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IMPERFECT STATE VERIFICATION AND FINANCIAL CONTRACTING

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ABSTRACT

Standard work on costly state verification, monitoring, and auditing generally assumes perfect signals about the underlying state, especially in questions about financial contracting. Relaxing that assumption has several intriguing consequences. Most imperfect audits turn out to be useless, and those that are useful cannot be ranked by conventional criteria such as Blackwell's information measure. Thus, the notion of "more" or "less" information becomes problematic.

1. Introduction

Costly state verification, monitoring, or auditing often shows up in models of informational economics and finance. Some papers use it to explain individual contracts, such as Townsend (1979), which introduces the concept, and Mookherjee and Png (1989), which explores random auditing. Others extend this work to look at financial institutions, such as Diamond (1984) and Williamson (1986). In all of these cases, the audit/monitoring technology, or state verification, is perfect in the sense that the true state is revealed with certainty. Much less work has been done using imperfect monitoring, where the signal gives only probabilistic information about the state. The reluctance stems in part from the belief that this generalization would be messy and complicated without yielding substantially new insights. For example, Dye (1986), in considering a principal-agent model, finds he can generalize his results to the case of imperfect monitoring with more restrictive and less plausible assumptions. Mookherjee and Png (p. 414) assert that "it may be verified, by continuity arguments, that our results extend to the case when there are 'small' errors in auditing."

The situation is in fact quite different for the standard costly state verification model. When signals about states are uncertain, there is generally no advantage to monitoring. Information in this class of models is useless unless it is perfect. Even in the exceptional cases, this result means that the natural metrics on information break down: It becomes difficult to rank monitoring schemes, even using such powerful tools as Blackwell's (1951, 1953) Measure of Informativeness.

This suggests that future investigations of imperfect monitoring may have to proceed on a case-by-case basis, to mix imperfect signals with perfect monitoring, or to rely on other types of imperfect monitoring, such as those used by Gorton and Haubrich (1987), where only a minimum effort level can be verified, or Lacker and Weinberg (1989), where some fraction of output can be hidden.

After presenting a simple two-state example, section 2 proves the general case and then discusses the exceptions. Section 3 shows the deficiencies of Blackwell's information measure in this model, and section 4 concludes.

2. Imperfect State Verification

In the model economy discussed here, a risk-averse (possibly risk-neutral) agent has private information about his income, and a risk-neutral uninformed principal has the right to audit or monitor the agent. For concreteness, I consider the agent as a borrower endowed with an investment project that needs one unit of funds to begin operation. The borrower has no funds and must raise them externally from a lender, who may invest in the project or in a riskless alternative asset with gross return R. A project that gets funded produces a random income level Y_i in state i. There are N possible income levels: $Y_1 < Y_2 <...< Y_N$. λ_i is the probability of state i occurring. The λ_i's thus describe the lender's prior distribution about the agent's income. The project's outcome is costlessly observed by the borrower, but this is private information. Without any sort of audit, the borrower can always claim that the bad state occurred and can transfer the minimum amount to the principal. To make the problem interesting, we assume that this amount isn't enough to keep the principal/investor happy, and that it falls below the reservation utility he gets from an alternative investment; in other words, $Y_1 < R$.

The lender may pay cost γ to obtain a signal s about the income level actually realized. We assume $0 < \gamma < Y_1$, so that income always covers audit costs. The conditional probability π_{is} relates the signal and the state; that is, it denotes the probability of getting signal s given state i. When income truly equals Y_i, $\pi_{is} = \Pr[s|i]$. Recall that by the definition of conditional probability, $\sum_s \pi_{is} = 1$. One very natural signal is the sort announcing "The true state is 5," which signals that Y_5 is actual income. The signal is of course uncertain, and π_{55} tells us the probability that if the state really is 5, we will receive the signal telling us that it is 5. The formulation, however, is more general. The number of possible signals, S, need not equal the number of income states, N. We might have four income states, but get a signal saying only that income was good or bad. Conversely, income may depend on whether it rained or not, signaled by a barometer reading of low, average, or high. The general signaling literature even allows continuous signals with finite states (Kihlstrom [1984]).

The sequence of events has two stages. In the first, the project owner proposes contracts specifying (ex post) state-contingent payoffs to the potential lender. The lender then decides between lending to the borrower or investing in the riskless asset. When the borrower obtains funding, the project produces its output and the borrower learns the state. In the next stage, the borrower announces a state, whereupon the lender flips a coin and with probability p_i conducts an audit. The borrower then makes a transfer to the lender. If an audit occurs, the transfer F_{is} depends on both the announced state and the signal received. If no audit occurs, the transfer T_i depends only on the announced state. In the second stage, lenders have no income of their own, so that all payments must come from the project, including not only transfer payments but also audit costs.

An eye toward financial contracting also results in some restrictions on transfers between the borrower and the investor (or the peasants and the lord). First, the borrower must have non-negative consumption. The lord cannot take more wheat than the farmer has grown. Second, the transfer must be non-negative: The investor never makes an interest payment to the entrepreneur, as the lord never gives wheat to the farmers -- he hasn't any to give.[1] This describes a world where the resources to be divided come from the agent's production, as in Border and Sobel (1987). Plausible alternative worlds exist. For example, in Mookherjee and Png (1989), negative transfers provide insurance against bad states.

Throughout this paper, I restrict attention to incentive-compatible direct mechanisms. Hence, transfers between the agent and principal depend only on the income level announced by the agent and on the signal received, or F_{is} , T_i if no signal is sent. This may represent a real restriction. Standard statements of the Revelation Principle (Harris and Townsend [1981]) do not allow messages that depend on the

[1] The agent can never claim to have more than the amount actually produced. Think of the claim as delivering bushels of wheat to the investor. The farmer may hold back (just as a businessman may hide profits), but he cannot deliver wheat that doesn't exist. Gale and Hellwig (1985) emphasize this point.

player's type. Green and Laffont (1986) show that in some cases using imperfectly verified information, the Revelation Principle does not hold.

The lender is risk neutral and the agent is weakly risk averse with von-Neumann-Morgenstern utility function u, so that u is strictly increasing, differentiable, and concave. The agent's consumption must be non-negative, with the convention that $u(0)=0$. This allows us to express the basic programming problem of the model as follows:

(1) $$\max_{\{F_{is},T_i,p_i\}} \sum_i \lambda_i \sum_s \pi_{is}[p_i u(Y_i - F_{is}) + (1-p_i)u(Y_i - T_i)]$$ (Expected utility of borrower)

Subject to

(2) $$V_i \equiv \sum_s \pi_{is}[p_i u(Y_i - F_{is}) + (1-p_i)u(Y_i - T_i)] \geq \sum_s \pi_{is}[p_h u(Y_i - F_{hs}) + (1-p_h)u(Y_i - T_h)] \;\forall\, i, h$$ (Incentive compatibility or reporting constraints)

(3) $$\sum_i \lambda_i \sum_s \pi_{is}[p_i F_{is} - p_i\gamma + (1-p_i)T_i] \geq R$$ (Expected profit for principal)

(4) $$Y_i - F_{is} \geq 0$$

(5) $$Y_i - T_i \geq 0$$ (Non-negativity constraints)

(6) $$F_{is} \geq 0$$

(7) $$T_i \geq \gamma .$$

The problem, then, is to choose audit probabilities p_i, along with transfers T_i and F_{is} dependent on the state, signal, and audit, to maximize the borrower's expected utility subject to i) the incentive compatibility constraints, ii) a participation constraint for the lender, and iii) the appropriate non-negativity constraints.

The non-negativity constraints (4-7) bound the set of possible F_{is} and T_i and, in conjunction with the form of the reporting constraints (2), guarantee a compact set. Maximizing a continuous function, the agent's expected utility (1) over a compact set has a solution by the Maximum Value Theorem (see Bartle [1976], section 22). Mookherjee and Png, who allow negative transfers, cannot invoke this theorem and provide a different existence proof. In either case, actually calculating the optimum is tricky. The non-convexity of the reporting constraints (2) generally precludes the use of Lagrange multipliers in these sorts of problems.

A simple 2x2 example exhibits the techniques and intuition. The key is the interaction between the incentive compatibility and non-negativity constraints. Let N=S=2, set the audit cost to zero, and ignore the possibility of random audits. In addition, let the conditional probability of each signal be strictly greater than zero, so that $\pi_{is} > 0$. Then, the reporting constraint (2) implies that for state 2, $V_2 = \pi_{21}u(Y_2 - F_{21}) + \pi_{22}u(Y_2 - F_{22}) \geq \pi_{21}u(Y_2 - F_{11}) + \pi_{22}u(Y_2 - F_{12})$. This says that the agent's expected utility from telling the truth -- correctly declaring I=2 -- exceeds his utility from falsely declaring i=1. The uncertainty arises because the signal may confirm (s=1) or contradict (s=2) the declared state. The non-negativity constraints force $F_{11} \leq Y_1$ and $F_{12} \leq Y_1$. Hence, $\pi_{21}u(Y_2 - F_{21}) + \pi_{22}u(Y_2 - F_{22}) \geq u(Y_2 - Y_1)$. The transfer to the principal can never exceed Y_1: The principal cannot receive more in the good state than in the bad state. If this form of the problem is to have any interest, the principal's expected profit (3) won't be met by this contract. Partial information does not help, as the incentive compatibility constraints conflict with the expected profit constraint.

This result has a very simple, straightforward, and intuitive explanation. The signal has shifted probabilities around, but it has not changed the nature of the problem. The principal cannot prove that the bad state did not occur and must therefore settle for the minimum possible payment.

The technique, and intuition, generalize to more incomes and more signals.

Proposition 1: Let there be N income states and S signals. If $\pi_{is} > 0$ (strictly) for all s, then for F_{is}, T_i, and p_i solving (1) subject to (2)-(7), we have that $\sum_i \lambda_i \sum_s \pi_{is}[p_i F_{is} - p_i \gamma + (1 - p_i T_i)] \leq Y_1$. That is, expected profits never exceed the output of the lowest state.

Proof: The incentive compatibility constraints (2) yield

$$V_i \ = \sum_s \pi_{is}[p_i u(Y_i - F_{is}) + (1 - p_i) u(Y_i - T_i)] \ \geq \ \sum_s \pi_{is}[p_h u(Y_i - F_{hs}) + (1 - p_h) u(Y_i - T_h)]$$

$\forall\ i,h$ for all i in $\{1, 2, \ldots N\}$. In particular,

$$V_i \geq \sum_s \pi_{is}[p_1 u(Y_i - F_{1s}) + (1 - p_1) u(Y_i - T_1)].$$

Thus, by the non-negativity constraints (4) and (5),

$$V_i \geq \sum_s \pi_{is}[p_1 u(Y_i - Y_1) + (1 - p_1) u(Y_i - Y_1)] \ = \ \sum_s \pi_{is}[u(Y_i - Y_1)] = u(Y_i - Y_1).$$

Risk aversion, via Jensen's inequality, implies that the expected utility of an uncertain transfer with expected value Y_1 will be lower than the expected utility of a certain transfer of Y_1. Hence, to satisfy the borrower's incentive compatibility constraint $V_i \geq u(Y_i - Y_1)$, the largest expected transfer to the principal (lender) cannot exceed Y_1.

Hence, the principal can never extract more than in the lowest output state Y_1. ■

Of course, with sufficiently low opportunity cost *R*, the principal would be satisfied with such a contract. But if we assume as above that $R > Y_1$, that is, if the bank demands more than the lowest possible output (or the landlord demands more than the worst possible harvest), no investment will take place. The principal cannot get an acceptable return from the project. We state this as:

Corollary: If $Y_1 < R$, then (1) subject to (2)-(7) has no solution.

An alternative statement, if we had included the initial investment decisions, would note that the only solution sets the initial investment to zero; the project does not get funded.

Imperfect information can help if some of the conditional probabilities π_{is} are zero. Receiving a particular signal may now definitely rule out some states and allow larger transfers. Returning to the simple two-state example will help to clarify this.

Let the matrix of π_{is} be $\begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix}$. The first thing to notice here is that the certain state is 2: If you get a signal saying state 2, you are in fact in that state. If the signal says state 1, you can't be sure. The reporting constraints are then

$$V_1 = u(Y_1 - F_{11}) \geq u(0)$$

$$V_2 = 0.5u(Y_2 - F_{21}) + 0.5u(Y_2 - F_{22}) \geq 0.5u(Y_2 - F_{11}) + 0.5u(Y_2 - F_{12}).$$

With F_{12} not pinned down by the reporting constraint for state 1, it can exceed Y_1, allowing F_{22} to exceed Y_1 -- a definite improvement. Of course, it's possible that the expected profit constraint is not met, but the zero in the first row is a move in the right direction.

The above example also shows that once we have added a zero in the first row, then more zeros can help -- in this case creating perfect information. This intuition generalizes. Putting a zero in the first row means that some signals will rule out the lowest state. Adding more zeros can rule out more states.

The sense in which information is generically useless in this class of models should now be clear. Unless there are zeros in the first row (that is, unless the probability of some signal given state 1 is zero), partial information adds nothing. Consider $\{\pi_{is}\}$ as a random vector in $\Re^{NS}$, drawn from an absolutely continuous distribution. Then it is only on a set of (Lebesgue) measure zero that partial information can improve upon the no-information case.

3. Blackwell's Information Criteria

Using imperfect information leads to a natural desire for quantification. Can we rank signals by how much information they provide? David Blackwell (1951, 1953) provides an affirmative answer by proving the equivalence of several natural measures of informativeness, such as every information user preferring one signal, or one signal being noisier than another (see McGuire [1986] or Kihlstrom [1984] for expositions). For our purposes, the most useful formulation is the one using Markov matrices. Let **P** and **Q** be the conditional probability matrices associated with two signals (p_{is} and q_{is}). Then **P** is more informative than **Q** (in the sense of Blackwell) if there exists a Markov matrix **M** ($\sum m_{st} = 1$ and $m_{st} \geq 0$) such that

(6) $$\mathbf{PM} = \mathbf{Q}.$$

Unfortunately, the Blackwell measure does not work for the problem at hand. First, notice that one signal may be more informative than another and yet be unable to improve upon the outcome if there are no zeros in the first row. An example would be

(7) $$\mathbf{P} = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.$$

Second, and conversely, adding zeros does not necessarily make the signal more informative in the sense of Blackwell. Working out these examples is not difficult. Note, however, that adding a zero, leaving all other rows unchanged, and redistributing only the probability mass from the matrix element reduced to zero does not guarantee a more informative signal in the sense of Blackwell. That is, we cannot find a Markov matrix **M** satisfying (6) when

(8) $$\mathbf{Q} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In this case, the element $m_{12} = -\frac{1}{8}$, so **M** cannot be Markov.

Blackwell's theorem on the comparison of experiments has two parts. The sufficiency part shows that any decisionmaker will prefer a more informative signal to a less informative one. The necessity part shows that if any decisionmaker prefers one signal to another, the preferred signal will be more informative.

In the monitoring model used in this paper, a weak form of sufficiency holds. Adding noise, in the sense of multiplying by a Markov matrix **M,** may remove a zero from the first row and hence make things worse. Even if adding noise does not remove the zero, a variant of the proof in Grossman and Hart's (1983) Proposition 13 shows that the added randomness does not help. Where signals are useless, adding noise will have no effect. Finally, adding noise will never put a zero in the first row unless the corresponding columns of the **M** matrix are all zero, which implies that all the corresponding columns of the transformed matrix must also contain only zeros. This merely drops one signal from consideration, which does nothing to help monitoring. Thus, a *less* informative signal cannot be better in the monitoring model.

The counterexamples (7) and (8) show that the necessity side of the Blackwell theorem fails in the monitoring model. Risk-averse agents prefer one signal to another, even though that signal is not more informative in the sense of Blackwell. This may not be surprising, as the monitoring model deals with incentives -- and thus with control -- in addition to estimation. Even so, it was only recently that Kim (1995) produced a counterexample for the principal-agent model.[2]

[2] See also the interesting work of Singh (1991).

4. Conclusion

Extending models of auditing, monitoring, and costly state verification to cases of imperfect signals seems a worthwhile goal. However, except in special cases, imperfect signals cannot improve upon the no-audit case. Likewise, except under special circumstances, Blackwell's information measure does not describe the quality of the information provided by the signal. The special cases may still be worth studying, however, since these extremes may be the scenarios most likely to occur in the real world (Friedman [1965]). Realistically, monitoring technology seems sufficiently advanced that a hugely profitable entrepreneur or farmer probably cannot appear truly destitute, even though he may hide or divert some funds. In future research, I hope to see whether the special cases have interesting applications. An alternative approach would be to consider some sort of two-stage audit. Then, imperfect information could provide a tighter distribution over the states, which in certain cases could then be verified perfectly.

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