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# Spatial Fixed Effects and Spatial Dependence

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**Abstract**

We investigate the common conjecture in applied econometric work that the inclusion of spatial fixed effects in a regression specification removes spatial dependence. We demonstrate analytically and by means of a series of simulation experiments how evidence of the removal of spatial autocorrelation by spatial fixed effects may be spurious when the true DGP takes the form of a spatial lag or spatial error dependence. In addition, we also show that spatial fixed effects correctly remove spatial correlation only in the special case where the dependence is group-wise, with all observations in the same group as neighbors of each other.

**Keywords**: spatial econometrics, spatial fixed effects, spatial autocorrelation, spatial externalities, spatial interaction, spatial weights

# 1 Introduction

The presence of spatial effects in the form of spatial heterogeneity and spatial dependence is increasingly acknowledged in both applied and theoretical econometric work (for recent overviews, see, e.g., Anselin 2006, 2010, Baltagi et al. 2007, Baltagi and Pesaran 2007, Arbia and Baltagi 2009, LeSage and Pace 2009, Pinkse and Slade 2010). In empirical applications, a common problem is the presence of unobserved local or regional variables that may give rise to spatial error correlation. In addition, some theoretical models of social and/or spatial interaction require the inclusion of spatial dependence in the regression specification. Estimation and inference of such models necessitates the application of specialized spatial econometric methods, typically based on maximum likelihood or on the use of generalized method of moments (e.g., Ord 1975, Anselin 1988, Kelejian and Prucha 1998, 1999, 2007, 2010, Conley 1999, Lee 2003, 2004, 2007).

In many empirical applications, rather than employing these methods, sometimes suggestions are formulated to "fix" the problem by other means. Typically, these do not involve advanced estimation methods and are based on the rationale that evidence of residual spatial autocorrelation is *removed* after the fix is applied. For example, McMillen (2003, 2010) sees spatial autocorrelation as a result of model misspecification (omitted variables) and advocates the use of semi-parametric modeling to remedy it rather than the use of spatial econometric methods. Others have suggested the inclusion of trend surface variables, i.e., polynomials in the coordinates of the observations (for an extensive discussion and counterexample, see Schabenberger and Gotway 2005, p. 234).

The most commonly cited remedy, however, is the inclusion of spatial fixed effects in the regression specification. In a recent study, Kuminoff et al. (2010, p. 148) reviewed a large number of empirical studies of hedonic house price models and reported that 23% of the analyses used spatial fixed effects to deal with spatial autocorrelation. In addition, they carried out an extensive series of simulation experiments to conclude that "spatial fixed effects are clearly the preferable strategy for addressing spatially correlated omitted variables in cross-section data" (Kuminoff et al. 2010, p. 158). Many recent hedonic house price analyses follow this practice, e.g., Pope (2008a,b), Horsch and Lewis (2009), Kovacs et al. (2011). This conjecture does not stand alone. For example, a number of papers dealing with agglomeration effects and spatial spill-overs refer to Ciccone (2002) as having suggested that "the introduction of increasingly detailed spatial fixed effects allows to control for spatially correlated omitted variables," e.g., recently in Dalmazzo and De Blasio (2007a,b), and De Blasio (2008, 2009).[1]

In this paper, we investigate this conjecture more closely. Specifically, we clarify that spatial fixed effects address a form of spatial heterogeneity, but

[1]Interestingly, although Ciccone (2002) employs spatial fixed effects to control for spatial externalities, his article actually does not make the recommendation cited in the subsequent papers.

not “true” spatial dependence. In practice, it may seem like spatial fixed effects remove spatial autocorrelation from a regression specification, but it turns out this may be spurious. If “true” spatial dependence is present, in general there is no reason why spatial fixed effects would remove this dependence. However, as we demonstrate in the paper, an exception to this general statement is the special case in which the spatial correlation takes on a group-wise structure.

In the remainder of the paper, we first present our formal argument, starting with a definition of spatial fixed effects. We then consider the connection between spatial dependence and model misspecification more closely, reviewing the spatial lag and spatial error models and spatially correlated omitted variables. In each of these cases, we show how spatial fixed effects do not in general correct for the presence of spatial correlation. We next focus on the special case of group-wise spatial dependence and demonstrate how this is the only setting where the inclusion of spatial fixed effects corrects for spatial correlation. We follow this formal discussion with an empirical illustration based on a number of Monte Carlo simulation experiments. We close with some recommendations for practice.

## 2 Spatial Fixed Effects

In so-called discrete spatial heterogeneity, the variability in the model is structured by grouping the observations into a small number of discrete categories. The definition or delineation of these categories should be related to spatial structure (Anselin 1990). A special case of discrete spatial heterogeneity is when only the constant term is allowed to vary between subgroups in the data. This specification is commonly referred to as *spatial fixed effects.*

The corresponding regression model includes an overall constant term $\alpha$ and expresses the spatial fixed effects as differences from the reference group, for each observation $i$ in group $j$ (with $j = 2, \ldots, G$) as:

$$y_{ij} = \alpha + \alpha_2 d_{i2} + \cdots + \alpha_G d_{iG} + \mathbf{x'}_i \beta + \epsilon_{ij}, \tag{1}$$

with $y$ as the dependent variable and $x'_i$ as the i-th row of the $N \times K$ matrix of explanatory variables $\mathbf{X}$, and $d_{ij}$ as an indicator variable ($d_{ij} = 1$ for $j = h$ when $i \in h$ and $d_{ij} = 0$ otherwise). In this expression, each separate intercept $\alpha_j$ measures the difference of the "level" or mean of the group $j$, as defined by a non-zero value for the dummy variable $d_{ij}$, relative to the reference group. The regression "slope" coefficients $\beta$ are common to all groups. The error terms are typically assumed to be *i.i.d.*

The spatial fixed effects specification in Equation 1 is only operational when the number of groups $G$ is small relative to the number of observations $N$. In addition, there should be sufficient observations in each subgroup. This ensures that enough degrees of freedom are available to estimate the effect parameter in each group. The fixed effects indicate the existence of different intercepts across groups, but the model does not *explain* why such differences may exist.

A spatial fixed effects specification is appropriate when individual observations are organized into well-delineated groups and some characteristics of the group are unobserved. For example, in hedonic analysis, when house sales are grouped by school districts, but no data are available to gauge the performance of the schools, a spatial fixed effects variable may capture how this is reflected in the house sales price. This simply removes the school district effect, rather than modeling it (Beron et al. 2001, p. 330).

There are at least three complications resulting from the use of spatial fixed effects. First, since the fixed effects are captured by a constant indicator variable, the effect is assumed to influence all observations in the group identically. If there is within group heterogeneity or interaction, this will be relegated to the error term, resulting in heteroskedastic and/or spatially correlated disturbances. Second, there is only a single indicator variable to capture all the omitted group effects, so multi-factor cases are excluded.

Most importantly, the spatial delineation of the groups is not always unambiguous. When the omitted effects pertain to clearly defined administrative units (such as school districts, or counties) this may be a reasonable assumption, but when the effects are designed to incorporate generic "neighborhood" effects (such as crime, views, air quality, etc.) that do not follow the administrative boundaries, this becomes problematic. Also, administrative districts (e.g, census tracts) are often used to delineate spatial areas out of convenience (or necessity), whereas there is no reason why various neighborhood effects would necessarily match these areal units. Incorrect delineation of the spatial extent of the groups will again result in spatially correlated and/or heteroskedastic error terms or potentially create additional model misspecification.

In order to assess the extent to which the inclusion of spatial fixed effects would eliminate spatial error correlation, we first take a closer look at spatial dependence and model misspecification.

# 3 Spatial Dependence and Model Misspecification

In spatial econometrics, the two main data generating processes (DGP) that incorporate spatial dependence into a regression specification are the spatial lag and the spatial error model (Anselin 1988). We consider in turn how model misspecification in the form of ignoring these spatial effects relates to the inclusion of spatial fixed effects. We also consider misspecification in the form of spatially correlated omitted variables.

## 3.1 Spatial Lag Model

The spatial lag model is a specification for so-called *substantive* spatial dependence, in the sense that it is a formal expression of the equilibrium outcome of a spatial interaction process (Brueckner 2003, Anselin 2006). This is typically implemented by including a spatially lagged dependent variable, spatially lagged explanatory variables, or a combination of these in the regression specification. This allows for modeling a range of global and local spatial multiplier effects (Anselin 2003). In its simplest form, a spatial lag model is then:

$$\mathbf{y} = \rho \mathbf{W} \mathbf{y} + \mathbf{X}\beta + \epsilon, \tag{2}$$

where, in the usual notation, $\mathbf{y}$ is a $N \times 1$ vector of observations on the dependent variable, $\mathbf{W}$ is a $N \times N$ spatial weights matrix, $\rho$ is the spatial autoregressive parameter, $\mathbf{X}$ is a $N \times K$ matrix of observations on explanatory variables, with associated $K \times 1$ coefficient vector $\beta$, and $\epsilon$ is an $N \times 1$ vector of error terms. For the sake of simplicity, we will take the errors as *i.i.d.*.

In this structural form of the model, the inclusion of the spatially lagged dependent variable $\mathbf{Wy}$ on the right hand side of the equation relates the value of the dependent variable at a location to the values at neighboring locations, where the neighbors are specified through the weights matrix $\mathbf{W}$. In principle, the weights matrix should reflect the spatial structure of the interaction process, although in practice this is not so obvious and the specification of the weights is often rather ad hoc. The presence of the spatially lagged dependent variable also reflects a degree of simultaneity or feedback in the model which requires the use of specialized estimation techniques (Ord 1975, Anselin 1988).

Ignoring the spatial lag term, or, ignoring a spatial interaction process when one is present, results in a misspecified model. Technically, the ignored spatial lag is an omitted variable and as a consequence any estimates obtained from the misspecified model will be biased and any inference will be misleading. Specification tests based on these estimates will indicate spatially correlated residuals.

An alternative look at the misspecification that results from ignoring the spatial interaction in the model is obtained from the reduced form of the spatial lag model:

$$\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}\beta + (\mathbf{I} - \rho\mathbf{W})^{-1}\epsilon, \tag{3}$$

or, using the familiar power expansion for the inverse matrix,

$$\mathbf{y} = (\mathbf{I} + \rho\mathbf{W} + \rho^2\mathbf{W}^2 + \dots)\mathbf{X}\beta + v, \tag{4}$$

where $v = (\mathbf{I} - \rho\mathbf{W})^{-1}\epsilon$, a spatially correlated and heteroskedastic error term.

In this expression, the omitted variables from using the standard regression specification consist of the sum of spatially lagged explanatory variables, scaled by powers of the spatial autoregressive coefficient:

$$\rho\mathbf{W}\mathbf{X}\beta + \rho^2\mathbf{W}^2 X\beta + \dots. \tag{5}$$

In order for spatial fixed effects to capture these omitted variables, they would have to match the structure of the power expansion. Unless that structure results in a set of group-wise constants that match the fixed effects, this will in general not be the case. We return to this special case in Section 4. In other words, in general, the inclusion of spatial fixed effects will *not* "fix" the misspecification resulting from the omission of a spatial interaction term.

## 3.2 Spatial Error Model

In contrast with the spatial lag model, the spatial error specification deals with dependence as a "nuisance," i.e., with whatever dependence remains after all the relevant variables have been included in the model. In other words, ignoring this form of dependence does not result in biased estimates but is primarily a problem of precision. Formally, this is expressed as the usual regression specification (using the same notation as before):

$$\mathbf{y} = \mathbf{X}\beta + \epsilon, \tag{6}$$

with a non-spherical error variance-covariance matrix:

$$\mathrm{E}[\epsilon\epsilon'] = \mathbf{\Sigma}. \tag{7}$$

The spatial structure of the variance-covariance matrix is the basis for referring to this as spatial correlation.

The consequences of ignoring spatial error correlation are well known: ordinary least squares regression does not result in biased estimates, but the variance of these estimates needs to be adjusted. In other words, this form of misspecification affects the second moment of the estimates (their precision).

A very general theoretical framework that provides a motivation for the presence of spatially correlated errors is contained in the "common shocks" perspective outlined in Andrews (2005). Unobserved effects are shared by pairs of observations and thus generate error correlation.

Formally, this can be expressed by considering an unobserved "factor" $f$, with $\mathrm{E}[f] = 0$ combined with an observation-specific "loading" $\delta_i$. The regression error term then consists of two components, one associated with the factor and its loading, the other with an idiosyncratic error:

$$\epsilon_i = \delta_i f + u_i. \tag{8}$$

Consequently, cross-sectional (spatial) correlation between errors in $i$ and $j$ follows from the presence of a common factor $f$, such that:

$$\mathrm{E}[\epsilon_i \epsilon_j] = \delta_i \delta_j \sigma_f^2. \tag{9}$$

This framework can be extended to multiple factors (as well as to a panel data setting) and encompasses a wide range of correlation structures, including most familiar forms of spatial autocorrelation as well as group effects.

In general, the complex structure expressed in the error term will not match the spatial fixed effects and therefore the latter will not "fix" the spatial correlation. We focus on the special case where this does occur in Section 4.

## 3.3 Spatially Correlated Omitted Variables

A common reason for the presence of spatial correlation in a cross-sectional regression is the existence of spatially correlated omitted variables. Under fairly general conditions, these are absorbed into the error term. The omitted variables should not be correlated with the included variables, otherwise the orthogonality of the $\mathbf{X}$ matrix and the error vector is violated.[2] Several spa-

[2] In LeSage and Pace (2009, pp. 27–28 and 60–68) omitted variables with spatial dependence are used as a motivation for a spatial regression model. However, in their examples,

tially correlated error structures are available that accommodate the notion of spatially correlated omitted variables.

The most commonly used specification is a spatial autoregressive model for the error term:

$$\epsilon = \lambda \mathbf{W} \epsilon + \mathbf{u}, \tag{10}$$

with $\lambda$ as the autoregressive coefficient, $\mathbf{W}$ as the spatial weights matrix and $\mathbf{u}$ as the idiosyncratic disturbance. We can therefore rewrite the regression specification as:

$$\mathbf{y} = \mathbf{X}\beta + \lambda \mathbf{W} \epsilon + \mathbf{u}. \tag{11}$$

Analogous to a panel data setting (e.g. Baltagi 2008), in order to correct for the spatial correlation, the fixed effects will need to conform to the structure of the random components in $\mathbf{W}\epsilon$. There is no formal argument to ensure that this would be the case in general.

# 4 Groupwise Spatial Dependence and Spatial Fixed Effects

A special structure for the spatial weights matrix, initially introduced in spatial econometrics in the work of Case (1991, 1992), organizes the observations into groups (e.g., locations within districts). All observations in the

the error term contains a component that is correlated with the explanatory variable in the model, and thus causes simultaneity bias in addition to the spatial correlation. We specifically exclude simultaneity bias.

same group are neighbors of each other, but there are no between-group interactions. The result is a block diagonal spatial weights matrix, with each block corresponding to a group. The elements of the row-standardized weights matrix for each group $k$ equal $1/(n_k - 1)$, where $n_k$ is the number of members of the group. As a consequence of the block structure, a spatially lagged variable takes on a particular form. In effect, the spatial lag term for each observation in the same group will be equal to the average of all values in the group, except the observation itself. This will be close to a constant, but not actually a constant, since $w_{ii} = 0$ by convention.

This particular model has received some attention in the theoretical spatial econometric literature. It is also formally related to the literature on spatial and social interaction, where, using the terminology of Manski (1993), the average of the members of the "reference group" enters into the model specification as a proxy for endogenous social effects (see also Brock and Durlauf 2001, Durlauf 2004, Li and Lee 2009). The econometric properties of this model were investigated by Lee (2002) and Kelejian and Prucha (2002). Specifically, Lee (2002) proved that OLS is a consistent estimator for a spatial lag specification that uses the group-wise weights. On the other hand, Kelejian and Prucha (2002) showed that both OLS and 2SLS are inconsistent in the case that only a single group is used.

We consider the weights structure more closely and extend the Kelejian and Prucha (2002) result to a situation with multiple blocks. Slightly adapting their notation, each block weight takes on the form:

$$\mathbf{W}_k = [1/(n_k - 1)]\mathbf{J}_{n_k} - [1/(n_k - 1)]\mathbf{I}_{n_k}, \tag{12}$$

where $\mathbf{J}_h = \iota_h \iota_h'$, with $\iota_h$ as a $h \times 1$ vector of ones and $\mathbf{I}_{n_k}$ as an identity matrix of dimension $n_k$. The complete weights matrix has a block-diagonal structure:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & 0 & \dots & 0 \\ 0 & \mathbf{W}_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mathbf{W}_G \end{bmatrix}. \tag{13}$$

for $G$ groups or blocks.

Kelejian and Prucha (2002, p. 694) show that the reduced form inverse for each block reduces to a special structure (in our notation):

$$(\mathbf{I} - \rho \mathbf{W}_k)^{-1} = \delta_1 \mathbf{J}_{n_k} + \delta_2 \mathbf{I}_{n_k}, \tag{14}$$

with $\delta_1$ and $\delta_2$ functions of $\rho$ and $n_k$. More importantly, they use this result to prove that the reduced form for the spatially lagged dependent variable asymptotically becomes a constant vector.

For a complete block structure as in Equation 13, this extends to the inverse matrix that corresponds to each block. More precisely, using this result, it follows that the spatially lagged dependent variable based on the group-wise weights will converge to a set of constant vectors, one for each group. In essence, this then corresponds to a spatial fixed effects structure, where each effect is constrained to a group.

The implications of this for removing spatial correlation are two-fold. If the dependence is of the spatial lag form, the inclusion of spatial fixed effects is asymptotically equivalent to a spatially lagged dependent variable with group-wise weights. On the other hand, if the dependence is of the spatial error form, the group-wise structure will result in error components that match the spatial fixed effects. Again, apart from a scale adjustment (in the constant term, since the error components have mean zero), this can be modeled by including spatial fixed effects in the regression specification.

These are the only special cases where there is a formal link between the spatial fixed effects and the specification of spatial dependence in the model. However, it should be kept in mind that the group-wise structure has some peculiar characteristics that do not correspond to received spatial theory. The equality of the weights within each block violates any notion of distance decay. Moreover, the lack of inter-block interaction is a serious constraint. Whether these limitations apply in practical contexts is largely an empirical question.

## 5 Empirical Evidence

To further illustrate the properties of the spatial fixed effects specification in the presence of spatial correlation, we carry out a series of Monte Carlo simulation experiments. We consider the two main spatial regression DGPs, the spatial autoregressive error model and the spatial lag model. The regression specification consists of a constant term ($\iota$) and one explanatory variable ($x$), drawn from a normal distribution with mean zero and standard deviation 1.4. The associated regression coefficients $\alpha$ and $\beta_1$ are both set to 1. The *i.i.d* idiosyncratic error terms are generated as standard normal.[3] With $\mathbf{u}$ as the vector of error terms and the matrix of explanatory variables as $\mathbf{X} = [\iota\ x]$, the $N \times 1$ vector of "observations" on the dependent variable under the spatial error DGP is obtained as:

$$\mathbf{y} = \mathbf{X}\beta + (\mathbf{I} - \lambda\mathbf{W})^{-1}\mathbf{u}, \tag{15}$$

with $\beta$ as a $2 \times 1$ vector of ones, $\lambda$ as the spatial autoregressive coefficient and $\mathbf{W}$ as the $N \times N$ spatial weights matrix. Under the the spatial lag DGP, the vector of observations on the dependent variable is obtained as:

$$\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}\beta + (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{u}, \tag{16}$$

with $\rho$ as the spatial autoregressive coefficient and the other notation as before.

We create increasing degrees of spatial autocorrelation by considering seven values for the autoregressive coefficient ($\lambda$ or $\rho$): 0.0 (null hypothesis on no misspecification), 0.01, 0.1, 0.25, 0.5, 0.75, 0.9 and 0.99. We consider two geographies. One consists of the 3084 contiguous counties in the U.S., the other of 5035 locations of single family residences for sale in the city of Seattle, WA in 1997 (for a detailed discussion of the sample, see Koschinsky et al.

[3]Under the null of no misspecification, this yields an average $R^2$ of 0.66.

2011). These reflect commonly encountered situations in empirical practice, such as growth convergence studies (counties or NUTS regions, e.g., Ciccone 2002) and hedonic house price models (house locations, e.g., Kuminoff et al. 2010).

Two types of spatial weights are introduced. One is derived from the commonly used criterion of contiguity. For the U.S. counties, this is based on so-called "queen" contiguity (i.e., two counties that have at least one point in common on their boundaries; this includes "four corner" situations). For the Seattle locations, we used a $k$-nearest neighbor criterion with 20 neighbors. These two approaches differ in that the former tends to result in a very sparse weights matrix with a small number of neighbors (6 on average), whereas the latter has a relatively large number of 20. In addition to these traditional spatial weights, we also include group-wise or "block" weights. For the U.S. counties, this is based on the states, with all counties in the same state being considered as neighbors (but no cross-state neighbors). This results in an average number of neighbors of 63 (ranging from 1 to 254). The Seattle blocks are based on census tracts, resulting in an average number of neighbors of 180 (ranging from 51 to 271). All spatial weights are used in row-standardized form.

The combination of geography (U.S. counties or Seattle locations), DGP (spatial lag or error), spatial weights (contiguity or block) and spatial autoregressive parameter values yields 56 different data setups. For each of these combinations, we generated 10,000 replications.[4]

We carried out three estimations for each data setup. First, we estimated the standard regression specification by means of OLS, ignoring any spatial effects. Second, we introduced spatial fixed effects into the specification and estimated the resulting model by means of OLS. The fixed effects corresponded respectively to the states and census tracts in the samples. Finally, we estimated the proper spatial model, following the DGP. For the spatial error model, we used the generalized moments estimator of Kelejian and Prucha (1999). The spatial lag model was estimated using the spatially lagged explanatory variable as an instrument for the spatially lagged dependent variable, as an application of spatial two stage least squares (Anselin 1988, Kelejian and Prucha 1998).

The results are summarized in Figures 1–4.[5] Each of the Figures contains six graphs depicting the empirical distribution of the $\hat{\beta}_1$ estimate over the 10,000 replications, for each value of the autoregressive parameter. The three graphs on the left, (a)–(c), use the contiguity-based spatial weights in the DGP, the three graphs on the right, (d)–(f), are based on block weights.

First, consider the spatial error case depicted in Figure 1 for the U.S. counties and in Figure 2 for the Seattle locations. The top row shows the distribution of the estimate using OLS. The familiar result depicts an in-

[4]The code is part of the so-called spatial econometric workbench in the PySAL open source library for spatial analysis (Rey and Anselin 2007). The PySAL code is available from http://pysal.org.

[5]Detailed tables with the full results are available from the authors.

creasing variance around the true value of 1 as the spatial autoregressive parameter gets larger, and quite dramatically so for the value of 0.99. The same pattern is seen in the two geographies. This fully conforms to the theoretical expectation that the OLS estimator remains unbiased, but its variance increases with increasing positive spatial error autocorrelation.

The second row of graphs shows the distribution of $\hat{\beta}_1$ for the spatial fixed effects estimator. In graph (b) the DGP is based on the contiguity weights, whereas in graph (e) the block weights are used. As argued above, the fixed effects estimator *does not* alleviate the variance-increasing influence of the error spatial autocorrelation when the latter is based on contiguity. In (b), while the variance is slightly smaller than for OLS, it does increase with larger $\lambda$. This is compatible with some of the evidence in practice that FE seem to eliminate the indication of spatial correlation. In fact, they do not, but the variance increases less with $\lambda$ than for OLS. However, as demonstrated in Section 4, when the fixed effects match the blocks used in the weights specification, the effect of spatial autocorrelation is negligible. For increasing values of $\lambda$, the distribution of $\hat{\beta}_1$ in graph (e) remains unaffected and identical to that under the null. The same result is found in Figure 1 and Figure 2. Finally, the use of the spatial GM estimator shown in the bottom graphs yields a result that is unaffected by the value of the spatial autoregressive parameter. In each case, the resulting distribution of the estimates is essentially the same as under the null.

In Figures 3 and 4 the DGP is the spatial lag model. The two graphs in the top row show the familiar result how the OLS estimate for $\beta_1$ becomes increasingly biased (moves away from the true value of 1.0) with larger spatial autoregressive parameters. In addition to the bias, the variance of the estimate increase with $\rho$ as well. The same result is obtained for the contiguity and block weights and for both geographies.

The second row of graphs illustrates the use of the spatial fixed effects. As in the case of the spatial error DGP, the result for the contiguity weights is largely unchanged from the base OLS case, with bias and variance growing with $\rho$, but at a less rapid rate. However, again as demonstrated in Section 4, the effect of $\rho$ on the $\hat{\beta}_1$ obtained with the fixed effects estimator is negligible. The estimates are centered on the true value of 1.0, with the variance essentially constant across values of $\rho$. The same result is found in the graphs of the bottom row, where the spatial two stage least squares estimator properly accounts for the effect of the spatial lag. In Figures 3 and 4, the distribution shown in graph (e) is essentially the same as those for the spatial estimators in graphs (c) and (f).

In sum, the simulation results confirm the theoretical expectations formulated in the early part of the paper. In general, the use of spatial fixed effects does not properly correct for the presence of spatial autocorrelation. However, when the true spatial correlation is of the group-wise form and the fixed effects exactly correspond to the groups in question, then the inclusion of spatial fixed effects is equivalent to the use of a spatial estimator to obtain estimates for the regression parameters.

# 6 Conclusions

We showed both analytically and in a series of simulation experiments how spatial fixed effects only control for spatial correlation when the DGP corresponds to a group-wise or block structure. While such a structure may be compatible with some of the social interaction literature, it violates the principal tenet of spatial interaction, namely Tobler's first law in which "everything depends on everything else, but closer things more so" (Tobler 1979). The implied distance decay is absent in the block structure.

In practice, since the true DGP is unknown, it remains largely an empirical matter which interaction structure is appropriate. However, unless there are strong theoretical or practical reasons why distance decay should be ruled out, the use of spatial fixed effects will not be sufficient to correct for the presence of spatial correlation. A careful assessment of alternative model specifications remains a prudent strategy, rather than the adoption of a one size fits all "fix."

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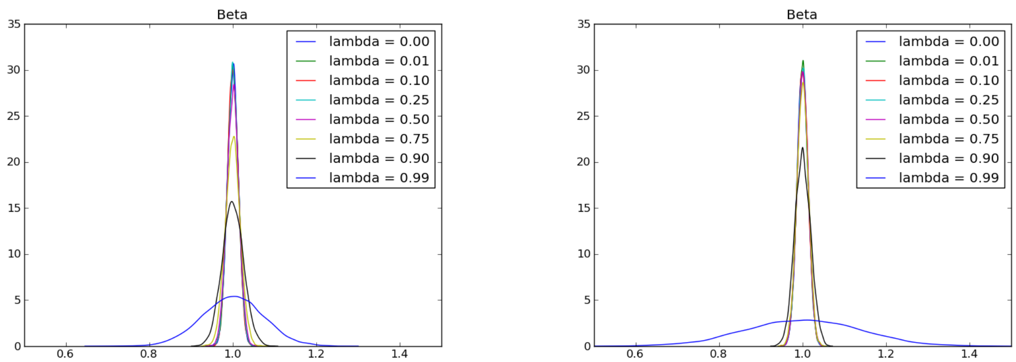


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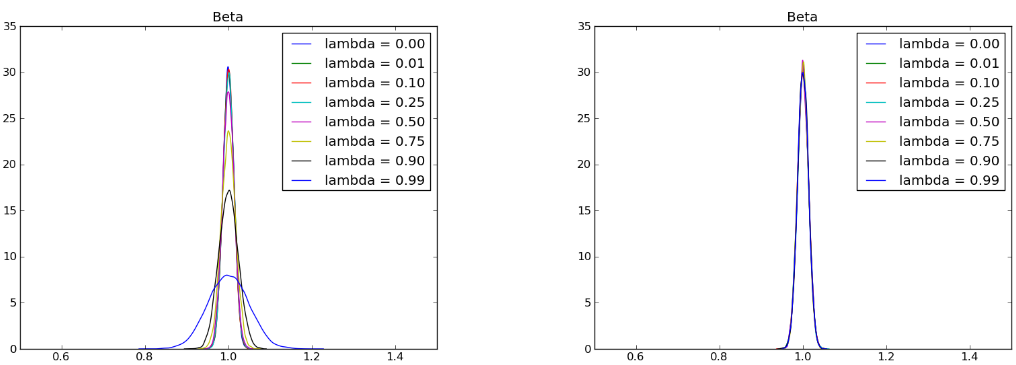
OLS




(b) Queen (e) Block

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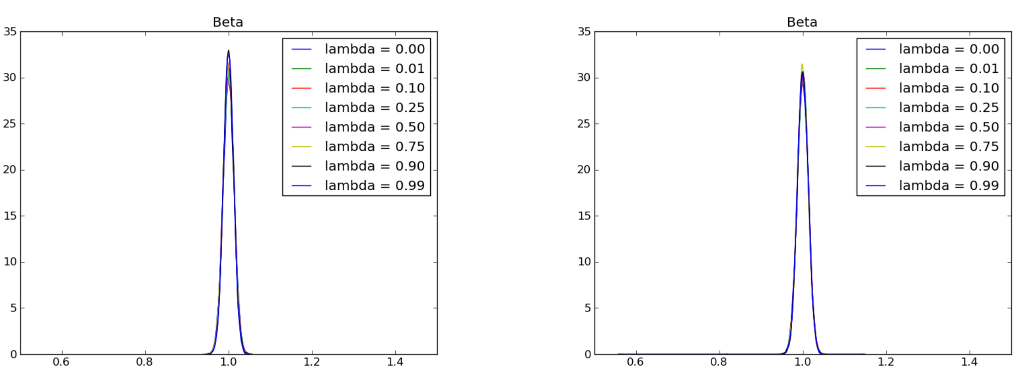


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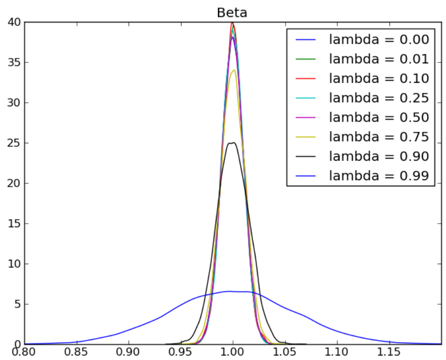
Error-GM

Figure 1: DGP Spatial Error Model – U.S. Counties

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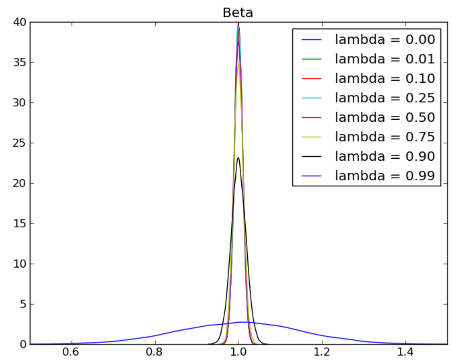


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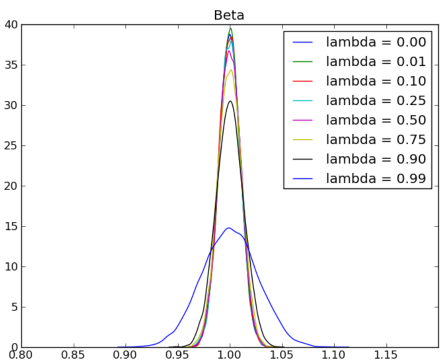


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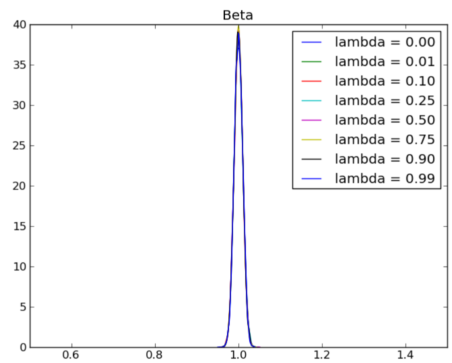


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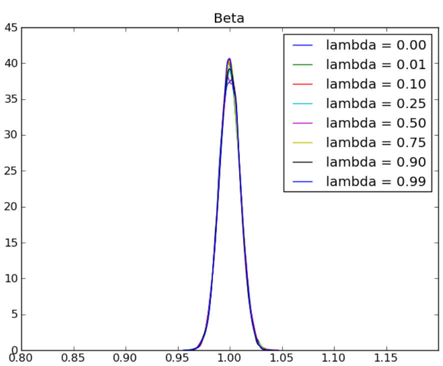


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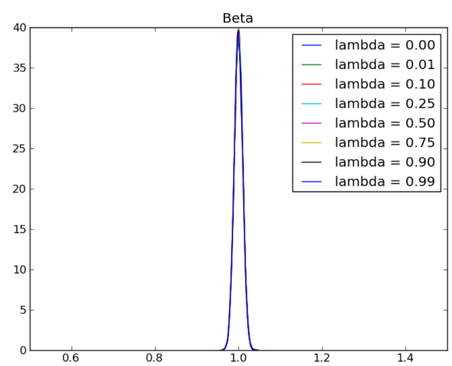
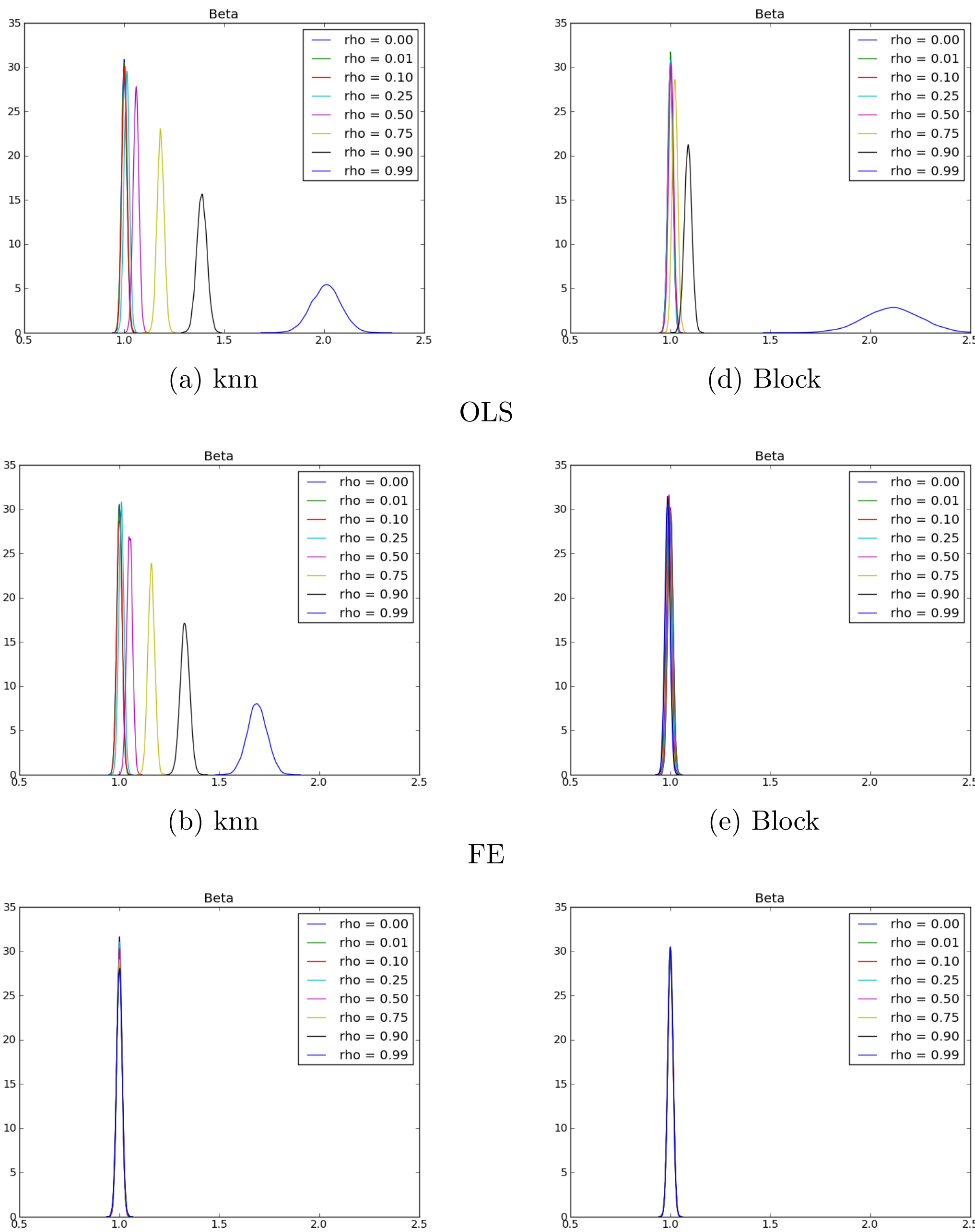



(f) Block

Error-GM

Figure 2: DGP Spatial Error Model – Seattle




(a) knn (d) Block

OLS

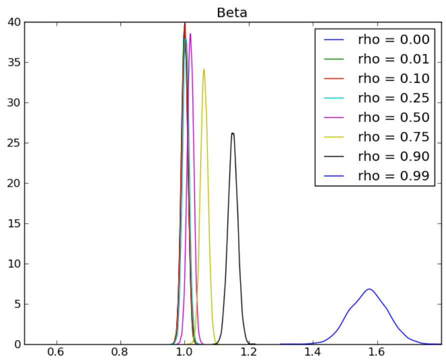
(b) knn (e) Block

FE

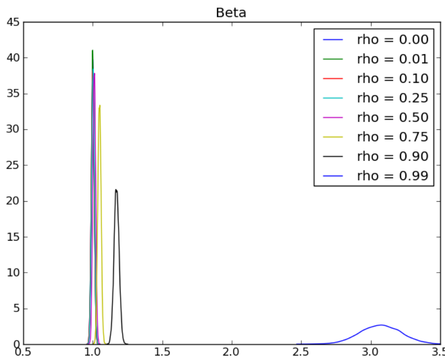
(c) knn (f) Block

S2SLS

Figure 3: DGP Spatial Lag Model – U.S. Counties




(a) knn

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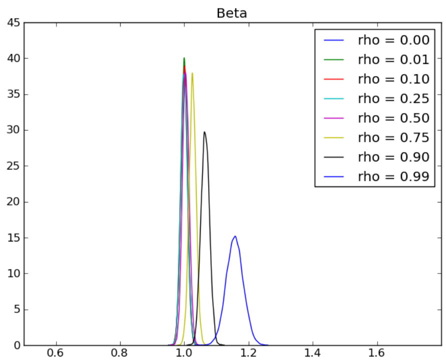
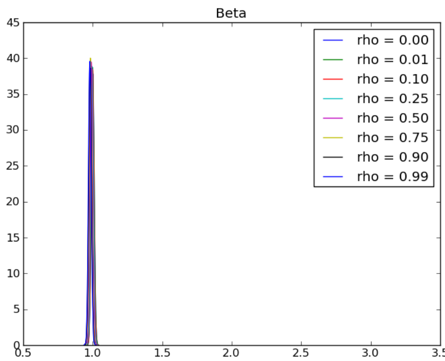



(b) knn

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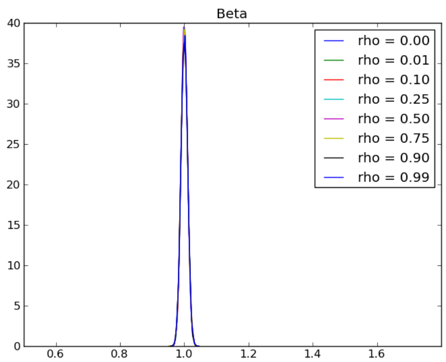


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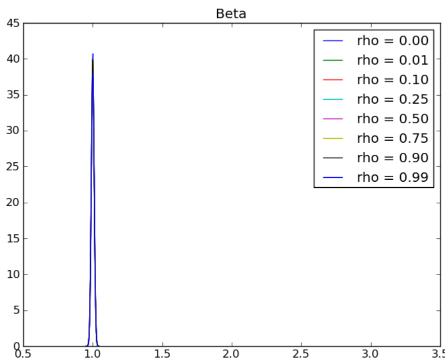


(f) Block

S2SLS

Figure 4: DGP Spatial Lag Model – Seattle