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The Engine of Growth

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THE ENGINE OF GROWTH

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Abstract

I develop a Schumpeterian model where the engine of growth is in the microeconomic structure of the patent races and derive new results on the determinants of growth. Under decreasing marginal productivity in the R&D sector, the equilibrium is characterized by small firms investing too little and the growth process is dynamically inefficient; the optimal policy for innovation always implies R&D subsidies. When the incumbent monopolists are leaders in the patent races, they engage in large R&D investment and their persistent leadership enhances growth. Other sources of growth may reduce investment inducing a paradoxical negative correlation between growth and R&D spending even if innovations are the main engine of growth. In the open economy, growth is driven by the largest country and increases with its relative size and openness. In a monetary economy, price stickiness induces an inverted U relation between inflation and long run growth.

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1 Introduction

The search for profits is what provides the incentives to invest and ultimately drives the economy. The new growth theory, starting with the works of Romer [1990] and Aghion and Howitt [1992]), has exploited this old Schumpeterian idea to formalize the link between innovation and long run growth. In this paper, I try to open the "black box" of the engine of growth, investigating its microeconomic organization in a more general and realistic way and examining the impact on investments in innovation of important factors like R&D policy, other sources of growth, globalization and monetary policy. This allows derivation of a number of new empirical predictions on the determinants of long run growth.

In the endogenous growth literature, investments for innovations are usually described in a very simple and empirically arguable way: the production function of new ideas is characterized by constant marginal productivity and a simple no-arbitrage condition pins down the equilibrium investment and, consequently, the rate of economic growth. This "minimalistic" approach does not allow to characterize the number and size of the firms investing in R&D, the relation between incumbent patentholders and outsiders and the effect of realistic R&D policies.

As noticed by Kortum [1993], Griliches [1994]), Cohen and Klepper [1996]) and other empirical works, investments in R&D are characterized by relevant fixed costs, decreasing marginal productivity at the firm level,[2] and wasteful duplications of resources between firms due to congestion reasons at the industry level. Hence, I introduce fully fledged "patent races" with the first two features in a Schumpeterian model due to Barro and Sala-i-Martin [1995] and derive the drastic consequences associated with the inefficiency in the market for innovation. Its organization is now characterized by a bias toward small firms investing too little in R&D. This inefficiency is amplified in the growth process, which becomes dynamically inefficient, in the sense that a country could increase its long run growth without reducing current consumption or *viceversa* increase consumption without reducing growth. Hence, contrary to the ambiguous results in the literature, the optimal R&D policy requires always subsidizing R&D.

Another important stylized fact about R&D investment is that a large part of it is done by dominant firms producing with the leading edge technologies.[3]

[2] One of the stilyzed facts pointed out by Cohen and Klepper [1996] is that the number of patents and innovations per dollar of R&D decreases with the level of R&D and this is well grounded empirically (see references cited there).

[3] For related and updated research on this topic see the International Think-tank on Innovation and Competition (www.intertic.org). For recent surveys of related theoretical and empirical advances on competition and growth see Aghion and Griffith [2005] and Faini *et al.*

As a recent *Economic Focus* of the *Economist* (May 22nd, 2004) put it, "many product innovations, in industries from razor blades to software, are made by companies that have a dominant share of the market. Most mainstream economists, however, have had difficulty explaining why this might be so. Kenneth Arrow, a Nobel prize-winner, once posed the issue as a paradox. Economic theory says that a monopolist should have far less incentive to invest in creating innovations than a firm in a competitive environment: experience suggests otherwise. How can this be so?". Following Etro [2004], I provide a rationale for investment by the patentholders based on their leadership and free entry in sequential patent races.[4] This paper goes beyond my previous research, where I sketched a dynamic model without solving it analytically, and provides new dynamic programming techniques to analytically solve for the equilibrium growth path and the endogenous value of technological leadership. Investment by incumbent patentholders leads to persistence of monopolies which in turn increases the value of developing a new technology: this is not just the value of the corresponding flow of profits (as traditionally in the literature), but that one together with the option value to a persistent monopolistic position. The new element increases the incentives to invest mostly for the monopolists but also for the outsiders, and hence it speeds up the growth process. In particular, when marginal productivity of R&D is constant (or close to constant), monopolists deter entry in the patent race remaining the only innovators, while in the more realistic case of decreasing marginal productivity, monopolists allow entry but still invest more than the outsiders.[5] This characterization of growth driven by dominant firms may help to explain the persistence of technological leadership especially for large corporations in high-tech sectors.

The framework developed in this paper allows to explore how different factors and policies can affect the engine of growth. I examine some of them deriving new results on the determinants of growth and new empirical predictions which could be analyzed in future research.

First, I augment the model with another generic source of growth, which

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[2006].

[4]Blundell *et al.* [1999] provide evidence from recent high-tech sectors which is consistent with this thesis. In a similar vein, Nicholas [2003] studies 1920s America and shows that "firms with high levels of market power tended to innovate more because they had strong incentives to do so pre-emptively". Ogilvie [2004] provides a broader hystorical perspective on the relationship between barriers to entry and innovation based on our approach.

[5]In a related paper, Segerstrom [2006] has developed a model where incumbent monopolists invest in R&D because they can use a different innovation technology from the one adopted by the other firms. This approach basically assumes cost advantages in the innovation activity for the monopolists and allows to study their persistence, but it does not explain its ultimate source. See also Denicolò and Zanchettin [2006].

may just be a traditional exogenous technological progress or it may be microfounded in some endogenous way. This allows to show a surprising result: an increase in growth due to other sources may reduce the incentives to innovate. Consequently, even if innovation is the main engine of growth (in the sense that it actually contributes to most of the growth rate), growth and investment in innovation may be negatively correlated (over time or across countries). This result is due to the increase in the interest rate associated with higher growth which may crowd out some firms from the innovation sector: this happens when the intertemporal elasticity of substitution is low enough. Such a result suggests that empirical tests of Schumpeterian growth theories should not look at the simple correlation between growth and R&D investment, but would need to separate the direct positive effect of innovation on growth from the feedback effect of growth on innovation.

Then, I consider a open economy version of the model showing that the technological frontier shifts toward the largest country, which develops a comparative advantage in the R&D sector. Even if in a stylized way, the world equilibrium is consistent with the growth experience of the last decades, and in particular with large R&D investments and technological progress in the US, high US imports of final goods which allowed other countries to import American technology and absorb its growth, and large capital flows toward the US financing its large current account deficits. The growth rate is positively related with the degree of openness and it exhibits relative (and not absolute) scale effects: an increase in the size of the leading economy compared to the rest of the world enhances growth. Moreover, R&D subsidization is again the optimal unilateral policy, but for different reasons than in a closed economy: it now allows to provide domestic firms with a strategic advantage in international competition for innovations and to conquer a leadership in the world markets, but without affecting world growth.

Finally, following the new-keynesian tradition [Blanchard and Kiyotaki, 1987; Obstfeld and Rogoff, 1996; Woodford, 2003] and in particular a recent contribution by Barro and Tenreyro [2006], I extend the model to a monetary economy drawing some new implications on the relationship between inflation and endogenous growth, a relationship rarely investigated in the recent theoretical literature. Price stickiness induces an inverted-U relation between inflation and long run growth which is broadly consistent with available empirical evidence: excessive inflation erodes the monopolistic profits of innovators reducing the aggregate incentives to invest and hence reducing growth. Within this context we can also derive a rule for optimal monetary policy. When price stickiness is negligible the optimal monetary rule tends toward a Friedman rule generalized to take endogenous growth into account. When pirce stickiness is relevant, the optimal monetary rule tends toward a price stabilization rule. Finally, this simple model can be used to provide an alternative newkeynesian framework where investment and growth are present.

The paper is organized as follows. Section 2 describes the model and Section 3 solves for the equilibrium organization of the R&D sector and for the general equilibrium discussing the optimal R&D policy. Section 4 endogenizes the persistence of monopolies. Section 5 extends the model to consider various determinants of growth and Section 6 concludes. Technical details are in the Appendix.

2 The Model

Let us consider an infinite horizon representative agent with isoelastic utility:

$$U = \int_0^\infty \frac{C^{1-\gamma}}{1-\gamma} e^{-\rho t} dt \quad \text{with} \quad \gamma > 0 \tag{1}$$

where $\rho > 0$ is the time preference rate. The representative agent earns income and chooses consumption and savings according to the usual optimality condition:

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\gamma} \tag{2}$$

which holds at each point in time. Using the intertemporal resource constraint, one can derive an expression for savings depending on the expected value of income and on the interest rate profile.

Output Y can be used for consumption C, production of intermediate goods X or investment in R&D activities providing a rate of return r, and it is produced according to a generalized version of the production function introduced by Barro and Sala-i-Martin [1995, Ch.7]:[6]

$$Y = A \left[\sum_{j=1}^{N} (q^{\kappa_j} X_j)^\theta \right]^{\frac{\alpha}{\theta}} L^{1-\alpha} \tag{3}$$

where A is Total Factor Productivity, L is the fixed labor force, X_j is the intermediate good j of quality κ_j, which is assumed non-durable for simplicity, N is the constant number of intermediate goods, $q > 1$ and $0 < \alpha \leq \theta \leq 1$. It can be easily verified that this function satisfies constant returns to scale and decreasing

[6] An advantage of this framework is that technological progress due to innovations realizes independently for each intermediate good: hence, it allows to think of small and frequent innovations rather than innovations for general purpose technologies.

marginal productivity of each input. The parameter α represent the factor share of income from intermediate goods while $1-\alpha$ is the labor share. The parameter θ reflects the elasticity of substitution between intermediate inputs, with $1/(1-\theta)$ approximating the elasticity of demand for each intermediate good. In the standard literature, these two parameters are equated setting $\alpha=\theta$, as in Barro and Sala-i-Martin [1995], while it is important to keep them separate.

The market for the final good, which is the *numeraire*, and the markets for labour and credit (to firms investing in R&D) are perfectly competitive. However some form of imperfect competition holds in the market for intermediate goods. Each intermediate good is produced at a unitary marginal cost and sold by a single producer until a newer version is on the market, which is a reasonable situation when the rate of creation of new products is fast enough.[7] For instance under both price and quantity competition, when the number of intermediate goods N is high enough (that strategic interactions are negligible), the equilibrium price of each intermediate good is $1/\theta$ when innovation are drastic. To be as general as possible, however, I will define $1+\mu$ as the price for each monopolistic producer (so that μ is the mark up). Then, the aggregate quantity produced of intermediate good j can be determined as:

$$X(\kappa_j)=\left(\frac{\alpha A}{1+\mu}\right)^{\frac{1}{1-\theta}} L^{\frac{1-\alpha}{1-\theta}} q^{\frac{\kappa_j \theta}{1-\theta}} Q^{-\frac{\theta-\alpha}{\theta(1-\alpha)}} \tag{4}$$

where we have introduced the Barro and Sala-i-Martin aggregate quality index $Q \equiv \sum_{j=1}^{N} q^{\frac{\kappa_j \alpha}{1-\alpha}}$. Substituting the quantity $X(\kappa_j)$ from (4), we obtain the output of final goods:

$$Y=\left(\frac{\alpha}{1+\mu}\right)^{\frac{\alpha}{1-\alpha}} A^{\frac{1-\theta+\alpha}{1-\theta}} L^{1+\frac{\alpha(\theta-\alpha)}{1-\theta}} Q^{\frac{\alpha(1-\theta)}{\theta(1-\alpha)}} \tag{5}$$

and the total amount of intermediate goods $X=\alpha Y/(1+\mu)$. Since TFP and labor force are constant, the growth rate of income must be:

$$g \equiv \frac{\dot{Y}}{Y}=\frac{\alpha(1-\theta)}{\theta(1-\alpha)} E\left[\frac{\dot{Q}}{Q}\right] \tag{6}$$

To describe the investment side of the economy, we need to describe the technology to create innovations. When an innovation for an intermediate good j generates the new quality rung κ_j, the innovator starts producing with the cutting-edge technology and obtains a flow of profits $\mu X(k_j)$. At the same

[7] According to the empirical evidence surveyed by Cohen and Klepper [1996], whether major or incremental and whether patented or not, innovations grant market advantage “within one year for many industries and within three years for most ”.

time the race to find out the subsequent innovation begins. To participate, any firm i has to pay a fixed cost $F(k_j)$, which may include an entry fee set by the government, and spend a flow of resources $z_i(k_j)$. Finally, for any unit of investment, there is a subsidy at rate s financed with lump sum taxes.

The technology for the invention of new goods allows for decreasing marginal productivity at the firm level. In particular the investment for firm i gives birth to the innovation k_j according to a Poisson process with arrival rate $p_i(k_j)$ given by a concave function of $z_i(k_j)$. To obtain closed form solutions, I assume the following specification:

$$p_i(k_j) = [\phi(k_j)z_i(k_j)]^{\epsilon} \tag{7}$$

where the function $\phi(k_j)$ expresses how difficult is to discover technology k_j and $\epsilon \in (0, 1]$ represents the degree of returns to scale in the innovation sectors or the elasticity of expected revenue with respect to the flow of investment. This parameter is unitary in the existent versions of the quality-ladder model, implying constant marginal productivity - equivalent to constant returns to scale since there is just one input - in the R&D sector, but empirical research, for instance by Cohen and Klepper [1996] and Kortum [1993] suggests an elasticity much smaller than 1.[8]

The arrival rate of innovation k_j, is the sum of the individual arrival rates of the $n(k_j)$ entrants plus the one of the incumbent, indexed with M, $p(\kappa_j) = \sum_{i=1}^{n(k_j)} p_i(k_j) + p_M(k_j)$. Using the properties of Poisson processes in a standard fashion, this implies that the expected discounted value of the profits with innovation k_j is:

$$V(k_j) = \frac{\mu X(\kappa_j)}{r + p(k_j)} \tag{8}$$

where $r + p(k_j)$ is a sort of effective discount factor. Moreover, the expected net profit of entrant i in the patent race in sector j when the current quality is k_j can be written as:

[8]Cohen and Klepper [1996] show that "the assumption of diminishing returns to R&D is well grounded empirically" for a broad sample of industries (from a theoretical point of view, notice that, while in most of the productive sectors there are good reasons to believe that doubling the amount of input total production will double, there are no reasons to believe that doubling the amount of inputs in the R&D activity will double the expected amount of innovations). Even Aghion and Howitt [1998, Ch.12] accept this as a stylized fact. Kortum [1993] suggests a range between 0.1 and 0.6 for this elasticity. Segerstrom [2005] assumes that decreasing returns hold just for the incumbent monopolist, while constant returns to scale characterize all the other firms. He solves the model through simulations and assumes $\epsilon = 0.3$ as the average between the values proposed by Kortum.

$$\Pi^i(k_j) = \frac{[\phi(k_j)z_i(k_j)]^\epsilon \mathbf{V}^M(k_j+1) - (1-s)z_i(k_j)}{r+p(k_j)} - F(k_j) \tag{9}$$

where $\mathbf{V}^M(k_j+1)$ is the value of being monopolist with the next technology k_j+1. Since also the incumbent monopolist with the technology k_j can invest to innovate, we need to consider its objective function, which is given by a Bellman equation defining the value of being a monopolist:[9]

$$\mathbf{V}^M(k_j) = \max_{z_M \geq 0} \left\{ \frac{[\phi(k_j)z_M]^\epsilon \mathbf{V}^M(k_j+1) - (1-s)z_M}{r+p(k_j)} + V(k_j) - F(k_j) \right\} \tag{10}$$

where the fixed cost is paid only if $z_M > 0$ and $V(k_j)$ is given by (8). This value is the core of the engine of growth, because what drives investment and growth is exactly the attempt to conquer it. In the standard literature, monopolists do not invest, hence the value of leadership is just the expected profit from the next innovation. However, as we will see later on, monopolists invest when they have a leadership in the patent race and in that case, the value of the innovation includes also the option value of a persistent monopoly, which fundamentally modifies the incentives to do research.

Finally, I assume that new ideas are more difficult to obtain when the scale of the sector increses, and that the fixed cost is a constant fraction of the expected cost of production with the new technology $\phi(k_j) = X(\kappa_j+1)^{-1}$ and $F(k_j) = \eta \int_0^\infty X(k_j+1)e^{-[r+p(k_j+1)]t}dt$ with $\zeta > 0$ and $\eta \in (0, \mu)$. I want to capture the idea that the larger is the scale of expected production of a firm, the larger are the costs necessary to discover and develop the associated technology (construction of prototypes and samples, new assembly lines and training of workers). These assumptions will deliver a balanced growth path and will avoid scale effects on the equilibrium growth rate, in line with the last generation of quality-ladder models - see Jones [1995] and Barro and Sala-i-Martin [2004].

To close the model, the expected growth rate of the quality index will be derived as:

$$E\left[\frac{\dot{Q}}{Q}\right] = p\left[q^{\frac{\theta}{1-\theta}} - 1\right] \approx \frac{p\theta \ln q}{1-\theta}$$

[9] Notice that the associated no arbitrage condition under optimal behaviour implies the standard equivalence between the riskless return rate and the expected return rate from R&D investment of a monopolistic firm:

$$r\left[\mathbf{V}^M(k_j) - F(k_j)\right] = p_M(k_j)\left[\mathbf{V}^M(k_j+1) - \mathbf{V}^M(k_j)\right] - \sum_{i=1}^{n(k_j)} p_i(k_j)\mathbf{V}^M(k_j) + \mu X(k_j) - (1-s)z_j(k_j)$$

where the right hand side includes the expected upgrade in the value of being a leader after a successfull innovation, the expected loss in case of innovation by a competitor and the current flow of profits net of the investment flow in R&D.

where where $p \equiv \left[\sum_{j=1}^{N} p(\kappa_j) q^{\frac{\kappa_j \theta}{1-\theta}}\right] / Q$ is a weighted average of the probability of innovations, and the growth rate of income wil be simply:

$$g \approx \frac{\alpha(1-\theta)}{\theta(1-\alpha)} \left(\frac{p\theta \ln q}{1-\theta}\right) = \frac{p\alpha \ln q}{1-\alpha} \tag{11}$$

Given this set-up, one can study different organizations of the market for innovations.[10] In this paper I will consider the traditional Nash case and the more realistic Stackelberg case, in which the current monopolist has a first mover advantage, with free entry.

3 Dynamic Inefficiency and R&D Policy

In this section I model competition in the market for innovation in the Nash fashion. As well known, under free entry the leader does not invest, because its best strategy is to stay out from the patent race and enjoy the profits from its current product until a new innovation will make it obsolete. Competition for innovations is just between outsiders and the scope of this section is to characterize the equilibrium organization of investment - the number of firms and the size of their investments - together with the usual macroeconomic variables and to derive the optimal R&D policy. A complete analysis of the optimal organization of the R&D sector is in the Appendix.

In each sector, the lack of investment by incumbents implies that the value of being a monopolist (10) boils down to $\mathbf{V}^M(k_j) = V(k_j)$. Each firm chooses its investment $z_i(k_j)$ to maximize (9) taking as given the strategies of the other firms, the value of the next innovation and the interest rate, while the free entry condition sets the expected profits (9) equal to zero providing the equilibrium number of entrants $n(k_j)$. Combining the optimality condition and the free entry condition we obtain the investment per firm:

$$z(k_j) = \epsilon^{\frac{1}{1-\epsilon}} \phi(k_j)^{\frac{\epsilon}{1-\epsilon}} \left(\frac{V(k_j+1) - F(k_j)}{1-s}\right)^{\frac{1}{1-\epsilon}} \tag{12}$$

which is increasing in the subsidy and in the value of the innovation net of the fixed cost of entry, while it is independent from the interest rate.

Using the endogenous value of innovation (8) and our functional form assumptions in (12) and substituting in the free entry condition, which sets (9)

[10] A related investigation is present in a work by Zeira [2003]. However, his interest is in the choice of innovators between simple innovations and more difficult but radical innovations and across multiple research strategies.

equal to zero, we can express the probability of innovation as a linearly decreasing function of the interest rate:[11]

$$\begin{aligned} p(k_j) &= \frac{[\phi(k_j)z(k_j)]^{\epsilon} V(k_j+1) - z(k_j)}{F(k_j)} - r = \qquad (13) \\ &= \frac{[\phi(k_j)X(k_j+1)]^{\frac{\epsilon}{1-\epsilon}} \left[\epsilon^{\frac{\epsilon}{1-\epsilon}} [\mu-\eta]^{\frac{\epsilon}{1-\epsilon}} \mu - \epsilon^{\frac{1}{1-\epsilon}} [\mu-\eta]^{\frac{1}{1-\epsilon}}\right]}{\eta \left[r + p(k_j)\right]^{\frac{\epsilon}{1-\epsilon}} (1-s)^{\frac{\epsilon}{1-\epsilon}}} - r = \\ &= \left[\frac{\epsilon(\mu-\eta)}{\bar{\zeta}(1-s)}\right]^{\epsilon} \left[\frac{\mu - \epsilon(\mu-\eta)}{\eta}\right]^{1-\epsilon} - r \quad \text{for any } k_j \end{aligned}$$

where $\bar{\zeta} = \zeta(1+\mu)^{1/(1-\theta)}$. Hence total investment in each patent race is decreasing in the interest rate as well. Using this to explicit the expected value of innovation (8), and substituting in (12) again, we obtain the equilibrium flow of investment per firm:

$$z(k_j) = \frac{\epsilon\eta(\mu-\eta)}{[\mu - \epsilon(\mu-\eta)](1-s)} q^{\frac{\theta}{1-\theta}} X(\kappa_j) \qquad (14)$$

which is increasing in the quality achieved in the single sector, since this implies higher demand and hence higher expected profits for the corresponding intermediate product, and increasing in the degree of returns to scale, ϵ, since this makes investment more productive. For a given scale of production, investment is also increasing in the mark up μ, which is exactly the core of the Schumpeterian idea that monopolistic profits drive investment of single firms.[12] Finally, the interest rate does not affect the individual investment of firms, while it negatively affects the number of firms and hence the total investment. This result is quite intriguing: any adjustment of R&D investment to variations in the interest rate goes through changes in the number of firms, not in their size.

The decentralized equilibrium does not optimizes the allocation of investment across firms in the sense that it does not minimizes R&D expenditure for a given probability of innovation. In particular, the investment per firm which minimizes total expected costs (fixed and variable) for a given probability of

[11] In the Barro and Sala-i-Martin [1995] model, the arbitrage equation for the patent race k_j pins down the investment in innovation in the patent race $k_j + 1$ with a weak economic intuition. Instead, here the free entry condition for the patent race k_j pins down the number of firms investing in innovation in the patent race k_j and, together with their profit maximising choices, their individual investments in the same patent race k_j.

[12] The effect of higher fixed costs on investment can be shown to be non monotonic, positive for η low but negative for η high enough: on one side high fixed costs reduce expected profits for a given life of the patent, but on the other, they reduce the innovation rate in the future so as to increase the expected life of the patent.

innovation is:[13]

$$z^*(k_j) = \frac{\epsilon\eta}{1-\epsilon} q^{\frac{\theta}{1-\theta}} X(\kappa_j) \tag{15}$$

which is larger than (14) in absence of subsidization: in a decentralized equilibrium firms tend to choose inefficiently small investments. The intuition relies on the fact that researchers do not internalize the effect of their choices on the entry decision, and entry creates wasteful duplication of R&D expenditures, in terms of fixed costs of research: hence, *ceteris paribus*, firms choose suboptimal investment. Since growth depends on the probability of innovation, the equilibrium is dynamically inefficient: the economy could achieve the same aggregate probability of innovation investing a smaller amount of total resources or increase the former at the same level of the latter. This leads to a crucial property of the equilibrium organization of the R&D sector:

PROPOSITION 1. *Under Nash competition in the market for innovations, the equilibrium implies a sub-optimal flow of investment in R&D per firm and dynamic inefficiency in the growth process.*

This form of dynamic inefficiency is absent in traditional models of endogenous growth, where the economy may grow above or below an optimal benchmark, but cannot increase the growth rate without giving up to some of the current consumption: when marginal productivity in the R&D sector is decreasing, the endogenous organization of this sector creates this inefficiency. Only a proper R&D policy can solve it and, using (14) and(15), one can easily derive the subsidy which induces the optimal investment per firm in the decentralized equilibrium:

$$s^* = \frac{\eta}{\mu(1-\epsilon)+\eta\epsilon} \in (0,1) \tag{16}$$

which is always positive, decreasing in μ and increasing in η, since higher effective markups already create larger investments.

On a balanced growth path with a constant interest rate, the resource constraint implies that income, investment and hence consumption must grow at the same rate. Equating (2) and (11) and using the aggregate probability of innovation common to all sectors and given by (13), we can derive all the equilibrium variables. In particular, the equilibrium growth rate is:

$$g = \frac{\left[\frac{\epsilon(\mu-\eta)}{\bar{\zeta}(1-s)}\right]^{\epsilon}\left[\frac{\mu(1-\epsilon)+\epsilon\eta}{\eta}\right]^{1-\epsilon} - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{17}$$

[13]Formally, if the investment flow is $\beta X(k_j+1)$ for each firm, the efficient organization chooses β to minimize expenditure $(\beta+\eta)X(k_j+1)$ for a given probability of innovation $[\phi(k_j)\beta X(k_j+1)]^{\epsilon}$. In the Appendix I show that the social planner solution results in the same organization.

As one could expect, the more costly are innovations (higher η and ζ), the lower is equilibrium growth, while the relation between the monopolistic mark up μ and growth has an inverted U-shape (but it is always positive for mark ups below the monopolistic level). Finally, the relation between growth and ϵ is U-shaped.

The dynamic inefficiency of the growth process shows how a country with an industrial structure characterized by small firms achieves inefficient results, and could grow more without losses in current consumption if its firms were increasing in size. This general conclusion may shed new light on the problems of countries that do not grow much and lack large and innovative corporations. This is the case of many European countries, most notably of Italy, whose industrial structure is characterized by a large number of small and medium size enterprises whose innovative capacity is quite limited.[14]

The equilibrium arrival rate of innovations is directly proportional to the above growth rate, while the equilibrium number of firms is:

$$n = \frac{\left[\frac{\mu-\epsilon(\mu-\eta)}{\eta}\right] - \rho\left[\frac{\bar{\zeta}[\mu-\epsilon(\mu-\eta)](1-s)}{\epsilon\eta(\mu-\eta)}\right]^{\epsilon}}{1+\alpha\gamma\ln q/(1-\alpha)} \tag{18}$$

hence higher size innovations are associated with higher growth but fewer firms (and less frequent innovations).

Clearly, a social planner would choose the number of firms investing in R&D and consequently the growth rate of the economy taking into account the social value of innovations, rather than their private value, and in the Appendix I show that in equilibrium both n and g are suboptimal at least for γ small enough. This result has a simple intuition: when the intertemporal elasticity of substitution is large (γ is low), it is optimal to choose a high growth rate of consumption, hence the social value of innovations is high. On the other side, the private value of innovations depends on market features which are independent from consumers preferences (except for an indirect channel going through the interest rate). Hence, for γ small enough, the social value of innovations is higher than the private value and the optimal number of firms becomes larger than the equilibrium one.

The optimal allocation of resources can be achieved with two policy tools derived in the Appendix, a positive R&D subsidy, which optimally allocates

[14]The reasons for the lack of growth in the size of Italian firms have been usually associated with credit rationing problems or with the family based structure of Italian capitalism, but the endogenous tendency toward small innovative firms suggested here may be part of the story (since R&D subsidization has been always limited in Italy compared to other western countries). Not by chance, the Italian endogenous response to this problem (without proper equivalents around the world) has been the delegation of innovative activities to "*industrial districts*", that is organizations of small firms investing in the same sector on a larger scale.

resources between investors and an entry fee increasing the cost of entry, which targets the optimal number of firms:[15]

Proposition 2. *The optimal R&D policy requires a positive R&D subsidy to investment and an entry fee (positive for γ large enough) to achieve the optimal organization of the market for innovations and the optimal growth rate.*

The message of this section is quite in contrast with the usual models of Schumpeterian growth, where the optimal R&D policy may imply taxation or subsidization of the R&D investment. When we take into account the organization of the market for innovations, we obtain a more intuitive result, for which innovating firms should always be subsidized to increase their size.

Before turning to a more realistic situation where incumbent monopolists compete with the outsiders to innovate, I want to stress that this general equilibrium model can be used for other interesting analysis, for instance to analyze qualitatively the effects of technological shocks on the investment in R&D.[16]

4 Growth Driven by Market Leaders

Many product innovations are due to dominant firms and a lot of the investment in R&D is actually done by both incumbent monopolists and new firms.[17] Existing models about innovation and growth are inconsistent with this simple fact, since under Nash competition and free entry, as we have seen also in the previous section, an incumbent monopolist has no incentives to invest in R&D.

Recent research has rationalized investment of the incumbents in a partial equilibrium framework showing that monopolists invest in R&D more than any other firm as long as they are leaders in the sense of Stackelberg in markets for

[15] Only in the limiting case of constant returns to scale, which is the traditional focus of this literature, the size and the number of firms do not matter and an R&D subsidy alone can achieve optimality. Notice that approaching constant returns to scale in our model (that is when $\epsilon \to 1$ and $\eta \to 0$), the investment by each firm and the number of firms become indeterminate, but the equilibrium growth rate converges to the traditional one (see Barro and Sala-i-Martin [2004]): $g \to \left[\mu/\bar{\zeta}(1-s)-\rho\right]/\left[\gamma+(1-\alpha)/\alpha\ln q\right]$.

[16] A temporary increase in TFP would increase production and wages in the short term. Consumption smoothing implies that part of the increase in income is saved and market clearing in the credit market requires that the interest rate goes down and investment in R&D increases through entry of new firms in the R&D sector. Consumption jumps up and keeps following the optimality condition (2), while income jumps up and then follows the dynamic of (11), but the temporary reduction in the interest rate implies that initially output grows more than consumption, creating a mechanism for the propagation of the shock.

[17] For empirical evidence on this classic Schumpeterian insight see for instance Blundell *et al.* [1999], Nicholas [2003] and Segerstrom [2006].

innovations where entry is free.[18] Actually, this behaviour of the leaders under free entry is a particular case of a much more general result established in Etro [2002,a; 2006,b] where I have shown that Stackelberg leaders are always aggressive (under quantity or price competition or in patent races as here) whenever entry is endogenous. Here, the requirement that incumbents are leaders in the market for innovations is realistic: after all, it is reasonable to imagine that they have a credible commitment to invest a certain amount of resources to improve their own products and protect their own rents. Otherwise, we can imagine that incumbent monopolists can undertake some preliminary investments which affect their profitability from engaging in R&D activity, like building laboratories, hiring researchers or borrowing to invest.[19]

Let us consider the market for innovation described in Section 2, where (9) and (10) are the objective functions of the entrants and the leader, and the latter has a first mover advantage (for simplicity, I will abstract from subsidies). In this set up, analyzed in more detail in the Appendix, the partial equilibrium for each sector is characterized as follows:

PROPOSITION 3. *Under Stackelberg competition in the market for innovations, when ϵ is large enough incumbent monopolists deter entry obtaining complete persistence of their leadership, while for smaller values of ϵ they allow entry but invest more than any outsider.*

When marginal productivity is close to constant, that is for ϵ close to 1, it is optimal for the monopolist to deter entry investing just enough in R&D to make unprofitable for any follower to engage in R&D activities, and this delivers complete persistence of monopolies. This case generates the equilibrium growth rate:

$$g = \frac{\left(\frac{\epsilon}{\zeta}\right)^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left(\bar{\psi}-\eta\right)-\rho}{\gamma+\left(1-\alpha\right)/\alpha\ln q} \tag{19}$$

where $\bar{\psi}$ is the return rate from leadership and is derived in the Appendix. This return rate is above μ since it includes the value of perpetual leadership, but, approaching constant returns to scale, it tends to μ: in such a case the growth rate approaches the same level as under Nash competition (the investment of the incumbent monopolist perfectly crowds out that of the outsiders). In what

[18] See Etro [2004]. For related theoretical works, see Wiethaus [2005], Zigic *et al.* [2006], Reksulak *et al.* [2006] and Denicolò and Zanchettin [2006].

[19] As I have shown in a more general context (see Etro [2006a,b]), this kind of strategic investment allows to reproduce similar outcomes to Stackelberg equilibria with free entry: leaders always overinvest strategically to be aggressive in the market for innovations afterward. See also Etro [2007] for a more general discussion on the link betweek the theory of market leaders and policy issues.

follows, however, I will focus on the more realistic scenario where entry by outsiders takes place and the persistence of monopolies is only partial: this requires that investment in R&D faces significant decreasing marginal productivity, i.e. ϵ is small enough.[20]

When marginal productivity is decreasing (enough), the free entry condition pins down the number of followers in each sector. In this case, it is easy to verify that their optimal strategy $z(k_j)$ is always independent from the one of the leader (while the number of followers decreases in the investment of the leader), hence the effective discount rate $r+p(k_j)$ must be also independent from the leader's strategy [Etro, 2004]. The leader chooses its investment $z_M(k_j)$ to solve the problem (10), where the effective discount rate is independent from its choice and hence taken as given. From the first order conditions of the leader and of the followers and the zero profit condition, we obtain:

$$z(\kappa_j) = \frac{\left\{\epsilon\phi(k_j)\left[\mathbf{V}^M(k_j+1) - F(k_j)\right]\right\}^{\frac{1}{1-\epsilon}}}{\phi(k_j)} < z_M(k_j) = \frac{\left[\epsilon\phi(k_j)\mathbf{V}^M(k_j+1)\right]^{\frac{1}{1-\epsilon}}}{\phi(k_j)} \tag{20}$$

Notice that the investment of each follower is increasing in the value of the leadership net of the fixed cost, while the investment of the leader is independent from the fixed cost. The characterization of the equilibrium is complicated from the fact that now we do not know what is the value of being a monopolist, since this is the solution to the Bellman equation (10). But this value is what drives the incentives to invest in innovation, that is the engine of growth. A contribution of this paper is to provide dynamic programming techniques to solve analytically this problem, which will emerge whenever one is dealing with Schumpeterian models of growth where incumbent monopolists engage in R&D activity - see also Segerstrom [2006]. To derive the balanced growth path and the equilibrium value function $\mathbf{V}^M(k_j)$, the functions $z(k_j)$ and $z_M(k_j)$ and the equilibrium values for g, r, p and n, we can adopt the method of undetermined coefficients. Let us guess a functional form for the value function as:

$$\mathbf{V}^M(k_j) = \mathbf{V}^M(k_j-1)q^{\frac{\alpha}{1-\alpha}} = \psi\frac{X(k_j)}{r+p} \tag{21}$$

where ψ is a coefficient to be determined which can be interpreted as the rate of return from leadership. This must be larger than μ, otherwise the value of being a leader investing in R&D would be smaller than the value of being a leader without investing (i.e.: it would be optimal to stay out of the patent race

[20] Segerstrom [2006] has criticized my approach for implying a low persistence of monopolies. However, my approach is even consistent with complete persistence (for ϵ high enough). Nevertheless, in a realistic set up monopolies should be persistent, not eternal.

for the leader). Substituting in (20), this implies:

$$z(k_j) = \left(\frac{\epsilon\,(\psi - \eta)}{\bar{\zeta}^{\epsilon}\,(r+p)} \right)^{\frac{1}{1-\epsilon}} q^{\frac{\alpha}{1-\alpha}} X(k_j), \quad z_M(k_j) = z(k_j) \left(\frac{\psi}{(\psi - \eta)} \right)^{\frac{1}{1-\epsilon}} \tag{22}$$

and, using this in the Bellman equation, we have:

$$\begin{aligned} \mathbf{V}^M(k_j) &= \frac{[\phi(k_j) z_M(k_j)]^{\epsilon}\,\mathbf{V}^M(k_j+1) - z_M(k_j)}{r+p} + V(k_j) - F(k_j) \\ &= \frac{X(k_j)}{r+p} \left[\mu + \left(\frac{\epsilon}{\bar{\zeta}} \right)^{\frac{\epsilon}{1-\epsilon}} \left(\frac{\psi}{r+p} \right)^{\frac{1}{1-\epsilon}} q^{\frac{\theta}{1-\theta}} (1-\epsilon) - \eta q^{\frac{\theta}{1-\theta}} \right] \end{aligned} \tag{23}$$

whose right hand side contains the sum of the mark up from the current innovation and another term which represents the option value of remaining monopolist after the next innovation: this option value is positive because of the leadership advantage. Using (21) and solving (23) for the effective discount rate we have:

$$r + p = \left(\frac{\epsilon}{\bar{\zeta}} \right)^{\epsilon} \left[\frac{(1-\epsilon)\, q^{\frac{\theta}{1-\theta}}}{\psi - \mu + \eta q^{\frac{\theta}{1-\theta}}} \right]^{1-\epsilon} \psi \tag{24}$$

which provides a negative relation between the effective discount rate $r+p$ and the rate of return from leadership ψ (for ψ small enough): the higher is the effective discount rate, the shorter is the lifetime of an innovation, and hence the lower is the value from being a leader.

Moreover, the zero profit condition for the followers provides another expression for the effective discount rate which is analogous to (13):

$$r + p = \left[\frac{\epsilon\,(\psi - \eta)}{\bar{\zeta}} \right]^{\epsilon} \left[\frac{\psi(1-\epsilon) + \epsilon\eta}{\eta} \right]^{1-\epsilon} \tag{25}$$

This is a positive relation between the effective discount rate $r+p$ and the rate of return from leadership ψ: the higher is the value of being a leader, the larger will be the investment in R&D and hence the probability of innovation and the effective discount rate.

Equating (24) and (25) we obtain the equilibrium value for ψ which provides all the equilibrium relations. An implicit expression for ψ is given by:

$$\psi = \mu + \frac{(1-\epsilon)\,\eta q^{\frac{\theta}{1-\theta}} \psi^{\frac{1}{1-\epsilon}}}{(\psi - \eta)^{\frac{\epsilon}{1-\epsilon}} \left[\psi(1-\epsilon) + \epsilon\eta \right]} - \eta q^{\frac{\theta}{1-\theta}} > \mu \tag{26}$$

which immediately implies:

Proposition 4. *Under Stackelberg competition in the market for innovations, the equilibrium rate of return from leadership is higher than under pure Nash competition because of the option value of monopoly persistence.*

Using (25), this implies that the effective discount rate and hence both the growth rate and the aggregate probability of innovation must be higher than under Nash competition.[21][22] The incumbency advantage adds power to the engine of growth because it endogenously increases the value of innovations associating with them an option to persistent leadership: this increases aggregate investment and hence growth. Moreover, we can easily verify that both the return from leadership ψ, the effective discount rate and hence the growth rate are increasing in the mark up μ. An increase in the fixed cost of innovation through η decreases the effective discount rate and hence the growth rate of the economy, but it has ambiguous effects on the value of being a leader. Finally, when innovations are more difficult to obtain because of an increase in ζ, the value of being a leader is unchanged, but the growth rate is ultimately reduced. In conclusion, for ϵ small enough, under Stackelberg competition in the market for innovations, monopolists invest in R&D more than any outsider and the equilibrium growth rate is:

$$g = \frac{\left(\epsilon/\bar{\zeta}\right)^{\epsilon} (1-\epsilon)^{1-\epsilon} \left(\eta + \frac{\psi-\mu}{q^{\theta/(1-\theta)}}\right)^{\epsilon-1} \psi - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{27}$$

where ψ, given by (26), is decreasing in ζ and η, and increasing in μ, and we have:

PROPOSITION 5. *Under Stackelberg competition in the market for innovations, the equilibrium growth rate is higher than under pure Nash competition.*

Clearly, when the engine of growth is given by persistent monopolistic positions as in this model, the investment by each firm increases, but it is still below the optimal level for both the incumbent monopolists and the outsiders (see the Appendix): the dynamic inefficiency is still present. It can be shown that the optimal allocation of resources can be achieved with a positive subsidy for the entrants, a smaller but positive subsidy for the incumbent monopolists and an appropriate entry fee to discipline entry.[23]

In conclusion, a model of Schumpeterian growth which incorporates some realistic features of the market for innovation like decreasing marginal productivity of investment, fixed costs and a first mover advantage for the incumbent

[21]This does not need to be the case when incumbents find optimal to deter entry (that is for high enough ϵ).

[22]Paradoxically, even if the rate of return from leadership is higher when growth is driven by monopolists, one can easily verify that the equilibrium value of innovation is smaller. The reason is that larger investments in R&D reduce the lifelenght of new intermediates.

[23]This is in contrast with the model of R&D investment by monopolists due to an exogenous technological advantage by Segerstrom [2006], which delivers the optimality of a negative R&D subsidy. This may show the importance of endogenizing monopoly persistence.

monopolist, delivers realistic implications for the patterns of innovation. Market leaders invest a lot in R&D and their leadership is somewhat persistent, but sooner or later they are replaced by new entrants. This environment enhances innovation and growth.

5 The Determinants of Growth

The above model can be used for a wide range of investigations on other determinants of growth and the way policies affect the internal mechanism of the engine of growth. I will analyze a few in this Section. First, I will study the relationship between growth due to other sources and growth due to innovations, showing a new causal relation between them. Then, I will examine innovation in a open economy context with trade and capital flows showing that the outcome is consistent with a number of features of the recent international experience. Finally, I will study how inflation and price frictions affect the engine of growth, deriving a long run relation between inflation and growth. Each one of these extensions has consequences for the empirical macroeconomic research which could be analyzed in the future. For simplicity, I will confine the rest of the analysis to the particular case with $\alpha = \theta$.

5.1 Innovation and growth

While a close attention has been paid to the effects of investments in innovation on growth, little interest has been captured by the opposite direction of causality: economic growth may affect the incentives to invest in R&D. This appears quite important for the empirical analysis on the determinants of growth (a serious critique to Schumpeterian growth models relies on the absence of a clear relation between R&D and growth) and also for growth accounting purposes.[24] I will address this issue augmenting the previous models with an external source of growth, which may just be the traditional exogenous technological progress or it may be endogenized through accumulation of human capital, public capital or other externalities increasing TFP. Then, I will evaluate the impact of this other source of growth on the incentives to invest and finally derive the nature of the relationship between overall growth and innovation. A surprising result emerges: even when innovation is the main engine of growth (in the sense that

[24]See the pathbreaking work by Jones [1995] and the survey by Aghion and Howitt [1998, Ch.12] on empirical tests of the Schumpeterian growth theory and Barro and Sala-i-Martin [2004, Ch. 10] on its implications for growth accounting.

it actually contributes to most of the growth rate), growth and investment in innovation may be negatively correlated.

Assume that our parameter A grows at an exogenous rate $x = \dot{A}/A$: even if we refer to this as TFP, we may think of any source of growth which does not derive from endogenous innovation. Now the expected discounted value of the profits with innovation k_j at time τ becomes:

$$V_\tau(k_j) = \frac{\mu \left(\frac{\alpha q^{k_j \alpha} A_\tau}{1+\mu} \right)^{\frac{1}{1-\alpha}} L}{r + p(k_j) - \frac{x}{1-\alpha}} \tag{28}$$

which is clearly increasing in x. Under this extension, the growth of output (5), as a function of the new source of growth, can be derived as $g(x) = \dot{Q}/Q + x/(1-\alpha)$, where the rate of technological progress is still defined by (11) but depends on x as well. As before, one can easily derive the equilibrium organization of the market for innovation under Nash or Stackelberg competition: now investment by each firm increases over time at a constant rate, while the number of firms is fixed and endogenously depends on the new source of growth. On the balanced growth path output must still grow at the same rate as consumption, which is given by the Euler equation (2). Hence, the equilibrium growth rate can be derived as:

$$g(x) = g(0) + \frac{x(1-\alpha+\alpha \ln q)}{(1-\alpha)(1-\alpha+\gamma\alpha \ln q)} \tag{29}$$

where $g(0)$ is the growth rate in absence of other sources of growth (the same derived in Section 3 or 4) and the right hand side is increasing in x.[25] Finally, we can residually derive the endogenous rate of technological progress as:

$$\frac{\dot{Q}}{Q} = g(0) + \frac{(1-\gamma)x\alpha \ln q}{(1-\alpha)(1-\alpha+\gamma\alpha \ln q)} \tag{30}$$

Hence, when $\gamma < 1$, an increase in growth due to other factors than innovation increases R&D investment: this happens through an increase in the number of firms investing in research (while the investment per firm does not change). However, when $\gamma > 1$, the increase in growth reduces entry and total investment in innovation. This surprising result is due to the effect of the endogenous adjustment of the interest rate on the value of innovations (28). When output growth increases, the interest rate must go up to raise savings and allow consumption growth to increase as well. The increase in the interest rate has

[25] A similar result would emerge if we were considering population growth: as well known from the last generation of endogenous growth models [Jones, 1995], per capita income growth would increase with the growth rate of population.

a negative affect on the value of innovations and hence on entry of firms investing in R&D, and this negative effect must be compared with the positive direct effect exerted by the increase in x on the value of innovation. When the intertemporal elasticity of substitution is high (γ is small), a small increase in the interest rate is needed to clear the credit market and R&D investment unambiguously increases, but when the intertemporal elasticity of substitution is low (γ is large), the opposite happens and an increase in x reduces investment and technological progress. Summarizing, we have:

Proposition 6. *An increase in the growth rate due to other factors than innovation reduces (increases) the number of firms investing in innovation and total investment in innovation when* $\gamma > (<)1$.

When the paradoxical negative relation between other factors of growth and innovation occurs, the empirical implications are quite drammatic: even if investment in innovation is the main engine of growth, the correlation between growth and investment/GDP ratio should be negative. Since realistic values for the intertemporal elasticity of substitution imply γ close to unity, we should not be surprised to find out that there is not a strong correlation between growth and R&D investment. In general, future empirical research on the relation between R&D and growth should take seriously the feedback effect of growth on R&D activity: the absence of a positive correlation between growth and R&D is not by itself a defeat of the Schumpeterian hypothesis.

5.2 Globalization and growth

In this section I will open up the economy to investigate how the engine of growth works in an international context with trade frictions.[26] I will show that growth is driven by the largest economy and enhanced by its relative size and by openness.

Let us consider two or more countries endowed with the same technology and preferences as above, but different TFP and population levels. Labour does not move across countries and final goods freely move across borders, while there are frictions in trade of intermediates: imagine that for 1 unit of intermediate good sent to a foreign country, $d \leq 1$ units arrive at destination because of iceberg transport costs or protectionism (but one may also think of losses due to incomplete protection of foreign IPRs): the parameter d can be interpreted as a measure of the degree of openness. These trade frictions imply that foreign demand is smaller than domestic demand for each intermediate good produced

[26] See Grossman and Helpman [1991] on many related issues. In Etro [2005] I develop further issues about R&D policy in the open economy framework sketched below. See also Impullitti [2006a,b].

at home, and the expected discounted value of the profits with innovation k_j by a firm in country i becomes:

$$V_i(k_j) = \frac{\mu \left(\frac{\alpha q^{k_j \alpha}}{1+\mu}\right)^{\frac{1}{1-\alpha}} \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f \neq i} A_f^{\frac{1}{1-\alpha}} L_f\right]}{r + p(k_j)} \tag{31}$$

When countries are homogeneous or market integration is perfect ($d = 1$), this value is the same everywhere and the allocation of R&D investment is indeterminate. Otherwise, the value of innovations is higher for larger economies.[27] As long as the productivity of the research efforts is the same in all countries, or higher in the larger ones, the endogenous allocation of R&D investment is always biased toward the larger country: under Nash competition in the market for innovations, only its firms will invest in R&D, while under Stackelberg competition incumbent firms from other countries may keep investing and retain the leadership, but they will loose it sooner or later in favour of firms from the largest country. Hence, for any initial allocation of the technological frontier, the engine of world growth is in the largest economy, which gradually conquers the technological leadership in all sectors through its innovative firms. Historically, a similar process realized during the XIX century with the Industrial Revolution in England and in the XX century when USA became first the largest world economy and then gradually conquered the technological leadership in most sectors; some observers would bet on China repeating this path during the XXI century.

The world economy must be characterized by a constant growth rate for all countries (differences would emerge reintroducing heterogeneity in TFP growth across countries). Growth increases in the relative size of the leading country $b = \max\left(L_f A_f^{1/(1-\alpha)}\right) / \left(\sum L_f A_f^{1/(1-\alpha)}\right)$. For instance, when $\epsilon \to 1$ the growth rate, as a function of the degree of openness, boils down to:[28]

$$g(d) = \frac{\frac{\mu}{\zeta}\left[b + d^{\frac{1}{1-\alpha}}(1-b)\right] - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{32}$$

Even if this model does not exhibit absolute scale effects as in the last generation of endogenous growth models (since Jones [1995]), "relative scale effects" emerge: the larger is the leading economy compared to the rest of the world,

[27]This specification is borrowed from Alesina and Barro [2002] who studied the relation between the size of countries and unions and globalization - see also Etro [2006c]. Notice that in case of different growth rates of TFP and population across countries, we should take into account the growth rates as well to compare country'sizes.

[28]For consistency, now I assume: $\phi(\kappa_j) = \left[\zeta\left(\alpha q^{\kappa_j \alpha}\right)^{1/(1-\alpha)}\left(\sum A_f^{1/(1-\alpha)} L_f\right)\right]^{-1}$.

the higher is the growth rate. A consequence of these relative scale effects is that the positive relation between openness and growth survives (see Barro and Sala-i-Martin [2004], on the related empirical evidence).

The equilibrium is characterized by intra-industry trade: even if the largest country has an absolute advantage in all sectors, it develops a comparative advantage in the intermediate goods sector and exports these goods and imports final goods in the long run.[29] Finally, notice that the world interest rate has to equate world savings and investment, hence savings from all the world finance investments in the leading country. This may help explaining the Lucas'paradox concerning why capital does not fly to poor countries, while often there is a flow of resources moving in the opposite direction. Summarizing, we have:

PROPOSITION 7. *In a open economy context with trade frictions, the largest country has a comparative advantage in the innovation sector and, in the long run, it leads alone the technological frontier, exports intermediate goods, imports final goods and attracts foreign capital to finance investment. Growth increases in the degree of openness and in the relative size of the largest country.*

Such an scenario may appear to extreme to be realistic, nevertheless, in a stylized way, it appears in line with the growth experience of the last decades. This was characterized by 1) large R&D investments and high rate of technological progress in the US, 2) high US imports of final goods which allowed other countries to grow as well, exporting final goods to the US and importing American technology, and by 3) impressive capital flows toward the US financing its large current account deficits and turning United States into the largest debtor country in the world.

As in the closed economy, also this decentralized international equilibrium is dynamically inefficient and a system of R&D subsidies and entry fees would be welfare improving. However, countries have conflicting interests. Since aggregate growth depends on international factors, every single country has a limited scope for intervening: shifting monopolistic profits toward domestic firms. In a more general context [Etro, 2002b], I have characterized the optimal unilateral policies for international competition, which in our framework require positive R&D subsidies.[30] Unfortunately, these policies just shift profits from one

[29]At each point in time, the trade surplus in intermediate goods, national savings and net capital inflows will have to be matched by a deficit in trade of final goods and by investments.

[30]More precisely, when the profit function of the subsidized domestic firm satisfies strategic complementarity, that is $\partial^2\Pi^i/\partial z_i \partial z_f > 0$, and marginal profitability increasing in the subsidy, that is $\partial^2\Pi^i/\partial z_i \partial s > 0$, as here for (9) with the value of innovation (31), 1) under barriers to entry there is always a unilateral incentive to tax the domestic firm (as well known in the trade literature), but 2) under free entry there is always a unilateral incentive to subsidize the domestic firm: this new result is derived in Etro [2002b] within a more general model and our framework is nested in that one (for related works on marker leaders and trade policy

country to another, crowding out investments by foreign firms in favour of the domestic subsidized firm, while aggregate growth is unaffected. For instance, under perfect market integration, the optimal R&D policy can be characterized as in the closed economy, but the optimal unilateral R&D policy by a single country (which maximizes profits of the domestic firms net of the subsidy cost, taking as given the world growth rate) is given by the following subsidy for any domestic firm:

$$\hat{s} = \frac{\eta(1-\epsilon)}{\left(\frac{\mu-\eta}{\mu}\right)^{\frac{\epsilon}{1-\epsilon}}\left[\mu(1-\varepsilon)+\eta\epsilon\right]-\epsilon\eta} \in (0,1)$$

In conclusion, in the open economy context, independent R&D policies do not promote growth,[31] suggesting a new case for international cooperation.

5.3 Inflation and growth

Endogenous growth models are well equipped to face monetary issues in growing economies, but they have been rarely used for this purpose. Here, building on an important contribution by Barro and Tenreyro [2006], I show that an inverted-U relation between inflation and long run growth emerges when monetary frictions affect the engine of growth.

Consider a closed economy, and introduce real money balances M/P, where M is nominal money issued at the growth rate $\sigma = \dot{M}/M$ and P is the price level for the final good, which is perfectly flexible and evolves with the rate of inflation $\pi = \dot{P}/P$. Adopting the Sidrausky approach with money in the utility function $C^{1-\gamma}/(1-\gamma) + \chi(M/P)^{1-\xi}/(1-\xi)$ with $\xi, \chi > 0$, standard optimization shows that the optimal consumption growth is still given by (2), while money demand is $M = P\left[\chi C^{\gamma}/(r+\pi)\right]^{1/\xi}$. Equating the latter and the money supply delivers the equilibrium price level at each point in time. This implies that on a balanced growth path the endogenous level of inflation is constant if and only if:

$$\pi = \sigma - \frac{\gamma g}{\xi} \tag{33}$$

In general equilibrium, monetary policy and hence inflation may affect the return rate and the growth rate. This is not the case when the prices of the intermediate goods perfectly adjust to changes in the price level of final goods: then inflation

see Boone *et al.* [2006] and Kovac and Zigic [2006]).

[31] One may wonder what would happen if all countries could optimally choose their R&D policy. Contrary to what happens in usual trade wars, here a world Nash equilibrium would be characterized by a limited number of countries choosing the subsidy $\hat{s}$ and all the other countries choosing not to subsidize R&D, again without impact on economic growth.

is superneutral. In such a case, the optimal monetary policy is always given by a generalized Friedman Rule which sets the nominal interest rate as close as possible to zero implying the optimal rate of money growth $\sigma = g\gamma\left(1-\xi\right)/\xi - \rho$ (which is positive for γ small enough).

However, when prices of intermediate goods are sticky in the newkeynesian tradition [Mankiw, 1985; Blanchard and Kiyotaki, 1987], new consequences emerge for equilibrium growth and we will focus on them. Before doing it, however, notice that, as emphasized by Barro and Tenreyro [2006], a surprise inflation temporary increases demand of intermediate goods and hence production. For instance under a price stabilization policy a discrete and permanent increase in money supply (or a temporary increse in the growth rate of money) increase the price level of final goods and decrease the relative price of intermediate goods (until firms change them), temporary boosting production. Moreover, consumption smoothing implies that part of the increase in income is saved, exerting a temporary downward pressure on the interest rate which, in turn, promotes investment in R&D, creating a novel mechanism for the propagation of the shock (compared to standard newkeynesian models).[32] The effect of inflation in such a model suggests problems of time-inconsistency for a policy of price stabilization, as in the Barro-Gordon framework. However, our model generates a good reason for commitment to rigorous monetary rules: the reason is that inflation creates other effects on investment and these effects have permanent consequences on long run growth.

There are many ways to formalize price stickiness. Following the newkeynesian literature [Woodford, 2003], it is convenient to assume staggered price-setting *à la* Calvo, where the opportunity to adjust prices follows a stochastic process for each firm. However, our model provides an easy and realistic way to endogenize price adjustments assuming that they coincide with the introduction of new goods in the market.[33] In particular, an innovator at time τ sets the price of its good at a level $P_\tau(1+\mu)$ and keeps it at this level until a new vintage is on the market with a new price. Then, the expected discounted value of the

[32]The qualitative analysis of these shocks is beyond the purpose of this paper. Nevertheless, a brief summary of them can be interesting. At the time of the shock, the real interest rate jumps down, while production and consumption jump up and new firms invest in R&D because of the lower interest rate: this implies that the growth rate of income is above the growth rate of consumption. While firms gradually change their prices, the impact of the shock vanishes: the interest rate goes up toward the initial level, the growth rate of consumption increases and the growth rate of income decreases, both slowly returning to the initial long run level.

[33]This is realistic if the average life-lenght of a product is short, that is if we are dealing with small and frequent innovations in each industry. One can see this as an extreme form of price stickiness, but we could obtain similar qualitative results when there is an exogenous probability of price adjustment at the cost of analytical complications.

profits with an intermediate good k_j at time τ can be derived as:

$$V_\tau(k_j) = \int_\tau^\infty \left(\frac{\alpha A q^{k_j \alpha} P_t}{(1+\mu) P_\tau}\right)^{\frac{1}{1-\alpha}} \frac{L\left[(1+\mu) P_\tau - P_t\right] e^{-[r+p(k_j+1)](t-\tau)}}{P_t} dt \quad (34)$$

where $P_t = P_\tau e^{\pi(t-\tau)}$. Developing the integral, it can be verified that this value is an inverted-U curve in the inflation rate and it is maximized at some level $\hat{\pi}$, which is positive if μ is large enough. Competition to innovate is driven by (34) as before (even if firms now decide their real investment flows and change the nominal ones over time with inflation). Since the equilibrium effective discount rate, the aggregate investment and the growth rate of output are directly related with this value of the intermediate goods (34) by the free entry condition in the market for each innovation, they inherit the same non-monotonic relation with inflation. Hence, price stickiness, generates an inverted-U curve linking the inflation rate and the endogenous growth rate:[34]

$$g = g(\pi) \quad \text{with } g'(\pi) \gtrless 0 \text{ for } \pi \lessgtr \hat{\pi} \quad (35)$$

The balance growth path is characterized by (33) and (35) for a given policy of constant money growth σ.[35] Summarizing, we have:

PROPOSITION 8. *In presence of price stickyness, there is a long run inverted-U relation between inflation and growth due to the effects of expected inflation on the incentives to invest.*

Notice that this result is in contrast with the Mundell-Tobin effect for which inflation stimulates investment and growth: here, this happens only for low levels of inflation, while high inflation erodes expected monopolistic profits and hence it reduces investment and growth. The last outcome may provide a channel for the negative relation between inflation and growth emphasized in the empirical literature at least for high levels of inflation - see Barro and Sala-i-Martin [2004].

[34] This effect may be quite relevant. Imagine that prices are constant for a year, mark up is at 20% and inflation at 5%. Then, assuming $\alpha = 0.5$, after one year the flow of profits is reduced by more than 20%. If the average life-lenght of an intermediate good is one year (or prices are changed every year), inflation reduces the value of innovation by about 10%. As in the newkeynesian literature on businness cycles, small price frictions imply that demand shocks can have large macroeconomic consequences: however, here the consequences are permanent.

[35] For intermediate levels of money growth there are two equilibrium growth paths and the inefficient one is characterized by high inflation and low growth. This self-fulfilling stagflation has a simple intuition: if high inflation is expected, firms reduce investment decreasing output growth, which generates a low growth rate of consumption and money demand, which creates high inflation. However, such a path is unstable, and in what follows I focus on the stable path.

While the newkeynesian theory has mainly focused on the short term consequences of inflation in presence of price frictions, little attention has been paid to the long run consequences, which can be even more important from a policy point of view. In our context, the utility maximizing policy can be quite complex, but if proper policies can solve the inefficiencies in the allocation of investment, the optimal inflation rate must be between the Friedman rule level and the growth maximizing level. Clearly, when $\chi \to 0$, so that the welfare costs from not implementing the Friedman rule under full price flexibility are negligible, the optimal policy tends to the growth maximizing one. Interestingly, the latter boils down to a policy of zero inflation when pricing strategies are optimally chosen by monopolists. To see this, notice that the optimal monopolistic mark up in sector j is:

$$\mu = \frac{(1-\alpha)\left[r + p(k_j+1)\right]}{\alpha\left[r + p(k_j+1) - \pi/(1-\alpha)\right]}$$

which is increasing in the inflation rate. The value of innovation then becomes:

$$V_\tau(\kappa_j) = \left(\frac{1-\alpha}{\alpha}\right)\left(\alpha^2 A q^{\kappa_j \alpha}\right)^{\frac{1}{1-\alpha}} LP_\tau \frac{\left[r + p(k_j+1) - \frac{\pi}{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}}}{\left[r + p(k_j+1) - \frac{\alpha\pi}{1-\alpha}\right]^{\frac{1}{1-\alpha}}}$$

which is maximized by $\hat{\pi} = 0$. If it is optimal to maximize the value of innovation and growth, the associated optimal rate of monetary growth is $\sigma^* = \gamma g(0)/\xi > 0$. Hence in this case we can conclude that when $\chi \to 0$ the optimal policy satisfies the Friedman rule under perfectly flexible prices but it requires price stabilization under sticky prices.[36]

For instance, an explicit solution for the long run growth rate as a function of the inflation rate can be obtained when $\alpha = 1/2$ and $\epsilon \to 1$:

$$g(\pi) = \frac{\pi + \frac{2}{\zeta(1-s)}\left[1 + \sqrt{1 - \zeta(1-s)\pi}\right] - \rho}{\gamma + 1/\ln q} \tag{36}$$

which holds for small enough inflation and is clearly maximized by a policy of price stabilization.

Finally, let us briefly consider the open economy model of the previous section. Purchasing power parity in the final goods would imply that domestic inflation is matched by depreciation of the home currency. Barro and Tenreyro [2006] have shown that in this environment inflationary surprises create

[36] Clearly, a characterization of optimal monetary policy is beyond the scope of this paper. However, this framework allows to formalize in a new and simple way the long run welfare costs of a positive inflation.

temporary expansions and transmit them abroad as well. Our general equilibrium extension would allow to study the effects of shocks in a simple way in the tradition of the new open economy macroeconomics [Obstfeld and Rogoff, 1996].[37]

More interesting for our purposes is to focus on the long run effects of inflation on growth for an open economy. These effects depend on the country we are looking at. In the leading economy with a comparative advantage in the investment sector, inflation will impact on growth in a similar way to the closed economy. However, in a small open economy whose growth rate depends on foreign innovations, inflation does not affect long run growth. Hence, in small open economies there is a stronger case for unilateral inflationary surprises (or unilateral competitive devaluations).

Even if here I focused on the relation between inflation and long run growth, I would like to stress as a collateral result that this framework could be usefully adopted in future research as a simple new open economy model with some advantages over the Obstfeld-Rogoff framework, which does not endogenize investment: this framework allows to take investment into the (current) account.

6 Conclusions

I developed a model of creative destruction where the engine of growth is in the microeconomic structure of the patent races leading to innovations. In conclusion, I want to summarize the most substantial findings of this research. Exploring a realistic organization of the market for innovations we found a new source of dynamic inefficiency of the growth process which is due to suboptimal investment by both incumbent patentholders and outside researchers and requires R&D subsidies as a general remedy. Moreover we have shown in a quite general framework that an increase in growth due to other sources than

[37]It is tempting to summarize quickly the effects of the same positive monetary shock as before in this open economy context. Domestic expected income increases and foreign expected income increases as well, but relatively less in case of trade frictions. The domestic exchange rate depreciates (but less than proportionally because of the relative increase in domestic money demand) and domestic and foreign savings increase. This implies that the international interest rate goes down until world investment increases enough to match world savings: hence, the temporary monetary shock boosts production and investment in the world economy. The current account surplus improves for small countries, but it deteriorates for the larger country (where investment takes place), contrary to what happens in the Mundell-Fleming and in the Obstfeld-Rogoff models. The difference with the latter derives from the absence of an investment channel for the propagation of the shock in the basic Obstfeld-Rogoff framework (where all short term effects go through intertemporal substitution in consumption and labour supply).

innovation can reduce the investment in R&D generating a negative correlation between growth and innovation. We also noticed that in a open economy context growth is driven by the largest country and increases with its relative size and with openness. Finally, in a monetary context price stickiness induces an inverted U relation between inflation and long run growth. Future empirical research may investigate some of these findings.

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Appendix

The Social Planner Solution. I derive the optimal organization of the R&D sector through an heuristic solution for the social planner problem. First of all, it is immediate to derive from the concavity of the arrival rate that it is optimal to allocate equal flows of investment between all the R&D laboratories. Consider linear investment flows in the future scale of production, as $z(k_j) = \beta X(k_j + 1) = \beta q^{\frac{\theta}{1-\theta}} X(k_j)$ for each firm in sector j, where the paremeter β must be chosen optimally. Let us keep the production of intermediates at the level chosen by the monopolist in the decentralized equilibrium. As well known, a social planner would not distort the choice of the input mix, hence we are basically solving for a second best allocation (the first best allocation would be obtained by subsidizing monopolists in such a way that their price equates marginal cost).

The resource constraint of the economy must take into account the fixed costs, which are paid only at the beginning of each new patent race. Without loss of generality, let us assume that the economy devotes a flow of resources for this purpose in each sector. If the number of sectors N is high enough, one can approximate this flows, say $f_j(k_j)$ with those equating their expected present value $f_j(k_j)/\left[r + p(k_j)\right]$ to the fixed cost $F(k_j)$, that is with $f_j(k_j) = \eta X(k_j + 1)$. Using the expressions for the quantity of intermediate goods and for the output, we can rewrite the resource constraint as:

$$\begin{aligned} Y &= \frac{X}{\alpha} = C + \sum_{j=1}^{N} X_j(k_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} z_i(k_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} f_j(k_j) = \\ &= C + X\left[1 + n(\beta+\eta) q^{\frac{\theta}{1-\theta}}\right] \end{aligned} \tag{37}$$

from which we derive an expression for consumption holding at each point in time. Under the optimal allocation of resources, growth is determined by the rate of innovation as:

$$g = n\left[\phi(k_j)z(k_j)\right]^{\epsilon}\left[q^{\frac{\theta}{1-\theta}} - 1\right] \approx \left(\frac{\beta}{\bar{\zeta}}\right)^{\epsilon}\frac{\theta n}{1-\theta}\ln q. \tag{38}$$

Given a constant growth rate of consumption, intertemporal utility is finite as long as $\rho > (1-\gamma)g$, and can be written as:

$$U = \int_0^{\infty}\frac{C_t^{1-\gamma}}{1-\gamma}e^{-\rho t}dt = \frac{C_0^{1-\gamma}}{(1-\gamma)\left[\rho-(1-\gamma)g\right]} \tag{39}$$

Finally, substituting (37) and (38) in (39), we can summarize the social planner problem as:

$$\max_{n,\beta}\frac{X_0{}^{1-\gamma}\left[\frac{1-\alpha}{\alpha} - n(\beta+\eta)q^{\frac{\theta}{1-\theta}}\right]^{1-\gamma}}{(1-\gamma)\left[\rho-(1-\gamma)\left(\frac{\beta}{\bar{\zeta}}\right)^{\epsilon}\frac{\theta n}{1-\theta}\ln q\right]} \tag{40}$$

which puts in clear evidence the basic trade-offs. A higher number of firms or a higher flow of investment per firm imply a higher growth rate of consumption but with a lower initial consumption level (and the time preference rate and the elasticity of substitution govern this trade-off in a standard fashion), but the weights on benefits and costs are different for the two choice variables. The higher is the fixed cost parameter η the more costly is to increase the number of firms rather than the flow of investment, while the opposite happens when the parameter ζ, which determines the arrival rate of innovations, is higher. Finally, also the size of the innovations q and the parameter θ characterizing the elasticity of demand of intermediate goods influence the trade-off.

If an interior solution exists, the first order conditions for the social planner problem (40) with respect to β and n can be divided one by the other to obtain $\beta^* = \epsilon\eta/(1-\epsilon)$, which implies the optimal flow of investment in R&D per firm (15) in the text. Let us now look at the number of firms investing in R&D. From the first order conditions we obtain the optimal number of R&D laboratories as:

$$n^* = \frac{\left[\left(\frac{1-\epsilon}{\eta}\right)\left(1-q^{-\frac{\theta}{1-\theta}}\right)\left(\frac{1-\alpha}{\alpha}\right) - \rho\left(\frac{(1-\epsilon)\bar{\zeta}}{\epsilon\eta}\right)^{\epsilon}\right]}{\gamma\left[q^{\frac{\theta}{1-\theta}}-1\right]} \tag{41}$$

which is decreasing in ϵ at least for ϵ high enough: this implies that when the marginal productivity of the investment is high enough, it is optimal to have just one laboratory investing in R&D. Finally, substituting β^* and n^* in our expression for growth (38), we obtain that the optimal growth rate is:[38]

$$g^* = \frac{1}{\gamma}\left[\left(\frac{\epsilon}{\bar{\zeta}}\right)^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left[1-q^{-\frac{\theta}{1-\theta}}\right]\left(\frac{1-\alpha}{\alpha}\right)-\rho\right] \tag{42}$$

[38]It can be verified that when A grows at the rate x, the optimal allocation of investment per firm is unchanged and the number of firms increases (decreases) in x for $\gamma < (>)1$.

which is higher than the equilibrium growth rate for any γ smaller than a cut-off (and tends to the Barro and Sala-i-Martin optimal growth rate approaching constant returns to scale, that is for $\epsilon \to 1$ and $\eta \to 0$). To derive the optimal R&D policy let us introduce an entry fee which is the fraction τ of expected production costs in each patent race. The equilibrium growth rate becomes:

$$g(s,\tau) = \frac{\left[\frac{\epsilon(\mu-\eta-\tau)}{\zeta(1-s)}\right]^{\epsilon}\left[\frac{\mu(1-\epsilon)+\epsilon(\eta+\tau)}{\eta}\right]^{1-\epsilon} - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{43}$$

The optimal R&D policy is given by (s^*, τ^*) such that $z(k)$ and $g(s,\tau)$ equate $z^*(k_j)$ and g^*, that is:

$$s^* = \frac{1}{\frac{\mu}{\eta+\tau^*}(1-\epsilon)+\epsilon} \in (0,1) \tag{44}$$

$$\tau^* = \frac{\left[\mu(1-\epsilon)+(\eta+\tau^*)\,\epsilon\right]^{\frac{1}{1-\epsilon}}\left[\frac{\epsilon}{\zeta(1-\epsilon)}\right]^{\frac{\epsilon}{1-\epsilon}}}{\left[1+(1-\alpha)/\gamma\alpha\ln q\right]\left[\left(\frac{\epsilon}{\zeta}\right)^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left(1-q^{-\frac{\theta}{1-\theta}}\right)\left(\frac{1-\alpha}{\alpha}\right)-\rho\right]+\rho} - \eta \tag{45}$$

which provide two unique optimal policy tools. Clearly $g^* < g(s,0)$ for γ high enough, in which case, the optimal entry fee is positive.

Growth driven by Market Leaders. I will now provide further details on the model with Stackelberg leadership for patentholders in the R&D sector. When there is entry of firms in the patent races, the equilibrium effective discount rate derived in the text allows to explicit the growth rate as:

$$g = \frac{r+p-\rho}{\gamma+(1-\alpha)/\alpha\ln q} \tag{46}$$

and then to derive the number of firms $n(\epsilon)$ as a function of ϵ and the equilibrium investments:

$$\begin{aligned} z(k_j) &= \frac{\epsilon\eta\,(\psi-\eta)}{\left[\psi-\epsilon\,(\psi-\eta)\right]} q^{\frac{\theta}{1-\theta}} X(\kappa_j), \\ z_M(k_j) &= \frac{\epsilon\left[\psi-\mu+\eta q^{\frac{\theta}{1-\theta}}\right]^{\frac{1}{1-\epsilon}}}{(1-\epsilon)q^{\frac{\theta}{1-\theta}}} q^{\frac{\theta}{1-\theta}} X(\kappa_j) \end{aligned}$$

which are both smaller than the first best level. Also in this case, the organization of the investment is dynamically inefficient. Efficiency would require $z(k_j) = z_M(k_j)$ and hence $(\psi-\eta)/(1-s) = \psi/(1-s_M)$, where s is the subsidy for the followers and s_M the one for the leaders. It is immediate to verify that the optimal subsidies would be:

$$s^* = \frac{\eta}{\psi(1-\epsilon)+\eta\epsilon} \in (0,1), \quad s_M^* = \frac{\eta\epsilon}{\psi(1-\epsilon)+\eta\epsilon} \in (0, s^*) \tag{47}$$

The investment of the monopolist is increasing with ϵ and actually converging to ∞ for $\epsilon \to 1$. This implies that there is a cut-off $\hat{\epsilon}$ such that $n(\hat{\epsilon}) = 1$. Then, for $\epsilon \geq \hat{\epsilon}$ the optimal strategy for the leader is pure entry deterrence - this applies a more general result by Etro [2002,a]. To derive the equilibrium under this regime of complete persistence of monopoly, notice that the investment of the leader must be slightly above the level at which the free entry condition allows entry by just one follower. Such an investment allows the leader to be alone in the patent race and, adopting the usual guess for the value function (21), it implies:

$$r + p = \left(\frac{\epsilon}{\bar{\zeta}}\right)^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}(\psi - \eta)$$

Using the equilibrium expression for the growth rate of consumption (2) and income (46), this provides:

$$\psi = \eta + \frac{p\left[1+\frac{\gamma\theta\ln q}{1-\theta}\right]+\rho}{\left(\frac{\epsilon}{\bar{\zeta}}\right)^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}}$$

which is a standard positive relation between the probability of innovation and the return from leadership. Since the leader is alone in the patent race, the probability of innovation is simply $p = [\phi(k_j)z_M(k_j)]^{\epsilon}$, which implies $z_M(k_j) = p^{(1/\epsilon)}/\phi(k_j)$. Now the Bellman equation expressing the value of leadership becomes:

$$\begin{aligned}
\mathbf{V}^M(k_j) &= \frac{[\phi(k_j)z_M(k_j)]^{\epsilon}\,\mathbf{V}^M(k_j+1) - z_M(k_j)}{r+p} + V(k_j) - F(k_j) = \\
&= \frac{X(k_j)}{r+p}\left[q^{\frac{\theta}{1-\theta}}p\frac{\psi}{r+p} + \mu - \bar{\zeta}q^{\frac{\theta}{1-\theta}}p^{\frac{1}{\epsilon}} - \eta q^{\frac{\theta}{1-\theta}}\right]
\end{aligned}$$

which, using the guess value for $\mathbf{V}^M(k_j)$, provides:

$$\psi = \left[\frac{\mu - \bar{\zeta}q^{\frac{\theta}{1-\theta}}p^{\frac{1}{\epsilon}} - \eta q^{\frac{\theta}{1-\theta}}}{\rho - \frac{(1-\gamma)p\theta\ln q}{1-\theta}}\right]\left\{\rho + p\left[1+\gamma\frac{\theta\ln q}{1-\theta}\right]\right\} \tag{48}$$

whose denominator must be positive under the transversality condition $\rho > (1-\gamma)\,g$, which requires γ large enough. This implies a negative relationship between the probability of innovation and the return from leadership due to the entry deterrence constraint: the larger is the investment in R&D needed to deter entry, the smaller is the value of being a leader. The two conditions above define the equilibrium values for $\psi = \bar{\psi}$ and the aggregate probability of innovation and hence the interest rate, the effective discount rate and the growth rate (19) in the text. In general, the leader is investing just enough to deter entry, while it could marginally reduce its investment and allow entry by one follower, which would increase the aggregate probability of innovation, the effective discount rate, and hence also the growth rate (this implies that in the regime of entry deterrence, the equilibrium growth rate could be below the one emerging without leaderships). However, approaching constant marginal productivity, the return from leadership $\bar{\psi}$ tends to μ and:

$$g\lfloor_{\epsilon\to 1,\eta\to 0}= \frac{\mu/\bar{\zeta}-\rho}{\gamma+(1-\alpha)/\alpha\ln q} \tag{49}$$

which is the same growth rate as under Nash competition in the market for innovations: when $\epsilon = 1$ the incumbent monopolist is actually indifferent between investing in R&D and crowding out outsiders' investment or not investing at all (with no changes in the aggregate equilibrium variables).