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Optimal use of scarce information: When partisan voters are socially useful*

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Abstract

We develop a model of simultaneous and sequential voting in a committee where members do not share their private information and do not have the same preferences. When objective functions differ, an optimal order in the sequential game can be found, leading to a unique socially optimal equilibrium. Our result rationalizes the presence of biased (i.e., partisan) voters in small committees as a way of reaching social optimality.

JEL Classification: D71, D72, D82.

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1 Introduction

Voting games are often played sequentially. It is not difficult to find examples in real life. The most evident case is probably the presidential election mechanism in the USA: the candidates are locally chosen through primaries and each local primary is set up on a different date. Another example is given by committees. Members often announce their vote, or opinion, in sequence. For instance, in U.S. Navy courts-martial, judges vote in a particular order, leaving the highest ranked ones at the end. In Italy, when expressing on particular topics, courts are composed by three members and, according to the code of penal procedure (art. 527), votes are gathered starting from the opinions of the less experienced members (seniority rule).

More specifically, this paper has been motivated by the observation of an important Italian institution, the "Corte Costituzionale" (Constitutional Court). One of the tasks of this court is to solve questions about the constitutionality of national and regional laws. This duty obviously requires a strong independence from, for instance, the parliament, which actually passes new laws. The fifteen members of this Court are appointed by three different institutional bodies (the president of the Republic, the parliament and high judges). It is fair to assume that members chosen by the parliament might be affected by political bias. We therefore question the rationale of this apparently inefficient choice, for an institution which is supposed to pursue social welfare[1].

We analyze a simultaneous and a sequential voting game where three members of a committee have different preferences. We show that, in the latter game, the order of voting is relevant and that optimality can be guaranteed. More surprisingly, it is the presence itself of a biased member which allows for uniqueness of the optimal equilibrium. The intuition for this result is that the bias provides certainty about the voter's strategy and removes an important (and inefficient) piece of uncertainty from the model.

To do this, we first explore a series of simpler voting games. They differ along several dimensions. First of all, we consider sequential and simultaneous voting games of players with truth purpose. Each member has private information, namely a correct signal about the true state of the world, and wants to maximize the expected (common) value of the election. Then, we change the players preferences and analyse what happens to their equilibrium strategies when they only want to maximize their own probability of being right (reputational purpose). Finally, we introduce heterogeneity and we study the voting behavior of a committee with three kinds of players.

From a theoretical point of view, the paper provides insights about the most efficient use of scarce information. One side of the problem is that one member (the biased one) find it optimal to destroy his information. Surprisingly, this choice provides uniqueness *and* optimality (given that a certain order of voting is imposed). Another side of the problem is about the behaviour of uninformed members when they are forced to vote (e.g., they are not allowed to abstain). They face (and solve) the dilemma of wrongly influencing the outcome

[1]Historical and political reasons are also provided (see section 5).

of an election.

The paper is organized as follows. After a review of the literature (section 2), in section 3 we introduce two simple voting games: a simultaneous and a sequential one. We provide intuitions for the results and discuss the emerging equilibria. In section 4, we change the players' preferences and solve the same games accordingly. A discussion about the Italian Constitutional Court introduces section 5, where we consider an heterogenous committee at work. In section 6, we briefly provide preliminary results and intuitions to some extensions of the basic model. Finally, in section 7 we discuss the limits of our model, suggest some possible developments of the research and draw our conclusions.

2 Review of the literature

First contributions analyzing voting behaviors date back to the work of Condorcet[2], more than two centuries ago. One of his main results is the Jury Theorem. It states that, in dichotomous elections[3], a large population of identical voters is more likely to select the best alternative than a single individual. In this contest, information is provided by (discrete) signals about the true state of the world. So, majority rules in elections allows for the best outcome. Still, until last decade, the traditional approach has considered mainly the mathematical and statistical properties of these situations. Only recently, voting games have been analyzed as strategic ones, that is, game theory has been used to solve them.

Two contributions in this area are particularly relevant for our paper. Feddersen and Pesendorfer (1996, 1997) analyze how well simultaneous voting elections can aggregate private information. Their results are worth discussing but the papers are not directly comparable with ours, as they focus on the asymptotic properties of their models (i.e., as the number of agents goes to infinity). In their former work, the authors want to explain the low rates of participation in many elections and emphasize the similarity between dichotomous elections with asymmetric information and auctions with a common value. In both cases, a player must condition his strategy not simply on his private information but also taking into account the fact that he may be decisive (i.e., he is the winner of the auction or the swing voter in the election). In equilibrium, uninformed independent voters, whenever they are indifferent between two options, prefer to abstain and not influence the outcome of the game. Informed independent voters choose according to their private information (a binary signal) whereas partisan voters always choose their favorite option. That is, no player plays a strictly dominated strategy. Mixed strategy equilibria (for the uninformed ones) are also possible. Surprisingly, they only involve a mix between abstention and voting for one candidate and not a mix between choosing one of the two options.

In the latter contribution, the authors rule out abstention (as we do in our model). They

[2] *"Essai sur l'application de l'analyse a la probabilite des decisions rendues la pluralite des voix"*, 1785.

[3] Elections with two opposite alternatives. For instance, when the choice is between rejecting or adopting an option.

discuss equilibrium strategies when voters play strategically and have different preferences. In this model, the symmetric Nash equilibria involve the informed independent voters playing according to their signal and the partisan ones playing their favourite option. Their main result is that (large) elections are decided by informed players and therefore they satisfy full information equivalence, that is, they successfully aggregate private information. Nevertheless, this is true as long as the informational structure is not very complex: with high uncertainty about the distribution of preferences and additional possible choices, the effectiveness of elections is weaker. In our paper, we confirm these equilibrium strategies for informed independent players and partisan ones; we also find optimal replies for independent uninformed players. Yet, our model deals with a small committee and uninformed statuses are never revealed. The outcome of the election can be determined even by uninformed voters and therefore information cannot be fully aggregated.

In the last decade, a growing literature about herd behavior has developed as well[4]. In economics, herding refers to the loss of information due to imitation of previous players' actions. From an information aggregation point of view, this is clearly inefficient. But eliciting information from the sequential structure may be in fact be desirable. So, there is a trade off which is worthy of analysis.

Most relevant references in herding are by Smith and Sørensen (2000) and Goeree, Palfrey and Rogers (2003). They both show how the importance of herding can be weaker when we introduce heterogenous preferences. Goeree, Palfrey and Rogers (2003) analyze the case with both private and common values (i.e.: preferences depending on the final outcome of the ballot but also on some ideological bias). Smith and Sørensen (2000) consider differences only along the common value dimension. The main assumption in this literature is that other players' actions do not influence each player's own payoffs. We show that this assumption does not necessarily hold in a sequential voting procedure.

Voting and herd behavior are naturally joined together in the analysis of sequential voting games.

Two main papers discuss sequential voting games[5]. Dekel and Piccione (2000) compare simultaneous and sequential voting games where individuals observe a signal about the true value of an option. Then, they decide whether to accept this option ("yes") or to stay with a status quo ("no"). Every player votes after observing his signal and previous players' choice. More than one individual can vote in each period. Their main result is that both simultaneous and sequential voting games are effective in aggregating information. Moreover, as far as symmetric equilibria are concerned, the sequential structure cannot do better than the simultaneous one. The intuition is not very difficult. Voters are behaving strategically and condition their actions on being pivotal in the election. As long as strategies are symmetric

[4]See the seminal contributions by Scharfstein and Stein (1990), Banerjee (1992) and Bikhchandany, Hirshleifer and Welch (1992, 1998).

[5]Sloth (1993) introduces and discusses sequential voting games, but only as a device to solve simultaneous ones. She shows that sophisticated equilibria in the former games are equivalent to subgame perfet equilibria in the latter.

and conditional, knowing who voted before you does not provide any useful information. This result applies to certain asymmetric equilibria as well (but they do not state a general rule). An interesting application is provided by elections with a common value for the alternative, that is, elections with the sole aim to aggregate private information. In (the unique) equilibrium, information is fully aggregated until an option is chosen; afterwards, the players do not need to vote informatively as they are no longer pivotal. This equilibrium recalls some characteristics of herd behavior. Of course, this result does not necessary hold in any setting. In an example, the authors show that it is possible to find a better (asymmetric strategies) equilibrium with sequential voting when there are more than two signals. This last point is important and leaves an open door to further research. Our own paper provides an analysis of cases where the best equilibrium strategy profiles are asymmetric ones. Nevertheless, we are not directly interested in how information is aggregated and consider players with heterogeneous preferences. In addition, even in games with homogeneous players, our environment is not totally symmetric: not necessarily every player observes a signal.

Ottaviani and Sørensen (2001) consider a game where members of a committee ("experts") sequentially reveal their opinions to a decision maker and finally a decision is taken. Members care about their reputation but not directly about the final decision. That is, they only want to show (to the market) that they make the right choice. Each of them observes a different signal but he does not know its quality. If the signal is very different from the common prior he shares with the other players, than he prefers to pool and play uninformatively. This lost of information parallels the findings of herding literature. The authors also try to endogenize the order of speech to find an optimal one. In general, better experts should speak first. As herding arises, some information may be lost. This is especially true with more than five members and with greater heterogeneity in the quality of the experts. With experts of unknown ability, there are equilibria in simultaneous games dominating equilibria in sequential ones. The opposite is true with known ability. Our paper may be viewed as a particular case where all the voters have the same known ability. We introduce heterogeneity in terms of preferences (and not ability) and find that the optimal order of voting actually depends on them.

3 Voting games with truth purpose

In this section, we study two voting games, both based on the following setting.

A committee must take a binary decision: it can accept an alternative or reject it. Suppose the number of members, n, to be 3 and the required majority to be 2. The status quo has a common value of 0. For any member m of the committee (with $m \in M = \{i, j, z\}$), the alternative can take two common values: $v_m \in \{1; -1\}$. Each value has the same prior (i.e., $\frac{1}{2}$) but a private signal about the true one can be observed with probability α. As we are dealing with "experts", we assume $\alpha \in [\frac{1}{2}, 1]$[6]. When observed, the signal is correct with probability

[6] This assumption is made without loss of generality.

β. For simplicity, we fix $\beta = 1$[7]. Every player expresses a vote and abstention is not possible. A strategy consists in a member's voting behavior. The usual notation applies: s_m is the generic player m's strategy and s_{-m} is everyone else's one. In this section, we assume that the players are mainly interested in the committee taking the right decision (*truth purpose*). Nevertheless, they are not willing to reveal an uninformed status to their fellow committee members, so any information transmission occurs solely through voting behavior (i.e., there is no pre-voting debate). We may think of this case as a situation where the public (*rectius*, who appoints the committee) observes only the final decision.

We now define optimality in our context.

Definition 1 (Optimality) *A generic player m's strategy s_m is individually optimal (s^*_m) when it maximizes the player m's own utility function, $u_m(s_m \mid s_{-m})$:*

$$s^*_m : \arg\max u_m(s_m \mid s_{-m})$$

A strategy profile $s = (s_i, s_j, s_z)$ is socially optimal (s^*) when it maximizes the probability that the committee takes the right decision (or, equivalently, the expected value of the election), given the available signals.

$$s^* : \arg\max E(v_m)$$

We need definition 1 to stress that optimality is here defined as a constrained concept. Due to lack of communication, full information aggregation is not always possible. This means that for members with truth purpose, individual and social optimum coincide. We will refer to our former constrained concept simply as *optimality* and to the latter as *full information optimality.* We say that the concept is constrained as with full information the probability of taking the right decision is always higher[8].

The probability of taking the right decision in the simultaneous voting game with truth purpose is therefore the only benchmark case we will refer to through the paper, when discussing the efficiency implications of different committee compositions or voting mechanisms.

At this point, it is worth noting the difference between herding literature and this case. Herd behaviour and cascades are a consequence of optimizing behavior of agents, whose utility is independent from followers' actions. They find it optimal to ignore their private information as they are not concerned with its effects on subsequent players.

Now, is this still true in voting games with truth purpose? Consider a committee of three members as described above and where votes occur sequentially. The first and third player play the following strategy: play the signal if observed, randomize if not. Then, assume the first chooses "yes". If the second player is uninformed, he assigns a probability $\alpha + \frac{1}{2}(1-\alpha) > \frac{1}{2}$ to "yes" being the true state. Indeed, he thinks that the first player voted "yes" either because he was informed (probability α) or because he randomized as uninformed (probability $1-\alpha$). We

[7] See section 6 for preliminary results when $\beta < 1$.

[8] See footnote 11.

may think that the second player should follow, as the probability of being right by imitating (left-hand side of the previous inequality) is higher than by randomizing (right-hand side). Is this our conclusion? If so, we would have a typical result of herd behavior. Well, this is not necessarily the case. He may optimize as well by doing exactly the opposite! In this way, the responsibility of the right decision is given to the third player, whose probability of being right is exactly the same as the first one: $\alpha + \frac{1}{2}(1 - \alpha)$.

Herding is not a straightforward conclusion in this sequential voting game. The reason is that a player's utility is no longer independent of the followers' actions.

Bearing in mind this example, we can now analyze two types of voting games: a simultaneous and a sequential one.

3.1 The simultaneous voting game

Two signals about the true state of the world are possible: $\omega_m = \{H; L\}$, and each player observes a signal with known and common probability $\alpha \in [\frac{1}{2}, 1]$[9]. If the true value of the alternative is $1(-1)$, then the only possible signal is $H(L)$. With a little abuse of notation, we say that the information set of the generic player m is simply $\Omega_m = \{\omega_m\}$ when he is informed, as he perfectly knows the true state of the world, which is equal to the signal he observes; on the contrary, when he is uninformed his information set is $\Omega_m = \{H, L\}$.

We also assume that players condition their strategy on being pivotal. Of course, any strategy is optimal when the player is not pivotal. So, we concentrate on weakly dominant strategies.

Proposition 1 introduces the first (and only) symmetric equilibrium of the game.

Proposition 1 *The simultaneous voting game with truth purpose has only one symmetric equilibrium. In this equilibrium, the generic player m's strategy s_m is:*

$$s_m = \left\{ \begin{array}{l} \textit{play } \omega_m \textit{ if } \Omega_m = \{\omega_m\}; \\ \textit{randomize } 50:50 \textit{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

Individual optimality requires that every player plays his signal if observed, as this is right with probability $\beta = 1$. It is not straightforward what happens if the signal is not observed. In the symmetric equilibrium, the uninformed player i is indifferent between accepting or rejecting the alternative. He also knows that:

$$s_{-i} = \left\{ \begin{array}{l} \text{play } \omega_{-i} \text{ if } \Omega_{-i} = \{\omega_{-i}\}; \\ \text{randomize } 50:50 \text{ if } \Omega_{-i} = \{H, L\} \end{array} \right\}, \text{ where } -i = j, z$$

As $\alpha \geq \frac{1}{2}$, i knows that it is very likely that at least one of the other two players observed a signal, so he should follow him. Unfortunately, this is not possible in a simultaneous game. As

[9] A model with three signals, where in each state the correct signal is observed with probability α and an uninformative one with probability $1 - \alpha$, is isomorphic to this one.

i is conditioning his strategy on being pivotal, he assumes that j and z played the opposite. Given their strategy, i is indifferent between following j or z. The best i can do is randomizing as well.

Nevertheless, this is not necessarily the only and the best possible equilibrium strategy profile. In appendix, we prove that there are several better equilibria if we allow for asymmetric strategies.

Consider the following strategy:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\}$$

that is, playing the signal if observed and playing "yes" with some probability if not. As we argued before, the first part of the strategy must be optimal, as the signal is always correct. As regards the second, we must solve for the utility maximizing probabilities. Before introducing and discussing proposition 2, which characterizes the set of equilibria of this game, we need definition 2.

Definition 2 (Compensation) *Two players are compensating each other when the following two conditions are satisfied:*

- *they are both uninformed;*
- *they play "yes" with probabilities whose sum is equal to 1. When these probabilities take extreme values (i.e., 0 or 1), then their actions involve "compensation in pure strategies".*

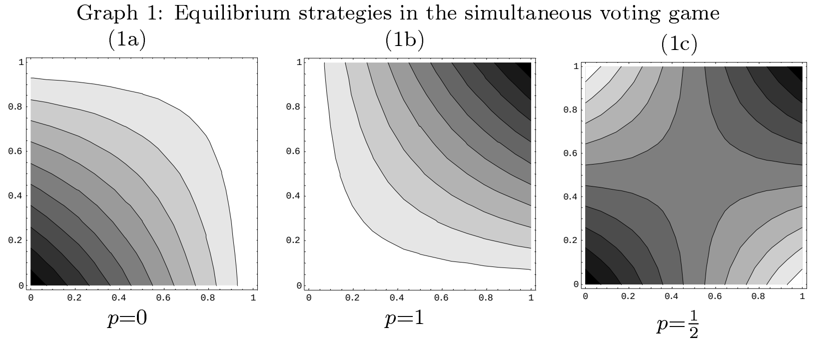
Proposition 2 *The simultaneous voting game with truth purpose has a continuum of equilibria in weakly dominant strategies. In any of the equilibrium strategy profile, every member plays according to the signal, if any, and plays "yes" with the following probabilities if not:*

$$\begin{aligned} \{p, q, r &\in [0,1]^3 : q + r = 1, qr = 0\}, \\ \{p, q, r &\in [0,1]^3 : p + q = 1, pq = 0\}, \\ \{p, q, r &\in [0,1]^3 : p + r = 1, pr = 0\}, \\ \{p, q, r &\in [0,1]^3 : p = q = r = \frac{1}{2}\}. \end{aligned}$$

The socially optimal equilibrium strategies are asymmetric ones and involve compensation in pure strategies.

We have already discussed the logic behind the internal solution. Using definition 2, we can state that there is only one equilibrium with compensation in mixed strategies and it does not maximize social welfare as we have defined it. The corner solutions are equilibria with compensation in pure strategies. In graph 1, we show the ranges of possible equilibria for fixed values of p. On the horizontal axis we measure q and on the vertical one r.

Graph 1: Equilibrium strategies in the simultaneous voting game



In the contour graphs, the lighter the colour the higher the value of the represented function (in this case, the committee's expected value). As we have shown, for the internal value $p = \frac{1}{2}$, an equilibrium is reached when $q = r = \frac{1}{2}$. Nevertheless, it evident that this is only a saddle point and so it is not optimal. For extreme values of p (graphs 1a and 1b), the maximum is reached by fixing either q or r in order to compensate p, leaving the other probability free to change in the set $[0, 1]$. For every internal value of p (i.e., graph 1c), q and r must compensate each other at extreme values (i.e., $q = 0; r = 1$) to maximize the social welfare.

Given $\alpha = \frac{1}{2}$, the maximum possible expected value of the election is $E(v_m) = \frac{3}{8}$. In graph 1c, this level is reached in all the combinations of q and r represented by a white area, that is $q = 0, r = 1$ and $q = 1, r = 0$.[10].

As long as individuals are uninformed, at least two of the players should compensate each other in order to minimize their influence on the outcome of the game. This compensation reaches the social optimum when it is played in pure strategies, that is, corner solutions dominate the symmetric one. For $p = q = r = \alpha = \frac{1}{2}$, the expected value of the election is indeed only $\frac{11}{32} < \frac{3}{8}$.

The difference between full information aggregation and our constrained concept of optimality can be effectively addressed here. It is probably better understood with an example. Suppose players i, j and z are playing the (constrained) optimal equilibrium strategy profile $s^* = \{s_i^*, s_j^*, s_z^*\}$, where:

$$s_i^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "no" if uninformed (i.e., } p = 0) \end{array} \right\};$$

$$s_j^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "yes" if uninformed (i.e., } q = 1) \end{array} \right\};$$

$$s_z^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "no" if uninformed (i.e., } r = 0) \end{array} \right\}.$$

[10] Unfortunately, some confusion may arise as the borders of the graphs are white as well.

Conditional on L being the true state, the decision is always correct. Anyway, conditional on H being the true state, the final decision may be wrong in two cases: either no one is informed or only j is informed. Both the cases has a probability of $\frac{1}{8}$ when $\alpha = \frac{1}{2}$ and the expected value of the election, as already stated, is $\frac{3}{8}$. If we could fully aggregate information, then the committee could be wrong only when no one was informed, as j could share his signal with the others. With full information aggregation, the expected value of the election would be $\frac{7}{16} > \frac{3}{8}$[11].

We believe that this compensation result has some similarities to the one in Feddersen and Pesendorfer (1996). In that case, the optimal strategy for uninformed members was to abstain. In our model, abstention is not possible but can be mimicked by compensation. The problem with compensation is that it may prevent full aggregation of information, as an uninformed player, when compensating, might in fact cancel out the vote of an informed player.

3.2 The sequential voting game

The model we develop in this subsection is slightly more complex. Followers (can) have different actions according to different histories of the game. This also means that the information set of the generic player m, Ω_m, contains the signal, if observed, and the voting history. The voting history is the collection of votes, or decisions d, expressed by members who have already played. Given an order of vote (e.g., i, j, z), we characterize the players' strategies as follows:

$$s_i : \left\{ \begin{array}{c} \text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\} \end{array} \right\},$$

as the first voter is in the same situation as in the simultaneous game;

$$s_j : \left\{ \begin{array}{c} \text{play } \omega_j \text{ if } \Omega_j = \{\omega_j, d_i\}; \\ \text{if } \Omega_j = \{H, L; d_i\}: \\ \text{play "yes" with probability } q_Y \text{ when } d_i = \text{"yes"} \\ \text{play "yes" with probability } q_N \text{ when } d_i = \text{"no"} \end{array} \right\},$$

that is, j votes according to his signal if observed, as this is always correct, independently on the vote casted by i. On the contrary, when j does not observe a signal, he conditions his vote

[11] See Appendix for a proof and a derivation of the expected value of the election with full information aggregation.

on i's decision. Finally:

$$s_z : \left\{ \begin{array}{c} \text{play } \omega_z \text{ if } \Omega_z = \{\omega_z, d_i, d_j\} \\ \text{if } \Omega_z = \{H, L; d_i, d_j\}\text{:} \\ \text{play "yes" with probability } r_{YY} \text{ when } d_i = d_j = \text{"yes"} \\ \text{play "yes" with probability } r_{YN} \text{ when } d_i = \text{"yes" and } d_j = \text{"no"} \\ \text{play "yes" with probability } r_{NY} \text{ when } d_i = \text{"no" and } d_j = \text{"yes"} \\ \text{play "yes" with probability } r_{NN} \text{ when } d_i = d_j = \text{"no"} \end{array} \right\},$$

which means that z always follows his signal, if any, and conditions his vote to the voting history when uninformed. Each player's strategy is no longer independent from previous voters' ones. From Dekel and Piccione (2000), we know that any symmetric equilibrium in the simultaneous voting game is an equilibrium in the sequential one. Hence, independently of history, playing "yes" with probability $\frac{1}{2}$ when uninformed is still an optimal strategy for each player. But the best equilibria we are dealing with are asymmetric ones. We are therefore interested in understanding whether they can be replicated in this different game and, above all, whether a better equilibrium strategy profile can be found.

The game is solved as before but for the presence of more variables (probabilities). The analysis is simplified if we note that r_{YY} and r_{NN} are irrelevant, as a decision has already been taken.

The results are presented in proposition 3, which is then discussed (and proved in appendix).

Proposition 3 *Every equilibrium strategy profile of the simultaneous game is also an equilibrium in the sequential one, but more equilibria are sustainable in the latter. These additional equilibria (in weakly dominant strategies) still involve compensation in pure strategies and are optimal. They are characterized by the following properties: when informed, members vote according to their signal; when uninformed, they play "yes" with probabilities satisfying the following conditions:*

$$\left\{ \begin{array}{c} r_{YN} = r_{NY} = 1 - p; \\ p r_{YN} = 0; \\ \forall q_Y, q_N \in [0,1] \end{array} \right\} \; or \left\{ \begin{array}{c} q_N = q_Y = 1 - p; \\ p q_Y = 0; \\ \forall r_{YN}, r_{NY} \in [0,1] \end{array} \right\}$$

in the first type of equilibrium profile, and:

$$\left\{ \begin{array}{c} r_{YN} = 1 - q_Y \neq r_{NY} = 1 - q_N; \\ r_{YN} q_Y = 0; r_{NY} q_N = 0; \\ \forall p \in [0,1] \end{array} \right\}$$

in the second one. The maximum expected value of the election is the same as in the simultaneous game.

Both the new equilibria are corner solutions. The first new equilibrium is a straightforward extension of the previous one, as once two players compensate each other, it does not make any difference whether the third is discriminating (as in the sequential game) or not (simultaneous game).

The last equilibrium profile is the most different one and all the players differentiate their actions. The intuition behind this last case is the following. Suppose $q_Y = 1$: this means that, after the first player i votes "yes", the second player j always plays "yes" if uninformed. Then, the third player z knows that, if he observes j voting "no", then it must be case that j is informed and "no" is the correct choice. The optimal answer for z is playing $r_{YN} = 0$. When $q_Y = 0$, then the optimal reply is $r_{YN} = 1$. Now z can not infer from observing j voting "no" whether he is informed or not: so the best he can do is compensating when uninformed. Of course, compensation is socially bad if j is actually informed. It is exactly the possibility of this information destruction which precludes full aggregation of information. A similar argument explains the relation between r_{NY} and q_N.

The range of equilibria is wider than in the simultaneous game. Compensation is still a condition but this may occur through in more complex ways, as, for instance, compensation conditional on the voting history involved in the last equilibrium strategy profile we discussed.

Quite surprisingly, the possibility of discriminating the action according to the history does not provide any improvement from a social welfare point of view. This means that the sequential structure does not add anything to the information aggregation process.

The only gain is that coordination appears to be more credible when players vote sequentially.

4 Voting games with reputational purpose

Suppose now the members in the same committee are only interested in appearing informed. They do not care about the final decision of the committee but want to show that they are right, even when they are uninformed. In this context, "being right" means voting according to the true state of the world. That is, voting "yes" when $v_m = 1$ and voting "no" when $v_m = -1$. The opposite is true for "being wrong".

In this game, voters clearly care about their own reputation (*reputational purpose*). A tension with the social optimal behavior might therefore emerge.

The rest of the model does not change: the players still face two states of the world and can observe one of the two possible signals with probability $\alpha \in [\frac{1}{2}, 1]$.

The new utility function of the generic member m is the following

$$u_m = \begin{cases} 1 & \text{when } m \text{ is right} \\ 0 & \text{when } m \text{ is wrong} \end{cases}$$

In this section we solve again a simultaneous and a sequential game, showing how the equilibrium strategies depend on the agents' utility functions.

4.1 The simultaneous voting game

A player's strategy no longer needs to be conditional on the fact that he is pivotal. Even though he knows he cannot influence the committee's final decision, a player is still maximizing his expected utility function.

Suppose m plays the usual generic strategy:

$$s_m : \left\{ \begin{array}{c} \text{play } \omega_m \text{ if } \Omega_m = \{\omega_m\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

that is, playing the signal if observed and playing "yes" with some probability if not. Then, it is straightforward to work out that m's expected utility is:

$$E(u_m) = \frac{1+\alpha}{2}$$

which is independent on the probability p. So, proposition 4 follows.

Proposition 4 *Any p is utility maximizing for the generic player m. His optimal strategy s_m^* is:*

$$s_m^* : \left\{ \begin{array}{c} \textit{play } \omega_m \textit{ if } \Omega_m = \{\omega_m\}; \\ \textit{play "yes" with any probability } p \in [0,1] \textit{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

The solutions found in the game of section 3.1 are only particular cases of these ones. As in this game any probability p, q and r is an equilibrium strategy, the social welfare is not necessarily maximized. We recall that social optimality requires compensation in pure strategies.

4.2 The sequential voting game

In the sequential version of this game, the first player is in the same situation as in the previous subsection. We still assume that members vote in this order: i, j, z.

So, player i plays "yes" with any $p \in [0,1]$ and is right with probability $\frac{1+\alpha}{2}$.

When uninformed, the second player should play as the first one, as $\frac{1+\alpha}{2} > \frac{1}{2}$. The left hand side of the inequality is the probability that the i is right, whereas the right hand side is the probability that j is right by randomizing. If informed, then j should play according to the signal.

The third player z faces two possible cases: i and j played the same or they played the opposite. As long as z has a signal, the difference does not matter, as he knows his signal is always correct and he plays it. But if he is uninformed, then he should follow j. Indeed, if j and i played the same, this is straightforward, as z is observing two voters doing the same. But if they did not, then the only reason why j played the opposite is that he was informed, so he must be followed.

According to our discussion, proposition 5 holds.

Proposition 5 *The individually optimal strategies of this game are:*

$$s_i^* : \left\{ \begin{array}{c} \textit{play } \omega_i \textit{ if } \Omega_i = \{\omega_i\}; \\ \textit{play "yes" with any probability } p \in [0,1] \textit{ if } \Omega_i = \{H, L\} \end{array} \right\}$$

$$s_{j(z)}^* : \left\{ \begin{array}{c} \textit{play } \omega_{j(z)} \textit{ if } \Omega_{j(z)} = \{\omega_{j(z)}; d_{i(j)}\}; \\ \textit{play as the previous player if } \Omega_{j(z)} = \{H, L; d_{i(j)}\} \end{array} \right\}$$

This equilibrium is clearly different from the ones above. As soon as an individual is uninformed, imitation arises. We cannot really talk of herding, as no information is lost in the process.

In the next subsection, we compare all these results and comment our findings.

4.3 Comments

In table 1, we report equilibrium strategies of the games we analyzed. In all of them, there is a common action, that is, "play the signal if any". For this reason, we focus only on the actions a player should take in equilibrium when he is uninformed. The table is bi-dimensional. On the horizontal line we discriminate about the utility function of the agents: the first column represents agents caring about the correct choice by the committee. On the vertical dimension, we consider the different voting games: the simultaneous and the sequential one.

In both the voting games with a common truth interest, herding is not a consequence. Actually, all the equilibria involve compensating behavior, i.e., destruction of possible cascades. The idea behind this result is that no uninformed member wishes to influence too much the final outcome of the game. As abstention is not possible, the only way to reach this result is through some coordination. As we pointed out, there are several equilibria in truth purpose games and the best ones involve pure strategy for two of the players.

Table 1: Equilibrium strategies

	Truth	Reputation
Simultaneous	(game A) Compensation	(game C) Any
Sequential	(game B) Compensation	(game D) Imitation

On the contrary, in voting games with reputational concerns, a difference emerges between the sequential and the simultaneous structure. The former weakly dominates the latter in social welfare terms.

The expected value of the election in game D is given by the function:

$$E(v_m) = \frac{1}{2} \left[Y(\cdot \mid v_m = 1) - Y(\cdot \mid v_m = -1) \right],$$

where the function $Y(\cdot)$ represents the probability that "yes" wins when either $v_m = 1$ or $v_m = -1$. In particular:

$$Y(p \mid v_m = 1) = [\alpha + (1-\alpha)p] + (1-\alpha)(1-p)\alpha$$
$$Y(p \mid v_m = -1) = (1-\alpha)^3 p + (1-\alpha)^2 \alpha p$$

When $v_m = 1$, the committee votes "yes" whenever the first player votes "yes" or when he does not but the second player is informed. When $v_m = -1$, the committee votes "yes" when nobody (or only the third player) is informed and the first votes the first "yes". For $\alpha = \frac{1}{2}$, we have:

$$E(v_m) = \frac{3}{8}, \quad \forall p \in [0,1]$$

which is the maximum value of the function for this particular value of α.

Every equilibrium in game D is socially optimal whereas only a few of the multiple equilibria in game C are. When $\beta = 1$ and players have reputational purposes, then imitation arises in the sequential game and it is an efficient behavior. The existence of equilibria which are all optimal is quite surprising as we are dealing with a game where social welfare is not the objective of the players. But this is not exact: actually, social welfare is not *necessarily* the objective of the players. When it is compatible with their individual aims, then it can be a result, as in this case.

5 Voting games with heterogeneous players

So far, we have focused on homogenous players: they all have the same ability (i.e., the probability that their signal is correct is the same) and the same preferences. We now introduce heterogeneity. In this context, heterogeneity can have two dimensions: on the one hand, it may concern the ability of the players; on the other hand, it may regard their objective functions. The former type of heterogeneity has been already widely analyzed (see, for example, Ottaviani and Sørensen, 2001). Quite surprisingly, no paper deals with the possibility of different preferences in sequential voting games (Feddersen and Pesendorfer does, but only in simultaneous games). We believe that the order of vote is relevant in the sequential game. This is one of the reasons for deciding to concentrate on this point.

The other reason is the fact that, in real life, we may expect to observe committees where members do not share exactly the same preferences. One possible example is the Italian Constitutional Court. According to the Italian Constitution (art. 135 catch 1), the 15 members of this court are experienced judges, one third of whom is appointed by the President of the Republic, one third by high levels of the magistrature and the remaining one third by the Parliament. It is not so strange to imagine that each of them could have, in principle, different preferences. We could suppose that the presidential ones are totally independent; the technical ones mainly status seeker and, finally, the Parliament ones are policy biased, in order to favour the party which appointed them. This assumption about types is very strong. We do not actually believe that a biased member is necessarily and exclusively concerned with satisfying his political side, neither that the status seeker has no interest in social welfare, nor that the truth seeker has no political opinions. A more accurate model would weight each of these three components for each of the members, but we reckon that this additional complexity would not add relevant insights to the current model.

Before introducing our main model, we devote a subsection to the study of this Italian institution, trying to understand historical and political reasons of his peculiar composition. Our model will then provide game theoretic foundations to it.

5.1 The Italian Constitutional Court

The Constitutional Court has been introduced in the Italian legal and political system by the Constitution of the Republic. This document was approved at the end of 1947 by the Constitutional Assembly, on the basis of a proposal made by a subcommittee, called the Committee of the 75s. The Court has been existing only since 1955 and its first sentence dates back to 1956.

Articles 134-137 of the Constitution are devoted to its organization; nevertheless, we are only interested in two of them. According to art. 134, the Court is in charge of solving:

- questions about the constitutionality of national and regional laws;
- conflicts among bodies of the State, among State and regions and among regions themselves;
- accusations against the President of the Republic.

In addition, the Court decides whether requests of referenda are admissible (constitutional law no. 1/53, art. 2). The duties of the Court are extremely important, as they strongly influenced the debate about its composition[12], which is our main interest here.

Indeed, two main opinions emerged in the Constitutional Assembly: for some members, the Court was mainly a political body and therefore needed to be controlled by a political organization. These people thought that the Parliament should have been in charge of appointing all its members and that membership itself should end after new political elections. For other members, the Court was basically a legal body; in particular, it should be independent of the political power exactly for his duty of "judge" of the Parliament itself, where the judgment is about the constitutional correctness of its decisions.

The proposal presented by the 75s (art. 127 of the proposal) stated that half of the Court was composed by judges, a quarter by lawyers and professors and a quarter by citizens. All of them were appointed by the Parliament, who could choose from a list of names (three times the required number of members), and were in office for nine years.

[12]The source for this debate is Falzone, Palermo and Cosentino (1976).

In the Assembly, the latter position was dominant and some catches were changed. In particular, according to art. 135 of the approved Constitution, the Court is composed by fifteen members, chosen only among judges (even if retired), lawyers with more than 20 years of experience and professors. Among these fifteen members:

- five are appointed by the President of the Republic;
- five by the Parliament;
- five by the highest judges.

As we can see, the political control has become milder, also considering the fact that the office of a member lasts nine years and the one of an MP no more than five (unless he is reelected).

The final decision of the Assembly was in favour of the "jurisdictional" view of the Court. Despite this and in line with the strong compromise behavior of its members, some political control has been guaranteed to the Parliament. The necessity of a balance among political and jurisdictional duties of the court eventually emerged.

Whether this choice is inefficient or not from a social welfare point of view is the question we wish to answer in the rest of this work.

5.2 A model of voting games with heterogeneous players

Trying to mirror the composition of the Italian "Corte Costituzionale", we study the behavior of a committee with heterogeneous players. For simplicity, we still assume $\beta = 1$ and the existence of the following three members. The first is a truth seeker one, a social optimizer chosen by the president of the Republic; the second is a status seeker one, mainly concerned about his reputation among other judges, who elected him. Finally, the third is a policy biased one, interested in satisfying the party who supported him. We label them as I (independent, with truth purpose), R (status seeker, with reputation purpose) and P (policy biased) respectively. The first two players' objective functions are already known:

$$u_I : E(v)$$

that is, the independent member maximizes the expected (common) value of the election; and:

$$u_R = \begin{cases} 1 & \text{when } R \text{ takes the right decision} \\ 0 & \text{when } R \text{ takes the wrong decision} \end{cases}$$

Additional specifications about P are necessary. His bias b can take two values: $b : \{Y, N\}$. We call him *conservative* if he is biased towards the *status quo* option ($b_P = N$), and *reformist* when he prefers the alternative ($b_P = Y$). He always supports his political opinion, systematically ignoring his private information. His utility is a function $u(o)$ of the final outcome of the game, o. When the alternative passes ("yes" wins), then $o = Y$; when the alternative looses ("no" wins), then we have $o = N$. Accordingly, $u(o)$ can assume two values. Suppose P is a reformist; then $u(Y) = 1$ and $u(N) = 0$. On the contrary, if he is a conservative, it is true that $u(Y) = 0$ and $u(N) = 1$.To summarize:

$$u_P : \begin{cases} 1 & \text{when } o = b \\ 0 & \text{when } o \neq b \end{cases}$$

Preferences are common knowledge among the players and we assume P to be a reformist[13].

It is worth stressing, especially if the reader thinks in terms of the example provided, that uninformativeness in this case should be interpreted as noisiness of available information. For instance, a judge is said to be uninformed when there are strong arguments supporting both the possibilities (rejecting or accepting).

The remaining assumptions of the model remain the same.

5.2.1 The simultaneous voting game

We first present the main result of the game and then we illustrate it.

Proposition 6 *In the simultaneous voting game with three heterogeneous players, multiple equilibria (in weakly dominant strategies) arise, including a unique socially optimal equilibrium.*

We now prove the above proposition. Suppose P is a reformist. Then he has a simple dominant strategy, which is as follows:

$$s_P^* : \{\text{always play "yes"}\},$$

as it maximizes his payoff function.

As for R, we know from previous discussion that in the simultaneous voting game his choice is independent from the other players' ones. So his utility maximizing strategy is still:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if } \Omega_R = \{\omega_R\}; \\ \text{play "yes" with any probability } q \in [0,1] \text{ if } \Omega_R = \{H, L\} \end{array} \right\}$$

Finally, the independent player's equilibrium strategy must look like the following:

$$s_I : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if } \Omega_I = \{\omega_I\}; \\ \text{play "yes" with probability } p \text{ if if } \Omega_I = \{H, L\} \end{array} \right\},$$

[13] In the thesis, we also allow for weaker biases, but the results do not substantially change.

as the signal is always correct. The optimal p can be easily found: player's I utility function is given by:

$$u_I : E(v) = \frac{1}{2}\left[Y(p,q \mid v=1) - Y(p,q \mid v=-1)\right]$$

where $Y(\cdot)$ is the probability that "yes" wins. As P's strategy is unique and certain (P always votes "yes"), u_I only depends upon q and p. In particular, the expected value of the election is negatively related to both p and q. When $v=1$, the committee expresses a positive vote when at least I or R is informed or, if uninformed, at least one of them votes "yes":

$$\begin{aligned} Y(p,q \quad | \quad & v=1): \\ & [\alpha + p(1-\alpha)][\alpha + q(1-\alpha)] + \\ & [\alpha + p(1-\alpha)(1-q)(1-\alpha) + \\ & (1-p)(1-\alpha)[\alpha + q(1-\alpha)] \end{aligned}$$

When $v=-1$, the committee expresses a positive vote when at least I or R is uninformed and votes "yes":

$$\begin{aligned} Y(p,q \quad | \quad & v=-1): \\ & (1-\alpha)^2 pq + \\ & (1-\alpha)p[(\alpha + (1-q)(1-\alpha)] + \\ & (1-\alpha)q[(\alpha + (1-p)(1-\alpha)] \end{aligned}$$

We obtain:

$$u_I : E(v) = \frac{1}{2}\left[\alpha p(\alpha-1) + \alpha q(\alpha-1) + \alpha(2-\alpha)\right]$$

The utility maximizing level of p must be 0. As I cannot choose q, an equilibrium does not need to be optimal. There is anyway one socially optimal equilibrium which is reached when R plays $q=0$ as well.

In other words, given P's dominant strategy, I's best reply is compensation in pure strategies, as in the truth purpose game. R's strategy is independent from the other's ones, so in general we should not expect a social optimal equilibrium profile to be selected.

The result is anyway striking in the sense that optimality is note ruled out, even if one player (namely, P) is completely ignoring his own private information. It is obtained only when there is a high compensation from R's and I's sides, that is, when they both play the same strategy and this strategy tries to offset P's strong bias. It is relatively easy to understand why this result is correct. Assume that the true state is H: then no information is lost by P. In particular, his action in this case (but not his strategy) recalls that one of the player who always votes "yes" when uninformed. To that action, I's best response is exactly the strategy we outlined here and, from a social welfare point of view, R can play any q. This profile has been already shown to be one of the optimal ones in section 3.1. Suppose now the true state is L: then, the right decision is always taken! If R and I are informed, then they take the correct decision by definition. But if they are not, then they both play "no", which is exactly the correct choice to take.

Recalling the example we provided in subsection 3.1, we can show the similarities between this game and the one with three truth seeker players. In the latter case, the committee was possibly wrong in two cases: conditional on H being the true state, either no player was informed or only j was informed. But this is exactly what happens here, except that we have player P instead of player j. So, for both the equilibrium strategy profiles, the probability that the final decision is wrong is the same.

Despite multiplicity of suboptimal equilibria, we know that there is a unique socially optimal equilibrium. In particular, both P and R plays a dominant strategy, to which I's best response is unique. Multiplicity only arises due to the fact that R can choose any level of $q \in [0, 1]$.

Introducing heterogeneity is therefore relevant for the characterization of the equilibria of the model. Even though equilibria are still multiple, their range is dramatically reduced.

5.2.2 The sequential voting game

In the sequential voting game, the order of vote turns out to be relevant. We link this result to the presence of heterogeneous players as, so far, no optimal order has ever emerged. In the following proposition, we present the main result of the game and then we illustrate it.

Proposition 7 *The sequential voting game with three heterogeneous players, has a unique socially optimal equilibrium when I votes before R. For other sequences, there are multiple equilibria, only one of which is socially optimal.*

To prove this proposition, we assume P to be a reformist. As his action is totally uninformative, his position is irrelevant. His strategy is:

$$s_P^* : \{\text{always play “yes”}\}$$

As regards R and I, we have two possibilities. If R votes before I, he has no additional information on which to base his action, so he behaves as in the simultaneous voting game and therefore his best strategy is still:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if any;} \\ \text{play “yes” with any probability } q \in [0,1] \text{ otherwise} \end{array} \right\}$$

As we know, the best I can reply at this point is to (partially) compensate P's behavior and so he plays:

$$s_I^* : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if any;} \\ \text{play “yes” with probability } p = 0 \text{ otherwise} \end{array} \right\},$$

like in the simultaneous voting game of the previous subsection.

We have already shown in the previous subsection that the first best is only achieved when $q = 0$. In general, there is a continuum of equilibria which are not optimal.

On the contrary, when I votes first, he still finds it optimal to play his usual strategy:

$$s_I^* : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if any;} \\ \text{play "yes" with probability } p = 0 \text{ otherwise} \end{array} \right\}$$

whereas R can now elicit some information from I's behavior and so he plays:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if any;} \\ \text{follow } I \text{ if } \Omega_R = \{H, L; d_I\} \end{array} \right\}$$

This strategy is optimal for R, as there is a positive probability that I is informed and therefore correct. But this strategy is socially optimal as well, as, when I is informed, then an uninformed R imitates the correct choice. And if I is uninformed as well, then he plays "yes" with probability $p = 0$ and R does the same, playing "yes" with probability $q = 0$. This is exactly the optimal equilibrium profile we found in the simultaneous voting game. Moreover, this equilibrium is clearly unique. So the sequential structure actually works as an "implementation mechanism" for the optimal strategy profile.

6 Possible extensions of the basic models

In this section we informally discuss two natural possible extensions of the basic models introduced in section 3 and 4. The first one regards the accuracy of the signal (the parameter β), while the second one concerns the number of players.

We refer again to the games as presented in table 1, which we reproduce below for simplicity.

Table 1 (revisited)

	Truth	Reputation
Simultaneous	Game A	Game C
Sequential	Game B	Game D

6.1 The effect of noisy information

We now assume $\beta \in (\frac{1}{2}, 1]$, that is, the signal observed by the players is not necessarily correct. As we are dealing with "experts", we find it reasonable to assume that the common β should be bigger than $\frac{1}{2}$. This choice is also without loss of generality, as, if $\beta < \frac{1}{2}$, then the utility maximizing behavior would be just the opposite. With a positive probability of wrong signals, we expect herding to arise, at least in the reputational sequential voting game.

The main result of the subsection is that noisy signals are either ineffective on the players' equilibrium strategies or, if they influence them, they are not source of inefficiency[14].

Game A: simultaneous voting with truth purpose Consider the usual strategy for player $i(j,z)$:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\}$$

The first part of the strategy is optimal as a signal, despite its noisiness, unambiguously shifts the player's prior (remember that $\beta > \frac{1}{2}$). We then solve for the utility maximizing p, q and r as before. We obtain that these probabilities are independent on the parameters α and β. Therefore equilibrium strategies are unaffected. The fact is that these two parameters influence only the expected value of election, which is indeed lower for lower values of α and β.

Before analyzing game B, we focus on games with reputational purpose, as some of the results will be useful for the remaining case.

Game C: simultaneous voting with reputational purpose We already know that, in this simultaneous game, a player's optimal strategy is independent from the others' ones and hence it is a dominant strategy. Consider the generic player i, playing the following strategy:

$$s_i : \left\{ \begin{array}{c} \text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\} \end{array} \right\}$$

Again, the first part of the strategy is optimal, as a signal shifts the player's prior (equal to $\frac{1}{2}$) about the true state of the world. We focus on p. It is straightforward to show that, again, p is independent on α and β. In particular, the expected utility of the election is given by:

$$E(u_i) = \frac{1 + 2\alpha\beta - \alpha}{2}$$

which is non negative for any $\alpha \in [\frac{1}{2}, 1]$ and $\beta \in (\frac{1}{2}, 1]$.

Game D: sequential voting with reputational purpose We begin with a definition, which clarifies what we mean by "following" and "herding" in the rest of the section. We recall that, with reputational purposes, the best choice is the one which is most likely to be correct.

Definition 3 *(Herding)We say that the generic player "j" is "following" the generic player "i" when this is his best choice according to his (lack of) information. In line with the literature, we say that "j" is "herding" when he disregards his own private signal.*

[14] We refer the interested readers to the thesis for formal proofs and only present some intuitions.

We must stress that a player always makes the best use of his available information. If he does not have any signal, then he finds it optimal to imitate who might have observed some signal. If he does have a signal, then he updates his prior according to all his information and decides whether to follow his signal or herd.

For simplicity, we assume that the three players are voting in this order: i, j, z. Simply by using Bayes' rule, it is easy to show that herding might arise only from the third player (z) onwards. In particular, herding requires four main conditions. First of all, the first two players (i and j) must vote for the same option. Secondly, z must be informed. Otherwise, he is not giving up any information and simply following (from definition 3). Then, his signal must contrast with the choice of i and j. Finally, the following condition about p, α and β must hold[15]:

Condition 1 *Herding condition:*

$$p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta),$$

where α and β are known parameters, respectively about the probability of observing a signal and his accuracy, and p is the probability that, when uninformed, i plays "yes".

This herding condition means that when α is very low ($\alpha \simeq \frac{1}{2}$), it is unlikely that both i and j observe a signal: for many values of the probability p, z trusts his signal and does not herd. When α grows, it's the term β which makes the difference. The higher the β, the lower the p sustaining herding: z has stronger confidence in his own signal. Finally, when $\alpha = 1$, herding is a straightforward conclusion for any β and p[16].

The conclusion of this discussion is that, in this case (with three players), herding is not inefficient from a social point of view, as it emerges only when the final decision has already been taken (z is not pivotal).

Game B: sequential voting with truth purpose This game mixes characteristics of the previous sequential one and of game A. So, we can easily conclude that, if herding arises, it is from the third player and it is therefore inefficient.

6.1.1 Comments

We start from a comparison between the general case when $\beta \in (\frac{1}{2}, 1]$ and the special one when $\beta = 1$. We can observe that nothing changes for players with truth purpose in a simultaneous game. The expected value is lower for smaller β, but the equilibrium strategies are the

[15] This condition is true when i and j vote "yes" and z observes the L signal. See the next footnote for the other case.

[16] In the symmetric case, where both i and j vote "no" but z has an H signal, the herding condition is $(1-p) < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$.

same. When players are status seekers, the situation is more similar to a typical model of herd behavior. For different values of α, β and p, herding may arise. The third player finds it profitable to give up his private information about the true state of the world when the probability that other players observed a signal is very high and the accuracy of this information is not very sharp.

Finally, the possibility of maximizing the social welfare is not dramatically reduced when we relax our assumption about β. This is clearly true for games A, B and C, as the equilibrium strategies do not change. In game D, herding may arise and it is inefficient from a social point of view. Though, in this particular case, a committee with three members can not really do worse than in the previous case ($\beta = 1$), as herding only arises when the voter is no longer pivotal (i.e., he cannot influence the final outcome of the game).

6.2 Changing the number of players

A further natural extension of our models is the analysis of committees composed of a different number of members.

First of all, we note that, as in game C the equilibrium strategy of the players is independent from the others, the number of players is not an issue. We provide some intuitions for the other cases, when $n = 2, 4, 5$, but leave their analytical development to future research.

Game A: simultaneous voting with truth purpose When $n = 2$ or, in general, with an even number of voters, we need a tie-breaking assumption. We assume that the alternative passes when it receives $\frac{n}{2} + 1$ votes, in other words, the *status quo* wins in case of parity. Given two players i and j, it is easy to work out their equilibrium strategy. Following our previous notation, we call p and q respectively the probability that i and j play "yes" when uninformed. Then the optimal probabilities are $p = q = 1$. Compensation between players does not emerge when $n = 2$. But we must stress the existence of a strong bias, that is, the tie-breaking rule favouring the *status quo*[17].

When $n = 5$, the optimal strategy requires four members compensating each other when uninformed, with the last one free to play "yes" with any probability. Unfortunately, we do not have a formal proof for this claim. Nevertheless, we can prove that the expected value of the election is higher with 5 players than with 3 when they play the (putative) optimal strategy. This result is in line with previous research stating that the more the players the finer the aggregation of information.

Game B: sequential voting with truth purpose When $n = 2$, it is very easy to work out the optimal equilibrium strategy. Actually, it fully replicates game A. Recall from above the definition of p, q_Y and q_N : with the tie-breaking rule we chose, q_N is irrelevant as a decision has already been taken and we reduce the problem to the one we have just solved above: $p = q_Y = 1$.

[17] With an opposite tie-breking rule, we obtain $p = q = 0$.

As the number of players grows, calculations become more and more difficult. We think the basic result of the 3 players case still holds, that is, the sequential structure can not improve the expected value of the election.

Game D: sequential voting with reputational purpose We know that for $n = 3$, herding may arise when the first two members (i, j) voted the same and the third player (z) has a signal in contrast with their choice. In particular, z herds when condition 1 is satisfied. But from a social point of view, herding is not a problem, as z is never pivotal when it occurs.

When $n = 4$ (and *a fortiori*, for higher values of n), the player after z (say, k) may face a similar situation: he observes i, j, z voting the same but has an opposite signal. Then, he should herd as well. The intuition is that, if condition 1 is enough for a cascade to start after two players, then it must be the case that it is enough to start after three. And we can show that herding always arises (or it continues). On the contrary, when k is informed but does not observe three similar votes, he knows that he is not the only one with that signal and therefore should consider it and play it.

With $n > 4$ players, herding might become dangerous from a social welfare point of view, as pivotal members of the committee are ignoring their own information.

7 Conclusions

As highlighted in the introduction, this paper tries to answer to two main questions. The first one is about the voting behaviour of socially oriented voters, when they are uninformed and cannot abstain. The second is about the rationale for the presence of biased members in committees whose aim is to maximize social welfare.

To do this, we solve and comment different voting games. We first discriminate among games played sequentially and games played simultaneously. Some papers have already discussed how, in the former case, the probability of herding may arise and the information aggregation process of the election may be inefficient. Indeed, by herding, players give up their private information and concur to create informational cascades.

We analyze the particular case of a committee composed of 3 members, which must take a binary decision. For instance, it may decide whether to accept an alternative or reject it and keep the status quo (which has a normalized value of 0). In case of acceptance, the alternative can take two values: either 1 or –1, according to the true state of the world. Abstention is not possible.

When players do not want to share their private information, the information aggregation process of the voting game may be flawed. This is not necessarily true in setting with simultaneous voting and a wide number of players but can be a serious consequence otherwise.

The traditional conclusions of the existing literature about sequential voting appear to be strongly dependent on the objective function of the players. For this reason, we also discriminate according to the members' objective functions. We show that herding still arises if the committee is composed of players with reputation objectives, that is, players who want to maximize their own probability of being right. But this is no longer true when players have a truth purpose, that is, when they want to maximize the social welfare (i.e., the probability that the committee takes the right decision). They might in fact decide to destroy possible informational cascades, playing compensating strategies instead of herding. In particular, optimality requires compensation in pure strategies. Compensation is an optimal behaviour if players cannot abstain. Nevertheless, it prevents full aggregation of available information and for this reason the optimality concept we refer to is defined as "constrained" one.

The main contribution of the paper relies in the analysis of a committee with heterogeneous members. Quite surprisingly, in the existing literature heterogeneity has exclusively regarded the ability of the players (i.e., the noisiness of their signals/private information). In our model, players with the same ability have in fact different preferences. We model our environment following the peculiar composition of the Italian Constitutional Court. According to the Italian Constitution, one-third of its members are appointed by the Parliament, one-third by the President of the Republic and one third by the highest judges. The historical and political reasons for that concern the necessity of a balance among political and jurisdictional power. We argue whether this composition is also efficient in terms of social welfare (probability of taking the right decision). Accordingly, we assume the existence of three members: a truth seeker, a status seeker (with reputation purpose) and a political biased one. The answer is very surprising: even with a political biased player, who constantly ignores his private information and favours his political party, there exists an equilibrium, which is socially optimum. In the simultaneous game, optimality requires some coordination, which may be not necessarily realized. On the contrary, in the sequential game both optimality and uniqueness are reached, simply given a particular order of voting.

8 Appendix: proofs

8.1 Proof of proposition 2

We provide a proof for proposition 2, which nests the proof for proposition 1 as well.

Proof. Consider the following strategy:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\},$$

The first part of the strategy must be optimal, as the signal is always correct. As regards the second, we want to solve for the profit maximizing probabilities. When the actual value is $v_m = 1$ (ex ante possible with probability equal to $\frac{1}{2}$), i plays "yes" with probability $\alpha + (1-\alpha)p$ (similarly for j and z), that is, the probability that he has the correct signal (H) plus the probability of playing "yes" when he has no signal. We define the probability for the committee as a whole to vote "yes" to be a function $Y(p, q, r)$, where the probability of voting "no" is then $1 - Y(p, q, r)$. The expected value when the actual value is 1 is then given by:

$$\begin{aligned} E(v_m|v_m &= 1) = 1\,[Y(p,q,r|v_m = 1)] + 0\,[1 - Y(p,q,r|v_m = 1)] \\ &= Y(p,q,r|v_m = 1) \end{aligned}$$

The alternative passes when it receives at least two votes. The probability that all the members vote "yes" is:

$$[\alpha + (1-\alpha)p]\,[\alpha + (1-\alpha)q]\,[\alpha + (1-\alpha)r] \tag{1}$$

The probability that only two members vote "yes" is:

$$\begin{aligned} &[\alpha + (1-\alpha)p]\,[\alpha + (1-\alpha)q]\,(1-r)(1-\alpha) + \\ &[\alpha + (1-\alpha)p]\,(1-q)(1-\alpha)\,[\alpha + (1-\alpha)r] + \\ &(1-p)(1-\alpha)\,[\alpha + (1-\alpha)q]\,[\alpha + (1-\alpha)r] \end{aligned} \tag{2}$$

Therefore:

$$E(v_m|v_m = 1) = Y(p,q,r|v_m = 1) = (1) + (2)$$

When the actual value is $v_m = -1$ (ex ante possible with probability equal to $\frac{1}{2}$), i plays "yes" with probability $(1-\alpha)p$ (similarly for j and z), as the only possible signal is L. The expected value is given by:

$$\begin{aligned} E(v_m|v_m &= -1) = -1\,[Y(p,q,r|v_m = -1)] + 0\,[1 - Y(p,q,r|v_m = -1)] \\ &= -Y(p,q,r|v_m = -1) \end{aligned}$$

The probability that all the members vote "yes" is:

$$(1-\alpha)^3 pqr \tag{3}$$

The probability that only two members vote "yes" is:

$$\begin{aligned} &(1-\alpha)^2 pq\,[\alpha + (1-r)(1-\alpha)] + \\ &(1-\alpha)^2 pr\,[\alpha + (1-q)(1-\alpha)] + \\ &(1-\alpha)^2 qr\,[\alpha + (1-p)(1-\alpha)] \end{aligned} \tag{4}$$

Therefore:

$$E(v_m|v_m = -1) = -Y(p,q,r|v_m = -1) = (3) + (4)$$

Finally, the expected value of the game is:

$$E(v_m) = \frac{1}{2}[Y(p,q,r|v_m = 1) - Y(p,q,r|v_m = -1)]$$

The equilibrium levels of p, q and r are obtained from the following:

$$\max_{p,q,r} E(v_m) = E(p,q,r)$$

The (simplified) first order conditions for this problem are independent on α and given by:

$$\begin{cases} \frac{\partial E(v_m)}{\partial p} : 1 - q - r = 0 \\ \frac{\partial E(v_m)}{\partial q} : 1 - p - r = 0 \\ \frac{\partial E(v_m)}{\partial r} : 1 - p - q = 0 \end{cases}$$

with the following solutions:

$$\begin{aligned} p &= \begin{cases} 1 & \text{if } 1-q-r>0 \\ 0 & \text{if } 1-q-r<0 \\ \in [0,1] & \text{if } 1-q-r=0 \end{cases} ; \\ q &= \begin{cases} 1 & \text{if } 1-p-r>0 \\ 0 & \text{if } 1-p-r<0 \\ \in [0,1] & \text{if } 1-p-r=0 \end{cases} ; \\ r &= \begin{cases} 1 & \text{if } 1-q-p>0 \\ 0 & \text{if } 1-q-p<0 \\ \in [0,1] & \text{if } 1-q-p=0 \end{cases} . \end{aligned}$$

The only interior solution is $\left\{r = q = p = \frac{1}{2}\right\}$, which is the symmetric equilibrium strategy profile. In addition, we have the following corner solutions:

$$\begin{aligned} \{p,q,r &\in [0,1]^3 : q+r=1, qr=0\}, \\ \{p,q,r &\in [0,1]^3 : p+q=1, pq=0\}, \\ \{p,q,r &\in [0,1]^3 : p+r=1, pr=0\}. \end{aligned}$$

■

8.2 Proof to proposition 4

Proof. Given the following order of vote: i, j, z, we characterize the players' strategies as follows:

$$s_i : \left\{ \begin{array}{c} \text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\} \end{array} \right\},$$

$$s_j : \left\{ \begin{array}{c} \text{play } \omega_j \text{ if } \Omega_j = \{\omega_j, d_i\}; \\ \text{if } \Omega_j = \{H, L; d_i\}: \\ \text{play "yes" with probability } q_Y \text{ when } d_i = \text{"yes"} \\ \text{play "yes" with probability } q_N \text{ when } d_i = \text{"no"} \end{array} \right\},$$

$$s_z : \left\{ \begin{array}{c} \text{play } \omega_z \text{ if } \Omega_z = \{\omega_z, d_i, d_j\} \\ \text{if } \Omega_z = \{H, L; d_i, d_j\}\text{:} \\ \text{play "yes" with probability } r_{YY} \text{ when } d_i = d_j = \text{"yes"} \\ \text{play "yes" with probability } r_{YN} \text{ when } d_i = \text{"yes" and } d_j = \text{"no"} \\ \text{play "yes" with probability } r_{NY} \text{ when } d_i = \text{"no" and } d_j = \text{"yes"} \\ \text{play "yes" with probability } r_{NN} \text{ when } d_i = d_j = \text{"no"} \end{array} \right\}$$

Recalling the definition of function $Y(\cdot)$, the problem we face is the following:

$$\max_{p, q_N, q_Y, r_{NY}, r_{YN}} E(v_m) = \frac{1}{2} \left[Y(p, q_N, q_Y, r_{NY}, r_{YN} \mid v_m = 1) - Y(p, q_N, q_Y, r_{NY}, r_{YN} \mid v_m = -1) \right]$$

First of all, we notice that if we fix $q_Y = q_N = q$ and $r_{YN} = r_{NY} = r$ (i.e., the players do not discriminate), then we go back to the previous case (simultaneous game) where the players' actions were not interdependent. So, every equilibrium profile in the simultaneous voting game is still an equilibrium in the sequential one. We avoid writing specific formulas for these functions, as they are straightforward extensions of the previous case. We focus on the first order conditions (after useful simplifications) in order to understand what the range of possible equilibria is:

$$\left\{ \begin{array}{l} \frac{\partial E(v_m)}{\partial p} : 2 - q_N - q_Y - r_{NY} - r_{YN} = 0 \\ \frac{\partial E(v_m)}{\partial q_N} : 1 - p - r_{NY} = 0 \\ \frac{\partial E(v_m)}{\partial q_Y} : 1 - p - r_{YN} = 0 \\ \frac{\partial E(v_m)}{\partial r_{NY}} : 1 - p - q_N = 0 \\ \frac{\partial E(v_m)}{\partial r_{YN}} : 1 - p - q_Y = 0 \end{array} \right.$$

As expected, the symmetric solution is still an equilibrium profile:

$$\left\{ r_{YN} = r_{NY} = q_Y = q_N = p = \frac{1}{2} \right\}.$$

In addition, it is still the only interior solution. Beside this, there are the following corner solutions:

$$\left\{ \begin{array}{c} r_{YN} = r_{NY} = 1 - q_Y = 1 - q_N; \\ q_N r_{YN} = 0; \forall p \in [0, 1] \end{array} \right\},$$

which is an equilibrium in the simultaneous game as well; and

$$\begin{array}{l} \{r_{YN} = r_{NY} = 1 - p; p r_{YN} = 0; \forall q_Y, q_N \in [0, 1]\} \\ \{q_N = q_Y = 1 - p; p q_Y = 0; \forall r_{YN}, r_{NY} \in [0, 1]\} \end{array}$$

which is a straightforward extension of the previous one, as once two players compensate each other, it does not make any difference whether the third is discriminating (as in the sequential game) or not (simultaneous game). Finally, we have the most different profile, where both the player differentiate their actions:

$$\left\{ \begin{array}{c} r_{YN} = 1 - q_Y \neq r_{NY} = 1 - q_N; \\ r_{YN} q_Y = 0; r_{NY} q_N = 0; \\ \forall p \in [0,1] \end{array} \right\}$$

■

8.3 Proof of condition 1

We begin with a definition, which clarifies what we mean by "following" and "herding" in the rest of the section. We recall that, with reputational purposes, the best choice is the one which is most likely to be correct.

Definition 4 *We say that the generic player "i" is "following" the generic player "j" when this is his best choice according to his own information. In line with the literature, we say that "i" is "herding" when he disregards his own private information.*

Proof. In the sequential voting game, the first player (say, i) is in the same situation as any generic player in game C. When informed, he plays the signal; when uninformed, he plays "yes" with any probability. As we show in the paper, he is correct with *ex ante* probability $\frac{1+2\alpha\beta-\alpha}{2}$.As regards following players, they can elicit some information from previous voters' actions. We use Bayes' rule to update the priors and we define the following events:

Y : the true state is "Yes";
X : the previous player(s) voted "Yes"
and the current player has a "No" signal.

Then, we introduce the herding condition:

$$\Pr[Y \mid X] > \frac{1}{2} \tag{5}$$

where, by definition:

$$\Pr[Y \mid X] = \frac{\Pr\left[Y \hat{} X\right]}{\Pr[X]} = \frac{\Pr[Y]\Pr[X \mid Y]}{\Pr[Y]\Pr[X \mid Y] + \Pr[\bar{Y}]\Pr[X \mid \bar{Y}]}$$

The herding condition in (5) states that, if the posterior belief (left hand side) is greater then the indifference point (right hand side), then the current player should ignore his own private information and follow the previous one(s).

- Starting from the second player (say, j), he faces three possible cases:
  - If he is uninformed, he should follow i, as he is right with probability $\frac{1+2\alpha\beta-\alpha}{2} \geq \frac{1}{2}$;
  - If he is informed and the signal is in line with i's choice, then there is a higher probability that the signal is correct. So, j should play his signal (or follow i, which is equivalent).
  - If he is informed but the signal contrasts with i's choice, j prefers to follow his signal. We do not need a tie breaking assumption (as, for instance, Banerjee, 1992). From our condition in (5):

$$\frac{\frac{1}{2}\left[\alpha\beta + p(1-\alpha)\right]\alpha(1-\beta)}{\frac{1}{2}\left[\alpha\beta + p(1-\alpha)\right]\alpha(1-\beta) + \frac{1}{2}\left[\alpha(1-\beta) + p(1-\alpha)\right]\alpha\beta} > \frac{1}{2}$$

    which is never true. This means that j should always play according to his own signal. He knows indeed that i could be uninformed whereas he knows he has a signal and put more weight on it then on i's choice.

- Finally, the third player (say, z), faces the following possible cases:
  - If i and j played the same and he has no signal, then z should follow.
  - If i and j played the same and he has a consistent signal, the probability that they are all right is certainly higher then $\frac{1}{2}$. Then he still should follow (or, equivalently, play his signal).
  - If i and j played the same and he has a different signal, then z may herd. This is the new result of the subsection, which is a direct consequence of introducing noisiness of the signal. Suppose both i and j play "yes" but z has an L signal, indicating "no" as the correct choice. Recalling that i plays "yes" with probability p when uninformed, we have from (5):

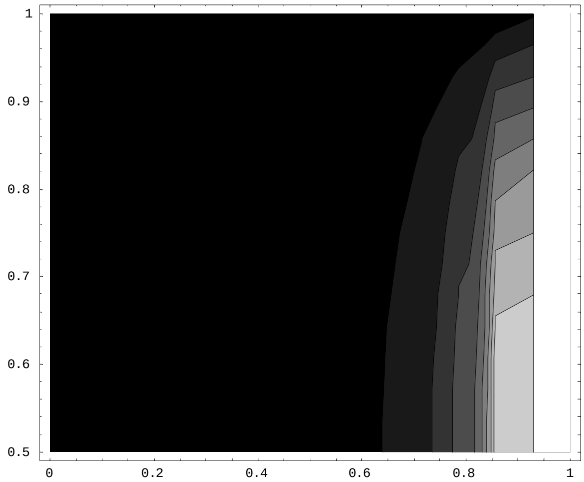
$$\frac{\frac{1}{2}\left[\alpha\beta + p(1-\alpha)\right]\left[\alpha\beta + 1 - \alpha\right]\alpha(1-\beta)}{\frac{1}{2}\left[\alpha\beta + p(1-\alpha)\right]\left[\alpha\beta + 1 - \alpha\right]\alpha(1-\beta) + \frac{1}{2}\left[\alpha(1-\beta) + p(1-\alpha)\right]\left[\alpha(1-\beta) + 1 - \alpha\right]\alpha\beta} > \frac{1}{2}$$

    in which it is important to note that, if uninformed, j always follows i. The condition is true if and only if

$$p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta) \tag{6}$$

    So, according to different values of p, herding may arise or not. Graph 2 presents a contour plot of the function $\left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$: lighter colors represent higher values of the function, and the black area its minimum.

Graph 2: Herding condition for z



On the horizontal axis we have α and on the vertical β. When α is very low ($\alpha \leq \frac{1}{2}$), it is very unlikely that i and j observed a signal: for any probability p, z trusts his signal and does not herd. When α grows, it's the term β which makes the difference. The higher β, the lower p which sustains herding: z has stronger confidence in his own signal. Finally, when $\alpha = 1$, herding is a straightforward conclusion for any β and p[18].

– If i and j played the opposite, this means that j was informed. If z is uninformed, then he should follow j, as $\beta \geq \frac{1}{2}$. If z has a consistent signal, then he should follow. Finally, if his signal his in line with i's choice, then he plays his signal. Again, this last result is obtained through Bayes' rule. The intuition is simply that z's signal is slightly reinforced by the possibility that i is informed as well and that j's signal is wrong. To prove this, we start by defining the following events:

$$\begin{aligned} Y &: \text{ the true state is "Yes";} \\ W &: \text{ the fist player voted "No", the second "Yes"} \\ &\quad\ \text{and the current player has a "No" signal.} \end{aligned}$$

Then, the herding condition requires:

$$\Pr[Y \mid W] > \frac{1}{2}$$

[18] In the symmetric case, where both i and j vote "no" but z has an H signal, the herding condition is $1 - p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$ and it is obtained with a simple modification of the condition in (5).

that is:

$$\frac{\frac{1}{2}\left[\alpha(1-\beta)+(1-p)(1-\alpha)\right]\alpha^2\beta(1-\beta)}{\frac{1}{2}\left[\alpha(1-\beta)+(1-p)(1-\alpha)\right]\alpha^2\beta(1-\beta)+\frac{1}{2}\left[\alpha(1-\beta)+(1-p)(1-\alpha)\right]\alpha^2\beta(1-\beta)}>\frac{1}{2}$$

which is never true.

■

8.4 Full information aggregation

We can imagine this situation as one where a single player (the *decision maker*) collects all the available information by the others and then takes a decision. It is straightforward to understand that the correct decision is always taken when at least one of the members has a signal, as this is always correct. When nobody is informed, the decision maker votes "yes" with a probability $p \in [0, 1]$, which is optimal, given the priors about the true state of the world.

The expected value of the election when the actual value is 1 is given by:

$$\begin{aligned} E(v_m|v_m &= 1) = 1\left[Y(p,q,r|v_m=1)\right] + 0\left[1 - Y(p,q,r|v_m=1)\right] \\ &= Y(p,q,r|v_m=1) = \alpha^3 + 3\alpha^2(1-\alpha) + 3\alpha(1-\alpha)^2 + p(1-\alpha)^3 \end{aligned}$$

The expected value when the actual value is -1 is given by:

$$\begin{aligned} E(v_m|v_m &= -1) = -1\left[Y(p,q,r|v_m=-1)\right] + 0\left[1 - Y(p,q,r|v_m=-1)\right] \\ &= -p(1-\alpha)^3 \end{aligned}$$

Therefore:

$$\begin{aligned} E(v_m) &= \frac{1}{2}[Y(p,q,r|v_m=1) - Y(p,q,r|v_m=-1)] \\ &= \frac{1}{2}\left[\alpha^3 + 3\alpha(1-\alpha)\right] \in \left[\frac{7}{16}, \frac{1}{2}\right] \text{ for } \alpha \in \left[\frac{1}{2}, 1\right] \end{aligned}$$

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