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# Limited Liability and the Development of Capital Markets

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We study the consequences of the introduction of widespread limited liability for corporations. In the traditional view, limited liability reduces transactions costs and enhances investment incentives for individuals and firms. But this view does not explain several important stylized facts of the British experience, including the slow rate of adoption of limited liability by firms in the years following legal reforms. We construct an alternative model that accounts for this and other features of the nineteenth century British experience. In the model, project risk is private information, and a firm's decision to adopt limited liability may be interpreted in equilibrium as a signal the firm is more likely to default. Hence less risky firms may choose unlimited liability or forego investments entirely. We show the choice of liability rule can lead to "development traps," in which profitable investments are not undertaken, through its effect on equilibrium beliefs of uninformed investors in the economy.

Key words: limited liability, capital markets, risk, investment.

JEL code: D24, D92.

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# 1 Introduction

The advent in the nineteenth century of the limited liability corporation is typically regarded as a key step in the development of modern capital markets and a precursor of large-scale industrialization. In the traditional view, limited liability reduced transactions costs associated with issuing marketable securities, which in turn permitted improved risk sharing and investment on a larger scale than previously.[1]

This perspective on the incorporation decision leaves unexplained a number of aspects of the historical experience with liability reform. In Great Britain, for example, firms were extremely slow to incorporate following the introduction of general limited liability in 1856; most large-scale firms in many industries remained partnerships until the end of the century, relying on private and internal sources of finance in lieu of new debt and equity issues. This decision apparently reflected a widespread belief that limited firms were riskier than average and would face higher costs of capital. Indeed, we present new evidence below that default rates were higher among the earliest limited firms. Moreover, the traditional view suggests that liability reform, in reducing transactions costs, should lead to a rise in economic activity and efficiency. In fact, British economic growth in the late nineteenth century appears to have lagged behind that of Germany, the U.S., and other countries that had introduced the corporate form much earlier.

In this paper, we analyze the effects liability rules can have on incorporation and investment decisions of firms. In the traditional view (Posner, 1976, e.g.), the transfer of risk from shareholders to creditors associated with incorporation should have no impact on the firm's cost of capital or level of investment, as yields on limited liability debt should rise to compensate bondholders for the risk of default. Equity investors therefore receive no transfer through the choice of liability regime, and the value of the firm remains unchanged. Thus limited liability was not a boon to shareholders, nor did it reduce the cost of capital for firms, except through its indirect effects on transactions costs in securities markets.

In contrast, we argue that liability rules can indeed have real effects on the economy, through their impact on the degree of adverse selection in corporate capital markets. In the model, as with Posner's argument, adoption of limited liability yields a risk premium on the firm's debt in equilibrium. In contrast, however, the equilibrium risk premium in the model reflects that

[1] For example, Manne (1967) discusses the difficulties in recovering debts from shareholders under a regime of unlimited liability.

of the average firm: low-risk firms therefore pay too much for debt, while high-risk firms are subsidized in equilibrium. We study how the resulting adverse selection among limited liability firms affects investment in equilibrium, and how the degree of adverse selection can in turn be influenced by the level of transactions costs firms must incur to obtain limited liability.

We summarize the results of the paper as follows. Section 2 discusses the introduction of the modern corporation in Great Britain in 1856. There we cite historical evidence to suggest that the rate of new incorporations following the reforms was extremely low, due at least in part to pessimistic beliefs of investors about the average quality of limited companies. We also produce new data which supports the adverse selection view: failure rates among the earliest limited companies were indeed significantly higher than for a comparable sample of unlimited companies.

Section 3 introduces our model of adverse selection in limited and unlimited liability credit markets. In Section 4 we show that, when firms may choose their liability status, credit markets can have multiple equilibria, each associated with different, self-fulfilling beliefs of investors about the average quality of limited companies and different levels of aggregate investment. Consequently, "animal spirits" of investors play a crucial role in avoiding development traps. We then study the effects of a reform that lowers the costs of incorporation for firms. We show that such a reform tends to increase the number of incorporations and the level of investment in the economy. Because multiple equilibria typically exist in the market, the timing of this response is difficult to predict. The effect of reforms on equilibrium interest rates and adverse selection is more ambiguous, however. We show that in some cases interest rates initially fall and then rise as the costs of incorporation decrease, as first high-quality then low-quality firms are induced to incorporate. Section 5 discusses the qualitative implications of our formal results, and section 6 concludes.

Since our interest is in incorporation and default, the body of the paper focuses on credit (bond) market equilibrium. However, nineteenth century corporations also raised funds through equity issues. In order to demonstrate that both our results and insights also apply to more general environments, in an appendix we extend the model so that entrepreneur can issue outside debt and/or equity. We find that although the existence of an equity market may attenuate the adverse selection effect in the incorporation decision, it cannot, in general, eliminate it.

# 2 Liability reform in Great Britain

## 2.1 Previous literature

General limited liability began in Great Britain with the Joint-Stock Companies Act of 1856, which permitted companies to limit the liability of equity owners for the company's debts to be no greater than their initial investment.[2] Limited liability was to be generally available to firms for a fee—a specific act of Parliament had previously been required for each firm desiring limited status—but new disclosure requirements would be imposed on limited firms.

A puzzling feature of the British experience is the exceptionally slow development of corporate capital markets in the period following the reforms. Many early firms established with limited liability were small and not particularly long-lived (Heaton, 1948). Retentions remained the most important source of finance throughout the latter half of the nineteenth century (Payne, 1978). This slow rate of incorporation was particularly evident among, but not confined to, family-operated firms. In this regard, the experience of English companies appears to have differed from that of companies in Germany, the United States, and other countries, during the period. As Forbes (1986) notes, the reluctance to adopt limited liability in England is particularly striking, given that liability reform occurred later and industrial capital demands were higher there than in the United States.[3]

The reluctance of investors to accept limited liability seems to have been associated with the belief that limited companies would be more prone to default, fraud, and other malfeasance. Contemporary accounts suggest that investors and policy-makers alike were concerned with the way liability rules affected the potential for malfeasance by stock promoters and the functioning of capital markets in general. To its opponents, liability reform would lead to fraud and excessive risk taking, according to Shannon (1931), and enterprise would function better if "kept within salutary bounds by dread of loss." (p. 374) The debates in *Hansard* in the period prior to 1856 indicated general concern that incorporation would undermine the reputation of British merchants with trading partners and general creditors. Hannah (1976) notes that "even those who accepted … its favorable effects

[2]Prior to 1856, limited liability could be obtained only by special act of Parliament, apparently a very expensive process undertaken by relatively few firms.

[3]Between 1863 and 1890, the number of corporate firms in England grew by 400 per cent, compared to almost 1400 per cent in the United States during the same period Forbes (1986).

on investment saw little future for the new companies…. Suppliers and customers, it was thought, would be reluctant to deal with them." (p. 20)

Indeed, these pessimistic beliefs appear largely to have been borne out in the experience of limited firms in the generation following the 1856 reform. Shannon (1932) reported that, of 4839 incorporations in the 1856-65 period, 50.1 per cent ceased operation within three years and 58.5 per cent within five years.[4] These failure rates are particularly high, given that the sample is restricted to publicly traded joint-stock companies, which presumably were among the most stable firms in the period. This record of failure was related to the reluctance of large firms to adopt limited liability. By the 1880s, less than 10 per cent of large-scale businesses had incorporated Forbes (1986).

## 2.2 New evidence

While Shannon's data are suggestive of high default risk among early limited liability corporations, they are not particularly conclusive. High default rates might have been common among all firms in the period, including partnerships and unlimited liability companies. For this reason, we collected new data for the period and compare failure rates of limited and unlimited corporations in the period immediately following the 1856 reform. To do so we exploit a "grandparent" provision of the 1856 legislation: joint-stock companies which had been registered under earlier legislation, and which by statute operated with unlimited liability, were required to register again under the new Act, claiming either limited or unlimited liability. Approximately one-third of these existing firms chose to adopt limited liability, while the rest continued to operate as before. Thus comparing default rates of the two classes of firms appears the most appropriate way to determine whether there was in fact adverse selection among early limited companies. Both groups operated at the same time and comprise approximately the same age-distribution of firms. Moreover, it is possible to control for innate differences in risk among industries in making the comparison.

The data for the study are derived from an 1864 report by the Registrar of Companies to Parliament,[5] which records the liability choices and op-

[4] Many of these companies failed very soon after incorporations, and so might not have raised much capital from outside investors. If these companies are excluded, more conservative estimates of the failure rates are obtained: 22.3 per cent after three years and 35.4 per cent after five years.

[5] *Return of Names, Places of Business, Date of Registration, Nominal Capital, and Number of Shares of Joint Stock Companies*, 1864 (452) LVIII.291.

Table 1: Failure rates, by industry and liability status

| Industry | Number of firms | Failure rate by liability status: Unlimited | Limited | Difference |
|---|---|---|---|---|
| Mining and smelting | 55 | 0·70 | 0·53 | −0·16 |
| | | | | (0·13) |
| Manufacturing | 309 | 0·11 | 0·20 | 0·09* |
| | | | | (0·04) |
| Trade and commerce | 76 | 0·05 | 0·28 | 0·23* |
| | | | | (0·08) |
| Transportation | 31 | 0·50 | 0·19 | −0·31 |
| | | | | (0·17) |
| Finance and real estate | 63 | 0·21 | 0·27 | 0·06 |
| | | | | (0·12) |
| All industries | 534 | 0·17 | 0·27 | 0·10* |
| | | | | (0·04) |

Notes: Standard errors of differences in parentheses. Differences marked with an asterisk are significant at the five per cent level.

erating status of corporations registering under the Act between 1856 and 1864. In the analysis, we focus on the subsample of 534 English companies which were in existence at the time of passage of the Act and which had re-registered prior to 1861; 186 of these adopted limited liability, while the remaining 348 retained unlimited liability.

Table 1 records the failure rates of firms in the sample by broad industry category and by chosen liability status. The table indicates that 27 per cent of limited firms and 17 per cent of unlimited firms had failed by 1864; the difference in aggregate failure rates is significantly different from zero at the five per cent level. Failure rates were higher for limited firms in three of our five broad industry groups. In two industries—manufacturing, which includes public utilities, and trade and commerce, which includes a number of overseas trading ventures—the difference is significant at the five per cent level. Arguably, problems of asymmetric information may have been greater among firms operating abroad, which could explain the greater evidence of adverse selection in the latter industry. Failure rates were in fact lower among limited firms in the remaining two industries, although not significantly so. The difference is more pronounced among firms in the

transportation sector, which includes a number of municipal and regional railways. Since the large railways had earlier been incorporated with limited liability by separate acts of Parliament, investors may have been less troubled by the risks of limited liability in this sector, so that no adverse selection occurred. Thus the data are broadly indicative of the notion that firms choosing limited liability were more likely to fail, and that adverse selection may have occurred.

In summary, historical evidence suggests that firms were slow to adopt limited liability in Great Britain, despite the significant liberalizations of the 1856 reforms. Most large, private firms continued to operate as partnerships, with unlimited liability, until late in the century. Contemporary accounts suggest the reluctance of firms to incorporate was strongly linked to the perception that limited firm posed greater risk of failure for creditors and equity investors. (Indeed, this perception appears to have played a role in the delayed introduction of limited liability in Great Britain, as many policy makers feared limited liability could lead to a crisis of confidence in capital markets.) Our data indicate this perception may have been accurate. In a small sample of highly comparable firms, limited liability companies failed with significantly greater probability than unlimited liability companies, even over a short period of time. Thus adverse selection among limited liability companies appears to have been a reality.

## 3 A model of debt finance

At date 0 an entrepreneur is endowed with an investment project that requires a capital outlay of $K$. With probability $p$, the project is successful and pays $A/p$ at date 1; with complementary probability it is unsuccessful and pays nothing. There are two types of investment projects. A *safe* project, $S$, is successful with probability $p_S$ and a *risky* project, $R$, is successful with probability $p_R > 0$, where $p_S > p_R$. Note that both the safe and risky projects have an expected payoff of $A$.

An entrepreneur can be either *wealthy*, $W$, or *poor*, $P$. A wealthy entrepreneur receives an exogenous private wealth payoff of $w > K$ at date 1, while a poor entrepreneur receives nothing. An entrepreneur is thus one of four possible types, where his type depends upon project risk, $S$ or $R$, and his wealth, $W$ or $P$. An entrepreneur's type is denoted by $t \in \{RP, SP, RW, SW\}$. The prior probability that the entrepreneur is wealthy is $\omega$, where $0 < \omega < 1$, and the prior probability that he is endowed with the safe project is $\sigma$, where $0 < \sigma < 1$. For simplicity, and without loss of generality, we assume that the

two characteristics that define the entrepreneur's type are independently distributed. So, for example, the probability that the entrepreneur is of type $RW$ is $(1-\sigma)\omega$.

The entrepreneur does not have any liquid assets that can be used to finance the investment opportunity at date 0. He must, therefore, raise funds in capital markets if he wishes to undertake the project. The entrepreneur can borrow funds from capital markets in one of two ways. First, the entrepreneur can issue *limited liability* debt. Historical evidence from both the UK and US indicates that it was costly to issue limited liability debt. We model this cost by assuming that the entrepreneur must incorporate in order to be able to issue limited liability debt and that the cost of incorporation is $B > 0$. Hence, an incorporated entrepreneur issues limited liability debt in exchange for $K + B$. Second, the entrepreneur can issue *unlimited liability* debt. Issuing unlimited liability debt does not require that the entrepreneur incorporate. Hence, an unincorporated entrepreneur will issue unlimited liability debt in exchange for $K$.

We assume that the incorporation cost $B$ is below the threshold level $\bar{B} = A - K$ at which projects are profitable even when undertaken by limited liability borrowing, i.e., $A - K - B \geq 0$ for $B \leq \bar{B}$.

If the entrepreneur issues limited liability debt, then the lender is only able to extract the promised debt payment from the project's payoff. That is, if the entrepreneur fails to make the promised payment, the creditor can only seize the value of the entrepreneur's project, up to the promised face value of debt. If the entrepreneur finances his investment project with unlimited liability debt, then the lender is able to extract the promised payment from both the project payoff *and* the entrepreneur's private wealth. If the entrepreneur fails to repay his unlimited liability debt in full because he lacks resources, then he suffers a large loss in utility.[6]

At date 0, the entrepreneur's type is private information. We assume

[6]Borrowers who failed to repay their debts were put in debtors' prison. Historical evidence suggests that this was not a very pleasant outcome. Alternatively, if one does not want to appeal to a large loss in utility associated with failure to repay unlimited liability debt, one can instead assume that the entrepreneur's date 1 wealth is *partially provable*. That is, if a entrepreneur is wealthy, he can prove that he is, in fact, wealthy by, for example, providing appropriate documentation. However, by withholding this documentation, a wealthy entrepreneur will be observationally equivalent to a poor entrepreneur at date 0. Under the assumption of partial provability, it can be shown, that investors will extend unlimited liability loans only to those entrepreneurs who can prove they are wealthy. Hence, assuming partially provability of wealth will generate identical equilibrium outcomes to assuming that anyone can borrow using unlimited liability debt, but failure to pay entails a large loss in utility.

that there exist at least two lenders in the capital market, each with enough resources to satisfy the entrepreneur's financing needs. At date 1, the lender can observe both the project payoff and the entrepreneur's wealth. The entrepreneur and lenders are risk neutral. For simplicity, we assume that personal discount rates and the riskless interest rate are zero.

The timing of the investment-financing game is as follows. At date 0, the entrepreneur chooses to: (i) incorporate and attempt to obtain $K+B$ in capital markets by issuing limited liability debt, (ii) remain unincorporated and attempt to obtain $K$ in capital markets by issuing unlimited liability debt, or (iii) completely forego the investment project and exit the market. Lenders observe these actions. If a lender decides to extend capital to the entrepreneur, then he offers a (gross) interest rate, $r$, to the entrepreneur. The entrepreneur will accept financing from the lender offering the lowest interest rate, $r$, and invests in the project. If the entrepreneur issues unlimited liability debt, then he receives $K$ at date 0 and promises to pay back $Kr$ at date 1; if he issues limited liability debt, then he receives $K+B$ at date 0 and promises to pay back $(K+B)r$ at date 1. At date 1, the investment project pays either $A/p$ or 0. If the entrepreneur is wealthy, he receives an additional payoff of $w$ at this time. Payments are exchanged between the entrepreneur and the lender as specified in their debt contract and the game ends.

The equilibrium concept that we will adopt is a Perfect Bayesian Equilibrium, (PBE). A PBE for our game specifies:

1. a strategy for each type of entrepreneur, where the entrepreneur's strategy is
   (a) to incorporate and seek limited liability funding; to remain unincorporated and seek unlimited liability funding; or to forego the investment project and exit the market
   (b) to choose a lender (if investment funds are forthcoming) on the basis of the gross interest rate offered by all lenders.
2. a strategy for each lender, where a lender's strategy is to offer to provide, or not, investment funding $K$ for an unincorporated entrepreneur and $K+B$ for an incorporated entrepreneur and, if funding is offered, a gross interest rate, $r$.
3. beliefs that lenders hold about entrepreneurial type.

Strategies and beliefs satisfy sequential rationality and, whenever possible, Bayes' rule.

# 4 Analysis with Debt Finance

The historical puzzle that we are addressing is that, loosely speaking, in the latter half of the nineteenth century the costs associated with incorporation—and, hence, the cost of issuing limited liability debt—dramatically fell in both the US and the UK. Borrowers in the US responded in great numbers by switching from unlimited liability to limited liability debt finance, while the in UK borrowers by and large continued to finance with unlimited liability debt. We are interested in understanding how changes in the cost of incorporation, $B$, affect the level of investment and the method of finance. In this section we will characterize the various equilibrium configurations that may arise and describe how an equilibrium is affected when the cost of incorporation is changed.

Since there is a large penalty associated with defaulting on unlimited liability debt, a poor entrepreneur's expected payoff will be negative if he borrows with unlimited liability debt. Therefore, any entrepreneur who finances with unlimited liability debt is wealthy. Competition among lenders implies that the gross rate of interest for an unlimited liability loan is equal to one, since such a loan carries no risk. The expected project payoff for a wealthy entrepreneur who finances his investment project with unlimited liability debt, $V^U$, is simply $V^U = A - K$.

Let $\pi$ represent a lender's belief about the success rate of a project that is financed with limited liability debt and let $r_\pi$ represent the gross interest rate charged for a limited liability loan by a lender with belief $\pi$. Competition implies that $r_\pi$ generates zero expected profits for lenders. The interest rate $r_\pi$ equates total amount loaned out, $K + B$, with the expected payoff associated with the loan, $\pi[r_\pi(K + B)]$. Zero expected profits for lenders implies that $r_\pi = 1/\pi$. Since $p_R \leq \pi \leq p_S$ , it will be the case that

$$\frac{1}{p_S} \leq r_\pi \leq \frac{1}{p_R}.$$

Denote the expected project payoff to an entrepreneur with success probability $p$ who finances with limited liability debt by

$$V^L(p) = A - pr(K + B) = A - \frac{p}{\pi}(K + B). \tag{1}$$

From (1), note that $V^L(p) \geq 0$ for all $p \leq \pi$. Therefore, the $RP$-type entrepreneur will always invest in the project—using limited liability debt—because the interest rate charged for a limited liability loan is $r_\pi \leq 1/p_R$

or $p_R/\pi \leq 1$; this implies that the project will generate a strictly positive expected payoff of at least $A-K-B$. Because the $RP$-type entrepreneur always borrows with limited liability debt, it will *always* be the case that $r_\pi > 1/p_S$. As a result, the $SW$-type entrepreneur will never finance with limited liability debt, since the expected payoff associated with this strategy is strictly less than $V^U = A-K$. Therefore, two of the four possible entrepreneurial types have dominant strategies: the $RP$-type entrepreneur always invests and finances with limited liability debt and the $SW$-type entrepreneur always invests and finances with unlimited liability debt.

The remainder of the analysis explores the investment and financing strategies of the $RW$- and $SP$-type entrepreneurs. The $RW$-type entrepreneur always invests in the project since he can obtain an expected project payoff of at least $V^U = A-K > 0$ by financing with unlimited liability debt. However, depending upon the interest rate that prevails for limited liability debt, the $SP$-type entrepreneur may or may not invest. So the analysis boils down to understanding the conditions under which

1. the $SP$-type entrepreneur chooses to invest (or not), and
2. the $RW$-type entrepreneur chooses to finance with limited or unlimited liability debt.

Before we characterize the various equilibrium outcomes, we can identify a situation that *cannot* arise in equilibrium when $B > 0$: one in which the $SP$-type entrepreneur exits the market and does not invest, while the $RW$-type entrepreneur finances his investment with limited liability debt. In this situation, the interest rate on limited liability debt will be equal to $1/p_R$ since only risky entrepreneurs—$RP$- and $RW$-types—issue limited liability debt. The project payoff to an $RW$-type entrepreneur is $A-K-B$ because he finances his investment with unlimited liability debt. But this entrepreneur can obtain an expected project payoff of $V^U = A-K$ if he finances with unlimited liability debt, since he avoids the incorporation cost, $B$, associated with limited liability debt.

The issue facing the $SP$-type entrepreneur is *whether or not* to invest; if he invests, then he always finances with limited liability debt. The $SP$-type entrepreneur will invest in the project if the expected project payoff is non-negative, i.e. $V^L(p_S) \geq 0$ or, using (1), if

$$\frac{A}{K+B}\frac{1}{p_S} \equiv r_{SP} \geq r_\pi \equiv \frac{1}{\pi};$$

otherwise he foregoes the investment and exits the market. The gross interest rate $r_{SP}$ represents the interest rate on a limited liability loan that makes the $SP$-type entrepreneur is indifferent between investing and exiting the market. If the lender charges less than this critical rate, then $SP$-type entrepreneur strictly prefers to invest in the project rather than exit the market. In what follows it will be convenient to define the parameters $R_I \equiv A/K > 1$ and $p_\Delta \equiv p_S/p_R > 1$. Using this notation, the condition for which the $SP$-type entrepreneur invests in the project can be rewritten as

$$1 + \frac{B}{K} \leq \frac{R_I}{p_\Delta}\frac{\pi}{p_R}, \tag{2}$$

where a strict inequality in (2) implies that $SP$-type entrepreneur invests with probability one. The parameter $R_I$ can be interpreted as a measure of the expected gross rate of return for the project and $p_\Delta$ as a measure of the dispersion in lenders' priors about project risk. We will say that $R_I$ measures the "investment effect" on the equilibrium, while $p_\Delta$ measures the "adverse selection effect." From (2) we can see that, holding all else constant, a higher value of $R_I$ will motivate the $SP$-type entrepreneur to invest. In contrast, inequality (2) tells us that, holding all else constant, a higher value of $p_\Delta$ will motivate the $SP$-type entrepreneur type to exit the market. Note also from (2) that, holding lender beliefs constant, smaller incorporation costs $B$ are conducive to investment by the $SP$-type entrepreneur.

The issue facing the $RW$-type entrepreneur is *how* to finance the investment project. If the $RW$-type entrepreneur finances with unlimited liability debt, then his project payoff is $V^U = A - K$. If, however, he finances using limited liability debt, then his expected project payoff is

$$V^L(p_R) = A - \frac{p_R}{\pi}(K + B).$$

The $RW$-type entrepreneur will finance with limited liability debt if $V^L(p_R) \geq A - K$ or if

$$\frac{K}{K+B}\frac{1}{p_R} \equiv r_{RW} \geq r_\pi \equiv \frac{1}{\pi};$$

otherwise he will finance with unlimited liability debt. The gross interest rate $r_{RW}$ represents the critical interest rate that makes the $RW$- type entrepreneur indifferent between financing with limited liability debt and financing with unlimited liability debt. Since financing with unlimited liability debt does not entail any incorporation costs, the $RW$-type entrepreneur will finance with limited liability debt only if the gross interest rate is strictly less than $1/p_R$. (Note that the above inequality tells us that $r_{RW} < 1/p_R$) As

above, it will be convenient to rewrite the condition under which the $RW$-type entrepreneur finances with limited liability debt as

$$1 + \frac{B}{K} \leq \frac{\pi}{p_R}, \tag{3}$$

where a strict inequality implies that the $RW$-type entrepreneur incorporates and issues limited liability debt with probability one. In contrast to (2), the investment effect $R_I$ is absent in (3), since the $RW$-type entrepreneur always invests. As in the case for the $SP$-type entrepreneur, a lower value for the incorporation cost $B$ makes limited liability finance more attractive. The $RW$-type entrepreneur's financing decision depends upon the degree of adverse selection in the market, which is clear when we rewrite $\pi/p_R$ as $\pi p_\Delta / p_S$. Intuitively, a higher value of $p_\Delta$ is favorable for the $RW$-type entrepreneur since the interest rate on limited liability debt puts some weight project success probability $p_S$, which lowers the cost of borrowing for a risky entrepreneur. So, in accordance with intuition, inequality (3) tells us that, holding all else constant, a higher value of $p_\Delta$ will motivate the $RW$-type entrepreneur to finance with limited liability debt.

Considering the various strategies that the $SP$- and $RW$-type entrepreneurs may adopt, we see that there are four qualitatively distinct equilibrium configurations that can arise:

1. an *underinvestment equilibrium* is one where the $SP$-type entrepreneur, with probability one, does not invest.

2. a *wealth separation equilibrium* is one where all entrepreneurs invest and all wealthy entrepreneurs finance with unlimited liability debt and all poor entrepreneurs finance with limited liability debt.

3. a *limited liability equilibrium* is one where all entrepreneurs invest and with positive probability the $RW$-type finances with *limited* liability debt.

4. an *limited liability-underinvestment equilibrium* is one where the $SP$-type entrepreneur invests with positive probability (less than one), all other entrepreneurial types invest with probability one, and the $RW$-type entrepreneur finances his investment with limited liability debt.

Note that there is an adverse selection effect operation in equilibrium configurations 1, 3 and 4. In equilibrium configuration 1, the $SP$-type entrepreneur has a valuable investment project but because the market interest rate is "high" he does not invest; in equilibrium configuration 3, the

*RW*-type entrepreneur increases the borrowing rate on limited liability debt; and in equilibrium 4, the *SP*-type entrepreneur does not invest with positive probability. In equilibrium configuration 2, wealthy entrepreneurs pay the appropriate risk adjusted borrowing rate and both types of poor entrepreneurs invest by using limited liability debt. Observe that these equilibrium configurations may be ranked (weakly) by the associated average level of investment and propensity of entrepreneurs to incorporate and issue limited liability debt. Equilibrium configuration 1 has the lowest levels of investment; equilibrium configuration 4 has higher levels of investment; and equilibrium configurations 2 and 3 have the highest. As the foregoing discussion suggests, an underinvestment equilibrium is most likely to arise when the gross productivity of investment projects is small relative to the potential for adverse selection, in the sense that $R_I$ is small relative to $p_\Delta$. We, therefore, proceed by characterizing the various equilibrium outcomes assuming first that the adverse selection effect dominates the investment effect, in the sense that $R_I \leq p_\Delta$, and then assuming the opposite, that $R_I > p_\Delta$. We emphasize that virtually all of the following analysis that follows simply considers the interaction of inequalities (2) and (3).

### 4.1 Equilibria: adverse selection effect dominates the investment effect

Intuitively, when the adverse selection effect dominates the investment effect, i.e., when $p_\Delta \geq R_I$, a likely outcome is that there will be market failure, i.e. underinvestment. The reasoning is as follows. The cost of capital, $r_\pi$, depends upon lenders' beliefs about project success, which, in equilibrium, depends on the actual probabilities of incorporation by the safe- and risky-type entrepreneurs. Investment by the *SP*-type entrepreneur creates a positive externality for the *RW*-type entrepreneur, to the extent that it lowers the cost of capital financed with limited liability loans. At the same time, however, incorporation by the *RW*-type entrepreneur creates a negative externality for the *SP*-type entrepreneur, which reduces the attractiveness of financing investment with limited liability debt relative to not investing at all. We now show that when the adverse selection effect dominates the investment effect, this latter externality is sufficiently strong to make the investment unprofitable for the *SP*-type entrepreneur.

If an underinvestment equilibrium exists, then the interest rate on limited liability debt must be equal to $1/p_R$ because the *RP*-type entrepreneur is the only entrepreneur that finances with limited liability debt. In this equilibrium, $\pi = p_R$ and, since $p_\Delta \geq R_I$, inequality (2) can never hold for strictly

positive incorporation costs, i.e., for all $B > 0$

$$1 + \frac{B}{K} \geq \frac{R_I}{p_\Delta} \frac{\pi}{p_R},$$

which implies that the $SP$-type entrepreneur strictly prefers not to invest. Since the interest rate on limited liability debt is equal to $1/p_R$ and it costs $B$ to issue limited liability debt, the $RW$-type entrepreneur has no incentive to incorporate and finance with limited liability debt. This establishes:

**Proposition 1:** *If $p_\Delta \geq R_I$, then an underinvestment equilibrium exists for all $B > 0$.*

Next, consider the possibility of a wealth separation equilibrium, in which the $RW$-type entrepreneur finances with unlimited liability debt, while the $SP$-type entrepreneur invests and finances with limited liability debt. Intuitively, if lenders believe the $SP$-type entrepreneur will invest, then the interest rate on limited liability debt will turn out to be sufficiently low so that the $RW$-type entrepreneur also prefers to finance with limited liability debt. But, an interest rate on limited liability finance that deters the $RW$-type entrepreneur from using it will also deter investment by the $SP$-type entrepreneur. To see this, suppose that a wealth separation equilibrium exists. Then inequality (2) must hold, and inequality (3) cannot, where $\pi = \sigma p_S + (1 - \sigma) p_R$. Combining the two inequalities, we get

$$\frac{\pi}{p_R} \leq 1 + \frac{B}{K} \leq \frac{R_I}{p_\Delta} \frac{\pi}{p_R}. \tag{4}$$

But these inequalities cannot simultaneously hold since $p_\Delta \geq R_I$ and $B > 0$. Hence,

**Proposition 2:** *If $p_\Delta \geq R_I$, then a wealth separation equilibrium does not exist for any $B > 0$.*

Consider now the limited liability equilibrium configuration, where the $SP$-type entrepreneur invests with probability one and the $RW$-type entrepreneur uses limited liability debt with probability $\mu$. Let $\pi(\mu)$ represent the lenders' belief about project success. Bayes' rule implies that,

$$\pi\left(\mu\right) = \frac{\sigma\left(1-\omega\right)}{1-\omega+\mu\omega\left(1-\sigma\right)} p_S + \frac{\left(1-\sigma\right)\left(1-\omega\right)+\mu\omega\left(1-\sigma\right)}{1-\omega+\mu\omega\left(1-\sigma\right)} p_R. \tag{5}$$

Note that $\pi(\mu)$ is decreasing in $\mu$: if the $RW$-type entrepreneur chooses limited liability finance with a higher probability, then the lender's belief about project success must fall. A limited liability equilibrium will exist if both (2) and (3) hold. But, since $p_\Delta > R_I$, if (2) holds, then (3) holds strictly, which implies that the $RW$-type entrepreneur strictly prefers to finance with limited liability debt whenever the $SP$-type entrepreneur invests. It follows that the only limited liability equilibrium in this case is one in pure strategies, and it suffices to consider lenders' equilibrium beliefs about project success for only $\mu = 1$. Simplifying (5), we get

$$\pi(\mu = 1) = \frac{\sigma - \sigma\omega}{1 - \sigma\omega} p_S + \frac{1 - \sigma}{1 - \sigma\omega} p_R = (1 - P(R|RW)) p_S + P(R|RW) p_R,$$

where $P(R|RW)$ denotes the posterior probability that a project financed with limited liability debt is a risky, given that the $RW$-type entrepreneur incorporates with probability one. A limited liability equilibrium will, therefore, exist for some $B > 0$ if the inequality in (2) strictly holds at $B = 0$, or equivalently if

$$R_I \frac{\pi(\mu = 1)}{p_S} > 1$$

Substituting for $\pi(\mu = 1)$ in the above inequality, we see that a limited liability equilibrium will exist if

$$\frac{1}{R_I} - 1 < \left( \frac{1}{p_\Delta} - 1 \right) P(R|RW). \tag{6}$$

However, since the proposed equilibrium has $p_\Delta \geq R_I$ and $P(R|RW) < 1$, it is not possible for condition (6) to hold; this, in turn, means that inequality (2) cannot hold for any $B > 0$, i.e.,

$$1 + \frac{B}{K} > \frac{R_I}{p_\Delta} \frac{\pi\left(\mu = 1\right)}{p_R}. \tag{7}$$

We can summarize this discussion by:

**Proposition 3:** *If $p_\Delta \geq R_I$, then a wealth separation equilibrium does not exist for any $B > 0$.*

Finally, a limited liability-underinvestment equilibrium requires that an $SP$-type entrepreneur be indifferent between investing and not and that the $RW$-type entrepreneur finance with limited liability debt with probability

one. A *necessary* condition for this equilibrium requires that

$$1+\frac{B}{K}=\frac{R_I}{p_\Delta}\frac{\pi\left(\phi\right)}{p_R}, \tag{8}$$

where

$$\pi\left(\phi\right)=\frac{\phi\sigma\left(1-\omega\right)}{\left(1-\sigma\right)+\phi\sigma\left(1-\omega\right)}p_S+\frac{1-\sigma}{\left(1-\sigma\right)+\phi\sigma\left(1-\omega\right)}p_R$$

represents the probability that the project is successful given that the $SP$-type entrepreneur invests with probability $\phi$. Condition (8) says that the $SP$-type entrepreneur is indifferent between investing and not investing. Note that $\pi\left(\phi\right)$ is increasing in $\phi$, and that $\pi\left(\phi=1\right)=\pi\left(\mu=1\right)$. If $\phi=1$, then it must be the case that the left-hand side of (8) exceeds the right-hand side, since $\pi\left(\phi=1\right)=\pi\left(\mu=1\right)$, i.e., see inequality (7). But because $\pi\left(\phi\right)$ is increasing in $\phi$, the left-hand side of (8) will exceed the right-hand side *for all* $0\leq\phi\leq1$; hence, condition (8) can never hold. We can summarize this discussion by:

**Proposition 4:** *If $p_\Delta\geq R_I$, then the limited liability-underinvestment equilibrium doess not exist for any $B>0$.*

In summary, propositions 1-4 imply that when the adverse selection effect dominates the investment effect, the equilibrium outcome is always characterized by underinvestment: the $SP$-type entrepreneur will exit the market and will not invest. As well, all wealthy entrepreneur finance their investment with unlimited liability debt and limited liability debt is "risky," i.e., $r=1/p_R$, as only the $RP$-type entrepreneur uses it.

## 4.2 Equilibria: investment dominates the adverse selection

When $R_I>p_\Delta$, the investment effect dominates the adverse selection effect, and the possibility of market failure is diminished. Unlike the analysis above, now an underinvestment equilibrium need not always exist. Such an equilibrium exists only if inequality (2) does not hold when the lender's beliefs satisfy $\pi=p_R$. That is, an underinvestment equilibrium exists only if

$$1+B/K\geq R_I/p_\Delta.$$

Clearly, for $B$ sufficiently "high," this inequality can exist. However, since $R_I>p_\Delta$, we can have $1+B/K<R_I/p_\Delta$ for $B$ sufficiently "small." At these low levels of $B$, the $SP$-type entrepreneur prefers to invest in the project even if

the cost of capital is $1/p_R$. This is because the gross rate of return on the investment is sufficiently high to offset even the maximal cost of adverse selection in the debt market. This establishes,

**Proposition 5:** *If $R_I > p_\Delta$, then an underinvestment equilibrium exists for all incorporation costs*

$$B > \frac{R_I - p_\Delta}{p_\Delta} K.$$

When the investment is sufficiently productive relative to adverse selection, i.e., when $R_I > p_\Delta$, the $SP$-type entrepreneur may be willing to invest even if the cost of capital is equal to $1/p_R$. As well, since the adverse selection effect is relatively small, the $RW$-type entrepreneur may not have an incentive to finance with limited liability debt even when both the $RW$- and $SP$-type entrepreneurs use this kind of finance. To see this, note that a wealth separation equilibrium—one where rich entrepreneurs use unlimited liability finance and poor entrepreneurs use limited liability—exists when inequality (2) holds and inequality (3) does not, where $\pi = \pi\left(\mu = 0\right) = \sigma p_S + (1-\sigma) p_R$, or in other words when

$$\frac{\pi\left(\mu = 0\right)}{p_R} \leq 1 + \frac{B}{K} \leq \frac{R_I}{p_\Delta} \frac{\pi\left(\mu = 0\right)}{p_R}.$$

(Note that this inequality is identical to inequality (4)). Since $R_I > p_\Delta$ and $\pi\left(\mu = 0\right) > p_R$, the interval is non-empty for some $B > 0$; hence,

**Proposition 6:** *If $R_I > p_\Delta$, then a wealth separation equilibrium exists for intermediate levels of the incorporation cost $B$, where*

$$B \in \left[ K\left( \frac{\pi\left(\mu = 0\right)}{p_R} - 1 \right), K\left( \frac{R_I}{p_\Delta} \frac{\pi\left(\mu = 0\right)}{p_R} - 1 \right) \right].$$

When a wealth separation equilibrium exists, the choice of financial instruments is "neutral" in the cross-section: the cost of capital is unrelated to the decision to incorporate, and all firms invest optimally in equilibrium.

In a limited liability equilibrium, the $SP$-type entrepreneur incorporates and invests with probability one, the $RW$-type entrepreneur incorporates and finances with limited liability debt with probability $\mu \in (0, 1]$, and lenders' beliefs are given by the expected default probability $\pi(\mu)$ defined in (5).

From (2) and (3), one can see that the necessary condition for this equilibrium to exist is

$$1 + \frac{B}{K} \leq \frac{\pi(\mu)}{p_R}.$$

That is, when the $RW$-type entrepreneur weakly prefers to finance with limited liability debt, the $SP$-type entrepreneur strictly prefers to invest because $R > p_\Delta$. Let $\hat{B}_0$ denote the highest incorporation cost consistent with the $RW$-type entrepreneur being indifferent between financing with limited or unlimited liability debt, i.e., $\hat{B}_0$ solves

$$1 + \frac{\hat{B}_0}{K} = \frac{\pi_0}{p_R}.$$

(Note that $\hat{B}_0$ is equal to the lower bound in the interval described in proposition 6.) Let $\hat{B}_1 < \hat{B}_0$ denote the lowest incorporation cost consistent with the $RW$-type entrepreneur being indifferent between financing with limited or unlimited liability debt, i.e., $\hat{B}_1$ solves

$$1 + \frac{\hat{B}_1}{K} = \frac{\pi_1}{p_R}.$$

In equilibrium, the probability with which the $RW$-type entrepreneur incorporates and finances with limited liability debt depends on the size of the incorporation costs, $B$. If $B = \hat{B}_0$, then the $RW$-type entrepreneur finances with unlimited liability debt with probability one; if $\hat{B}_1 < B < \hat{B}_0$, then the $RW$-type entrepreneur finances his investment with limited liability debt with probability $\mu \in (0,1)$, where $\mu$ solves $1 + B/K = \pi(\mu)/p_R$, and, with probability $1-\mu$, he finances with unlimited liability debt; finally if $B \leq \hat{B}_1$, then the $RW$-type entrepreneur finances with limited liability debt with probability one. When $B \leq \hat{B}_0$, in equilibrium, the $SP$-type entrepreneur invests with probability one. The interest rate that lenders' offer for financing limited liability debt is $1/\pi(\mu)$ for $\hat{B}_1 \leq B \leq \hat{B}_0$; when $B < \hat{B}_1$, the interest rate on limited liability debt is $1/\pi(\mu = 1)$. Note that on the interval $(\hat{B}_1, \hat{B}_0)$, the probability that the $RW$-type entrepreneur incorporates is a strictly decreasing function of $B$, which in turn strictly decreases the expected default rate on limited liability debt. Thus we have,

**Proposition 7:** *If $R_I > p_\Delta$, then a limited liability equilibrium exists if the incorporation cost, $B$, is less than $\hat{B}_0$; the $RW$-type entrepreneur will incorporate with probability one if $B \leq \hat{B}_1$ and will incorporate with a probability less than one, but greater than zero if $\hat{B}_1 < B < \hat{B}_0$.*

Although financial structure is irrelevant in this equilibrium in the sense that all profitable investments are undertaken under some organizational form, from the perspective of an individual entrepreneur it is not, and local changes in the cost of incorporation have predictable effects on the equilibrium cost of capital and the default rate of corporate firms.

Finally, an unlimited liability-underinvestment equilibrium will exist if: (i) the *SP*-type entrepreneur is indifferent between investing and not, i.e., condition (8), and (ii) the *RW*-type entrepreneur finances with unlimited liability debt, i.e., if

$$1 + \frac{B}{K} \leq \frac{\pi(\phi)}{p_R}. \tag{9}$$

But since $R_I > p_\Delta$, it is not possible that conditions (8) and (9) can simultaneously hold. Therefore,

**Proposition 8:** *If $R_I > p_\Delta$, then the limited liability-underinvestment equilibrium does not exist for any $B > 0$.*

Figure 1 characterizes the relationship between incorporation costs and the equilibrium interest rate on limited liability debt in the various possible equilibrium configurations. Proposition 5 showed that an underinvestment equilibrium exists for $1 + B/K \geq R_I/p_\Delta$; in this situation the equilibrium interest rate on a limited liability loan is $1/p_R$. Proposition 6 showed that a wealth separation equilibrium with $r = 1/\pi(\mu = 0)$ exists when

$$\frac{\pi(\mu = 0)}{p_R} \leq 1 + \frac{B}{K} \leq \frac{R_I}{p_\Delta}\frac{\pi(\mu = 0)}{p_R}$$

Note that since $\pi(\mu = 0)/p_R > 1$, there always exists values of $B$ at which an underinvestment equilibrium and a wealth separation equilibrium coexist.[7] Proposition 7 showed that a limited liability equilibrium, where the *RW*-type randomizes between limited and unlimited liability debt, exists when

$$\frac{\pi(\mu = 1)}{p_R} \leq 1 + \frac{B}{K} \leq \frac{\pi(\mu = 0)}{p_R}$$

Again, an underinvestment equilibrium may coexist with a limited liability equilibrium at a particular value of $B$ or not, depending on the value of $R_I/p_\Delta$ compared to $\pi(\mu = 1)/p_R$ and $\pi(\mu = 0)/p_R$.

[7] In the figure, we assume $\pi_0/p_R > R/p_\Delta$, so that there exist parameter values at which the underinvestment and limited liability equilibria both exist. However, this is arbitrary.

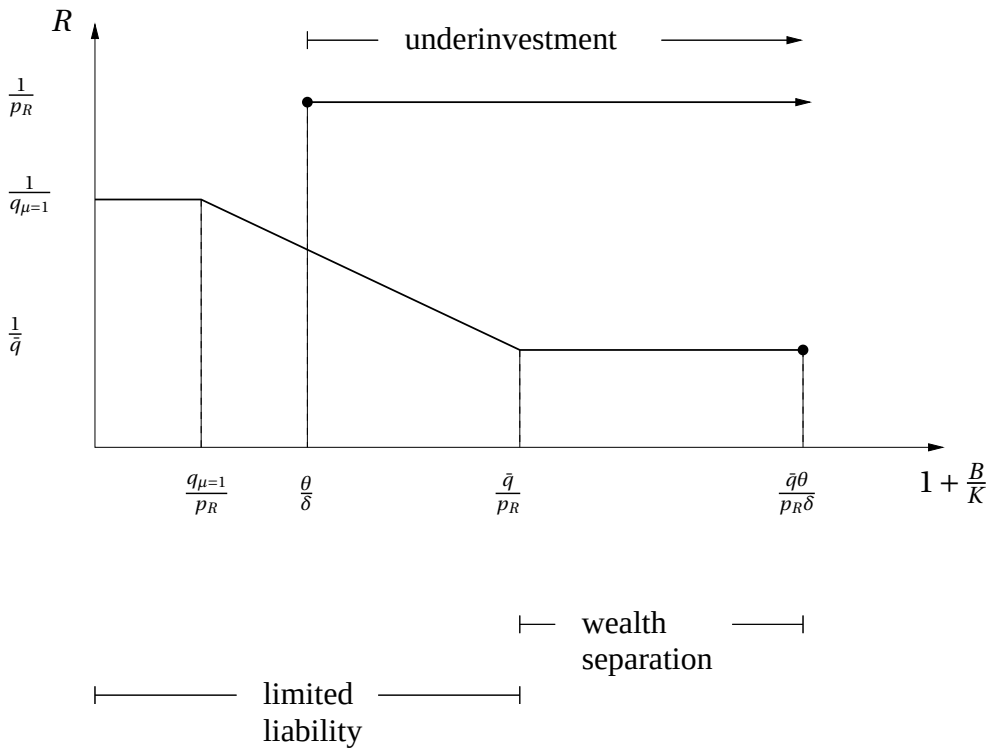


Figure 1: The equilibrium correspondence when $\delta < \theta$.

## 5 Discussion

In terms of addressing the historical puzzle—a dramatic fall in the cost of issuing limited liability debt in the US and UK resulted in a large increase in the use of limited liability debt in the US and essentially no change in the UK with similar outcomes for investment—our model can provide both a "fundamental-driven" explanation and a "belief-driven" explanation.

The fundamental-driven explanation would start with the observation that the capital/labor ratio in Britain was greater than that of the US in the mid-$19^{th}$ century. Hence, the rate of return to capital in the US exceeded that in the UK. Supposing that the adverse selection effect, $p_\Delta$, is the same in both countries, then it is quite possible—owing to the differences in the rate of return on capital—that in the UK the adverse selection effect dominates the investment effect and in the US the investment effect dominates the adverse selection effect in the US. Therefore, in the UK, the

only possible outcome is the underinvestment equilibrium, where the use of limited liability debt is constrained and the level of investment is "low." In the US, however, as the cost of using limited liability falls, i.e., $B$ falls, then, the wealth separation and the limited liability equilibria can be supported, where in both cases the level of investment will be "high" supported by the use of limited liability debt.

The following definitions will be useful for a belief-driven explanation. Lenders will be said to have "optimistic beliefs" if they hold highest possible belief of project success for a limited liability financed entrepreneur that is consistent with an equilibrium and will be said to have "pessimistic beliefs" if they hold the lowest possible belief. We will consider how equilibrium outcomes are altered when the cost of incorporation falls under the scenarios where lenders have optimistic beliefs and lenders have pessimistic beliefs.

If lenders have optimistic beliefs, then the level of investment and the number of incorporations will increase as incorporation costs fall. This can be seen in figure 1: As incorporation costs fall, first there is an underinvestment equilibrium, then a wealth separation equilibrium and finally a limited liability equilibrium. On the other hand, if lenders have pessimistic beliefs, the economy can experience stagnation, i.e., the economy is "stuck" in an underinvestment equilibrium, even as the cost of incorporation become very low. In figure 1, if investors have pessimistic beliefs, the incorporation costs have to decline significantly—below $\theta/\delta$— before the economy cannot be in an underinvestment equilibrium.

The predicted relationship between incorporation costs and interest rates on limited liability debt is however more subtle. When investment is relatively high profitability and investors have optimistic beliefs, observe that interest rates initially fall and then rise as the costs of incorporation decrease, as first high-quality then low-quality firms are induced to incorporate. Thus an apparent rise in interest rates following the reform need not signal an unfavorable climate for investment.

These observations are consistent with an "animal spirits" theory of institutional reform and economic development. When investors hold optimistic beliefs, the introduction of limited liability may induce a sharp decline in the cost of capital and a concomitant rise in investment, even when the cost of incorporation remains relatively high, while low investment may persist after the reform under pessimistic beliefs. Of course, such beliefs are self-fulfilling: a willingness of investors to hold limited liability debt is rewarded by better-quality issuance on average, and so empirical default rates that keep interest rates low. In this sense, the different experience of Great Britain and the United States with liability reform might be interpreted as

equilibrium phenomena, even without differences in economic fundamentals in the two countries.

Some corporations in the mid-nineteenth century raised funds by issuing both debt and equity. Does the introduction of equity finance affect any of our results or insights? We explore this question in an appendix and find that, qualitatively speaking, all of our results remain intact. In fact, by redefining variables in the environment where entrepreneurs can raise funds by issuing debt and/or equity, the conditions for the existence of the various equilibria, as well as the diagrammatic characterization of these equilibria are observationally equivalent to the those that are characterized in the main text, (i.e., when entrepreneurs can only raise funds by issuing either limited or unlimited liability debt).

# 6 Conclusion

While our research was motivated by the evidence of development traps in the period following British liability reforms, the model presented is quite general. It is therefore natural to ask whether similar concerns arise in the modern era. Indeed, there is evidence that entrepreneurs continue to self-select in choosing liability status. Thus Horvath and Woywode (1996) report that, for a sample of modern German firms, limited liability is more likely to be adopted by firms that are riskier and that have greater capital demands, which is consistent with our model. While self-selection apparently persists, outside investors are considerably better informed now than in the past, so that adverse selection problems are attenuated. For example, there were very few reporting requirements for publicly traded companies in nineteenth-century Britain. Moreover, our model suggests that when the transactions costs incurred to obtain limited liability are small relative to total capital demands, underinvestment need not occur in equilibrium. Specifically, an equilibrium with full investment always exists for $B/K$ sufficiently small. Moreover, when gross returns to investment are large relative to potential adverse selection costs (in the notation of the model, $R > p_\Delta$) the full investment equilibrium is unique when $B/K$ is small. Thus development traps are unlikely to occur in the model for the parameter values that best represent the modern economic environment. We conclude that the problems of liability choice we have identified are probably of greatest relevance in periods of transition, such as nineteenth century Britain.

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# Appendix

Thus far we have restricted financing to take the form of debt. In the nineteenth century, firms also had the option to finance their investment projects by issuing equity. In this section we examine how our insights and results are affected when firms may issue equity in addition to debt. Since equity, like limited liability debt, does not guarantee a certain payoff to the holder, we assume that a firm that issues equity must become incorporated. We have to slightly alter the model in the main text to allow for both equity and debt finance.

## A1 A Model with Debt and Equity

We consider a minimal extension of the model in the main text that incorporates an equity market. To make the debt-equity decision a non-trivial one, we introduce a moral hazard problem: at the beginning of date 1—before the project pays off—the entrepreneur must choose either to "work hard" ($e = 1$) or "shirk" ($e = 0$). With probability $p$ the project is successful and pays $[eA + (1-e)A']/p$ at date 1 and with complementary probability is unsuccessful and pays nothing. Thus a successful project pays $A/p$ if the entrepreneur works hard ($e = 1$) and pays $A'/p$ if the entrepreneur shirks ($e = 0$), where $A' < A$. When the entrepreneur works hard he incurs a private cost equal to $c > 0$ for each unit of expected project payoff that exceeds $A'$. Hence, the total cost that the entrepreneur incurs for working hard is $c(A - A')$.[8]

The timing of the investment-financing game is as follows. At date 0, the entrepreneur chooses to:

(i) incorporate and attempt to obtain $K + B$ in capital markets by either

   (a) issuing limited liability debt in exchange for $D \in [0, K + B]$ units of capital and issuing equity in exchange for $K + B - D$ units of capital.

   (b) issuing unlimited liability debt in exchange for $D \in [0, K + B]$ units of capital and issuing equity in exchange for $K + B - D$ units of capital.

[8] If this moral harzard problem was introduced into the "only debt finance" environment of the main text, it would have no effect on analysis.

(ii) remain unincorporated and obtain $K$ in capital markets by issuing unlimited liability debt

(iii) completely forego the investment project and exit the market.

Investors—potential creditors and equity holders—observe these actions. If an investor decides to extend capital $D \in [0, K+B]$ to the entrepreneur in exchange for debt, then the investor offers a (gross) interest rate to the entrepreneur. If an investor decides to extend capital $K+B-D$ to the entrepreneur in exchange for equity, then the investor offers a fraction $(1-\alpha)$ to the entrepreneur. The fraction $(1-\alpha)$ represents the share of the project's payoff, net of any debt payments, that goes to the equity holder. The entrepreneur will accept debt financing from the lender who offers the lowest interest rate and will accept equity financing from the investor who offers the smallest fraction $(1-\alpha)$. After financing has been arranged, the project is put into place. At date 1, the entrepreneur makes his effort decision: he either works hard or shirks. The investment project pays either $A/p$ or 0 if the entrepreneur works hard, or either $A'/p$ or 0 if he shirks. If the entrepreneur is wealthy, he receives his private wealth payoff of $w$ at this time. Payments are exchanged between the entrepreneur and the investors as specified in their debt and equity contracts and the game ends.

If the entrepreneur incorporates, issues *unlimited* liability debt and equity, and retains $\alpha$ ownership, $0 \leq \alpha \leq 1$, then the payoff to (outside) equity holders is $(1-\alpha)(P-F)$, where $P$ represents the project payoff and $F$ represents the promised payment to creditors. Note that it may be the case that $(1-\alpha)(P-F) < 0$.[9] If the entrepreneur gets incorporated, issues *limited* liability debt and equity, and retains $\alpha$ ownership, then the payoff to (outside) equity holders is $(1-\alpha)\max\{P-F, 0\}$.

To focus on interesting cases, we assume:

$$1 > c > \frac{A-K}{A}. \qquad \text{((A1))}$$

The first inequality implies $A - c(A-A') > A'$; that is, hard work maximizes the value of the project. The second inequality gives the moral hazard problem some bite in equilibrium: Suppose that the project were undertaken under limited liability with 100 per cent equity finance. If investors belief that the entrepreneur will work hard, then in exchange for $K+B$ units of

[9] For convenience, we will continue to assume that each investor has resources at least equal to $K+B$.

capital, the investor would receive the fraction $(1-\alpha^*)$ of date 1 output, where

$$\alpha^* = \frac{A-K-B}{A}.$$

Since under (A1), $\alpha^* \leq (A-K)/A < c$, we have

$$\alpha^* A - c(A-A') < \alpha^* A',$$

which implies that it is optimal for the entrepreneur to shirk at date 1 if he obtains 100 per cent equity finance. To focus on the simplest, and most interesting, case, we will assume that

$$A' < K. \qquad \text{((A2))}$$

Inequality (A2), in conjunction with inequalities (A1) implies that most debt and equity will be required for investment financing. (Inequality (A2) also implies that shirking cannot occur in any perfect Bayesian equilibria of the game.)

## A2 Equilibrium

Let $\pi(D)$ represent a lender's belief about the success rate of the project when the entrepreneur raises $D$ units of capital via limited liability debt and raises the remainder, $K+B-D$, by issuing equity. In equilibrium, competition among investors will induce each lender to offer a gross interest rate $r_{\pi(D)} = 1/\pi(D)$ for limited liability debt.

The expected residual or equity value of a corporation that has borrowed $D$ units of capital *as calculated by an investor with beliefs* $\pi(D)$ is

$$\bar{A}(e) - \left[\pi(D) r_{\pi(D)} D + (1-q(D)) \cdot 0\right] = \bar{A}(e) - D,$$

where $\bar{A}(e) = eA + (1-e)A'$. Because lenders and equity investors share common beliefs in equilibrium, the expected transfer from creditors to shareholders through default is zero. This implies that a corporation's equity will never be mispriced.[10] If $D < K+B$, then the entrepreneur must issue (outside) equity and, in equilibrium, competition among investors will induce each investor to offer the fraction $(1-\alpha)$ that solves $(1-\alpha)\left(\bar{A}(e)-D\right) = K+B-D$, or

$$\alpha = \frac{\bar{A}(e)-K-B}{\bar{A}(e)-D}. \qquad \text{((A3))}$$

[10] An implication of equity not being mispriced is that any problem which arises with the introduction of limited liability is fundamentally about debt, and not about equity.

An entrepreneur who incorporates and finances with limited liability debt, in the amount of $D$, and retains $\alpha$ ownership in the corporation, will work hard, i.e., will choose $e = 1$, if

$$\alpha\left(A - pr_{\pi(D)}D\right) - c(A - A') \geq \alpha\left(A' - pr_{\pi(D)}D\right)$$

or if

$$\alpha \geq c.$$

Hence, in order for the entrepreneur to be induced to work hard, his stake in the firm, $\alpha$, can not be diluted below $c$. The minimum amount of debt finance that is consistent with the entrepreneur working hard, denoted $D^{\min}$, is given by setting the right hand side of (A3) equal to $c$ and is given by

$$D^{\min} \equiv \frac{K + B - (1 - c)A}{c}.$$

Since we assume that $c > (A - K)/K$, i.e., inequality (A1), it is necessarily the case that $D^{\min} > 0$.[11] Note that the minimum fraction of firm ownership, $\alpha = c$, and the minimum amount of debt, $D = D^{\min}$, required to induce the entrepreneur to work hard, is independent of whether the entrepreneur is risky or safe. Define $D_0^{\min} = (K - (1 - c)A)/c$, i.e., $D_0^{\min}$ is simply $D^{\min}$ evaluated at $B = 0$.

In any equilibrium the amount borrowed, $D$, must satisfy $D \in \left[D^{\min}, K + B\right]$. If $D < D^{\min}$, then the entrepreneur will shirk. But if the entrepreneur shirks, the corporation will generate an expected payoff that is less than the required project investment of $K + B$ since $K > A'$; investors will not provide funds to the entrepreneur under these conditions. In what follows, we will assume that if an entrepreneur finances his investment project with limited debt, he will borrow $D^{\min}$ units of capital and finance the remainder with equity.[12] Since we will assume that $D = D^{\min}$ for the remainder of the paper, we will denote $\pi(D^{\min})$ simply as $\pi$.

[11] Since $A - (K + B) > 0$, it is also the case that $D^{\min} < K + B$. It is straight forward to demonstrate that $D^{\min} < A' < K$.

[12] The investment-financing game with debt and equity will typically have many equilibria *for a given belief of the investors*. For example, suppose that there exists a perfect Bayesian equilibrium where (i) both *SP*- and *RW*-type entrepreneurs incorporate and finance the project raising $D^{\min}$ funds from limited liability debt and the rest from equity, and (ii) the *SP*-type entrepreneur's expected payoff is strictly greater than zero. Then there exists an interval $[D^{\min}, D^{\max}]$ such that for *every* $D \in [D^{\min}, D^{\max}]$, there is perfect Bayesian equilibrium where both *SP*- and *RW*-type entrepreneurs incorporate and finance the project by raising $D$ funds from limited liability debt and the rest from equity. Unfortunately, the Cho and Kreps (1987) intuitive refinement does not have much power in terms of eliminating equilibria here. The *SP*-type entrepreneur would like to minimize that amount of debt that

As above, both the $RP$- and $SW$-type entrepreneurs have dominant strategies. The $RP$-type entrepreneur will use limited liability debt to borrow $D^{\min}$units of capital and will obtain the rest by issuing equity. Since it costs $B$ to incorporate and the interest rate on limited liability debt is always greater than $r_S$, the $SW$-type will always fund his project using unlimited liability debt. We can now turn to the investment and funding decisions of the $SP$- and $RW$-type entrepreneurs. The issues facing these entrepreneurs are the same as above: The $SP$-type entrepreneur must decide whether to obtain funding for his investment project or to exit the market; $RW$-type entrepreneur must decide whether to use unlimited liability debt or to incorporate and fund his project with a mix of limited liability debt and equity.

The $SP$-type entrepreneur will invest in the project if the expected project payoff is non-negative, i.e., if

$$p_S c\left(\frac{A}{p_S} - \frac{1}{\pi} D^{\min}\right) - c\left(A - A'\right) \geq 0,$$

(since $\alpha = c$ when $D = D^{\min}$) which can be simplified to read

$$D^{\min} \leq \frac{A'\pi}{p_S}.$$

Multiplying and dividing this inequality by appropriately chosen 1's, we can rewrite it as

$$\frac{D^{\min}}{D_0^{\min}} \leq \frac{A'/K}{\left(p_S/p_R\right)\left(D_0^{\min}/K\right)} \frac{\pi}{p_R}.$$

Recall that in all debt model of Section 3, we interpreted $A/K$ as an investment effect. In a model with equity finance and costly entrepreneurial effort, we will now interpret the investment effect as $A^{'}/K$, and will denote this investment effect as $R'$. Since only a fraction of investment finance is

he holds since he effectively subsidies the risky entrepreneurs debt and because equity is appropriately priced. Suppose that the $SP$-type entrepreneur defects from a proposed equilibrium by offering a level of debt that is lower than the proposed equilibrium value (but greater than $D^{\min}$). If the investor treats this defector as a $SP$-type entrepreneur, then the entrepreneur would receive financing interest rate of $r_S$. If defections were treated this way, then the $RW$-type entrepreneur may also have an incentive to defect in this manner. In what follows, we will effectively focus on perfect Bayesian equilibria that are consistent with the Cho and Kreps (1987) D1 refinement of out-of-equilibrium beliefs since this refinement will eliminate *all* capital structures that have more than $D^{\min}$ capital raised by limited liability debt. Hence, it is for this reason that we restrict the analysis to capital structures where the entrepreneur raises exactly $D^{\min}$ from debt finance and the remainder from issuing equity.

raised through debt, the adverse selection effect will now only be a fraction of $p_\Delta = p_S/p_R$. (Recall that there is no adverse selection effect associated with equity since it is not mispriced.) We will now represent the adverse selection effect as the maximum potential adverse selection effect, $p_\Delta = p_S/p_R$, times the fraction of total funds raised by debt when incorporation costs are zero and will denote this (new) adverse effect parameter by $p_\Delta^0$, i.e., $p_\Delta^0 = \left(D_0^{\min}/K\right) p_\Delta$. Finally, the left hand side of the above equality can be simplified to read,

$$\frac{D^{\min}}{D_0^{\min}} = \frac{\frac{K+B-(1-c)A}{c}}{\frac{K-(1-c)A}{c}} = 1 + \frac{B}{K-(1-c)A} \equiv 1 + \frac{B}{K_\alpha},$$

where $K_\alpha \equiv K-(1-c)A$. Hence, condition that has the $RP$ entrepreneur investing in this project can be expressed as

$$1 + \frac{B}{K_\alpha} \leq \frac{R'}{p_\Delta^0}\frac{\pi}{p_R}. \qquad \text{((A4))}$$

The $RW$-type entrepreneur will always invest in his project. If he finances with limited liability debt, his expected payoff will be

$$c\left(A - \frac{p_R}{\pi}D^{\min}\right) - c\left(A - A'\right);$$

if he finances with unlimited liability debt, his expected payoff will be

$$A - K - c\left(A - A'\right).$$

An $RW$-type entrepreneur will finance with limited liability debt if

$$\frac{D^{\min}}{D_0^{\min}} \leq \frac{\pi}{p_R},$$

(since $D_0^{\min} = (K-(1-c)A)/c$) which can be rewritten as

$$1 + \frac{B}{K_\alpha} \leq \frac{\pi}{p_R}. \qquad \text{((A5))}$$

Note that the inequality that describes the $SP$-type entrepreneur's investment decision when investment is financed with debt and equity, inequality (A4), is "identical" to that when the investment is financed by debt alone, inequality (2), when $K_\alpha$ is replaced with $K$, $R'$ is replaced by $R$ and $p_\Delta^0$ is replaced with $p_\Delta$. Similarly, the inequality that describes the $RW$-type entrepreneur's financing decision when investment can be financed with both

debt and equity, inequality (A5), is "identical" to that when the investment can only be financed by debt, inequality (3). The characterization of the various equilibrium outcomes along the lines that the "adverse selection effect dominates the investment effect," i.e., $p_\Delta^0 > R'$, and "the investment effect dominates the adverse selection effect," i.e., $R' > p_\Delta^0$, when the entrepreneur can finance with debt and equity are (qualitatively) identical to the equilibrium outcomes when the entrepreneur can only finance with debt. So when $p_\Delta^0 > R'$, the unique equilibrium is the underinvestment equilibrium; when $R' > p_\Delta^0$ if in figure 1 $R$ is replaced by $R'$, $p_\Delta$ is replaced by $p_\Delta^0$ and $K$ is replaced by $K_\alpha$, then this amended figure also describes the equilibrium outcomes in a world where the entrepreneur uses debt and equity to finance his investment project. Hence, introducing equity into the analysis does not at all affect the insights and results obtained from the all debt finance model. Because shareholders and bondholders have common beliefs about default risk, equity is never mispriced in our model, but debt may be. Consequently, reforms affecting the cost of incorporation may influence the degree of adverse selection in the limited liability credit market and hence the investment opportunities of firms, in a way precisely analogous to the debt-only model of our main text.