Revising Claims and Resisting Ultimatums in Bargaining Problems

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Abstract

We propose a mechanism which implements a unique solution to the bargaining problem with two players in subgame-perfect equilibrium. Players start by making claims and accept a compromise only if they cannot gain by pursuing their claim in an ultimatum. The player offering the lowest resistance to his opponent's claim can propose a compromise. The unique solution depends on the extent to which claims can be revised. If no revisions are allowed, compatible claims implement the Nash solution. If all revisions are allowed, maximal claims implement the Kalai-Smorodinsky solution.

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1 Introduction

Negotiations in conflicts often share the following two features. First, players commonly revise their initial claims during negotiations in order to find a compromise. Their willingness or ability to make revisions depends on the issue and may differ among players. Second, concessions may be induced by the threat of an ultimate take-it-or-leave-it offer. However, negotiators discourage such uncompromising behavior by adopting a firm posture, which allows them to walk away from negotiations without agreement when facing such an ultimatum.

These two features also appear in practical guides for negotiators, as in the defense procurement and acquisition guidelines by the US Department of Defense:[1] "Aim high" but "Give yourself room to compromise" and "Be willing to walk away from or back to negotiations". They correspond to the two approaches of creating value and claiming value described in the negotiation literature (Sebenius, 1992; Lewicki et al., 1994). Whereas negotiation analysis tends to ignore the interaction in the negotiators' choices between these two strategies, the game-theoretic bargaining literature has not yet tried to incorporate these two features in its analysis.

We try to fill this gap by analyzing the outcome of a stylized mechanism in subgame-perfect equilibrium. At the same time, we contribute to the Nash program by showing that the mechanism implements a class of solutions to the bargaining problem as in Miyagawa (2002), rather than a unique solution as in Moulin (1984), Binmore et al. (1986) and Howard (1992). The single distinguishing feature is the extent to which initial claims can be revised. The mechanism nests the solution of Nash (1950) and the solution of Kalai and Smorodinsky (1975) in the extreme cases excluding or admitting all revisions.

The restrictions on revising claims can be justified in many ways. Restrictions can arise from costs of revising plans or frustration from unfulfilled expectations raised by the claims. The general feature of loss aversion appears as concession aversion in the context of negotiations (Kahneman and Tversky, 1995). The extent to which claims can be revised can also be explicitly specified in the mandate given to the negotiator by his principal. Alternatively, it may depend on the mediator who steers the negotiations. We show that a player can never lose in equilibrium by becoming less restricted in the revisions of his initial claim. Negotiators who can suppress their frustration or do not fear to disappoint their principals obtain better agreements.

The mechanism has four stages. In the first stage, both players make claims defining their demands. In the second stage, they bid resistance to the uncompromising opponent, who pursues his demand in a take-it-or-leave-it offer. In the third stage, the mediator gives leadership to the player with the lowest resistance. The leader proposes a compromise within the set of feasible compromises which depends on the initial claims

[1] The Contract Pricing Reference Guides (Vol5, Ch6) of the DPAP of the US Department of Defense, http://www.acq.osd.mil/dpap/contractpricing/vol5chap6.htm#6.10.

but remains beyond his control in all other respects. In the final stage, the opponent accepts or rejects the compromise. If he rejects, then he obtains his demand in an ultimatum unless he meets resistance to which the proposer is committed by the second stage.

We characterize the unique outcome which is implemented in subgame-perfect equilibrium in two steps. Take any pair of incompatible claims in a first step. The resistance chosen by the negotiator in the second stage reflects his determination to resist a take-it-or-leave-it offer in the final stage. The follower will pursue his demand in an ultimatum and reject any feasible compromise when the leader's resistance falls short of a minimum level. Above this minimum, the leader's payoff in his best achievable compromise is increasing in his resolve. As in the first-price sealed-bid auction, the mediator gives leadership to the negotiator showing the lowest resistance in order to avoid impasses, leaving no acceptable compromises for two belligerent opponents. However, the same outcome would be obtained in an unmediated conflict if building up resistance against an uncompromising opponent requires time. Assume that both players build up their resistance at the same pace and that a player obtains leadership by stopping this process, as in the Dutch auction. Stopping this process gives leadership for certain. Yet, early stopping reduces the leader's payoff for lower resolve. Competition for leadership induces a player to take the lead as soon he is confident that he can block a take-it-or-leave-it offer by an acceptable compromise within his mandate. In this way, the right to make an offer is endogenized.[2]

Schelling (1956) argues that negotiators, as a bargaining tactic, purposely create uncertainty by reducing the scope of their own authority: union officials stir up excitement and determination on the part of the membership and governments arouse public opinion by statements that permit no concessions to be made. As a result, by rejecting the leader's compromise and pursuing his claim, the follower runs the risk of an undesired outcome, such as a strike or a war. This risk is under the leader's control. It is increased by his time-consuming effort of mobilizing the rank and file or of putting other deterrents in place. Since the player with insufficient room to make concessions needs too much time to deter a take-it-or-leave-it offer, he will lose the contest for leadership. Similarly, in a modification of the game with alternating offers of Rubinstein (1982) in which the responder can stop negotiations in an ultimatum, the proposer needs more time to deter such ultimatum for a better deal. This moves the outcome of the negotiations towards the one of the four-stage mechanism.

The competition for leadership, given a pair of claims, has two consequences. First, if the proportional solution is feasible, then it is implemented. The proportional solution is the Pareto-efficient outcome for which the players' concessions are proportional to their claims. Therefore, both players need the same minimal resolve for imposing

[2] The desirability of endogenizing the right to make a bargaining offer is emphasized in Perry and Reny (1993), Sakovics (1993) and Board and Zwiebel (2007).

that solution as a leader. This minimum guarantees the payoff of the proportional solution in equilibrium. Since leadership is given to the lowest bidder, both players bid that minimum. Second, if the proportional solution is not feasible, then the revised demands, in which concessions are maximal, are incompatible. The equilibrium outcome is the revised demand of the strong player, whose claim defines the largest extended Nash product, since the strong player needs lower resistance to impose his own revised demand. The extended Nash product of a player's claim is the product of his claim and his opponent's payoff in his revised demand. It is equal to the Nash product in case demands and revised demands are equal.

These two observations determine equilibrium claims in a second step. If the proportional solution is not feasible, the weak player may become strong and impose his own revised demand for a reduced claim. If the revised demands remain incompatible, this deviation is profitable. We distinguish between two cases.

In the first case, the strategic advantage of being strong induces players to exhibit restraint in the formulation of claims. This competition equalizes the extended Nash products for the equilibrium claims. Either revised demands are equalized as well, or they remain incompatible. If revised demands are equal, then the concessions are proportional to the claims. The proportional solution for this pair of claims is implemented. If claims cannot be revised, then equalized (revised) demands for equalized (extended) Nash products define the Nash solution, which maximizes the Nash product. If revised demands remain incompatible, then some player's extended Nash product is maximized and the other player's revised demand is implemented.

In the complementary case, some player is strong for his maximal claim in all pairs defining incompatible revised demands. For this maximal claim, his opponent becomes strong by reducing his claim only in pairs defining compatible revised demands. For these pairs, the proportional solution is implemented, so that the opponent loses by reducing his claim. Hence, no player has reason to exhibit restraint in the formulation of his claim. Unless the strong player prefers his revised demand for his maximal claim, the Kalai-Smorodinsky solution is implemented. In this Pareto-efficient solution, both players make maximal claims and make proportional concessions.

Harsanyi (1977) justified the Nash solution in a two-stage demand game. Players make demands in the first stage. Demands cannot be revised. The player with the highest risk limit imposes his demand in the second stage. A player's risk limit is the minimum level of resistance of his opponent for which he accepts his opponent's demand rather than pursuing his own demand in a take-it-or-leave-it offer. A player's risk limit for the opponent's revised demand is higher if his extended Nash product is higher. Hence, the 4-stage mechanism justifies the strategic advantage for the strong player in a two stage-demand game, whether or not claims are revisable.

Moulin (1984) justified the Kalai-Smorodinsky solution in an auction game in which players are assumed to make maximal claims and revisions are unrestricted. Stages two to four of the 4-stage mechanism recast Moulin's auction game, giving first-mover advantage to the player bidding the lowest resistance. By augmenting the auction game with a demand stage, the assumption that players make maximal claims is justified if the revision of claims is unrestricted.

The paper is organized as follows. The next section defines the bargaining problem and the 4-stage mechanism. Section 3 introduces a simple mechanism to characterize the equilibrium claims in the first stage of the 4-stage mechanism. Section 4 introduces risk limits and discusses the equilibrium strategies in the later stages. Section 5 establishes the equivalence between the simple mechanism and the 4-stage mechanism and shows how the 4-stage mechanism justifies simplifying assumptions in Harsanyi's demand game and Moulin's auction game. The robustness of the mechanism is tested in section 6 and some examples of revision procedures are given in section 7. The final section concludes.

2 The model

In this section, we define the bargaining problem and a mechanism for selecting a solution in the bargaining set.

2.1 The bargaining problem

The bargaining problem is defined as follows. Let $N = \{1, 2\}$ be the set of players, Ω be the set of jointly feasible outcomes, and $v_i(\omega)$ be the bounded utility of the outcome $\omega \in \Omega$ for each player $i \in N$. The players are male and the mediator is female. Player $-i$ is player i's opponent. The players and the mediator are completely informed about the players' private valuations of the outcomes.

Let $\Delta\Omega$ be the set of lotteries defined over Ω. For $x, y \in \Delta\Omega$ and for $i \in N$, we write $x \succeq_i y$ if the lottery x is at least as desirable as y and $x \succ_i y$ for strict preference. These preferences are represented by $u_i(x)$, player i's von Neumann-Morgenstern expected utility of the lottery $x \in \Delta\Omega$. Let $u(x) = (u_1(x), u_2(x))$. We write $u(x) \geq u(y)$ if $u_i(x) \geq u_i(y)$ for all $i \in N$. The outcome $\omega_0 \in \Omega$ obtains when no agreement can be reached. The dictatorial outcome x_i of player $i \in N$ is the Pareto-efficient outcome in $\arg\max_{x \in X^*} u_i(x)$. As a normalization, $v_i(\omega_0) = u_i(x_0) = 0$ in the degenerate lottery x_0 and $u_i(x_i) = 1$. Let $\mathbf{0} = (0, 0)$ and $\mathbf{1} = (1, 1)$. The set of individually rational lotteries is

$$X^* = \{x \in \Delta\Omega |\, u(x) \geq \mathbf{0}\}.$$

The bargaining set is

$$S^* = \{u(x) |\, x \in X^*\} \subset D = [0, 1] \times [0, 1].$$

Let $\varphi^*_{-i}(u_i) = \sup \{u_{-i} |\, u \in S^*\}$. Since S^* is convex by construction, the function φ^*_{-i}

is concave and decreasing on $[u_i(x_{-i}), 1]$ for all $i \in N$. The bargaining problem is comprehensive if, in addition, φ^*_{-i} is constant on $[0, u_i(x_{-i})]$ for all $i \in N$. The set of Pareto-efficient utility pairs in S^* is

$$PO(S^*) = \{ u \in [u_1(x_2), 1] \times [u_2(x_1), 1] |\, u_{-i} = \varphi^*_{-i}(u_i), i \in N\}.$$

The outcome $x \in PO(X^*)$ is Pareto efficient if and only if $u(x) \in PO(S^*)$. Also, $\dot{S}^* = S^* \setminus PO(S^*)$.

2.2 The mechanism

The mechanism Γ^B for selecting a solution in S^* is defined as follows. It has four stages.

The first stage is a demand stage, inspired by Harsanyi's demand game. Both players simultaneously formulate their claims $p \in D$. The second stage is a bidding stage, inspired by Moulin's auction game. Both players simultaneously submit bids $q \in D$. The bid q_i is the probability that player i stops negotiations in disagreement in the fourth stage, if his compromise were rejected by his opponent. The third stage is the compromising stage. The player with the lowest bid is selected by the mediator as the leader L, who makes a compromise proposal. The fourth and final stage is the approval stage. The follower F accepts the compromise or pursues his initial claim in a take-it-or-leave-it offer.

The claims of the first stage serve a double purpose. First, player i's claim p_i defines his demand $z_i(p_i)$ which gives him his claim $p_i = u_i(z_i(p_i))$ and his opponent $\varphi^*_{-i}(p_i) = u_{-i}(z_i(p_i))$ as payoffs. The demand $z_F(p_F)$ would be pursued by F in a take-it-or-leave-it offer in stage 4 if F rejects L's compromise. In that case, L's resistance to F's demand would result in disagreement with probability q_L. Hence, the payoffs of rejection of a compromise proposal are given by $(1 - q_L)\, u(z_F(p_F))$.

Second, player i's claim determines the set $X^B_i(p_i)$ of compromises which player i must consider when his opponent proposes in stage 3. The mapping z^B_i:$[0, 1] \rightarrow PO(X^*)$ defines player i's revised demand. It is the maximal concession which can be reconciled with an excessive claim ($z_i(p_i) \succeq_i z^B_i(p_i)$). It is the maximal remuneration which is consistent with a modest claim ($z_i(p_i) \prec_i z^B_i(p_i)$). We assume that

$$X^B_i(p_i) = \left\{ x \in X^* |\, u_i(z^B_i(p_i)) \leq u_i(x) \leq \max\left[p_i, u_i(z^B_i(p_i))\right] \right\}.$$

This implies that $PO(X^B_i(p_i)) = \{z^B_i(p_i)\}$ for modest claims. We also assume that a player can propose only those compromises as a leader which are also feasible for his opponent as a leader.[3] Hence, the set of feasible compromises is

$$X^B_1(p_1) \cup X^B_2(p_2).$$

[3] This assumption is relaxed in section 6.

Restricting the opponent's room for maneuver also confines the own freedom in finding a compromise. The correspondence linking the set of feasible compromises to each pair of claims is selected by the mediator, depending on the idiosyncrasies of the players' relationship, costs of revising claims and frustration, among others. It is beyond the control of the players and the single distinguishing feature of each mechanism Γ^B.

The player, who made the lower bid in the second stage and who will show lower resistance to his uncompromising opponent in the final stage, is rewarded by the mediator with first-mover advantage. In case of equal bids, leadership is given to the strong player $\xi(p)$, as defined in the next subsection. Hence,

$$L(q,p) = \begin{cases} i \text{ if } q_i < q_{-i}, \\ \xi(p) \text{ if } q_1 = q_2. \end{cases}$$

The rules of the mechanism Γ^B can be summarized as follows:

- **Stage** 1. All $i \in N$ formulate claims $p = a^1 \in D$.
- **Stage** 2. All $i \in N$ submit bids $q = a^2 \in D$.
- **Stage** 3. $L(q,p)$ proposes the compromise $c = a^3 \in X_1^B(p_1) \cup X_2^B(p_2)$.
- **Stage** 4. $F \in N \setminus \{L(q,p)\}$ chooses $a^4 \in \{Y, N\}$.

The payoffs are

$$\begin{aligned} u(c) \text{ if } a^4 &= Y, \\ (1-q_L)u(z_F(p_F)) \text{ if } a^4 &= N. \end{aligned}$$

The mediator announces the set of feasible compromises for each claim in stage 0 and guarantees that players abide by the rules of the mechanism. She designates the leader and verifies whether his compromise proposal is feasible. She also imposes disagreement with the probability chosen by L and limits F's payoff to his claim if F rejects L's compromise proposal.

2.3 The revision procedures

We consider revision procedures in which the revised demand z_i^B can be related to a convex subset S_i^B of D. If S_i^B is convex, then

$$\varphi_{-i}^B(p_i) = \sup \left\{ p_{-i} | \, p \in S_i^B \right\}$$

is a concave function.

Definition

The convex subset $S_i^B \subset D$ *defines a comprehensive revision procedure for the bargaining problem* S^* *if and only if* $\varphi_{-i}^B(1) \geq \varphi_{-i}^*(1)$ *and* $\varphi_{-i}^B(p_i) = \varphi_{-i}^B(0) > 0$ *whenever* $\varphi_{-i}^B(p_i) \leq \varphi_{-i}^*(p_i)$.

The revised demand $z_i^B(p_i) \in PO(X^*)$ of player i is the outcome for which

$$u_i(z_i^B(p_i)) = \varphi_i^* \left(\varphi_{-i}^B(p_i)\right) \text{ for } p_i \in [0,1] \, .$$

Hence, p_i and $u_{-i}(z_i^B(p_i))$ define a point in $PO(S_i^B)$. The revised demand is insensitive to the claim for modest demands.

We next give a definition of the strong player, who is the leader in stage 3 of Γ^B in case of equal bids.

Definition

Player ξ *is strong and his opponent* ζ *is weak if and only if* $p_\xi u_\zeta(z_\xi^B(p_\xi)) \geq p_\zeta u_\xi(z_\zeta^B(p_\zeta))$ *for* $\xi = 1$ *and* $p_\xi u_\zeta(z_\xi^B(p_\xi)) > p_\zeta u_\xi(z_\zeta^B(p_\zeta))$ *for* $\xi = 2$.

We call $p_i u_{-i}(z_i^B(p_i))$ the extended Nash product of player i's claim. The extended Nash product is equal to the Nash product $u_i(z_i(p_i)) \times u_{-i}(z_i(p_i))$ of the demand $z_i(p_i)$ if claims cannot be revised. Nash (1950) proposed $c^N \in \arg\max_{x \in X^*} u_1(x)u_2(x)$ as a solution to the bargaining problem.

The reduced bargaining problem $S(p)$ for $p \notin \dot{S}^*$ is defined by

$$S(p) = \{ u \in S^* | \, u \leq p \} \, .$$

Kalai and Smorodinsky (1975) proposed the Pareto efficient outcome $c^{KS}(p)$ for which $u_1(c^{KS}(p))/p_1 = u_2(c^{KS}(p))/p_2$ as a solution to the comprehensive bargaining problem $S(p)$ for $p \notin \dot{S}^*$. In this solution, the concessions are proportional to the claims and the payoffs are increasing in the own claim. Moreover, the extended Nash products are equal for p if the revised demands for p meet exactly in $c^{KS}(p)$.

3 Simple mechanisms

In this section, we define a simple two-stage demand game $\hat{\Gamma}^B$. Players formulate claims in the first stage as in Γ^B and the strong player ξ imposes a compromise in $\hat{X}_\xi^B(p)$ in the second stage. We define $\hat{X}_\xi^B(p)$ such that the strong player's preferred outcome is equal to the equilibrium outcome in Γ^B for each pair of claims. Hence, the simple demand game highlights the trade-off for each player i between increasing his claim p_i which increases his payoffs in $\hat{X}_i^B(p)$ and exhibiting restraint to become strong and impose his preferred outcome in this set. We characterize the equilibrium claims and outcome for the class of comprehensive revision procedures.

The simple mechanism $\hat{\Gamma}^B$ for a comprehensive bargaining problem S^* is defined as follows:

- **Stage** *a***.** All $i \in N$ formulate claims $p \in D$.

- **Stage** b**.** Player $\xi(p)$ selects an outcome in $\hat{X}^B_{\xi(p)}(p)$,

$$\hat{X}^B_i(p) = \begin{cases} \{z^B_i(p_i), c^{KS}(p)\} & \text{if } p \notin \dot{S}^*, \\ \{z_1(p_1), z_2(p_2)\} & \text{if } p \in \dot{S}^*. \end{cases}$$

Stage a of $\hat{\Gamma}^B$ is equivalent to stage 1 of Γ^B and stages 2 to 4 of Γ^B are compressed in stage b of $\hat{\Gamma}^B$. Notice that $\hat{\Gamma}^B$ nests Harsanyi's demand game, in which the player, whose claim has the largest Nash product in stage a, proposes $z_1(p_1)$ or $z_2(p_2)$ in stage b. In this simple demand game, claims cannot be revised, so that $z^B_i(p_i) = z_i(p_i)$. In that case, $z_i(p_i) \succeq_i c^{KS}(p) \succeq_i z_{-i}(p_{-i})$ for $p \notin \dot{S}^*$ and $z_1(p_1) = z_2(p_2) = c^{KS}(p)$ for $p \in PO(S^*)$. Hence, player i selects in stage b the same outcome in $\{z_1(p_1), z_2(p_2)\}$ as in $\hat{X}^B_i(p)$.

The payoff of z^B_i is non-decreasing in the claim by comprehensiveness. The payoff of c^{KS} is increasing in the own claim and decreasing in the opponent's claim. It follows that for $p, p' \notin \dot{S}^*$ and $p'_i \geq p_i$,

$$\min_{x \in \hat{X}^B_{-i}(p)} u_i(x) \leq \max_{x \in \hat{X}^B_i(p)} u_i(x) \leq \max_{x \in \hat{X}^B_i(p')} u_i(x) \qquad (\bigstar)$$

By the first inequality, it is advantageous to be strong and to select the outcome for the pair p of claims. The advantage is strict, unless both players prefer proportional concessions and the identity of the strong player does not matter. By the second inequality, the strong player will never be worse off by increasing his claim if he remains strong. The two inequalities together imply that the strong player cannot be better off by reducing his claim.

The privilege of the strong player is given to the player whose claim defines the largest extended Nash product. The extended Nash product is unimodal in p_i if S^B_i is comprehensive. It reaches its maximum in $\hat{p}_i$. If there exists a pair p of claims such that the extended Nash products are equal for $p \geq \hat{p}$, we write $(\check{p}_1(p_2), p_2) = (p_1, \check{p}_2(p_1))$. The players' preferences on $\hat{X}^B_i(p)$ for incompatible claims with equal extended Nash products are as in Remark 1.

Remark 1. *If S^* is strictly comprehensive and S^B_1 and S^B_2 are comprehensive, then for $p = (p_1, \check{p}_2(p_1)) \notin \dot{S}^*$ and for all $i \in N$,*

$$z^B_{-i}(p_{-i}) \succeq_i (\prec_i)\, z^B_i(p_i) \text{ if and only if } c^{KS}(p) \succeq_i (\prec_i)\, z^B_i(p_i).$$

Proof *See appendix.*

We characterize the solution c^B of $\hat{\Gamma}^B$ for a partition of the set of comprehensive revision procedures into three subsets. In the first subset, some player is strong for his maximal claim, whatever his opponent claims. In the second subset, some player is strong for the maximal claim of his opponent, but only for claims defining compatible revised demands. The last subset contains all other comprehensive revision procedures.

For this partition, the following labelling convention guarantees that, for any claim of player 2, player 1 is strong for claims belonging to a closed interval. In each of the three subsets, the labelling is such that $i = 1$.

$$\exists i \text{ such that } \quad u_{-i}(z_i^B(1)) \geq \hat{p}_{-i} u_i(z_{-i}^B(\hat{p}_{-i})), \tag{3.1}$$

$$\text{or } \exists i \text{ such that } \quad u_i(z_{-i}^B(1)) \geq u_i(z_i^B(\check{p}_i(1))) \text{ and } \hat{p}_i < \check{p}_i(1), \tag{3.2}$$

or if (3.1) and (3.2) do not hold

$$\exists i \text{ such that } \hat{p}_i u_{-i}(z_i^B(\hat{p}_i)) \geq \hat{p}_{-i} u_i(z_{-i}^B(\hat{p}_{-i})) \text{ and } \hat{p}_i < 1.$$

In the first two elements of the partition, for which respectively (3.1) and (3.2) hold, none of the players gains by showing restraint in the formulation of their claims. This is obvious in case (3.1) holds. If a player is always strong for his maximal claim, then he cannot gain by reducing his claim by the second inequality of (★). If (3.2) holds, a player can become strong, if he is weak for the pair of maximal claims, but only for claims for which both players prefer proportional concessions to their revised demands by Remark 1. By the monotonicity of the proportional solution, this player can even lose by showing restraint. His claim remains inconsequential as long as the strong player prefers his revised demand for his maximal claim to the proportional solution. In both cases, the unique equilibrium outcome c^B is player $\xi(\mathbf{1})$'s preferred outcome in $\{z_{\xi(\mathbf{1})}^B(1), c^{KS}(\mathbf{1})\}$ as in the first part of Proposition 1.

The second part of the proposition characterizes the solution c^B for revision procedures in the third element of the partition for which (3.1) and (3.2) do not hold. In that case, competition for the privilege enjoyed by the strong player induces restraint in the formulation of claims by (★). Although the payoffs in $\hat{X}_i^B(p)$ are non-increasing in the own claim p_i, only the strong player can impose his preferred solution. This equalizes the extended Nash products if the conditions of Lemma 1 hold. Let P be the set of claims in stage a for which an equilibrium is reached in $\hat{\Gamma}^B$. This set of equilibrium claims will never contain compatible claims defining unequal demands.

Lemma 1. *In any subgame-perfect equilibrium of $\hat{\Gamma}^B$ for which S^* is strictly comprehensive and S_1^B and S_2^B are comprehensive, $P \cap \dot{S}^* = \emptyset$. Assume, furthermore, that (3.1) and (3.2) do not hold and that $p \in P$. Then $\mathbf{1} > p = (p_1, \check{p}_2(p_1)) \geq \hat{p}$ or $p = (\check{p}_1(1), 1) \geq \hat{p}$.*

Proof *See appendix.*

By Lemma 1, it suffices to consider cases in which $p_1 = \check{p}_1(p_2)$ for $p \notin \dot{S}^*$. Equilibria for which $c^{KS}(p) \succ_\xi z_\xi^B(p_\xi)$ are excluded, since the weak player gains by a larger claim for which he remains weak and for which the strong player chooses proportional concessions. By Remark 1, revised demands must be incompatible or equal. As long as revised demands are incompatible and $p_\zeta > \hat{p}_\zeta$, the weak player has a reduced claim $p'_\zeta < p_\zeta$ for which he is strong such that $z_\zeta^B(p'_\zeta)$ is preferred to $z_\zeta^B(\check{p}_\xi(p_\zeta))$.

In equilibrium, either $p_\zeta = \hat{p}_\zeta$ or revised demands are equal. In the first case, revised demands are incompatible for the pair $(\check{p}_\xi(\hat{p}_\zeta), \hat{p}_\zeta) \notin \dot{S}^*$. The weak player cannot become strong by changing his claim. If the weak player claims $\hat{p}_\zeta$, the strong player cannot claim more than $\check{p}_\xi(\hat{p}_\zeta)$ without becoming weak. Nor can he gain by claiming less than $\check{p}_\xi(\hat{p}_\zeta)$ by the second inequality of ($\bigstar$). Hence, $c^B = z_\xi^B(\check{p}_\xi(\hat{p}_\zeta))$ is imposed in equilibrium. In the second case, the revised demands are compatible for the pair $(\check{p}_1(\hat{p}_2), \hat{p}_2) \notin \dot{S}^*$. A unique pair of claims p exists such that $z_1^B(p_1) = z_2^B(p_2)$ for $p = (\check{p}_1(p_2), p_2)$. Since compatible and incompatible revised demands are excluded, this is the unique pair of equilibrium claims, which implements $c^B = z_1^B(p_1) = z_2^B(p_2)$. The identity of the leader does not matter since the revised demands $z_1^B(p_1)$ and $z_2^B(p_2)$ and the proportional solution all coincide by Remark 1. If some player decreases his claim, he cannot gain by the second inequality of ($\bigstar$). If some player increases his claim, the same outcome will be imposed.

Proposition 1. *Assume that c^B is the outcome implemented in a subgame-perfect equilibrium of $\hat{\Gamma}^B$ for which S^* is strictly comprehensive and S_1^B and S_2^B are comprehensive.*

- *If* (3.1) *or* (3.2) *hold, then c^B is player $\xi(\mathbf{1})$'s preferred outcome in $\{z_{\xi(\mathbf{1})}^B(1), c^{KS}(\mathbf{1})\}$.*
- *If* (3.1) *and* (3.2) *do not hold, then $p = (\check{p}_1(p_2), p_2)$ and*

$$c^B = \begin{cases} z_1^B(p_1) = z_2^B(p_2) = c^{KS}(p) & \text{if } z_1^B(\check{p}_1(\hat{p}_2)) \preceq_1 z_2^B(\hat{p}_2), \\ z_1^B(\check{p}_1(\hat{p}_2)) & \text{if } z_1^B(\check{p}_1(\hat{p}_2)) \succ_1 z_2^B(\hat{p}_2). \end{cases}$$

Proof *See appendix.*

If $S_i^B = D$ for all $i \in N$ and S^* is comprehensive, then $u_{-i}(z_i^B) = 1$. The extended Nash products reach a maximum for the maximal claim. Hence player 1 is strong for all claims of player 2 and (3.1) holds. Since player 1 prefers $c^{KS}(\mathbf{1})$ to his revised claim, the Kalai-Smorodinsky solution $c^{KS}(\mathbf{1})$ of S^* is implemented.

If $S_i^B = S^*$ for all $i \in N$, then $z_1^B = z_1$ and $z_1^B(\check{p}_1(\hat{p}_2)) = z_2^B(\hat{p}_2)$ for $\hat{p}_2 = u_2(c^N)$. At the same time, $z_1^B(p_1) = z_2^B(p_2) = c^{KS}(p)$ for $p = u(c^N) \in PO(S^*)$. This is also the case for $z_1^B \neq z_1$, but for $p \notin S^*$, in the following example.

Example

Let $\varphi_i^*(p_{-i}) = [1-(1-\sqrt{1-\sqrt{p_{-i}}})^2]^2$, $\varphi_2^B(p_1) = [1-(1-\sqrt{1-p_1})^2]^2$ and $\varphi_1^B(p_2) = (\varphi_2^B)^{-1}(p_2)$. Hence, $S^* = \{u|\, u_1 \in [0,1], u_2 \in [0, \varphi_2^*(u_1)]\}$ and $S_1^B = S_2^B = S^B = \{u|\, u_1 \in [0,1], u_2 \in [0, \varphi_2^B(u_1)]\}$, as in Figure 1. The thick and thin curve represent respectively $\varphi_2^*(p_1)$ and $\varphi_2^B(p_1)$ for $p_1 \in [0,1]$. The payoff pairs achieved in the set of feasible compromises $X_1^B \cup X_2^B$ belong to the shaded area. The claims $p = (.73, .81)$ are incompatible and define the same revised demand. If player 1 claims $p_1 = .73$, then $u(z_1^B(p)) = (.53, .59)$. If player 2 claims $p = .81$, then $u(z_2^B(p)) = (.53, .59)$. Also the

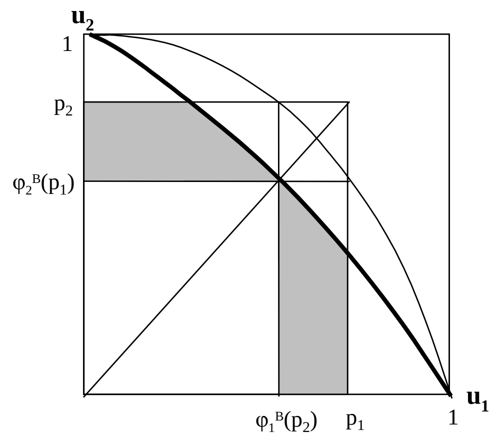

Figure 1: Bargaining Set with Revision Procedures

extended Nash products $p_i \varphi^B_{-i}(p_i)$ are equal for $i \in N$. Therefore, the revised demands meet exactly after proportional concessions, $c^{KS}(p) = z^B_1(p) = z^B_2(p)$ by Remark 1.■

4 Risk limits and bidding strategies

In the simple demand game, the strong player was granted the privilege of dictating his preferred solution in $\hat{X}^B_\xi$. In this section, we justify this assumption, solving for the equilibrium strategies in Γ^B from the second stage on. We first define risk limits, introduced by Zeuthen (1930) and used by Harsanyi (1977) to justify the Nash solution. The risk limit defines F's response in stage 4 to L's compromise proposal in stage 3 and determines the compromise proposal as in a Stackelberg game. We also relate the extended Nash products to the risk limits of the two players, before we derive the bidding strategies in the second stage. We distinguish between subgames with compatible claims and subgames with incompatible claims in case the demands of both players are Pareto-efficient and excessive. We also derive the bidding strategies for Pareto-inefficient or modest demands, which never pay in equilibrium.

4.1 Risk limits

In the approval stage, player F chooses between the compromise c and the compound lottery d which has disagreement x_0 and his demand z_F as outcomes. The probability of disagreement equals the leader's resistance q_L. If $u_F(z_F) > 0$, the risk limit of player F is defined as

$$r_F(c) = \max \left[\frac{u_F(z_F) - u_F(c)}{u_F(z_F) - u_F(x_0)}, 0 \right].$$

The risk limit measures the resolve of F in pursuing z_F when c is proposed as a compromise. Given the resistance $q_L \in [0, 1]$ of L to the uncompromising F, the risk limit of F defines his response in the last stage as

$$a_F^4 = \begin{cases} \text{Y if } q_L \geq r_F(c) \text{ and } u_F(c) > 0, \\ \text{N otherwise.} \end{cases} \tag{4.1}$$

That is, F will accept L's proposal c if L's resistance q_L to F's demand is at least equal to F's risk limit. A proposal becomes acceptable by reducing F's resolve in an improved compromise or by increasing the resistance to F's demand. In particular, F always accepts $c = z_F$ (because $r_F(z_F) = 0$).

If c is rejected, the outcome $d = q_L x_0 + (1 - q_L) z_F$ obtains. It is assumed that F accepts $c \in X^B$ in a tie. Hence, if $u_F(c) = u_F(d)$, F accepts c. We add d to the set of feasible compromises. For a pair p of claims, L can choose from

$$X^B(p) = X_1^B(p_1) \cup X_2^B(p_2) \cup \{d\}.$$

Hence, if there is no acceptable c in $X_1^B(p_1) \cup X_2^B(p_2)$, then L proposes d so that F always responds with Y (because $r_F(d) = q_L$). The response of F determines the compromise proposed by L as in

$$a_L^3 \in \arg\max_{x \in X^B} [u_L(x) |\, a_F^4(z_F, x, q_L) = \text{Y}]. \tag{4.2}$$

Let $\sigma = (\sigma_1, \sigma_2)$ be a strategy profile for Γ^B. The history $h^{s-1} \in \mathcal{H}^{s-1}$ at stage $s = 1, ..., 4$ is recursively defined by $h^s = \left(a^s, h^{s-1}\right)$ and $h^0 = \emptyset$. The strategy of player i at stage s in the subgame for the history h^{s-1} in σ is denoted by $a_i^\sigma(h^{s-1})$.

Lemma 2. *Assume that σ is a subgame-perfect equilibrium of Γ^B and that S_1^B and S_2^B are comprehensive. Then for all $h^2 \in \mathcal{H}^2$, $c = a_L^\sigma(h^2) = a_L^3$ satisfies condition* (4.2) *and, for all $\hat{c} \in X^B$, $a_F^\sigma(\hat{c}, h^2) = a_F^4$ satisfies condition* (4.1) *. In particular, if the pair of incompatible claims p defines Pareto-efficient and excessive demands, then c is always accepted. Moreover,*

- (i) *if $q_L = 0$, then $c = z_F$,*
- (ii) *if $q_L \in \left[0, r_F(z_F^B)\right] \cup \left[r_F(z_L^B), r_F(z_L)\right]$, then $r_F(c) = q_L$ and $u_L(c)$ is increasing in q_L on each of these intervals,*
- (iii) *if $r_F(z_F^B) \leq q_L < r_F(z_L^B)$, then $c = z_F^B$*
- (iv) *if $r_F(z_L) \leq q_L$, then $c = z_L$.*

Proof *See appendix.*

We now recast the definition of the strong and the weak player in terms of risk limits. The follower is indifferent between accepting z_L^B and the lottery d if player L has submitted the bid

$$q_L^B(p) = r_F(z_L^B(p_L)) \text{ for } L \in N. \tag{4.3}$$

Since $p_{-i} = u_{-i}(z_{-i})$ and $u_{-i}(x_0) = 0$, $q_i^B(p) = 1 - \dfrac{u_{-i}(z_i^B)}{p_{-i}}$. Hence, we find the equivalence

$$p_i u_{-i}(z_i^B(p_i)) \geq p_{-i} u_i(z_{-i}^B(p_{-i})) \text{ if and only if } q_i^B(p) \leq q_{-i}^B(p).$$

The strong player needs less resistance to implement his revised demand. Since the less militant player is rewarded with first-mover advantage, the strong player's greater risk limit gives him strategic advantage. Any player can make sure that the proportional solution $c^{KS}(p)$ is implemented, if it is feasible. This result is driven by the proportionality property of $c^{KS}(p)$. If he leads with resistance q_L equal to

$$q^{KS}(p) = r_i(c^{KS}(p)) \text{ for } i \in N,$$

the opponent accepts $c^{KS}(p)$, since

$$u(c^{KS}(p)) = (1 - q^{KS}(p))p. \tag{4.4}$$

Moreover, the opponent cannot gain by taking the lead. By choosing his resistance below $q^{KS}(p)$, all proposals which he prefers to $c^{KS}(p)$ will be rejected. However, only the strong player can guarantee that no proposals in X_ζ^B are made or accepted when $X_1^B \cap X_2^B = \emptyset$. If the strong player bids $q_\xi = q_\xi^B(p)$ in stage 2, the weak player accepts z_ξ^B as a follower. The weak player becomes leader only by bidding $q_\zeta < q_\xi^B(p) \leq q_\zeta^B(p)$. In that case, his resistance to z_ξ is too low for imposing his revised demand z_ζ^B.

4.2 Incompatible claims

We characterize the equilibrium bidding strategies for incompatible claims defining Pareto-efficient and excessive demands. They depend on whether the strong player prefers c^{KS} to his revised demand z_ξ^B by the following remark.

Remark 2. *Assume that $z_i \in PO(X^*)$ for $p \notin \dot{S}^*$ and that S_i^B is comprehensive. For all $i \in N$,*

$$c^{KS}(p) \succ_i (\preceq_i) z_i^B(p_i) \textit{ if and only if } q^{KS}(p) > (\leq) q_i^B(p).$$

Proof *See appendix.*

Case 1. $z_\xi^B(p_\xi) \prec_\xi c^{KS}(p)$.

If the strong player strictly prefers c^{KS} to z_ξ^B for incompatible claims, then $c^{KS} \in X_\xi^B$. Competition for leadership puts downward pressure on the bids if the leader were to win with $q_L > q^{KS}(p)$. The follower would lead the compromise stage by undercutting his opponent with resistance exceeding $q^{KS}(p)$. He could propose an acceptable compromise preferred to the proportional solution rather than being forced to accept a compromise that is worse than the proportional solution. Conversely, no player would want to lead the compromise stage for $q_L < q^{KS}(p)$, since the leader's compromise must be preferred to the proportional solution by the follower to be accepted. Profitable deviations are excluded, only if both players bid resistance equal to $q^{KS}(p)$. Since both players propose the proportional solution, the identity of the leader is unimportant. Hence, $c^{KS}(p)$ is the unique outcome implemented in subgame-perfect equilibrium.

Lemma 3. *Assume that σ is a subgame-perfect equilibrium of Γ^B for which S_1^B and S_2^B are comprehensive. For all $h^1 \in \mathcal{H}^1$ with $p \notin S^*$ defining Pareto-efficient and excessive demands for which $c^{KS}(p) \succ_\xi z_\xi^B(p_\xi)$, $c^{KS}(p)$ is proposed by ξ and accepted by ζ and*

$$q_i = a_i^\sigma(h^1) = q^{KS}(p) \text{ for all } i \in N. \tag{4.5}$$

Proof *See appendix.*

Case 2. $z_\xi^B(p_\xi) \succeq_\xi c^{KS}(p)$.

If, for $p \notin S^*$, the strong player weakly prefers z_ξ^B to c^{KS}, then c^{KS} does not belong to the feasible set of compromises X^B, unless $c^{KS} = z_\xi^B$ by Remark 1. Hence, the revised demands are equal or incompatible. In any case, the strong player can guarantee himself his revised demand z_ξ^B by choosing resistance within $[q_\xi^B, q_\zeta^B]$. Since the resistance of ξ to z_ζ exceeds q_ξ^B, ζ will accept ξ's proposal z_ξ^B. Since the resistance of ζ to z_ξ as a leader falls short of q_ζ^B, ξ will reject any proposal of ζ in X_ζ^B. If claims cannot be revised, $z_\xi = z_\xi^B$ will be proposed and accepted, independently of the weak player's bid. If claims can be revised, then by bidding within $[r_\xi(z_\xi^B), q_\xi^B]$, the best proposal z_ξ^B for ζ in X_ξ^B is accepted by ξ since ζ's resistance to z_ξ exceeds $r_\xi(z_\xi^B)$. Moreover, ξ is a leader only if $q_\xi = q_\zeta = q_\xi^B = r_\zeta(z_\xi^B)$ and ξ will propose z_ξ^B since ζ will reject any proposal of ξ in $X_\xi^B \setminus \{z_\xi^B\}$. Hence, $z_\xi^B(p_\xi)$ is the unique outcome in subgame-perfect equilibrium.

Lemma 4. *Assume that σ is a subgame-perfect equilibrium of Γ^B for which S_1^B and S_2^B are comprehensive. For all $h^1 \in H^1$ with $p \notin S^*$ defining Pareto-efficient and excessive demands for which $z_\xi^B(p_\xi) \succeq_\xi c^{KS}(p)$, $z_\xi^B(p_\xi)$ is proposed and accepted with*

$$\begin{aligned} q_\xi &= a_\xi^\sigma(h^1) \in \left[q_\xi^B(p), q_\zeta^B(p)\right], \\ q_\zeta &= a_\zeta^\sigma(h^1) \in \begin{cases} [0,1] \text{ if } z_\xi^B(p_\xi) = z_\xi(p_\xi), \\ [r_\xi(z_\xi^B(p_\xi)), q_\xi^B(p)] \text{ if } z_\xi^B(p_\xi) \neq z_\xi(p_\xi). \end{cases} \end{aligned} \tag{4.6}$$

Proof *See appendix.*

4.3 Compatible claims

For subgames with compatible claims defining Pareto-efficient and excessive demands, the opponent's demand is the best outcome for each player in the set of feasible compromises. Moreover, the opponent always accepts his own demand as a follower, regardless of the opposition of the leader. Hence, competition for first-mover advantage excludes opposition of the leader to the opponent's demand. Player 1 bids no resistance and proposes player 2's demand as a leader. Compatible claims only occur in equilibrium if claims cannot be revised.

Lemma 5. *Assume that σ is a subgame-perfect equilibrium of Γ^B in which S_1^B and S_2^B are comprehensive and that $p \in S^*$ defines Pareto-efficient and excessive demands for $h^1 \in \mathcal{H}^1$. If $p \in PO(S^*)$, then $q = a^\sigma(h^1) \in D$. If $p \in \dot{S}$, then $q_1 = a^\sigma(h^1) = 0$ and $q_2 = a^\sigma(h^1) \in [0,1]$.*

Proof *See appendix.*

4.4 Inefficient or modest demands

Up to now, claims defining Pareto-inefficient or modest demands were excluded. Low claims may result in Pareto-inefficient demands when S^* is not strictly comprehensive. They may also result in modest demands when $S^* \backslash S_i^B \neq \emptyset$, even if S^* is strictly comprehensive. We complete the analysis of bidding strategies by characterizing the outcomes implemented in equilibrium for these cases in Lemma 6.

Lemma 6. *Assume that $q = a^\sigma(h^1)$ in a subgame-perfect equilibrium σ for Γ^B, when S_1^B and S_2^B are comprehensive. First, assume that there exists $k \in N$ for $h^1 \in H$ such that z_k is Pareto inefficient. If $z_\zeta \notin PO(S^*)$, then player ξ's preferred outcome in $X^B \cup \{z_\zeta\}$ is implemented. If $z_\zeta \in PO(S^*)$, then z_ζ is implemented. Second, assume that z_k is Pareto efficient for all $k \in N$ and that there exists $i \in N$ for $h^1 \in H$ such that $z_{-i}^B \succ_{-i} z_{-i}$. If $z_i \succ_i z_{-i}$ and $c^{KS} \in X_\xi^B$, c^{KS} is implemented.* If $z_i \succ_i z_{-i}$ and *$c^{KS} \notin X_\xi^B$, the preferred outcome of the weak player in $\{z_\xi^B, z_\xi\}$ is implemented. If $z_{-i} \succeq_i z_i$, z_ζ is implemented.*

Proof *See appendix.*

5 Implementing bargaining solutions

We characterized the strategies in subgame-perfect equilibrium following any pair of initial claims in any mechanism Γ^B for the bargaining problem S^*. The pairs of jointly feasible and individually rational payoffs belong to a convex set and the revision procedure S_i^B for each player is comprehensive. It remains to determine the claims in the first stage for the complete characterization of a subgame-perfect equilibrium for Γ^B. We show that they implement the same solution as the simple mechanism $\hat{\Gamma}^B$. These findings are related to the bargaining theory of Moulin (1984) for $S_1^B = S_2^B = D$ and Harsanyi (1977) for $S_1^B = S_2^B = S^*$.

5.1 The solution to Γ^B

It follows from Lemma 3 and Lemma 4 that in Γ^B, as in $\hat{\Gamma}^B$, the preferred outcome of the strong player in $\{z_\xi^B(p_\xi), c^{KS}(p)\}$ is implemented in equilibrium for any pair of incompatible claims, defining excessive and efficient demands. Moreover, from Lemma 5, $z_\zeta(p_\zeta)$ is implemented for any pair of compatible claims, defining excessive and efficient demands. The mechanisms Γ^B and $\hat{\Gamma}^B$ will have the same solution for strictly comprehensive bargaining problems. Since no different solutions are implemented in equilibrium by claims defining Pareto-inefficient and modest demands in Γ^B, the equilibrium outcome still coincides with the equilibrium outcome for the mechanism $\hat{\Gamma}^B$, in which only claims defining Pareto-efficient and excessive demands are feasible. Hence, modesty never pays.

Proposition 2. *Assume that σ is a subgame-perfect equilibrium for the mechanism Γ^B in which S_1^B and S_2^B are comprehensive. Then σ implements the solution c^B of Proposition* 1.

Proof *See appendix.*

5.2 Harsanyi's augmented demand game

Corollary 1. *Any subgame-perfect equilibrium σ of Γ^B for which S_1^B and S_2^B are comprehensive and $S_1^B = S_2^B = S^*$ implements the Nash solution to S^*.*

Proof If $S_i^B = S^*$, then $z_i^B = z_i$. Since condition (3.1) and (3.2) do not hold, by Lemma 1, $p_1 = \check{p}_1(p_2)$ so that $z_1^B(p_1) = z_2^B(p_2) = c^{KS}(p)$ if and only if $z_1(p_1) = z(p_2) = c^N \in \arg\max_{x \in X^*} u_1(x)u_2(x)$. By Proposition 2, c^N is the unique outcome implemented in subgame-perfect equilibrium.■

Harsanyi (1977) derived the Nash solution c^N as an equilibrium for a demand game in which, according to the conjecture of Zeuthen (1930), the player with the lowest risk limit submits. By augmenting Harsanyi's demand game, Zeuthen's conjecture is confirmed.

The inability of the leader to revise his incompatible demand in the compromising stage gives veto power to the strong player. By his militantness, the weak player accepts his demand. By his moderation, he dissuades militantness of the weak player which could compel him to accept the weak player's demand. By keeping his resistance within bounds, the strong player imposes his demand on the weak player and the weak player cannot impose his demand on the strong player. A player is therefore vulnerable if he has a lower risk limit, as in Zeuthen's conjecture. By reducing his claim, each player reduces the opponent's resoluteness in withstanding his demand. In equilibrium, equal demands imply zero risk limits and the own risk limit falls below the opponent's risk limit after any improvement in the own demand.

5.3 Moulin's augmented auction game

Corollary 2. *Any subgame-perfect equilibrium σ of Γ^M for which $S_1^B = S_2^B = D$ implements the Kalai-Smorodinsky solution to S^*.*

Proof: If $S_i^B = D$, then $z_i^B = x_{-i}$, so that $\hat{p}_i u_{-i}(x_{-i}) = 1$ and $1 = u_2(x_1) \geq \hat{p}_i u_{-i}(x_{-i})$. Since (3.1) holds and $c^{KS}(\mathbf{1}) \succeq_1 z_1^B(1) = x_2$, $c^{KS}(\mathbf{1})$ is implemented by Proposition 2.[4]■

The stages 2 to 4 of Γ^B recast the mechanism proposed by Moulin (1984). The non-cooperative justification of the Kalai-Smorodinsky solution relies on two features of the negotiation procedure. First, the player showing lower resistance to an uncompromising opponent is given first-mover advantage. Although moderation is rewarded with first-mover advantage, militantness improves the own payoff in a compromise proposal, as in Moulin's model in which each player demands $z_i = x_i$ for his maximal claim $p_i = 1$. Let

$$c_i(q_i) = \arg\max_{x \in X^*} \left[u_i(x) | \, r_{-i}(x) \leq q_i \right].$$

Since $u_{-i}(x_{-i}) = 1$ and $u_{-i}(x_0) = 0$,

$$u_{-i}(c_i(q_i)) = 1 - r_{-i}(c_i(q_i)) = 1 - q_i \text{ for } u(c_i(q_i)) \in PO(S^*).$$

The winning bid, determining the probability of disagreement, is the lowest bid. If player i wins the auction, then $u_i(c_i(q_i))$ is his payoff. If his opponent wins the auction, then his payoff is $1 - q_{-i} \geq 1 - q_i$. Hence, $\min\left[u_i(c_i(q_i)), 1 - q_i\right]$ is a lower bound for his payoff. Militantness increases $u_i(c_i(q_i))$, but decreases $1 - q_i$. Moulin proposes

$$q_i^* \in \arg\max_{q_i} \min\left\{u_i(c_i(q_i)), 1 - q_i\right\} \text{ for } i \in N$$

as bidding strategies. For each $i \in N$,

$$u_i(c_i(q_i^*)) = \max_{u \in S^*}[u_i | \, u_{-i} \geq 1 - q_i^*] = 1 - q_i^* = u_{-i}(c_i(q_i^*)).$$

Since the Pareto efficient compromise $c_i(q_i^*)$ gives the same payoff to each player in the normalized bargaining problem, it is the Kalai-Smorodinsky solution to the bargaining problem S^*. Hence, both players submit the same bids for Moulin's bidding strategies. If player i were to increase his bid, then his opponent wins with q_{-i}^* and proposes $c_{-i}(q_{-i}^*)$. If player i were to reduce his bid, then he wins and he has to reduce his payoff in a compromise. Moulin's bidding strategies constitute a Nash equilibrium in the bidding of resistance when each player demands his dictatorial outcome. The same argument holds for any pair of incompatible claims $p \notin S^*$ in the reduced bargaining problem $S(p)$.

The second feature of the negotiation procedure implementing the Kalai-Smorodinsky

[4] The trivial bargaining problem with $S^* = D$ is not excluded.

solution is $X^B(p) = X(p)$. Since in the Kalai-Smorodinsky solution for $S(p)$, $u_i(c^{KS}(p))$ is monotone in p_i, nobody shows restraint in the formulation of claims. By augmenting Moulin's model with a demand stage, Γ^B justifies Moulin's assumption that players make maximal claims for $S_1^B = S_2^B = D$.

6 Robustness

In this section, we justify some of the simplifying features of the mechanism Γ^B.

6.1 Unmediated leadership

First of all, we assumed that the mediator gives leadership to the player exhibiting the lowest resistance. If the mediator were to give leadership to the highest bid, then she would encourage militantness. Hence both players would make maximal claims and the labelling would make a dictator of one of the players. The incentives to limit resistance against a take-it-or-leave-it offer can be endogenized in an unmediated mechanism, if we assume that building up resistance to be armed against a take-it-or-leave-it offer needs time. Arms races are an example. Alternatively, if building up resistance takes no time, then the time needed by the negotiator to convince his principal or himself that concessions are necessary in an acceptable proposal may be assumed to be proportional to the resistance. In the second stage of the mechanism, both players increase resistance q, simultaneously and at the same pace, until one of the players stops and proposes a compromise. The resistance built by the proposer at that point is the disagreement probability to which he is committed when his opponent stops negotiations with a take-it-or-leave-it offer. The equilibrium strategies when players bid for leadership or when resistance is built until one player takes the lead are the same. The strategy space and the allocation of leadership do not change. The equivalence between bidding and building resistance is the same as the equivalence between a sealed-bid first-price auction and a Dutch auction.

6.2 Unmediated revisions

We also assumed that $X_1^B(p_1) \cup X_2^B(p_2)$ is the set of feasible compromises. That is, any opportunity for compromise available to one player is also made available to the other player by the mediator. For incompatible claims in unmediated revisions in the mechanism $\tilde{\Gamma}^B$, each player i makes compromises within his own set $X_i^B(p_i)$. This will be the case when costs of revising plans or frustration restrict the revision of claims, as in Example 1 and 2. If the proportional solution is unfeasible under this alternative assumption, the strong player will always lead the negotiations. He will wait for making a compromise until the weak player's opposition is high enough to make an acceptable compromise in $X_\zeta^B(p_\zeta)$. The strong player's strategic advantage is enhanced if claims define unequal extended Nash products. If the proportional solution is feasible or the extended Nash products are equal for the equilibrium claims in Γ^B, the outcome implemented in the equilibrium of $\tilde{\Gamma}^B$ remains unchanged.

In the equilibrium of $\tilde{\Gamma}^B$ for any pair of incompatible claims, defining Pareto-efficient and excessive demands, the strong player implements his preferred solution in $\tilde{X}^B_\xi(p) = \{y^B_\xi(p), c^{KS}(p)\}$, where

$$u_\zeta(y^B_\xi(p)) = \max\{(1 - q^B_\zeta(p))p_\zeta, u_\zeta(z_\xi(p_\xi))\}.$$

By Lemma 2 and Remark 2, $q^B_\xi(p) \leq q^B_\zeta(p)$ implies that $y^B_\xi(p) \succeq_\xi z^B_\xi(p_\xi)$. Moreover, $q^{KS}(p) < q^B_\zeta(p)$ implies that $y^B_\xi(p) \succeq_\xi c^{KS}(p) \succ_\xi z^B_\zeta(p_\zeta)$ and $c^{KS}(p) \notin X^B_\zeta(p_\zeta)$, and $q^{KS}(p) \geq q^B_\zeta(p)$ implies that $c^{KS}(p) \succeq_i z^B_i(p_i)$ and $c^{KS}(p) \in X^B_i(p_i)$ for all $i \in N$. Hence, we distinguish between two cases. If $q^{KS}(p) \geq q^B_\zeta(p)$, the proportional solution is feasible for both players. It is implemented and the equilibrium strategies remain the same. By stopping before $q^{KS}(p)$, the leader is worse off than in the proportional solution. By stopping after $q^{KS}(p)$, the leader would be better off. However, the follower could gain by stopping and becoming leader at an earlier time. If $q^{KS}(p) < q^B_\zeta(p)$, the strong player waits until $q^B_\zeta(p)$ is reached and proposes $y^B_\xi(p)$. By stopping before $q^B_\zeta(p)$, the strong player is worse off than in $y^B_\xi(p)$. By stopping after $q^B_\zeta(p)$, he gives the opportunity to the weak player to stop at $q^B_\zeta(p)$ and to impose $z^B_\zeta(p_\zeta)$. As in Γ^B, both players compete for the strategic advantage of the strong player. Hence, the equilibrium claims are the same in $\tilde{\Gamma}^B$ and Γ^B, unless condition (3.1) or (3.2) holds and $y^B_\xi(\mathbf{1}) \succ_\xi c^{KS}(\mathbf{1})$. In that case, the equilibrium claim of the weak player $\zeta(\mathbf{1})$ is unique, in contrast with Γ^B. Since

$$u_\zeta(y^B_\xi(1, p_\zeta)) = p_\zeta u_\xi(z^B_\zeta(p_\zeta)) \leq \hat{p}_\zeta u_\xi(z^B_\zeta(\hat{p}_\zeta)),$$

the weak player's payoff in $y^B_\xi(1, p_\zeta)$ increases for a reduced claim $p_\zeta \geq \hat{p}_\zeta$. However, his payoff in $c^{KS}(p)$ decreases. Since the equilibrium outcome equals the strong player's preferred outcome in $\tilde{X}^B_\xi(1, p_\zeta)$, the unique equilibrium claim $p^\blacklozenge_\zeta$ solves

$$\min_{p_\zeta} \max\left\{u_\xi(y^B_\xi(1, p_\zeta)), u_\xi(c^{KS}(1, p_\zeta))\right\}. \tag{$\blacklozenge$}$$

Notice that if condition (3.1) holds, $p^\blacklozenge_\zeta = \hat{p}_\zeta$. Hence, $y^B_\xi(1, p^\blacklozenge_\zeta)$ is implemented in $\tilde{\Gamma}^B$ instead of the strong player's preferred outcome in $\{z^B_1(1), c^{KS}(\mathbf{1})\}$ in Γ^B.

The equilibrium of a bargaining problem in which the feasible revisions are player specific satisfies the following property. The more a negotiator is susceptible to feelings of frustrations from unfulfilled expectations, the less proficient he will be in negotiating. The higher the negotiator's fear of disappointing his principal, the less ambitious the targets set by the principal and the less favorable the resulting agreement. In general, a player's payoff in the equilibrium outcome can never decrease if larger concessions become feasible for this player. If a player can make larger concessions for a given claim, then his extended Nash product for that claim increases. He may become strong and impose his own preferred solution for an even larger claim. Even if he remains weak, he may be able to propose an acceptable compromise for a lower level of resistance, forcing his opponent to take the lead at an earlier time.

Proposition 3. *Assume that the mechanisms $\tilde{\Gamma}^X$ and $\tilde{\Gamma}^Y$ only differ in the comprehensive revision procedure of player i with*

$$\varphi_{-i}^X(p_i) \geq \varphi_{-i}^Y(p_i) \text{ for all } p_i \in [0,1],$$

then player i weakly prefers the unique equilibrium outcome c^X to c^Y.

Proof *See appendix.*

6.3 Alternating offers

The mechanism allows for take-it-or-leave-it offers, possibly resulting in disagreement. This contrasts with the alternating offer game proposed by Rubinstein (1982) in which players respond to a compromise by accepting or proposing a new compromise. We propose a modification of the mechanism $\Gamma^B(\varepsilon)$ in line with Rubinstein's alternating offer game which shows that this contrast is essential. As in Γ^B, both players start making claims simultaneously. After the initial claims, players take turns in making proposals until one player accepts his opponent's proposal or pursues his initial claim in a take-it-or-leave-it offer. At his turn, each player i builds up resistance q_i before he proposes a compromise in $X_i^B(p_i)$. Building up resistance to block a take-it-or-leave-it response of his opponent to his compromise requires time. The depreciation of the payoffs in this time interval is $\exp(-\varepsilon q_i)$.

For any pair of incompatible claims $p(\varepsilon)$, there exists $\varepsilon_0 > 0$ such that for any $\varepsilon < \varepsilon_0$, a unique pair of bargaining solutions $(c_i(\varepsilon), c_{-i}(\varepsilon))$ and levels of resistance $q(\varepsilon)$ solves

$$\begin{aligned} u_i(c_{-i}(\varepsilon)) &= \exp(-\varepsilon q_i(\varepsilon))u_i(c_i(\varepsilon)) \\ &= (1 - q_{-i}(\varepsilon))p_i(\varepsilon) \text{ for all } i \in N. \end{aligned}$$

By the first equality, accepting his opponent's offer is as good as waiting for the own proposal. By the second equality, accepting his opponent's offer is as good as stopping with a take-it-or-leave-it offer. If $c_i(\varepsilon) \in X_i^B(p(\varepsilon))$ for all $i \in N$, then each player i proposes $c_i(\varepsilon)$ at his turn and accepts any compromise $c \succeq_i c_{-i}(\varepsilon)$ at the opponent's turn in equilibrium for $p(\varepsilon)$.

If impatience $\rho_i = q_i(\varepsilon)$ were exogenously determined, as in Rubinstein's alternating offer game, then, by the first equality,

$$\frac{\ln u_i(c_{-i}(\varepsilon)) - \ln u_i(c_i(\varepsilon))}{\varepsilon} = \rho_i \text{ for all } i \in N.$$

For $\varepsilon \to 0$, $c_i(\varepsilon)$ and $c_{-i}(\varepsilon)$ converge to c^*. By the chain rule,

$$\frac{d \ln \varphi_i^*(u_{-i}(c^*))}{d \ln u_{-i}} = \frac{\rho_i}{\rho_{-i}}.$$

This defines the generalized Nash bargaining solution, which is equal to the Nash solution c^N for equally impatient players. The former is preferred to the latter by the less impatient player i for whom $\rho_i < \rho_{-i}$. However, by the second equality,

$$q_{-i}(\varepsilon) = 1 - \frac{u_i(c_i(\varepsilon))}{p_i(\varepsilon)} = r_i(c_{-i}(\varepsilon))|_{p_i(\varepsilon)} \text{ for all } i \in N.$$

The impatience of the players is endogenized by the choice of resistance. Since player i's payoff is increasing in his claim p_i, his claim will be maximal or such that $c_i(\varepsilon)$ is constrained by his revised demand. If revisions are unrestricted, players make maximal claims so that only take-it-or-leave-it offers distinguish $\Gamma^B(\varepsilon)$ from Rubinstein's alternating offer game. In equilibrium, $p(\varepsilon) = p^* = \mathbf{1}$ and $c_i(\varepsilon)$ and $c_{-i}(\varepsilon)$ converge to c^* which satisfies

$$\frac{d \ln \varphi_i^*(u_{-i}(c^*))}{d \ln u_{-i}} = \frac{r_{-i}(c^*)}{r_i(c^*)}.$$

With unrestricted revisions in Γ^B, the risk limits and, therefore, the resistance levels are equal in equilibrium, so that $c^B = c^{KS}(\mathbf{1})$ is implemented. However, if c^N and $c^{KS}(\mathbf{1})$ do not coincide, $\frac{d \ln \varphi_i^*(u_{-i}(c^{KS}(\mathbf{1})))}{d \ln u_{-i}} \neq 1$. Hence, $c^{KS}(1)$ cannot be a solution to $\Gamma^B(\varepsilon)$. Neither can c^N, since $\frac{r_{-i}(c^N)}{r_i(c^N)} \neq 1$. In c^N, the player with the lower risk limit is forced to take more time to build up the necessary resistance for obtaining c^N. However, his higher impatience inhibits him to obtain a compromise as good as c^N in equilibrium. The introduction of take-it-or-leave-it offers in the alternating offer game $\Gamma^B(\varepsilon)$ moves the equilibrium outcome from the Nash solution towards the equilibrium outcome $c^B = c^{KS}(\mathbf{1})$ in Γ^B.[5]

7 Examples

We clarify the two Propositions by specifying $\psi_1^B(u)$ and $\psi_2^B(u)$. If $\psi_i^B(u) = 0$, then $p_i = u_i$ is player i's claim and $\varphi_{-i}^B(p_i) = u_{-i}$ is his opponent's utility after his maximal

[5] The same result may hold for restricted revisions when $c^* = z_1^B(p_1^*) = z_2^B(p_2^*)$ for $p(\varepsilon) \to p^*$ such that $c_i(\varepsilon) = z_i^B(p_i(\varepsilon))$ for all $i \in N$. If some player i decreases his claim $p_i < p_i(\varepsilon)$, $(c_i(\varepsilon), c_{-i}(\varepsilon))|_{(p_i, p_{-i}(\varepsilon))}$ remains feasible and reduces player i's payoff. If some player i increases his claim $p_i > p_i(\varepsilon)$, $c_{-i}(\varepsilon)|_{(p_i, p_{-i}(\varepsilon))} \notin X_{-i}^B(p_{-i}(\varepsilon))$. The increased gap between the revised demands allows player $-i$ to propose $z_{-i}^B(p_{-i}(\varepsilon))$ and refuse any feasible compromise of his opponent in an equilibrium. This excludes all profitable deviations.

concession given this claim. The indicator function $\delta(u|D)$ equals 0 if $u \in D$ and ∞ if $u \notin D$.

Example 1: Costs of Revising

$$\psi_1^B(u) = \psi_2^B(u) = \chi(u) + \delta(u|D) - \beta,$$

$$\text{with } \chi(u) = \begin{cases} \inf\{|u-y||\, y \in S^*\}, \\ \text{or } \inf[\lambda \geq 0|\, u \in \lambda S] - 1. \end{cases}$$

The distance of a pair of claims to the set S^*, measured by the Euclidian distance or the gauge, restricts the ability of revising these claims. Obtain c^N for $\beta = 0$ and c^{KS} for all β above some threshold $\hat{\beta}$. All Pareto efficient outcomes in between these two solutions can be obtained for some intermediate value of β.

In many applications, negotiators are agents defending interests of their principal. Crawford (1982) refers to costs for negotiators who retreat on a position that they have agreed to defend. Negotiators have limited authority and are accountable to their principals. If revising targets must be justified, revisions of claims will be limited. Moreover, plans must be well documented before they are workable and the duties of the parties must be correctly specified before they can be written in a contract for complex relationships. The costs of replanning become prohibitive for major revisions after setting the target, which will be pursued in a take-it-or-leave-it offer.□

Example 2: Frustration

$$\psi_i^B(u) = u_{-i} - [\alpha\varphi_{-i}^*(\mu_i(u_i))^\kappa + (1-\alpha)\lambda_{-i}\left(\varphi_{-i}^*(u_i)\right)^\kappa]^{\frac{1}{\kappa}} + \delta(u|D),$$

$$\text{with } \begin{cases} \mu_i(u_i) \leq u_i \text{ an increasing convex function,} \\ \lambda_{-i}(u_{-i}) \geq u_{-i} \text{ an increasing concave function,} \\ \kappa \leq 1, 0 \leq \alpha \leq 1. \end{cases}$$

If $\alpha = 1$, the largest concession of player i for his claim p_i is $p_i - \mu_i(p_i) > 0$. This yields $\varphi_{-i}^B(p_i) = \varphi_{-i}^*(\mu_i(p_i))$ as an upper bound for the opponent's revised payoff. If $\alpha = 0$, $\varphi_{-i}^B(p_i) = \lambda_{-i}\left(\varphi_{-i}^*(p_i)\right)$ is the upper bound for the opponent's revised payoff. This yields $\varphi_i\left(\lambda_{-i}\left(\varphi_{-i}^*(p_i)\right)\right) < p_i$ as a lower bound for player i's payoff. By combining the two approaches, the upper bound to the opponent's payoff is a weighted average ($\kappa = 1$) or converges to the minimum ($\kappa \to -\infty$) of the payoffs obtained in the two cases. Since $\varphi_{-i}^*(\mu_i(p_i))$ and $\lambda_{-i}\left(\varphi_{-i}^*(u_i)\right)$ are decreasing concave functions of p_i, the increasing convex function ψ_i^B defines a convex set S_i^B which contains S^*.

Example 2 can be used to model frustration or concession aversion (Kahneman and Tversky, 1995). Assume that for $\bar{\mu}_i < 1, \bar{\lambda}_{-i} < 1$ and $\kappa \to -\infty$, the opponent's payoff in the maximal concession of player p_i is the minimum of the payoffs obtained for $\mu_i(u_i) = \bar{\mu}_i u_i$ and $\lambda_{-i}(u_{-i}) = \bar{\lambda}_{-i} + u_{-i}$. Player i would be frustrated if his payoff were below μ_i or his opponent's payoff were above λ_{-i} after the revision of a claim.[6] Larger $\bar{\mu}_i$ and lower $\bar{\lambda}_{-i}$ reduce the set of feasible compromises for which player i avoids frustrations. This results in a less favorable outcome for him by Proposition 3. Before the own proportional loss and the opponent's absolute gain is evaluated, one may transform the normalized payoffs into monetary payoffs by the inverse utility function v_i^{-1}. If each player i exhibits risk aversion on $\left[0, m_i^* = \nu_i^{-1}(1)\right]$, then $\mu_i = \nu_i \circ (1-\bar{\mu}_i)\nu_i^{-1}$ and $\lambda_{-i} = \nu_{-i} \circ \left(\bar{\lambda}_{-i} m_{-i}^* + \nu_{-i}^{-1} \circ \varphi_{-i}^*\right)$ are respectively increasing convex and decreasing concave function such that $\psi_i^B(u)$ is convex.□

Example 3: Modest Demands

$$\psi_i^B(u) = u_{-i} - \beta_{-i} + \delta(u|\, D).$$

Each player's revised demand is independent of his initial claim. The payoff of player i in a compromise in $X_i^B(p_i)$ cannot be lower than $\varphi_i^*(\beta_{-i})$. Claims below $\varphi_i^*(\beta_{-i})$ are modest. As an example, β_{-i} could be the right of player $-i$ in a claims problem in which the sum of rights β_1 and β_2 of the two players exceed the estate (Thomson 2003). The mediator does not allow to make concessions in which the opponent obtains more than his right. The player with the lowest minimal right is strong for his maximal claim and for all claims of his opponent. He proposes the Kalai-Smorodinsky solution in equilibrium, because his opponent's claim exceeds 1/2.

The approaches in the three examples can be combined by taking the intersection of the convex sets S_i^B obtained in each of these examples.□

Example 4: Social Welfare Functions

$$\psi_1^B(u) = \psi_2^B(u) = u_1 u_2^c + u_2 u_1^c - \max_{u \in S^*}\left[u_1 u_2^c + u_2 u_1^c\right].$$

Let $\hat{S}^B$ be the half-space below the hyperplane $\left\{u \in \mathcal{R}^2 | u_1 u_2^c + u_2 u_1^c = \max_{u \in S^*}\left[u_1 u_2^c + u_2 u_1^c\right]\right\}$ supporting S^*. It follows that $S_1^B = S_2^B = [0,1] \times [0,1] \cap \hat{S}^B$. The revision process specifies a pair of claims $p^c \in \arg\max_{u \in S^*}\left[u_1 u_2^c + u_2 u_1^c\right]$ for which no revisions are feasible, and a constant exchange rate u_{-i}^c / u_i^c belonging to the subdifferential $-\partial \varphi_{-i}^*(p_i^c)$ for player i's other claims. The exchange rate measures player i's ability to revise an increased claim. If $u^c = (u_1^c, u_2^c) \in PO(S^*)$, then for some $\lambda \geq 1$,

$$\lambda u^c \in \arg\max_{u \in \hat{S}^B} u_1 u_2.$$

For the claims $p_i = u_i^c(2\lambda - 1)$ for $i \in N$, the revised demands are equal (i.e. $u_i(z_i^B(p_i)) = u_i^c$ for $i \in N$) and both players have the same strength. Therefore, if $\max_{i \in N}(u_i^c(2\lambda -$

[6] Instead of requiring that L proposes $c \in C^B(p)$ in the compromising stage, a sufficiently large frustration cost can be introduced which penalizes L in the compromising stage if $c \notin C_L^B(p)$ and F in the acceptance stage if $c \notin C_F^B(p)$ is accepted. The players will adjust their claims so that these costs are always avoided.

1) ≤ 1, u^c is implemented in equilibrium by Proposition 1. In particular, for the exchange rate $u_{-i}(c^N)/u_i(c^N)$ for player i, c^N is implemented ($\lambda = 1$), and, for the same exchange rate for the two players ($u^c_{-i}/u^c_i = 1$), $c^{KS}(\mathbf{1})$ is implemented. Obtain intermediate solutions by varying the exchange rate between these bounds. If player i prefers c^N to $c^{KS}(\mathbf{1})$, his exchange rate has to be increased to move the equilibrium outcome from c^N to $c^{KS}(\mathbf{1})$. The higher the exchange rate, the lower his willingness to make concessions for larger claims and the lower his final success in the negotiation process. The advantage of making larger concessions for one player in Proposition 3 is confirmed in this example. As long as the claims defined by $u^c(2\lambda - 1)$ are feasible, the mediator, who steers the negotiations by specifying the exchange rate in the revision procedure, can implement any bargaining solution that maximizes a quasi-concave social welfare functions, as in Miyagawa (2002).

8 Conclusion

We analyzed a simple, intuitive mechanism that implements a unique solution to the bargaining problem with two players. The mechanism is also a parsimonious model for dealing with the Negotiators' Dilemma (Lax and Sebenius, 1986) in negotiation analysis. A cooperative approach may be self-defeating if one of the negotiators takes advantage of his opponent in a value-taking approach. We showed that, for completely informed players, the value-creating approach prevails in equilibrium without giving up the value-taking approach. In this way, this paper contributes to narrowing the gap between the game-theoretic approach and the behavioral approach to negotiations (Sebenius, 1992).

In line with Rubinstein, Safra and Thomson (1992), our approach offers an intuitive interpretation of a bargaining solution in terms of the primitives of the bargaining problem, provided that preferences are expected utility preferences. Consider a player's appeal against an alternative by an objection in his advantage. The objection to the alternative is *sensible* for some rejection rate, if the objection is feasible for the player's demand and if the rejection rate deters the opponent's pursuit of the alternative in an ultimatum. A player's *relevant* alternative for a solution is his preferred demand for which the solution remains a feasible compromise. A solution is robust if, for each player, it is a sensible objection to his opponent's relevant alternative for a rejection rate not exceeding the one which makes the solution a sensible objection for the player's opponent to his own relevant alternative.

In this interpretation, we generate a whole family of solutions by varying the extent to which demands in the value-claiming approach can be revised in the value-creating approach. The Nash solution is the unique robust solution, if the relevant alternative is the solution itself. The ability to revise claims was assumed to be beyond the control of the negotiators in the course of negotiations. However, if a negotiator were to suppress his feelings of frustration or if he did not fear to disappoint his principal by making large concessions, he would achieve better deals. In the evaluation of the performance of a negotiator, results loom larger than circumstances under which his results were achieved. Hence, it seems plausible that professional negotiators will strive for more room to maneuver. Similarly, principals will learn by experience to give discretionary power to their negotiators as to decide which concessions have to be made. If maximal concessions yield compatible revised demands, restraint in the formulation of claims will disappear. The predicted outcome in conflicts between experienced negotiators would be the Kalai-Smorodinsky solution. As in its original presentation, the players start by demanding their dictatorial outcomes, but are still able to find a compromise in which the proportional concessions are the same. Of course, players must still be able to reconcile their claims in compatible concessions after contemplating the prospect of their dictatorial outcome.

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9 Appendix

9.1 Proofs of Propositions

Proof of Proposition 1

Assume that (3.1) or (3.2) holds. We show that $\mathbf{1}$ is a pair of equilibrium claims. For any unilateral deviation from $\mathbf{1}$, the pair p of claims does not belong to $\dot{S}^*$, hence $X^B_{\xi(p)} = \{z^B_{\xi(p)}(p_{\xi(p)}), c^{KS}(p)\}$. By the labelling convention, player 1 is strong for any pair of claims if (3.1) holds, player 2 is strong for the pair of maximal claims if (3.2) holds. By ($\bigstar$), player $\xi(\mathbf{1})$ cannot gain by reducing his claim. Gains for player $\zeta(\mathbf{1})$ are excluded as well. If (3.1) holds, player 2 cannot become strong by reducing his claim. If (3.2) holds, player 1 becomes strong for $p_1 \in [\hat{p}_1, \check{p}_1(1))$. We consider two subcases. In the first case, $z^B_2(2) \succ_2 c^{KS}(1)$. Player 2 proposes the same outcome $z^B_2(1)$ as long as $p_1 \geq \max[p_1^+(1), \check{p}_1(1)]$ with $c^{KS}(1, p_1^+(1)) = z_2^B(1)$. If player 1 remains weak for $p_1 < p_1^+(1)$ or becomes strong for $p_1 < \check{p}_1(1)$, $c^{KS}(p_1, 1)$ is proposed and player 1 is worse off. In the second case, $c^{KS}(\mathbf{1}) \succeq_2 z_2^B(1)$. Either player 1 remains weak and player 2 proposes $c^{KS}(p_1, 1) \prec_1 c^{KS}(1)$. Or he becomes strong for $p_1 \in [\hat{p}_1, \check{p}_1(1))$. Then, by the second inequality of ($\bigstar$), his payoff cannot exceed $\max_{x \in \hat{X}_1^B(\check{p}_1(1),1)} u_1(x)$**.** By (3.2) and (3.3), this upper bound cannot exceed the payoff of $c^{KS}(\check{p}_1(1), 1) \prec_1 c^{KS}(1)$**.**

We conclude the proof of the proposition's first part by showing that for (3.1) or (3.2), player $\xi(\mathbf{1})$'s preferred outcome in $\{c^{KS}(\mathbf{1}), z^B_{\xi(\mathbf{1})}(1)\}$ is implemented for any $p \in P$. If (3.1) holds and $z^B_{\xi(\mathbf{1})}(1) = z^B_{\xi(\mathbf{1})}(0) \succeq_{\xi(\mathbf{1})} c^{KS}(\mathbf{1})$, there may be claims below 1 for which player $\xi(\mathbf{1})$ is strong whatever player $\zeta(\mathbf{1})$ claims and for which $z^B_{\xi(\mathbf{1})}(1)$ is implemented. If $c^{KS}(\mathbf{1}) \succ_{\xi(\mathbf{1})} z^B_{\xi(\mathbf{1})}(1)$ or $z^B_{\xi(\mathbf{1})}(1) \succ_{\xi(\mathbf{1})} z^B_{\xi(\mathbf{1})}(0)$ and either (3.1) or (3.2) holds, $p_{\xi(\mathbf{1})} = 1$ is the unique equilibrium claim for $\xi(\mathbf{1})$. Assume that $p_{\xi(\mathbf{1})} < 1$ in equilibrium. If (3.1) holds ($\xi(\mathbf{1}) = 1$), player 1 is strong whatever player 2 is claiming. He gains by increasing his claim. If (3.2) holds ($\xi(\mathbf{1}) = 2$) and $p_2 < 1$**,** $p = (p_1, \check{p}_2(p_1)) \geq \hat{p}$ for $p \in P$ by the same argument as in Lemma 1. If not, either $p_1 = 1$ and player 2 remains or becomes strong by making the maximal claim $p_2 = 1$, which makes him strictly better off or $p_1 < 1$ and the strong player could remain strong for a higher claim and strictly increase his payoff. However for $p_2 < 1$ and $p = (p_1, \check{p}_2(p_1))$, $z_2^B(p_2) \succeq_1 z_2^B(\check{p}_2(1)) \succeq_1 z_1^B(1) \succ_1 z_1^B(p_1)$. By (3.3), $c^{KS}(p) \succ_i z_i^B(p_i)$ is selected for compatible revised demands. However, this cannot be an equilibrium, because both players can gain for some $p'_i > p_i$. Hence, if player 2 claims $p_2 = 1$, from above, his preferred outcome in $\{c^{KS}(\mathbf{1}), z_2^B(1)\}$ is implemented.

In order to prove the proposition's second part, assume that (3.1) and (3.2) do not hold. If $p \in P$, then by Lemma 1, either $\hat{p}_2 = 1$ and $p = (\check{p}_1(1), 1) \geq \hat{p}$ or $1 > p = (p_1, \check{p}_2(p_1)) \geq \hat{p}$. We first consider the existence of incompatible revised demands in equilibrium. If $z_1^B(\check{p}_1(p_2)) \succ_1 z_2^B(p_2)$, then $c^{KS}(p) \prec_i z_i^B(p_i)$ for all $i \in N$ by (3.3)) and player 1 selects $z_1^B(p_1)$. We distinguish between two subcases. In the first subcase, there exists p' such that $p'_2 < p_2$, $p'_1 = \check{p}_1(p_2)$ and $z_2^B(p'_2) \succ_2 z_1^B(p'_1)$. Since player 2 is strong for p' and since $z_2^B(p'_2)$ is feasible, player 2 gains by claiming p'_2. In this subcase, $p \notin P$. In the complementary subcase, player 2 remains weak for all his claims if player 1 claims $\check{p}_1(p_2)$. Hence $p_2 = \hat{p}_2$. It follows that $z_1^B(\check{p}_1(\hat{p}_2))$ is implemented in equilibrium if $z_2^B(\hat{p}_2) \prec_1 z_1^B(\check{p}_1(\hat{p}_2))$. By increasing his claim, player 1 becomes weak and player 2 selects $z_2^B(\hat{p}_2)$. His payoff cannot increase. By reducing his claim, player 1 remains strong. By monotonicity of his payoffs in $\hat{X}_1^B(p)$ for $p \notin \dot{S}^*$, he cannot gain if his reduced claim remains incompatible. Since he selects $z_2(p_2)$ for a reduced compatible claim and since $z_2(p_2) \preceq_2 z_2^B(p_2)$ for $p_2 \geq \hat{p}_2$, gains are excluded as well. For any other claim, player 2 remains weak. Since the Pareto-efficient outcomes in $\hat{X}_1^B$ for incompatible claims do not give a lower payoff to player 1 and since player 2 cannot gain for a reduced compatible claim for the same reason as player 1, player cannot gain for any other claim. It follows that $z_1^B(\check{p}_1(\hat{p}_2))$ is the unique outcome if

$z_1^B(\check{p}_1(\hat{p}_2)) \succ_1 z_2^B(\hat{p}_2)$.[7]

We next consider the case for which $z_1^B(\check{p}_1(\hat{p}_2)) \preceq_1 z_2^B(\hat{p}_2)$. Since (3.2) does not hold, $z^B_{\zeta(\mathbf{1})}(\check{p}_{\zeta(\mathbf{1})}(1)) \prec_{\xi(\mathbf{1})} z^B_{\xi(\mathbf{1})}(1)$. By comprehensiveness of S_1^B and S_2^B, there exists p_2^* such that $z_1^B(\check{p}_1(p_2^*)) = z_2^B(p_2^*)$. By (3.3), $z_2^B(p_2^*) = c^{KS}(\check{p}_1(p_2^*), p_2^*)$. For $p_2 < p_2^*$, $z_1^B(\check{p}_1(p_2)) \prec_1 z_2^B(p_2)$, so that by (3.3), $c^{KS}(p) \succ_i z_i^B(p_i)$ for all $i \in N$. Player 1 is strong and selects $c^{KS}(p)$. Player 2 claims $p_2' > p_2$ such that $c^{KS}(p_1, p_2') = c^{KS}(p)$ and gains. For $p_2 \in (p_2^*, \check{p}_2(1)]$, $z_1^B(\check{p}_1(p_2)) \succ_1 z_2^B(p_2)$, so that by (3.3), $c^{KS}(p) \prec_i z_i^B(p_i)$ for all $i \in N$. Player 1 is strong and selects $z_1^B(p_1)$. Player 2 becomes strong and gains by claiming $p_2' < p_2$ with $z_2^B(p_2') \succeq_2 c^{KS}(p) \succ_2 z_1^B(p_1)$. Hence, $(\check{p}_1(p_2^*), p_2^*)$ is the unique equilibrium pair. By increasing his claim, a player becomes or remains weak. Since his opponent's payoff in c^{KS} is reduced, his opponent chooses his unchanged revised demand. By decreasing his claim, a player becomes or remains strong. His payoff in c^{KS} is reduced and his payoff in his revised demand is not increased.■

Proof of Proposition 2

In case of incompatible demands, defining Pareto-efficient and excessive demands, $c^{KS}(p)$ is proposed and accepted if $z_\xi^B(p_\xi) \prec_\xi c^{KS}(p)$ by Lemma 3 and $z_\xi^B(p_\xi)$ is proposed and accepted if $z_\xi^B(p_\xi) \succeq_\xi c^{KS}(p)$ by Lemma 4. That is, the strong player implements his preferred outcome in $\{z_\xi^B(p_\xi), c^{KS}(p)\}$. This set coincides with $\hat{X}^B_{\xi(p)}(p)$ for $p \notin \dot{S}^*$ for $\hat{\Gamma}^B$. In case of compatible claims, defining Pareto-efficient and excessive demands, the strong player implements his preferred outcome in $\{z_\xi(p_\xi), z_\zeta(p_\zeta)\}$ in equilibrium by Lemma 5. This set coincides with $\hat{X}^B_{\xi(p)}(p)$ for $p \in \dot{S}^*$ for $\hat{\Gamma}^B$. The choice set for p in any subgame $h^1 \in \mathcal{H}$ of Γ^B and $\hat{\Gamma}^B$ coincide, except for Pareto-inefficient and modest demands. However, as long as $p < \mathbf{1}$ and some claim defines a Pareto-inefficient or modest demand, at least one player can gain by increasing his claim from Lemma 6. Maximal claims define Pareto-efficient and excessive demands. Hence, Pareto-inefficient or modest demands can occur in equilibrium, only if the player, whose claim $p_\zeta < 1$ defines a Pareto-inefficient or modest demand, can never become stronger than the opponent whose claim is maximal, so that the claim p_ζ is inconsequential for the equilibrium outcome. That is, if (3.1) holds and $z_\xi^B(1) \succ_\xi c^{KS}(1, p_\zeta)$. In this case, $z_\xi^B(1)$ is the unique outcome implemented in equilibrium in Γ^B, which coincides with the strong player's preferred outcome in $\hat{X}^B_{\xi(1,p_\zeta)}(1, p_\zeta)$.

This proves that for any equilibrium claim $p = a^\sigma(h^0)$ in Γ^B, the equilibrium outcome coincides with the strong player's preferred outcome in $\hat{X}^B_{\xi(p)}(p)$. Hence, σ implements the solution c^B of Proposition 1.■

Proof of Proposition 3

Assume that p^Y is the pair of equilibrium claims in $\tilde{\Gamma}^Y$. By assumption, (1) $\hat{p}_i^X u_i(z_i^X(\hat{p}_i^X)) \geq \hat{p}_i^Y u_i(z_i^Y(\hat{p}_i^Y))$ and (2) $\tilde{p}_i^X u_{-i}(z_i^X(\tilde{p}_i^X)) = p_i^Y u_{-i}(z_i^Y(p_i^Y))$ for $\tilde{p}_i^X \geq$

[7]Notice that $z_1^B(\check{p}_1(1)) \succ_1 z_2^B(1)$ for $\hat{p}_2 = 1$ when (3.1) and (3.2) do not hold. Hence, $P = \{(\check{p}_1(1), 1)\}$.

$\max[\hat{p}_i^X, p_i^Y]$ or $u_{-i}(z_i^X(1)) > p_i^Y u_{-i}(z_i^B(p_i^Y))$. We start by considering two cases in which $c^X = c^Y$ for any change in the revision procedure. In the first case, $c^Y = c^{KS}(\mathbf{1}) \in X_1^Y \cap X_2^Y$. Since $X_1^Y \cap X_2^Y \subset X_1^X \cap X_2^X$, c^Y remains feasible and is implemented in $\tilde{\Gamma}^X$. In the second case, $c^Y = y_i^Y(1, p_{-i}^{\blacklozenge Y})$. Since $y_i^X(1, p_{-i}^{\blacklozenge Y}) = y_i^Y(1, p_{-i}^{\blacklozenge Y})$, $c^X = c^Y$. Notice that in this case, player i's payoff would be reduced in Γ^X. We next consider two cases for which $c^X \succeq_i c^Y$. In the first case, $c^Y = y_{-i}^Y(1, p_i^{\blacklozenge Y})$, so that, by ($\blacklozenge$), $y_{-i}^X(1, p_i^{\blacklozenge X}) \succeq_i y_{-i}^Y(1, p_i^{\blacklozenge Y}) = c^Y$. Either $c^X = y_{-i}^X(1, p_i^{\blacklozenge X})$ or player $-i$ has to reduce his claim $p_{-i}^X < 1$ in equilibrium. In both situations, $c^X \succeq_i c^Y$. In the second case, the extended Nash products for the players' claims are equal for p^Y in $\tilde{\Gamma}^Y$ and player i may become strong for p^Y in $\tilde{\Gamma}^X$. Either, player i chooses $c^{KS}(\mathbf{1})$ or $y_i^B(1, p_{-i}^{\blacklozenge X})$ weakly preferred to c^Y. Or the extended Nash products in $\tilde{\Gamma}^X$ are equal for the equilibrium pair p^X with $p_{-i}^X \leq p_{-i}^Y$. Then, $c^X \succeq_i c^Y$ by (1) if $c^X = z_{-i}^X(p_{-i}^X)$ and by (2) if $c^X = z_i^X(p_i^X)$.■

9.2 Proofs of Lemmas

Proof of Lemma 1

Since S^* is strictly comprehensive, $c^{KS}(p)$ is well-defined for $p \notin \dot{S}^*$. Also $z_{-i}(p_{-i}) \succ_i z_i(p_i)$ for $p \in \dot{S}^*$ and for all $i \in N$. It follows that $P \cap \dot{S}^* = \emptyset$. Assume, to the contrary, that $p \in \dot{S}^*$ is an equilibrium. There exists $p' \in \dot{S}^*$, $p'_\xi = p_\xi$ and $p'_\zeta > p_\zeta$, such that $z_\zeta(p'_\zeta) \succ_\xi z_\xi(p_\xi)$. If $\xi(p)$ remains strong for p', he chooses $z_\zeta(p'_\zeta)$. If $\zeta(p)$ becomes strong for p', he chooses $z_\xi(p_\xi)$. Hence, ζ has a profitable deviation, a contradiction. In the remainder of the proof $p \notin \dot{S}^*$.

If (3.1) and (3.2) do not hold, by the labelling convention, $\hat{p}_1 < 1$. Either $\hat{p}_2 = 1$ or $\hat{p} < 1$. If $\hat{p}_2 = 1$, then $\zeta(1) = 1$ by the labelling. Moreover, $p = (\check{p}_1(1), 1) \in P$ and $z_1^B(p_1)$ is implemented in equilibrium. By ($\bigstar$), player 1 cannot gain by reducing his claim, which remains incompatible for $p_2 = 1$. Since player 1 becomes weak by increasing his claim to p'_1 and since $z_2^B(1) \succ_2 c^{KS}(p) \succ_2 c^{KS}(p'_1, 1)$, his payoff is reduced when $z_2^B(1)$ is selected. Since player 2 remains weak for all his claims and for $\check{p}_1(1)$, he cannot gain. It remains to proof the lemma for $\hat{p} < 1$.

By comprehensiveness of S_i^B, $z_i^B(p_i) \succ_i z_i^B(\hat{p}_i)$ for any $p_i \in (\hat{p}_i, 1]$ and for all $i \in N$ when $\hat{p} < \mathbf{1}$. It follows that $\hat{X}_i(p)$ satisfies strict monotonicity for $p_i \in [\hat{p}_i, 1]$. That is, the payoff of player i in each outcome of $\hat{X}_i(p)$ is strictly increasing in p_i on $[\hat{p}_i, 1]$. Moreover, if $p_{\xi(p)} \in [0, \hat{p}_{\xi(p)})$, then there exists $p'_{\xi(p)} \in (\hat{p}_{\xi(p)}, 1]$, for which $\xi(p)$ remains strong and which strictly dominates $p_{\xi(p)}$. Hence, if $p_{\xi(p)} < 1$ for $p \in P$, then $p_1 u_2(z_1^B(p_1)) = p_2 u_1(z_2^B(p_2))$. Assume, to the contrary, that for $\xi = \xi(p)$ and $\zeta = \zeta(p)$, $p_\xi u_\zeta(z_\xi^B(p_\xi)) > p_\zeta u_\xi(z_\zeta^B(p_\zeta))$ for $p \in P$. Since player ξ remains strong for some $p'_\xi \in (\hat{p}_\xi, 1]$, he has a profitable deviation by strict monotonicity of $\hat{X}_\xi(p)$ for $p_\xi \in [\hat{p}_\xi, 1]$ or because p'_ξ strictly dominates $p_\xi \in [0, \hat{p}_\xi)$. Hence $p \notin P$, a contradiction. But then, $p_i \geq \hat{p}_i$ for $p \in P$ and for all $i \in N$. Assume, to the contrary, that some player claims $p_i < \hat{p}_i$ for $p \in P$. This player remains or becomes strong for some $p'_i \in (\hat{p}_i, 1]$, since $p_i u_{-i}(z_i^B(p_i)) = p_{-i}u(z_{-i}^B(p_{-i}))$ and $\hat{p} < \mathbf{1}$. By strict monotonicity of $\hat{X}_i(p)$ for $p_i' \in (\hat{p}_i, 1]$, the second weak inequality in (★) is strict. Hence, (★) implies a profitable deviation for player i and $p \notin P$, a contradiction. It follows that $p = (p_1, \check{p}_2(p_1)) \geq \hat{p}$ for $p \in P$.

The proof is completed by showing that $p_{\xi(p)} < 1$ for $p \in P$ when (3.1) and (3.2) do not hold. Assume, to the contrary, that $p_{\xi(p)} = 1$ for $p \in P$. It must be that $z_{\xi(p)}^B(1)$ is selected by $\xi(p)$. If $p_{\zeta(p)} = 1$, then $\xi(p) = \xi(\mathbf{1})$ and $z_{\xi(\mathbf{1})}^B(1) \succ_{\xi(\mathbf{1})} c^{KS}(\mathbf{1})$, since (3.2) does not hold. If $p_{\zeta(p)} < 1$, then $\zeta(p)$ gains by increasing his claim if $z_{\xi(p)}^B(p_{\xi(p)})$ were not selected by $\xi(p)$. Since profitable deviations are excluded for $p \in P$, $z_{\xi(p)}^B(1)$ is selected by $\xi(p)$. Either $\xi(p) = \xi(\mathbf{1})$ or $\xi(p) = \zeta(\mathbf{1})$. In the former case, there exists $p_{\zeta(\mathbf{1})}' < \check{p}_{\zeta(\mathbf{1})}(1)$ for which $\zeta(\mathbf{1})$ becomes strong and gains by imposing $z_{\zeta(\mathbf{1})}^B(p_{\zeta(\mathbf{1})}'(1))$. In the latter case, $\xi(\mathbf{1})$ becomes strong and selects $z_{\xi(\mathbf{1})}^B(1)$ for the pair $\mathbf{1}$ of claims. In both cases $\zeta(p)$ has a profitable deviation, which is excluded for $p \in P$. Therefore, if (3.1) and (3.2) do not hold, either $p_{\zeta(p)} = 1$ and $p = (\check{p}_1(1), 1) \geq \hat{p}$ for $p \in P$ or $p_{\zeta(p)} < 1$ and $\mathbf{1} > p = (p_1, \check{p}_2(p_1)) \geq \hat{p}$ for $p \in P$.■

Proof of Lemma 2

The choices of L after the history h^2 and of F after the history $h^3 = (\hat{c}, h^2)$ in σ are determined by respectively (4.1) and (4.2) as in a Stackelberg two-stage game. Since X^B is non-empty, closed and bounded, the maxima in condition (4.2) are well defined.

For all $i \in N$, the unrestricted compromise

$$c_i(q_i) = \arg\max_{x \in X^*} [u_i(x) | \, r_{-i}(x) \leq q_i]$$

is a uniquely defined Pareto efficient outcome in X^*. Hence, $u_i(c_i(.))$ and $-u_{-i}(c_i(.))$ are strictly increasing. It follows that F accepts c if and only if $c \in X^B$ and $c \succeq_F c_L(q_L)$. Therefore, L proposes $c \in PO(X^B)$ such that for all $c(q_L') \succ_L c$, either $q_L' < q_L$ or $c(q_L') \notin X^B$.

By comprehensiveness of S_1^B and S_2^B, z_1^B and z_2^B belong to $PO(X^*)$. In particular, if $z_F \succeq_F z_L$ and $z_F \in PO(X^*)$ is an excessive demand, the outcomes $c_L(q_L)$ in X_F^B and in X_L^B are obtained by varying q_L on, respectively, $\left[r_F(z_F), r_F(z_F^B)\right]$ and $[r_F(z_L^B), r_F(z_L)]$. For any q_L in these intervals, $c = c_L(q_L)$ will be proposed by L and accepted by F. The interior of these intervals is non-empty if, respectively, $z_F \succ_F z_F^B$ or $z_L \succ_L z_L^B$. Notice that $r_F(z_F) = 0$. These imply (i) and (ii). If $X_1^B \cap X_2^B = \emptyset$, then z_L^B will be rejected by F and z_F^B is proposed and accepted for $r_F(z_F^B) < q_L < r_F(z_L^B)$. Finally, for $q_L \geq r_F(z_L)$, the best outcome z_L for L in X^B is proposed and accepted. These imply (iii) and (iv).■

Proof of Lemma 3

Since $z_\xi \succ_\xi c^{KS}$ for $p \notin S^*$, $c^{KS} \succ_\xi z_\xi^B$ implies that c^{KS} belongs to the interior of $X_\xi^B \setminus \{z_\xi^B\}$. From Remark 1, it also follows that $q^{KS} > q_\xi^B = \min[q_1^B, q_2^B]$. Hence, there exists an open neighborhood N_q around q^{KS} in $\left[q_\xi^B, r_\zeta(z_\xi)\right]$ such that, by (ii) in Lemma 2, $u_L(c(q_L))$ (and $-u_F(c(q_L))$) are strictly increasing in q_L on N_q. Any player can guarantee himself the payoff of the proportional solution c^{KS} by bidding q^{KS}. From Lemma 2, c^{KS} is proposed and accepted in the subgame with $q_L = q^{KS}$ and $u_F(c(q_L)) > u_F(c^{KS})$ with $q_L < q^{KS}$. Since $c^{KS} \in PO(X^*)$, no other compromise $c \in X^* \setminus \{c^{KS}\}$ exists such that $u_i(c) \geq u_i(c^{KS})$ for all $i \in N$. Hence, c^{KS} is the unique outcome implemented in equilibrium. If $q_L < q^{KS}$ or $q_F > q^{KS}$, L strictly gains by increasing his bid. Hence, $q_L = q_F = q^{KS}$ are the unique equilibrium bids.

■

Proof of Lemma 4

For $p \notin S^*$, $z_\xi \succeq_\xi z_\xi^B \succeq_\xi c^{KS}$ implies that $q^{KS} \leq q_\zeta^B \leq q_\xi^B$ by Lemma 2 and that $r_\xi(z_\xi^B(p_\xi)) \leq q^{KS} \leq q_\xi^B(p)$. The bidding strategies in (4) are well defined. We show that $z_\xi^B(p_\xi)$ is proposed and accepted for these bidding strategies. If ζ is leader, then $q_\zeta < q_\xi \leq q_\zeta^B$ and ξ rejects $c \in X_\zeta^B$. By Lemma 2, ζ proposes in X_ξ^B. Since $q_\zeta \geq r_\xi(z_\xi^B)$, the best outcome z_ξ^B in X_ξ^B for ζ is accepted by ξ. If ξ is leader, then $q_\zeta \geq q_\xi \geq q_\xi^B$ implies that z_ξ^B is accepted by ζ. If $z_\xi^B = z_\xi$, then z_ξ^B is the best outcome for ξ. If $z_\xi^B(p_\xi) \neq z_\xi(p_\xi)$, then $q_\zeta \leq q_\xi^B$ implies that $q_\xi = q_\xi^B$ and that $c \in X_\xi^B \setminus \{z_\xi^B\}$ would be rejected by ζ because $r_\zeta(c) > q_\xi^B(p)$.

We show that for all other bids, some player has a profitable deviation. If $q_\xi > q_\zeta^B$ and if ζ were leader for a bid $q_\zeta \in \left[q_\zeta^B, q_\xi\right[$, then ξ would accept z_ζ^B. If $q_\xi < q_\xi^B$, then ζ would reject any proposal in $X_\xi^B(p)$ and ξ would propose in $X_\zeta^B(p)$ by Lemma 2. In both cases, ξ would gain by a bid in $[q_\xi^B, q_\zeta^B]$. Since there are no deviations for ζ when $z_\xi^B(p_\xi) = z_\xi(p_\xi)$, it suffices to consider $z_\xi^B(p_\xi) \neq z_\xi(p_\xi)$. If $q_\zeta > q_\xi^B$, ξ is leader for $q_\xi = q_\zeta > q_\xi^B$ and ζ accepts compromises in $X_\xi^B(p) \setminus \{z_\xi^B(p_\xi)\}$ giving him a lower payoff than $z_\xi^B(p_\xi)$. If $q_\zeta < r_\xi(z_\xi^B(p_\xi))$, then ζ is leader and the compromise accepted by ξ in $X_\xi^B(p) \setminus \{z_\xi^B(p_\xi)\}$ gives him a lower payoff than $z_\xi^B(p_\xi)$. In both cases, ζ would gain by a bid in $[r_\xi(z_\xi^B(p_\xi)), q_\xi^B]$.■

Proof of Lemma 5

By assumption, $z_i \in PO(X^*)$ and $z_i \in X^B$ and for all $i \in N$. If $q_L = 0$, then $c = z_F$ by (i) of Lemma 2. If $z_1 = z_2$, then $r_F(z_L) = 0$. By (iv) of Lemma 2, it is implied that $c = z_1 = z_2$. If $p \in \dot{S}$ and $q_1 > q_2 \geq 0$, then $z_1 \succ_2 z_2$ and player 2 is leader by the bidding rules. Since $r_1(z_1, z_1) = 0$, $c = z_1$ by (ii) of Lemma 2. However, player 1 is leader for a zero bid and $c = z_2$ by (i) of Lemma 2. Hence, $q_1 > q_2 \geq 0$ cannot occur in equilibrium. If $p \in \dot{S}$ and $q_2 > q_1 \geq 0$, then any change of player 2's bid does not change the outcome. This proves Lemma 5.■

Proof of Lemma 6

Since $z_i^B \in PO(X^*)$ for comprehensive revision procedures, claims defining inefficient demands are modest. Hence, by assumption, $PO(X_i^B) = \{z_i^B\}$ for some $i \in N$.

If player L is leader for $q_L = 0$ and if $z_F \in PO(S^*)$, then player L makes sure that z_F is implemented. There exists a proposal $c \in X^B$ for player L which is worse than z_F for player F. Since c is unacceptable for player F, c is rejected. For $q_L = 0$, $d = z_F$ is implemented. Player L cannot achieve a better outcome in X^B than z_F. If player L prefers x to z_F in X^B, then player F prefers z_F to x and rejects x, so that $d = z_F$ is always implemented for $q_L = 0$.

If $z_k \notin PO(S^*)$ for some $k \in N$, then $r_k(z_k, x) = 0$ for all $x \in PO(X^B)$. Therefore, if $z_F \notin PO(S^*)$, player L who leads with $q_L = 0$ can implement all $x \in PO(X^B) \cup \{z_F\}$. We consider two cases. If $z_\zeta \notin PO(S^*)$ in the first case, player ξ's preferred outcome in $PO(X^B) \cup \{z_\zeta\}$ is implemented for any pair of equilibrium bids q. Bidding $q_\xi = 0$ is weakly dominant for player ξ, since it allows him to achieve his best outcome. Player ζ's bid is inconsequential, so no player has a profitable deviation. If $z_\zeta \in PO(S^*)$ in the second case, it follows that player ξ is always leader for $q_\xi = 0$ and that z_ζ is always implemented. Player ζ bids $q_\zeta = 0$ if $z_\zeta \succ_\zeta z_\zeta^B$, $q_\zeta \in [0, r_\zeta(z_\xi^B))$ if $z_\xi^B \succ_\xi z_\zeta \succeq_\xi z_\zeta^B$ and $q_\zeta \in [0,1]$ otherwise. For $q_\xi > 0$, player ζ would have a profitable deviation, by becoming leader for $q_\zeta = 0$ and proposing his preferred outcome in $PO(X^B) \cup \{z_\xi\}$. This proves the first part of the lemma if at least one of the demands is inefficient.

For the second part of the lemma, assume that both demands are efficient, but that $PO(X_{-i}^B) = \{z_{-i}^B\}$ for some $i \in N$. We consider two cases. In the first case, the demands are incompatible so that c^{KS} is well-defined. If $c^{KS} \in X_i^B$, then by the argument of Lemma 3, (4.5) defines q and $c^{KS} \in X_i^B$ is proposed and accepted. If $c^{KS} \notin X_i^B$, then by bidding in $[q_\xi^B, q_\zeta^B]$, as defined in (4.6), the strong player can propose z_ξ^B as a leader and discourages the weak player of proposing z_ζ^B. If $z_\xi \succeq_\xi z_\xi^B$, then by the argument of Lemma 4, the weak player bids $q_\zeta \in [0,1]$ and z_ξ^B is implemented. If $z_\xi \succ_\zeta z_\xi^B$, then $PO(X_\xi^B) = \{z_\xi^B\}$. The weak player makes sure that z_ξ is implemented by bidding $q_\zeta = 0$. If player ζ were to make a positive bid or if $q_\xi \notin \left[q_\xi^B, q_\zeta^B\right]$, then some player would have a profitable deviation. If initial claims are compatible in the second case, the strong player makes sure that z_ζ is implemented. To avoid that the strong player gains by increasing opposition, $q_\zeta = 0$ if $z_\zeta^B \succ_\xi z_\zeta$ and $q_\zeta \in [0, r_\zeta(z_\xi^B))$ if $z_\xi^B \succ_\xi z_\zeta$. For $q_\xi > 0$, player ζ would have a profitable deviation, by becoming leader for $q_\zeta = 0$ and by proposing z_ξ. This proves the second part of the lemma if at least one of the Pareto-efficient demands is modest.■

9.3 Proofs of Remarks

Proof of Remark 1

If $p = (p_1, \check{p}_2(p_1))$, then $p_1/p_2 = u_1(z_2^B(p_2))/u_2(z_1^B(p_1))$. By definition, $p_1/p_2 = u_1(c^{KS}(p))/u_2(c^{KS}(p))$. It follows that $u_i(z_{-i}^B(p_{-i})) \geq (<)\, u_i(z_i^B(p_i))$ if and only if

$$u_i(z_i^B(p_i))/u_i(c^{KS}(p)) \leq (>)\, u_{-i}(z_i^B(p_i))/u_{-i}(c^{KS}(p)).$$

From the last inequality, $z_i^B(p_i) \succ_i (\preceq_i) c^{KS}(p)$ implies $z_i^B(p_i) \succ_{-i} (\prec_{-i}) c^{KS}(p)$, which must be excluded since $c^{KS}(p) \in PO(X^*)$. Hence, $z_{-i}^B(p_{-i}) \succeq_i (\prec_i) z_i^B(p_i)$ if and only if $c^{KS}(p) \succeq_i (\prec_i) z_i^B(p_i)$.■

Proof of Remark 2

By comprehensiveness of S_i^B, $z_i^B \in PO(X^*)$. Since the proportional solution and the revised demands are efficient, it follows immediately that $r_{-i}(c^{KS}) > (\leq) r_{-i}(z_i^B)$ if and only if $c^{KS} \prec_{-i} (\succeq_{-i}) z_i^B$. Since $q_\xi^B \leq q_\zeta^B$, the proportional solution c^{KS} belongs to the feasible set of compromises if and only if it is preferred by the strong player to his revised demand z_ξ^B.■