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## MAGNIFICATION EFFECTS AND ACYCLICAL REAL WAGES

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## I. Introduction

Real business cycle (RBC) theory has been successful in simulating the variability of and comovements among aggregate variables, such as output, consumption, and investment. However, in order to generate the observed movements in employment over the business cycle, RBC models have had to produce highly procyclical real wages. ${ }^{1}$ This is inconsistent with data showing that real wages are either slightly procyclical or acyclical.

This paper presents a one-period, two-sector model that reconciles large movements in employment and output with acyclical real wages. The two sectors, which can be thought of as durables and services, differ in their cyclical sensitivities. The shocks to both sectors are positively correlated, but the shock to durables has a larger variance than the shock to services. In this type of stochastic environment, workers move from the durables sector to the service sector during downturns and from the service sector to the durables sector during upturns.

The model presented here is motivated by Rogerson (1986), who studies an infinite-horizon two-sector model. One of the sectors is high growth (low cyclical sensitivity, interpreted as services), while the other is low growth (high cyclical sensitivity, interpreted as durables). Rogerson shows that this

[^0]type of economy experiences a sectoral reallocation of workers from durables to services during downturns. Empirical evidence reported by Loungani and Rogerson (1988) supports this finding.

We modify Rogerson's environment to account for the acyclical behavior of wages. For simplicity, we present a one-period, twosector model that captures the sectoral reallocation process discussed in the earlier study. However, unlike Rogerson's model, ours assumes that firms face a fixed cost of hiring workers, which in turn generates a magnification effect. ${ }^{2}$ In other words, the size of the sectoral shock needed to generate a given amount of sectoral reallocation is smaller in the presence of a fixed hiring cost. This "magnification effect" causes wages to be less procyclical, because additional workers flowing into the service sector tend to depress wages there.

If the economy experiences a negative productivity shock, workers move from the durable goods sector to the service sector, and as they do, more firms in the service sector find it worthwhile to incur the fixed cost of hiring some of the incoming workers. With an increase in the number of service-sector firms hiring, each firm hires fewer workers, thus mitigating the real wage decline in that sector. Because the real wage in the service sector declines by less than it would in the absence of a fixed cost of hiring, additional workers find it advantageous to

[^1]move from durable goods to services. Therefore, the increase in the number of workers leaving the durables sector causes the real wage in that sector to decline by less than it would have without the presence of fixed costs.

The magnification effect generated by our model is similar to that reported by Chatterjee and Cooper (1988, 1989). Their work is specifically aimed at generating large output effects from small shocks in economies with endogenous entry and exit, and is part of a growing body of literature that demonstrates how economies can exhibit Keynesian-type features such as multiple equilibria and magnification effects (see Diamond [1988] and Cooper and John [1988]).

Although our model exhibits magnification effects, it differs from Chatterjee and cooper's work in that we do not construct an economy with multiple equilibria. Our eventual goal is to embed this type of model into an RBC model in order to generate data that can be matched with real-world observations.

This paper is organized as follows. Section II describes the model. Section III presents simulations of the model and analyzes the results. Section IV discusses extensions of our framework and concludes.

## II. The Model

This section presents a one-period, two-sector, general equilibrium model in which the firm's decision to hire and the worker's decision to stay or to relocate are endogenous. Labor
moves freely from one sector to the other, but firms face a fixed hiring cost. ${ }^{3}$

The existence of this fixed cost produces a nonconvexity in the firm's hiring decision; that is, firms will not hire additional workers until the payoff exceeds the fixed cost. Once this threshold is exceeded, the firm hires workers until wages equal the marginal productivity of labor. This nonconvexity leads to both the magnification effect and a real wage that is less procyclical.

The timing of decisions in this economy is as follows. Workers are initially allocated to one of the two sectors: durables or services. One can think of this initial allocation as being determined by the demand conditions in the previous period. For simplicity, we assume that N workers are allocated to each firm and that there are an equal number of firms in each sector. Prior to production, firms and workers observe the state of the economy, i.e., they observe the productivity shocks as well as the demand conditions in each sector. ${ }^{4}$ Simultaneously, workers decide where to work and firms decide whether to incur the fixed cost of hiring. Once these variables are determined,

[^2]production and consumption take place. We consider types of shocks such that, in equilibrium, workers will move from the durable goods sector to the service sector, and only firms in the service sector hire additional workers.

We repeat this one-period game for several different realizations of the productivity shocks to each sector. We then compute the variances of wages, output, and employment and compare these to the case in which firms do not face a fixed cost of hiring.

## Consumption

We assume that the economy is populated by a large number of identical consumers, each of whom lives for one period. A representative consumer is initially located in one of the two sectors: the $S$-sector (services) or the D-sector (durables). Prior to production, the consumer observes the productivity shock to each sector and decides whether to stay and produce or move to the other sector and produce. There are no fixed costs associated with moving; however, the worker who moves faces an exogenous probability of finding a production opportunity (employment) in the other sector.

A representative worker chooses a sector to work in and the quantities of the $S$-good ( $C_{s}$ ) and the $D$-good ( $C_{D}$ ) in order to maximize the following utility function:

$$
\begin{equation*}
\mathrm{U}\left(\mathrm{C}_{\mathrm{S}}, \mathrm{C}_{\mathrm{D}}\right)=\gamma \mathrm{C}_{\mathrm{S} \delta} / \delta+\mathrm{C}_{\mathrm{D}}^{\delta} / \delta, \tag{1}
\end{equation*}
$$

where $0 \leq \delta \leq 1$ and $\gamma \geq 1$. We restrict $\gamma$ to be greater than or equal to one because we only consider movements from durables to services.

The representative worker faces the usual constraint that the sum of the quantities $C_{s}$ and $C_{D}$ multiplied by their respective prices is less than or equal to the wage. For simplicity, we assume that individuals in this economy have no utility for leisure. ${ }^{5}$

## Production

Production in both sectors is carried out by a large number of perfectly competitive firms. Each firm is initially endowed with N workers, and labor is the only input in the production process. The production functions in both sectors exhibit diminishing marginal product and are identical except for a multiplicative shock.

A representative firm faces the decision of whether to produce with its initial allocation of workers or to incur a fixed cost and hire additional workers. Formally, the profit function for the representative firm in the s-sector that decides not to hire any additional workers is as follows:

$$
\begin{equation*}
\pi_{\mathrm{s}}{ }^{\mathrm{nh}}=\mathrm{P}_{\mathrm{s}} \epsilon_{\mathrm{s}} \mathrm{~N}^{\alpha}-\mathrm{W}_{\mathrm{S}} \mathrm{~N}, \tag{2}
\end{equation*}
$$

${ }^{5}$ The implications of relaxing this assumption are explored in section IV.
where $W_{s}$ is the wage paid in the $S$-sector to existing workers, $P_{s}$ is the price of output in the $S$-sector, and $\epsilon_{\mathrm{s}}$ is a random productivity shock.

Firms in the s-sector that decide to hire additional workers have the following profit function:

$$
\begin{equation*}
\pi_{s}{ }^{h}=p_{s} \epsilon_{s}[N(1+\theta q / h)]^{\alpha}-W_{S} N(1+\theta q / h)-p_{s} k \tag{3}
\end{equation*}
$$

where $\theta$ is the proportion of $D$-sector workers who move to the $S$ sector, $q$ is the exogenous probability of finding employment in the $s$-sector, $h$ is the fraction of firms in the $S$-sector that decide to incur the fixed cost in order to hire the incoming workers, and $k$ is the fixed cost of hiring measured in units of the s -good. The quantity $\theta \mathrm{q} / \mathrm{h}$ is thus the number of additional workers that will be employed by those firms in the s-sector that decide to hire.

Since we consider only movements from the $D$-sector to the $S$ sector, the profit function for the representative D-sector firm is simply:

$$
\begin{equation*}
\pi_{D}=P_{D} \epsilon_{D}[N(1-\theta)]^{\alpha}-W_{D} N(1-\theta), \tag{4}
\end{equation*}
$$

where $\epsilon_{D}$ is the productivity shock to the $D$-sector, $W_{D}$ is the wage paid in the $D$-sector, and $P_{D}$ is the price of output in the D-sector.

Since output and labor markets are perfectly competitive and
consumers are all identical, this problem can be reduced to a social planner's problem. In order to maximize the utility of a representative consumer subject to the production constraints, the social planner chooses consumption in the $S$-sector ( $\mathrm{C}_{\mathrm{s}}$ ), consumption in the $D$-sector $\left(C_{D}\right)$, the proportion of workers moving from the D-sector to the s-sector ( $\theta$ ), and the proportion of firms hiring in the s-sector (h). Formally:

$$
\begin{align*}
& \max U\left(C_{S}, C_{D}\right)=\gamma C_{s}^{\delta} / \delta+C_{D}^{\delta} / \delta  \tag{5}\\
& \left\{C_{S}, C_{D}, \theta, h\right\}
\end{align*}
$$

subject to

$$
\begin{align*}
& C_{S}=h \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha}+(1-\mathrm{h}) \epsilon_{\mathrm{S}} \mathrm{~N}^{\alpha}-\mathrm{hk}, \text { and }  \tag{6}\\
& \mathrm{C}_{\mathrm{D}}=\epsilon_{\mathrm{D}}[\mathrm{~N}(1-\theta)]^{\alpha} . \tag{7}
\end{align*}
$$

Constraint (6) shows that the total amount of the $s$-good produced (per consumer) is equal to the proportion produced by the hiring firms $\left\{h \epsilon_{S}[N(1+\theta q / h)]^{\alpha}\right\}$ and the nonhiring firms [(1h) $\left.\epsilon_{\mathrm{S}} \mathrm{N}^{\alpha}\right]$ less the fixed cost of hiring (hk). We normalized the number of firms to be equal to one. Constraint (7) is simply total output of firms in the D-sector.

Carrying out this maximization yields the following first order conditions:

$$
\begin{equation*}
\gamma C_{s}{ }^{\delta-1}=\lambda_{1}, \tag{8}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{C}_{\mathrm{D}}{ }^{\delta-1}=\lambda_{2},  \tag{9}\\
& \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha}-\epsilon_{\mathrm{S}} \mathrm{~N}^{\alpha}=\alpha \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha-1} \mathrm{~N} \theta q / \mathrm{h}+\mathrm{k},  \tag{10}\\
& \lambda_{1} \alpha \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha-1} \mathrm{q}=\lambda_{2} \alpha[\mathrm{~N}(1-\theta)]^{\alpha-1},  \tag{11}\\
& \mathrm{C}_{\mathrm{S}}=\mathrm{h} \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha}+(1-\mathrm{h}) \epsilon_{\mathrm{S}} \mathrm{~N}^{\alpha}-\mathrm{hk}, \text { and }  \tag{12}\\
& \mathrm{C}_{\mathrm{D}}=\epsilon_{\mathrm{D}}[\mathrm{~N}(1-\theta)]^{\alpha}, \tag{13}
\end{align*}
$$

where $\lambda_{1}$ and $\lambda_{2}$ are the Lagrange multipliers associated with constraints (6) and (7), respectively. First order conditions (8) and (9) are the usual marginal utility conditions.

To understand equations (10) and (11), consider the following decentralized version of the social planner's problem in which we interpret $\lambda_{1}$ to be the price of the $S$-good, $P_{S}$, and $\lambda_{2}$ to be the price of the $D$-good, $P_{D}$. Using this convention, first order condition (11) states that expected real wages (measured in utility units) are equal across sectors.

Equation (10) is the result of maximizing with respect to the number of firms hiring in the s-sector, $h$. To see the intuition behind this condition, consider the outcome of the maximization problem faced by an individual firm in the economy. Firms that choose to incur the fixed cost of hiring do so up to the point where the return from hiring additional workers is equal to the cost:

$$
\begin{equation*}
P_{S} \epsilon_{S}[N(1+\theta q / h)]^{\alpha}-W_{S}[N(1+\theta q / h)]-\left(P_{S} \epsilon_{S} N^{\alpha}-W_{S} N\right)=P_{S} k \tag{14}
\end{equation*}
$$

The left side of this equation is the difference between
profits if the firm hires or does not hire. The right side is simply the fixed cost of hiring. Multiplying equation (10) by $\lambda_{1}$ ( $\mathrm{P}_{\mathrm{S}}$ ) and recognizing that $\mathrm{W}_{\mathrm{S}}=\lambda_{1} \alpha \epsilon_{\mathrm{S}}[\mathrm{N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha-1}$, this condition can be solved to yield first order condition (10). Equations (10) and (14) state that profits of firms that do hire must be equal to profits of firms that do not hire, or $\pi_{s}{ }^{\mathrm{h}}=\pi_{\mathrm{s}}{ }^{\mathrm{nh}}$. Although the structure of this model is quite simple, the first order conditions are highly nonlinear and cannot be solved for reduced-form expressions. As an alternative to an analytical solution, we parameterize this economy and simulate the behavior of wages, employment, and output for various realizations of the productivity shocks $\epsilon_{\mathrm{S}}$ and $\epsilon_{\mathrm{D}}$.

## III. Simulations

The purpose of this section is twofold. First, we briefly describe the technique used to simulate the model presented in section II. Second, we present the results from several simulations of the model and the intuition behind them. ${ }^{6}$

As shown in the appendix, first order conditions (8)-(13) can be reduced to a system of two equations in two unknowns: $\theta$ and $h$. To obtain a measure of the variability of wages, employment, and output in this economy, we simulate the solutions to $\theta$ and $h$ for numerous realizations of $\epsilon_{\mathrm{S}}$ and $\epsilon_{\mathrm{D}}$ drawn from a random number generator. The general forms for $\epsilon_{\mathrm{S}}$ and $\epsilon_{\mathrm{D}}$ are as follows:

[^3]\[

$$
\begin{aligned}
& \epsilon_{\mathrm{S}}=\mathrm{m}_{\epsilon \mathrm{S}}+\mathrm{B}_{\epsilon \mathrm{S}} \operatorname{ran} 1, \text { and } \\
& \epsilon_{\mathrm{D}}=\mathrm{m}_{\epsilon \mathrm{D}}+\mathrm{B}_{\epsilon \mathrm{D} 1} \operatorname{ran} 1+\mathrm{B}_{\epsilon \mathrm{D} 2} \operatorname{ran} 2,
\end{aligned}
$$
\]

where ranl and ran2 are independently distributed uniform random variables with mean zero and unit variance. The parameters $m_{e s}$ and $m_{e D}$ are the means of $\epsilon_{S}$ and $\epsilon_{D}$, respectively. In table 1 , we choose $m_{\epsilon S}>m_{\epsilon D}$ to generate movements from the $D$-sector to the $S$ sector. The random variable ranl can be interpreted as an aggregate productivity shock. The parameters $B_{\epsilon S}$ and $B_{\epsilon D 1}$ measure the sensitivity of productivity in the $s$ - and $D$-sectors to changes in aggregate productivity. The random variable ran 2 is a sector-specific shock, with $\mathrm{B}_{\epsilon \mathrm{D} 2}$ measuring the importance of that shock to the determination of productivity in the D-sector, $\epsilon_{\mathrm{D}}$.

In table 1, under the subheading "aggregate shocks," we set $B_{\epsilon \mathrm{S}}<\mathrm{B}_{\epsilon \mathrm{D} 1}$ and $\mathrm{B}_{\epsilon \mathrm{D} 2}=0$ to simulate an economy in which productivity in both sectors is subject to an aggregate productivity shock, but the variance of the productivity shock is greater in the $D$ sector than in the $S$-sector. Since $m_{\epsilon S}>m_{\epsilon D}$, workers continually relocate from the $D$-sector to the $S$-sector; however, the bulk of this sectoral reallocation occurs during cyclical downturns.

In table 1 , under the subheading "sectoral shocks," we set $\mathrm{B}_{\epsilon \mathrm{S}}$ $<B_{\epsilon D 2}$ and $B_{\epsilon D 1}=0$ to simulate an economy in which productivity in each sector is determined by an independent shock. Since $\mathrm{m}_{\epsilon \mathrm{S}}>$ $\mathrm{m}_{\epsilon}$, workers continually relocate from the D -sector to the S sector. When a "bad" shock hits the D-sector (or a good shock
hits the $s$-sector), workers relocate from the D-sector to the S sector.

Table 2 repeats these experiments with $m_{\epsilon S}=m_{\epsilon D}$, but $\gamma>1$. This causes workers to move from the D-sector to the S -sector because of an increased demand for services.

Tables 1 and 2 present the results from a subset of the simulations performed. Before examining them in detail, we present a brief overview of the parameter settings for each set of simulations. The utility function parameter, $\delta$, is set equal to . 5, the probability of employment, $q$, is set equal to 1 , and the number of workers per firm, $N$, is set equal to 1 . We experimented with several different values of these parameters and found the results to be qualitatively similar to the results presented here.

The simulations presented in tables 1 and 2 show the effects of changing the production function parameter, $\alpha$, and the fixed cost of hiring, k. The simulations are presented in pairs: For each value of $\alpha$, we present the simulation results when $k=0$ and k > 0. To measure the relative variability of wages, employment, and output, we compute the coefficient of variation for each of these variables for 50 independent draws of the productivity shocks $\epsilon_{\mathrm{S}}$ and $\epsilon_{\mathrm{D}}$.

The real wage and real output are measured in utility units. ${ }^{7}$

[^4]Real wages in both sectors ( $W_{S}$ and $W_{D}$ ) exhibit the same variability in all simulations. This follows from the fact that $q=1$. As seen in equation (11), the wages in both sectors are always equal when the probability of employment (q) is 1. We therefore present the single measure of wage variability under the column heading "Wage." Real output is denoted by y. In addition to the coefficients of variation, we also present the regression coefficients obtained from regressing the real wage in each sector (deviation from mean) on real output (deviation from mean). Again, only one regression coefficient is presented, since the coefficients for each wage regressed on real output are identical. This coefficient is found in the far-right columns of tables 1 and 2.

It should be clear from the structure of the model presented in section II that sectoral movements in employment ( $\theta>0$ ) can be generated either by differences in the preferences for the $s$ good and D-good, or by differences in the productivity shocks to each sector. The simulations presented in table 1 show the effects of different mean productivities in each sector. In particular, all of these simulations are for the case where $m_{e S}$ $=2$ and $m_{\epsilon D}=1$. In addition, the utility function parameter $\gamma$ is set equal to 1 to eliminate any effects from preferences. In this case, sectoral movement occurs because workers wish to move from the low-productivity D-sector to the high-productivity $s$ sector.

The simulations presented in table 2 are for the case in which
the sectoral movements in employment are caused by differences in the preferences for the $S$-good and the D-good. In particular, mean productivities $m_{\epsilon S}$ and $m_{\epsilon D}$ are set equal to 1 , and the utility function parameter $\gamma$, which measures the weight given to the consumption of the $s-g o o d, ~ i s ~ s e t ~ e q u a l ~ t o ~ 1.2 . ~$

In table 1, the simulations under "aggregate shocks" are for the case where $B_{c D 2}=0$, so that only the common shock ranl generates movements in productivity in both sectors. In addition, to capture the relative variability of shocks to the $s$ and D-sectors, we set $B_{\epsilon S}=.05$ and $B_{\epsilon D 1}=.1$. In other words, we assume that $D$-sector productivity is twice as variable as S sector productivity. The simulations under "sectoral shocks" are for the case where the D-sector experiences a productivity shock that is independent of the aggregate shock. For these simulations, the productivity shock parameters are as follows: $B_{\epsilon S}=.05, B_{\epsilon D 1}=0$, and $B_{\epsilon D 2}=.1$. The shocks for the simulations in table 2 are identical in terms of $B_{\epsilon S}, B_{\epsilon D 1}$, and $B_{\epsilon D 2}$; the only difference is that $m_{\epsilon S}=m_{\epsilon D}=1$.

## Results

All of the simulations exhibit both the magnification effect and dampening of the real wage in the presence of a fixed hiring cost. For example, in table 1, the first pair of simulations under the heading "aggregate shocks" shows that increasing the fixed costs from 0 to .01 leads to a reduction in the regression coefficient on real output from 1.00 to .75. The variability of
the wage falls from . 0075 to .0056 (a 25 percent reduction). In addition, the variability of employment, as measured by $\theta$, increases from . 0297 to . 0388 (a 31 percent increase). These results are consistent throughout the tables. The presence of a fixed cost of hiring reduces the regression coefficient and the variability of the real wage and increases the variability of employment.

The results for the sector-specific shocks show the same general pattern; however, the regression coefficients and the variation in real wages are smaller, while the variation in employment is larger. For example, comparing simulations (7) and (8) to (1) and (2) shows that the regression coefficient fell 47 percent instead of 25 percent when $k$ was increased from 0 to .01. Simulations (7) and (8) show that increasing the fixed cost from 0 to . 01 causes the coefficient of variation for wages to fall from .0057 to .0037 (a 37 percent decline), while employment variability, as measured by $\theta$, rises from . 0460 to . 0630 (a 37 percent increase). The regression coefficients in this case differ from the coefficients of variation because of the independence of the shocks to both sectors.

As seen in table 1, simulations (5) and (6), the real wage and variability of employment effects are not sensitive to our choice of k. In fact, the coefficient of variation for the real wage in the case of $k=.001$ is actually smaller than in the case where $k$
$=.01 .^{8}$ Throughout the simulations, we find that arbitrarily small values of $k$ generate the dampened real wage and magnification of employment effects.

The impact of increasing the production function parameter ( $\alpha$ ) can be seen by examining the difference between simulations (2) and (4). It is clear that the dampening of the real wage and the magnification of employment are lessened by increasing $\alpha$. This result makes sense, since it is the existence of a positive producer's surplus, due to the decreasing returns-to-scale technology, that gives firms the incentive to pay the fixed cost in order to hire additional workers. Notice that with fixed costs (positive k), a competitive equilibrium does not exist when $\alpha=1 .{ }^{9}$

The simulations in table 2 , where preferences drive the sectoral dispersion, show the same general results. Simulations (11) and (12) demonstrate that increasing the fixed cost from 0 to . 01 leads to a reduction in the regression coefficient on real output from 1.0 to .91 . The variability of the real wage then falls from .0099 to .0091 (an 8 percent reduction) and the variability of $\theta$ increases from . 0386 to 0.1819 (a 471 percent increase). In general, we find that the magnification effects and the dampening of the real wage are present whether we generate sectoral dispersion through differences in the average

[^5]productivities in each sector or through differences in preferences for the two goods. The latter mechanism, however, consistently produces dramatic employment magnification effects.

## IV. Conclusion

We attempt to show the effects of a small, fixed hiring cost on the relative variability of output, employment, and real wages in the face of productivity shocks. The model presented in section II, although very basic, illustrates that if firms face a fixed cost of hiring, then for a given size shock the real wage response will be smaller and the employment response will be larger than if there were no fixed cost.

The intuition behind this result is clear. When a recession begins, workers leave the sector most affected by the recession (durable goods) for the service sector. As more workers relocate ( $\theta$ increases), the return to hiring an additional worker rises. As the number of firms hiring increases, the tendency for the real wage to fall in the high-demand sector (services) is dampened (increasing $h$ decreases $\theta / h$, which leads to an increase in the real wage). This leads to a further rise in the number of workers moving from the low-demand sector to the high-demand sector, which leads to a further dampening of the real wage. At the same time, the outflow of workers from the low-demand sector mitigates the wage decline in that sector.

As stated in the introduction, the results of this analysis suggest a possible mechanism for generating increased employment
variability and decreased real wage variability within an RBC framework. In order to do this, the model must be extended to incorporate some dynamic elements. Although we are still in the process of formulating this extension, we anticipate that a dynamic version of this model will yield similar results. In addition, a dynamic version will yield different implications for the adjustment of employment and wages to permanent versus transitory productivity shocks. The firm is most likely to incur the fixed cost of hiring (and the worker is most likely to move) if the shock is perceived to be permanent. In that case, we expect to observe the dampened real wage and the magnification of employment effects. This would make sense, since the model presented here considers only permanent shocks. Thus, a possible way to test for this effect would be to examine the differential response of real wages to permanent versus transitory changes in productivity. Our model predicts that the real wage response to a permanent productivity shock will be less than the real wage response to a temporary productivity shock.

In addition to adding dynamics, any extension of this framework should also consider adding leisure to the utility function. This would reinforce the effects illustrated above, because there would then be two sources of adjustment to a given shock: movements in and out of the labor force and movements from one sector of the economy to the other.

Our model generates the dampening real wage and magnification of employment effects within a sectoral-shifts framework. The
basic results, however, could easily be generated in a more general framework in which individuals face the choice of not working (producing in the household sector) or working in the production sector. In such a framework, a positive productivity shock to the production sector will cause individuals to leave the household sector. As individuals flow into the labor force, more firms will find it worthwhile to hire workers, leading to an increase in the real wage and thereby inducing additional workers to relocate.

## Appendix

## Solution Technique

Normalizing $\epsilon_{\mathrm{D}}=1$, the first order conditions (8)-(13) can be solved to yield two equations and two unknowns $Z$ and $h$, where $\mathrm{Z}=\theta / \mathrm{h}:$

$$
\begin{equation*}
(1+\mathrm{Zq})(1-\alpha \mathrm{Zq})=\mathrm{kN}^{-\alpha} / \epsilon_{\mathrm{s}} \tag{A1}
\end{equation*}
$$

$$
\begin{align*}
& \gamma q \epsilon_{\mathrm{S}}\left[\mathrm{~h} \epsilon_{\mathrm{S}}(1+\mathrm{Zq})^{\alpha}+(1-\mathrm{h}) \epsilon_{\mathrm{S}}-\mathrm{hkN}\right]^{-\alpha-1}= \\
& (1-\mathrm{Zh})^{\alpha \delta-\alpha}(1+\mathrm{Zq})^{1-\alpha} . \tag{A2}
\end{align*}
$$

Given values for $N, \alpha, \gamma, \delta, \epsilon_{S}$, and $q$, equation (A1) is solved using Newton's method (see Press [1988]). The value of $Z$ that solves equation (A1) is then plugged into equation (A2). Next we apply Newton's method to (A2) to obtain solutions for $Z$ and $h$, which are then used to calculate wages, employment, and output.

Expressions for Real Wages and Real Output

$$
\begin{array}{ll}
\mathrm{W}_{\mathrm{S}}=\lambda_{1} \alpha \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha-1} & \text { S-sector wage } \\
\mathrm{W}_{\mathrm{D}}=\lambda_{2} \alpha \epsilon_{\mathrm{D}}[\mathrm{~N}(1-\theta)]^{\alpha} & \text { D-sector wage } \\
\mathrm{Y}=\lambda_{1} \mathrm{C}_{\mathrm{S}}+\lambda_{2} C_{D} & \text { Real output } \\
\mathrm{C}_{\mathrm{S}}=\mathrm{h} \epsilon_{\mathrm{S}}[\mathrm{~N}(1+\theta \mathrm{q} / \mathrm{h})]^{\alpha}+(1-\mathrm{h}) \epsilon_{\mathrm{S}} \mathrm{~N}^{\alpha}-\mathrm{hk} & \text { S-sector output } \\
\mathrm{C}_{\mathrm{D}}=\epsilon_{\mathrm{D}}[\mathrm{~N}(1-\theta)]^{\alpha} & \text { D-sector output }
\end{array}
$$

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Table 1: Productivity ( $\gamma=1$ )
I. Aggregate Shocks

$$
\begin{aligned}
& \epsilon_{\mathrm{s}}=2+.05 \cdot \operatorname{ran} 1 \\
& \epsilon_{\mathrm{D}}=1+0.1 \cdot \operatorname{ran} 1
\end{aligned}
$$

Regression
Coefficient of Variation Coefficient on $Y$

|  | $\mathbf{k}$ | $\alpha$ | Wage | Y | $\boldsymbol{\theta}$ | Wage/Y | Wage |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 0 | 0.5 | 0.0075 | 0.0075 | 0.0297 | 1.0 | 1.0 |
| $(2)$ | .01 | 0.5 | 0.0056 | 0.0075 | 0.0388 | 0.75 | 0.75 |
| $(3)$ | 0 | 0.9 | 0.0071 | 0.0071 | 0.0289 | 1.0 | 1.0 |
| $(4)$ | .01 | 0.9 | 0.0067 | 0.0071 | 0.0312 | 0.94 | 0.94 |
| $(5)$ | 0 | 0.5 | 0.0075 | 0.0075 | 0.0297 | 1.0 | 1.0 |
| $(6)$ | .001 | 0.5 | 0.0053 | 0.0074 | 0.0298 | 0.72 | 0.72 |

## II. Sectoral Shocks

$$
\begin{aligned}
& \epsilon_{\mathrm{s}}=2+.05 \cdot \operatorname{ran} 1 \\
& \epsilon_{\mathrm{D}}=1+0.1 \cdot \operatorname{ran} 1
\end{aligned}
$$

Regression
Coefficient of Variation Coefficient on $Y$

|  | k | $\alpha$ | Wage | Y | $\theta$ | Wage/Y | Wage |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $(7)$ | 0 | 0.5 | 0.0057 | 0.0057 | 0.0460 | 1.0 | 1.0 |
| $(8)$ | .01 | 0.5 | 0.0037 | 0.0057 | 0.0630 | 0.65 | 0.53 |
| $(9)$ | 0 | 0.9 | 0.0052 | 0.0052 | 0.0450 | 1.0 | 1.0 |
| $(10)$ | .01 | 0.9 | 0.0047 | 0.0052 | 0.0490 | 0.90 | 0.89 |

Source: Authors' simulations.

Table 2: Preferences

$$
(\gamma=1.2)
$$

I. Aggregate Shocks

$$
\begin{aligned}
& \epsilon_{\mathrm{s}}=2+.05 \cdot \operatorname{ran} 1 \\
& \epsilon_{\mathrm{D}}=1+02 \cdot 1 \cdot \operatorname{ran} 1
\end{aligned}
$$

Regression
Coefficient of Variation Coefficient on $Y$

|  | k | $\alpha$ | Wage | Y | $\boldsymbol{\theta}$ | Wage/Y | Wage |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $(11)$ | 0 | 0.5 | 0.0099 | 0.0099 | 0.0386 | 1.0 | 1.0 |
| $(12)$ | .01 | 0.5 | 0.0091 | 0.0099 | 0.1819 | 0.91 | 0.91 |
| $(13)$ | 0 | 0.9 | 0.0098 | 0.0098 | 0.0384 | 1.0 | 1.0 |
| $(14)$ | .01 | 0.9 | 0.0097 | 0.0098 | 0.0492 | 0.98 | 0.98 |

II. Sectoral Shocks

$$
\begin{aligned}
& \epsilon_{\mathrm{s}}=1+.05 \cdot \operatorname{ran} 1 \\
& \epsilon_{\mathrm{D}}=1+0.1 \cdot \operatorname{ran} 1
\end{aligned}
$$

Regression
Coefficient of Variation Coefficient on $Y$

|  | k | $\alpha$ | Wage | Y | $\theta$ | Wage/Y | Wage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(15)$ | 0 | 0.5 | 0.0068 | 0.0068 | 0.100 | 1.0 | 1.0 |
| $(16)$ | .01 | 0.5 | 0.0064 | 0.0069 | 0.5965 | 0.92 | 0.71 |
| $(17)$ | 0 | 0.9 | 0.0066 | 0.0066 | 0.1008 | 1.0 | 1.0 |
| $(18)$ | .01 | 0.9 | 0.0064 | 0.0066 | 0.1365 | 0.95 | 0.94 |

Source: Authors' simulations.


[^0]:    ${ }^{1}$ As pointed out by Barro (1989, p. 8), "... the (RBC) models tend to overstate the procyclical patterns of hours, productivity, real interest rates and real wage rates."

[^1]:    ${ }^{2}$ Alternatively, this effect can be generated by assuming that the number of firms producing is exogenous and that there exists a fixed cost of entering into production (see Chatterjee and Cooper [1988, 1989]).

[^2]:    ${ }^{3}$ Examples of a fixed cost of hiring are placing an ad in Job openings for Economists, renting a hotel room for interviewing at the American Economic Association meetings, or hiring a personnel service.

    4 Our model allows sectoral reallocation to take place either as a result of productivity differences or of taste differences between the sectors. The latter is what we refer to as the demand conditions.

[^3]:    ${ }^{6}$ See the appendix for a discussion of the simulation technique.

[^4]:    ${ }^{7}$ The expressions for real wages and output are contained in the appendix.

[^5]:    ${ }^{8}$ Wage variability increases with $k$ once $k$ is positive because the marginal utility of consumption of the $s$-good is increasing with k .
    ${ }^{9}$ This can be seen from equation (14) when $\alpha=1$ and $k=0$.

