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Productivity and the Real Euro-Dollar Exchange Rate

by

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DISCUSSION
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Productivity and the real euro-dollar exchange rate

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Abstract

This paper analyses empirically how changes in productivity affect the real euro-dollar exchange rate. We consider the two-sector new open macro model in Benigno and Thoenissen (2003). The model predictions are used, in the form of sign restrictions, to identify productivity shocks in a structural vector autoregression. We estimate economy-wide and traded sector productivity shocks, controlling for demand and nominal factors. Our results show that productivity shocks are much less important in explaining the variation in the euro-dollar exchange rate than are demand and nominal shocks. In particular, productivity can explain part of the appreciation of the dollar in the late 1990s only to the extent that it created a boost to aggregate demand in the US. We find an insignificant contribution of the Balassa-Samuelson effect.

Keywords: real exchange rate, productivity, VAR, sign restrictions
JEL classification: F41, F31

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1 Introduction

In the first two years of its existence, the euro depreciated by almost 30% in real terms against the dollar. The slide of the euro has been linked to the more rapid productivity growth in the US relative to the euro area in the late 1990s[1]. We address this issue empirically, using insights from the new open macro literature.

The theoretical link between productivity and exchange rates is not straightforward. In an open economy with two sectors, the effect of a productivity shock on the real exchange depends on its location. An *economy-wide* productivity boom at home raises the supply of home-produced goods. As a consequence, home-produced tradables become cheaper relative to imports: the terms of trade deteriorate. Through this terms of trade effect, the real exchange rate depreciates.

A productivity shock *concentrated in the tradable goods sector* has the same effect on the terms of trade. But now there is an additional channel through which the real exchange rate may appreciate. As workers in the export sector become more productive, they earn higher wages. Through intersectoral labour mobility, wages increase also in the nontraded goods sector, where productivity has not changed. Prices of nontraded goods rise in line with marginal costs. The result is a rise in the relative price of nontradable goods, known as the Balassa-Samuelson effect, which - in isolation - results in a real appreciation. The net effect of a traded sector productivity shock is a combination of the terms of trade effect and the Balassa-Samuelson effect and can go in either direction. Our estimation method identifies these two channels and lets the data determine their relative importance.

Another explanation for the dollar's real appreciation during the new economy boom is a rise in aggregate demand, possibly a wealth effect stemming from the rise in equity prices[2]. Finally, currency movements are also influenced by nominal disturbances, by which we mean money market shocks or exchange rate changes that are not driven by fundamentals.

The 'new open macro'-type model in Benigno and Thoenissen (2003) formalises the concepts outlined above. We use the model predictions to identify a vector autoregression with four variables: output, traded goods prices, the real exchange rate and the relative price of nontradables. We identify four kinds of shocks: economy-wide and sector-specific shocks to productivity, demand shocks, and nominal shocks. The effect of productivity shocks on the real exchange rate is left unrestricted, which enables us to check if the model predictions are reasonable when compared with data. In addition, we estimate the importance of each type of shock for variation in the euro-dollar rate, in the whole sample (1981-2003) and in specific periods. This allows us to find out, for example, to what extent the dollar

[1] See, for example, Bailey, Millard and Wells (2001), Alquist and Chinn (2002).
[2] For a discussion, see Meredith (2001).

appreciation of the late 1990s was driven by productivity.

This paper is new in that it estimates the effect of productivity on the real euro-dollar exchange rate, distinguishing between economy-wide and sector-specific shocks. We filter out the Balassa-Samuelson effect and estimate its contribution to the appreciation of the dollar in the late 1990s. Up to now, research on the Balassa-Samuelson effect has has concentrated on small countries.[3] Our approach takes into account that for large countries, prices of tradables are endogenous. Finally, the use of sign restrictions to identify a VAR is fairly recent and can be regarded as less stringent than short run or long run zero restrictions.[4]

The paper is structured as follows. In section 2, we briefly outline the theoretical model by Benigno and Thoenissen, which we use to justify our sign restrictions. We decompose the real exchange rate, illustrating how the terms of trade effect and the Balassa-Samuelson effect work. The identification scheme and estimation results are given in section 3. Section 4 concludes.

[3] See, for example, De Gregorio and Wolf (1994).

[4] This is explained thoroughly in Canova and de Nicoló (2002) and Peersman (2003).

2 The Benigno and Thoenissen new open macro model

In this section, we look at how the real exchange rate is determined in the context of a two-country optimising sticky price model. For this purpose we use the paper by Benigno and Thoenissen (2003). Although the model cannot be solved analytically, we can extract impulse responses using numerical techniques. In section 3, we use the signs of the expected short run responses of selected variables as restrictions in a vector autoregression.

2.1 Model setup

We have a two-sector model with imperfectly competitive product and labour markets. Home and foreign agents all consume three types of goods: home-produced tradables, foreign-produced tradables and nontradables. Each country produces a continuum of tradables and a continuum of nontradable goods. Home agents, as well as home-produced goods, are indexed by $[0, n]$, while foreign agents and foreign-produced goods are indexed by $[n, 1]$. The parameter n indicates relative country size.

Consumers are infinitely lived and maximise the present discounted value of lifetime utility. Utility depends positively on consumption, C_t^i, and real money holdings, M_t^i/P_t, and negatively on labour supply, L_t^i.

$$U_t^i = E_t \sum_{s=t}^{\infty} \beta^{(s-t)} \left[U\left(C_s^i\right) + N\left(\frac{M_s^i}{P_s}\right) - V\left(L_s^i\right) \right] \tag{1}$$

The representative home individual, indexed by i, consumes traded goods, $C_{T,t}$, and non-traded goods, $C_{N,t}$. His consumption basket is given by

$$C_t = \frac{C_{T,t}^{\gamma} C_{N,t}^{(1-\gamma)}}{\gamma^{\gamma} (1-\gamma)^{(1-\gamma)}}$$

where γ is the relative weight that Home individual puts on traded goods. The consumption-based price index (the price of the consumption basket C_t) in the Home country is derived as

$$P_t = P_{T,t}^{\gamma} P_{N,t}^{(1-\gamma)}$$

where $P_{T,t}$ is the price of the basket of traded goods and $P_{N,t}$ is the price of the basket of nontraded goods. Consumption of tradables is divided into domestically produced tradables, indexed by H, and imports, indexed by F. Tradable goods consumption is given by the following subindex

$$C_{T,t} = \frac{C_{H,t}^{\nu} C_{F,t}^{(1-\nu)}}{\nu^{\nu} (1-\nu)^{(1-\nu)}}$$

where ν is the relative weight that Home individual puts on domestically produced traded goods. $P_{H,t}$ is a price subindex for the home-produced tradables goods and $P_{F,t}$ is the price subindex for the foreign-produced traded goods, expressed in the domestic currency.

$$P_{T,t} = P_{H,t}^{\nu} P_{F,t}^{(1-\nu)}$$

The relative price of home imports in terms of home exports is called the (inverse) terms of trade, ToT_t. Usually it is defined as $P_{F,t}/S_t P_{H,t}^*$, where S_t is the nominal exchange rate. However, we use a different definition of the (inverse) terms of trade, namely the home currency price of imports divided by the home currency price of home-produced tradables.

$$ToT_t^{-1} = \frac{P_{F,t}}{P_{H,t}}$$

In a world where the law of one price does not hold, the two are not the same. However, it can be shown that in the steady state, one is just a linear combination of the other. When ToT_t^{-1} increases, we speak of a terms of trade deterioration as imports become dearer relative to exports (conventional definition), or as imports become more expensive relative to home-produced tradables (our definition).

Home bias arises when at any given relative price, Home residents consume more Home-produced tradables (relative to foreign-produced tradables) than do foreign consumers. I.e., $\frac{C_{H,t}}{C_{F,t}} > \frac{C_{H,t}^*}{C_{F,t}^*}$ at any given relative price, which requires that $\nu > \nu^*$.

$$\begin{aligned} \frac{C_{H,t}}{C_{F,t}} &= \frac{\nu}{1-\nu}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-1} \\ \frac{C_{H,t}^*}{C_{F,t}^*} &= \frac{\nu^*}{1-\nu^*}\left(\frac{P_{H,t}^*}{P_{F,t}^*}\right)^{-1} \end{aligned}$$

Domestic agents produce nontradables, N, tradables for the home market, H, and tradables for the export market, H^*. Similarly, foreign agents produce nontradables, N^*, tradables for their own market, F^*, and tradables for the export market, F. Let σ^j and σ^{j^*} denote the demand elasticities for home-produced goods and foreign-produced goods respectively, where $j = N, H, H^*$ and $j^* = N^*, F, F^*$. We introduce the following consumption subindices for home- and foreign-produced goods.

$$\begin{aligned} C_{j,t} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma^j}} \int_0^n c_t(z)^{\frac{\sigma^j-1}{\sigma^j}} dz\right]^{\frac{\sigma^j}{\sigma^j-1}} \\ C_{j^*,t} &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma^{j^*}}} \int_n^1 c_t(z)^{\frac{\sigma^{j^*}-1}{\sigma^{j^*}}} dz\right]^{\frac{\sigma^{j^*}}{\sigma^{j^*}-1}} \end{aligned}$$

We assume that $\sigma^H \neq \sigma^{H^*}$ and $\sigma^F \neq \sigma^{F^*}$ ie., home and foreign agents have different demand elasticities for the same good, which allows producers to price discriminate between the two countries.

Labour supply is assumed to be immobile between countries and perfectly mobile between sectors. Individual labour supply, $L_t(i)$, is divided between the two sectors. For the individual agent, working in the nontraded or the traded sector is equivalent. Household unions bundle individual labour supply.

$$L_t^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n L_t^j(i)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}}$$

The elasticity of substitution between labour inputs is denoted by ϕ and is assumed to be the same across sectors. Let W_t^j be the price of labour inputs in sector j and let $W_t^j(i)$ be the nominal wage of individual i in sector j. Firms choose labour $L^j(i)$ to maximise profits, given the household union's labour supply. We assume that labour is the only input and that the production function is characterised by constant returns to scale:

$$Y_{j,t} = A_{j,t} L_{j,t} \tag{2}$$

where $j = H, N, F, N^*$. With a linear production function, the production decision in one market does not affect the marginal cost in the other market and we can look at the production decision in the two markets separately. Total demand for agent i's labour supply is then given by

$$L_t^j(i) = \frac{1}{n} \left[\frac{W_t^j(i)}{W_t^j} \right]^{-\phi} L_t^j \tag{3}$$

The household maximises lifetime utility, equation (1), subject to the following budget constraint:

$$\begin{aligned} & P_{T,t} C_{T,t}^i + P_{N,t} C_{N,t}^i + M_t^i - M_{t-1}^i + \frac{B_{H,t}^i}{1+i_t} + \frac{S_t B_{F,t}^i}{(1+i_t^*)\,\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} \\ \leq \quad & B_{H,t-1}^i + S_t B_{F,t-1}^i + T_t^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \frac{\int_0^n \Pi_{H,t}^i di}{n} + \frac{\int_0^n \Pi_{N,t}^i di}{n} \end{aligned} \tag{4}$$

In addition to money, there are two assets available for consumption smoothing, home bonds denominated in domestic currency, $B_{H,t}^i$, and internationally traded foreign bonds, $B_{F,t}^i$. In order to trade in the foreign bond market, the home individual has to pay an intermediation cost, denoted by $\Theta(\cdot)$, which is a function of the foreign asset position of the whole economy.

On the right hand side of the budget constraint, we have bond holdings carried over from the previous period, lump-sum government transfers, T_t^i, labour income and firm profits,

$\Pi^i_{j,t}$, which are assumed to be shared equally. On the left hand side, we have consumption spending on tradables and nontradables, the reduction in money holdings, $M^i_t - M^i_{t-1}$ and the purchase of home and foreign bonds.

A fraction $(1-\varepsilon)$ of firms set prices in a forward-looking way, knowing that every period, they are able to change prices with a fixed probability $1-\alpha_p$ (Calvo pricing). We assume local currency pricing, which means that each firm producing tradable goods sets two prices, one for the domestic market and one for the foreign market. For a firm producing good H for the home market, for example, we have the following price setting equation, which is derived by maximising utility of the representative agent with respect to the price of good H in the home country, $P^f_H(i)$.

$$E_t\left\{\sum_{k=0}^{\infty}\left(\alpha_p^H\beta\right)^k\left[\left(1-\Phi_H\right)P^f_{H,t}(i)-\frac{W_t}{A_{H,t}}\right]\frac{\widetilde{Y}_t^{Hd}(i)}{P_{H,t}}\frac{U_c\left(C_{t,t+k}\right)}{P_t}\right\}=0 \qquad (5)$$

where $P^f_{H,t}(i)$ is the price set optimally in period t, $\widetilde{Y}_t^{Hd}(i)$ is the home demand for home-produced tradables at time t and $1-\Phi_H$ is the degree of monopolistic distortions in the domestic tradable sector.

$$1-\Phi_H = 1-\frac{1}{\sigma^H}$$

A fraction ε of firms are backward-looking and set a price $P^b_{H,t}$. The (log-linearised) index of prices set at time t, $\overline{\overline{p}}_t$, is therefore a weighted average of the forward-looking price and the backward-looking price.

$$\overline{\overline{p}}_{H,t} = (1-\varepsilon)\,p^f_{H,t} + \varepsilon p^b_{H,t}$$

Backward-looking firms set prices equal to last period's price index, $\overline{\overline{p}}_{H,t-1}$, corrected for lagged inflation.

$$p^b_{H,t} = \overline{\overline{p}}_{H,t-1} + \pi_{t-1}$$

Under this assumption, inflation has a forward-looking component but also depends on its lagged value. This is a way to introduce inflation persistence, which is a stylised fact present in the data.

The monopolistic distortion in the labour market is given by $(1-\Phi_w)$, where

$$1-\Phi_w = 1-\frac{1}{\phi}$$

Household unions set wages as a markup over the marginal rate of substitution between consumption and labour supply. As in product markets, we assume that wages are set according to Calvo-contracts i.e., each period, they are adjusted with a probability of $(1-\alpha_w)$.

Household unions maximise utility with respect to the individual wage rate $W_t(i)$, taking as given labour demand (3). This implies the following first order condition for wage setters.

$$E_t\left\{\sum_{k=0}^{\infty}(\alpha_w\beta)^k\left[(1-\Phi_w)\frac{W_t^o(i)}{P_{t+k}}-\frac{V_l\left(L_{t+k}^i\right)}{U_c\left(C_{t,t+k}\right)}\right]U_c\left(C_{t,t+k}\right)L_{t+k}(i)\right\}=0 \qquad (6)$$

where $W_t^o(i)$ is the wage rate set optimally at time t.

At time t, the government budget constraint equates seignorage revenues to transfers:

$$\int_0^n\left[M_t(i)-M_{t-1}(i)\right]di=\int_0^n T_t(i)\,di$$

The resource contraint for the whole economy consolidates the public and private sector budget constraints, where the latter is derived by aggregating the individual budget constraints over all home agents. This gives us the current account equation.

$$\frac{S_tB_{F,t}}{P_t\left(1+i_t^*\right)\Theta(\cdot)}=\frac{S_tB_{F,t-1}}{P_t}+\frac{P_{H,t}Y_{H,t}^d}{P_t}+\frac{S_tP_{H,t}^*Y_{H,t}^{d*}}{P_t}-\frac{P_{T,t}C_{T,t}}{P_t}$$

where $Y_{H,t}^d$ and $Y_{H,t}^{d*}$ denote the aggregate demand for domestic goods coming from home and abroad.

Monetary policy takes the form of a Taylor rule with interest-rate smoothing, according to which the nominal interest rate is set in response to current inflation, the output gap (output less its flexible-price level) and the lagged interest rate. We introduce nominal shocks to the interest rate rule as ε_t^M and ε_t^{M*}. In this model, ε_t^M and ε_t^{M*} are interpreted as monetary policy shocks. More generally, we could think of ε_t^M and ε_t^{M*} as exogenous disturbances equivalent to a loosening of monetary conditions, e.g. a nonfundamental depreciation of the home currency. The loglinearised policy reaction functions at home and abroad are given by

$$\begin{aligned} i_t &= \Gamma_\pi\pi_t+\Gamma_y\left(y_t-y_t^{flex}\right)+\Gamma_{i-1}i_{t-1}-\varepsilon_t^M \\ i_t^* &= \Gamma_\pi^*\pi_t^*+\Gamma_y^*\left(y_t^*-y_t^{flex*}\right)+\Gamma_{i-1}^*i_{t-1}^*-\varepsilon_t^{M*} \end{aligned}$$

Purchasing power parity (PPP) holds when the consumption basket costs the same in two countries. The real exchange rate, S_t^R, is defined as the cost of a basket of goods in the home country relative to the foreign country. Absolute PPP implies that $S_t^R=1$. When S_t^R increases (decreases), this is called a depreciation (appreciation).

$$S_t^R=\frac{S_tP_t^*}{P_t}$$

S_t denotes the nominal exchange rate, which is the price of foreign currency in terms of domestic currency. The price of a basket of goods may vary between countries due to the presence of nontraded goods, home bias in consumption or international market segmentation. The real exchange rate can be rewritten to show the three channels of deviations from PPP, the market segmentation channel, the home bias channel (which is a function of the terms of trade) and the internal real exchange rate channel. We use the price indices to express the real exchange rate in terms of the relative price of traded goods and the relative prices of nontraded goods in the two countries.

$$S_t^R = \frac{S_t P_t^*}{P_t} = \frac{S_t P_{T,t}^*}{P_{T,t}} \frac{\left(\frac{P_{N,t}^*}{P_{T,t}^*}\right)^{(1-\gamma^*)}}{\left(\frac{P_{N,t}}{P_{T,t}}\right)^{(1-\gamma)}}$$

The first part is the real rate of exchange for traded goods, the second part is the relative prices of nontraded goods in the two countries (the internal real exchange rate). The first part can be split into two separate components: deviations from the law of one price for traded goods (market segmentation channel) and differences in preferences of Home and foreign consumers (home bias channel).

$$\frac{S_t P_{T,t}^*}{P_{T,t}} = \left(\frac{S_t P_{H,t}^*}{P_{H,t}}\right)^{\nu^*} \left(\frac{S_t P_{F,t}^*}{P_{F,t}}\right)^{(1-\nu^*)} \left(\frac{P_{F,t}}{P_{H,t}}\right)^{(\nu-\nu^*)}$$

Total real exchange rate decomposition

$$S_t^R = \left[\left(\frac{S_t P_{H,t}^*}{P_{H,t}}\right)^{\nu^*} \left(\frac{S_t P_{F,t}^*}{P_{F,t}}\right)^{(1-\nu^*)}\right] \left[\left(\frac{P_{F,t}}{P_{H,t}}\right)^{(\nu-\nu^*)}\right] \left[\frac{\left(\frac{P_{N,t}^*}{P_{T,t}^*}\right)^{(1-\gamma^*)}}{\left(\frac{P_{N,t}}{P_{T,t}}\right)^{(1-\gamma)}}\right] \tag{7}$$

The first part is the market segmentation component, the second part the home bias component and the third part is the internal real exchange rate component. PPP holds if each of the three components is equal to one.

1. Unless all goods are traded, $\gamma, \gamma^* \neq 1$ and so the internal real exchange rate channel is different from one.

2. With home bias in consumption, $\nu > \nu^*$ and so the second component is different from one.

3. Assume that the two countries have different price elasticities of demand for good H. If firms in the home country can price discriminate between the two countries, the law of one price fails and so $S_t P_{H,t}^*/P_{H,t} \neq 1$. Similarly for the foreign tradable good F. Then the market segmentation component is different from one.

2.2 Signs of impulse responses

Benigno and Thoenissen calibrate the model and analyse the dynamic adjustment to productivity shocks. Here, we do no reproduce their findings but instead explain the expected short run responses of selected variables to four types of shocks: traded sector productivity shocks, economy-wide productivity shocks, demand shocks and nominal shocks. In section 3, we show how these expected responses can be used to identify a vector autoregression.

2.2.1 Traded sector productivity shocks

In this model, productivity improvements are shared between consumers (in terms of lower prices) and workers (in terms of higher wages). The shares depend on the relative size of the monopolistic distortions in the labour and product markets.

From the production function (2), we see that a positive shock to productivity in the traded goods sector, $A_{H,t}$, raises the amount of goods that can be produced with a given labour input. Due to the monopolistic distortion in the product markets, firms set prices as a markup over marginal cost. The increase in productivity directly reduces their marginal cost, and therefore prices in the traded goods sector must fall as we move down the demand curve.

The labour market is also imperfectly competitive. Household unions set the wage rate as a markup over the marginal rate of substitution between labour and consumption. As firms produce more to meet the increased demand, consumption of home tradables rises and the marginal utility of consumption falls accordingly. From equation (6) we see that wages rise.

Because labour is mobile between the two sectors, wages increase in the whole economy. In the nontradables sector, where productivity has not changed, these higher wages imply higher marginal costs and therefore higher prices. As the prices of nontradable goods rise relative to those of tradable goods, the internal real exchange rate appreciates. Since the internal real exchange rate is a component of the (overall) real exchange rate, as we can see from equation (7), this effect in isolation would lead to a real appreciation. However, an increase in the supply of home tradables leads to a deterioration in the terms of trade, which we define as the price of home tradables relative to the price of foreign tradables. In terms of equation (7), $P_{F,t}/P_{H,t}$ increases. Therefore, the real exchange rate depreciates via the home bias channel. The net effect of tradable sector productivity shocks on the real exchange rate is ambiguous.

2.2.2 Economy-wide productivity shocks

The same arguments apply to an increase in productivity in the nontradable goods sector, $A_{N,t}$. Due to the assumption of a linear production function, an economy-wide increase in productivity can be derived from summing the responses to shocks to traded sector and nontraded sector productivity shocks. In this scenario, there are no spillover effects between sectors and the Balassa-Samuelson effect does not arise. We expect a reduction in prices, an increase in total output and a real depreciation. In addition, a productivity shock across both sectors leads to a depreciation in the internal real exchange rate, as nontraded goods become cheaper relative to the basket of imported and home-produced tradables. This is because although the ratio of home-produced tradables to non-tradables prices, $P_{H,t}/P_{N,t}$, remains unchanged, both prices fall relative to the price of foreign-produced tradables, $P_{F,t}$. So $P_{N,t}$ falls more than $P_{T,t}$.

2.2.3 Consumption shocks

We can extend the Benigno-Thoenissen model to incorporate demand shocks. Consider an exogenous permanent rise in demand for home-produced goods. As home output increases to accommodate this extra demand, the prices of home-produced goods, $P_{H,t}$ and $P_{N,t}$, go up. We see from equation (7) that the real exchange rate appreciates via the home bias channel. The internal real exchange rate is expected appreciate too, as $P_{N,t}$ rises more than $P_{T,t}$.

2.2.4 Nominal shocks

Positive nominal shocks, which are incorporated as an exogenous variable, ε_t^M, in the Taylor rule, lower the nominal interest rate. This has the effect of increasing current consumption at the expense of future consumption. Current output increases to meet demand. As the consumption price index rises, the real exchange rate depreciates. A nominal shock should increase all prices in the same way, and should therefore not affect the internal real exchange rate.

3 Empirical analysis

We estimate a structural vector autoregression in order to examine the effect of productivity shocks (in the whole economy as well as in the traded sector) on the real exchange rate, while controlling for demand shifts and nominal factors. To identify the structural shocks to the system, we use sign restrictions[5] on short run responses, building on the model outlined in the previous section. This technique was pioneered by Uhlig (1999) and Canova and De Nicoló (2002). Using sign restrictions to identify a VAR avoids some problems that arise in the context of short run or long run zero restrictions. See Canova and De Nicoló (2002) for details.

Peersman and Farrant (2004) use short run sign restrictions based on the model predictions in Clarida and Gali (1994) to identify supply, demand and nominal shocks in a three-variable VAR model of output, prices and the real exchange rate. Our estimation exercise extends the Peersman-Farrant approach. In addition to identifying productivity shocks to the whole economy, we want to control for the effect of sector-specific productivity shocks. We do this by adding the relative price of non-tradables to tradables in the two countries, the internal real exchange rate. This variable allows us to distinguish between productivity shocks that affect the whole economy and those that are limited to the tradable goods sector.

3.1 Data

Our sample runs from 1981Q1 to 2003Q1. Our four data series[6], shown in figure (1), are the ratio of US to euro area real GDP, Y_t, the ratio of US to euro area traded goods prices, $P_{T,t}$, the real euro-dollar exchange rate, S_t^R, and the euro-dollar internal real exchange rate, $IRER_t$, as defined in section (2). The data sources are given in the appendix. The US is regarded as the home country. The data series for the euro area before 1999Q1 are from the ECB Working Paper 'An area-wide model for the euro area' by Fagan, Henry and Mestre. All data thereafter are from IMF International Financial Statistics. Nontraded goods prices are proxied by the consumer price index, traded goods prices are proxied by the producer price index. The nominal euro-dollar exchange rate is constructed as described in Schnatz, Vijselaar and Osbat (2003): it is computed as the geometric weighted average of the dollar exchange rates of the euro legacy currencies. The real exchange rate is the nominal euro-dollar exchange rate multiplied by the ratio of consumer prices in euro-area and the US.

[5] I thank Gert Peersman for sharing his programming code.

[6] We take logs of all variables and rescale them by a factor 100, which gives us the percentage difference between the US and the euro area.

3.2 Preliminary analysis and unrestricted VAR

We conduct augmented Dickey-Fuller tests with four lags, a constant and a trend, on all series in levels and in first differences. The null hypothesis of nonstationarity cannot be rejected at the 5% significance level for all of the levels series. The first differences of relative output, relative traded goods prices and the internal real exchange rate are stationary at the 5% level, the first difference of the real exchange rate is stationary at the 10% level. We proceed assuming that all four variables are integrated of order 1 and estimate a VAR in first differences. The lag length of the VAR was set equal to three as selected by the likelihood ratio test. We therefore estimate the following model.

$$\Delta \mathbf{x}_t = c + \beta(L)\,\Delta \mathbf{x}_{t-1} + e_t$$

where $\mathbf{x}_t = \left(\log Y_t, \log P_{T,t}, \log S_t^R, \log IRER_t\right)$, and $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2$ and c is a vector of constants and linear trends.

3.3 Identification of the structural shocks

Having estimated the unrestricted VAR, we obtain reduced form residuals, e_t, which are a linear combination of the four underlying structural shocks, ε_t. The unknown matrix A links the two types of shocks is: $e_t = A\varepsilon_t$. Imposing the normalisation that all structural shocks have unit variance and are uncorrelated, we have the following relation:

$$\Sigma_e = AA' \tag{8}$$

where Σ_e is the covariance matrix of the residuals. We want to obtain estimates of the orthogonal structural shocks, ε_t. Therefore, our aim is to find a matrix A for which equation (8) holds. Since the number of possible matrices A is infinitely large, we impose further restrictions. The impulse responses arising from the structural shocks should have the signs given in table (1). Using the method in Peersman (2003), explained in detail in the appendix, we search over the space of orthogonalisations and check the signs of the impulses responses each time. If they match our priors, we save them. We order the resulting impulse response functions and variance decompositions and report the median, as well as the 16th and 84th percentile (one standard deviation) error bands. For each type of shock, the signs of the short run responses are summarised in the table below.

We identify the four shocks as follows. In the short run, both economy-wide productivity shocks and traded sector productivity shocks lead to a rise in output and a fall in traded sector goods prices, but can be discriminated by their effect on the internal real exchange rate. Nominal shocks and demand shocks raise prices and also increase total output in the

Table 1: **Signs of responses used for identification**

	Variable			
	Y_t	$P_{T,t}$	S_t^R	$IRER_t$
Type of shock				
overall productivity shock	$\geqslant 0$	$\leqslant 0$	?	$\geqslant 0$
demand shock	$\geqslant 0$	$\geqslant 0$	$\leqslant 0$	?
nominal shock	$\geqslant 0$	$\geqslant 0$	$\geqslant 0$	?
traded sector productivity	$\geqslant 0$	$\leqslant 0$	?	$\leqslant 0$

short run. We distinguish between the two shocks through the restriction that demand shocks reduce the real exchange rate, while nominal shocks increase it.

The horizon over which the sign restriction is binding, is set equal to 4 quarters for output, traded goods prices and the internal real exchange rate and to 1 quarter for the real exchange rate. The idea behind this is that the real exchange rate is a more flexible variable than output or prices. Setting a higher value reduces the number of plausible decompositions.

The sign of the response of the real exchange rate to productivity shocks (economy-wide and in the traded sector) is not restricted, but is instead determined by the data. Similarly, the response of the internal real exchange rate to demand and nominal shocks is unrestricted.

3.4 Estimation results

3.4.1 Impulse response functions

Graphs of the impulse response functions are given in the appendix, figure (2). The median response to a positive shock is given by the continuous black line, while the dotted lines represent the 16th and 84th percentile error bands. Impulse responses are significant in the cases where the upper and lower error bands have the same sign. On the x-axis, the forecast horizon is given in quarters.

First of all, we note that the impulse responses of output and prices make sense economically. The impulse responses for output show that productivity and demand shocks lead to significant increases in output in the long run, which is consistent with many macroeconomic models. Nominal shocks, by construction, lead to temporary booms, but are insignificant at longer horizons. Prices of tradable goods rise permanently following a nominal shock and fall permanently in reponse to productivity shocks. Demand shocks raise traded goods prices only at short horizons.

In response to sectoral productivity shocks, the real exchange rate appreciates significantly in the first quarter. The effect at longer horizon is, however, uncertain. A permanent appreciation of the *internal* real exchange rate indicates that the Balassa-Samuelson effect plays a role in the determination of relative prices, but this does not necessarily translate into a permanent appreciation of the (overall) real exchange rate.

In the case of economy-wide productivity shocks, we expect the real exchange rate to depreciate[7]. The corresponding impulse response function suggests that an appreciation is more likely, although the result is not significant. This is a puzzling result. Based on our macroeconomic model, we have filtered out the only channel through which productivity shocks may cause an appreciation, which is the Balassa-Samuelson effect. Yet, even controlling for this effect, we still find evidence that an overall productivity improvement makes a currency stronger. We conclude that the standard macroeconomic model outlined above cannot capture the link between productivity and the real exchange rate very well. In other words, productivity shocks affect exchange rates in ways that are missing in the standard macro model. Of course, this result holds only for the euro-dollar exchange rate and for the period under study. Further research on other exchange rates and sample periods should make out if this result holds more widely.

Another striking finding, which confirms Peersman and Farrant (2004) is that nominal shocks have significant effects on the real exchange rate at long horizons. Many empirical studies using VARs impose the restriction that nominal shocks have no permanent effect on real variables, reflecting long run money neutrality. Our result might just demonstrate that this restriction is rather stringent in small samples, especially if the long run turns out to be very long. Nevertheless, the result that nominal shocks have permanent effects is rather worrying.

We label any shock that increases output, prices and the real exchange rate as a nominal shock. It is conceivable that instead of monetary policy shocks, we have identified positive non-fundamental shocks to the real exchange rate, which have the same effects: they boost output through increased exports and raise import prices, which enter the general price level. Peersman and Farrant find that these 'pure exchange rate shocks' explain a substantial amount of the real exchange rate variability in the very short run. Finally, demand shocks result in significant long run appreciations in the real exchange rate.

3.4.2 Variance decompositions

Decompositions of the forecast error variances are given in table (3). Again, we report the median value and the 16th and 84th percentile error bands. Notice that the variance decompositions do not sum to one, as they would in the case of a single decomposition.

As we would expect, productivity shocks explain most of the output variation in the long run. Traded goods prices are affected mostly by sectoral productivity shocks and, at

[7]Even though this is an unambiguous result of a number of macroeconomic models, many empirical papers find a perverse supply effect of productivity on the real exchange rate, i.e., an appreciation. See, for example, Clarida and Gali (1994) and other papers that have estimated their model.

longer horizons, by nominal shocks.

Demand and nominal shocks explain most of the variation in the real euro-dollar exchange rate. Nominal shocks dominate at short horizons. The importance of nominal shocks contrasts with earlier findings of researchers who use long run neutrality restrictions to identify shocks. Empirical papers estimating the Clarida-Gali (1994) model tend to find only a small role for nominal disturbances. The results of estimating of structural VARs are very sensitive to the type of restrictions imposed.

Productivity shocks explain little of the observed variation in the real exchange rate. Productivity shocks in the traded sector accounts for 9.8% (33.4% and 1.3%) and economy-wide ones account for 9.7% (34.7% and 1.4%) of the variation in the long run real exchange rate (upper and lower error bands given in brackets).

One problem with our approach is that we identify only uncorrelated shocks. However, an increase in productivity might itself boost demand, as people expect to earn more in the future. Then the positive demand effect on prices may overwhelm the negative price effect of the productivity increase. In that case, we only identify this as a "demand shock", while the underlying cause is an anticipated rise in productivity.

3.4.3 Historical contributions

Having identified the structural shocks to our system, we can divide each data series into a base projection (the path that the variable would have followed had there been no exogenous shocks) and the various shock component series, reflecting the deviations from the base projection. This allows us to compute the median contribution of each shock to the path of, say, the real exchange rate in a particular period. For example, we can address the question what caused the dollar's appreciation against the euro in 1999/2000.

For this purpose, we look at three sub-periods in our sample, which were characterised by large and continuous movements in the real euro-dollar rate. The chosen sub-periods (and the corresponding change in the deviation of the euro-dollar rate from base projection) are 1982Q1-1984Q4 (-35%), 1985Q1-1988Q1 (+59%), 1999Q1-2000Q4 (-26%). Graphs showing the decomposition are given in the appendix, figures (4) to (6). On these graphs, the real exchange rate is shown in deviations from the baseline projectioin. Note that the sum of the shock components and the baseline projection is not equal to the real exchange rate series, as it would be in the case of a single Monte Carlo draw.

Focussing on the period 1999-2000, we note that nominal shocks alone accounted for a 15% appreciation of the dollar, while demand shocks were responsible for a 11% appreciation. Economy-wide productivity shocks had hardly any effect on the real exchange rate, while sectoral productivity shocks accounted for a 2% *de*preciation of the dollar. Similar pictures emerge from the other graphs; the relative importance of each shock is about the same throughout our sample.

Our findings show that higher productivity growth in the US relative to the euro area cannot be directly responsible for the appreciation of the dollar against the euro at the beginning of Economic and Monetary Union. It may have played a role to the extent that it triggered an aggregate demand shock. Nominal disturbances weigh even stronger than a US demand shock and swamp any productivity effects. Since monetary policy has not been very different in the US compared with the euro area, the only remaining explanation is that there were (and are) exogenous shocks to the exchange rate, which are unrelated to fundamental variables.

4 Conclusion

The motivation for this paper is the conjecture that productivity differentials are at the origin of the dollar's appreciation at the end of the 1990s. We analyse the effect of productivity on the real euro-dollar exchange rate, using a structural VAR. Our identifying restrictions build on the 'new open macro'-type model in Benigno and Thoenissen (2003), which suggests that productivity shocks in the tradable goods sector can lead to a real appreciation through the Balassa-Samuelson effect. We identify economy-wide and sector-specific productivity shocks, demand shocks and nominal shocks.

We find that the Balassa-Samuelson effect on the relative price of nontradables has not translated into a significant real appreciation of the dollar during the 1980s and 1990s. Consequently, this cannot explain the behaviour of the exchange rate in 1999-2001. In general, changes in productivity account for only a small fraction of the variation in the real exchange rate, in contrast with demand and nominal shocks. Our findings indicate that productivity shocks affect the exchange rate largely indirectly, through aggregate demand. They also show that the effect of macroeconomic shocks are outweighed by exogenous exchange rate fluctuations not driven by fundamental variables.

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5 Appendix

5.1 Data

The sample period is 1981Q1 to 2003Q1. All indexed series have base year 1995.

For each euro legacy currency, the *nominal exchange rate* series (national currency per US dollar) is taken from the IFS (line rf) and divided by its fixed euro conversion rate with the euro. The synthetic euro-dollar exchange rate before 1999Q1 is computed by taking a geometrically weighted average of the euro legacy currencies' exchange rates vis-à-vis the dollar. The weights are those proposed in Schnatz, Vijselaar and Osbat (2003).

Taking the US as the home country, the *real exchange rate* is computed as the inverse of the nominal exchange rate, multiplied by the ratio of euro area to US consumer price index. Sources of the consumer prices series are given below.

Consumer prices before 1999Q1 are taken from the area-wide model (AWM), and after 1999Q1 from the IFS (line 64h). For the US, producer prices are from the IFS (line 64).

For the US, the euro member countries before 1999Q1 and the euro area thereafter, *producer or wholesale prices* are from the IFS (line 63). Euro area producer prices before 1999Q1 are computed as a geometrically weighted average of the producer price indices of the euro members, using the same trade weights as proposed in Schnatz, Vijselaar and Osbat (2003). Due to a lack of data, Portugal is not included in the euro area producer price index.

The *relative price* ratio between the nontraded and traded goods sector is proxied by consumer prices divided by producer prices.

GDP series for the US is line 11199BVRZF... from the IFS. For the euro area, data before 1999Q1 is from the AWM, series YER. From 1999Q1, data from the IFS is used (line 16399BVRZF...)

5.2 Methodology

A vector autoregression (VAR) is a system of equations in which every endogenous variable is a function of all lagged endogenous variables[8]. Consider the vector of the n endogenous variables $\mathbf{x}_t = (x_{1t}, x_{2t}, \ldots, x_{nt})'$ and the vector of n unobservable structural disturbances $\mathbf{z}_t = (z_{1t}, z_{2t}, \ldots, z_{nt})'$. In its structural form, the VAR can be written

$$B\mathbf{x}_t = C(L)\,\mathbf{x}_{t-1} + \mathbf{z}_t \tag{9}$$

where $C(L) = C_0 + C_1L + C_2L^2 + \cdots + C_qL^q$, L is the lag operator, the $(n \times n)$ matrix B comprises the parameters on the contemporaneous endogenous variables. A non-zero element in B indicates that an endogenous variable has a contemporaneous effect on another. Assuming that B is invertible, we derive the reduced form VAR by multiplying the structural form by B^{-1}

$$\mathbf{x}_t = B^{-1}C(L)\,\mathbf{x}_{t-1} + B^{-1}\mathbf{z}_t \tag{10}$$

If we model all structural disturbances as unit root processes, then $\Delta\mathbf{z}_t = \mathbf{z}_t - \mathbf{z}_{t-1} = \varepsilon_t$. If the variables in $\mathbf{x}_t$ are I(1) and not cointegrated, it is valid to estimate the VAR in first differences. Applying the first difference operator $\Delta = (1 - L)$ to equation (10), we have

$$\Delta\mathbf{x}_t = \beta(L)\,\Delta\mathbf{x}_{t-1} + e_t \tag{11}$$

where $\beta(L) = \beta_0 + \beta_1L + \beta_2L^2 + \ldots + \beta_qL^q$ and $e_t = B^{-1}\varepsilon_t$. The lag length q can be determined using, for example, a sequential likelihood ratio test or the Akaike information criterion. An equation-by-equation OLS regression of the reduced form (11) yields estimates of the coefficients, $\beta(L) = B^{-1}C(L)$ and the reduced form residuals $e_t = B^{-1}\varepsilon_t$, as well as the variance-covariance matrix of the residuals, Σ_e.

5.2.1 VAR identification with sign restrictions

We want to identify shocks that are mutually orthogonal and have unit variance, i.e. they should have $\Sigma_\varepsilon = E(\varepsilon_t\varepsilon_t) = I$. Define $A = B^{-1}$; then $\Sigma_e = AA'$. Without imposing any restrictions, there are infinitely many possible decompositions of Σ_e. For example, using the eigenvalue-eigenvector decomposition, we have $\Sigma_e = PDP'$, where D is a diagonal matrix of eigenvalues and P consists of the eigenvectors of Σ_e and thus $A = PD^{\frac{1}{2}}$. However, any decomposition of Σ_e, such that $\Sigma_e = AQQ'A'$, where Q is orthonormal (i.e. $QQ' = I$) is also valid. We use the decomposition $Q = \prod_{m,n} Q_{m,n}(\theta_i)$, where $Q_{m,n}(\theta_i)$ are rotation

[8] This exposition follows Keating (1992). For simplicity, we do not consider deterministic variables such as constants, time trends or seasonal dummies.

matrices of the following form.

$$Q_{m,n}(\theta_i) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cos\theta_i & \cdots & -\sin\theta_i & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ 0 & \cdots & \sin\theta_i & \cdots & \cos\theta_i & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & \cdots & \cdots & 0 & \cdots & \cdots & 1 \end{bmatrix}$$

where the subscript (m,n) indicates that rows m and n are rotated by an angle θ_i, $0 < \theta_i < \pi$ and $i = 1, \ldots, 6$. With four variables, the number of possible bivariate rotations is six ($C_2^4 = 6$). We have:

$$Q = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_2 & -\sin\theta_2 \\ 0 & 0 & \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \cos\theta_4 & 0 & 0 & -\sin\theta_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\theta_4 & 0 & 0 & \cos\theta_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_5 & -\sin\theta_5 & 0 \\ 0 & \sin\theta_5 & \cos\theta_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_6 & 0 & -\sin\theta_6 \\ 0 & 0 & 1 & 0 \\ 0 & \sin\theta_6 & 0 & \cos\theta_6 \end{bmatrix}.$$

Any rotation can be produced by varying the angles θ_i in the range $[0, \pi]$. A "rotation" amounts to drawing numbers for θ_i from a uniform distribution: the probability of drawing one particular number is constant over the range. Because the six θ_i's can take on infinitely many values in this range, we divide it into intervals separated by $M = 12$ equally spaced points, such that we have a finite number of rotations over which to search. With six θ_i's and twelve possible values for each θ_i, i.e. twelve possible values for Q_i, there are 12^6 possible rotation matrices Q.

5.2.2 Computing error bands with Monte Carlo simulations

In the following[9], we explain how to compute standard error bands using Monte Carlo integration. This is a Bayesian method in which we make draws from the posterior distribution of the impulse response functions.

In Bayesian statistics, we try to improve upon our estimates by incorporating prior information (i.e., information we have before observing the sample) into our analysis. The true value of our estimator, θ, is regarded as a random variable. Reflecting the uncertainty we have about θ, inference therefore takes the form of a probability statement. Any prior information about θ is represented by the prior density function, $f(\theta)$. In this framework, the sample likelihood is the density of y conditional on a particular on the value of the random variable θ, denoted $f(y|\theta)$. The marginal density multiplied by the prior density equals the joint density.

[9] Discussion based on Hamilton (1994), Uhlig (1999) and RATS 5 user's guide by Estima (2000).

$$f(y,\theta) = f(y|\theta) \cdot f(\theta)$$

Probability statements about θ, once the data y have been observed, are made on the basis of the posterior density function, given by

$$f(\theta|y) = \frac{f(y,\theta)}{f(y)}$$

In the case of a linear regression model given by $y_t = \mathbf{x}_t'\boldsymbol{\beta} + e_t$, prior information about $\boldsymbol{\beta}$ can be represented by a $N\left(m, \sigma^2 M\right)$ distribution, where m is the best guess of the true coefficient vector $\boldsymbol{\beta}$ and $\sigma^2 M$ is the uncertainty surrounding this guess. This assumes that we know the true variance σ^2. However, since σ^2 is unknown, we need to assume a prior distribution for it. The gamma distribution[10] lends itself to this application.

Let's look at Bayesian inference in a VAR framework. Write the vector autoregression with n variables as

$$\mathbf{y}_t = (\mathbf{I}_n \otimes \mathbf{X}_t)\,\boldsymbol{\beta} + \mathbf{u}_t \qquad t = 1, \ldots, T.$$

where $\mathbf{y}_t$ is $(n \times 1)$, $\mathbf{X}_t$ is $(1 \times k)$, $\boldsymbol{\beta}$ is $(kn \times 1)$, the coefficient matrix in its columnwise vectorised form, and k is the number of coefficients per equation. Assume that the errors are independently and identically distributed as $\mathbf{u}_t \sim N(0, \Sigma)$, where Σ is $(n \times n)$, and the likelihood function is conditional upon the values of $\mathbf{y}_t$ for t less than 1. Let $\mathbf{b}$ and $\mathbf{S}$ be the OLS estimates of $\boldsymbol{\beta}$ and Σ. With a joint prior distribution for β, Σ given by

$$f(\boldsymbol{\beta}, \Sigma) \propto |\Sigma|^{-(n+1)/2}$$

the posterior distribution of Σ is Normal-inverse Wishart[11], with

$$\Sigma^{-1} \sim Wishart\left((T\mathbf{S})^{-1}, T\right) \text{ and, given } \mathbf{S}, \tag{12}$$

$$\beta|\Sigma \sim N\left(\mathbf{b}, \Sigma \otimes (\mathbf{X}\prime\mathbf{X})^{-1}\right) \tag{13}$$

where $\mathbf{X}$ is $(T \times k)$.

Step 1: Draws for Σ, $\mathbf{S}_{MC}$, can therefore be obtained by drawing from a Wishart distribution centred on the identity matrix, inverting and pre- and postmultiplying by the factor matrices for Σ.

[10] Let $\{Z_i\}_{i=1}^{N}$ be a sequence of i.i.d. $N\left(0, \tau^2\right)$ variables. Then $W = \sum_{i=1}^{N} Z_i^2$ is said to have a gamma distribution with N degrees of freedom and scale parameter λ, indicated $W \sim \Gamma(N, \lambda)$, where $\lambda = 1/\tau^2$. W has the distribution of τ^2 times a $\chi^2(N)$ variable.

[11] If X_i for $i = 1, ..., m$ has a multivariate normal distribution with mean vector $\mu = 0$ and covariance matrix Σ, and X denotes the $m \times p$ matrix composed of the row vectors X_i, then the $p \times p$ matrix $X'X$ has a Wishart distribution with scale matrix Σ and degrees of freedom parameter m. The Wishart distribution is most typically used when describing the covariance matrix of multinormal samples.

Step 2: To get draws for β, the covariance matrix in (13) is factored into

$$(P_\Sigma \otimes P_{XX})\,(P_\Sigma \otimes P_{XX})' \text{ where } P_\Sigma P_\Sigma' = S \text{ and } P_{XX}P_{XX}' = (\mathbf{X}'\mathbf{X})^{-1}$$

where P_Σ is $(n \times n)$ and P_{XX} is $(k \times k)$.
Premultiplying a $(kn \times 1)$ draw of random Normals, $vec\,(V)$, by $(P_\Sigma \otimes P_{XX})$ gives the desired deviation from the OLS coefficients. The structure of the Kronecker product can be exploited[12] to simplify this to $P_{XX}VP_\Sigma'$ where V is a $k \times n$ matrix of Normal draws. This produces a $k \times n$ coefficient matrix with the distribution we want. Draws for the VAR coefficients, $\mathbf{b}_{MC}$, are then computed as the sum of the OLS coefficients $\mathbf{b}$ and the deviations from the OLS coefficients, $P_{XX}VP_\Sigma'$.

Step 3: With draws for the covariance matrix, $\mathbf{S}_{MC}$, and the coefficients, $\mathbf{b}_{MC}$, and choosing a particular decomposition matrix A, we can obtain impulse response functions, variance decompositions and historical decompositions.

Steps 1 to 3 constitute one Monte Carlo draw. Repeating these three steps, say, 1000 times, we obtain posterior distributions of the parameters of interest. The problem we face is that for each Monte Carlo replication, we ought to try 12^6 possible rotations for the decomposition matrix A, check the impulse response functions and save the ones that meet our sign restrictions. This would imply computing and checking 12^6*1000 decompositions. Since this is computationally too demanding, we use the method proposed by Peersman (2003). For each Monte Carlo draw, we try one possible rotation for the decomposition matrix A and check the signs of the impulse responses, saving the solutions that match our restrictions. We continue until we have 1000 valid decompositions.

[12] $(P_\Sigma \otimes P_{XX})\,vec\,(V) = vec\,(P_{XX}VP_\Sigma')$

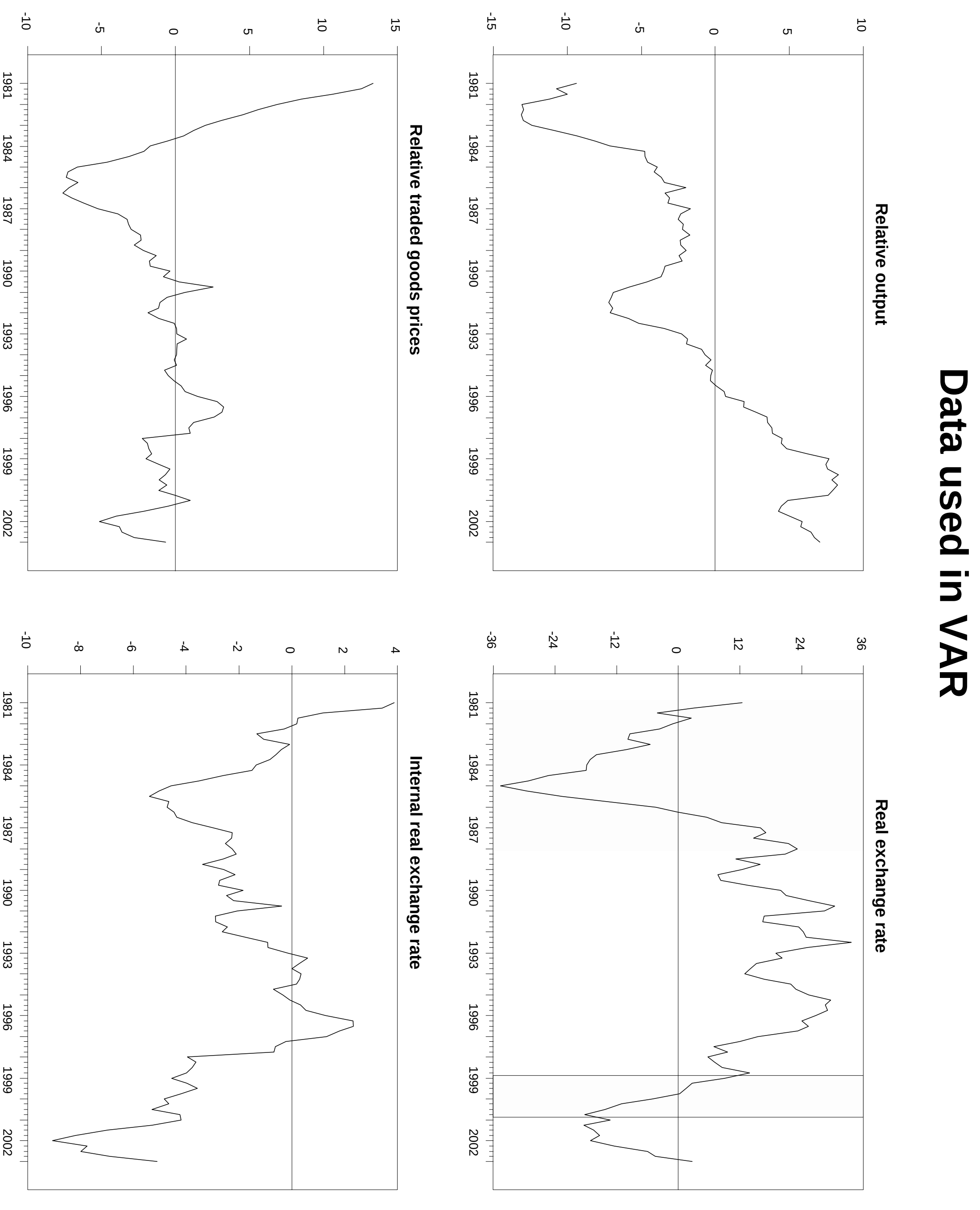

Figure 1: Data series

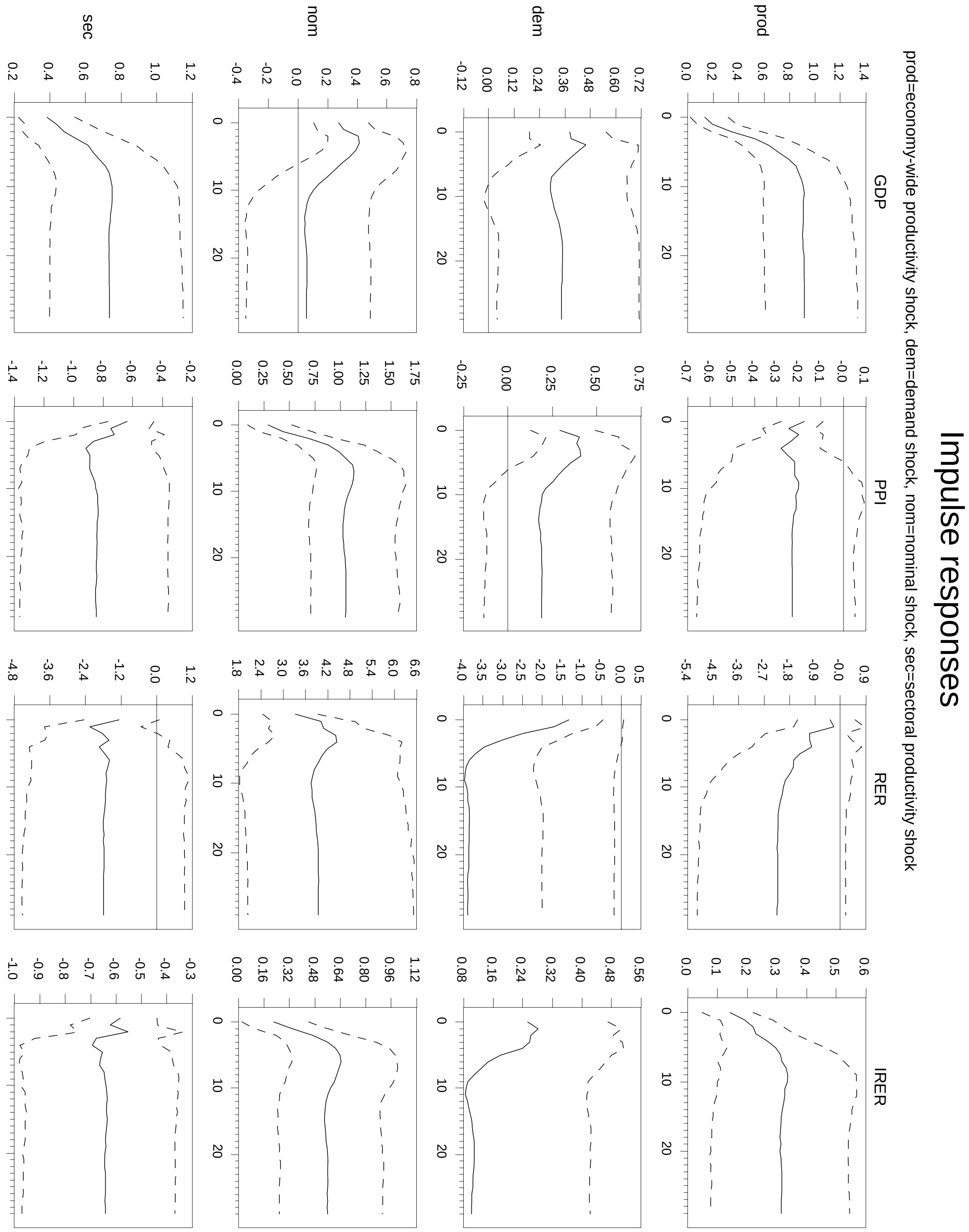

Figure 2: Impulse response functions

Forecast error variance decomposition for relative output

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
horizon	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*
1 quarter	0,036	0,204	0,001	0,306	0,615	0,075	0,153	0,447	0,023	0,290	0,590	0,106
1 year	0,142	0,359	0,061	0,221	0,503	0,062	0,156	0,439	0,042	0,275	0,549	0,090
5 years	0,343	0,564	0,173	0,107	0,319	0,027	0,086	0,317	0,027	0,302	0,535	0,109
10 years	0,438	0,675	0,236	0,077	0,265	0,015	0,054	0,191	0,014	0,311	0,540	0,104

Forecast error variance decomposition for relative traded goods prices

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
horizon	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*
1 quarter	0,045	0,117	0,011	0,123	0,355	0,023	0,119	0,385	0,011	0,599	0,831	0,312
1 year	0,040	0,093	0,013	0,111	0,289	0,032	0,303	0,598	0,128	0,464	0,701	0,178
5 years	0,031	0,099	0,009	0,059	0,200	0,015	0,456	0,740	0,222	0,367	0,632	0,104
10 years	0,032	0,138	0,006	0,036	0,161	0,007	0,504	0,777	0,233	0,329	0,620	0,066

Forecast error variance decomposition for the real exchange rate

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
horizon	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*
1 quarter	0,030	0,144	0,000	0,101	0,274	0,015	0,652	0,855	0,358	0,101	0,349	0,010
1 year	0,040	0,181	0,008	0,169	0,356	0,046	0,554	0,768	0,259	0,121	0,383	0,022
5 years	0,066	0,250	0,012	0,270	0,486	0,087	0,411	0,679	0,149	0,102	0,347	0,020
10 years	0,097	0,347	0,014	0,279	0,536	0,083	0,340	0,625	0,100	0,098	0,334	0,013

Forecast error variance decomposition for the internal real exchange rate

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
horizon	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*	*median*	*upper*	*lower*
1 quarter	0,034	0,087	0,004	0,115	0,400	0,011	0,090	0,347	0,006	0,621	0,856	0,343
1 year	0,053	0,113	0,020	0,099	0,293	0,013	0,236	0,540	0,056	0,518	0,748	0,238
5 years	0,076	0,175	0,019	0,066	0,205	0,014	0,321	0,644	0,085	0,441	0,703	0,173
10 years	0,095	0,253	0,014	0,044	0,182	0,010	0,322	0,656	0,072	0,415	0,692	0,147

Figure 3: Variance decomposition

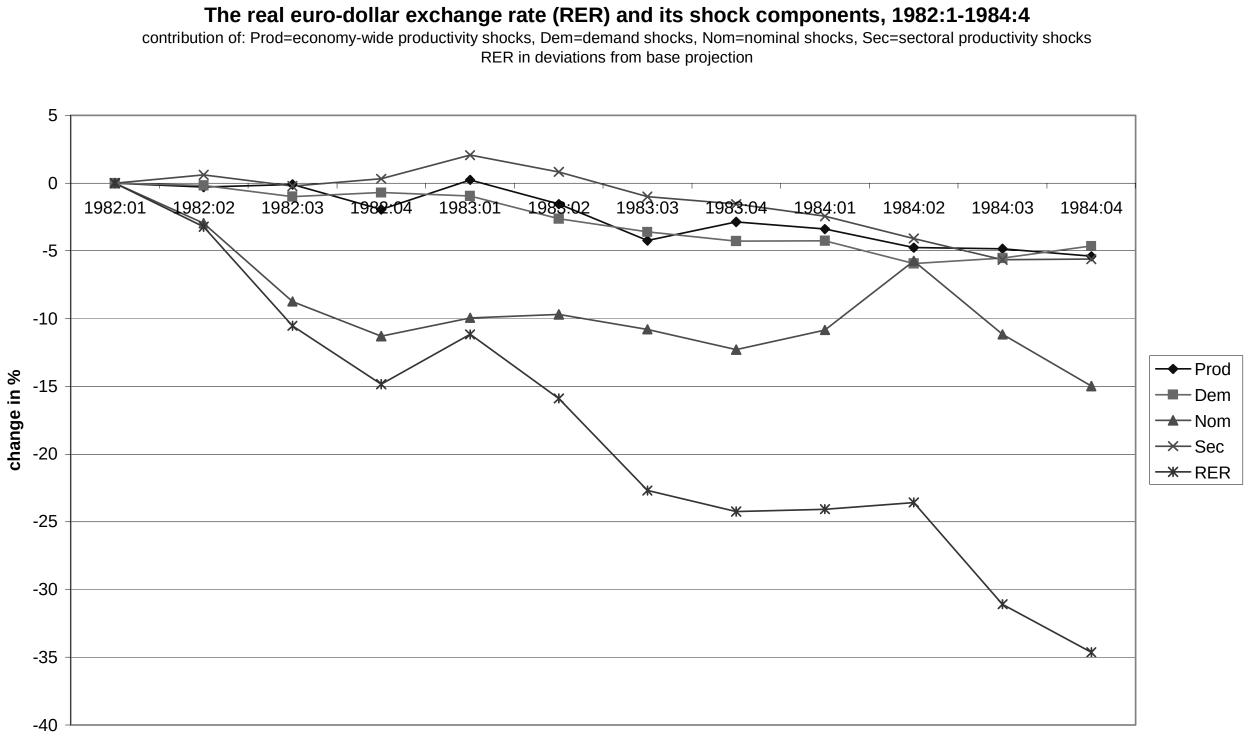

Figure 4: Historical decomposition, 1982:1-1984:4

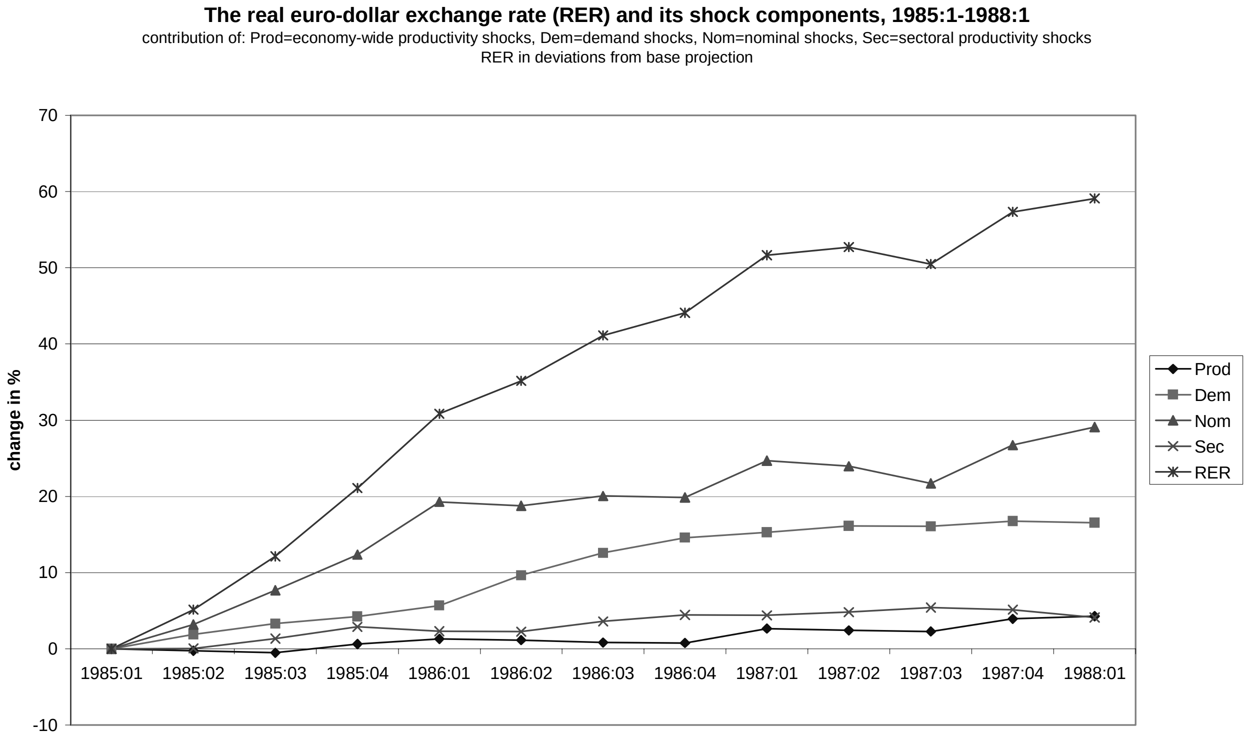

Figure 5: Historical decomposition, 1985:1-1988:1

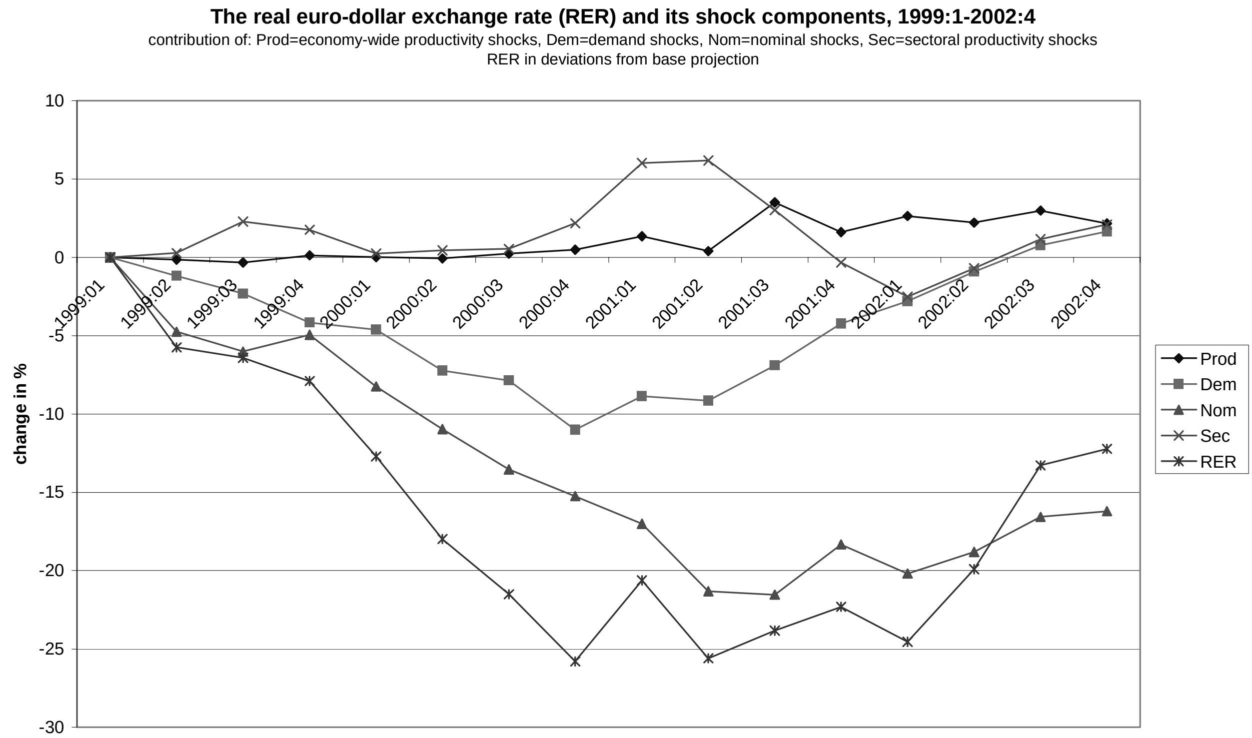

Figure 6: Historical decomposition, 1999:1-2002:4