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Strategies for search on the housing market and their implications for price dispersion

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Abstract

When an household needs to change its home, a new house must be bought and the old one must be sold. In order to complete these two transactions, the household can adopt either a sequential or a simultaneous search strategy. In sequential strategies, it first buys (or sells) and only after tries to sell (or buy), to avoid either being homeless or holding two houses, respectively. In the simultaneous strategy, the household tries to buy and sell simultaneously. If the household adopts the simultaneous strategy, it can reduce its search costs, but becomes exposed to the risk of becoming a homeless renter or the owner of two houses. The literature generally considers only the sequential search strategy. However, we show in this article that the simultaneous strategy is (i) generally welfare improving for households, (ii) sometimes the sole equilibrium strategy, and (ii) at the origin of price dispersion on the housing market.

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1 Introduction

It is widely recognized that price dispersion exists in real estate markets. Herein, a new explanation of this phenomenon is presented that is based on the search strategies of households. This explanation does not rely on any form of heterogeneity, either in dwellings or in households, but considers the presence on the housing market of households that simultaneously try to buy a new home and to sell their current one. This explanation has a significant degree of importance, given the puzzling existence of price dispersion in real estate markets.

A significant part of price dispersion can be attributed to the heterogenous nature of real estate assets. Properties differ according to their structural attributes (size, building period, ...) and their location (submarkets, local amenities, ...). The large empirical literature on hedonic price (see Rosen, 1974) provides evidence of the impact of each of these qualitative factors on the selling price and enables a quantitative assessment of the price gap between two almost similar dwellings that differ only in one attribute or, more importantly, their location (i.e. different geographical submarkets)[1]. Recently, a significant strand of research has been devoted to the *remaining* part of real estate price dispersion: the amount of volatility that cannot be attributed to the heterogeneity of assets. It appears that two similar dwellings (that have the same attributes and that are located close to each other) can be valued differently at the same time and that this *residual* heterogeneity is empirically non negligible. For example, Leung, Leong and Wong (2006) trace the evolution of quality-controlled price dispersion on the Hong Kong housing market over time and show that the amount of volatility differs from zero and is connected to macroeconomic factors[2].

The question is, how does it come about that two seemingly identical assets may be sold at different prices? The literature on the real estate market provides two typical answers, one of which relies on unob-

[1] For example, Gabriel, Marquez and Wascher (1992) document how large the regional dispersion in house prices is in the U.S. and show that this spatial variability is linked to interregional migration of households. On the theoretical side, the role of housing supply regulation (Glaeser, Gyourko and Saks, 2005, 2006), income distribution (Gyourko, Mayer and Sinai, 2006) or differential in productivity gains across metropolitan areas (Glaeser, Scheinkman and Shleifer, 1992, or Van Nieuwerburgh and Weill, 2009) in explaining spatial dispersion in house prices have been explored.

[2] Their work complements earlier contributions on the housing market (see Harding, Knight and Sirmans, 2003) or on other durable goods markets (see for example Goldberg and Verboven 2001, on the car market).

served (good or agent) heterogeneity, the other of which relies on the liquidity dimension of housing markets. According to the first view, remaining price differentials may be caused by either missing variables concerning the attributes of the good that is to be sold (unobserved good heterogeneity) or *ex ante* heterogeneity on the part of buyers and/or sellers, or both. This view has intuitive appeal: some structural attributes are not observable, difficult to measure, and consequently not included in hedonic estimates even if they might affect selling prices. Moreover, households entering the housing market can differ across tastes, information (first-time buyers vs experienced buyers) or search costs. These factors affect the bargaining power of the protagonists and then the final price. For example, Read (1991) proposes a theoretical setup with search costs on the housing market in which agents have exogenously distributed preferences concerning the choice of location. This assumption induces equilibrium price dispersion as well as positive vacancy rates. Harding, Rosenthal and Sirmans (2003) adds some variables concerning the buyers' socio-economic profile in standard hedonic house price equations. They show that transaction prices may differ according to the buyer's age or marital status. Moreover, they build a theoretically founded proxy to evaluate the respective bargaining powers of both buyers and sellers. This proxy appears to have a significant effect on the valuation of the property.

According to the second view, the concept of market liquidity - i.e. the time a dwelling has been on the market - provides an explanation for price dispersion. By means of an empirical study, Merlo and Ortalo-Magné (2004) not only show that the time that a property remains on the market and the final sale price are correlated (which is a widely documented empirical fact in the real estate literature); they also provide evidence that properties with higher listed prices will take longer to sell, but will be sold at a higher price than properties with lower listed prices. The time to sell is linked negatively to the ratio of the listed price to the sale price. This suggests that *ex ante* identical sellers who put similar goods on the market and adopt different listing price strategies may sell at a different price. This is the *liquidity* assumption: price dispersion does not come solely from the heterogenous nature of assets or the *ex ante* heterogeneity of agents, but also from the uncertainty in time to sale inherent in a standard search process. *Ceteris* *paribus*, the longer a property remains on the market, the more likely the listed (and reservation) price will be revised downward and consequently the lower the sale price. From a theoretical point of view, Fisher, Gatzlaff, Geltner and Haurin (2003) working in a search setup, suppose that (*ex ante*) identical agents are affected by an exogenous source of shocks that cause a continuous distribution of reservation prices for both buyers and sellers on (commercial) real estate markets. Even if the purpose of their contribution is to propose a liquidity-adjusted price index, this distribution is also responsible for varying liquidity and finally for part of the dispersion in sale prices. In a recent contribution with a matching model of the housing market, Albrecht, Anderson, Smith and Vroman (2007) suppose that *ex ante* identical agents who currently have properties on the market are affected by an exogenous disturbance: they may move from a relaxed to a desperate state (with high costs of being unmatched) at a Poisson rate. This generates multiple price equilibria, because each desperate or relaxed seller can meet a desperate or relaxed buyer. The authors show that the variance in price is affected by the length of time that a property has been on the market.

The main shortcoming of all the above-mentioned views is that price dispersion comes either from *ex ante* (deterministic) heterogeneity (i.e. different socio-economic profiles, tastes, or search costs), or from an exogenous idiosyncratic disturbance (i.e. an exogenous move from a relaxed to a desperate state). Price heterogeneity is not due solely to the endogenous functioning of the market. This contrasts with the theoretical literature on price dispersion, which has sought to provide an endogenous explanation of this phenomenon. For example, Burdett and Judd (1983) propose a nonsequential search model with identical agents where price dispersion is purely endogenous. The noisy nature of the search process leads to *ex post* heterogeneity in agents' information and consequently to price heterogeneity. Without relying on a search setup, Salop and Stiglitz (1982) prove that equilibrium price dispersion can be attained in an homogenous commodity market with *ex ante* identical agents and no exogenous disturbances. They show that a two-price equilibrium exists in a competitive setup with no auctioneer and with costly information. More recently, for the labor market, Burdett and Mortensen (1998) provide a model in which wage dispersion may exist in equilibrium with perfectly identical workers and firms. Single market-wage equilibria are ruled out by firms posting wages strategically and workers searching for jobs strategically.

The main goal of this paper is to propose an original model that explains dispersion in housing prices without relying on *ex ante* heterogeneity on the part of agents, or on any source of exogenous idiosyncratic noise. We intend to present a model of search in which price dispersion is purely endogenous, i.e. due to the very specific nature of the search process *on the housing market.* A striking feature of the housing market is the existence of agents who are *simultaneously* on the two sides of the market. When an household wants to move house for family or professional reasons, a new house needs to be bought and the old one needs to be sold. There are several ways in which this can be accomplished: *(i)* sell the old home first and then rent (and perhaps buy another house later), *(ii)* buy a new house (and temporarily own two properties) and then try to sell the old one, and *(iii)* enter the market as both a seller and a buyer and try to conduct both transactions (if possible at the same time). The third strategy is potentially optimal because it may result in agents' avoiding having to rent a property temporarily (attributes of dwellings on the rental market are generally of a lesser quality than on the owner-occupied market) and may also result in their avoiding owning two houses simultaneously (which incurs substantial financial costs and risks (for example, fixed-term bridge loans in some European countries) for indebted households). Hence, households entering the market as both a buyer and a seller may hope to get two simultaneous matches and acquire a suitable new home without extra costs. Nevertheless, due to the uncertain nature of the search process, these agents may not escape a transient state, namely the homeless/renter state (Buyer Only agents) or the two-house-owner state (Seller Only agents) and consecutive additional costs with respect to finance (for owners of two houses) and search (when they still have a house to buy or to sell). Consequently, there are different kinds of matches on the market. A buyer-and-seller household can match with another buyer-and-seller or a seller-only household to buy a new dwelling. Similarly, it can match with another buyer-and-seller household or a buyer-only household to sell the old one. All these agents get different flow values from a conclusive transaction that leads to a multiple price equilibrium, without relying on any exogenous factor. Price dispersion occurs only as a result of the history of the occupational status of each household.

We base our setup on Wheaton's (1990) seminal search model for the housing market. In this model, only the sequential strategy *(ii)* is considered, i.e. households first buy their new dwelling and then try to sell the old one. Hence, only one kind of match is possible: households that are trying to buy their new house always meet two house owners who are trying to sell their old one. No price dispersion equilibrium is possible in this context. We extend Wheaton's setup and compare equilibria where households adopt either a sequential or a simultaneous strategy when they enter the housing market. The complexity of our model prevents us from basing our comparison fully on analytic properties, so we use numerical analysis to characterize some of the positive and normative properties of the equilibria. The comparison highlights the importance of taking the simultaneous strategy into consideration. The first lesson drawn from our numerical analysis is that multiple equilibria (where both simultaneous and sequential strategies exist and are stable) occur for a wide range of parameter values. When the equilibria are Pareto-ranked[3], the simultaneous strategy often dominates the sequential one. Moreover, we do not find a range of parameter values for which the sequential strategy is the unique equilibrium strategy. On the contrary, for some ranges of parameters, the simultaneous strategy is the only possible equilibrium outcome. These results lead us to conclude that the simultaneous strategy is a credible behaviour on the housing market and therefore also a credible explanation of the price dispersion induced by this strategy. We end our analysis by describing price dispersion in the model. We prove that there are three equilibrium prices if the bargaining processes are symmetric and four otherwise, and use numerical analysis to discuss the determinants of the amplitude of the price dispersion.

The remainder of the paper is organized as follows. The environment is presented in Section 2, the model is presented and solved in Section 3 with the sequential strategy and in Section 4 with the simultaneous strategy. The two equilibria are compared on the basis of numerical simulations in Section 5. Section 6 concludes.

[3] Equilibria are ranked with respect to the steady-state welfare of a matched household, which exists outside the housing market. We do not compute the transitional dynamics between steady-state equilibria.

2 The Environment

The economy is populated by two types of households, the singles and the couples, which differ only in their house preferences. Let h_1 denoted the mass of singles and h_2 the mass of couples, with $2h$ the total number of households that satisfies $2h = h_1 + h_2$. The transition between family types is exogenous. Without loss of generality, we assume that the transition rate β between the two states is symmetric. The associated laws of motions are then

$$h'_1 = h_1 + \beta h_2 - \beta h_1 \tag{1}$$

$$h'_2 = h_2 + \beta h_1 - \beta h_2 \tag{2}$$

where the symbol $'$ denotes the next period value of the variable. The symmetry of the transition rate implies the equality of the two masses of households at the steady state : $h_1 = h_2 = h$.

On the housing market, each type of household can live in two types of dwellings, i.e. small and large. Single (resp. couples) households are matched when they live in small (resp. large) units and mismatched otherwise. Hence, each previously matched household hit by a demographic shock will become mismatched and will have to enter the housing market to get a convenient dwelling. More precisely, this household needs to achieve two transactions: sell its current unit and buy a new (appropriate) one. To do so, mismatched households can proceed in different ways:

- Enter the market as a "Buyer-First" (BF), i.e. first search for a new dwelling. Once the household succeeded in buying a convenient new house, it then puts its old unit up for sale (and becomes a "Seller-Only", SO). It will own two houses and bear the consequent financial costs until the sale transaction is concluded. This strategy exactly corresponds to the one described by Wheaton (1990).
- Enter the market as a "Seller-First" (SF), i.e. first put his current asset up for sale. Such a strategy is perfectly symmetric to the preceding one. Once the household succeeded in selling its asset, it then starts searching for a new one. During the spell between the two transactions, the household owns no

house (or live in a rented dwelling) and becomes a "Buyer Only" (BO).

- Enter the market as both a buyer and a seller (BS), i.e. simultaneously search for a buyer and for a new house. The potential interest of this third strategy is obvious: if it manages to realize both transactions at the same time, then it will escape a costly state of "Seller-Only" with two houses or of "Buyer-Only" with no house. Nevertheless, if buying (resp. selling) transaction occurs first, then the household will encounter a "Seller-Only" (resp. "Buyer-Only") spell and bear the subsequent search and/or financial costs.

Following Wheaton (1990), we assume that while a household is mismatched, it can still be hit by a demographic shock. Typically, a couple with a small house, searching for a large house on the market, can decide to separate and then becomes matched again without concluding any transaction. In our setup, "Buyer-First", "Seller-First" or "Buyer-and-Seller" households will therefore be matched again after a demographic shock. On the contrary, households with either two ("Seller-Only") or no ("Buyer-Only") house will not be affected by a demographic shock. SO households will still have to sell one dwelling and BO households to buy one.

[**Insert Figs 1 to 5**]

In the following, we will propose a full description of the first and third strategies only. Due to the perfect symmetry of the first two strategies, our reasoning could be easily extended to the "Seller First" strategy. Figure 2 (resp. 3) provides complete pictures of possible occupational status of a mismatched household entering the housing market as a BF (resp. a BS). The figures are completed by Figure 4 and 5 for households becoming Buyer-Only or Seller-Only. Table 1 complements the description of the environment with a summary of the costs associated with the states on the housing market.

State\Costs	Cost of living in an inconvenient dwelling	Cost of living in a rented dwelling	Search cost to buy	Search cost to sell	Financial costs of holding two dwellings
Buyer Only		×	×		
Seller Only				×	×
Buyer First	×		×		
Buyer Seller	×		×	×	

Table 1. States on the housing market and associated costs

In the following sections, we expose with full details the sequential strategy (the "Wheaton" case) and the simultaneous strategy the ("BS" case) and establish conditions of existence of both kind of equilibria in steady-state. Due to the homogeneity assumption, we can solve independently the two strategies. Indeed, all mismatched households in the model will choose the same strategy, sequential or simultaneous (mixed strategies are ruled out). Moreover, it is straightforward to see that since there are no state variable (no aggregate source of disturbance) in the model, households have no interest in changing their strategies. For example, a mismatched household entering the market as a "Buyer First" at a specific period and which did not succeed in buying a new house in that period (and is not hit by a demographic shock), will remain a BF next period rather than become a BS, since the household problem is not time-dependent. Consequently, in our setup, all households choose the same strategy and keep it until they become matched. Our set of potential strategies is complete. Therefore, for each strategy, we discuss the presence of price dispersion. For range of parameters where both equilibria exist, we proceed to Pareto ranking.

3 The Sequential Strategy Equilibrium

3.1 Dynamics

In a "Buyer-First" equilibrium, each household entering the housing market first decides to buy a new dwelling corresponding to its new family structure and then to sell the old one. Hence there are only three possible states : matched (M), buyer first (BF) and seller only (SO). There is no rental market. Time is discrete. The time sequence is as follows: the BF household first observes if it has a match on the housing market. This occurs with probability q^b. Notice that the only type of sellers on the market are SO households, which have a match on the housing market with probability q^s. Hence transactions always occurs between BF and SO households. Both buyers and sellers decide whether or not to accept the transaction and negotiate the price. Let h_M, h_{BF} and h_{SO} respectively denote the mass of matched households, buyers and sellers (for each family type, singles or couples), with $h = h_M + h_{BF} + h_{SO}$. Let s denote the constant stock of housing of each type (small and large dwellings) in our economy. Following Wheaton (1990), we assume a permanent excess of housing, i.e. $v = s - h > 0$. This is a necessary condition to ensure the equilibrium existence: M and BF households own exactly one house while SO households own two houses. Hence the vacancy rate has to be permanently positive.

The mass of BF households evolves according to

$$h'_{BF} = \left(1 - q^b\right)(1 - \beta)\, h_{BF} + \beta\,(h - h_{BF} - h_{SO}) + q^s \beta h_{SO} \tag{3}$$

The first term of the RHS of equation (3) is the mass of BF households of the last period which did not have a match – which occurs with probability $\left(1 - q^b\right)$ – and did not support a change in their family type – with probability $(1 - \beta)$, i.e. no demographic shock – and consequently remain on the market as BF next period. The second term is the mass of matched households of the last period hit by a demographic shock – with probability β – which become unmatched and enter the market as BF. The last term is the mass of sellers which get a match and sell their old asset – with probability q^s – but are immediately hit by a demographic shock and have to reenter the market as BF.

The dynamics of the mass of SO households obeys

$$h'_{SO} = (1 - q^s)\, h_{SO} + q^b h_{BF} \tag{4}$$

The first term of the RHS of equation (4) is the mass of SO households last period that did not get a match. Since SO households own two houses, they stay in that state even if they change their family types: they still have one home to sell. The second term is the mass of BF which get a match and become a SO. Similarly, their status is not modified by an eventual demographic shock. The dynamics of the mass of matched households is easily deduced from equations (3) and (4). To complete the presentation of the dynamic system of mass of households, we need to introduce an aggregate specification of the rate of meeting of agents on the housing market. We adopt the same strategy as Wheaton (1990) and treat q^b as an exogenous parameter[4] and deduce the value of $q^s = q^b \times (h_{BF}/h_{SO})$.

3.2 Value functions

To define the value functions associated with the states on the housing market, we have to introduce additional parameters. The household discount factor is $0 < \delta < 1$. The per-period flow of utility depends on several factors. Let $\overline{u}$ denote the instantaneous flow of value of living in a convenient home. Moreover, if the household searches (as a buyer or a seller) on the housing market, it pays the cost $\kappa_x > 0$ that depends on his state x for $x = BF,\ SO$. κ_{BF} includes the cost of being unmatched and living in a non suitable home. The value of κ_{BF} also includes search costs for a new house, while κ_S comprises the search costs of a buyer as well as potential financial costs of owning two houses (capital expenditures, mortgage bridge

[4] Notice that the introduction of a Cobb-Douglas matching function: $q^b = q\left(\frac{h_{SO}}{h_B}\right)^{1-\theta}$ and $q^s = q\left(\frac{h_B}{h_{SO}}\right)^{\theta}$ where q and θ are exogenous paremeters is not straightforward and may produce multiple equilibria outcomes.

loans for the time spell between buy date and sell date). If the household makes a transaction, the utility flow $u(p) = bp$ is added ($b > 0$, this flow is positive in the case of a sell and negative in the case of a buy). Since we restrict our study to the steady state, we do not introduce time subscripts. The value function of a matched household is:

$$\mathcal{V}_M = \overline{u} + \delta \left[\beta \mathcal{V}_{BF} + (1 - \beta) \mathcal{V}_M\right] \tag{5}$$

A matched household earns the instantaneous utility from $\overline{u}$ (and pays no additional costs) and get the discounted weighted values of staying matched with probability $(1 - \beta)$ (i.e. no demographic shock) or becoming unmatched and entering the market as a BF with probability β. The value function of a BF is:

$$\mathcal{V}_{BF} = \overline{u} - \kappa_{BF} + q^b u(-p_B) + \delta \left[q^b \mathcal{V}_{SO} + \left(1 - q^b\right) \left(\beta \mathcal{V}_M + (1 - \beta) \mathcal{V}_{BF}\right)\right] \tag{6}$$

The BF household pays the search cost κ_{BF}. With probability q^b, the household matches and then buys its new house at price p_B. In this case, the next period value function is $\delta \mathcal{V}_{SO}$ since the household becomes a SO. Conversely, with probability $\left(1 - q^b\right)$, the BF household does not match and buys no house. Its next period value function is $\delta \mathcal{V}_M$ if it is hit by a demographic shock (and becomes matched) or $\delta \mathcal{V}_{BF}$ otherwise (household stays in its BF state). The value function of a SO household is:

$$\mathcal{V}_{SO} = \overline{u} - \kappa_{S0} + q^s u(p_B) + \delta \left[q^s \left(\beta \mathcal{V}_{BF} + (1 - \beta) \mathcal{V}_M\right) + (1 - q^s) \mathcal{V}_{SO}\right] \tag{7}$$

The SO household pays the search/financial costs κ_S. With probability q^s, the household matches and sell its old house at price p_B. Notice that this price is the same as in equation (6) since the sole buyers on the market are BF households. Next period, the SO becomes matched if its family type does not change or get back to the market as a BF otherwise. With probability $(1 - q^s)$, the SO households does not match and stays in this state next period.

3.3 Bargaining process

Prices are the outcome of a Nash bargaining process between buyers and sellers. Each household assesses its benefit in the case of bargaining success compared with the case of bargaining failure. The unique price p_B is determined according to

$$p_B : \arg\max \left\{ [\mathcal{B}\mathcal{V}_{BF}]^{\gamma} [\mathcal{S}\mathcal{V}_{SO}]^{1-\gamma} \right\} \tag{8}$$

with $\mathcal{B}\mathcal{V}_{BF} = \mathcal{V}_{BF}|_{q^b=1} - \mathcal{V}_{BF}|_{q^b=0}$ and $\mathcal{S}\mathcal{V}_{SO} = \mathcal{V}_{SO}|_{q^s=1} - \mathcal{V}_{SO}|_{q^s=0}$. $\mathcal{B}\mathcal{V}_{BF}$ is the value increase for the buyer in case of a bargaining success and $\mathcal{S}\mathcal{V}_{SO}$ is the value increase for the seller in case of a bargaining success. No equilibrium price exists if one of these two terms is negative, since no transaction could occur. In the following, we will only consider range of parameters where $\mathcal{B}\mathcal{V}_{BF}$ and $\mathcal{S}\mathcal{V}_{SO}$ are positive to keep the bargaining process consistent with the dynamic equations (3) and (4). γ is the parameter governing value surplus sharing. Simple manipulations lead to the following expression for the negotiated price:

$$p_B = \frac{\delta}{b} (1-\gamma) [\mathcal{V}_{SO} - (\beta \mathcal{V}_M + (1-\beta) \mathcal{V}_{BF})] - \frac{\delta}{b} \gamma [\beta \mathcal{V}_{BF} + (1-\beta) \mathcal{V}_M - \mathcal{V}_{SO}] \tag{9}$$

The first term in brackets is the value surplus of a successful match for the buyer. It becomes a SO in case of transaction rather than stays either a BF with probability $(1-\beta)$ or a matched household with probability β in case of no transaction. The second term in brackets is the equivalent value surplus of a conclusive transaction for a seller. Overall, the price p_B is increasing in $\mathcal{V}_{SO}$: for large values of $\mathcal{V}_{SO}$, buyers would accept to pay a higher price to become SO and sellers would request a higher price to leave this state. Conversely, p_B is decreasing in $\mathcal{V}_{BF}$ and $\mathcal{V}_M$, since the higher these value functions the lower the price buyers would be ready to pay for smaller surplus and the lower the price sellers would request for larger surplus. Finally, the price is logically decreasing with the buyer's bargaining power in the negotiation γ. Details of the calculations leading to equation (9) are given in Appendix A.1.

3.4 Equilibrium

The steady-state equilibrium volumes of transactions and masses of households in each state (M, BF, SO) are entirely determined by the structural parameters that govern the demographic and transaction processes, namely β, s, q^b and h. Parameters governing the price bargaining process (γ) or utility flows ($\overline{u}$, b, κ_{BF}, κ_S) do not impact these quantities for range of parameters where all matches lead to a transaction (recall that otherwise the equilibrium is meaningless since no transaction ever occurs).

Definition 1 *The equilibrium price p_B, the value functions $\{\mathcal{V}_M, \mathcal{V}_{BF}, \mathcal{V}_{SO}\}$ and masses $\{h_M, h_{BF}, h_{SO}\}$ solve*

1. *the Nash bargaining outcome (9);*
2. *the definition of the value functions (5)-(6)-(7);*
3. *the dynamic equations (3)-(4) considered at the steady state and the residual equation $h = h_M + h_{BF} + h_{SO}$;*
4. *the following equilibrium existence condition $\mathcal{V}_{BF} > \mathcal{V}_{BS}$ where $\mathcal{V}_{BS}$ is the value function of a (unmatched) household entering the market as a Buyer/Seller rather than a Buyer First. The Nash equilibrium exists if no agent has interest in deviating from the equilibrium decisions and adopting a different strategy.*

   Appendix A.2 gives the complete set of equilibrium equations.

The whole system of equilibrium equations is linear subject to the inequality constraint (condition 4) given in the above definition. Consequently equilibrium existence and uniqueness is straightforward to establish as long as $\mathcal{V}_{BF} > \mathcal{V}_{BS}$. The final equilibrium expression of the value function of a BF is simply

$$\mathcal{V}_{BF} = \frac{-\kappa_{BF}}{(1-\delta)(1+\omega)} + \frac{\overline{u}}{(1-\delta)} \tag{10}$$

with $\omega = \delta \left[\beta + q^b \gamma \left(1 - 2\beta\right)\right] / \left(1 - \delta \left(1 - \beta\right)\right)$. As expected, $\mathcal{V}_{BF}$ is decreasing with κ_{BF}. If $\beta < 1/2$ (households change their family type at an average periodicity above two periods which seems highly plausible at an annual or quarterly frequency), $\mathcal{V}_{BF}$ also increases with γ (the bargaining power of the buyer) and q^b (the probability of a match for a buyer). Nowadays, the larger β the lesser the impact of γ and q^b since the gains of a conclusive match are lower if the probability of becoming matched without any transaction is higher. Finally notice that the direct impact of β on $\mathcal{V}_{BF}$ is ambiguous, since it impacts positively the probability of becoming matched immediately for a BF, but, in case of a conclusive transaction, impacts negatively the value function of the new SO household (which could get back to the market just after selling). Interestingly, notice that $\mathcal{V}_{BF}$ does not depend on κ_S even if BF households may become SO in one period. It appears that price p_B acts as a buffer against the search/financial of SO households. Price increases as κ_S decreases. But this does not impact the value of BF households since (without considering demographic shocks), they will buy and then sell a dwelling at similar price p_B in the future.

To characterize the existence condition $\mathcal{V}_{BF} > \mathcal{V}_{BS}$ in the equilibrium definition, we express the value function of a household deviating from other households on the market and entering the market as a buyer seller:

$$\begin{aligned} \mathcal{V}_{BS} \;=\; & \overline{u} - \kappa_V + q^s b p^1_{BF} - q^b b p^2_{BF} + \delta \Big[q^s q^b \left(\beta \mathcal{V}_{BS} + \left(1 - \beta\right) \mathcal{V}_M\right) + q^b \left(1 - q^s\right) \mathcal{V}_{SO} \\ & + q^s \left(1 - q^b\right) \mathcal{V}_{BO} + \left(1 - q^s\right) \left(1 - q^b\right) \left(\beta \mathcal{V}_M + \left(1 - \beta\right) \mathcal{V}_{BS}\right) \Big] \end{aligned} \tag{11}$$

where $\overline{u} - \kappa_V$ is the instantaneous utility flow and κ_V is the cumulative search costs of buying and selling. The probability of a match with a buyer (resp. a seller) is q^s (resp. q^b). In case of a sell, the household gets price p^1_{BF} and in case of a buy, it pays price p^2_{BF}. These prices are specific to the deviating household and will be determined later[5]. If the deviating household succeeds – with probability $q^s q^b$ – in both selling and buying in the current period, it becomes a matched household (if no change in family type occurs) or

[5] Since the deviating household is atomistic, its presence on the housing market is neglected by the other households.

get back to the market (if a demographic shock occurs). If it succeeds only in buying a new dwelling – with probability $q^b(1-q^s)$ – it becomes a seller-only whose value function is given by equation (7). At this point, the deviating household becomes identical to all other SO households and will sell its dwelling at the price p_B given by (9) in case it gets a match. If it succeeds only in selling a new dwelling – with probability $q^s(1-q^b)$ – it becomes a buyer only[6]. Finally, if it does not succeed in buying nor selling, it stays on the market if its family type does not change. The value function of the deviating household if it becomes a Buyer-Only (BO) is

$$\mathcal{V}_{BO} = \overline{u} - \kappa_B - q^b b p_{BF}^3 + \delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M\right) + \left(1-q^b\right)\mathcal{V}_{BO}\right] \tag{12}$$

The BO pays the same search costs as a BF but also bears the costs of being homeless (or on the rental market with lesser quality housing). Hence, κ_B is intuitively higher than κ_{BF}. The BO buys a new house with probability q^b and pays price p_{BF}^3 (which is a priori different from p_{BF}^2 the price paid by a BS). In this case, it becomes matched (if no demographic change occurs). Otherwise, it stays in the BO state. To complete the problem of the deviating household, we have to determine the Nash bargaining processes governing prices p_{BF}^1 (match between a BF and a BS), p_{BF}^2 (match between BS and a SO) and p_{BF}^3 (match between a BO and a SO). The value surpluses of these three bargainings are determined in a way similar to equation (8) and we keep parameter γ as the share of the buyer in the negotiation process. From the whole resolution of these price negotiations provided in Appendix A.3, we can prove that $\mathcal{V}_{BS}$ is strictly decreasing with κ_V. Therefore, since $\mathcal{V}_{BF}$ is independent on κ_V, we can define $\underline{\kappa_V}$ such that the single equilibrium exists (e.g. $\mathcal{V}_{BF} > \mathcal{V}_{BS}$) if $\kappa_V > \underline{\kappa_V}$.

Proposition 1 *A necessary and sufficient for the existence of the Single Equilibrium is* $\kappa_V > \underline{\kappa_V}$. *See Appendix A.4 for the explicit form of* $\underline{\kappa_V}$.

[6] The deviating household is the sole buyer only of the economy.

The above proposition is a re-expression of existence condition 4 in the equilibrium definition. It proves that we can always find a range of parameters (i.e. a sufficiently large value for κ_V) ensuring the BF equilibrium existence (recall that the system is linear). For given values of κ_{BF} and κ_S, the larger κ_V the lower the incentive for agents to deviate from the equilibrium strategy (which does not depend on κ_V).

4 The Simultaneous Strategy Equilibrium

4.1 Dynamics

In the simultaneous strategy equilibrium, each mismatched household enters the housing market as a Buyer and a Seller (BS), i.e. decides to search simultaneously for a new dwelling and for a buyer for its old dwelling. If the mismatched household managed to sell its house but still not to buy the new one, it becomes a Buyer-Only (BO), i.e a homeless (or a renter) seeking to buy a new asset. Conversely, if the mismatched household has already bought its new house but still not sold the old one, it becomes a Seller-Only (SO), i.e. it owns two homes and still tries to sell one on the housing market. Hence, there are four possible states in the simultaneous strategy equilibrium: matched (M), buyer-and-seller (BS), buyer only (BO) and seller only (SO). Let h_M, h_{BS}, h_{BO} and h_{SO} respectively denote the mass of matched households, BS, BO and SO (for each family type, singles or couples), with $h = h_M + h_{BF} + h_{BO} + h_{SO}$.

The time sequence is as follows: the BS household observes if it has a match with a seller (either a BS or a SO) which occurs with probability q^b and if it has a match with a buyer (either a BS or a BO) which occurs with probability q^s. Similarly, BO – respectively SO – households observe if they have a match with a seller (either a BS or a SO) – respectively with a buyer (either a BS or a BO) – which occurs with probability q^b – respectively q^s. Notice that the matching probabilities are the same for BS, BO and SO since we suppose the search process is blind, i.e. agents cannot seek for a specific status of their counterpart. After observing if they have a match (or two matches for a BS) or not, both buyers and sellers decide whether or not to accept the transaction and negotiate the price. We solve the dynamics of the population under the hypothesis that each household which get a match on the housing market, accepts the transaction (whatever the type of the household it has matched). As will be later explained, this assumption will be *ex post* checked when solving the equilibrium.

The next period mass of BS household is given by

$$\begin{aligned} h'_{BS} &= \beta \times (h - h_{BS} - h_{BO} - h_{SO}) + \beta \times \left(q^b q^s h_{BS}\right) + (1-\beta) \times (1-q^s) \left(1-q^b\right) h_{BS} \\ &+ \beta \times \left(q^b h_{BO} + q^s h_{SO}\right) \end{aligned} \tag{13}$$

The first term is the mass of matched households (h_M) which experience a demographic shock (with probability β). The second term is the mass of BS households (h_{BS}) which succeed in buying and selling its houses during the period (with probability $q^b q^s$), but are afterwards hit by the demographic shock (with probability β). Consequently, they have to return on the housing market as BS households. The third term is the mass BS households (h_{BS}) who neither sell nor buy during the period (with probability $(1-q^s)\left(1-q^b\right)$) and do not experience a demographic shock (with probability $(1-\beta)$). The four term is the sum of the masses of BO and of SO ($h_{BO} + h_{SO}$) which succeed in buying and selling, respectively with probability q^b and q^s, but experience a demographic shock (with probability β). Consequently, they have to return on the housing market as BS households.

The dynamic equations for masses of SO and BO households are simpler than for the BS households because their objectives do not change after a demographic shock. Whatever its demographic type, a household with two homes (a SO) wants to sell one these assets (possibly the last one it bought if it recently experienced a demographic shock) and a household without a house (a BO) wants to buy a new one. A demographic shock changes the home type needed, but not the state of the household on the housing market. Consequently, the next period mass of SO households is simply given by

$$h'_{SO} = (1-q^s)\, h_{SO} + q^b (1-q^s)\, h_{BS} \tag{14}$$

that is the mass of SO households which have not sold (with probability $(1-q^s)$) plus the mass of BS households which have bought but not sold (with probability $q^b(1-q^s)$). This equation is almost equivalent to equation (4) in the single equilibrium, except that BF households are replaced by BS households. Similarly, the next period mass of BO households is given by

$$h'_{BO} = \left(1-q^b\right) h_{BO} + q^s \left(1-q^b\right) h_{BS} \tag{15}$$

that is the mass of BO households which has not bought (with probability $\left(1-q^b\right)$) plus the mass of BS households which has sold but not bought (with probability $q^s\left(1-q^b\right)$).

4.2 Matches and prices

Table 2 summarizes the type of matches in the economy.

Type of matches	The buyer	The seller	The price
1	Buyer and Seller	Buyer and Seller	p_1
2	Buyer Only	Seller Only	p_2
3	Buyer and Seller	Seller Only	p_3
4	Buyer Only	Buyer and Seller	p_4

Table 2. The type of matches with the simultaneous search strategy

The four states in the housing market give rise to potentially four different transaction prices. In this setup, we limit ourselves to consider only one price outcome for each kind of match. This means that a match between two BS households for example always conduct to a conclusive transaction at unique price p_1 whatever their situation on the housing market *regarding their other match*. When a BS acting as a buyer meet a BS acting as a seller, we suppose that whether the first one has a match to sell its asset or not (if yes, with a BS or a BO household) and the second one a match to buy or not (if yes with a BS or a SO) has no influence on their bargaining. This "limited information" assumption keeps the number of equilibrium prices limited to four since agents engaged in a bargaining only use the information regarding this sole negotiation. We leave open for future research the "full information" case where agents starting a bargaining process know if they have a match besides and the status of the other household they met[7],[8].

This price dispersion occurs without relying on any source of exogenous disturbance: all mismatched households enter the market as perfectly identical BS. Their status may have changed over time (they become matched, SO or BO), but this is due to the functioning of the market. As we said, the only disturbance (i.e. the demographic shock) only forces agents to enter the market but does not create heterogeneity in *mismatched* agent per se. Had we suppressed demographic disturbances for households on the market, the price dispersion would not have disappeared.

4.3 Value functions

We define the value functions associated with each state on the housing market: $\mathcal{V}_M$ for a matched household, $\mathcal{V}_{BS}$ for a Buyer-and-Seller household, $\mathcal{V}_{BO}$ for a Buyer-Only and $\mathcal{V}_{SO}$ for a Seller-Only. Compared to the preceding subsection, the whole set of parameters remains unchanged.

The value function of matched households is close to its single equilibrium counterpart

$$\mathcal{V}_M = \overline{u} + \delta \left[\beta \mathcal{V}_{BS} + (1-\beta) \mathcal{V}_M \right] \tag{16}$$

With probability β the current home becomes non suitable and the household enters the market as a BS. Otherwise, the household remains matched with its convenient dwelling. The value function of a BS

[7] Moreover, simple algebra suggests that besides multiplying the number of prices, the "full information" case may drive into multiple equilibria outcome while our "limited information" assumption conducts to linear steady state equilibria as will be seen later.

[8] The full information hypothesis could induce some agents to reject a transaction because they did not find a counterpart on the "other side" of the market. Such a setup would be linked to the "housing chains" literature (see Rosenthal, 1997) where Agent A cannot buy the property of agent B if the latter cannot buy its new house to a third agent, etc.

household is

$$\mathcal{V}_{BS} = \overline{u} - \kappa_V + U(p_1,p_3,p_4) + \delta\left[q^s q^b(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M) + q^b(1-q^s)\mathcal{V}_{SO} + q^s\left(1-q^b\right)\mathcal{V}_{BO} + (1-q^s)\left(1-q^b\right)(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS})\right] \quad (17)$$

Its per period utility flow is $\overline{u}$, less the search cost κ_V, plus the expected utility associated with the transaction prices $U(p_1,p_3,p_4)$, defined below. Four future issues are feasible at the next period. If the household manages to buy and sell in the period (with probability $q^s q^b$), it either remains a BS in the case of demographic change (and gets $\delta\mathcal{V}_{BS}$ with probability β), or becomes matched (and gets $\delta\mathcal{V}_M$ with probability $(1-\beta)$) in the next period. If the household does not buy nor sell (with probability $(1-q^s)\left(1-q^b\right)$), it either remains BS in the absence of demographic change (then gets $\delta\mathcal{V}_{BS}$ with probability $(1-\beta)$), or becomes matched (then gets $\delta\mathcal{V}_{BS}$ with probability β). If the household buys its new home without selling his old one (with probability $q^b(1-q^s)$), it gets $\delta\mathcal{V}_{SO}$, the expected value of being SO and if it sells its old home without buying (with probability $q^s\left(1-q^b\right)$), it gets $\delta\mathcal{V}_{BO}$, the expected value of being BO.

The utility function associated with prices is based on the same structure of event probabilities. We introduce two additional variables

$$\theta^b = h_{BS}/(h_{BS}+h_{BO})\,;\ \theta^s = h_{BS}/(h_{BS}+h_{SO}) \quad (18)$$

θ^b (respectively θ^s) is the share of BS households among the mass of buyers (respectively sellers) households. These ratio are taken as the probabilities for a seller and a buyer, if matched, of being matched with a BS household. The utility function associated with prices is then defined as follows

$$\begin{aligned} & U(p_1,p_3,p_4) \quad (19) \\ = \; & q^s q^b\left[\theta^b\theta^s u(0) + \theta^s\left(1-\theta^b\right)u(p_4-p_1) + (1-\theta^s)\theta^b u(p_1-p_3) + \left(1-\theta^b\right)(1-\theta^s)u(p_4-p_3)\right] \\ & + q^s\left(1-q^b\right)\left[\theta^b u(p_1) + \left(1-\theta^b\right)u(p_4)\right] + q^b(1-q^s)\left[\theta^s u(-p_1) + (1-\theta^s)u(-p_3)\right] + \left(1-q^b\right)(1-q^s)u(0) \end{aligned}$$

If the household succeeds in matching twice (as a buyer and a seller, with probability $q^s q^b$) four matches are feasible with different payoffs. If it matches twice with a BS (with probability $\theta^b \theta^s$), the prices of its sale and its purchase are the same and the household gets $u(p_1 - p_1) = u(0)$, where p_1 is price negotiated by a pair of BS. With probability $\theta^s \left(1-\theta^b\right)$, the seller is a BS (the agent pays p_1) and the buyer a BO (the agent earns p_4). With probability $(1-\theta^s)\,\theta^b$, the seller is a SO (the agent pays p_3) and the buyer is a BS (the agent earns p_1). If it matches with a BO and a SO, with probability $\left(1-\theta^b\right)(1-\theta^s)$, the agent pays p_3 and earns p_4. If the household succeeds only in selling (with probability $q^s\left(1-q^b\right)$), it earns p_1 in the case of a match with a BS household (with probability θ^b) and p_4 in the case of a match with a BO household (with probability $\left(1-\theta^b\right)$). If the household succeeds only in buying (with probability $q^b(1-q^s)$), it pays p_1 in the case of a match with a BS household (with probability θ^s) and p_3 in the case of a match with a SO household (with probability $(1-\theta^s)$). Finally, with probability $\left(1-q^b\right)(1-q^s)$ the household does not buy nor sell.

The value function of a BO is

$$\mathcal{V}_{BO} = \overline{u} - \kappa_B + q^b\left[\theta^s u(-p_4) + (1-\theta^s)\,u(-p_2)\right] + \delta\left[q^b\left(\beta \mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M\right) + \left(1-q^b\right)\mathcal{V}_{BO}\right] \tag{20}$$

The search cost of buying for BO households is κ_B. With probability q^b, the household matches and then buys its new house at the price p_4 if it is matched with a BS (with probability θ^s) or the price p_2 if it is matched with a SO (with probability $(1-\theta^s)$). In this case, the next period value of function is $\delta\mathcal{V}_M$ if no change in family type occurs (with probability $(1-\beta)$) and $\delta\mathcal{V}_{BS}$ if it occurs (with probability β). If the household does not buy during the period, with probability $\left(1-q^b\right)$, it remains a BO in the next period whatever its family type.

The value function of a SO is

$$\mathcal{V}_{SO} = \overline{u} - \kappa_S + q^s\left[\theta^b u(p_3) + \left(1-\theta^b\right) u(p_2)\right] + \delta\left[q^s\left(\beta \mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M\right) + (1-q^s)\,\mathcal{V}_{SO}\right] \tag{21}$$

The search cost of selling for households which own two houses is κ_S. With probability q^s, the household matches and then sells its house at price p_3 if it is matched with a BS (with probability θ^b) or at price p_2 if it is matched with a BO (with probability $\left(1-\theta^b\right)$). In this case, the next period value of function is $\delta\mathcal{V}_M$, if the household does not change of family type (with probability $(1-\beta)$) and $\delta\mathcal{V}_{BS}$ if it does (with probability β). If the household does not sell its home during the period, with probability $(1-q^s)$, it remains a SO at the next period whatever the realization of the demographic shock.

4.4 Bargaining process

Prices are the outcome of a Nash bargaining process between the two households. Each household assesses its benefits in the case of bargaining success compared to the case of bargaining failure. Since this benefit varies with the household's state, we have to define and to solve all the feasible matches on the housing market. We begin with the case of two BS households. The price p_1 is the outcome of

$$p_1 : \arg\max \left\{ \left[\mathcal{B}_{BS}\mathcal{V}_{BS}\right]^{\gamma} \left[\mathcal{S}_{BS}\mathcal{V}_{BS}\right]^{1-\gamma} \right\}$$

where $\mathcal{B}_{BS}\mathcal{V}_{BS} = \mathcal{V}_{BS}|_{q^b=1,\theta^s=1} - \mathcal{V}_{BS}|_{q^b=0}$ is the increase in value for a BS to buy to a BS and $\mathcal{S}_{BS}\mathcal{V}_{BS} = \mathcal{V}_{BS}|_{q^s=1,\theta^b=1} - \mathcal{V}_{BS}|_{q^s=0}$ is the increase in value for a BS to sell to a BS. The equilibrium housing price solution of the Nash bargaining process p_1 satisfies

$$\begin{aligned} bp_1 = {} & (1-\gamma)\,\delta\left[q^s\left(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M\right) + (1-q^s)\,\mathcal{V}_{SO} - q^s\mathcal{V}_{BO} - (1-q^s)\left(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BS}\right)\right] \\ & -\gamma\delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M\right) + \left(1-q^b\right)\mathcal{V}_{BO} - q^b\mathcal{V}_{SO} - \left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BS}\right)\right] \end{aligned} \tag{22}$$

Obviously, the price increases with the surplus in value of the buyer (first term in brackets) and decreases with the surplus in value of the seller (second term in brackets). Consequently, p_1 is increasing with $\mathcal{V}_{SO}$ and decreasing with $\mathcal{V}_{BO}$: the higher $\mathcal{V}_{SO}$, the higher the gain of a buy for a BS (since it could become a SO with probability $(1-q^s)$) and the lower the gain of a sell for a BS (since it escapes a SO state). Similarly, the higher $\mathcal{V}_{BO}$, the lower the gain of a buy for a BS (it can no longer become a BO) and the higher the gain of a sell for a BS (it can become a BO with probability $\left(1-q^b\right)$). The effects of $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ on p_1 are ambiguous and depend on the bargaining power of each agent. They have a positive effects on surplus of buyers and sellers because a conclusive match increase the probability of reaching these states. This impact is lowered by the fact that BS households could become matched (or stay BS) without transacting and then lose this opportunity once they become matched. As will be seen below, these effects of $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ drop out when $\gamma = 1/2$ (symmetric Nash bargaining) and price p_1 only depends on $\mathcal{V}_{SO}$ and $\mathcal{V}_{BO}$.

Price p_2 is the outcome of bargaining process between a BO and a SO households

$$p_2 : \arg\max \left\{ \left[\mathcal{B}_{SO}\mathcal{V}_{BO}\right]^{\gamma} \left[\mathcal{S}_{BO}\mathcal{V}_{SO}\right]^{1-\gamma} \right\}$$

where $\mathcal{B}_{SO}\mathcal{V}_{BO} = \mathcal{V}_{BO}|_{q^b=1,\theta^s=0} - \mathcal{V}_{BO}|_{q^b=0}$ is the increase in value for a BO to buy to a SO and $\mathcal{S}_{BO}\mathcal{V}_{SO} = \mathcal{V}_{SO}|_{q^s=1,\theta^b=0} - \mathcal{V}_{SO}|_{q^s=0}$ is the increase in value for a SO to sell to a BO. We impose the same bargaining power parameter γ than for the negotiation between two BS. The equilibrium housing price solution of the Nash bargaining process p_2 satisfies

$$p_2 = (1-2\gamma)\frac{\delta}{b}\left[\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M\right] + \gamma\frac{\delta}{b}\left[\mathcal{V}_{SO} - \left(\frac{1-\gamma}{\gamma}\right)\mathcal{V}_{BO}\right] \tag{23}$$

A similar argument as for price p_1 shows that p_2 increases with $\mathcal{V}_{SO}$ and decreases with $\mathcal{V}_{BO}$. The direct impact of $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ on gains in value is positive for both BO and SO since a conclusive match increase their probability of reaching these states. Consequently the total effect depends on the respective bargaining power of the BO and the SO : positive for $\gamma < 1/2$ (higher seller bargaining power) and negative for $\gamma > 1/2$. Once again, their impact is null in case of a symmetric Nash bargaining.

The price p_3 is the outcome of bargaining process between a SO and a BS households

$$p_3 : \arg\max \left\{ \left[\mathcal{B}_{SO}\mathcal{V}_{BS}\right]^{\gamma} \left[\mathcal{S}_{BS}\mathcal{V}_{SO}\right]^{1-\gamma} \right\}$$

The increase in value of selling to a BS for a SO is defined by $\mathcal{S}_{BS}\mathcal{V}_{SO} = \mathcal{V}_{SO}|_{q^s=1,\theta^b=1} - \mathcal{V}_{SO}|_{q^s=0}$ and the increase in value of buying to a SO for a BS is given by $\mathcal{B}_{SO}\mathcal{V}_{BS} = \mathcal{V}_{BS}|_{q^b=1,\theta^s=0} - \mathcal{V}_{BS}|_{q^b=0}$. The equilibrium housing price solution of the Nash bargaining process p_3 satisfies

$$\begin{aligned} bp_3 &= (1-\gamma)\,\delta\left[q^s\left(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M\right) + (1-q^s)\,\mathcal{V}_{SO} - q^s\mathcal{V}_{BO} - (1-q^s)\left(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BS}\right)\right] \\ &-\gamma\delta\left[\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right] \end{aligned} \tag{24}$$

The first term in brackets is the same as in equation (22). The second term in brackets in the surplus in value of a match for a SO. It increases with $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ since the SO is ensured to be in one of these two states if it succeeds in selling and (compared to a BS) is not concerned with these two states if it does not sell. The surplus of a SO decreases with $\mathcal{V}_{SO}$ because it will leave this state in case of a conclusive match. In contrast with a BS, the SO is not directly concerned with $\mathcal{V}_{BO}$ because it cannot reach this state in one period. Consequently, the positive effect of $\mathcal{V}_{SO}$ is stronger on p_3 than on p_1 and the negative effect of $\mathcal{V}_{BO}$ is smaller on p_3 than on p_1. Such a differential explains the gap between p_1 and p_3 in equilibrium and generates price dispersion.

The price p_4 is the outcome of bargaining process between a BO and a BS households

$$p_4 : \arg\max \left\{ \left[\mathcal{B}_{BS}\mathcal{V}_{BO}\right]^{\gamma} \left[\mathcal{S}_{BO}\mathcal{V}_{BS}\right]^{1-\gamma} \right\}$$

The increase in value of buying to a BS for a BO is defined by $\mathcal{B}_{BS}\mathcal{V}_{B0} = \mathcal{V}_{BO}|_{q^b=1,\theta^s=1} - \mathcal{V}_{BO}|_{q^b=0}$ and the increase in value of buying to a SO for a BS is given by $\mathcal{S}_{BO}\mathcal{V}_{BS} = \mathcal{V}_{BS}|_{q^s=1,\theta^b=0} - \mathcal{V}_{BS}|_{q^s=0}$. The equilibrium housing price solution of the Nash bargaining process p_4 satisfies

$$bp_4 = (1-\gamma)\,\delta\left[\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{BO}\right] \tag{25}$$
$$-\gamma\delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right) - \left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BS} - \mathcal{V}_{BO}\right)\right]$$

The first term in brackets is the surplus in value of a match for a BO. The first term in brackets is the same as in equation (22). We use the reverse argument as for price p_3. The positive effect of $\mathcal{V}_{SO}$ is stronger on p_1 than on p_4 and the negative effect of $\mathcal{V}_{BO}$ is smaller on p_1 than on p_4. This explains the gap between p_1 and p_4 in equilibrium. It now clearly appears that price dispersion in our setup is directly related to the probability of reaching the states BO and SO. Joining equations (24) and (25), we expect p_1 to lie between p_3 and p_4, the ordering depending on the relative values of $\mathcal{V}_{SO}$ and $\mathcal{V}_{BO}$.

4.5 Equilibrium

Lemma 1 *The demographic structure and the transaction process imply the equality of (i) the matching probabilities of buyers and sellers:* $q^b = q^s = q$*, and of (ii) the proportions of buyer-and-seller households in the masses of buyers and of sellers:* $\theta^b = \theta^s = \theta$*, where* θ *is a function of the structural parameters* q *and* β*.*

Proof. *See appendix B.1.* ■

The equilibrium quantity of transactions and the mass of households are entirely determined by the structural parameters that govern the demographic and transaction processes, namely q, β, and h. The equilibrium prices do not influence these quantities. This property of our model proceeds from the assumption that all matches lead to a transaction and that home supply is fixed. Such a property would be unsatisfactory if we were concerned with the size of the housing market, rather than price dispersion. We leave open for future works the issue of the influence of price dispersion on the equilibrium quantities on the housing market.

Definition 2 *The steady state equilibrium price set* $\{p_i\}_{i=1}^4$ *and value functions* $\{\mathcal{V}_{SO}, \mathcal{V}_{BO}, \mathcal{V}_{BS}\}$ *solve*

1. *the four Nash bargaining outcomes (22)-(23)-(24)-(25);*

2. *the definition of the value functions (17)-(20)-(21);*

3. *the definition of* q *and* θ *introduced in Lemma 1;*

4. a*ll value surpluses from a match* $\mathcal{B}_{BS}\mathcal{V}_{BS}, \mathcal{S}_{BS}\mathcal{V}_{BS}, \mathcal{B}_{SO}\mathcal{V}_{BO}, \mathcal{S}_{BO}\mathcal{V}_{SO}, \mathcal{B}_{SO}\mathcal{V}_{BS}, \mathcal{S}_{BS}\mathcal{V}_{SO}, \mathcal{B}_{BS}\mathcal{V}_{BO}$, *and* $\mathcal{S}_{BO}\mathcal{V}_{BS}$ *are positive.*

5. *the existence condition :* $\max\{\mathcal{V}_{BF}, \mathcal{V}_{SF}\} < \mathcal{V}_V$. *The equilibrium exists if no agent has interest in deviating from the equilibrium decisions. For a matched household hit by a demographic shock, to deviate means enter on the housing market as a buyer first (and get* $\mathcal{V}_{BF}$*) or a seller first (and get* $\mathcal{V}_{SF}$*) rather than as a buyer-seller.*

   Appendix B.2 gives the complete set of equilibrium equations.

The point 4 of the equilibrium's definition guarantees that all negotiations conduct to a transaction. If one of the value surplus terms is negative, at least one of the four type of transactions never occurs which reduces the number of equilibrium prices. Consequently, the dynamics of masses of households would be modified and so the proportion of BS households among buyers (θ^b) or sellers (θ^s). In the next section, the positivity of value gains will be numerically checked. We do not put much more focus since as will be seen below, cases where one of the value surpluses are negative are quite scarce.

The point 5 of the equilibrium's definition 2 guarantees that all matched households hit by a demographic shock have no interest to enter the housing market with a different strategy: they should enter as a buyer-and-seller for the simultaneous equilibrium to exist. The appendix B.4 gives the complete expression of the payoffs $\{\mathcal{V}_{BF}, \mathcal{V}_{SF}\}$, which computation is quite complex since it requires to solve the Nash Bargaining process for matches with a single agent deviating from the equilibrium strategy. The condition of existence will be checked in all further equilibria comparisons and numerical experiments. As in the Sequential Equilibrium case, the steady state equilibrium is linear subject to conditions 4 and 5.

4.6 Price dispersion

We first characterize the price dispersion for symmetric bargaining power ($\gamma = 1/2$) and then for the asymmetric case ($\gamma \neq 1/2$).

Proposition 2 *For symmetric Nash bargaining programs, the equilibrium prices satisfy*

$$\frac{2b}{\delta}p_1 = \frac{2b}{\delta}p_2 = \mathcal{V}_{SO} - \mathcal{V}_{BO} = \frac{\kappa_B - \kappa_S}{1-\delta} \tag{26}$$

$$\frac{2b}{\delta}p_3 = -\left(1-q\right)\left(\mathcal{V}_{BS} + \mathcal{V}_M\right) - q\mathcal{V}_{BO} + \left(2-q\right)\mathcal{V}_{SO} \tag{27}$$

$$\frac{2b}{\delta}p_4 = \left(1-q\right)\left(\mathcal{V}_{BS} + \mathcal{V}_M\right) + q\left(\mathcal{V}_{SO} + \mathcal{V}_{BO}\right) - 2\mathcal{V}_{BO} \tag{28}$$

Corollary 1 *For symmetric Nash bargaining programs, there are three distinct prices on the housing market. The price p_1 is equal to p_2 and to the mean of prices p_3 and p_4 :*

$$p_1 = p_2 = \left(p_3 + p_4\right)/2 \tag{29}$$

Prices p_1 and p_2 are linear and increasing with of the differential in value functions for a SO household and for a BO household. If these value functions are equal, the equilibrium price is null. In this case, the buyer and the seller have the same expected value for the next period and then the same threat point in the bargaining process. Prices are positive if the value of being SO exceeds the value of BO, therefore prices are positive if homelessness is the worse state, i.e. $\kappa_B - \kappa_S > 0$. The link between these value functions and the prices proceeds from a compensation mechanism between the utility drawn from price and the utility associated with the expected change of situation on the housing market. The household accepts the transaction if the payoff is high enough. All things being equal, a damaging in future expected situation for the seller has to be compensated by a higher price in the transaction to ensure his acceptation.

To analyze the determinants of these prices, we first comment the match composed of SO and BO. The two households face the same probability of being BS or matched at the next period. With identical power in the bargaining process ($\gamma = 1/2$), the associated effects of value functions ($\mathcal{V}_{BS}$ and $\mathcal{V}_M$) cancel and the price depends only on the difference between $\mathcal{V}_{SO}$ and $\mathcal{V}_{BO}$. The higher the value of being seller only, the higher the negotiated price. Reciprocally, the lower the value of being buyer only, the higher the negotiated price. The negotiated price ensures the consent of agents to buy or to sell. If the value of being a SO is high, a high price is needed to obtain the seller's consent. If the value of being a BO is low, the buyer is ready to accept a high price.

Another interesting result of our model is that the price is the same for a match composed of two BS households than for match composed of one BO and one SO. As for the match of type 1, the equality of the bargaining power of agents cancels out the effects of value functions associated with the states of BS and matched. The two agents are in a symmetric position for these states. They face the same probability of being BS or matched in the future. However, they are in asymmetric position with regards to the state of SO and BO. The BS which sells its house avoids the risk of owning two houses in the next period, but exposes more heavily to the risk of being a homeless. On the contrary, the BS which buys its house avoid the risk of being a homeless, but exposes himself to the risk of being a two houses owner. Consequently, a rise in the value of being BO (or a fall in the value of SO) increases the utility associated with the expected change of situation for the seller which then accepts a lower price.

Finally, a pair of BS households behaves similarly to a pair of BO and SO households even if they have radically different perspectives on the housing market (except if we consider asymmetric bargaining power as shown below). If we restrict our analysis to these types of matches, allowing for buyer-and-seller strategy does not improve our understanding of price dispersion on the housing market. The conclusion is radically different when we consider the matches composed of a BS household with either a BO or a SO household. These matches lead to different equilibrium prices, which depend on all the four value functions.

Let us first comment the price p_3, which is increasing with the value function of SO and decreasing with the value functions of BS, BO, and matched. For the SO household, selling its house solves its problem on the housing market. At the end of the period, it is matched with a convenient home (but is exposed to demographic risk). Utilities $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ are discounted by $\beta\delta$ and $(1-\beta)\,\delta$, respectively. The BS household is in a more complex situation. First, its probability to solve its housing problem during this period is lower than for a SO (or a BO) household. It depends on the probability q of selling its house. Second, if it does not sell its house, buying the house prevents from benefiting from a reversal of demographic situation that would solve its housing problem without buying nor selling. Utilities $\mathcal{V}_{BS}$ and $\mathcal{V}_M$ are discounted by $[\beta q-(1-q)(1-\beta)]\,\delta=[\beta-(1-q)]\,\delta<\beta\delta$ and $[(1-\beta)\,q-(1-q)\,\beta]\,\delta=(q-\beta)\,\delta<(1-\beta)\,\delta$, respectively. Consequently, an increase in $\mathcal{V}_{BS}$ or $\mathcal{V}_M$ rises the gains of a conclusive match for the SO more heavily than the gains of the BS. The seller accepts to sell at a lower price as asked by the buyer. The effects of values $\mathcal{V}_{SO}$ and $\mathcal{V}_{BO}$ are direct. By selling, the SO household looses value $\mathcal{V}_{SO}$ whereas buying the house exposes the BS to the risk of being SO (in case it does not sell during the period). An increase in $\mathcal{V}_{SO}$ diminishes the payoff for the SO and increases the payoff for the BS, which implies a rise in the price p_3. Finally, the utility $\mathcal{V}_{BO}$ impacts only the payoff of the BS. An increase in $\mathcal{V}_{BO}$ increases the payoff for the BS. Since the BS is acting a a buyer, this lowers the sale price p_3. A similar argument applies for price p_4.

Corollary 2 *For symmetric Nash bargaining programs, the amount of price dispersion, i.e.* p_4-p_1*, on the housing market is*

$$p_4-p_1=\frac{\delta}{2b}\,(1-q)\,[(\mathcal{V}_{BS}+\mathcal{V}_M)-(\mathcal{V}_{SO}+\mathcal{V}_{BO})] \tag{30}$$

The RHS of equation (30) is the difference in (future) value gains to buy for a BO compared to the gains for BS. If transaction occurs, the BO is ensured not to become a SO next period (whereas such a risk is supported by a BS seeking to buy) and is ensured to leave its current BO state (while this risk is anyway null for a BS). Hence, the lower $\mathcal{V}_{BO}$ or $\mathcal{V}_{SO}$, the larger the gains of a match for a BO compared to those of a BS. The reverse is true from a BS point of view. Moreover, the lower $\mathcal{V}_{BS}$ or $\mathcal{V}_{M}$, the larger the gains of a match for a BS compared to those of a BO, since the latter encounters a higher probability being in these states next period than the former if a (buying) transaction occurs. When $\mathcal{V}_{SO} + \mathcal{V}_{BO}$ is low (or $\mathcal{V}_{BS} + \mathcal{V}_{M}$ is large) the gains of conclusive match with a BS are higher for a BO than for a BS. Hence, the BO is ready to accept a high price (p_4) to ensure the transaction occurs and $p_4 - p_1$ is strictly positive.

To end the study of price dispersion, it is worth mentioning that for asymmetric Nash Bargaining a fourth price appears in the economy as shown in the following proposition.

Proposition 3 *For asymmetric Nash bargaining programs, the equilibrium prices satisfy*

$$p_2 - p_1 = \frac{\delta\,(1-2\gamma)}{b}\,(1-q)\,[(\mathcal{V}_{BS} + \mathcal{V}_{M}) - (\mathcal{V}_{SO} + \mathcal{V}_{BO})] \tag{31}$$

Interestingly the sign of $p_2 - p_1$ depends on the sign of $(\mathcal{V}_{BS} + \mathcal{V}_{M}) - (\mathcal{V}_{SO} + \mathcal{V}_{BO})$ which also determines the amount of price dispersion $(p_4 - p_1)$ in the symmetric case. When $\mathcal{V}_{SO} + \mathcal{V}_{BO}$ is low ($\mathcal{V}_{BS} + \mathcal{V}_{M}$ is large) the gains of a conclusive match are higher for a BO than for a BS (acting as a buyer) as already explained. But similarly, these gains are also higher for a SO than for a BS (acting as a seller), since the former encounters a higher probability becoming a BS or a matched household if the transaction is concluded. Consequently, if the respective bargaining powers of BO and SO households, which determine price p_2, are the same (i.e. $\gamma = 1/2$), both surplus in value compared to BS-BS transactions cancel out and $p_2 = p_1$. Evidently, for large (resp. low) values of bargaining power of buyers i.e. when γ is above (resp. below) 1/2, the BO household can extract a larger (resp. lower) part of this gain in value than a SO household and price decreases $p_2 < p_1$. The opposite argument applies when $(\mathcal{V}_{SO} + \mathcal{V}_{BO})$ is high and above $(\mathcal{V}_{BS} + \mathcal{V}_{M})$.

5 Numerical analysis

We proceed to numerical experiments to determine the range of parameters for which the sequential strategy equilibrium, the simultaneous strategy equilibrium or both of them exist. When multiple equilibria occur, they are ranked with respect to the steady-state welfare of matched households. Then, we discuss the existence and the amount of price dispersion.

5.1 Calibration

We suppose time is annual. In this numerical analysis, some parameters will not be allowed to vary: we impose $\delta = 0.95$ which suggests a 5% annual discount rate. Without loss of generality, the population size h is equal to one. The utility flow of being matched and of being paid for a home are scale parameters: we impose $\overline{u} = 1$ and $b = 1$. Moreover, we try to reduce the disparities between the two equilibria by assuming similar matching probabilities. In the simultaneous strategy equilibrium, we impose $q = 0.7$ (this value will be allowed to vary) which means that the average duration before getting matched is $1/0.7 = 1.43$ years. In the sequential strategy equilibrium, we adjust the vacancy rate v/h such as $q^s = q^b = q = 0.7$. Finally we impose $\beta = 0.10$. The probability of a change in family type is 10% in a year. The other parameters, which correspond to the states' costs, will be allowed to vary: the costs of being a BS (κ_{BS}), a BF (κ_{BF}), a SO (κ_S) and a BO (κ_B).

5.2 Comparing equilibria

In equilibrium definitions 1 and 2, we determine the respective existence conditions of both kind of equilibria. They exist if no agent has interest in refusing to conclude a transaction, nor to deviate from equilibrium decisions of other agents. In line with Wheaton's (1990) results, the sequential strategy equilibrium is characterized by a unique price, while the simultaneous strategy equilibrium is characterized by potential price dispersion. Different cases are possible for each set of parameters:

- No equilibrium exists. No price.
- The sequential strategy equilibrium exists, but the simultaneous strategy equilibrium does not exist. Price is unique.
- The sequential strategy equilibrium does not exist, but the simultaneous strategy equilibrium exists. Multiplicity of prices occurs.
- Multiple equilibria exist. Price dispersion is a possible outcome. In such a case, steady state equilibria are Pareto ranked according to the value of a matched household ($\mathcal{V}_M$) in both economies. Matched households experience the same probability β of a change in their family type and must choose between the sequential strategy (enter as a BF) and the simultaneous strategy (enter as a BS) if such event occurs. If their discounted flows of utility is higher in the second case, it suggests that the simultaneous strategy may shorten the unmatched spell and that price dispersion has an indirect positive impact on welfare at steady-state[9].

5.3 Existence and ranking of equilibria

Figures 6 and 7 provide a comparison of the equilibria with sequential or simultaneous strategy for span of values of κ_{BS}, κ_B and κ_S. In this experiment, we impose $\kappa_{BF} = \kappa_{BS}$, i.e. we assume that the costs of simultaneously searching for a buyer and a seller are not fundamentally greater than those of just searching for a seller.

[Insert Figs 6, 7]

A crucial point in our numerical analysis is that the results qualitatively depend on the *relative* positions of κ_{BS}, κ_B and κ_S. Recall that κ_{BS} summarizes the costs of being unmatched (to live in a non

[9]The comparison of $\mathcal{V}_M$ in both equilibria should only be seen as a proxy of a Pareto ranking procedure, since we directly compare *steady state* equilibria and leave out the transient dynamics before joining the steady state.

convenient home) and the costs of searching for both a seller and a buyer, κ_B is the cost of being homeless (or renter) and searching for a new home, κ_S is the cost of searching for a buyer plus the potential financial costs. Hence we can reasonably suppose that $\kappa_B > \max\{\kappa_{BS}, \kappa_S\}$. Being a homeless is the worse state in our economy[10]. Without loss of generality, we arbitrarily choose a reference value for κ_B (we impose[11] $\kappa_B = 8$). The relative position of κ_{BS} and κ_S is more ambiguous. The SO household lives in a convenient home (which is not the case for a BS), but bears financial costs that could potentially outweigh the search costs of a BS. As a reference we impose $\kappa_{BS} = 5$ and $\kappa_S = 2$ but we will allow cases where $\kappa_S > \kappa_{BS}$.

Overall, the figures show that for large spans of values of κ_{BS}, κ_B and κ_S, the simultaneous strategy equilibrium robustly exists, which is not the case for the sequential strategy equilibrium. Figure 6 provides an analysis of the existence of the two equilibria for different values of κ_B ($> \kappa_{BS}$) and κ_S ($< \kappa_B$). In this figure, κ_{BS} is set to its reference value (i.e. $\kappa_{BS} = 5$). For low values of the cost of being homeless (i.e. κ_B close to κ_{BS}) the sequential strategy equilibrium does not exist. Therefore, since the simultaneous equilibrium still exists even for very large values of κ_B (more than three times the search costs of a BS household): the larger κ_B, the more frequent the occurrence of multiple equilibria. This result is natural since an unmatched buyer adopting the sequential strategy is ensured not to experience a spell of BO in the future. On the contrary, a BS household incurs the risk of becoming a BO and bearing costs κ_B if it sells its home but simultaneously does not find a seller. Consequently, large values of κ_B prevent households from deviating from the sequential strategy. When multiple equilibria exist, the simultaneous strategy remain Pareto dominant over the sequential strategy ($\mathcal{V}_M > \mathcal{V}_{BF}$) as long as κ_B is not too large (for example $\kappa_B < 13.36$ for the reference value $\kappa_{BS} = 5$). This suggests that the main gain of being a matched household adopting the simultaneous strategy instead of the sequential one – shorter expected unmatched spells – overwhelms the losses (positive probability of becoming a homeless/renter in the BO state). In the simultaneous strategy equilibrium, the average duration of an unmatched spell is (in years) $d_{\text{simultaneous}} =$

[10] This is consistent with our focus on the ownership market, without an explicit modelling of the rental market.
[11] We need to impose such a large value for κ_{BO} to guarantee that prices will remain positive in all our numerical experiments, see Proposition 2.

$(3-2q) / \left[\beta + 2(1-2\beta)\left(2q - q^2\right)\right]$ which is lower than its counterpart with sequential strategy $d_{\text{sequential}} = 2/(\beta + q - 2\beta q)$ for a large span of values of β and q. For example, for our reference values of β ($\beta = 0.10$), the average time spent on the housing market with the sequential strategy for $q = 0.7$ is more than 3 years ($d_{\text{sequential}} = 3.03$), but only slightly above 1 year with the simultaneous strategy[12].

Figure 6 also provides an analysis of the existence of the two equilibria for different values of κ_S. Results are very similar to those obtained for different span of values of κ_B since, as noticed in section 3, the value function of a BF household does not depend on κ_S which impacts only the equilibrium price in the sequential strategy equilibrium. Consequently, the higher κ_S, the lower the interest for household in deviating from the sequential strategy and the higher the probability to have multiple equilibria since, as for κ_B, the simultaneous equilibrium always exists. Both equilibria exist when $\kappa_S \in [\kappa_{BS}, \kappa_B]$. On the contrary, for low values of κ_S, the simultaneous strategy is the only equilibrium outcome. In this case, the risk of becoming a SO when adopting the simultaneous strategy is smaller than the gains consecutive to a shorter expected unmatched spell. Finally, notice that the simulteanous stragegy is Pareto dominant over the sequential one if $\kappa_S < 7.36$ when κ_B is set at its reference value.

Finally, Figure 7 numerically confirms the robustness of the simultaneous equilibrium existence for a large span of values of $\{\kappa_B, \kappa_{BS}\}$. This figure suggests that the positive effect of κ_B on the occurence of multiple equilibria lowers as κ_{BF} ($= \kappa_{BS}$) grows. The sequential equilibrium does not exist for large span of values of κ_{BF} (even in sensible cases where $\kappa_{BF} < \kappa_B$). The higher κ_{BF}, the lower the welfare value of a BF in the sequential equilibrium. Consequently, households have a greater interest in deviating and adopting the simultaneous strategy because their probability to leave their current state (with probability $q^2 + 2q(1-q)$, apart from the demographic chock) is larger than with the sequential strategy (with probability q), since, in the first case, they only have to sell *or* buy to leave their current BS state. Notice that all these results appear to be robust for other plausible calibrations of parameter β (i.e. $0 < \beta < 0.2$) and q (i.e. $0.5 < q < 0.9$).

[12] This gap is mitigated for larger values of β: the higher β the higher the probability of becoming matched directly with a demographic shock rather than with conclusive transactions. Hence, the lower the relative interest of a simultaneous strategy compared to the sequential one.

In these cases, the simultaneous strategy always exists and multiple equilibria only occurs for low values of κ_{BF} and/or large values of κ_B.

All these results (the stable existence of the equilibrium with simultaneous strategy and its numerically large Pareto dominance over the equilibrium with sequential strategy) confirms the robustness of price dispersion in our setup.

5.4 Price dispersion

Let us now focus on the amount of price dispersion in the equilibrium with simultaneous strategy. Once again, we restrict our attention to the symmetric Nash bargaining program ($\gamma = 1/2$). As noticed in Proposition 2, $p_1 = p_2$ in the symmetric case and p_1 is the mean of $[p_3, p_4]$. So there are only three different prices in equilibrium. Figure 8 provides the values of p_1, p_3 and p_4 for different values of κ_B, κ_S together with the rate of relative dispersion, i.e. $(p_4 - p_1)/p_1$. Notice that we restrict our analysis to parameter values where the simultaneous equilibrium with price dispersion *does exist* (i.e. we do not consider too large values for κ_B).

[**Insert Fig 8**]

Panel (a) and (c) of the Figure 8 show the impact of search costs on the price level. In line with Equation (26), the price p_1 is decreasing with the search cost of seller only and decreasing with the search cost of buyer only. Prices p_3 and p_4 logically increase as p_1 increases. Panel (b) and (d) of the Figure 8 show that price dispersion on the housing market rises with the two search costs. It is particularly impressive with the search cost of seller-only that increases the price dispersion from near 1% to more than 400%. Price dispersion is also growing with the search cost of buyer-only, but without the same amplitude. As shown with Equation (31), price dispersion proceeds from the gap between two sums of value function, $(\mathcal{V}_{BS} + \mathcal{V}_M)$ and $(\mathcal{V}_{SO} + \mathcal{V}_{BO})$. For these simulations, the two sums fall as search costs grow. The rising of price dispersion comes form the fact that $(\mathcal{V}_{BS} + \mathcal{V}_M)$ decreases at a lower rate than $(\mathcal{V}_{SO} + \mathcal{V}_{BO})$.

To conclude this numerical analysis, the sequential strategy generates a substantial price dispersion highly sensitive to search costs in the housing market. It is worth mentioning here that these search costs include, for example, the prospect of information but also financial costs induced by the ownership of two houses. Our model can then explain how modifications in the economic environment could lead to changes in both the level and the dispersion of price on the housing market.

6 Conclusion

Every year many households change their residential location either for personal or professional reasons. Moving home is a tricky operation, which can be costly for households in case of difficulties in concluding transactions. The households can experience homeless (rental market) spell, if they sell their old dwelling before buying the new one, high financial costs, if they are stuck with two houses, or the annoyance of living in a non convenient dwelling for a long time. These features of the housing market are well-known and have already been considered in the literature, notably through the presence of matching frictions and search costs. However, few attention has been devoted to the search strategy of households. This is especially the case for the simultaneous strategy where the household tries to simultaneously buy and sell. Indeed, the literature examplified by Weathon (1990) generally considers only a sequential strategy where the household first buys the new dwelling before selling the old one.

The comparison of the two strategies provided in this article shows however that the simultaneous strategy is more stable than the sequential strategy and may deliver higher welfare. The numerical analysis of our theoretical model indicates that for a large range of parameters values, the equilibrium with simultaneous strategy always exists contrary the equilibrium with sequential strategy. In the case of multiple equilibria, each of the two strategies can be associated with the higher steady-state welfare according to the parameters values. Besides its implications on the equilibrium (existence, uniqueness, and welfare), the simultaneous strategy provides also a rational explanation for the puzzling price dispersion observed on the housing market. When households enter the housing market both as buyer and seller, the transactions are done by pairs of households which are at different stages of their housing market path (buyer and seller, seller only, or buyer only). Since these households have different prospects on the housing market, the bargained prices are different. It is worth mentioning that this price dispersion proceeds only from the endogenous functioning of the housing market and does not require any exogenous form of heterogeneity either on dwellings or on households. Our model has been kept sufficiently simple to provide analytical proof of the existence of price dispersion. Its simplicity may also limit its scope. For example, it doesn't allow to link prices (and their dispersion) with the volume of transactions on the housing market. Similarly, we do not tackle the challenge of solving both the bargaining processes and the strategy's choice while taking the existence of chains on the housing market into account. However, we think that the model we proposed in this article improves our understanding of the housing market and could be fruitfully enriched in these directions in future researches.

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Appendix

A The equilibrium with sequential strategy

A.1 The equilibrium price

The bargaining process (8) leads to the standard sharing rule

$$\gamma \mathcal{S}\mathcal{V}_{SO} = (1-\gamma)\,\mathcal{B}\mathcal{V}_{BF} \tag{A.1}$$

where the inputs of the bargaining process are calculated using (6)-(7) and $u(p) = bp$

$$\mathcal{S}\mathcal{V}_{SO} = \mathcal{V}_{SO}|_{q^s=1} - \mathcal{V}_{SO}|_{q^s=0} = bp_B + \delta\,(\beta\mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M) - \delta\mathcal{V}_{SO} \tag{A.2}$$

$$\mathcal{B}\mathcal{V}_{BF} = \mathcal{V}_{BF}|_{q^b=1} - \mathcal{V}_{BF}|_{q^b=0} = -bp_B - \delta\,(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BF}) + \delta\mathcal{V}_{SO} \tag{A.3}$$

Therefore, it is straightforward to deduce the expression of p_B given by (9) by introducing Equations (A.2)-(A.3) in the sharing rule (A.1).

A.2 The system of equations

The set of endogenous variables of the equilibrium with sequential strategy $\{p_B, \mathcal{V}_M, \mathcal{V}_{BF}, \mathcal{V}_{SO}, h_{BF}, h_{SO}\}$ solves the following system of equations

$$p_B = \frac{\delta}{b}(1-\gamma)\left[\mathcal{V}_{SO} - (\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BF})\right] - \frac{\delta}{b}\gamma\left[\beta\mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right] \tag{A.4}$$

$$\mathcal{V}_M = \overline{u} + \delta\left[\beta\mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M\right] \tag{A.5}$$

$$\mathcal{V}_{BF} = \overline{u} - \kappa_{BF} + q^b u\,(-p_B) + \delta\left[q^b\mathcal{V}_{SO} + \left(1-q^b\right)(\beta\mathcal{V}_M + (1-\beta)\,\mathcal{V}_{BF})\right] \tag{A.6}$$

$$\mathcal{V}_{SO} = \overline{u} - \kappa_{S0} + q^s u\left(p_B\right) + \delta\left[q^s\left(\beta \mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M\right) + (1-q^s)\,\mathcal{V}_{SO}\right] \tag{A.7}$$

$$h'_{BF} = \left(1 - q^b\right)(1-\beta)\,h_{BF} + \beta\left(h - h_{BF} - h_{SO}\right) + q^s \beta h_{SO} \tag{A.8}$$

$$h'_{SO} = (1-q^s)\,h_{SO} + q^b h_{BF} \tag{A.9}$$

A.3 The equilibrium prices with a deviating household

If an household deviates from the sequential strategy and enters the market as a buyer and seller, three additional types of match are feasible on the market with three associated prices. The three matches are defined in the Table A.3.

Type of Match	The buyer	The seller	The price
1	A Buyer First	The unique Buyer and Seller	p^1_{BF}
2	The unique BS	A Seller Only	p^2_{BF}
3	BO who has been BS first	A Seller Only	p^3_{BF}

Table A.3. Type of match with a deviating household.

To establish the condition of existence of the equilibrium given by the Proposition 1, we need to find the value of these prices to calculate the net payoff of deviation. Prices are the solutions of Nash bargaining programs, which inputs are the increases in value given by the transaction.

- The price p^1_{BF} solves the negotiation process

$$p^1_{BF} : \arg\max \left\{ \left[\mathcal{BV}_{BF}\right]^{\gamma} \left[\mathcal{SV}_{BS}\right]^{1-\gamma} \right\} \tag{A.10}$$

where the expression of $\mathcal{BV}_{BF}$ is given by (A.3), with p^1_{BF} instead of p_B, and $\mathcal{SV}_{BS}$ is calculated using

(11)

$$\mathcal{SV}_{BS} = \mathcal{V}_{BS}|_{q^s=1} - \mathcal{V}_{BS}|_{q^s=0} \tag{A.11}$$
$$= bp^1_{BF} + \delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{SO}\right) - \left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS} - \mathcal{V}_{BO}\right)\right]$$

The sharing rule $\gamma\mathcal{SV}_{BS} = (1-\gamma)\mathcal{BV}_{BF}$ gives the equilibrium price

$$bp^1_{BF} = \delta(1-\gamma)\left[\mathcal{V}_{SO} - (\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BF})\right] - \gamma\delta q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{SO}\right) \tag{A.12}$$
$$+\gamma\delta\left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS} - \mathcal{V}_{BO}\right)$$

- The price p^2_{BF} solves the negotiation process

$$p^2_{BF} : \arg\max\left\{\left[\mathcal{BV}_{BS}\right]^{\gamma}\left[\mathcal{SV}_{SO}\right]^{1-\gamma}\right\} \tag{A.13}$$

where the expression of $\mathcal{SV}_{SO}$ is given by (A.2), with p^2_{BF} instead of p_B, and $\mathcal{BV}_{BS}$ is calculated using (11)

$$\mathcal{BV}_{BS} = \mathcal{V}_{BS}|_{q^b=1} - \mathcal{V}_{BS}|_{q^b=0} \tag{A.14}$$
$$= -bp^2_{BF} + \delta\left[q^s\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{BO}\right) - (1-q^s)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS} - \mathcal{V}_{SO}\right)\right]$$

The sharing rule $\gamma\mathcal{SV}_{SO} = (1-\gamma)\mathcal{BV}_{BS}$ gives the equilibrium price

$$bp^2_{BF} = (1-\gamma)\delta q^s\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{BO}\right) \tag{A.15}$$
$$-(1-\gamma)\delta(1-q^s)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS} - \mathcal{V}_{SO}\right) - \gamma\delta\left[\beta\mathcal{V}_{BF} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{SO}\right]$$

- The price p_{BF}^3 solves the negotiation process

$$p_{BF}^3 : \arg\max \left\{ [\mathcal{BV}_{BO}]^{\gamma} [\mathcal{SV}_{SO}]^{1-\gamma} \right\} \tag{A.16}$$

where the expression of $\mathcal{SV}_{SO}$ is given by (A.2), with p_{BF}^3 instead of p_B, and $\mathcal{BV}_{BO}$ is calculated using (12)

$$\mathcal{BV}_{BO} = \mathcal{V}_{BO}|_{q^b=1} - \mathcal{V}_{BO}|_{q^b=0} = -bp_{BF}^3 + \delta \left[\beta \mathcal{V}_{BS} + (1-\beta) \mathcal{V}_M - \mathcal{V}_{BO} \right] \tag{A.17}$$

The sharing rule $\gamma \mathcal{SV}_{SO} = (1-\gamma) \mathcal{BV}_{BO}$ gives the equilibrium price

$$bp_{BF}^3 = (1-\gamma) \delta \left[\beta \mathcal{V}_{BS} + (1-\beta) \mathcal{V}_M - \mathcal{V}_{BO} \right] - \gamma \delta \left[\beta \mathcal{V}_{BF} + (1-\beta) \mathcal{V}_M - \mathcal{V}_{SO} \right] \tag{A.18}$$

A.4 The condition of existence

Proposition 1 gives a necessary and sufficient condition of existence of the equilibrium with sequential strategy. This proposition relies on a threshold value for the search cost of buyer and seller, $\underline{\kappa_V}$, such that the equilibrium exists if $\kappa_V > \underline{\kappa_V}$ and doesn't exist otherwise. This property is intuitive. Since κ_V is paid only by the deviating household which tries to simultaneously buy and sell, one can always find a value for κ_V (eventually very high if necessary) such that deviating from the sequential strategy is not optimal.

To confirm this intuition, we only need to prove that the value function of the deviate household ($\mathcal{V}_{BS}$) is strictly decreasing with κ_V since the value function of the buyer first ($\mathcal{V}_{BF}$) is not impacted by κ_V. By inspecting equation (11), it seems evident that $\mathcal{V}_{BS}$ is decreasing with κ_V. The difficulty comes from the fact that this parameter impacts also the prices $\{p_{BF}^i\}_{i=1}^3$ and the value function of the deviating household if it becomes BO. Therefore, we start with the definition of $\mathcal{V}_{BS}$ given by equation (11) and re-express it as a function of the structural parameters using the equilibrium expression of $\{p_{BF}^i\}_{i=1}^3$, equations (A.12)–(A.15)–(A.18), and of $\mathcal{V}_{BO}$, equation (12). Finally, notice that the value functions associated with the single strategy ($\mathcal{V}_M$, $\mathcal{V}_{BF}$, and $\mathcal{V}_{SO}$) do not depend on κ_V. Consequently, they are treated as exogenous in this proposition.

We begin with the prices $\{p_{BF}^i\}_{i=1}^3$, which equilibrium expressions are transformed as follows

$$\begin{aligned} bp_{BF}^1 &= A + \gamma\delta\left(1-\beta-q^b\right)\mathcal{V}_{BS} - \gamma\delta\left(1-q^b\right)\mathcal{V}_{BO} && \text{(A.19)}\\ bp_{BF}^2 &= B + (1-\gamma)\,\delta\left[(\beta-(1-q^s))\,\mathcal{V}_{BS} - q^s\mathcal{V}_{BO}\right] && \text{(A.20)}\\ bp_{BF}^3 &= C + (1-\gamma)\,\delta\left[\beta\mathcal{V}_{BS} - \mathcal{V}_{BO}\right] && \text{(A.21)} \end{aligned}$$

where A, B, and C are independent from κ_V and given by

$$\begin{aligned} A &= \delta(1-\gamma)\left[\mathcal{V}_{SO} - \beta\mathcal{V}_M - (1-\beta)\,\mathcal{V}_{BF}\right] - \gamma\delta\left[q^b\left((1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right) - \left(1-q^b\right)\beta\mathcal{V}_M\right] && \text{(A.22)}\\ B &= (1-\gamma)\,\delta\left[q^s(1-\beta)\,\mathcal{V}_M - (1-q^s)\,(\beta\mathcal{V}_M - \mathcal{V}_{SO})\right] - \gamma\delta\left[\beta\mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right] && \text{(A.23)}\\ C &= (1-\gamma)\,\delta\,(1-\beta)\,\mathcal{V}_M - \gamma\delta\left[\beta\mathcal{V}_{BF} + (1-\beta)\,\mathcal{V}_M - \mathcal{V}_{SO}\right] && \text{(A.24)} \end{aligned}$$

We do a similar transformation for $\mathcal{V}_{BO}$, which satisfies

$$\mathcal{V}_{BO} = D + \gamma\delta q^b\beta\mathcal{V}_{BS} + \delta\left(1-\gamma q^b\right)\mathcal{V}_{BO} \tag{A.25}$$

where D is also independent from κ_V and given by

$$D = \overline{u} - \kappa_B - q^bC + \delta q^b(1-\beta)\,\mathcal{V}_M \tag{A.26}$$

We are now in position to compute the value $\mathcal{V}_{BS}$ as a function of κ_V

$$\mathcal{V}_{BS} = -\frac{\kappa_V}{1-\delta F} + \frac{E}{1-\delta F} + \frac{\delta q^s(1-\gamma)}{1-(1-\gamma q^b)\,\delta}\frac{D}{1-\delta F} \tag{A.27}$$

where E and F are two additional block of parameters and value functions that do not depend on κ_V

$$F = q^s q^b \beta + (1-q^s)\left(1-q^b\right)(1-\beta) + q^s\gamma\left(1-\beta-q^b\right) - q^b(1-\gamma)(\beta-(1-q^s)) \quad \text{(A.28)}$$
$$+\left(q^s\left(1-q^b\right)+q^b(1-\gamma)q^s - q^s\gamma\delta\left(1-q^b\right)\right)\frac{q^b\gamma\delta\beta}{1-(1-\gamma q^b)\delta}$$

$$E = \overline{u} + q^s A - q^b B + \delta\left[q^s q^b(1-\beta)\mathcal{V}_M + q^b(1-q^s)\mathcal{V}_{SO} + (1-q^s)\left(1-q^b\right)\beta\mathcal{V}_M\right]$$

The condition of existence of the equilibrium with sequential strategy is deduce from the condition $\mathcal{V}_{BF} > \mathcal{V}_{BS}$. Using the equations (6) and (A.27) for the value functions, we obtain

$$\kappa_V > \underline{\kappa_V} = E - \frac{(\overline{u}-\kappa_{BF})(1-\delta F)}{1-\delta(1-q^b)(1-\beta)} + \frac{(1-\delta F)q^b b}{\delta(1-q^b)(1-\beta)}p_B - \frac{(1-\delta F)\delta q^b}{\delta(1-q^b)(1-\beta)}\mathcal{V}_{SO} \quad \text{(A.29)}$$
$$-\frac{(1-\delta F)\delta\left(1-q^b\right)}{\delta(1-q^b)(1-\beta)}\beta\mathcal{V}_M + \frac{\delta q^s(1-\gamma)}{1-(1-\gamma q^b)\delta}D$$

The expressions of $D, E, F, p_B, \mathcal{V}_{SO}$ and $\mathcal{V}_M$ do not depend on κ_V.

B The equilibrium with simultaneous strategy

B.1 The demographic structure

Proof. Proof of Lemma 1: A the steady-state the mass of BO households and SO households are respectively

$$h_{BO} = \frac{q^s(1-q^b)}{q^b}h_{BS} \quad \text{(B.30)}$$

$$h_{SO} = \frac{q^b(1-q^s)}{q^s}h_{BS} \quad \text{(B.31)}$$

Since we simply assume an implicit one-for-one matching function, we have

$$q^s = q \times \left(\frac{h_{BS}+h_{BO}}{h_{BS}+h_{SO}}\right) \quad \text{(B.32)}$$

The probability of a match for each seller exogenously depends on the fraction q of the total mass of buyers. Similarly

$$q^b = q \times \left(\frac{h_{BS} + h_{SO}}{h_{BS} + h_{BO}} \right) \tag{B.33}$$

At the equilibrium, the total mass of sales $q^s (h_{BS} + h_{SO})$ must equal the total mass of bought $q^b (h_{BS} + h_{BO})$, hence, using the above expressions, $q^s = q^b$. Plugging this results in equations (B.30) and (B.31), we find

$$h_{BO} = (1-q)\, h_{BS} = h_{SO} \tag{B.34}$$

It directly follows that

$$\theta^s = \frac{h_{SO}}{h_{BS} + h_{SO}} = \frac{h_{BO}}{h_{BS} + h_{SO}} = \theta^b \tag{B.35}$$

■

B.2 The equilibrium system of equations

The sets of prices $\{p_i\}_{i=1}^4$ and of value functions $\{\mathcal{V}_M, \mathcal{V}_{BS}, \mathcal{V}_{BO}, \mathcal{V}_{SO}\}$ solution of the equilibrium with sequential strategy solve the following system of equations.

$$[1 - (1-\beta)\,\delta]\, \mathcal{V}_M = \overline{u} + \delta\beta\mathcal{V}_{BS} \tag{B.36}$$

$$\begin{aligned} & \left[1 - \left(q^2\beta + (1-q)^2 (1-\beta)\right) \delta\right] \mathcal{V}_{BS} \qquad \text{(B.37)} \\ = \ & \overline{u} - \kappa_V + q\,(1-\theta)\, b\,(p_4 - p_3) + \left[q^2 (1-\beta) + (1-q)^2 \beta\right] \delta\mathcal{V}_M + q\,(1-q)\,\delta\,(\mathcal{V}_{SO} + \mathcal{V}_{BO}) \end{aligned}$$

$$[1 - (1-q)\,\delta]\, \mathcal{V}_{BO} = \overline{u} - \kappa_B - qb\,[\theta p_4 + (1-\theta)\, p_2] + \delta\,[q\,(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M)] \tag{B.38}$$

$$[1 - (1-q)\,\delta]\, \mathcal{V}_{SO} = \overline{u} - \kappa_S + qb\,[\theta p_3 + (1-\theta)\, p_2] + \delta\,[q\,(\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_M)] \tag{B.39}$$

$$\frac{b}{\delta}p_1 = [(1-\gamma) - q(1-2\gamma)]\mathcal{V}_{SO} - [\gamma + (1-2\gamma)q]\mathcal{V}_{BO} - (1-2\gamma)[(\beta-q)\mathcal{V}_M + (1-q-\beta)\mathcal{V}_{BS}] \quad \text{(B.40)}$$

$$\frac{b}{\delta}p_2 = (1-2\gamma)[\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M] + \gamma\left[\mathcal{V}_{SO} - \left(\frac{1-\gamma}{\gamma}\right)\mathcal{V}_{BO}\right] \quad \text{(B.41)}$$

$$\frac{b}{\delta}p_3 = (1-\gamma)[q(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M) + (1-q)\mathcal{V}_{SO} - (1-q)(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS}) - q\mathcal{V}_{BO}] - \gamma[\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{SO}] \quad \text{(B.42)}$$

$$\frac{b}{\delta}p_4 = (1-\gamma)[\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{BO}] - \gamma q(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M - \mathcal{V}_{SO}) + \gamma(1-q)(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS} - \mathcal{V}_{BO}) \quad \text{(B.43)}$$

These equations are deduced from equations (16)-(17)-(20)-(21)-(22)-(23)-(24)-(25) together with the Lemma 1.

B.3 The equilibrium price dispersion

This appendix provides the detailed calculus associated with the Proposition 2. It is straightforward to deduce the equality of the price p_1 with the price p_2 for symmetric bargaining programs. Setting $\gamma = 1/2$ in Equations (B.40) and (B.41) gives

$$\frac{b}{\delta\gamma}p_1 = \frac{b}{\delta\gamma}p_2 = \mathcal{V}_{SO} - \mathcal{V}_{BO} \quad \text{(B.44)}$$

To get an expression of these prices as a function of structural parameters and not value functions, we compute the difference between $\mathcal{V}_{SO}$ and $\mathcal{V}_{BO}$ using (B.38) and (B.39) and obtain

$$[1 + (1-q)\delta](\mathcal{V}_{SO} - \mathcal{V}_{BO}) = \kappa_B - \kappa_S + qb[\theta p_3 + 2(1-\theta)p_2 + \theta p_4] \quad \text{(B.45)}$$

For the expression of p_2 given by (B.44), we have the following expression for the difference in value functions

$$\mathcal{V}_{SO} - \mathcal{V}_{BO} = \frac{\kappa_B - \kappa_S + qb\left(\theta p_3 + \theta p_4\right)}{1 + (1-q)\,\delta - q\,(1-\theta)\,\delta} \tag{B.46}$$

and we deduce the following expression for equilibrium prices p_1 and p_2

$$p_1 = p_2 = \frac{\delta\gamma}{b}\left(\frac{\kappa_B - \kappa_S + qb\theta\left(p_3 + p_4\right)}{1 + (1-q)\,\delta - 2qb\,(1-\theta)}\right) \tag{B.47}$$

The next step consists in the determination of the equilibrium value of $(p_3 + p_4)$, which is obtained from equations (B.42) and (B.43)

$$p_3 + p_4 = \frac{\delta}{b}\left(\mathcal{V}_{SO} - \mathcal{V}_{BO}\right) = 2 \times p_1 \tag{B.48}$$

We then conclude that the price p_1 is mean of prices p_3 and p_4 The exact value of p_1 reported in the Proposition 2 is obtained by plugging (B.48) in (B.47).

B.4 The equilibrium with a deviating household

If a household deviates from the simultaneous strategy and enters the market as a buyer first, two additional types of match are feasible on the market with two additional associated prices. After the household succeeds in buying its house, it becomes a seller only similar to other households with two dwellings. The two matches are defined in the Table B.4.

Type of Match	The buyer	The seller	The price
1	The unique Buyer First	A Buyer and Seller	p_{BS}^{BF}
2	The unique Buyer First	A Seller Only	p_{BO}^{BF}

Table B.4. Type of match with a deviating household.

- The value function of the deviating household who first buy is

$$\mathcal{V}_{BF} = (\overline{u} - \kappa_V) + U\left(p_{SO}^{BF}, p_{BS}^{BF}\right) + \delta\left[q^b \mathcal{V}_{SO} + \left(1-q^b\right)(1-\beta)\mathcal{V}_{BF} + \left(1-q^b\right)\beta\mathcal{V}_M\right] \tag{B.49}$$

where the utility drawn from price transactions is

$$U\left(p_{SO}^{BF}, p_{BS}^{BF}\right) = q^b\theta^s u\left(-p_{BS}^{BF}\right) + q^b\left(1-\theta^s\right)u\left(-p_{SO}^{BF}\right)$$

- The Nash bargaining program of the first type of match (with a buyer and seller household) is

$$p_{BS}^{BF} : \arg\max\left\{\left[\mathcal{B}_{BS}\mathcal{V}_{BF}\right]^{\gamma}\left[\mathcal{S}_{BF}\mathcal{V}_{BS}\right]^{1-\gamma}\right\} \tag{B.50}$$

where the increase in value for the seller is

$$\begin{aligned} \mathcal{S}_{BF}\mathcal{V}_{BS} &= u\left(p_{BS}^{BF}\right) \\ &+\delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M\right) + \left(1-q^b\right)\mathcal{V}_{BO} - q^b\mathcal{V}_{SO} - \left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS}\right)\right] \end{aligned} \tag{B.51}$$

which is computed using Equation (11) and for the buyer is

$$\mathcal{B}_{BS}\mathcal{V}_{BF} = \left.\mathcal{V}_{BF}\right|_{q^b=1,\theta^s=1} - \left.\mathcal{V}_{BF}\right|_{q^b=0} = u\left(-p_{BS}^{BF}\right) + \delta\left[\mathcal{V}_{SO} - (1-\beta)\mathcal{V}_{BF} - \beta\mathcal{V}_M\right]$$

which is computed using Equation (B.49). The equilibrium price satisfies the sharing rule, $\gamma\mathcal{B}_{BS}\mathcal{V}_{BF} = (1-\gamma)\mathcal{S}_{BF}\mathcal{V}_{BS}$, that is

$$\begin{aligned} 2bp_{BS}^{BF} &= \delta\left[\mathcal{V}_{SO} - (1-\beta)\mathcal{V}_{BF} - \beta\mathcal{V}_M\right] \\ &-\delta\left[q^b\left(\beta\mathcal{V}_{BS} + (1-\beta)\mathcal{V}_M\right) + \left(1-q^b\right)\mathcal{V}_{BO} - q^b\mathcal{V}_{SO} - \left(1-q^b\right)\left(\beta\mathcal{V}_M + (1-\beta)\mathcal{V}_{BS}\right)\right] \end{aligned} \tag{B.52}$$

- The Nash bargaining program of the second type of match (with a seller only household) is

$$p_{SO}^{BF} : \arg\max \left\{ \left[\mathcal{B}_{SO}\mathcal{V}_{BF}\right]^{\gamma} \left[\mathcal{S}_{BF}\mathcal{V}_{SO}\right]^{1-\gamma} \right\} \tag{B.53}$$

where the increase in value for the seller is

$$\mathcal{S}_{BF}\mathcal{V}_{SO} = u\left(p_{SO}^{BF}\right) + \delta\left[\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_{M} - \mathcal{V}_{SO}\right] \tag{B.54}$$

which is computed using Equation (7) and for the buyer is

$$\mathcal{B}_{SO}\mathcal{V}_{BF} = \mathcal{V}_{BF}|_{q^b=1,\theta^s=0} - \mathcal{V}_{BF}|_{q^b=0} = u\left(-_{SO}^{BF}\right) + \delta\left[\mathcal{V}_{SO} - (1-\beta)\,\mathcal{V}_{BF} - \beta\mathcal{V}_{M}\right] \tag{B.55}$$

which is computed using Equation (B.49). The equilibrium price satisfies the sharing rule, $\gamma\mathcal{B}_{SO}\mathcal{V}_{BF} = (1-\gamma)\,\mathcal{S}_{BF}\mathcal{V}_{SO}$, that is

$$2bp_{SO}^{BF} = \delta\left[\mathcal{V}_{SO} - (1-\beta)\,\mathcal{V}_{BF} - \beta\mathcal{V}_{M}\right] - \delta\left[\beta\mathcal{V}_{BS} + (1-\beta)\,\mathcal{V}_{M} - \mathcal{V}_{SO}\right] \tag{B.56}$$

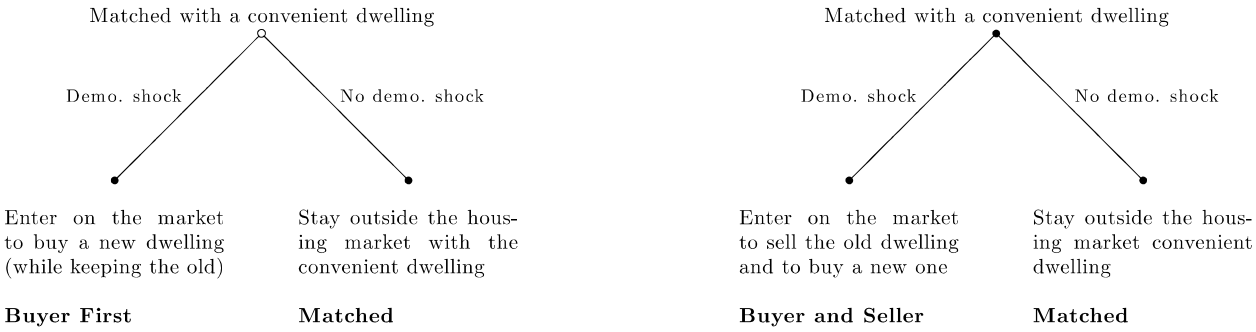

Figure 1. **The two alternative strategies in the economy.** The household enters the housing market as a buyer first with the sequential strategy (left panel) and as a buyer and seller with the simultaneous strategy (right panel))

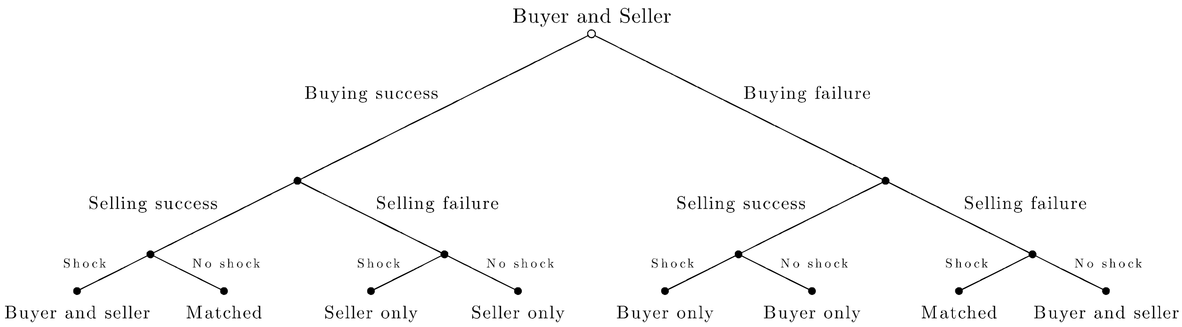

Figure 2. **The buyer and seller outcomes**. At the end of the period, the buyer and seller can be either a buyer and seller, a matched, a seller only or a buyer only household according to its success and/or failure in buying and selling and the realization of the demographic shock.

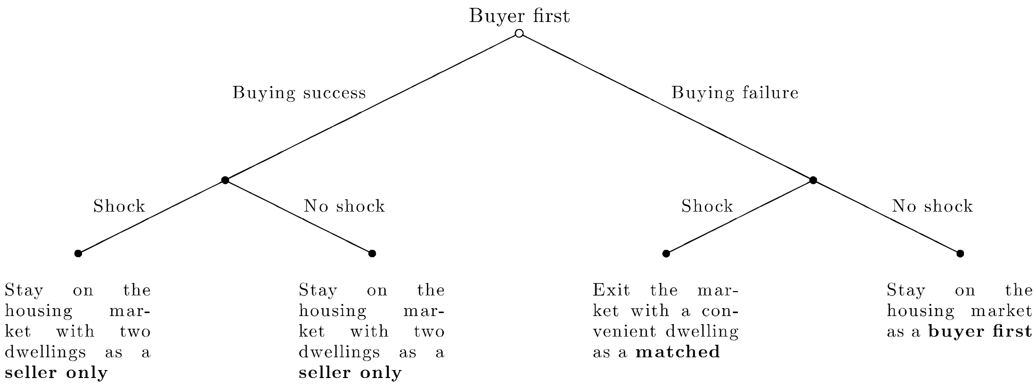

Figure 3. **The buyer first outcomes**. At the end of the period, the buyer first can be either a buyer first, a matched, or a seller only household according to its success or failure in buying and the realization of the demographic shock.

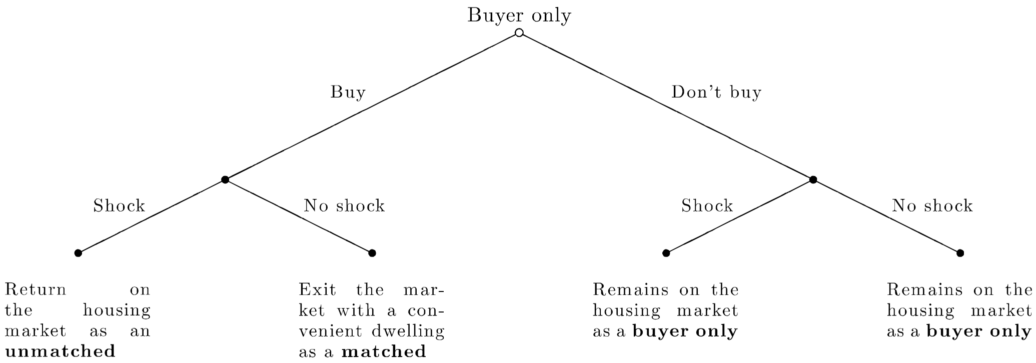

Figure 4. **The buyer only outcomes**. At the end of the period, the buyer only can be either an unmatched, a matched, or a buyer only household according to its success or failure in buying and the realization of the demographic shock.

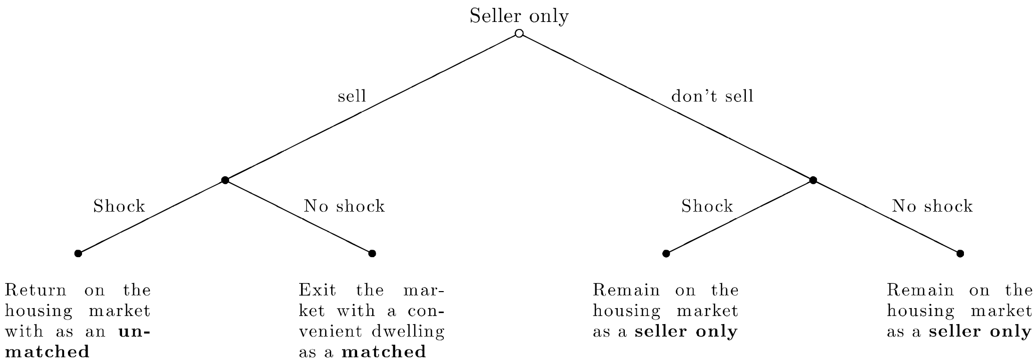

Figure 5. **The seller outcomes**. At the end of the period, the seller only can be either a buyer first, a matched, or a seller only household only according to its success or failure in buying and the realization of the demographic shock.

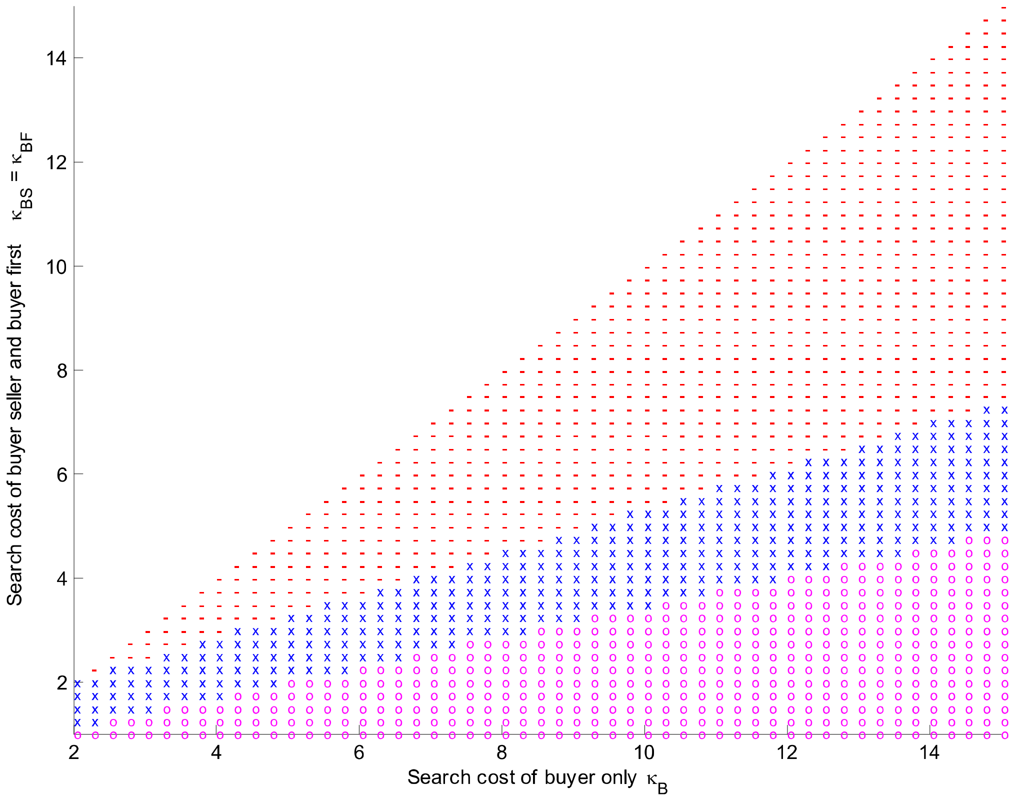

Figure 6. **Existence of Equilibria and Pareto Ranking.** The symbol (-) is for cases where the simultaneous strategy is the unique equilibrium. For multiple equilibria, symbol (x) means that the welfare is higher with the simultaneous strategy, whereas higher with the sequential strategy for symbol (o).

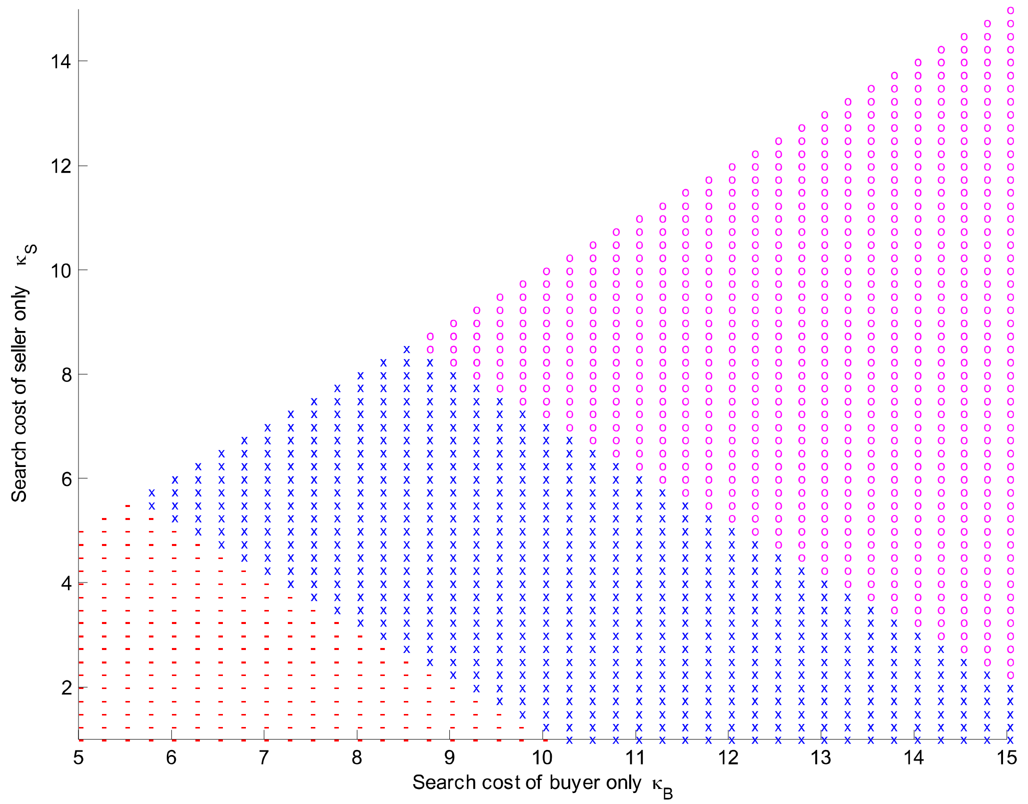

Figure 7. **Existence of Equilibria and Pareto Ranking.** The symbol (-) is for cases where the simultaneous strategy is the unique equilibrium. For multiple equilibria, symbol (x) means the welfare is higher with the simultaneous strategy, whereas higher is with the sequential strategy for symbol (o).

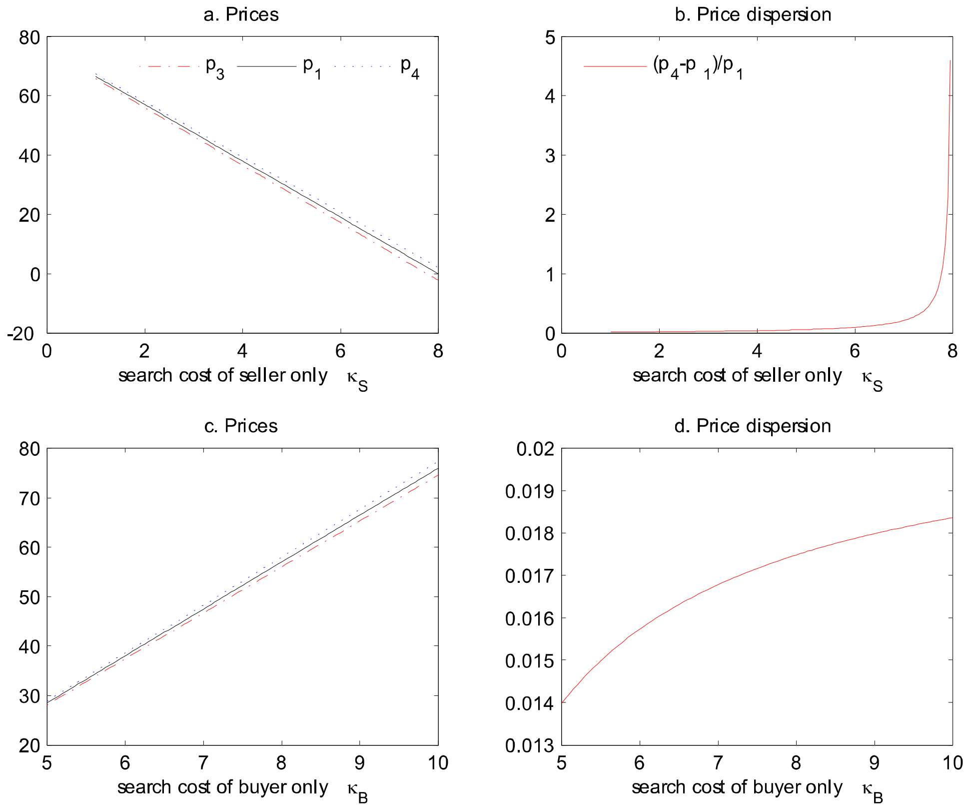

Figure 8. Equilibrium prices and price dispersion in the simultaneous strategy equilinirum. Panels (a) and (c) show the equilibrium prices for various values of κ_S and κ_B, respectively. Panels (b) and (c) give the associated rate of price dispersion on the housing market.