FACULTY OF ECONOMICS AND
APPLIED ECONOMIC SCIENCES
CENTER FOR ECONOMIC STUDIES
ENERGY, TRANSPORT & ENVIRONMENT

KATHOLIEKE
UNIVERSITEIT
LEUVEN

WORKING PAPER SERIES
n°2006-05

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March 2006

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TRIP CHAINING: WHO WINS WHO LOSES?

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March 2006

Abstract

In this paper we study how trip chaining affects the pricing and equilibrium number of firms. We use a monopolistic competition model where firms offer differentiated products as well as differentiated jobs to households who are all located at some distance from the firms. Trip chaining means that shopping and commuting can be combined in one trip. The symmetric equilibriums with and without the option of trip chaining are compared. We show analytically that introducing the trip chaining option can, in the short run, only decrease the profit margin of the firms and will increase welfare. The welfare gains are however smaller than the transport cost savings. In the long run, with free entry, the number of firms decreases but welfare with trip chaining possible is still higher than when it is excluded. A numerical illustration gives orders of magnitude of the different effects.

Keywords: trip chaining, discrete choice model, general equilibrium model, imperfect competition, wage competition.

JEL Classification: D43, L13, R3

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1. INTRODUCTION

Trip chaining is considered to be a growing phenomenon in travel and activity behaviour, as individuals try to reduce the amount of travel time needed to complete daily activities, given the limitations of their time budget. Most trip chaining research has concentrated on the demand side taking the prices of products and wages as given. In this paper we pursue a different avenue of research and examine the effect that the trip chaining options by households has on the pricing and wage setting decisions of firms. Do trip chaining possibilities increase or decrease the profit margins, does this in the end also lead to more or fewer firms and how does it affect welfare?

Our starting point is an analytical, symmetric model of a city (de Palma and Proost, 2006), in which households live in the city centre and shop and work in equidistant subcentres. Each subcentre offers a different variety of the product and offers a different workplace variety. Households' consumer and labour supplier choices are modelled using a logit model. The unique firm located in each subcentre maximizes profits by setting a price and a wage that attracts the optimal number of customers and attracts the necessary workers to supply the demand addressed to it. In this model, there exists a unique symmetric short term Nash equilibrium in prices and wages and a free entry equilibrium. In the original model individuals made separate working and shopping trips. Here we relax this assumption and allow consumers to shop at the sub centre where they work.

For this model, we show that a symmetric short term and a free entry Nash equilibrium exist when the trip chaining option is introduced. We present four key results. First, in the short run, trip chaining will increase competition between subcentres and decrease mark ups as long as the love for variety in the product space is strictly different from the love for variety in the workplace space. Second, allowing trip chaining benefits consumers and increases welfare but the gain is smaller than the savings in transport costs. Third, in the free entry equilibrium, the trip chaining option decreases the number of firms. Finally, the welfare of the free entry equilibrium is higher with the trip chaining option than without the trip chaining option.

The effect of the trip chaining option on the degree of competition has been studied by Claycombe (1991), Claycombe and Mahan (1993) and Raith (1996). In their approach the workplace and the wage are fixed and they concentrate on the shopping market only. The shopping market is represented by a Hotelling type of model where evenly spaced shops offering identical products are placed along one infinite line. Evenly distributed consumers have to make exogenously determined commuting trips of a given distance and may stop for shopping on their commuting trip. Raith proves that an increasing commuting distance means that more shops will be encountered on the trip to work and this implies more intensive price competition. Our approach is different on three accounts. First we use a monopolistic competition model with differentiated products which allows the effect of the love for variety to be studied. Second, we model the two trip purposes simultaneously as we have a general equilibrium model with differentiated workplaces and differentiated products, this is important as both markets interact. Third the number of firms can be endogenous in our approach as we also study the free entry equilibrium

The model is first briefly described in Section 2 and the short-run and free equilibrium with trip chaining and without trip chaining are compared. In Section 3 we look at the welfare implications of trip chaining. A small numerical illustration to show the relative importance of different parameters is included in Section 4 and Section 5 concludes.

2. THEORETICAL FRAMEWORK

2.1. Model Setting

Imperfect competition in a city both with and without congestion has been analysed recently for a closed economy by de Palma and Proost (2006). In their model, all households live in the city centre and make trips to work and shop at *n* subcentres ($n \geq 2$) that are located at identical travel cost from the city centre. In each subcentre there is one firm that offers a differentiated product and a differentiated work place. The firms compete in wages on the labour market to attract workers and compete in prices to attract customers. Households are constrained to make separate trips for shopping and working, so trip-chaining is de facto not permitted. In the current paper we relax this assumption and allow residents to shop at their work location without making a separate journey. The model set-up is still symmetric but, in contrast to de Palma and Proost (2006), we do not include congestion in order to focus solely on the effect trip-chaining has on the price equilibrium. In this section we provide a brief description of the model set-up and derive the relevant expressions for the symmetric price equilibrium without congestion but with trip chaining.

All trips are between the city centre and the subcentres. Residents cannot travel directly from one subcentre to another. In the original model of de Palma and Proost, every work and shopping trip was a separate trip. In this paper we allow households to combine a working and a shopping trip. Residents first choose where to work and then decide whether to shop at their work location or at another subcentre; however residents can only travel between the centre and each subcentre and not between subcentres.

A homogeneous good is produced in the city centre and used as an intermediate input for the differentiated good produced in the subcentres. A quantity *c* of homogenous good is necessary per unit of the differentiated good. Each of the *N* households supplies θ units of homogeneous labour for the production of the homogeneous good in the city centre. Each household also buys exactly one unit of the differentiated good and supplies exactly one unit of differentiated labour for the production of differentiated goods in the subcentre. Summing up, in order to produce its variety of the differentiated good, each of the *n* sub centres needs four inputs: the intermediate inputs (*c* per unit), one unit of differentiated labour, a fixed set up cost (*F* units of the homogenous good) and a public capital good (roads, parking etc.) that requires *K* units of the homogenous good.

The total production possibilities of an economy with *N* households and *n* firms can then be expressed in terms of the following identity for total labour supply and demand:

$$(1+\theta)N = D + cD + nF + nK + \left[\alpha^{w} + (1-\delta)\alpha^{d} + \alpha^{h}\right]tD + G\,, \qquad (1)$$

The left hand side represents the total labour supply that is fixed: for each household we have one unit of labour supplied to the subcentres and θ units of labour supplied to the production of homogenous goods. The total demand for the differentiated good is given by *D*.

Both firms and households incur travel costs. Households have to make trips from the centre to the subcentres for working and shopping and firms have to bring the intermediate input from the centre to the subcentre. We assume that the transport cost per trip is *t* (measured in units of homogenous labour foregone per trip). The total transportation costs for commuting, shopping and shipping goods to the sub centres are given by $\left[\alpha^w + (1-\delta)\alpha^d + \alpha^h\right]tD$. The parameters α^w, α^d and α^h represent, respectively, the number of commuting and shopping trips the consumer[1] undertakes per unit of production (respectively consumption) of the differentiated good and the number of shipping trips that are necessary to deliver the intermediate good to the subcentre. The parameter $\delta\,(\leq 1)$ effectively represents the proportion of consumers who take advantage of trip-chaining. When trip chaining is not possible each unit of the differentiated good that is bought requires $\alpha^d tD$ in terms of transport costs and δ equals zero. When trip chaining is an option, transport costs for shopping can be lower and total $(1-\delta)\alpha^d tD$, where δ is endogenous as it depends on the extent of trip chaining.

The last term in (1), *G*, represents the total quantity of the homogenous consumption good that will be available after all other production costs and transport costs related to the differentiated goods have been accounted for. Inspection of this equation can give us a flavour of the trade-offs involved in trip chaining. Firstly, increasing the proportion of trip chaining δ reduces transport costs and allows higher consumption possibilities. Secondly, some consumers may give up their preferred product variant in order to save on transport costs: this means that welfare gains may be smaller than the saved transport costs. Finally, trip chaining may, by affecting the profit margins, also affect the number of subcentres in equilibrium and affect welfare in the long term. A lower number of subcentres saves on fixed costs but leads to a loss of diversity that itself has a welfare cost.

As the household preference for variety plays a key role in the trip chaining process, we first define the specification of the working and shopping preferences of the households. Next we address the behaviour of the firm and we conclude with an analysis of the market equilibrium.

In order to make the model complete, we define the government budget equilibrium and the ownership of the firms. The only role of government in this model is to supply the fixed public inputs (*K* per subcentre) and to finance this supply via a head tax on households *T* and a fixed levy *S* per firm. The government budget equilibrium requires $nK = nS + NT$. The ownership of all firms and their net profits are divided evenly between the *N* individuals.[2]

2.2. Household Preferences

Household utility is represented by a linear function of the utility obtained from consumption of the differentiated and homogeneous goods and the disutility of supplying labour to the production of these goods. Each of the *N* households is paid a wage, w_i, for working at

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[1] In the following we will use household and consumer interchangeably as it is easier to consider the household as a single worker or shopper.

[2] As *N* is large this means that every firm will be directed by its shareholders to maximise net profits.

subcentre *i* and buys one unit of variant *k* at price, p_k. Both prices and wages will be determined by the model. Homogeneous labour is supplied at the centre for a unit wage. The price of the homogenous consumption good, the price of intermediate deliveries and the price of delivering the fixed private and public infrastructure are all also equal to one. Using the household budget equation to substitute for consumption of the homogeneous good, an indirect conditional utility function can be derived to express household preferences. This utility function is only defined in as far as one unit of the differentiated good is consumed and one unit of differentiated labour is supplied. In the absence of trip chaining, the following utility function represents the preferences of a household that buys differentiated good *k* and supplies labour to subcentre *i*:

$$U_{ik} = \underbrace{\tilde{h}_k - p_k - \delta\alpha^d t_k}_{A} + \underbrace{w_i - \tilde{\beta}_i - \alpha^w t_i}_{B} + \underbrace{\theta(1-\beta) + \frac{1}{N}\sum_l \pi_l - T}_{C}. \tag{2}$$

The first three terms (*A*) represent the net utility from consuming differentiated good *k* with intrinsic quality component or willingness to pay $\tilde{h}_k$, that is bought at a price, p_k. and this requires a travel cost $\alpha^d t$ to subcentre *k*. Note that for the consumer who trip chains, this travel cost is zero ($\delta = 0$). The next three terms (*B*) are related to the supply of differentiated labour to subcentre *i*. This generates a wage w_i but has a disutility $\tilde{\beta}_i$ and requires a travel cost $\alpha^w t$. The three last terms (*C*) have to do with the consumption of the homogenous good (before subtracting the transport costs). As the disutility of homogenous labour equals β, the first term in *C* represents the net utility derived from his supply of θ units of homogenous labour for a unit wage. The second one represents the consumption possibilities derived from his equal share in total profits (π) and the last term is the head tax. The net utility derived from the consumption of the homogenous good equals the terms in (*C*) plus $w_i - \alpha^w t_i - p_k - \delta\alpha^d t_k$, the net wage received from the supply of differentiated labour in *i* minus the costs of buying differentiated good *k*. Since the travel time required for shopping activities, t_k, is zero if this consumer trip chains, this translates into a higher consumption of the homogenous good.

We will concentrate on the symmetric case where all subcentres are equidistant from the centre, so that commuting and shopping travel times are identical and positive ($t_k = t_i = t > 0$). Moreover, we assume all households value the quality of all product variants in the same way and experience the same disinclination to work at all subcentres. We set these valuations to zero without loss of generality. However, the households will still vary in their tastes. The utility of consumption of differentiated product variant *k* is then simply given by a stochastic component: $\mu^d \varepsilon_k$, such that

$$\tilde{h}_k = \mu^d \varepsilon_k \tag{3}$$

and the disutility of labour at subcentre *i* is similarly given by

$$\tilde{\beta}_i = -\mu^w \varepsilon_i. \tag{4}$$

The parameters ε_i and ε_k represent the intrinsic heterogeneity of household preferences and are assumed to be *i.i.d.* double exponentially distributed with mean normalised to zero and unit variance. The degree of heterogeneity of preferences is determined by μ^w and μ^d.

Substitution of (3) and (4) in the utility formulation (2) results in a random utility function for which the choice probabilities can be determined using the nested logit model.

2.3. Nested logit model

We use a heuristic approach to derive the probabilities of working and shopping at a given subcentre: the resident first selects his workplace and then chooses where to shop. In order to apply the nested logit approach, consistency requires that $0 < \mu^d \leq \mu^w$, so that households' preferences for their choice of workplace are at least as strong as their preferences for shopping location.[3] The consumer surplus associated with the resident's shopping alternatives, given his work location, affects his initial workplace choice. A full derivation of the choice probabilities can be obtained using the Generalised Extreme Value (GEV) approach of McFadden (1978). The decision tree for the nested logit is shown in Figure 1 below.

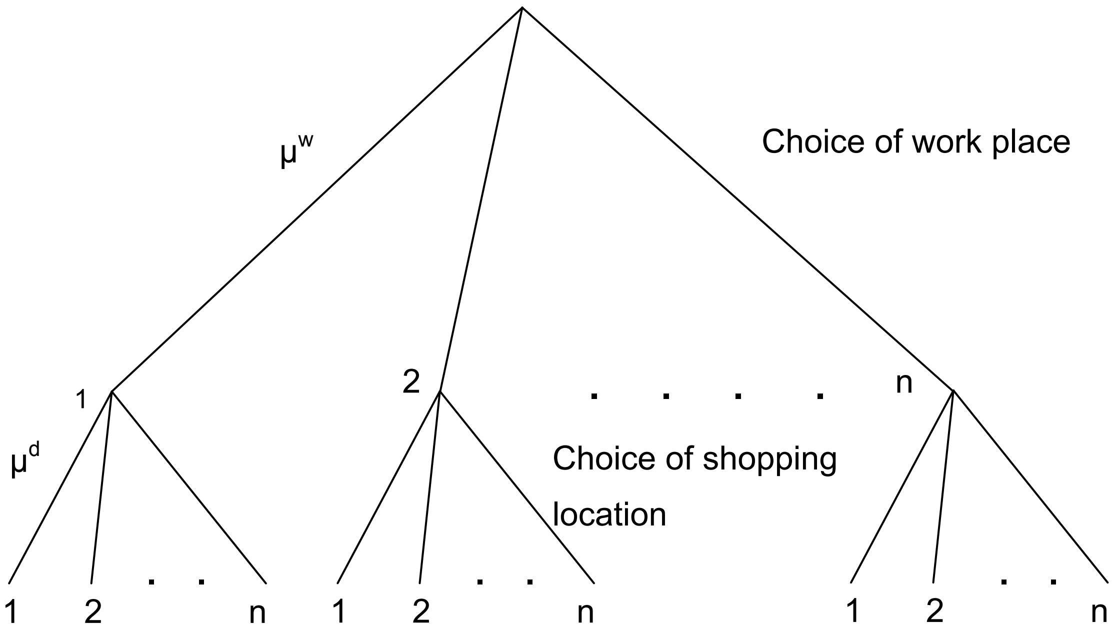

Figure 1 Nested logit

Recall from Section 2.1 that it is assumed that every household consumes one unit of the differentiated good and that the production of every unit of differentiated good (produced by one firm at one subcentre) requires one unit of labour, which is provided by one household. Assuming that the labour market clears, this means that the proportion of residents who decide to work at a given subcentre must equal the proportion of residents who shop there.

In order to simplify the exposition we will concentrate on the price p_1 and wage w_1 set by firm 1 and assume that the prices and wages set by all other firms are identical and equal p^* and w^* respectively.

[3] In the extreme case where individuals do not care where they work ($\mu^w = 0$), everybody trip chains in our model. When $\mu^w < \mu^d$ we could reformulate the full model and define a nested utility function where the shopping decision comes first. All the results of this paper would carry over.

2.4. Shopping model

We first consider the shopping model. The probability of a resident shopping at subcentre *1* that charges a price p_1, given he works there and given that all other subcentres charge a price p^*, can be expressed as

$$P^s_{1|1}(p_1, p^*) = \exp\left(-p_1/\mu^d\right)\Big/D^s_{1|1}\,, \tag{5}$$

where $D^s_{1|1} = \exp\left(-p_1/\mu^d\right) + (n-1)\exp\left(\left(-p^* - \alpha^d t\right)/\mu^d\right)$. The exponents of the terms in (5) represent the utility derived from shopping by the resident who works at subcentre *1*. Thus, the first term of $D^s{}_{1|1}$ (and the numerator of (5)) refers to a resident who trip chains, working and shopping at subcentre *1* (where price p_1 is charged), while the second refers to the resident who works at subcentre *1* but shops elsewhere. The consumer surplus associated with the shopping decisions of a resident who works at subcentre *1* can be calculated from the expected maximum utility derived from his shopping activities. This can be shown to be the log sum of $D^s_{1|1}$ (Anderson et al., 1992, Ben-Akiva and Lerman, 1979). Hence the consumer surplus takes the form

$$CS^s_1 = \mu^d \log\left[D^s_{1|1}\right]. \tag{6}$$

The corresponding probability of a resident shopping at subcentre *1*, given he does not work there is given by

$$P^s_{1|-1}(p_1, p^*) = \mathrm{e\,xp}\left(\left(-p_1 - \alpha^d t\right)/\mu^d\right)\Big/D^s_{1|-1}\,, \tag{7}$$

where $D^s_{1|-1} = \exp\left(\left(-p_1 - \alpha^d t\right)/\mu^d\right) + \exp\left(-p^*/\mu^d\right) + (n-2)\exp\left(\left(-p^* - \alpha^d t\right)/\mu^d\right)$. Since all subcentres with the exception of subcentre *1* are identical, only one expression is needed. In this case the terms in $D^s_{1|-1}$ cover the options of: a) shopping at subcentre *1* but working elsewhere so there is a travel cost; b) shopping and working at some subcentre ($k \neq 1$ say); and c) shopping at *k* but working at subcentre *j* ($j \neq k\ or\ 1$), so there is a travel cost component. The resident has to travel to subcentre *1*, so *t* appears in the numerator. Again the consumer surplus associated with shopping activities is calculated from the log sum of $D^s_{1|-1}$.

The consumer surplus, CS^s_{-1}, for the shopping activity of a resident who shops at *1* but works at any other subcentre *k*, ($k \neq 1$) is

$$CS^s_{-1} = \mu^d \log\left[D^s_{1|-1}\right]. \tag{8}$$

2.5. Employment model

The utility of an individual working at subcentre *1* is

$$U^w_1 = w_1 - \alpha^w t + CS^s_1 + \mu^w \varepsilon_1\,,$$

where the parameter ε_1 represents the intrinsic heterogeneity of household preferences for working at subcentre *1* (see Section 2.2). The probability of working at subcentre *1* is given by a nested logit model, as follows:

$$P_1^w(w_1, w^*) = \exp\left(\frac{w_1 - \alpha^w t + CS_1^s}{\mu^w}\right) \Big/ D_{1|1}^w , \tag{9}$$

where $D_{1|1}^w = ex\,\mathrm{p}\left(\left(w_1 - \alpha^w t + CS_1^s\right)/\mu^w\right) + (n-1)\exp\left(\left(w^* - \alpha^w t + CS_{-1}^s\right)/\mu^w\right)$ and CS_1^s is defined in (6). The probability of working at a subcentre other than subcentre *1* is given by

$$P_{-1}^w(w_1, w^*) = \exp\left(\frac{w^* - \alpha^w t + CS_{-1}^s}{\mu^w}\right) \Big/ D_{1|1}^w . \tag{10}$$

The denominator ($D_{1|1}^w$) is the same as in (9) since the consumer still has the same chance of working at subcentre *1* and being paid w_1 or working at another subcentre and being paid w^*. CS_{-1}^s is defined in (8).

2.6. Market clearing

Let N_1^w be the proportion of households that work at subcentre *1* and N_1^s the proportion that shop there. Then, by market clearing we need, for each firms, that the number of workers equals total sales[4] so that

$$N_1^w = N_1^s . \tag{11}$$

We can further express the number of shoppers frequenting subcentre *1* as

$$N_1^s = N_1^w P_{1|1}^s + N(1 - P_1^w) P_{1|-1}^s , \tag{12}$$

where P_1^w is the probability of working at subcentre *1*, and $P_{1|1}^s$ and $P_{1|-1}^s$ respectively denote the probability of a resident shopping at subcentre *1*, given that he does or does not work there. Then, since by definition $N_1^w = NP_1^w$, (12) simplifies to

$$P_1^w\left[1 - P_{1|1}^s + P_{1|-1}^s\right] - P_{1|-1}^s = 0 , \tag{13}$$

which provides an implicit relation between the price p_1 and wage w_1 set by firm *1*. This relation means that in an equilibrium, a firm that wants to cut its price and gain market share will need to increase its wage in order to produce the extra goods for sale. Equation (13) is still in implicit form; its implications for the behaviour of the firm are explained in the following sections.

2.7. Behaviour of firms

In general, the profit of firm *i* can be written

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[4] Remember the assumption that the production of one unit of the differentiated good requires one unit of differentiated labour.

$$\pi_i(w,p) = (p_i - w_i - c^1 - \alpha^h t)NP^w_i - (F+S) \ \forall i = 1...n, \tag{14}$$

where the demand $D_i = NP^d_i = NP^w_i$ under market clearing conditions[5].

Firms compete in a non-cooperative Nash game with their own prices and wages as the strategic variables. Since from (13) we know that p_1 determines w_1 and vice versa, we take the wage as the strategic variable for firm *1* and write $p_1 = g_1(w_1)$. Note that all firms other than firm *1* charge *p** for their product and pay wage *w**. Then, further assuming that firm *1* takes the prices and wages of the other firms as given, the first order condition for profit maximisation by this firm is given by

$$\frac{d\pi_1}{dw_1} = \left[\left(\frac{dg_1}{dw_1} - 1\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(\frac{1-P^w_1}{\mu^w}\right)\right]NP^w_1 = 0. \tag{15}$$

In Section 2.9, we derive an expression for the key strategic term dg_1/dw_1 in the case with trip chaining. As we want later to compare the equilibrium with and without trip chaining, in the next section, we recall some properties of the non trip chaining equilibrium derived in de Palma and Proost (2006).

2.8. Market equilibrium without trip chaining

Recall Propositions *1* and *3* in de Palma and Proost (2006) for the no congestion case.

Proposition (price) *When no trip chaining is permitted, there exists a unique symmetric short run Nash equilibrium in prices and wages. The price-wage equilibrium is given by*

$$p^*_{ntc} - w^*_{ntc} = c^1 + \alpha^h t + (\mu^w + \mu^d)\frac{n}{n-1}. \tag{16}$$

In the next section (see discussion after Proposition 1) we will show that this proposition is a limiting case of the equilibrium with trip chaining.

Proposition (free entry) *In the free entry symmetric Nash equilibrium with no trip chaining permitted and when firms pay a levy equal to the public infrastructure cost there is one subcentre too many.*

In this equilibrium, trip-chaining does not occur and households make separate working and shopping trips, although these may be to the same destination. We denote this as reference equilibrium and in later sections; we compare the results for the reference equilibrium with the results for the trip-chaining equilibrium.

2.9. Short run market equilibrium with trip chaining

We will show that the symmetric price equilibrium with trip chaining is indeed a short run Nash equilibrium. In order to show the properties of this equilibrium we start with the symmetric price equilibrium with trip-chaining in which all firms charge p^* and pay w^*. *W*e consider single price

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5 P^w_i and P^d_i are the probability of working and shopping at any subcentre *i*, respectively.

and wage deviations from this equilibrium. We first suppose that firm *1* deviates and sets price p_1 for its product and pays its workers a wage w_1. All other firms continue to charge p^* and pay w^*. Market clearing in fact precludes any other possible deviations since, as we have seen, a change in wage by one firm must be accompanied by a price alteration at the same firm. Analysis of the behaviour of firm *1* allows us to derive the conditions for the symmetric price equilibrium with trip-chaining.

Since firm *1* deviates, his profit becomes

$$\pi_1(w_1, w^*, p_1, p^*) = (p_1 - w_1 - c^1 - \alpha^h t) N P_1^w - (F + S). \tag{17}$$

In order to derive an expression for a candidate Nash equilibrium from the profit maximisation condition and prove its existence, we first need to determine the derivative of the price at firm *1* with respect to its wage (dg_1/dw_1 in (15)). Defining $\mu \equiv \mu^d / \mu^w \leq 1$ *and* $\Phi\left(p_1, p^*\right) \equiv P_{1|1}^{\ s} - P_{1|-1}^{\ s}$ *> 0,* we have

Lemma 1 $\dfrac{dg_1}{dw_1} = \dfrac{-\mu}{1+(1-\mu)\Phi} < 0$.

Proof. See Appendix A1.

To understand Lemma 1, take first the case without trip chaining. Then $dg_1/dw_1\big|_{ntc} = -\mu$, which means that the more consumers are loyal to their variety of product, the larger the price cut needed to sell the extra production brought about by the workers attracted by a wage increase. When trip chaining is permitted, the necessary price cut is smaller because the extra workers attracted by a wage increase will actually trip chain themselves, so fewer new customers need to be attracted. There is a greater probability of trip chaining than of working and shopping in separate locations.

Substitution of dg_1/dw_1 from Lemma 1 in (15) and replacing P_1^w in terms of the conditional shopping probabilities ($P_{1|1}^s$ and $P_{1|-1}^s$) from (13), we obtain

$$\left[\left(\frac{-(1+\mu)-(1-\mu)\Phi}{1+(1-\mu)\Phi}\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(\frac{1-P_{1|1}^{\ s}}{\mu^w\left[1-\Phi\right]}\right)\right] N \frac{P_{1|-1}^{\ s}}{\left[1-\Phi\right]} = 0. \tag{18}$$

Now, at equilibrium in the symmetric case, $p_1 = p^*$ and we can therefore rewrite the conditional shopping probabilities (5) and (7) as

$$P_{1|1}^{\ s} = \frac{1}{1+(n-1)\lambda} > \frac{1}{n}, \tag{19}$$

and

$$P_{1|-1}^{\ s} = \frac{\lambda}{1+(n-1)\lambda} < \frac{1}{n}, \tag{20}$$

where $\lambda \equiv e^{-\alpha^d t/\mu^d} \in \left(0,1\right)$ from our model assumptions. λ can be seen as a trip chaining cost parameter. When travel costs are high and shopping and variety preferences are not strong, λ is small and there is more frequent trip chaining. Note that total transport costs for shopping amount to $\left(1-P_{1|1}^{\ s}\right)\alpha^d tN$ (see (1) where $\delta = P_{1|1}^{\ s}$ and *N=D*). Thus, we can write

$$\Phi = \frac{1-\lambda}{1+(n-1)\lambda}. \tag{21}$$

Substitution of expressions (19), (20) and (21) in (18) allow us to specify the candidate Nash equilibrium.

Proposition 1 *When trip chaining is permitted, there exists a unique symmetric short run Nash equilibrium in prices and wages. The price-wage equilibrium is given by*

$$p^* - w^* = c^1 + \alpha^h t + (\mu^w + \mu^d)\frac{n}{n-1} - \frac{n\mu^d}{n-1}\left\{\frac{(1-\mu)(1-\lambda)}{[2-\mu+(n-2)\lambda+\mu\lambda]}\right\}. \tag{22}$$

Proof. See Appendix A2.

From (14), in equilibrium, a firm's profit per household is

$$\pi^* = \frac{(\mu^w + \mu^d)}{n-1} - \frac{\mu^d}{n-1}\left[\frac{(1-\mu)(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda)}\right] - \frac{F+S}{N}. \tag{23}$$

Using the fact that $\mu < 1$ and $\lambda < 1$, it can be verified that the gross profit (neglecting fixed costs) is positive. The comparative statics result is straightforward and left to the reader. The relationship between the mark-up (price minus wage), profit and the parameters $n, \alpha^d, \mu^w, \mu^d, t$ and λ is discussed in Section 4 using a numerical example.

It is possible to perform the same analysis, within the nested logit framework, for the case where consumers have to perform two single purpose trips, even if they work and shop at the same subcentre. This is the reference equilibrium without trip chaining, which is the same as the equilibrium which can be derived when working and shopping decisions are taken independently (see (16)), with the restriction $\mu^d \leq \mu^w$ for the nested logit approach **(**Anderson et al., 1992**).** In this case profits only depend on the household heterogeneity parameters and number of firms. We can now compare the symmetric short run trip chaining equilibrium, (22), with the symmetric short run, reference equilibrium, (16).

Proposition 2 *The symmetric short run firm mark-up when households can trip chain cannot exceed the mark-up when households can only perform single purpose trips. The mark-ups are equal when $\mu^d = \mu^w$. The difference in mark-up is given by*

$$\left(p^* - w^*\right) - \left(p^*_{ntc} - w^*_{ntc}\right) = -\frac{n\mu^d(1-\mu)}{n-1}\left[\frac{(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda)}\right] < 0. \tag{24}$$

The intuition why trip chaining decreases margins is not obvious given the complexity of the RHS of (24). The dominant mechanism can be seen as follows. Compared to the no trip chaining case, the same price decrease will attract more customers because a relatively large part *(> 1/n*) of the necessary labour to produce it trip chains and adds to the group of customers. This means there is a larger reward for a price cut (a flatter demand curve) and this will lead to more price cuts and ultimately lower profit margins.

2.10. The long run free entry equilibrium with trip chaining

Using (23) we can write the difference in profit per household as

$$\pi^* - \pi^*_{ntc} = -\frac{\mu^d}{n-1}\left[\frac{(1-\mu)(1-\lambda)}{\left(2-\mu+(n-2)\lambda+\mu\lambda\right)}\right] < 0\,. \tag{25}$$

It follows directly from (25) that, for any given number of firms *n*, the profit level of firms present in the market when trip chaining is possible always lies below the corresponding level when trip chaining is not possible. At free entry, the profit of all firms in the market is zero. Hence this must occur at a smaller value of *n* when there is trip chaining.

Proposition 3 *The symmetric long run Nash equilibrium with free entry has a smaller number of firms when trip chaining is possible than when trip chaining is not possible:* $n^f < n^f_{ntc}$.

We will illustrate this difference in the entry of firms numerically in Section 4.

3. WELFARE ANALYSIS

3.1. Consumer Surplus in the short run

Using the expected maximum utility approach described in Section 2.4, the total consumer surplus, from working and shopping activities associated with households working at subcentre *1*, can be obtained from the log sum of the denominator of (9). In the symmetric equilibrium, this is indeed the consumer surplus of households working at any subcentre and can be written as

$$CS = \mu^w \log\left[n \exp\left(\frac{w^* - \alpha^w t + CS^s - \beta}{\mu^w}\right)\right], \tag{26}$$

where $CS^s = h - p^* + \mu^d \log\left[1+(n-1)\lambda\right]$ is the consumer surplus[6] derived by households from shopping activities ((6) and (8)) in the symmetric equilibrium with trip chaining. An equivalent expression can be obtained for the reference case without trip chaining (see Appendix A3).

Proposition 4 *In the symmetric short run Nash equilibrium, the consumer surplus when households can trip chain is larger than the consumer surplus when households must perform only single purpose trips. The difference in consumer surplus is given by*

$$CS - CS_{ntc} = \left(p^*_{ntc} - w^*_{ntc}\right) - (p^* - w^*) + \mu^d \log\left[1 + \frac{\lambda^{-1}-1}{n}\right] > 0\,. \tag{27}$$

Proof. See Appendix A3.

A higher mark up (p-w) for the firm means a higher price and lower wage for the differentiated good so a lower consumer surplus for the household. The second travel cost component can be clarified by further subdividing this term into the travel time saving for households who do not change their behaviour between the two equilibria and the cost for a household that changes its shopping behaviour to take advantage of trip chaining. Hence

$$\mu^d \log\left[1+\frac{\lambda^{-1}-1}{n}\right] = \frac{\alpha^d t}{n} + \mu^d \log\left[\lambda^{\frac{1}{n}}\left(1+\frac{\lambda^{-1}-1}{n}\right)\right] \tag{28}$$

[6] Consumer surplus and welfare are calculated per household.

The first term on the RHS (28) of the above equation is equal to the transportation cost saving when an individual works in a subcentre and has his most preferred good in the same subcentre (this event happens with probability *1/n*). The second term is the saving from economising on transport cost when the first best choice is not at the same location as the work place but close enough in the idiosyncratic preference space (see Anderson et al., 1989). This second term, which translates the quality adjustment of the consumer, is strictly positive and converges to zero when the travel time goes to zero and λ goes to one.

Substitution of the mark-up terms from (24) into (27) allows us to perform a comparative statics exercise on the model parameters. It can easily be shown that the difference in consumer surplus is decreasing in the trip chaining cost λ and *n*. These results are verified by the numerical exercise in Section 4.

3.2. Welfare effects in the short run

For a quasi linear utility function, welfare difference is the sum of the difference in consumer surplus from the consumption and supply of differentiated product and differentiated labour plus difference in producer surplus from the supply of the differentiated product. Using expression (27) and the fact that profits are redistributed equally to households, we obtain:

Proposition 5 *In the symmetric short run Nash equilibrium, welfare*[6] *is greater when households can trip chain, than when they have to perform only single purpose trips. The difference in welfare is given by*

$$W - W_{ntc} = \mu^d \log\left[1 + \frac{\lambda^{-1} - 1}{n}\right]. \tag{29}$$

Clearly the difference in welfare between the equilibria with and without trip chaining is equal to the difference in consumer surplus minus the difference in profits. Equation *(29)* is positive since with trip chaining the individuals have more options (to use or not use the trip chaining scheme). Therefore *(29)* represents, in a sense, the option value associated with trip chaining. Note that the fact that prices are adjusted by trip chaining is irrelevant for the welfare analysis, since price changes are pure transfers with no social impacts.

3.3. Welfare effects in the long run

When we analyse the welfare effects of trip chaining in the long run, there are three conflicting forces to consider. For an identical number of firms (short run) we know that the gain in transport costs is larger than the loss in consumer diversity (Proposition 5). But, we also know from Proposition 3 that the free entry number of firms is smaller with trip chaining than without trip chaining and this means a loss of diversity (welfare loss) and a decrease in fixed costs (welfare gain).

We will be able to show that trip chaining is indeed welfare enhancing even with free entry but this will require a few intermediate steps contained in Lemmas 2 and 3. These are proved in Appendices A4 and A5.

Lemma 2 The welfare maximising number of firms is smaller when the trip chaining option is possible: $n^0 < n^0_{ntc}$.

The next intermediate result we need is to show that with the trip chaining option, the free entry equilibrium has always too many firms compared to the optimal number of firms.

Lemma 3 When trip chaining is possible, the free entry number of firms is larger than the optimal number of firms: $n^f > n^0$.

We can now prove the following proposition, which extends Proposition 5 to the long run.

Proposition 6 *In the free entry Nash equilibrium, the welfare is higher when trip chaining is possible than when it is not.*

$$W(n^0) \geq W(n^f) \geq W_{ntc}(n^0_{ntc}) \geq W_{ntc}(n^f_{ntc}) . \tag{30}$$

Proof

Recall from Section 2.10 that the profit curve for firms when trip chaining is possible always lies below that when trip chaining is not possible, as shown on Figure 2. Further, from Proposition 3 the free entry number of firms with trip chaining is smaller than without trip chaining ($n^f < n^f{}_{ntc}$). Using Lemma 2 and Lemma 3, we also have $n^0 \leq n^f \leq n^f_{ntc}$. We know from *(29)* (Proposition 5) that the welfare curve when there is trip chaining always lies above that when trip chaining is not possible. These curves are also illustrated in Figure 2. We can then show that (30) holds. The first inequality follows from the definition of an optimum, the next inequality follows from the concavity of *W*, the last inequality follows from Proposition 5. QED.

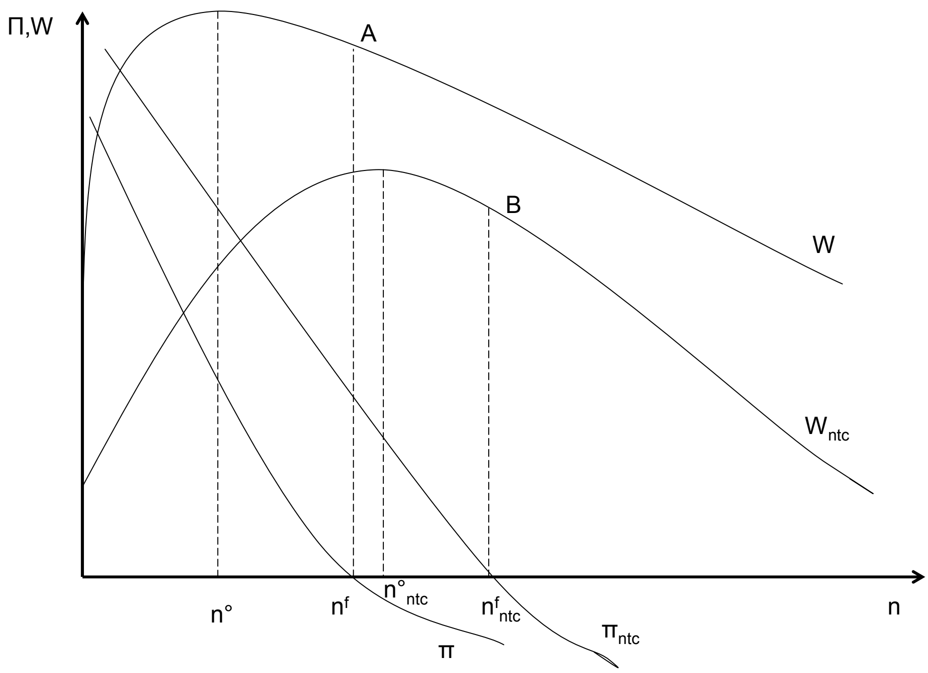

Figure 2 Welfare and profit functions with and without the possibility of trip chaining

4. NUMERICAL EXAMPLE

The trip chaining equilibrium in price and wages, (24), depends in a complex way on a number of parameters: in particular $\mu^w, \mu^d, \alpha^d, n$, $\lambda \equiv e^{-\alpha^d t/\mu^d}$ and travel time, *t*. The following numerical exercise illustrates the effect of each of these parameters on the price-wage equilibrium and also on profit, consumer surplus and welfare.

We use the simple, stylised example of an economy of one day. As a reference, for the short run equilibrium, we assume there are three firms offering the differentiated good. Each resident makes one commuting trip and one shopping trip per day, giving a total transport time of one hour. He also supplies 7.5 hours of labour, of which one hour is spent on the production of the differentiated good. Truck deliveries are such that each truck contains sufficient intermediate good to produce 50 units of the differentiated good. One unit of the differentiated good requires an intermediate input that can be produced using 0.1 units of homogeneous labour. Finally, we set the fixed costs[7] per firm at 0.5 hours of labour per capita, as these do not affect the short-run equilibria or welfare analysis, and present gross profits per household.

In Table 1 we examine, for the short run equilibrium with a fixed number of firms, the effect on price minus wage and gross profit (*π*) of varying exogenous factors like the consumers' preference for work and shopping locations (μ^w and μ^d, respectively), number of shopping trips (α^d) and travel time, for the equilibria with and without trip chaining. In the last line, we also look at the effect of increasing the number of firms. The short run equilibria (with given number of firms) are presented in the second part of Table 1. The long run equilibrium number of firms are given in the last two columns.

We first examine the short run equilibria. When consumers can trip chain, profits increase as μ^w increases since the strong preference for working location means that a firm can pay lower wages (or charge higher prices) without losing workers. Similarly, a weak preference for shopping location (small μ^d) necessitates firms charging lower prices to retain shoppers. Profits also decrease when there are more firms due to increased competition. Similar effects are also seen for changes in these parameters in the no trip-chaining reference case.

Exogenous parameters					*Short run equilibria*						*Free entry*	
μ^d	μ^w	α^d	*t* *(hrs)*	*n*	λ	p^*-w^*	$p^*_{ntc}-w^*_{ntc}$	π^*	π_{ntc}	$\Delta\pi^*$ (%)†	n^f	n^f_{ntc}
1	2	1	0,5	3	0,61	4,488	4,610	1,459	1,500	2,7	6	7
1	5	1	0,5	3	0,61	8,923	9,110	2,938	3,000	2,1	12	13
0,1	2	1	0,5	3	0,01	3,188	3,260	1,026	1,050	2,3	5	5
1	2	0,2	0,5	3	0,90	4,585	4,610	1,492	1,500	0,6	6	7
1	2	2	0,5	3	0,37	4,379	4,610	1,423	1,500	5,1	6	7
1	2	1	2	3	0,14	4,259	4,640	1,373	1,500	8,5	6	7
1	2	1	0,25	3	0,78	4,543	4,605	1,479	1,500	1,4	6	7
1	2	1	0,5	10	0,61	3,410	3,443	0,330	0,333	1,0		

Table 1 Comparative statics with and without trip chaining in the short run and in the long run

[7] Fixed costs here include levies (S) which are assumed equal to the fixed public inputs (K) since there are no head taxes (T).

† The difference in profit is calculated as a percentage of the symmetric case without trip chaining.

Interestingly, however, we see that, when consumers can work and shop at the same subcentre, the number of shopping trips they make (α^d) plays a role. If consumers do not make frequent shopping trips then firms can make higher profits. A small value of α^d means that the travel cost for shopping trips is low, a smaller proportion of workers trip chain and profits increase. Decreasing or increasing the travel time from the city centre to the subcentres has the same effect on profits as does α^d. A longer travel time means higher travel costs and, in this case, a higher proportion of the workforce prefers to trip-chain to minimise these costs. The demand curve is consequently flatter. For the no trip-chaining case, the price mark-up over wage does depend on travel time because of travel costs for the intermediate good but profits are independent of travel cost. Note also that, for the trip chaining case, profit increases with the trip chaining cost parameter for households, λ.

It is clear from Table 1 that when consumers can trip chain, firms cannot make greater profits than when consumers can only make single purpose trips. The magnitude of the difference in profits obviously depends on the values of the input parameters but the difference is large for long travel time or high frequency of shopping trips.

With free entry, we see that trip chaining reduces the number of firms. The effect of travel time and trip frequency is much smaller than that of consumer heterogeneity and is not apparent when integer numbers of firms are considered, as is the case here.

In Table 2 we present the difference in consumer surplus and welfare (per household) between the two equilibria in the short and long run. For the short-run equilibrium, as expected, consumer surplus decreases with *n* (and λ). The largest gains in consumer surplus and welfare with trip chaining are seen when travel costs are high or when consumers have a low preference for shopping location, so they are more likely to trip chain and firms also charge lower prices. With respect to shorter travel time or reduced trip frequency, consumer surplus also increases as firms are able to increase prices and profits but these increases are smaller than in the reference case without trip chaining so the gain in consumer surplus from trip chaining is reduced.

The total reduction in travel costs per capita due to trip chaining is given by $\delta\alpha^d t$. This can be larger or smaller than the gain in consumer surplus due to trip chaining, depending on the exogenous model parameters (see Section 3.1).

Exogenous parameters					Short run equilibria				Free entry
μ^d	μ^w	α^d	t (hrs)	n	$\delta\alpha^d t$	$CS - CS_{ntc}$	$W - W_{ntc}$	$W - W_{ntc}$ (%GDP)	$W(n^f) - W_{ntc}(n^f{}_{ntc})$ (%GDP)
1	2	1	0,5	3	0,23	0.32	0,20	2,3	1.65
1	5	1	0,5	3	0,23	0.38	0,20	2,3	0.9
0,1	2	1	0,5	3	0,49	0.46	0,39	4,6	4.0
1	2	0,2	0,5	3	0.04	0.06	0,03	0,4	0.6
1	2	2	0,5	3	0.58	0.68	0,45	5,3	3.4
1	2	1	2	3	1.57	1.52	1,14	13,4	9.0
1	2	1	0,25	3	0.10	0.15	0,09	1,1	1.0
1	2	1	0,5	10	0,08	0.10	0,06	0,7	

Table 2 Welfare effects with and without trip chaining

In the long run with free entry, welfare is always greater when trip chaining is possible, as shown in Proposition 6. In this case the largest gains in welfare are also seen when travel costs are high or when consumers have a low preference for shopping location. However, when consumer preference for working location is strong the benefits of trip chaining are smaller. Preference for work location has an impact on welfare gains from trip chaining in the long run because there are fewer firms in the long run equilibrium with trip chaining. Consumers have less choice of working location which reduces welfare. In the short run, there are the same number of firms in the equilibria with and without trip chaining, so μ^w does not play a role.

5. CONCLUSIONS

In this paper we studied the effect of trip chaining on the profits and number of firms as well as on welfare. We found that firms make smaller profits when the trip chaining is permitted because price cuts tend to generate higher demand responses. The higher demand responses come from the workers that are also partly consumers of the firm. In the symmetric equilibrium, lower profit margins imply that a smaller number of firms can survive in the free entry case. Welfare does unambiguously increase in the short run and with free entry because the gain in transport costs is not fully offset by the loss of variety of firms.

Trip chaining is beneficial to consumers in the short and long run. On the contrary, firms collectively loose when trip chaining is possible and would therefore not support legislation promoting it. However, the net impact (i.e. the impact on welfare) is positive, which obviously suggests such legislation should be encouraged (via parking policies, pricing and information systems, for example).

In this paper we compared a symmetric equilibrium where trip chaining is allowed with a symmetric equilibrium where it is not. We can add one step to the game and consider trip chaining as an option to be decided unilaterally by each firm. Consider a situation where *n-m* firms allow trip chaining and the remaining *m* do not allow trip chaining. We look at the incentive for one of the *m* firms to change its policy and allow trip chaining. The profits of a firm that decides to allow trip chaining change for two reasons. First, keeping its price fixed, its demand increases as the transport cost reduction increases its attractiveness. Second, there will be new price equilibriums since the competition will decrease their price and since goods are strategic substitutes, the deviant firm will also decrease its price. We conjecture the latter effect is dominated by the former and as a result the profit of the deviant firm increases while the profit of the other *n-1* firms decreases. We are typically facing a prisoners' dilemma situation and as a result all firms will accept trip chaining one after another and, as we have seen from Proposition 2, all firms will be worse off compared to the situation when trip chaining is not allowed.

One may therefore expect that industry associations will lobby against trip chaining, for example, by relocating far enough from the business district to make trip chaining unfeasible. This is the same phenomenon as the opening hours' discussion where each firm has an incentive to deviate and steal markets from its competitors by staying open longer (Rouwendal and Rietveld, 1999). However, at least when demand is inelastic, all firms will be worse off with extended opening hours.

APPENDICES

Appendix A1: Proof of Lemma 1.

Recall from (13) that

$$P_1^w\left[1-P_{1|1}{}^s+P_{1|-1}{}^s\right]-P_{1|-1}{}^s=0\,. \tag{31}$$

We will denote the left hand side of (31) by X so that $X=0$. This expression is constant and can be differentiated implicitly to give

$$\frac{dg_1(w_1)}{dw_1}=-\frac{\partial X/\partial w_1}{\partial X/\partial p_1}\,. \tag{32}$$

We next evaluate the numerator and denominator of the right hand side of (32) by differentiating (31).

A) $\dfrac{\partial X}{\partial w_1}=\dfrac{\partial P_1^w}{\partial w_1}\left[1-\Phi\right]$ (Recall $\Phi\equiv P_{1|1}{}^s-P_{1|-1}{}^s$.)

We can substitute $\dfrac{\partial P_1^w}{\partial w_1}=\dfrac{1}{\mu^w}\left[1-P_1^w\right]P_1^w$ to obtain

$$\frac{\partial X}{\partial w_1}=\frac{1}{\mu^w}\left[1-P_1^w\right]P_1^w\left[1-\Phi\right]. \tag{33}$$

B) $$\frac{\partial X}{\partial p_1}=\frac{\partial P_1^w}{\partial p_1}\left[1-\Phi\right]-P_1^w\frac{\partial P_{1|1}{}^s}{\partial p_1}-(1-P_1^w)\frac{\partial P_{1|-1}{}^s}{\partial p_1}\,. \tag{34}$$

To evaluate this expression, we consider each of the partial derivatives on the right hand side in turn. Firstly

$$\frac{\partial P_1^w}{\partial p_1}=\frac{1}{\mu^w}\frac{\partial CS_1}{\partial p_1}\left[1-P_1^w\right]P_1^w-\frac{1}{\mu^w}\frac{\partial CS_{-1}}{\partial p_1}(n-1)P_{-1}{}^wP_1^w\,. \tag{35}$$

The derivatives of the consumer surplus with respect to price are given by

$\dfrac{\partial CS_1}{\partial p_1}=\mu^d\exp\left(-p_1/\mu^d\right){D^s}_{1|1}{}^{-1}\left(\dfrac{-1}{\mu^d}\right)=-P_{1|1}{}^s$ and

$\dfrac{\partial CS_{-1}}{\partial p_1}=\mu^d\exp\left(\left(-p_1-\alpha^dt\right)/\mu^d\right){D^s}_{1|-1}{}^{-1}\left(\dfrac{-1}{\mu^d}\right)=-P_{1|-1}{}^s$ with

${D^s}_{1|1}=\exp\left(-p_1/\mu^d\right)+(n-1)\exp\left(\left(-p*-\alpha^dt\right)/\mu^d\right)$ and

${D^s}_{1|-1}=\exp\left(\left(-p_1-\alpha^dt\right)/\mu^d\right)+\exp\left(-p*/\mu^d\right)\left[1+(n-2)\exp\left(-\alpha^dt/\mu^d\right)\right]$. These are the expressions for Roy's identity in the case of a discrete choice model. Substituting these derivatives in (35) leads to

$$\frac{\partial P_1^w}{\partial p_1} = -\frac{\left[1-P_1^w\right]P_1^w}{\mu^w}\Phi, \tag{36}$$

where we have also used $\left(1-P_1^w\right)=(n-1)P_{-1}{}^w$.

Next we have

$$\frac{\partial P_{1|1}{}^s}{\partial p_1} = -\frac{1}{\mu^d}\left[1-P_{1|1}{}^s\right]P_{1|1}{}^s \tag{37}$$

and

$$\frac{\partial P_{1|-1}{}^s}{\partial p_1} = -\frac{1}{\mu^d}\left[1-P_{1|-1}{}^s\right]P_{1|-1}{}^s. \tag{38}$$

Substituting (36), (37) and (38) in (35), we obtain

$$\frac{\partial X}{\partial p_1} = -\frac{\left[1-P_1^w\right]P_1^w}{\mu^w}\Phi\left[1-\Phi\right] + \frac{P_1^w}{\mu^d}\left[1-P_{1|1}{}^s\right]P_{1|1}{}^s + \frac{\left(1-P_1^w\right)}{\mu^d}\left[1-P_{1|-1}{}^s\right]P_{1|-1}{}^s. \tag{39}$$

Finally, substituting (33) and (39) in (32) leads to

$$\frac{dg_1}{dp_1} = -\frac{\frac{1}{\mu^w}\left[1-P_1^w\right]P_1^w\left[1-\Phi\right]}{-\frac{\left[1-P_1^w\right]P_1^w}{\mu^w}\Phi\left[1-\Phi\right] + \frac{P_1^w}{\mu^d}\left[1-P_{1|1}{}^s\right]P_{1|1}{}^s + \frac{\left(1-P_1^w\right)}{\mu^d}\left[1-P_{1|-1}{}^s\right]P_{1|-1}{}^s}. \tag{40}$$

We can now use our original expression (31) to eliminate P_1^w from (40). Then, dividing numerator and denominator by $\left[1-\Phi\right]$ to simplify the equation, we obtain

$$\frac{dg_1}{dw_1} = \frac{-\mu}{1+(1-\mu)\Phi} \tag{41}$$

where $\mu \equiv \frac{\mu^d}{\mu^w}$. QED.

Appendix A2: Proof of Proposition 1.

Recall from Lemma 1 that at the candidate equilibrium

$$\frac{dp_1}{dw_1} = \frac{-\mu}{1+(1-\mu)\Phi}, \tag{42}$$

where $\mu \equiv \frac{\mu^d}{\mu^w}$ and $\Phi \equiv P_{1|1}{}^s - P_{1|-1}{}^s$. This expression is negative and single valued, so that there exists a one-to-one relationship between p_1 and w_1. Hence the set of prices is a convex, compact set and the equilibrium exists. Further (42) is constant, since *μ, t* and *n* are all exogenous.

Since a candidate equilibrium exists, we need only show that the profit function is quasi-concave to guarantee that the candidate equilibrium is the unique Nash solution.

At any extremum

$$\frac{d\pi_1}{dw_1} = \left[\left(\frac{dp_1}{dw_1} - 1\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(\frac{1 - P_1^w}{\mu^w}\right)\right] N P_1^w = 0 . \tag{43}$$

The corresponding second order condition is given by

$$\begin{aligned}
\frac{d^2\pi_1}{dw_1^{\,2}} &= N P_1^w \frac{d^2 p_1}{dw_1^{\,2}} + N P_1^w \left[\left(\frac{dp_1}{dw_1} - 1\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(-\frac{P_1^w}{\mu^w}\right)\left(\frac{1 - P_1^w}{\mu^w}\right)\right] + \\
&\quad \left[\left(\frac{dp_1}{dw_1} - 1\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(\frac{1 - P_1^w}{\mu^w}\right)\right] N P_1^w \left(\frac{1 - P_1^w}{\mu^w}\right) \\
&= N P_1^w \frac{d^2 p_1}{dw_1^{\,2}} + N P_1^w \left(\frac{1 - P_1^w}{\mu^w}\right)\left[2\left(\frac{dp_1}{dw_1} - 1\right) + \left(p_1 - w_1 - c^1 - \alpha^h t\right)\left(\frac{1 - 2P_1^w}{\mu^w}\right)\right].
\end{aligned} \tag{44}$$

From (43) we can replace $\left(p_1 - w_1 - c^1 - \alpha^h t\right)$ in (44) to get

$$\begin{aligned}
\frac{d^2\pi_1}{dw_1^{\,2}} &= N P_1^w \frac{d^2 p_1}{dw_1^{\,2}} + N P_1^w \left(\frac{1 - P_1^w}{\mu^w}\right)\left(\frac{dp_1}{dw_1} - 1\right)\left[2 - \left(\frac{1 - 2P_1^w}{1 - P_1^w}\right)\right] \\
&= N P_1^w \frac{d^2 p_1}{dw_1^{\,2}} + N P_1^w \left(\frac{1 - P_1^w}{\mu^w}\right)\left(\frac{dp_1}{dw_1} - 1\right)\left(\frac{2 - 2P_1^w - 1 + 2P_1^w}{1 - P_1^w}\right) \\
&= N P_1^w \frac{d^2 p_1}{dw_1^{\,2}} + N\left(\frac{P_1^w}{\mu^w}\right)\left(\frac{dp_1}{dw_1} - 1\right).
\end{aligned} \tag{45}$$

Now

$$\begin{aligned}
\frac{d^2 p_1}{dw_1^{\,2}} &= \frac{d}{dw_1}\left[\frac{-\mu}{1 + (1-\mu)\Phi}\right] \\
&= \frac{-\mu^2}{\left[1 + (1-\mu)\Phi\right]^2} \frac{\partial\Phi}{\partial p_1} \frac{dp_1}{dw_1} \\
&= \frac{\mu^3}{\left[1 + (1-\mu)\Phi\right]^3}\left[\frac{\partial P_{1|1}{}^s}{\partial p_1} - \frac{\partial P_{1|-1}{}^s}{\partial p_1}\right] \\
&= \frac{\mu^3}{\left[1 + (1-\mu)\Phi\right]^3}\left[-\frac{1}{\mu^d}\left[1 - P_{1|1}{}^s\right] P_{1|1}{}^s + \frac{1}{\mu^d}\left[1 - P_{1|-1}{}^s\right] P_{1|-1}{}^s\right] \\
&= \frac{-\mu^3}{\mu^d\left[1 + (1-\mu)\Phi\right]^3} \Phi\left[1 - P_{1|1}{}^s - P_{1|-1}{}^s\right].
\end{aligned} \tag{46}$$

From our model assumptions $0 < \mu \le 1$ and $\mu^d > 0$. Further, we know that, at the candidate symmetric equilibrium, $\Phi > 0$, $P_{1|1}{}^s = \dfrac{1}{1 + (n-1)\lambda}$ and $P_{1|-1}{}^s = \dfrac{\lambda}{1 + (n-1)\lambda}$, where

$\lambda \equiv e^{-\alpha^d t/\mu^d} > 0$. Hence $\left[1 - P_{1|1}{}^s - P_{1|-1}{}^s\right] = \dfrac{(n-2)\lambda}{1+(n-1)\lambda}$ is non-negative for $n \geq 2$. Thus (46) is non-positive.

Substituting from (46) in (45) means that the first term on the right hand side of (45) is non-positive. We also know from (42) that $dp_1/dw_1 < 0$, so the second term in (45) is negative. Hence $\dfrac{d^2\pi_1}{dw_1{}^2}$ is strictly negative at any extremum (solution of the first-order equations) and thus the profit is quasi-concave. As a consequence, the candidate Nash equilibrium is a Nash equilibrium. QED.

Appendix A3: Proof of Proposition 4

Recall from (26) that with trip chaining

$$CS = \mu^w \log\left[ne\frac{w^* - \alpha^w t + CS^s - \beta}{\mu^w}\right], \tag{47}$$

where $CS^s = h - p^* + \mu^d \log\left[1+(n-1)\lambda\right]$ and $\lambda \equiv e^{-\alpha^d t/\mu^d}$. By substitution, the above equation can be reformulated as

$$CS = \mu^w \log n + w^* - p^* + h - \beta - \alpha^w t + \mu^d \log\left[1+(n-1)\lambda\right]. \tag{48}$$

Note that, for the case without trip chaining

$$CS^s{}_{1_{ntc}} = CS^s{}_{-1_{ntc}} = h - \alpha^d t + \mu^d \log\left[D^s{}_{1|1_{ntc}}\right] \tag{49}$$

and

$$P_{1_{ntc}}^w(w_1, w^*) = \exp\left(\frac{w_1 - \alpha^w t + CS^s{}_1}{\mu^w}\right) D^w{}_{1|1_{ntc}}{}^{-1}, \tag{50}$$

where $D^w{}_{1|1_{ntc}} = e\,\mathrm{xp}\left(\left(-\alpha^w t + CS^s{}_1\right)/\mu^w\right)\left[\exp\left(w_1/\mu^w\right) + (n-1)\exp\left(w^*/\mu^w\right)\right]$.

Similarly using (49) and the log sum of the denominator of (50) both evaluated at the symmetric equilibrium, we obtain

$$CS_{ntc} = \mu^w \log\left[ne\frac{w^*{}_{ntc} - \alpha^w t + CS^s{}_{ntc} - \beta}{\mu^w}\right] \tag{51}$$

for the reference case without trip chaining, where $CS^s{}_{ntc} = h - p^*_{ntc} - \alpha^d t + \mu^d \log n$. Substituting for $CS^s{}_{ntc}$ in (51) and subtracting the resulting equation from (48) leads to

$$\begin{aligned} CS - CS_{ntc} &= (p^*{}_{ntc} - w^*{}_{ntc}) - (p^* - w^*) + \alpha^d t + \mu^d \log n + \mu^d \log\left[1+(n-1)\lambda\right] \\ &= (p^*{}_{ntc} - w^*{}_{ntc}) - (p^* - w^*) + \mu^d \log\left[1 + \frac{\lambda^{-1} - 1}{n}\right] \end{aligned} \tag{52}$$

QED.

Appendix A4: Proof of Lemma 2

Call the welfare optimum number of firms without trip chaining n_{ntc}^{0} and the optimal number of firms when trip chaining is possible n^{0}.

The optimal number of firms in the absence of trip chaining maximizes the following welfare function (per household):

$$W(n) = \Psi - \frac{n}{N}(F+K) + (\mu^{d} + \mu^{w})\log n,$$

where $\Psi = \theta(1-\beta) - c^{1} + \alpha^{h}t + h - \alpha^{d}t - \beta - \alpha^{w}t$,so the optimal number of firms satisfies the following first order condition:

$$\frac{\mu^{w}}{n_{ntc}^{0}} + \frac{\mu^{d}}{n_{ntc}^{0}} = \frac{F+K}{N}.$$

When trip chaining is allowed, the welfare function (per household) to be maximised is

$$W(n) = \Psi' + \mu^{w}\log n + \mu^{d}\log\left[1+(n-1)\lambda\right] - \frac{n}{N}(F+K),$$

where $\Psi' = \Psi + \alpha^{d}t$. This leads to the first order condition

$$\frac{\mu^{w}}{n^{0}} + \frac{\mu^{d}\lambda}{1+(n^{0}-1)\lambda} = \frac{F+K}{N}.$$

Comparing both first order equations, the solutions must satisfy: $n_{ntc}^{0} \geq n^{0}$. Q.E.D.

Appendix A5: Proof of Lemma 3

We first show that in the symmetric equilibrium with trip chaining allowed, the profit per firm is a decreasing function of the number of firms. We limit ourselves to the case where $n \geq 2$. Taking the derivative of the profit equations for firms when trip chaining is possible, (23), with respect to n we find:

$$\frac{\partial \pi}{\partial n} = -\frac{(\mu^{w}+\mu^{d})}{(n-1)^{2}} + (1-\mu)(1-\lambda)$$
$$\left[\frac{\mu^{d}}{(n-1)^{2}(2-\mu+(n-2)\lambda+\mu\lambda)} + \frac{\mu^{d}}{(n-1)}\frac{\lambda}{(2-\mu+(n-2)\lambda+\mu\lambda)^{2}}\right]$$
$$sign\frac{\partial \pi}{\partial n} = sign\left\{-(\mu^{w}+\mu^{d}) + \left[\mu^{d} + \frac{\lambda(n-1)\mu^{d}}{(2-\mu+(n-2)\lambda+\mu\lambda)}\right]\frac{(1-\mu)(1-\lambda)}{2-\mu+(n-2)\lambda+\mu\lambda}\right\}$$

The first term on the right hand side is negative. Because μ and λ are both smaller than one, we know that the term $\frac{(1-\mu)(1-\lambda)}{2-\mu+(n-2)\lambda+\mu\lambda}$ is at most equal to one, we will use therefore the upper bound for this term and put this term equal to one.

It is therefore sufficient to show that the following expression is negative:

$$sign\left\{-\mu^w\left[2-\mu+(n-2)\lambda+\mu\lambda\right]+\lambda(n-1)\mu^d\right\}$$
$$=sign\left\{-2\mu^w+\mu^d-\mu^w(n-2)\lambda-\mu^d\lambda+\lambda(n-1)\mu^d\right\},$$
$$=sign\left\{-\mu^w+(\mu^d-\mu^w)+(n-2)\lambda(\mu^d-\mu^w)\right\}$$

where we have used the definition $\mu \equiv \mu^d / \mu^w \leq 1$ and this last expression is indeed negative.

The optimal number of firms can be found by using the first order condition for a maximum of the welfare function

$$\frac{\mu^w}{n^0}+\frac{\mu^d\lambda}{1+(n^0-1)\lambda}-\frac{F+K}{N}=0 \quad (53)$$

and the equation that determines the free entry equilibrium number of firms is (using (23) and the zero profit condition)

$$\frac{(\mu^w+\mu^d)}{n-1}-\frac{\mu^d}{n-1}\left[\frac{(1-\mu)(1-\lambda)}{\left(2-\mu+(n-2)\lambda+\mu\lambda\right)}\right]-\frac{F+S}{N}=0\,. \quad (54)$$

We know from the above that the left hand side of (54) is a decreasing function of *n*. Moreover we know that the profit goes to infinity (or at least a very large number) when *n* approaches one. The left hand side (LHS) of (53) is however finite when *n* approaches one. This means that starting from a value of *n=1*, the LHS of (53) is initially always smaller than the LHS of (54). Since both LHS are decreasing, it is sufficient to prove that the LHS of (53) and (54) can never be equal to know that the solution of (54) is always larger than the solution of (53).

We now show that there is no value of *n >0* that satisfies the LHS of both equations. Equating the left hand sides of (53) and (54) and rearranging, we have

$$\frac{\mu^w}{n}=\mu^d\left\{\frac{-1}{1+(n-1)\lambda}+\frac{(1-\mu)(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda}\right\},$$

which can be rewritten as

$$\frac{\mu^w}{n}=\mu^d\left\{\frac{-\left[2-\mu+(n-2)\lambda+\mu\lambda\right]+(1-\mu)(1-\lambda)\left[1+(n-1)\lambda\right]}{\left[1+(n-1)\lambda\right]\left[2-\mu+(n-2)\lambda+\mu\lambda\right]}\right\}. \quad (55)$$

The LHS is always positive and the denominator of the RHS is always positive in (55), so it is sufficient to prove that the numerator is always negative to prove our result. The numerator of the LHS of (55)can be simplified to $-1-\lambda(n-1)\left\{\mu-\lambda\mu+\lambda\right\}$. This is always negative given that μ and λ are both smaller than one. Q.E.D.

ACKNOWLEDGEMENTS

We would like to thank the Belgian Program on Interuniversity Poles of Attraction initiated by the Science Policy Office of Belgium, the French research contract “Systèmes complexes en SHS : « Economie spatiale et dynamique non linéaire » and the Flanders fund for Scientific Research (contract G0076-04). We also thank G Giuliano for comments on a first version of the paper.

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