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Indeterminacy and Stabilization Policy

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Abstract

It has been shown that a one-sector real business cycle model with sufficient increasing returns in production may possess an indeterminate steady state that can be exploited to generate business cycles driven by “animal spirits” of agents. This note shows how an income tax schedule that exhibits a progressivity feature can ensure saddle path stability in such a framework and thereby stabilize the economy against sunspot fluctuations. Conversely, an economy that exhibits a flat or regressive tax schedule is more susceptible to indeterminacy.

Keywords: *Indeterminacy*, *Stablization Policy, Business Cycles*

JEL Classification: E32, E62

1 Introduction

It has been shown that a one-sector real business cycle (RBC) model with sufficient increasing returns in production, as in Benhabib and Farmer (1994) and Farmer and Guo (1994), may possess an indeterminate steady state (i.e., a sink) that can be exploited to generate business cycle fluctuations driven by "animal spirits" of agents.[1] This note shows how an income tax schedule that exhibits a progressivity feature can ensure "saddle path stability" in such a framework and thereby stabilize the economy against sunspot fluctuations. Conversely, an economy that exhibits a flat or regressive tax schedule is more susceptible to indeterminacy.

The basic intuition for the existence of multiple equilibria in the Benhabib-Farmer-Guo model is straightforward. Suppose, for example, that households become optimistic about market returns on labor and decide to work harder. To justify such optimism with higher employment as an equilibrium, a higher wage rate is called for. If externalities induced by increased aggregate employment result in higher labor productivity, then competitive firms will in fact pay higher wages in equilibrium. As a result, households' expectations of higher wages become self-fulfilling. Furthermore, increased labor effort raises the marginal product of capital, which in turn stimulates investment and contributes to the boom. This example illustrates how "animal spirits" can be a driving force of business cycle fluctuations.

By emphasizing expectations as an independent source of shocks, these models create an opportunity for Keynesian-type stabilization policies designed to suppress fluctuations driven by animal spirits. The key for the success of such policies is to dampen the mechanism in the model that makes for multiple equilibria. In this note, we consider an income tax schedule that exhibits a progressivity feature, that is, the household's tax rate is increasing in its own level of income. This yields the property that the marginal tax rate—the rate applied to the last dollar of income—is

[1] In this note, we use the terms "animal spirits", "sunspots" and "self-fulfilling beliefs" interchangeably. All refer to any randomness in the economy that is not related to uncertainties about economic fundamentals such as technology, preferences and endowments. The term "animal spirits" was introduced by Keynes (1936) and has been revived by Howitt and McAfee (1988), (1992). Since the early work of Shell (1977), there is a large literature exploring the relationship of animal spirits to dynamic general equilibrium models. Notable examples include Azariadis (1981), Cass and Shell (1983), Woodford (1986), (1991), Chiappori and Guesnerie (1991) and Farmer and Woodford (1997).

higher than the average tax rate. Hence, when agents expect to receive a higher wage, they also realize that they will face a higher tax rate. This reduces their expected after-tax return and prevents expectations from becoming self-fulfilling. Thus, a progressive tax schedule operates like a classic automatic stabilizer that mitigates business cycle fluctuations. Moreover, it renders the equilibrium unique.[2] From a welfare standpoint, the desirability of this stabilization policy depends on the relative strength of two opposing factors: concavity of utility and increasing returns in production. Concavity of utility implies that fluctuations reduce welfare, while the increasing returns effect goes in the opposite direction.

We derive the necessary and sufficient condition on the tax schedule to ensure saddle path stability in the model economy. It turns out that the slope of the tax schedule (which indexes the degree of progressivity) must be set high enough to make the equilibrium labor demand schedule *flatter* than the labor supply curve, that is, to overturn the relative steepness criteria derived by Benhabib and Farmer (1994). Our result implies that an economy with a flat or regressive tax schedule will be more susceptible to indeterminacy, that is, belief-driven fluctuations can occur for a lower degree of increasing returns.

Our model allows for a rich set of possibilities regarding the links between tax policy and indeterminacy and helps bring together some recent results in the literature. For example, Schmitt-Grohé and Uribe (1997) show that a standard RBC model with a constant returns-to-scale technology can exhibit indeterminacy if tax rates are determined by a balanced budget rule with a pre-set level of revenue. Under this type of budget rule, the government is forced to adjust tax rates downward as per capita income (the tax base) rises. This resembles our model with a regressive tax schedule. The general point is that regressive taxes can reduce the required degree of increasing returns needed for indeterminacy.

Christiano and Harrison (1996) show that indeterminacy can be prevented using a stabilization policy in which the tax rate is an increasing function of labor effort,

[2] Guo and Lansing (1997) explores an alternative policy mechanism that can achieve this result. We consider a fiscal policy that creates an environment with offsetting income and substitution effects so that current decisions are independent of expectations. As a result, expected higher future returns cannot be self-fulfilling and the mechanism for sunspot fluctuations is completely shut down.

as opposed to income. This resembles our model with a progressive tax schedule. In their model, households ignore the impact of their actions on the tax rate. However, the necessary and sufficient condition that we derive to ensure saddle path stability is the same, regardless of whether households take the progressivity of the tax schedule into account.

The remainder of this note is organized as follows. Section 2 describes the model and the tax schedule. The dynamics of the model are analyzed in section 3. Section 4 derives the necessary and sufficient condition on the tax schedule to ensure saddle path stability and briefly discusses the welfare implications. Section 5 concludes.

2 The Economy

2.1 Firms

The model introduces government fiscal policy into the continuous time framework of Benhabib and Farmer (1994). There is a continuum of identical competitive firms, with the total number normalized to one. The aggregate production function is Cobb-Douglas, given by

$$Y = K^{\alpha} L^{\beta}, \qquad \alpha,\ \beta > 0, \tag{1}$$

where Y is total output, K represents the aggregate stock of capital and L is aggregate labor hours.[3] Unlike the standard RBC model, we allow the parameters α and β to sum to more than one. Benhabib and Farmer (1994) describe two production environments that can reconcile (1) with a competitive theory of income distribution. For expositional simplicity, we choose to present the model with productive externalities.[4]

Equation (1) represents the *social* technology which explains how aggregate output would respond if all firms were to expand their capital and labor inputs simultaneously. This function is conceptually distinct from the *private* technology that

[3] For ease of notation, the time dependence of all variables is suppressed throughout this note. For example, Y represents $Y(t)$ and so on for other variables.

[4] An alternative specification incorporates internal increasing returns at the firm level in an imperfectly competitive economy. This yields an aggregate production function with the same form as (1).

exhibits constant returns to scale:

$$Y_i = K_i^a L_i^b X, \qquad a + b = 1, \tag{2}$$

where the subscript i indexes the individual firm. The term X represents productive externalities that are taken as given by each firm. We postulate that externalities take the form

$$X = \left(K^a L^b\right)^\chi, \qquad \chi \geq 0. \tag{3}$$

When $\chi = 0$, the model collapses to a standard RBC formulation with aggregate constant returns to scale. In a symmetric equilibrium, all firms take the same actions such that $K_i = K$ and $L_i = L$, for all i. As a result, (3) can be substituted into (2) to obtain the social technology (1) by defining $\alpha \equiv a(1+\chi)$ and $\beta \equiv b(1+\chi)$. We restrict our attention to the case where externalities are not strong enough to generate sustained endogenous growth. This implies an upper limit on α in conjunction with another parameter that governs the slope of the tax schedule. We will elaborate further on this point in our discussion of the equilibrium conditions.

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r = \frac{aY}{K}, \tag{4}$$

$$w = \frac{bY}{L}, \tag{5}$$

where r is the rental rate of capital and w is the real wage. Notice that the parameters a and b represent the shares of national income received by capital and labor, respectively.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time. Each household maximizes

$$\int_0^\infty e^{-\rho t} \left\{ \log c - A \frac{\ell^{1+\gamma}}{1+\gamma} \right\} dt, \qquad A > 0, \tag{6}$$

where c and ℓ are the individual household's consumption and hours worked, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution for labor supply, and $\rho > 0$ is the discount rate. The budget constraint faced by the representative household is

$$\dot{k} = (1-\tau)(rk + w\ell) - \delta k - c, \qquad k(0) \text{ given}, \tag{7}$$

where k is the household's capital stock, δ is the capital depreciation rate and τ represents the income tax rate, which is specified more completely below. Households derive income by supplying capital and labor services to firms, taking factor prices r and w as given. We assume that there are no fundamental uncertainties present in the economy.

2.3 Condition for Indeterminacy

In a version of the above model without government ($\tau = 0$), Benhabib and Farmer (1994) show that the necessary and sufficient condition for an indeterminate steady state is

$$\beta - 1 > \gamma, \tag{8}$$

which is equivalent to the statement that the equilibrium labor demand schedule slopes up as a function of real wage and is *steeper* than the labor supply curve.[5] When this condition is satisfied, the economy no longer exhibits a unique perfect foresight equilibrium. For any given initial stock of capital, there will be a continuum of equilibrium trajectories each converging towards the steady state (a sink). In a stochastic version of the model, (8) implies that there exist a continuum of stationary rational expectations equilibria in which self-fulfilling beliefs of agents can be a driving force of business cycle fluctuations. Farmer and Guo (1994) calibrate a discrete time version of this model economy and show that the business cycle properties of post-war U.S. time series data are broadly consistent with the predictions of a model driven

[5] Since $\gamma \geq 0$, the condition (8) requires $\beta - 1 > 0$. Recent empirical studies by Burnside (1996) and Basu and Fernald (1997) suggest that the U.S. economy is not characterized by such a high degree of increasing returns. However, further theoretical work by Benhabib and Farmer (1996), Harrison (1997), Perli (1997), and Weder (1997) shows that a two-sector version of the RBC model with a conventional downward sloping labor demand curve ($\beta - 1 < 0$) can exhibit indeterminacy for a much lower degree of increasing returns.

solely by *i.i.d.* sunspot shocks. In what follows, we maintain the hypothesis that condition (8) is satisfied.

2.4 Government

The government chooses the tax policy τ and balances its budget at each point in time. The instantaneous government budget constraint is

$$G = \tau Y. \tag{9}$$

We assume that tax revenues are used to finance government spending on goods and services G that do not contribute to either production or household utility.[6] The aggregate resource constraint for the economy is given by

$$C + \dot{K} + \delta K + G = Y, \tag{10}$$

where C represents aggregate consumption. We postulate that the government sets τ according to the following tax schedule:

$$\tau = 1 - \eta \left(\frac{\bar{y}}{y} \right)^{\phi}, \qquad \eta \in (0, 1], \quad \phi \in \left(\tfrac{\alpha - 1}{\alpha}, 1 \right), \tag{11}$$

where $y = rk + w\ell$ represents the individual household's taxable income and $\bar{y}$ denotes a base level of income that is taken as given. Here, we set $\bar{y}$ equal to steady-state level of per capita income.[7] The parameters η and ϕ govern the level and slope of the tax schedule. When $\phi > (<)\, 0$, the tax rate τ increases (decreases) with the household's taxable income, that is, households with taxable income above $\bar{y}$ face a higher (lower) tax rate than those with income below $\bar{y}$. When $\phi = 0$, all households face the same tax rate $1 - \eta$ regardless of their taxable income. In making decisions about how much to consume, work, and invest over their lifetimes, households take into account the way in which the tax schedule affects their earnings.

To understand the progressivity feature of (11), it is useful to distinguish between the average and marginal tax rates. The average tax rate τ is defined as the tax paid

[6] Our analysis remains unaffected if G enters the household utility function (6) in an additively separable way.

[7] In a model with technological progress, (11) must be modified to replace $\bar{y}$ by $z\bar{y}$, where z measures the level of technology that grows over time. This formulation ensures that τ remains stationary along the balanced growth path.

by each household T divided by its taxable income y. The marginal tax rate τ_m is defined as the change in taxes paid divided by the change in taxable income, that is, the rate applied to the last dollar earned. The expressions for the various tax rates are

$$T = \tau y = y - \eta \bar{y}^{\phi} y^{1-\phi}, \tag{12}$$

$$\tau = \frac{T}{y} = 1 - \eta \left(\frac{\bar{y}}{y}\right)^{\phi}, \tag{13}$$

$$\tau_m = \frac{\partial T}{\partial y} = 1 - \eta(1-\phi) \left(\frac{\bar{y}}{y}\right)^{\phi}. \tag{14}$$

We require the average tax rate τ to be non-negative. A negative average tax rate implies the existence of an income subsidy that could only be financed with a lump-sum tax. Here, we restrict our attention to an environment in which the government does not have access to lump-sum taxes or transfers. Furthermore, the average tax rate must be strictly less 100 percent to prevent the government from confiscating all productive resources. We also require the marginal tax rate τ_m to be strictly less than 100 percent so that households have an incentive to supply labor and capital services to firms. Finally, to guarantee the existence of a steady state, the economy's equilibrium after-tax interest rate must be strictly decreasing in K. This imposes a lower bound on τ_m that depends on the magnitude of the technology parameter α. In the steady state, the above considerations imply that $\eta \in (0, 1]$ and $\phi \in \left(\frac{\alpha-1}{\alpha}, 1\right)$.

Since $\tau_m = \tau + \eta\phi \left(\frac{\bar{y}}{y}\right)^{\phi}$, the marginal tax rate is above the average tax rate when $\phi > 0$. In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the value $1 - \eta$ and the tax schedule is said to be "flat".[8] When $\phi < 0$ the tax schedule is "regressive". In our model, a regressive tax schedule implies that τ is a decreasing function of household income y, which is similar to the specification used in Schmitt-Grohé and Uribe (1997). In their economy, the government must balance its budget at all times (as we also require

[8] A flat tax can also exhibit progressivity features through the appropriate use of a standard personal deduction. For simplicity, we abstract from any deductions here. An early proposal for a flat tax can be found in Friedman (1962, p. 174). For a more detailed proposal, see Hall and Rabushka (1995).

here), but public expenditures are assumed to be fixed outside of the government's control. With this type of budget rule, tax rates are decreasing in the level of *per capita* income. As per capita income rises, the tax base expands and the government is forced to balance its budget through downward adjustments in the tax rates.

2.5 Perfect Foresight Equilibrium

We focus on a symmetric perfect foresight equilibrium in which households maximize utility and firms make zero profits. The equilibrium is symmetric since each firm uses the same amount of capital and labor inputs in production. In equilibrium, the aggregate consistency condition requires $y = Y$, $\bar{y} = \bar{Y}$, $k = K$, $c = C$, and $\ell = L$. As a result, the first-order conditions that characterize the equilibrium are given by

$$ACL^{\gamma} = \underbrace{\eta(1-\phi)\left(\tfrac{\bar{Y}}{Y}\right)^{\phi}\tfrac{bY}{L}}_{(1-\tau_m)w}, \tag{15}$$

$$\frac{\dot{C}}{C} = \underbrace{\eta(1-\phi)\left(\tfrac{\bar{Y}}{Y}\right)^{\phi}\tfrac{aY}{K}}_{(1-\tau_m)r} - \rho - \delta, \tag{16}$$

$$\dot{K} = \underbrace{\eta\bar{Y}^{\phi}Y^{1-\phi}}_{(1-\tau)Y} - \delta K - C, \qquad K(0) \text{ given}, \tag{17}$$

$$\lim_{t\to\infty} e^{-\rho t}\frac{K}{C} = 0, \tag{18}$$

where (15) equates the slope of the representative household's indifference curve to the after-tax real wage and (16) is the consumption Euler equation. Notice that these first-order conditions are influenced by the level of the marginal tax rate τ_m. The lower bound on ϕ implies $\alpha(1-\phi) < 1$ so that the after-tax interest rate $(1-\tau_m)\,r$ in (16) is strictly decreasing in K. This restriction guarantees the existence of a steady state. Equation (17) is the law of motion for the capital stock and (18) is the transversality condition. When $\eta = 1$ and $\phi = 0$, the above conditions collapse to those derived by Benhabib and Farmer (1994).

3 Analysis of the Dynamics

To facilitate our analysis, we make the following logarithmic transformation of variables. Let $\hat{Y} \equiv \log(Y)$, $\hat{K} \equiv \log(K)$, $\hat{C} \equiv \log(C)$ and $\hat{L} \equiv \log(L)$. With this transformation, the two dynamic equations, (16) and (17), can be rewritten as

$$\dot{\hat{C}} = a\eta(1-\phi)\bar{Y}^{\phi}e^{(1-\phi)\hat{Y}-\hat{K}} - \rho - \delta, \tag{19}$$

$$\dot{\hat{K}} = \eta\bar{Y}^{\phi}e^{(1-\phi)\hat{Y}-\hat{K}} - \delta - e^{\hat{C}-\hat{K}} \tag{20}$$

For (19) and (20) to be an autonomous pair of differential equations, we need to eliminate $\hat{L}$ by expressing $(1-\phi)\hat{Y} - \hat{K}$ in terms of $\hat{C}$ and $\hat{K}$ only. Combining the two static equations (1) and (15) yields:

$$(1-\phi)\hat{Y} - \hat{K} = \lambda_0 + \lambda_1\hat{K} + \lambda_2\hat{C}, \tag{21}$$

where

$$\lambda_0 = \frac{-\beta(1-\phi)\left\{\log\left[\frac{b\eta(1-\phi)\bar{Y}^{\phi}}{A}\right]\right\}}{\beta(1-\phi)-1-\gamma}, \tag{22}$$

$$\lambda_1 = \frac{-\beta(1-\phi)-(\gamma+1)[\alpha(1-\phi)-1]}{\beta(1-\phi)-1-\gamma}, \tag{23}$$

$$\lambda_2 = \frac{\beta(1-\phi)}{\beta(1-\phi)-1-\gamma}. \tag{24}$$

Substituting (21) into (19) and (20), we obtain

$$\dot{\hat{C}} = a\eta(1-\phi)\bar{Y}^{\phi}e^{\lambda_0+\lambda_1\hat{K}+\lambda_2\hat{C}} - \rho - \delta, \tag{25}$$

$$\dot{\hat{K}} = \eta\bar{Y}^{\phi}e^{\lambda_0+\lambda_1\hat{K}+\lambda_2\hat{C}} - \delta - e^{\hat{C}-\hat{K}}. \tag{26}$$

An equilibrium is defined as a trajectory $\{\hat{C}(t), \hat{K}(t)\}_{t=0}^{\infty}$ that solves (25) and (26) subject to the initial condition $\hat{K}(0)$ and the transversality condition (18). Since

$\alpha(1-\phi) < 1$, it is straightforward to show that the above dynamical system possesses a unique interior steady state.[9] We can therefore compute the Jacobian matrix of the system defined by (25) and (26), evaluated at the steady state. The trace and the determinant of the Jacobian are given by

$$\text{Tr} = [\lambda_1 + a(1-\phi)\lambda_2]\left[\frac{\rho+\delta}{a(1-\phi)}\right] + \frac{\rho+b\delta}{a(1-\phi)}, \tag{27}$$

$$\text{Det} = \frac{-a(\gamma+1)(1-\phi)[\alpha(1-\phi)-1]}{\beta(1-\phi)-1-\gamma}\left[\frac{\rho+\delta}{a(1-\phi)}\right]\left[\frac{\rho+b\delta}{a(1-\phi)}\right]. \tag{28}$$

Notice that the level parameter of the tax schedule η does not appear in either (27) or (28), and thus has no impact on the model's local stability properties. We now derive the necessary and sufficient condition on the slope parameter ϕ to prevent indeterminacy.

4 Necessary and Sufficient Condition

Proposition. *The necessary and sufficient condition for the model to exhibit saddle path stability is*

$$\beta(1-\phi) - 1 < \gamma. \tag{29}$$

Proof. Saddle path stability requires the eigenvalues of the dynamical system (25) and (26) to be of opposite sign, that is, the determinant of the Jacobian as in (28) must be negative. The requirement that the marginal tax rate be strictly less than 100 percent implies $\phi < 1$. Given the fact that $a, b, \rho > 0$ and $\gamma, \delta \geq 0$, the determinant will be negative if and only if $\frac{[\alpha(1-\phi)-1]}{\beta(1-\phi)-1-\gamma} > 0$. Since $\alpha(1-\phi) < 1$ to guarantee the existence of a steady state, $\beta(1-\phi)-1-\gamma$ governs the sign of the determinant. When $\beta(1-\phi)-1 < \gamma$, we have $\frac{[\alpha(1-\phi)-1]}{\beta(1-\phi)-1-\gamma} > 0$ and Det < 0. ■

The intuition for condition (29) can be understood by taking logarithms of both sides of the labor market equilibrium condition (15) to obtain

$$\hat{C} + \gamma\hat{L} = \hat{\omega} = \left\{\log\left[\frac{b\eta(1-\phi)\bar{Y}^{\phi}}{A}\right]\right\} + \alpha(1-\phi)\hat{K} + [\beta(1-\phi)-1]\hat{L}, \tag{30}$$

[9] Christiano and Harrison (1996) study an endogenous growth version of this model with $a = \frac{1}{3}$ and $\chi = 2$ such that the social technology (1) is $Y = KL^2$. Thus, the dynamics of their model collapse to a first-order Euler equation in L. In contrast, our model exhibits the standard second-order dynamics.

where $\hat{\omega}$ represents the logarithm of the after-tax real wage. Notice that the slope of the equilibrium labor demand schedule is given by $\beta(1-\phi)-1$, and the slope of the labor supply curve is γ. For the government to stabilize the economy against sunspot fluctuations, condition (29) requires the slope parameter of the tax schedule ϕ must be high enough to make the equilibrium labor demand schedule *flatter* than the labor supply curve, that is, to overturn the relative steepness criteria of the Benhabib-Farmer condition (8).

To provide a sense of the required magnitude of ϕ that satisfies saddle path stability, consider the following parameterization used by Benhabib and Farmer (1994): the labor share of national income $b = 0.7$, the degree of externalities in production $\chi = 0.5$, and the labor supply elasticity parameter $\gamma = 0$ (i.e., the *indivisible* labor specification of Hansen [1985] and Rogerson [1988]). By substituting these values into condition (29), namely, $\phi > 1 - \frac{1+\gamma}{\beta}$, we find that the tax policy described by (11) renders a unique determinate equilibrium for $\phi > 0.048$.[10]

Since there is no upper bound on the parameter γ, condition (29) can be satisfied by a wide range of values for ϕ, provided $\frac{\alpha-1}{\alpha} < \phi < 1$. When the tax schedule becomes more progressive (higher ϕ), saddle path stability is more likely to occur even for an economy with strong increasing returns. Conversely, as the tax schedule becomes flatter or more regressive (smaller ϕ), the economy is more susceptible to indeterminacy, that is, (29) can be violated at a lower degree of increasing returns.

It turns out that if households ignore the influence of their actions on the tax rate, the necessary and sufficient condition given by the Proposition remains unchanged. In this case, the expression for the determinant becomes

$$\text{Det} = \frac{-a(\gamma+1)[\alpha(1-\phi)-1]}{\beta(1-\phi)-1-\gamma}\left(\frac{\rho+\delta}{a}\right)\left(\frac{\rho+b\delta}{a}\right), \tag{31}$$

which again shows that $\beta(1-\phi)-1-\gamma$ governs the sign of the determinant. The crucial feature in being able to prevent self-fulfilling expectations is for the policy to "tax away" the higher returns from belief-driven labor or investment spurts. This

[10] Cassou and Lansing (1996) use a nonlinear least squares regression to approximate the the U.S. federal income tax schedule by the function $\tau = 0.1823(y/\bar{y})^{0.2734}$, where $y/\bar{y}$ is household taxable income divided by its mean level. Comparing this expression to (11) yields $\eta = 0.8177$ and $\phi = 0.0609$. This value of ϕ exceeds the critical value of 0.048 needed to ensure saddle path stability. In contrast, the same economy with a flat tax ($\phi = 0$) will exhibit an indeterminate steady state.

occurs regardless of whether agents have taken into account the impacts of their decisions on the tax rate.

The welfare consequences of the tax policy given by (11) depend on a number of important factors. First, our model does not require the tax schedule parameters η and ϕ to be set according to any welfare maximizing criterion. Second, as pointed out by Christiano and Harrison (1996), the desirability of stabilizing the economy against sunspot fluctuations is determined by the relative magnitude of two opposing factors. Concavity of utility implies that fluctuations reduce welfare, whereas increasing returns in production works in the opposite direction. As a result, we cannot argue *a priori* that a stabilization policy which eliminates indeterminacy is desirable from a welfare standpoint. Hence, the welfare effects of the policy (11) will depend crucially on the model parameterization and assumptions regarding the availability of other policy instruments, such as lump-sum taxes or transfers.

5 Conclusion

We have shown how a simple progressive tax schedule can stabilize the economy against belief-driven fluctuations in a one-sector real business cycle model with strong increasing returns. Conversely, an economy that exhibits a flat or regressive tax schedule may be more susceptible to such fluctuations.

This note can be extended in several directions. In particular, it would be interesting to more thoroughly explore the empirical plausibility of indeterminacy in an economy like that of U.S. which is characterized by a progressive tax schedule. Given the findings of recent empirical studies, such an analysis would best be conducted in a two-sector model in which the required degree of increasing returns for indeterminacy is much less stringent. Moreover, it would be worthwhile to introduce a richer tax structure that allows the model to capture differences in progressivity between the personal and corporate tax schedules in the U.S. economy. With these modifications, a discussion of the magnitude of the tax schedule slope relative to the degree of increasing returns becomes more relevant. We hope to pursue this project in the near future.

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