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REGULATORY TAXES, INVESTMENT, AND FINANCING DECISIONS FOR INSURED BANKS

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Abstract

This article develops a two-factor model of bank behavior under credit and interest rate risk. In addition to flat-rate government deposit guarantees, we assume banks possess charter values that are lost if audits reveal that their tangible assets cannot cover their liabilities. Within this framework, we investigate the effects of interest rate and credit risk on optimal capital structure and investment decisions. We then show that with no uncertainty in interest rates, capital regulation will reduce the risk of the assets in the bank. However, with interest rate uncertainty, the impact of regulation may be detrimental and raise the risk of the deposits as well as the government subsidies to the shareholders of the bank.

I. Introduction

Most models of deposit insurance assume that the volatility of a bank's asset prices is exogenously provided and derives from a single source. In this framework, the relative merits of the firm increasing volatility can be easily explored.[1] This approach, however, does not provide a rich enough structure for equityholders to compare alternative capital structures and investment policies under a fixed-rate deposit insurance regime. In this study, we extend the analysis of Merton [1977] and Marcus [1984] by allowing for two sources of asset risk: credit risk, which arises from economic uncertainty; and interest rate risk, which emanates from a duration mismatch between the bank's assets and liabilities. We also assume that a bank possesses a valuable growth option embodied in its charter. The presence of a charter and multiple sources of uncertainty provides a rich enough framework for examining the consequences of alternative capital structure and investment decisions of the bank. Our objective is to explore the bank's investment and financing strategies that maximize shareholder interests in a model that incorporates both government-subsidized deposit insurance, the charter, and regulatory constraints.

In our model, banks have incentives to increase the value of fixed rate deposit insurance by maximizing risk. Extreme risk taking, however, may not be optimal because it increases the likelihood of regulatory interference and charter-related bankruptcy costs. To reduce the moral hazard problem associated with deposit insurance, we follow Buser, Chen, and Kane [1981] and assume the deposit insurer has two tools at its disposal to limit the value of its insurance. The first is through charter regulation. By limiting the supply of charters and by implementing regulations intended to limit competition in banking markets, the government seeks to increase charter values and hence reduce the risk-taking incentives provided by deposit insurance. The second is through capital regulation. Under interest rate certainty, *capital regulation* as embodied in the current risk-based capital standards and *charter regulation* are substitute policies. That is, the deposit insurer can use capital regulation to offset declines in charter values. This result, however, does not necessarily obtain under uncertain interest rates.

Our study is not the first to consider the implications of interest rate risk on shareholder wealth and on the value of deposit insurance. Similar analyses have been conducted by McCulloch [1981] and Crouhy and Galai [1991]. McCulloch's primary objective is to explore the impact of interest rate risk on the value of deposit insurance. Crouhy and Galai's main focus is to investigate the impact of capital regulation and bank reserve account regulations when deposit rates reflect the risk of the asset portfolio. Neither study investigates the impact of interest rate risk on optimal investment and financing decisions for insured banks. In contrast, our primary focus is how interest rate risk interacts with asset risk to alter the optimal investment and capital structure decisions, and the attendant implications for capital regulation.

[1] The literature on deposit insurance using an option pricing framework was pioneered by Merton [1977]. For a review of the literature, see Flood (1990).

The paper proceeds as follows. Section II develops a model of an insured bank in which financing and investment decisions are predetermined. Uncertainty is represented by interest rate risk and by credit risk in the loan portfolio. Section III investigates the shareholders' optimization problem under interest rate certainty. As in Marcus [1984] and Ritchken, Thomson, DeGennaro, and Li [1993], the capital structure and investment decisions reflect the tradeoff between maximizing the charter value and the deposit insurance subsidy. Without interest rate uncertainty, extreme point solutions for the investment solution dominate. However, the optimal financing decision may involve the shareholders supplying some capital. In this case, capital regulation and charter regulation are substitute policies for limiting the value of deposit insurance. Section IV rederives the optimal investment and capital structure decisions when banks face interest rate risk. In this case, we show that the second source of risk allows for diversification effects, which may make interior investment decisions optimal. Moreover, with interest rate risk present, the effects of capital regulation on shareholder behavior can lead to counterproductive results. Indeed, we show that capital regulation can result in increased risk taking by banks, thereby increasing the value of deposit insurance rather than reducing it. The implication of using capital regulation to offset declining bank charter values is then explored. Section V concludes the paper.

II. A Model of an Insured Bank

We assume that the market for default-free bonds is a competitive one in which banks are price-takers. Banks do have a comparative advantage in evaluating credit risks, however, and therefore can invest in positive net present value loans. We assume owners of the bank are also its managers. At date 0 they fund the asset portfolio with α dollars of equity and $D(0) = 1 - \alpha$ dollars of deposits fully insured by a government agency. The agency charges the bank a flat-rate premium per dollar of deposits. The net present value of deposit insurance at time 0, denoted by $G(0)$, can be viewed as government-contributed capital. The insurance provides depositors with full protection over the time period $[0, T]$, at which time it is renewed if the bank is solvent. The insurer is assumed to strictly enforce the closure policy at date T. Specifically, if at date T, the market value of the assets of the bank is below the deposit base, the bank is immediately closed.

In order to operate, the bank requires a charter. Charters are valuable because, by rationing them, the government grants some degree of monopoly power to banks in both loan and deposit markets. Keeley [1990] argues that this power allows banks to earn rents in the form of higher risk-adjusted loan rates and lower deposit rates than in competitive markets. These rents continue as long as the bank remains solvent. The value of the charter is further enhanced because of growth options the bank possesses. These options arise because of the ability of banks to identify, on an ongoing basis, new loans with positive net present value.[2] A third source of charter value derives from longstanding customer banking relationships. Kane and Malkiel [1965] argue that such relationships have value because they lower the information and contracting costs associated with

[2] These strategic growth options are discussed by Myers [1977] and Herring and Vankudre [1987].

doing business. The reduction of costs associated with servicing long-term customers is available only to the servicing bank and is a source of future business opportunities. Reputation capital, as discussed in Diamond [1989], is a fourth source of charter value. In a world where information is costly, a high level of reputation capital reduces the cost of external equity and debt capital. Finally, as discussed in Kane (1985) and Kane and Unal (1990), bank charter values incorporate the value of the deposit insurance subsidy in future periods.

The charter can be viewed as a bundle of options whose value to equityholders fluctuates with the health of the bank. Let $C(0)$ represent its value at time 0. As the bank's condition deteriorates, the value of the charter that derives from the growth options as well as from the long-standing customer relationships is eroded by increased regulatory taxes and by funding constraints. For a bank that fails the audit, the deadweight costs of bankruptcy exceed any residual charter value. For a bank that passes the audit, its charter value increases with its health, eventually saturating at a point that reflects minimal probability of ongoing default. Rather than modeling the payoffs of this claim by a complex nonlinear function, we capture its main attributes by a step function. In particular, we follow Marcus [1984] and model the value of this claim at time T by:[3]

$$C(T) = \begin{cases} g\,D(T), & \text{if } V(T) \geq D(T), \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Here, $V(T)$ represents the tangible value of the asset portfolio at date T and $D(T)$ is the level of deposits at date T. The government can induce banks to take on less risk by rationing charters and enacting regulations designed to limit competition between banks and from nonbank financial intermediaries. Through charter regulation, the government increases the size of potential monopoly rents that banks can continue to capture as long as they remain solvent. The parameter g in equation (1) represents the size of the monopolistic rents as a percent of $D(T)$.[4]

Dating back to the work of Merton [1977], most models of insured banks do not explicitly incorporate the charter value. By treating deposits as insured debt, such models lead to shareholder interests being best served by extreme portfolio and capital structure decisions. With the addition of the above charter, incentives are established for shareholders to move away from their extreme risk-maximizing positions.

Since the charter includes the capitalized value of the spread earned on deposits, without loss of

[3] The claim on the charter corresponds to that of a digital option. Such options are encountered in over-the-counter markets and are characterized by discontinuous payoffs where either a constant or zero is received subject to the value of the underlying stochastic variable.

[4] While Marcus argues that the magnitude of the charter value of a solvent bank should be modeled as some fraction, g, of the deposit base, this assumption is not essential for our analysis. What is important is the assumption that bankruptcy costs and charter losses increase in value as the bank slides towards bankruptcy. For simplicity, we have modeled this as a digital option.

generality we shall assume that the deposit base grows at the riskless Treasury rate. In particular,

$$D(T) = \frac{1-\alpha}{P(0,T)} \tag{2}$$

where $P(0,T)$ is the time 0 value of a default-free pure discount bond.

The bank controls the capital structure and investment decision. Initially the bank has 1 dollar available for investment. The bank invests fraction q dollars in a risky loan portfolio and the remaining $(1-q)$ dollars in Treasury bonds of maturity s. The date s equals or exceeds the audit date, T.[5] The risky loan portfolio provides a net present value of L_q, where

$$L_q = q(e^{\delta(q)} - 1) \, . \tag{3}$$

$\delta(q)$ is usually assumed to be non-negative and concave.[6] For most of our analysis, we shall choose $\delta(q)$ to be independent of q.

Let $V(0)$ represent the initial value of the loan portfolio. Then

$$V(0) = 1 + L_q \, . \tag{4}$$

The bank's balance sheet at time zero can be summarized as follows:

Assets		**Liabilities & Net Worth**	
Tangible Assets			
Treasury Bonds	$1-q$	Deposits	$1-\alpha$
Loan Portfolio	$q+L_q$	Shareholder Equity	$e(0)$
Intangible Assets			
Government Subsidy	$G(0)$		
Claim on Future Rents	$C(0)$		
Total	$1+G(0)+C(0)+L_q$	**Total**	$1-\alpha+e(0)$

[5] Clearly, if banks were allowed to choose s , they could eliminate interest rate risk by choosing $s=T$. However, since we are interested in the effects of interest rate risk on optimal decisions, we restrict $s>T$. For many financial institutions, regulation implicitly imposes a similar restriction. An example of this is the qualified thrift lender test, which requires thrifts to invest 80 percent of their assets in mortgages.

[6] This functional form reflects the fact that the bank can detect only a limited number of good loans. For further discussion of the net present value function, see Gennotte and Pyle [1991] and McDonald and Siegel [1984].

The time zero equity value, $e(0)$, exceeds the capital supplied by the shareholders. This difference comes from the government subsidy, the charter, and the loan portfolio. Thus

$$e(0) = \alpha + G(0) + C(0) + L_q \ . \tag{5}$$

If the liquidation value of the tangible assets, $V(T)$, is greater than or equal to the deposit base $D(T)$, the bank is declared solvent. Otherwise, the bank is declared insolvent. The terminal claim on the charter value, insurance, and equity at the audit date T are

$$C(T) = \begin{cases} g\,D(T) & \text{if } V(T) \geq D(T), \\ 0, & \text{otherwise.} \end{cases} \tag{6a}$$

$$G(T) = \begin{cases} 0, & \text{if } V(T) \geq D(T), \\ D(T) - V(T) & \text{otherwise.} \end{cases} \tag{6b}$$

$$e(T) = \begin{cases} V(T) - D(T) + g\,D(T), & \text{if } V(T) \geq D(T), \\ 0, & \text{otherwise.} \end{cases} \tag{6c}$$

The value of the tangible assets of the bank at date T will depend on the risk that drives the value of the loan portfolio and on the evolution of interest rates. From equations (6a-c), we see that these claims are complex contracts subject to interest rate and loan uncertainties.

To model the risk derived from the loan portfolio, we assume the originator of the loan captures the full net present value. Hence, the resale value of the loan is set to yield a zero net present value. Once originated, the dynamics of each dollar investment in the loan portfolio is given by

$$\frac{dS(t)}{S(t)} = \mu_S dt + \sigma_S dz(t), \quad S(0) = 1. \tag{7}$$

Since the resale value of the loan is set to yield a zero net present value, the drift term, μ_S, corresponds to that of a traded security of equivalent risk. The accrued q dollar investment over the time period $[0,T]$ is given by $qe^{\delta(q)}S(T)$.

Now consider interest rate uncertainty. Let $P(t,s)$ be the date t price of a default-free pure discount bond that pays \$1 at date s. Let

$$P(t,s) = e^{-\int_t^s f(t,x)\,dx}$$

where $f(t,x)$ is the instantaneous forward rate at time t for the time increment $[x, x+dx]$. Forward rates are assumed to follow a diffusion process of the form

$$df(t,s) = \mu_f(t,s)dt + \sigma_f(t,s)dw(t), \tag{8}$$

with the forward rate function, $f(0,\cdot)$, initialized to the observed value. Here, $\mu_f(t,s)$, $\sigma_f(t,s)$ and $dw(t)$ are the drift, the volatility structure and the Wiener increment, respectively, and

$\mathrm{E}[dw(t)dz(t)] = \varrho dt$. We follow Heath, Jarrow and Morton [1992] and assume that $\sigma_f(t,\cdot)$ is an exponentially dampened function of the form

$$\sigma_f(t,s) = \sigma\, e^{-\kappa(s-t)}$$

where $\sigma, \kappa \geq 0$.[7] In this model $\mu_f(t,\cdot)$ is chosen so as to avoid riskless arbitrage opportunities from arising among bonds of different maturities.

The initial investment of q dollars in risky loans and $(1-q)$ dollars worth of bonds appreciates to a value $V(T)$ at date T, where

$$V(T) = q\, e^{\delta(q)} \frac{S(T)}{S(0)} + (1-q)\frac{P(T,s)}{P(0,s)}\,. \tag{9}$$

The initial values of the charter, government subsidy, and the equity can be computed once the unique martingale measure under which all securities are priced is identified. Using standard arbitrage arguments the martingale measure can be readily obtained, and the initial fair values of these claims are given by:[8]

$$C(0) = g\,(1-\alpha)\,\Phi_0(z_2^*) \tag{10a}$$

$$G(0) = q\, e^{\delta(q)}[\Phi_1(z_1^*) - 1] + (1-q)[\Phi_2(z_2^*) - 1] - (1-\alpha)[\Phi_0(z_2^*) - 1] \tag{10b}$$

$$e(0) = q\, e^{\delta(q)}\, \Phi_1(z_1^*) + (1-q)\,\Phi_2(z_2^*) - (1-\alpha)(1-g)\Phi_0(z_2^*)\,. \tag{10c}$$

where

$$\Phi_0(z_2^*) = \int_{-\infty}^{z_2^*} N\Big(\Delta_2(z)\Big)\, n(z)\, dz + N(-z_2^*)\,,$$

$$\Phi_i(z_i^*) = \int_{-\infty}^{z_i^*} N\Big(\Delta_i(z)\Big)\, n(\sigma_i - z)\, dz + N(\sigma_i - z_i^*) \quad i = 1,2$$

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[7] Given this structure, Heath, Jarrow and Morton show that bond prices at a future date T can be related to current bond prices through the relationship

$$P(t,s) = A(t,s)\, e^{-\beta(t,s)\, r(t)}$$

where

$$A(t,s) = \left[\frac{P(0,s)}{P(0,t)}\right] e^{-\frac{1}{2}\beta^2(t,s)\phi^2(t) + \beta(t,s) f(0,t)}$$

$$\beta(t,s) = \frac{1}{\kappa}\left(1 - e^{-\kappa(s-t)}\right)$$

and

$$\phi^2(t) = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa t}\right).$$

[8] For further discussion of this point, see appendices 1 and 2.

$$\Delta_1(z) = \frac{\ln\left[\frac{q_2}{1-\alpha-q_1 e^{\sigma_1 z}}\right] - \rho\sigma_2 z}{\sigma_2\sqrt{1-\rho^2}}$$

$$\Delta_2(z) = \frac{\ln\left[\frac{q_1}{1-\alpha-q_2 e^{\sigma_2 z}}\right] - \rho\sigma_1 z}{\sigma_1\sqrt{1-\rho^2}}$$

$$q_1 = q e^{\delta(q)-\frac{1}{2}\sigma_1^2}$$

$$q_2 = (1-q)e^{-\frac{1}{2}\sigma_2^2}$$

$$z_i^* = \frac{1}{\sigma_i}\ln\left[\frac{1-\alpha}{q_i}\right] \quad i = 1, 2.$$

The exact formulas for σ_1, σ_2 and ρ are given in appendix 2. σ_1^2 is the variance of the logarithmic returns of the risky loan over the period $[0, T]$, while σ_2^2 is the variance of the logarithmic return on the default-free bond over the same period. Finally, ρ is the correlation between these two logarithmic returns.

$\Phi_0(z_2^*)$ is the probability of passing the audit under the risk-neutralized probability distribution. For any given investment mix, q, the smaller the shareholder supplied equity, the higher the probability that the bank will be declared insolvent. The shareholders' excess, $e(0) - \alpha$, is affected by the value of the charter, the government subsidy, and the net present value of the loan portfolio. These, in turn, are influenced by the bank's capital structure and investment decisions, α and q.

The value of the charter depends on shareholder equity, α, and on the probability of passing the audit. As the equity supplied capital, α, declines, the threat of insolvency rises. This, in turn, places the charter at risk and thus imposes costs on equityholders. By raising the equity supplied capital, α, the charter is protected. However, beyond some critical point, the benefits resulting from a reduction in the probability of insolvency are dominated by the erosion of the charter value stemming from a smaller deposit base.

Now consider the value of the government-subsidized put option, $G(0)$. In competitive markets, as the proportion of capital supplied by shareholders declines, bondholders would normally demand higher returns to compensate for the reduction in bond quality. Deposit insurance, however, protects the bondholders' capital and ensures that the bonds are riskless. Since the bonds are fairly priced, the cost borne by the government in providing this insurance, $G(0)$, is a benefit that accrues to the shareholders. Further, as the deposit base expands, the incidence of insolvency rises and the value of this subsidy expands.

From the above discussion, as equityholders contribute more capital, the charter value initially rises while the government subsidy declines. Ignoring, for the moment, the net present value feature of the loan portfolio (that is, taking $\delta(q) = 0$) and assuming $\alpha = 0$, equityholders will contribute capital as long as the marginal increase in $C(0)$ exceeds the marginal decrease in $G(0)$, with the optimum α obtaining when $\partial C(0)/\partial\alpha = -\partial G(0)/\partial\alpha$. In the case where there is no deposit insurance, equityholders supply capital up to the point where $\partial C(0)/\partial\alpha = 0$. Clearly, for flat-rate deposit insurance $\partial G(0)/\partial\alpha \leq 0$, and hence for any given investment mix, the optimal amount of capital supplied is lowered by the existence of deposit insurance. This is the classical moral hazard problem.

The values of the government subsidy and the charter are also affected by the investment mix, q. In particular, the investment mix directly affects the probability of default. As the incidence of default declines, the value of the charter rises. At the same time, the value of the government subsidy declines. Maximizing the subsidy involves raising the probability of default and runs counter to the objective of maximizing the charter. Nonetheless, the existence of deposit insurance creates incentives to take on additional investment risk.

The government can induce banks to take on less risk by creating additional barriers to entry, thereby raising g. By tightening the rationing of charters, the government provides existing banks with the ability to capture larger monopolistic rents, which continue as long as the banks remain solvent. An alternate approach to force banks to reduce their risk is to impose capital-based regulatory constraints. Under these constraints, as the bank's investment in risky loans rises, equityholders are required to contribute more capital. For example, one type of regulatory constraint that is employed is

$$\alpha \geq \text{Max}(wq, k), \tag{11}$$

where w is the capital weight applied to risky loans and k is the minimum capital requirement.[9] By requiring that equityholders contribute more capital than they would otherwise, it is to be expected that the value of the government subsidy will be reduced. In the next section we show that in an economy with no interest rate risk this intuition is correct. However, when interest rates are uncertain, then the minimum risk position may involve a diversified portfolio and a capital requirement that falls below the required standards. We show that in some circumstances, the optimal equityholders' response is to move to a feasible position that involves creating riskier investments. This may raise the value of the government subsidy and run counter to the intent of the regulatory standard.

III. Optimal Shareholder Decisions with no Interest Rate Uncertainty

Let $Z(\alpha, q)$ represent the shareholder surplus. Then

$$Z(\alpha, q) \;=\; e(0) - \alpha \;=\; G(0) + C(0) + L(0). \tag{12}$$

[9] In practice, w is 8 percent and k is 4 percent for U.S. banks. For a description of the new international risk-based capital standards, see Avery and Berger [1991]. For a derivation of optimal capital weights in a world without interest rate risk, see Kim and Santomero [1988].

Equation (12) clearly illustrates the trade-off faced by shareholders. Specifically, in selecting the optimal capital and investment decisions, the shareholders trade off the claim on the charter, government subsidy, and their ability to capture projects with positive net present values. Let α^* and q^* represent optimal financing and investment decisions. That is,

$$Z(\alpha^*, q^*) \; = \; \operatorname*{Max}_{\alpha, q \in [0,1]} \Big[Z(\alpha, q) \Big]. \tag{13}$$

To focus on the trade-offs between the conflicting objectives of protecting the claim on the charter and maximizing the government subsidy, we assume that the benefits of the loan portfolio are independent of the scale of the investment; that is, $\delta(q) = \delta$. Setting the volatility of interest rates to zero results in equations ($10a$-c) simplifying to[10]

$$C(0) \; = \; \begin{cases} g\,(1-\alpha)\,N(d_2), & \text{if } q > \alpha, \\ g\,(1-\alpha), & \text{otherwise.} \end{cases} \tag{14a}$$

$$G(0) \; = \; \begin{cases} (q-\alpha)\,N(-d_2) - q\,e^{\delta}\,N(-d_1), & \text{if } q > \alpha, \\ 0, & \text{otherwise.} \end{cases} \tag{14b}$$

$$e(0) \; = \; \begin{cases} q\,e^{\delta}\,N(d_1) - [q-\alpha-g\,(1-\alpha)]\,N(d_2), & \text{if } q > \alpha, \\ g\,(1-\alpha) + q\left(e^{\delta}-1\right) + \alpha, & \text{otherwise.} \end{cases} \tag{14c}$$

where

$$\begin{aligned} d_1 \; &= \frac{\ln\left(\frac{q}{q-\alpha}\right) + \frac{1}{2}\sigma_S^2 T + \delta}{\sigma_S \sqrt{T}} \\ d_2 \; &= d_1 - \sigma_S \sqrt{T}\,. \end{aligned}$$

For $q \geq \alpha$, $N(d_2)$ can be viewed as the probability of passing the audit. From equation (14c) for $q \leq \alpha$, the shareholders' excess, $Z(\alpha, q)$, increases linearly in the investment mix, q. For any given α, the optimal q value is in the interval $[\alpha, 1]$. Now consider the behavior of $Z(\alpha, q)$ along any line $\alpha = \omega q$ where $0 \leq \omega \leq 1$ is a constant. Along this ray $Z(\alpha, q)$ is a linear function of q. This result implies that the global maximum of $Z(\alpha, q)$ will occur at either $q = 0$ or $q = 1$, and the optimal capital, α^*, and investment, q^*, are obtained by solving the following optimization problem:

$$Z(\alpha^*, q^*) \; = \; \operatorname{Max}\left\{ \operatorname*{Max}_{0 \leq \alpha \leq 1} [Z(\alpha, 0)], \operatorname*{Max}_{0 \leq \alpha \leq 1} [Z(\alpha, 1)] \right\},$$

where

$$\begin{aligned} Z(\alpha, 0) \; &= (1-\alpha)\,g \\ Z(\alpha, 1) \; &= N(d_1^*)\,e^{\delta} - (1-\alpha)\,(1-g)\,N(d_2^*) - \alpha \end{aligned}$$

[10] When $\delta = 0$, these equations ($14a$-c) reduce to expressions derived by Marcus [1984] and by Ritchken, Thomson, DeGennaro, and Li [1993].

and

$$
\begin{aligned}
d_1^* &= \frac{-\ln(1-\alpha) + \frac{1}{2}\sigma_S^2 T + \delta}{\sigma_S \sqrt{T}} \\
d_2^* &= d_1^* - \sigma_S \sqrt{T}\,.
\end{aligned}
$$

The investment policy is extreme because, with no interest rate risk, the benefits of portfolio diversification are not available. Hence, an extremely valuable charter is worth protecting and equityholders respond by investing the funds in the risk-free asset. On the other hand, if the charter is not that valuable, equityholders will strive to maximize the government subsidy by investing all the funds in the risky loan portfolio. By controlling the value of the charter through g, the government can influence the optimal investment choice.

Table 1 illustrates the optimal α and q values for a range of potential charter values, g. For the example below, annual audits were considered ($T = 1$), the annual volatility of the loan portfolio, σ_s, was set at 10% and all loans were considered to be zero net present value ($\delta = 0$).[11]

If the government's regulatory policies produce a high charter value, g, then shareholders will take actions to protect the value of their claim on the charter (rather than solely maximize the value of the insurance subsidy) by choosing safe rather than risky portfolios. If, however, market forces erode the effectiveness of charter regulation, then g falls. The optimal response by banks to declining charter values is to increase the value of the deposit insurance put by bearing more risk.

In practice, the bank's investment and financing decisions are constrained by regulation. Buser, Chen and Kane [1981] argue that as a condition for receiving deposit insurance, banks subject themselves to regulation. The cost associated with regulation in turn reduces the value of the government subsidy. Our model permits us to explicitly establish both the cost to the shareholders and the benefit to the regulators of the regulatory constraint. Consider, for example, the risk-based capital standard introduced earlier in equation (11). The shareholders' objective function in equation (13) is now replaced by the following constrained optimization problem

$$
Z(\alpha_R^*, q_R^*) = \operatorname*{Max}_{\alpha, q}[Z(\alpha, q)] \quad \text{subject to} \quad \alpha \geq \operatorname{Max}(wq, k).
$$

The difference between the unconstrained and constrained optimization problems yields the implicit cost of regulation to the shareholders. Let ΔZ represent this difference. Also let $\Delta\phi_0$ represent the corresponding changes in the probability of solvency. That is,

$$
\begin{aligned}
\Delta Z &= Z(\alpha^*, q^*) - Z(\alpha_R^*, q_R^*) \\
\Delta\phi_0 &= \phi_0(\alpha^*, q^*) - \phi_0(\alpha_R^*, q_R^*).
\end{aligned}
$$

11 In Table 1, the optimal solutions are extreme because $\delta = 0$. If positive net present value projects are available then, while q remains extreme, interior solutions for α may arise.

Clearly, $\Delta Z \geq 0$. Finally, let $\eta(\alpha, q) = \frac{G(0)}{(1-\alpha)}$ represent the value of the government subsidy per deposit dollar insured, and let $\Delta\eta$ be given by

$$\Delta\eta = \eta(\alpha^*, q^*) - \eta(\alpha_R^*, q_R^*).$$

$\Delta\eta$ represents the change in the value of the government subsidy per dollar insured.

Proposition 1

When interest rate risk is not present, risk-based capital standards reduce the likelihood of a bank failing an audit. Moreover, the value of the government subsidy per dollar insured decreases with regulation.

Proof

The unconstrained optimum mix, q^*, is either zero or unity. First, assume $q^* = 1$. Then, the impact of the capital constraint cannot increase risk, and the requirement that shareholders place a minimum amount of capital will usually result in decreased risk that lowers the probability of failing the audit.[12] Second, consider the alternative value of q^*, namely, $q^* = 0$. In this case, since no risk is borne, the unconstrained optimum equals the constrained optimum, and the probability of closure is unchanged at zero. All that remains to be shown is that the government subsidy per dollar insured decreases as α increases. To confirm this, substitute for $G(0)$ from equation (14b) and when $q^* = 1$. This yields

$$\eta(\alpha, 1) = N(-d_2^*) - y\,N(-d_1^*)$$

where

$$\begin{aligned} y &= \frac{e^\delta}{1-\alpha} \\ d_1^* &= \frac{\ln(y) + \frac{1}{2}\sigma_s^2 T}{\sigma_S\sqrt{T}} \\ d_2^* &= d_1^* - \sigma_S\sqrt{T} \end{aligned}$$

$$\frac{\partial\eta(\alpha,1)}{\partial\alpha} = \frac{\partial\eta}{\partial y}\frac{\partial y}{\partial\alpha}.$$

Now note that $\partial y/\partial\alpha \geq 0$ and that $\partial\eta/\partial y \leq 0$. Hence, $\partial\eta/\partial\alpha \leq 0$. That is, as α increases, the government subsidy per deposit dollar decreases. □

[12] Formally, for $q = 1$, the probability of closure is $N(-d_2)$ and $\partial N(-d_2^*)/\partial\alpha \leq 0$.

The above proposition indicates that part of the loss of wealth for the shareholders, caused by the regulatory constraint, leads to a reduction in the size of the wealth transfer from the government. From a policy perspective, tightening capital requirements has the same effect on risk taking as tightening control of issuing charters to prospective banks.

IV. Optimal Shareholder Decisions with Interest Rate Uncertainty

In the presence of interest rate risk, diversification provides an additional risk management option, and the optimal unconstrained solution to the shareholders' optimization problem is more likely to contain interior solutions than when interest rates were deterministic. In particular, the minimal risk portfolio may not occur when $q = 0$, but, due to the diversification effect, may arise at an interior point. Indeed, if the interest rate and asset risk exposures are of a similar magnitude, and if these risks are uncorrelated, then one would expect diversification to be very important, especially if charter values are high.

Table 2 shows the behavior of the optimal q^* and α^* values to changes in g. For the case parameters selected, investment decisions become riskier (q^* increases), and incentives for shareholders to supply equity capital diminish, as the effects of charter regulation weaken.

Figures 1 and 2 show the sensitivity of optimal decisions to changes in the volatility of interest rates and the correlation between interest rates and risky loans. For the case parameters, figure 1 shows that as the volatility of the bond increases, the optimal response by equityholders is to change their investment and capital structure decisions by increasing their investment in risky loans, and reducing the probability of losing their charter, by supplying additional capital. Of course, the nature of these results depends on the magnitude of the charter, g, and on the size of the net present value factor, δ, and on the magnitude of the correlation, ρ.

Figure 2 illustrates how the correlation can affect optimal decisions. In the example, as the correlation increases, shareholders are prepared to supply more capital. With perfect correlation, $\rho = 1.0$, there is no natural hedge, and to protect the valuable claim on the charter, shareholders supply the most capital.

As table 2 shows, if the charter value is large relative to the government subsidy, incentives exist for the firm to reduce risk. By diversifying between risky loans and bonds, overall risk is reduced, and the likelihood of retaining the charter is improved. However, when the bond returns are not perfectly negatively correlated with loan portfolio returns, risk cannot be completely eliminated. Hence, to further reduce the probability of default, the infusion of additional equity capital may be optimal. To illustrate this point, consider a solvent bank whose charter value is 5 percent of the deposit base. The volatility of the loan portfolio, σ_1, is 5 percent, the volatility of the long-term bond, σ_2, is also 5 percent, and the correlation, ρ, is zero. The loan portfolio is fairly priced, that is, $\delta(q) = 0$. For this problem, the optimal capital is near 7 percent. This is in contrast to the optimal capital structure of $\alpha = 0$ or 1 that would have been obtained if interest rate risk were ignored.

The introduction of interest rate uncertainty into the economy has consequences for the role of regulation in general and for the capital requirements constraint in particular. While the constrained shareholders' optimization problem leads to a wealth loss, this loss could indeed come from a loss in the claim on the charter, rather than a loss in the government subsidy. Indeed, the constrained optimal investment and capital structure may be more risky than the unconstrained optimal solutions. As a result, this regulatory constraint may result in increasing, rather than decreasing, government subsidies.

Proposition 2

The impact of regulation is indeterminate. In particular, regulation may induce banks to increase their risk exposure and the likelihood of failing the audit. Moreover, the value of the government subsidy per dollar insured may increase.

The proposition is proved by an example which illustrates that capital regulation can be counterproductive. Assume that the positive net present value factor, $\delta(q)$, is 1 percent, that the charter value is 6 percent of the deposit base, and that the correlation between the risky bond and the loan portfolio is -0.75. The instantaneous volatilities of the bond and loan portfolio are 8 and 5 percent, respectively. The regulatory reserve requirement parameter values for k and w are 3 and 8 percent, respectively.

The optimal solution for the unconstrained problem occurs at $(\alpha^*, q^*) = (0.0601, 0.8353)$, with shareholder surplus, $Z(\alpha^*, q^*) = 0.06419$ and the deposit subsidy per dollar insured, $\eta(\alpha^*, q^*) = 0.00016$. For the constrained problem, $(\alpha^*_R, q^*_R) = (0.08, 1.0)$, with $Z(\alpha^*_R, q^*_R) = 0.06412$ and $\eta(\alpha^*_R, q^*_R) = 0.00062$. These results are summarized in figure 3. Notice that regulation reduces shareholder wealth by 0.109 percent. The value of the government subsidy, however, grows 290 percent. This increase in the subsidy arises because the constrained bank's leveraged portfolio is riskier in spite of the additional capital that is required. [12]

To illustrate the potential importance of the minimum capital requirement constraint, k, on shareholder wealth and deposit insurance, we consider a second example in which loans are fairly priced ($\delta(q) = 0$); the charter value is 5 percent of the deposit base; the risky bond and the loan portfolios are uncorrelated; the instantaneous volatilities of the bond and loan portfolio are 5 and 10 percent, respectively; and w is 8 percent.

[12] These results are similar to those of Koehn and Santomero [1980] and Gennotte and Pyle [1991], who find that for insured banks, higher capital requirements may increase the probability of bankruptcy. However, neither paper looks directly at how changes in capital regulation affect deposit insurers' risk exposure.

The optimal solution for the unconstrained problem is $(\alpha^*, q^*) = (0, 1)$, with $Z(\alpha^*, q^*) = 0.05901$ and $\eta(\alpha^*, q*) = 0.03983$. For the constrained problem with $k = 0$, $(\alpha_R^*, q_R^*) = (0, 0)$, with $Z(\alpha_R^*, q_R^*) = 0.03997$, and $\eta(\alpha_R^*, q_R^*) = 0.02034$. However, when the minimum reserve of $k = 3$ percent is added, the new constrained optimum moves to $(\alpha_R^*, q_R^*) = (0.08, 1)$, with $Z(\alpha_R^*, q_R^*) = 0.03978$, and $\eta(\alpha_R^*, q_R^*) = 0.0117$. The results are shown in figure 4.

The example shows that the introduction of minimum capital requirements reduces insurance costs. Indeed, in this example, the 3 percent minimum capital requirement reduced dollar insurance costs by almost one-half (from 2.03 to 1.17 percent) without lowering the shareholder surplus very much (from 3.997 to 3.978 percent). This example illustrates the importance of the minimum capital constraint. Without it, a risk-based capital standard that considers only asset risk may result in the deposit insurance fund having a large risk exposure. However, the minimum capital constraint implicitly taxes interest rate risk and therefore changes the relative cost of regulation associated with asset risk. Thus, when interest rate risk is present, the minimum capital requirement may significantly reduce the exposure of the deposit insurance fund.

V. Conclusion

This article develops a two-factor model of bank behavior under credit and interest rate risk. Optimal investment and financing decisions for the bank are explored in a regime where a government agency provides a flat-rate guarantee on all deposits. Since the bank possesses a valuable charter that is eroded if an audit reveals that the liquidation value of the tangible assets does not exceed the deposit base, maximizing risk may not be optimal. Nonetheless, the government subsidy still provides an incentive for banks to bear more risk than they would if their deposits were uninsured.

We investigate the moral hazard problem by explicitly identifying the bank's optimal capital structure and investment decisions. The government agency can reduce moral hazard by regulating capital requirements. Within the framework of our models, we can explore the policy implications of such regulations. We show that without interest rate risk, diminishing charter regulations can be offset by an increasing capital constraint. However, in an economy where interest rate risk exists, increasing capital regulation may not produce the same results as increasing charter regulation. Indeed, we note that increasing capital regulation may induce some banks to bear more risk and hence may raise the cost of the subsidy provided by the government agency.

We investigate optimal shareholders' policies and the impact of their actions on the value of government-subsidized insurance. We also explore the effect of interest rate risk and credit risk (and their correlation) on deposit insurance and look at how regulation has affected optimal shareholder policies. In some cases, regulation increased banks' holdings in the loan portfolio, thus magnifying the value of the government-subsidized put option.

The model presented is a single period model in which the time remaining before an audit is certain. It remains for future work both to assess how an uncertain audit date would alter the findings and to generalize the class of functions used to characterize the charter value and the positive net present value from the loan portfolio.

Appendix 1

Let

$$\begin{aligned}\frac{dS(t)}{S(t)} &= \mu_S(t)dt + \sigma_S dz(t) \quad ; \quad S(0) = 1 \\ df(t,T_L) &= \mu_f(t,T_L)dt + \sigma_f(t,T_L)dw(t) \quad ; \quad \forall T_L \geq t\end{aligned}$$

with $f(0,T_L)$ given, $\sigma_f(t,T_L) = \sigma e^{-\kappa(T_L - t)}$, and $\mu_f(t,T_L)$ curtailed so as to avoid riskless arbitrage opportunities. Further, let $\varrho dt = E[dw(t).dz(t)]$ denote the correlation between the two stochastic disturbances. Let $M(0)$ be the value of a claim at date 0 that has terminal payouts at date T_L, fully determined by the asset value $S(T_L)$ and the term structure at date T_L. Then

$$M(0) \;=\; P(0,T_L)\,\widetilde{E}_0\{M(T_L)\}$$

where the expectation is taken under the joint normal distribution of the spot rate, $r(T_L)$, and the logarithm of the asset price, $S(T_L)$, given by

$$\begin{aligned}
\widetilde{E}_0\{\ln[S(T_L)]\} &= -\ln[P(0,T_L)] - \frac{1}{2}\sigma_1^2 \\
\widetilde{Var}_0\{\ln[S(T_L)]\} &= \sigma_1^2 \;=\; \int_0^{T_L} [\sigma_S^2 + \sigma_p^2(t,T_L) - 2\varrho\sigma_S\sigma_p(t,T_L)]dt \\
\widetilde{E}_0\{r(T_L)\} &= f(0,T_L) \\
\widetilde{Var}_0\{r(T_L)\} &= \phi^2(T_L) \;=\; \frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa T_L}\right) \\
\widetilde{Cov}_0\{r(T_L),\ln[S(T_L)]\} &= \sigma_{12} \;=\; \int_0^{T_L} \sigma_f(t,T_L)[\varrho\sigma_S - \sigma_p(t,T_L)]dt \\
\sigma_p^2(t,T_L) &= \sigma^2\beta^2(t,T_L) \;=\; \frac{\sigma^2}{\kappa^2}\left(1 - e^{-\kappa(T_L - t)}\right)^2.
\end{aligned}$$

For a derivation of the above martingale measures, see Ritchken and Sankarasubramanian [1991].

Appendix 2

Theorem

The fair values of the charter, the government subsidy and the equity in the bank are given by

$$
\begin{aligned}
C(0) &= g\,(1-\alpha)\,\Phi_0(z_2^*) \\
G(0) &= q\,e^{\delta(q)}[\Phi_1(z_1^*)-1] + (1-q)[\Phi_2(z_2^*)-1] - (1-\alpha)[\Phi_0(z_2^*)-1] \\
e(0) &= q\,e^{\delta(q)}\,\Phi_1(z_1^*) + (1-q)\,\Phi_2(z_2^*) - (1-\alpha)(1-g)\Phi_0(z_2^*)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_0(z_2^*) &= \int_{-\infty}^{z_2^*} N\Big(\Delta_2(z)\Big)\,n(z)\,dz + N(-z_2^*)\,, \\
\Phi_i(z_i^*) &= \int_{-\infty}^{z_i^*} N\Big(\Delta_i(z)\Big)\,n(\sigma_i - z)\,dz + N(\sigma_i - z) \quad i = 1,2 \\
\Delta_1(z) &= \frac{\ln\left[\frac{q_2}{1-\alpha-q_1 e^{\sigma_1 z}}\right] - \rho\sigma_2 z}{\sigma_2\sqrt{1-\rho^2}} \\
\Delta_2(z) &= \frac{\ln\left[\frac{q_1}{1-\alpha-q_2 e^{\sigma_2 z}}\right] - \rho\sigma_2 z}{\sigma_2\sqrt{1-\rho^2}} \\
\sigma_2^2 &= \sigma^2\frac{1}{4\kappa^3}\left(1-e^{-\kappa(T-t)}\right)^2\left(1-e^{-\kappa T}\right)^2 \\
\rho &= \frac{\sigma_{12}}{\sigma_1\sigma_2} \\
q_1 &= qe^{\delta(q)-\frac{1}{2}\sigma_1^2} \\
q_2 &= (1-q)e^{-\frac{1}{2}\sigma_2^2} \\
z_1^* &= \frac{1}{\sigma_1}\ln\left[\frac{1-\alpha}{q_1}\right] \\
z_2^* &= \frac{1}{\sigma_2}\ln\left[\frac{1-\alpha}{q_2}\right]
\end{aligned}
$$

and where σ_{12} and σ_1 are as defined in Appendix 1.

Proof The value of the portfolio at date T is

$$V(T) \;=\; q\,e^{\delta(q)}\frac{S(T)}{S(0)} + (1-q)\frac{P(T,s)}{P(0,s)}\;.$$

Now substituting for $P(T,s)$ and rearranging yields

$$V(T) = \frac{1}{P(0,T)}\left[qe^{\delta+\ln[P(0,T)]+\ln[S(T)]} + (1-q)A^*(T,s)e^{-\beta(T,s)r(T)}\right] \qquad (A2.1)$$

where $\qquad A^*(T,s) = e^{-\frac{1}{2}\beta^2(T,s)\phi^2(T)+\beta(T,s)f(0,s)}.$

Under the martingale measure (see Appendix 1)

$$\begin{aligned} \ln[S(T)] &\sim N\Big(-\ln[P(0,T)] - \frac{1}{2}\sigma_1^2, \sigma_1^2\Big) \\ r(T) &\sim N\Big(f(0,T), \phi^2(T)\Big). \end{aligned}$$

Now let

$$\begin{aligned} Z_1 &= \frac{\ln[S(T)] + \ln[P(0,T)] + \frac{1}{2}\sigma_1^2}{\sigma_1} \\ Z_2 &= \frac{r(T) - f(0,T)}{\phi(T)}\ . \end{aligned}$$

Then substituting into $(A2.1)$ we obtain

$$V(T) = \frac{1}{P(0,T)}\left[q_1 e^{\sigma_1 Z_1} + q_2 e^{\sigma_2 Z_2}\right].$$

Here σ_1^2 is the variance of the logarithmic returns on the loan portfolio over $[0,T]$ and σ_2^2 is the variance of the logarithmic returns of the bonds over $[0,T]$, viewed from time 0. (See Appendix 1.)

Now the bank will pass the audit if $V(T) > D(T)$ or equivalently if

$$q_1 e^{\sigma_1 Z_1} + q_2 e^{\sigma_2 Z_2} \geq 1 - \alpha\ .$$

Equivalently, the bank passes the audit if

$$Z_1 \geq \gamma_1(Z_2)$$

or if

$$Z_2 \geq \gamma_2(Z_1)$$

where

$$\begin{aligned} \gamma_1(z_2) &= \frac{1}{\sigma_1}\ln\left[\frac{1-\alpha-q_2 e^{\sigma_2 z_1}}{q_1}\right] \\ \gamma_2(z_1) &= \frac{1}{\sigma_2}\ln\left[\frac{1-\alpha-q_1 e^{\sigma_1 z_2}}{q_2}\right]. \end{aligned}$$

The probability of solvency is therefore given by

$$\begin{aligned}
Prob[Z_1 \geq \gamma_1(Z_2)] &= \int_{-\infty}^{\infty} Prob[z_1 \geq \gamma_1(z_2) | z_2 = z_2]\, n(z_2)\, dz_2 \\
&= \int_{-\infty}^{z_2^*} \int_{z_1 \geq \gamma_1(z_2)} \left[\frac{e^{-\frac{1}{2(1-\rho^2)}(z_1 - \rho z_2)^2}}{\sqrt{2\pi(1-\rho^2)}} \right] n(z_2)\, dz_1\, dz_2 + \int_{z_2^*}^{\infty} n(z_2)\, dz_2 \\
&= \int_{-\infty}^{z_2^*} N\left[\frac{\rho z_2 - \gamma_1(z_2)}{\sqrt{1-\rho^2}} \right] n(z_2)\, dz_1\, dz_2 + \int_{z_2^*}^{\infty} n(z_2)\, dz_2 \\
&= \int_{-\infty}^{z_2^*} N[\Delta_2(z_2)]\, n(z_2)\, dz_2 + N[-z_2^*] \\
&= \Phi_0(z_2^*).
\end{aligned}$$

By symmetry, the probability of solvency can also be expressed as

$$Prob[Z_2 \geq \gamma_2(Z_1)] = \int_{-\infty}^{z_1^*} N[\Delta_1(z_1)]\, n(z_2)\, dz_1 + N[-z_1^*].$$

The value of the claim on the charter at date T is

$$C(T) = \begin{cases} g\,(1-\alpha)\, D(T), & \text{if } V(T) \geq D(T) \\ 0, & \text{otherwise.} \end{cases}$$

Substituting for $V(T)$ and $D(T)$, we obtain

$$C(T)\, P(0,T) = \begin{cases} g\,(1-\alpha), & \text{if } Z_1 \geq \gamma_1(Z_2) \\ 0, & \text{otherwise.} \end{cases}$$

Computing expectations leads to

$$C(0) = E\{C(T)\, P(0,T)\} = g\,(1-\alpha)\, \Phi_0(z_2^*). \tag{A2.2}$$

The value of the equity at date T is

$$e(T) = \begin{cases} V(T) - D(T) + C(T), & \text{if } V(T) \geq D(T) \\ 0, & \text{otherwise.} \end{cases}$$

Substituting for $V(T)$, $D(T)$ and $C(T)$, we obtain

$$e(T)\, P(0,T) = \begin{cases} q_1 e^{\sigma_1 z_1} + q_2 e^{\sigma_2 z_2} - (1-\alpha)(1-g), & \text{if } Z_1 \geq \gamma_1(Z_2) \\ 0, & \text{otherwise.} \end{cases}$$

Hence,

$$e(0) = \tilde{E}_0 \Big(q_1 e^{\sigma_1 z_1} + q_2 e^{\sigma_2 z_2} - (1-\alpha)(1-g) \Big)\, \mathcal{I}_{z_1 \geq \gamma_1(z_2)} \tag{A2.3}$$

where $$\mathcal{I}_{z_1 \geq \gamma_1(z_2)} = \begin{cases} 1, & \text{if } Z_1 \geq \gamma_1(Z_2) \\ 0, & \text{otherwise.} \end{cases}$$

Now note that

$$\begin{aligned}
\tilde{E}_0\left(e^{\sigma_2 z_2}\,\mathcal{I}_{z_1 \geq \gamma_1(z_2)}\right) &= \int_{-\infty}^{\infty} \int_{z_1 \geq \gamma_1(z_2)} e^{\sigma_2 z_2}\, n(z_1|z_2)\, dz_1\, n(z_2)\, dz_2 \\
&= \int_{-\infty}^{z_2^*} e^{\sigma_2 z_2} \int_{z_1 \geq \gamma_1(z_2)} \left[\frac{e^{-\frac{1}{2(1-\rho^2)}(z_1-\rho z_2)^2}}{\sqrt{2\pi(1-\rho^2)}}\right] n(z_2)\, dz_1\, dz_2 + \int_{z_2^*}^{\infty} e^{\sigma_2 z_2}\, n(z_2)\, dz_2 \\
&= \left(\int_{-\infty}^{z_2^*} N[\Delta_2(z_2)]\, n(\sigma_2 - z_2)\, dz_2 + N[\sigma_2 - z_2^*]\right) e^{\frac{1}{2}\sigma_2^2} \\
&= \Phi_2(z_2^*) e^{\frac{1}{2}\sigma_2^2}. \qquad (A2.4)
\end{aligned}$$

Further, by symmetry, we also obtain

$$\begin{aligned}
\widetilde{E}_0\left(e^{\sigma_1 z_1}\,\mathcal{I}_{z_1 \geq \gamma_1(z_2)}\right) &= \widetilde{E}_0\left(e^{\sigma_1 z_1}\,\mathcal{I}_{z_2 \geq \gamma_2(z_1)}\right) \\
&= \left(\int_{-\infty}^{z_1^*} N[\Delta_1(z_1)]\, n(\sigma_1 - z_1)\, dz_1 + N[\sigma_1 - z_1^*]\right) e^{\frac{1}{2}\sigma_1^2} \\
&= \Phi_1(z_1^*) e^{\frac{1}{2}\sigma_1^2}. \qquad (A2.5)
\end{aligned}$$

Substituting (A2.2), (A2.4) and (A2.5) into (A2.3) and rearranging then leads to the equity equation. The government subsidy equation then follows by substituting for $C(0)$ and $e(0)$ into equation (5). □

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Table 1

Optimal Capital Structure and Investment Decisions for Different Charter Values

g	α	q
0	0	1
.025	0	1
.05	0	1
.075	0	1
0.10	1	0
0.125	1	0
0.150	1	0

Table 1 shows the optimal capital structure (α) and investment decisions (q) for different charter values (g). The annual volatility of the loan portfolio, σ, is 10 percent. All loans are zero-net-present-value projects. If g = 0.0767, then any capital structure and investment decisions are optimal. The extreme-point nature of decisions arises because all projects are fairly priced.

SOURCE: Authors.

Table 2

Optimal Capital Structure and Investment Decisions for Different Charter Values

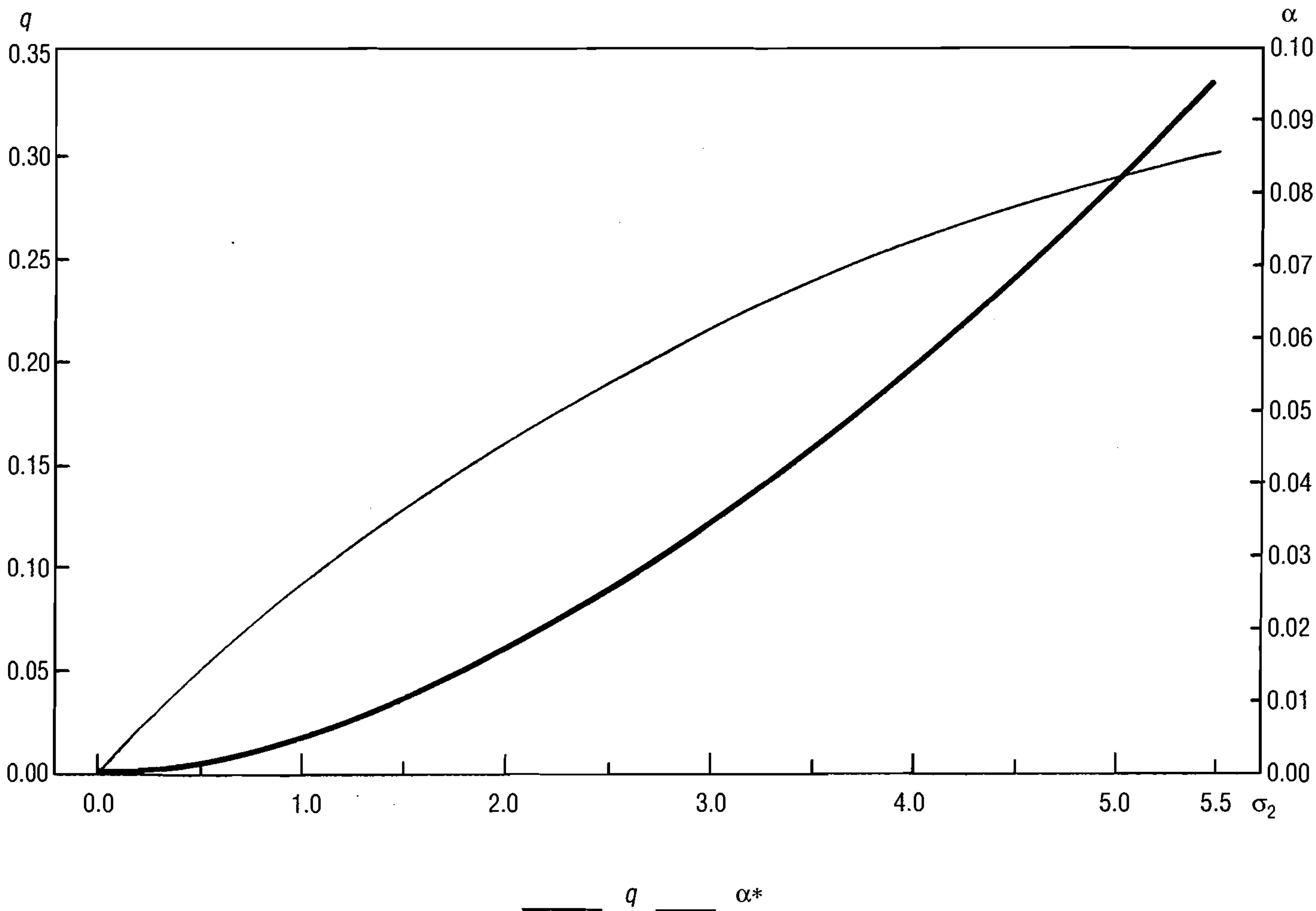
g	α	q
0	0	1
.025	0	1
.05	.0830	0.1584
.075	.0891	0.1577
0.10	.0914	0.1576
0.125	.0925	0.1575
0.150	.0932	0.1575

Table 2 shows the optimal capital structure (α) and investment decisions (q) for different charter values (g). The annual volatilities of the risky loan and the default-free bond portfolio are 8 and 5 percent, respectively. The correlation is 0.4. All risky projects have zero net present value (that is, $\delta = 0$). Notice that with interest rate risk, interior solutions may be optimal.

SOURCE: Authors.

Figure 1

Optimal Investment and Financing Decisions as a Function of the Volatility of Bonds



As the volatility of bonds, σ_2, increases, the optimal portfolio decision involves allocating more funds to risky loans. Also, shareholders increase their capital, α. In this example, $\delta = 0.01$, $p = -0.5$ and the charter value, g, is 0.06. The sensitivity of optimal decisions to changes in the volatility of bonds is quite sensitive to these parameters. In the above diagram, σ_2 is expressed in percentage form.

SOURCE: Authors.

Figure 2

Optimal Investment and Financing Decisions as a Function of the Loan Return and Bond Return Correlation

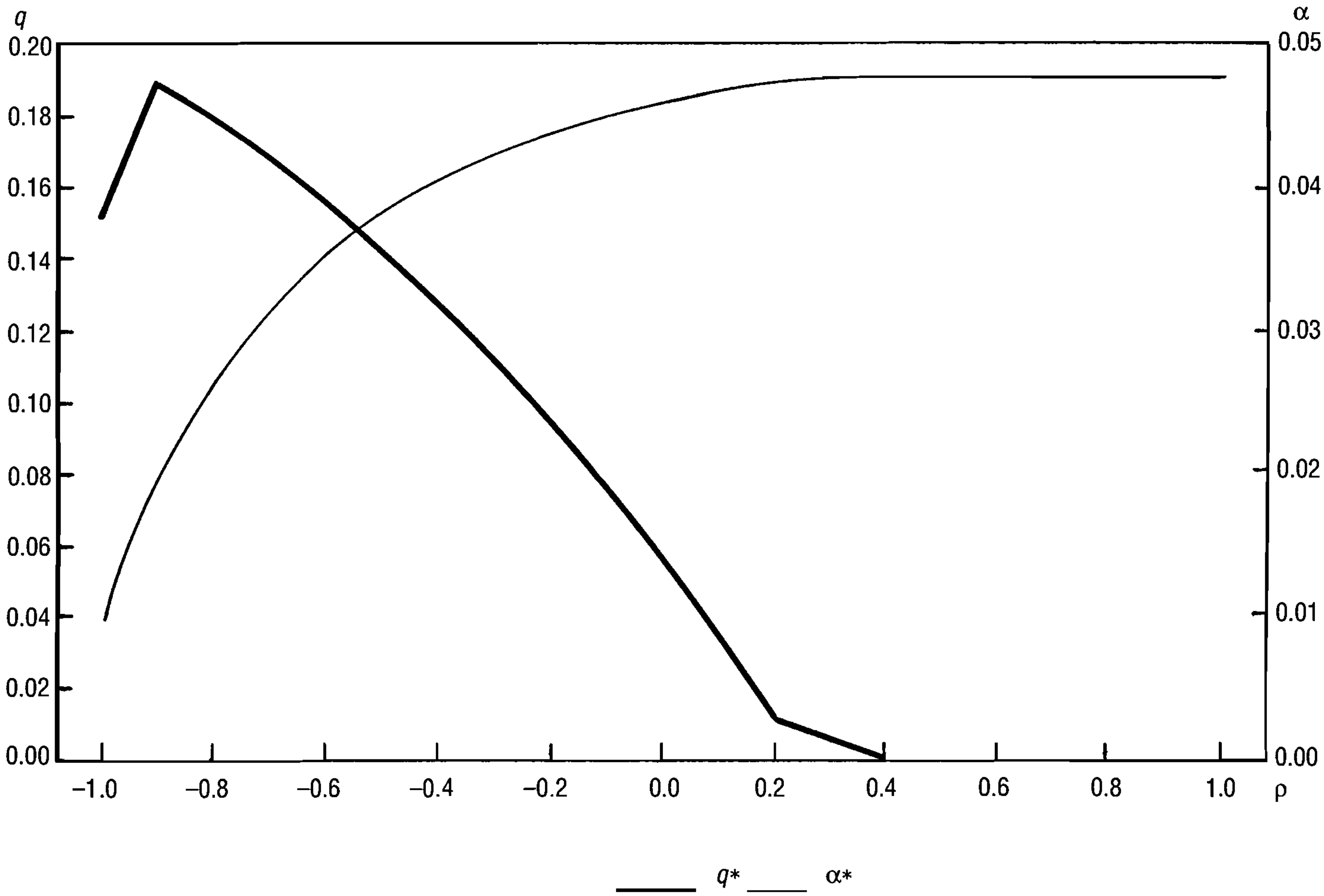


Figure 2 shows the sensitivity of the optimal decisions, q^* and $\alpha*$, to changes in the correlation, ρ. As ρ increases toward 1, the optimal q^* value drops to zero. At the same time, the optimal $\alpha*$ value converges to 0.047. The case parameters are the same as in figure 1.

SOURCE: Authors.

Figure 3

Restricted and Unrestricted Optimal Capital Structure and Investment Decisions

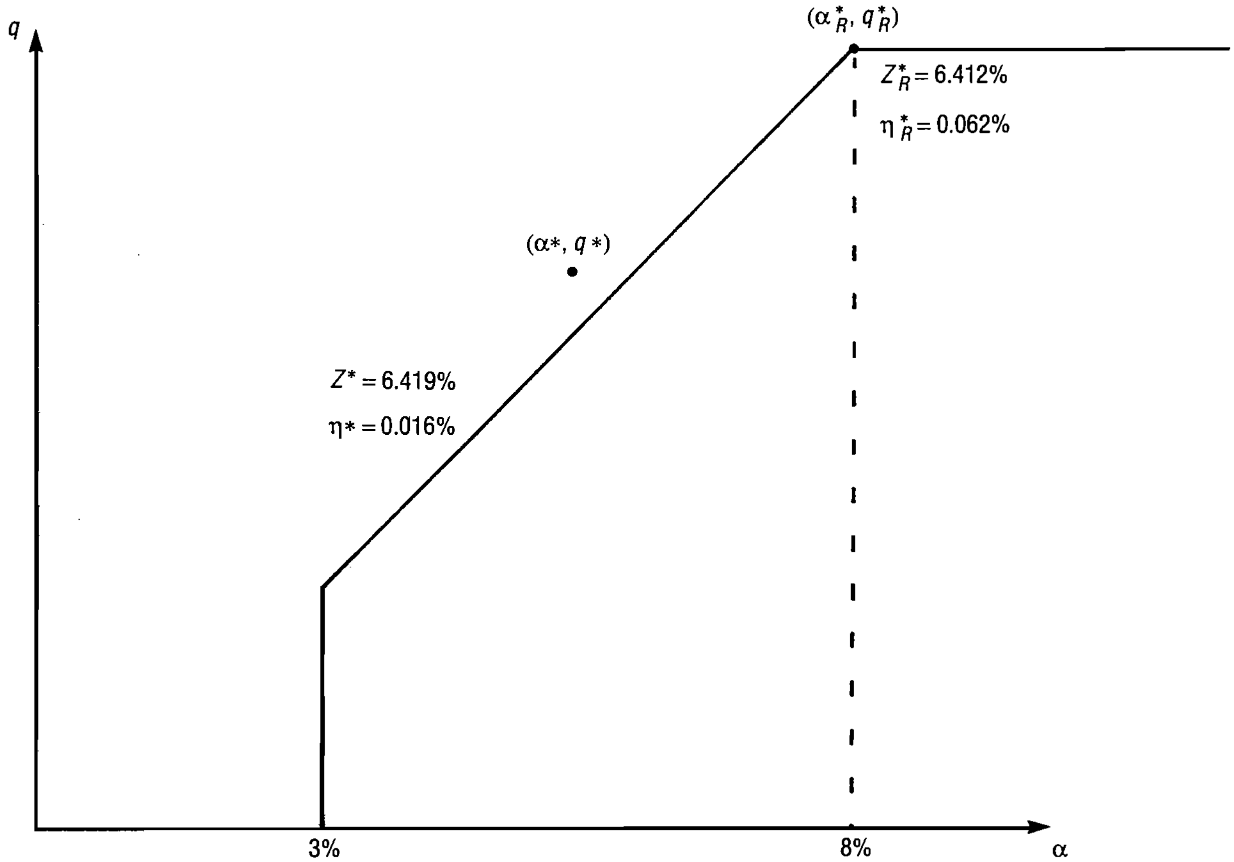


Figure 3 shows the unrestricted optimal solution and the restricted optimal solution in $\alpha - q$ space. Notice that the unrestricted optimum violates the capital constraint. The restricted optimum has a higher capital requirement and a higher risky loan investment component. Notice also that the cost of deposit insurance under the constrained optimal solution, η^*_R, exceeds the unconstrained optimum value, η^*. In this example, $\delta(q)$ is 1 percent, the charter value is 6 percent of the deposit base, the correlation between the risky bond and the loan portfolio is –0.75, and the volatilities of the bond and loan portfolio are 8 and 5 percent, respectively. The regulatory parameters are k = 3 percent and w = 8 percent.

SOURCE: Authors.

Figure 4
Restricted and Unrestricted Optimal Capital Structure and Investment Decisions

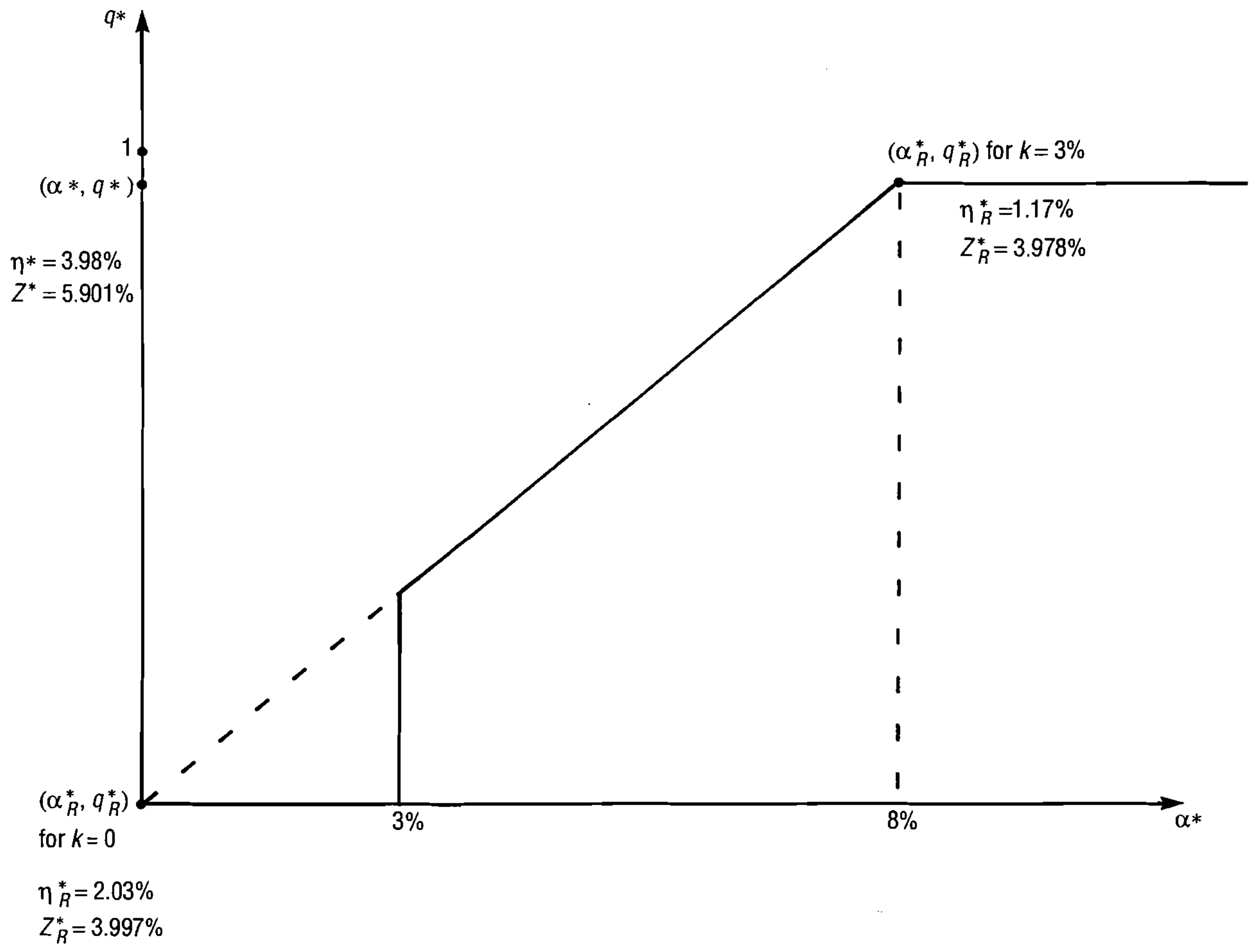


Figure 4 shows the unrestricted optimal solution and two restricted optimal solutions, the first for $k = 0$ and the second for $k = 3$ percent. In this example, the charter value is 5 percent of the deposit base, the risky bonds and loan portfolios are uncorrelated, the instantaneous volatilities of the bond and loan portfolios are 5 and 10 percent, and w is 8 percent. The example illustrates the sensitivity of the optimal capital and investment decisions to the capital constraint parameters, k and w.

SOURCE: Authors.