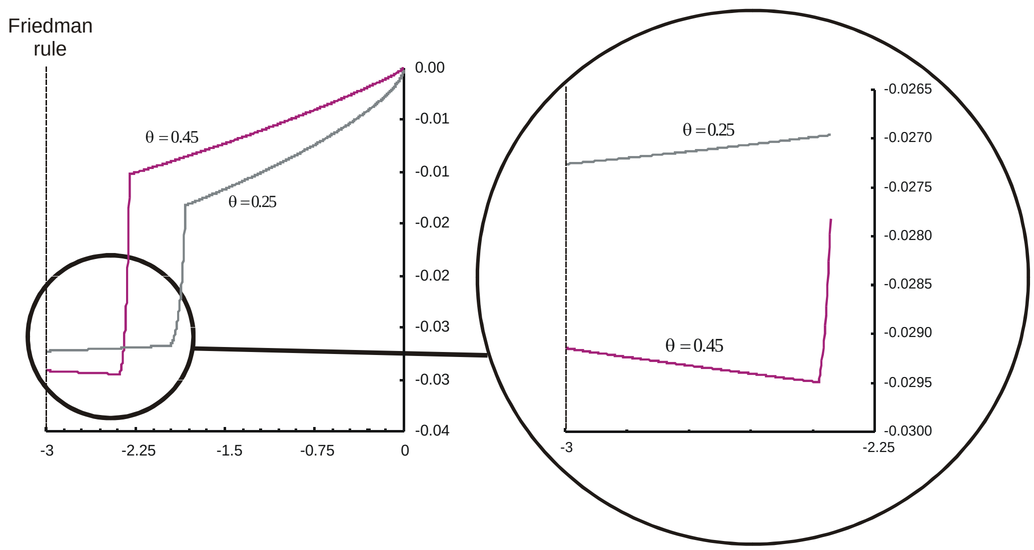


Δ as a function of π (%) $\qquad$ Δ as a function of π (%)

To summarize, the presence of nominal rigidities can explain why a small positive inflation can generate a higher welfare than price stability. Assuming prices are sticky, as inflation increases buyers are able to extract larger gains from trades. As a consequence, they have incentives to increase their real balances which affects welfare positively. However, when looking at the optimal monetary policy, deflation does better than inflation. This contrasts with our result for cashless economies. The reason for this difference is that our model incorporates a monetary wedge. In the presence of sticky prices, deflation reduces sellers' market power, just as inflation does. In addition, deflation reduces the cost of holding real balances.

4.4 The Friedman rule

We have seen that in presence of nominal rigidities and a monetary wedge, the optimal inflation rate is negative. However, it is not clear from the previous Figures whether the Friedman rule is optimal or not. Close to the Friedman rule, the economy behaves similarly to the cashless economy. In particular, the number of sellers decreases as π decreases (equivalently, the deflation rate decreases). If the number of sellers is inefficiently high at the Friedman rule, the Friedman rule is optimal since an increase of the inflation rate makes the number of sellers even higher. The case where there are too many sellers because of congestion externalities corresponds to low values of θ. In the example below, the Friedman rule is optimal for $\theta = 0.25$. In contrast, if the number of sellers is too low at the Friedman rule, a deviation from the Friedman rule can be optimal because an increase in inflation raises the entry of sellers. The case where there are too few sellers corresponds to large values of θ. In the example below, the Friedman rule is not optimal for $\theta = 0.45$. We found that the threshold for θ above which the Friedman rule is no longer optimal is slighty above 1/3, the threshold for θ above which a deviation from price stability is suboptimal in cashless economies. So the extent of search externalities matters in cashless economies to explain the optimality of price stability while it matters in monetary economies to explain the optimality of the Friedman rule.



Δ as a function of π. (Neighborhood of Friedman rule)

5 Conclusion

We considered a model of monetary policy that incorporates realistic frictions in the trading process and price formation. We studied first a version of the model in which there is no transaction demand for money. We established a simple condition under which deviations from price stability could raise welfare. In the presence of menu costs, inflation reduces sellers' market power, which is beneficial to society when there are congestion effects in the goods market. We have also shown that inflationary policies dominate deflationary ones. So this model provides some rationale for why positive inflation should be preferred to price stability.

We then extended the model to allow for monetary exchange. When prices are flexible, the optimal monetary policy is given by the Friedman rule, but equilibrium is still inefficient. In the presence of menu costs, and for non-negative inflation rates, welfare is a hump-shaped function of the inflation rate. Again, inflation erodes sellers' market power, which has a positive effect on individual real balances, and in some cases, inflation also ameliorates congestion. However, deflation is better than inflation since it also reduces the cost of holding real balances. The optimal monetary policy may or may not correspond to the Friedman rule depending on the extent of search externalities. The results are summarized in the following table.

	Flexible prices	**Sticky prices**
No monetary wedge	π is neutral	$\pi^* = 0$ if θ high $\pi^* > 0$ if θ low
Monetary wedge	$\pi^* = -\rho$	$\pi^* > -\rho$ if θ high $\pi^* = -\rho$ if θ low

Summary of the results

On a methodological note, we have shown how models of monetary exchange can be extended to incorporate the sticky prices that some economists think matter for monetary policy. Introducing a cost to changing nominal prices provides a reason why deviations from the Friedman rule might be efficient – something which is difficult to achieve in the previous models of monetary theory. Our analysis has also illustrated how the combined effects of trading and nominal frictions matter for the design of monetary policy.

Appendix

A1. Derivation of (2)

The seller's value function $W^s(h)$ obeys the following Bellman equation:

$$W^s(h) = \Pr\left[T_s \leq \tau - h\right]$$
$$\times \mathbb{E}\left[\int_0^{T_s} e^{-\rho t}(-k)\,dt + e^{-\rho T_s}\left[G\left[p(h+T_s)\right] + W^s(h+T_s)\right] \middle| T_s \leq \tau - h \right]$$
$$+ \Pr\left[T_s > \tau - h\right] \left\{ \int_0^{\tau - h} e^{-\rho t}(-k)\,dt + e^{-\rho(\tau-h)} W^s(\tau) \right\}, \tag{27}$$

where τ is the length of the period of time between two price adjustments, which has the following interpretation. If $T_s \leq \tau - h$, then the seller meets a buyer before he readjusts his price, and his expected surplus is $G\left[p(h+T_s)\right]$, where $p(h+T_s)$ is the price posted by the seller. If $T_s > \tau - h$, then no trade occurs before the seller's real price hits the trigger point s.

The distribution for T_s conditional on $T_s \leq \tau - h$ is a truncated exponential distribution. Thus, $\Pr\left[T_s \leq \tau - h\right] = 1 - e^{-\frac{\alpha(n)}{n}(\tau - h)}$. Consequently, (27) can be rewritten

$$W^s(h) =$$
$$\int_0^{\tau-h} \frac{\alpha(n)}{n} e^{-\frac{\alpha(n)}{n}t} \left[\frac{(-k)\left(1 - e^{-\rho t}\right)}{\rho} + e^{-\rho t}\left[G\left[p(h+t)\right] + W^s(h+t)\right] \right] dt$$
$$+ e^{-\frac{\alpha(n)}{n}(\tau-h)} \frac{(-k)}{\rho}\left(1 - e^{-\rho(\tau-h)}\right) + e^{-\left(\rho + \frac{\alpha(n)}{n}\right)(\tau-h)} W^s(\tau). \tag{28}$$

Using the change of variable $u = h + t$, (28) yields

$$W^s(h) =$$
$$\int_h^{\tau} \frac{\alpha(n)}{n} e^{-\frac{\alpha(n)}{n}(u-h)} \left[\frac{(-k)\left(1 - e^{-\rho(u-h)}\right)}{\rho} + e^{-\rho(u-h)}\left[G\left[p(u)\right] + W^s(u)\right] \right] du$$
$$+ e^{-\frac{\alpha(n)}{n}(\tau-h)} \frac{(-k)}{\rho}\left(1 - e^{-\rho(\tau-h)}\right) + e^{-\left(\rho + \frac{\alpha(n)}{n}\right)(\tau-h)} W^s(\tau). \tag{29}$$

Differentiate (29) with respect to h to obtain

$$\frac{\partial W^s(h)}{\partial h} = -\frac{\alpha(n)}{n} G\left[p(h)\right] + k + \rho W^s(h).$$

A2. Proof of Lemma 1.

We prove Lemma 1 for the case where buyers hold real balances z and cannot use credit. The distribution of real balances across buyers is $F(z)$.

Case 1: $\gamma = 0$. The seller chooses p in order to maximize

$$G(p) \equiv (1-\theta) \int \max_{q \leq z/p} \left[pq - c\left(q\right)\right] \mathbf{1}_{\{p \leq u'\}} dF(z), \tag{30}$$

where $\mathbf{1}_{\{p \leq u'\}}$ is an indicator function that is equal to 1 if $p \leq u'$ and 0 otherwise. According to (30), in each match the seller chooses the quantity q to produce, subject to the constraint that q is not greater than the buyer's demand. If $p > u'$ the buyer's demand is 0, and if $p \leq u'$ then the buyer's demand is z/p. Denote $\xi(z)$ the Lagrange multiplier corresponding to $pq \leq z$. The seller's problem can be rewritten as

$$\max_p \int \max_q \left\{ \left[pq - c\left(q\right)\right] \mathbf{1}_{\{p \leq u'\}} + \xi(z) \left[z - pq\right] \right\} dF(z). \tag{31}$$

Assume $p \leq u'$ so that $\mathbf{1}_{\{p \leq u'\}} = 1$. The first-order condition for $q(z)$ is

$$p\left[1 - \xi(z)\right] = c'\left[q(z)\right]. \tag{32}$$

Differentiate $G(p)$ and use (32) to obtain

$$G'(p) = (1-\theta) \int \frac{q(z) c'\left[q(z)\right]}{p} dF(z) > 0, \quad \forall p \leq u'.$$

Therefore, the optimal price is $p = u'$.

Case 2: $\gamma > 0$ and $\pi > 0$. First, we show $S = u'$ using a proof by contradiction. Assume that the optimal (S, s) rule is such that $S > u'$. Using the fact that $G(p) = 0$ for all $p > u'$, (4) yields

$$W^s(0) = -\int_0^{\ln(S/u')/\pi} e^{-\rho t} k dt + \int_{\ln(S/u')/\pi}^{\ln(S/s)/\pi} e^{-\rho t} \left[-k + \frac{\alpha(n)}{n} G(Se^{-\pi t})\right] dt. \tag{33}$$

The first term on the right-hand side corresponds to the interval of time during which the seller's price is above u'. Since this term is negative, we have

$$W^s(0) < \int_{\ln(S/u')/\pi}^{\ln(S/s)/\pi} e^{-\rho t} \left[-k + \frac{\alpha(n)}{n} G(Se^{-\pi t}) \right] dt,$$

which can be re-expressed as

$$W^s(0) < e^{-\rho \frac{\ln(S/u')}{\pi}} \int_{\ln(S/u')/\pi}^{\ln(S/s)/\pi} e^{-\rho\left[t - \frac{\ln(S/u')}{\pi}\right]} \left[-k + \frac{\alpha(n)}{n} G(Se^{-\pi t}) \right] dt.$$

Since $e^{-\rho \frac{\ln(S/u')}{\pi}} < 1$ we have

$$W^s(0) < \int_{\ln(S/u')/\pi}^{\ln(S/s)/\pi} e^{-\rho\left[t - \frac{\ln(S/u')}{\pi}\right]} \left[-k + \frac{\alpha(n)}{n} G(Se^{-\pi t}) \right] dt.$$

Adopt the change of variable $\tilde{t} = t - \frac{\ln(S/u')}{\pi}$ to rewrite the previous inequality as

$$W^s(0) < \int_0^{\ln(u'/s)/\pi} e^{-\rho \tilde{t}} \left[-k + \frac{\alpha(n)}{n} G(u' e^{-\pi \tilde{t}}) \right] d\tilde{t}.$$

Consequently, a profitable deviation is to set $S = u'$ while keeping s unchanged. Therefore, $S > u'$ is not optimal. Consider next a (S, s) rule such that $S < u'$. Since $G(Se^{-\pi t})$ is increasing in S, it can be checked from (4) that for given τ, $W^s(0)$ is a strictly increasing function of S for all $S \leq u'$. Consequently, $S < u'$ is not optimal. Second, to determine the optimal length of the (S, s) cycle, differentiate the right-hand side of (4) with respect to τ to obtain (7).

Case 3: $\gamma > 0$ and $\pi < 0$. The reasoning for $S = u'$ is similar to the one for the case $\pi > 0$. To determine the optimal τ, express $W^s(0)$ as

$$W^s(0) = \int_0^{\tau} e^{-\rho(\tau - t)} \left[-k + \frac{\alpha(n)}{n} G(u' e^{\pi t}) \right] dt. \tag{34}$$

Differentiate (34) with respect to τ and use the fact that $W^s(0) = \gamma$ to obtain (7).

A3. Proof of Proposition 1

We consider three cases.

Case 1: $\gamma = 0$. The seller sets $p = u'$ and $\tau = \infty$. From (5),

$$-k + \frac{\alpha(n)}{n} G(u') = 0, \tag{35}$$

where $G(u') = (1-\theta)[u'q^* - c(q^*)] > 0$. Since $\alpha(n)/n$ is continuous and strictly decreasing, $\lim_{n\to 0} \alpha(n)/n = \infty$ and $\lim_{n\to\infty} \alpha(n)/n = 0$, there exists a unique n that satisfies (35) and it is such that $\partial n/\partial \pi = 0$.

Case 2: $\gamma > 0$ and $\pi > 0$. Using $p(t) = u'e^{-\pi t}$, Equations (5) and (7) yield

$$\max_{\tau \geq 0} \int_0^\tau e^{-\rho t} \left[-k + \frac{\alpha(n)}{n} G(u'e^{-\pi t}) \right] dt = \gamma, \tag{36}$$

where

$$G(p) = (1-\theta) \max_q [pq - c(q)] \quad \text{if } p \leq u'$$

and $G(p) = 0$ otherwise. Let n^f be the value of n that satisfies (35). The left-hand side of (36) tends to ∞ as n approaches 0, and it is equal to 0 for all $n \geq n^f$. Furthermore, for all $n \in \left(0, n^f\right)$, the left-hand side of (36) is strictly decreasing in n. Consequently, there exists a unique $n \in \left[0, n^f\right]$ that satisfies (36). Given n, price dispersion $\pi\tau$ is determined by (7). Totally differentiate (36) to obtain

$$\frac{\partial n}{\partial \pi} = \frac{-n \int_0^\tau e^{-\rho t} \left[t u' e^{-\pi t} G'(u'e^{-\pi t}) \right] dt}{[1 - \eta(n)] \int_0^\tau e^{-\rho t} G(u'e^{-\pi t}) dt} < 0,$$

where $\eta(n) = -\alpha'(n)n/\alpha(n)$. From (7), $\partial \pi\tau/\partial \pi > 0$.

Case 3: $\gamma > 0$ and $\pi < 0$. Using the fact that $p(t) = u'e^{\pi(\tau - t)}$, (5) and (7) can be rewritten as

$$\max_{\tau \geq 0} \int_0^\tau e^{-\rho t} \left[-k + \frac{\alpha(n)}{n} G\left(u'e^{\pi(\tau - t)} \right) \right] dt = \gamma. \tag{37}$$

Following the reasoning of Case 2, it is easy to show that there exists a unique $n \in \left[0, n^f\right]$ that satisfies (37). Furthermore, $\partial n/\partial \pi > 0$ and $\partial |\pi| \tau/\partial \pi < 0$.

A4. Proof of Proposition 2.

In the case $\gamma = 0$, $\mathcal{W} = \mathcal{W}^b = \alpha(n)[u(q) - c(q)] - kn$. The efficient value for n satisfies $\alpha'(n)[u'q^* - c(q^*)] = k$, whereas the equilibrium value for n satisfies $\alpha(n)(1-\theta)[u'q^* - c(q^*)] = nk$. The two coincide iff $\alpha'(n)n/\alpha(n) = 1 - \theta$.

A5. Proof of Proposition 3

Consider the case $\gamma > 0$ and $\pi > 0$. The relationship between n and $\pi\tau$ is given by (7), i.e.,

$$\alpha(n)(1-\theta)\max_q[u'e^{-\pi\tau}q - c(q)] = nk. \tag{38}$$

The welfare metric is

$$\mathcal{W}^b = \alpha(n)\left\{\theta\left[u'q^* - c(q^*)\right] + (1-\theta)\int_0^1 u'q_s(l\pi\tau) - u'e^{-l\pi\tau}q_s(l\pi\tau)dl\right\}.$$

Differentiate and take the limit as $\pi\tau \to 0$ to obtain

$$\frac{d\mathcal{W}^b}{d\pi\tau} = \alpha'(n_0)\theta\left[u'q^* - c(q^*)\right]\frac{dn}{d\pi\tau} + \alpha(n_0)(1-\theta)\frac{u'q^*}{2}. \tag{39}$$

Substitute (**??**) into (39) to get

$$\frac{d\mathcal{W}^b}{d\pi\tau} = \alpha(n)u'q^*\left\{\frac{(1-\theta)}{2} - \frac{\theta\eta(n)}{[1-\eta(n)]}\right\}.$$

It can be checked that $d\mathcal{W}^b/d\pi\tau > 0$ iff $\theta < [1-\eta(n)]/[1+\eta(n)]$. The proof for the deflation case is analogous to the one for the inflation case and is therefore omitted.

A6. Proof of Proposition 4

The proof is by contradiction. Assume the optimal inflation rate is $\pi^* < 0$. From Proposition 1 there is a one-to-one relationship between $\pi\tau$ and π for all $\pi > 0$, and there is a one-to-one relationship between $-\pi\tau$ and π for all $\pi < 0$. Furthermore, since $\gamma \approx 0$, $|\pi|\tau$ approaches 0 as π tends to 0 . Let $\tilde{\pi}$ be the positive inflation rate such that price dispersion $|\pi|\tau$ at $\pi = \tilde{\pi}$ is equal to price dispersion at $\pi = \pi^*$. From (7) the measure of sellers satisfies $n(\tilde{\pi}) > n(\pi^*)$. Therefore, from (8), $\mathcal{W}^b(\tilde{\pi}) > \mathcal{W}^b(\pi^*)$. A contradiction.

A7. Derivation of (12)

Since T_b is exponentially distributed, the maximand on the right-hand side of (9) can be rewritten as

$$RHS = \int_0^\infty\left[\int_0^t e^{-\rho s}x(s)ds + e^{-\rho t}V^b[z(t)]\right]\alpha(n)e^{-\alpha(n)t}dt. \tag{40}$$

Equation (40) yields

$$RHS = \int_0^\infty \int_0^\infty e^{-\rho s} x(s) \alpha(n) e^{-\alpha(n)t} \mathbf{1}_{\{s \leq t\}} ds dt$$
$$+ \int_0^\infty V^b \left[z(t)\right] \alpha(n) e^{-[\rho+\alpha(n)]t} dt. \tag{41}$$

Interchange the order of integration in the repeated integral to get

$$RHS = \int_0^\infty e^{-\rho s} x(s) \int_s^\infty \alpha(n) e^{-[\alpha(n)]t} dt ds$$
$$+ \int_0^\infty \alpha(n) V^b \left[z(t)\right] e^{-[\rho+\alpha(n)]t} dt. \tag{42}$$

Finally, substitute $e^{-[\alpha(n)]s}$ for $\int_s^\infty \alpha(n) e^{-[\alpha(n)]t} dt$ into (42) to obtain

$$\begin{aligned} RHS &= \int_0^\infty x(s) e^{-[\alpha(n)+\rho]s} ds + \int_0^\infty \alpha(n) V^b \left[z(t)\right] e^{-[\rho+\alpha(n)]t} dt, \\ &= \int_0^\infty e^{-[\rho+\alpha(n)]t} \left\{ x(t) + \alpha(n) V^b \left[z(t)\right] \right\} dt. \end{aligned}$$

A8. Proof of Lemma 2

Let λ be the current value costate variable associated with z. The current value Hamiltonian is

$$\mathcal{H}(x, z, \lambda) = x + \alpha(n) V^b (z) + \lambda \left(-x + L - \pi z\right). \tag{43}$$

Assuming an interior solution for z, the necessary conditions from Pontryagin's maximum principle are

$$\lambda = 1, \qquad \forall t \tag{44}$$
$$\rho\lambda = \alpha(n) \left[V_z^b (z) - \lambda\right] - \pi\lambda + \dot{\lambda}, \tag{45}$$

where V_z^b is the derivative of the value function $V^b(z)$. We add the following transversality condition from the Mangasarian sufficiency theorem

$$\lim_{t\to\infty} e^{-[\rho+\alpha(n)]t} \lambda(t) z(t) = 0. \tag{46}$$

To get (13), combine (44) and (45). Assuming $V^b(z)$ is concave, the Hamiltonian $\mathcal{H}(x,z,\lambda)$ is jointly concave in (x,z). Since the transversality condition is satisfied for the solution given by (44) and (45), it is a maximum. Furthermore, (10) implies $\dot{\lambda}=0$ and z satisfies (13). The uniqueness of the solution to (13) follows from the strict concavity of $V^b(z)$. Note that the state variable z jumps to the solution of (13) instantly.

A9. Proof of Proposition 5

Part 1. There is no monetary equilibrium if $\theta=0$ or $\theta=1$. From (21), if $\theta=0$, then $q=0$ or $n=\infty$. From (22), if $n=\infty$, then $q=0$. As a consequence, $q=0$. From (22), if $\theta=1$, then $n=0$, which from (21) implies $q=0$. In both cases, money is not valued in exchange.

Part 2. If $\theta\in(0,1)$, there exists $\bar{\pi}>-\rho$ such that a monetary equilibrium exists for all $\pi<\bar{\pi}$. At the equilibrium with the highest z, $\partial z/\partial\pi<0$ and $\partial n/\partial\pi<0$. Equilibrium condition (21) can be reexpressed as

$$q=q(n;\pi)\equiv c'^{-1}\left[\frac{u'\alpha(n)\theta}{\rho+\pi+\alpha(n)\theta}\right], \tag{47}$$

with $q(0;\pi)=0$ and $q(\infty;\pi)=q^*$. Furthermore, $q(n;\pi)$ is strictly increasing in n and strictly decreasing in π. Using (47), equilibrium condition (22) can be reformulated as $\Gamma(n;\pi)=0$ with

$$\Gamma(n;\pi)\equiv(1-\theta)\frac{\alpha(n)}{n}\left\{c\left[q(n;\pi)\right]-c\left(\frac{c\left[q(n;\pi)\right]}{u'}\right)\right\}-k. \tag{48}$$

We first show that under the Friedman rule ($\pi=-\rho$) a monetary equilibrium always exists. From (47), $q(n;-\rho)=q^*$ for all $n>0$. Therefore, given that $c(q^*)<u'q^*$, $\Gamma(0;-\rho)=\infty$ and $\Gamma(\infty;-\rho)=-k$. Consequently, if $\pi=-\rho$ there exists a $n>0$ that satisfies $\Gamma(n;\pi)=0$.

Consider next $\pi>-\rho$. For all $\pi>-\rho$, $\Gamma(\infty;\pi)=-k$. For all $n>0$, $\Gamma(n;\pi)$ is continuous and decreasing in π. Using the continuity of $\Gamma(n;\pi)$ one can deduce that there is a threshold $\bar{\pi}>-\rho$ such that for all $\pi\in(-\rho,\bar{\pi})$ there exists $n>0$ such that $\Gamma(n;\pi)=0$.

Finally, let us show that $\partial z/\partial\pi<0$ and $\partial n/\partial\pi<0$ at the equilibrium with the highest z. Equations (21) and (22) give two positive relationships between z and n. Furthermore, at the equilibrium with the highest z the curve (21) cuts the curve (22) by below in the space (z, n). An increase in π moves the curve (21) upward leading to a decrease of both z and n.

A10. Proof of Proposition 6

The two measures of welfare $\mathcal{W}$ and $\mathcal{W}^b$ are the same in the flexible price economy,

$$\mathcal{W} = \alpha(n) \left\{ \theta \left[u' q_b - c(q_b) \right] + (1-\theta) \left[u' q_s - c(q_s) \right] \right\} - nk. \tag{49}$$

From (22) and (49), social welfare in equilibrium reduces to $\mathcal{W} = \alpha(n)\,\theta\,[u'q - c(q)]$. Given that an increase in π reduces both q and n, the optimal monetary policy is the Friedman rule.

Let us turn to the second part of the proposition. Since $z = c(q_b)$ and $c'(q^*) = p = u'$, $q^s = \min[q^*, c(q_b)/u']$. From (18), for all $z \leq z^*$, $q_b \leq q^*$ which implies $c(q_b) \leq c(q^*) < u'q^*$. Therefore, $q_s = c(q_b)/u' < q^*$.

A11. Proof of Proposition 7

From (18), $q_b(z) = q^*$ for all $z \in [c(q^*), u'q^*]$. Consequently, from (21), any $z \in [c(q^*), u'q^*]$ corresponds to an equilibrium when $\pi = -\rho$. Seller's welfare is 0 and buyer's welfare is $\mathcal{W}^b = \alpha[n(z)]\{u'q^* - c(q^*)\}$ where $n(z)$ is the value of n that satisfies (22). Since $n(z)$ is strictly increasing in z, buyer's welfare is strictly increasing in z so that equilibria with higher values for z Pareto-dominate equilibria with lower values for z. The first-best allocation is such that $q_b = q_s = q^*$ and n satisfies $\alpha'(n)[u'q^* - c(q^*)] = k$. From (18) and (19) this requires $z = u'q^*$, and from (22),

$$1 - \theta = \frac{\alpha'(n^*)n^*}{\alpha(n^*)}.$$

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