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DSFM fitting of Implied Volatility Surfaces

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Abstract

Implied volatility is one of the key issues in modern quantitative finance, since plain vanilla option prices contain vital information for pricing and hedging of exotic and illiquid options. European plain vanilla options are nowadays widely traded, which results in a great amount of high-dimensional data especially on an intra day level. The data reveal a degenerated string structure. Dynamic Semiparametric Factor Models (DSFM) are tailored to handle complex, degenerated data and yield low dimensional representations of the implied volatility surface (IVS). We discuss estimation issues of the model and apply it to DAX option prices.

JEL classification codes: C14

1. Introduction

The Black-Scholes formula (BS) for calculating the price of a European plain vanilla option is one of the most recognized results in modern quantitative finance. The price is given as a function of the price of the underlying, a strike price, the interest rate, time to maturity and an unobserved volatility. By plugging the observed option price into the BS formula it is straightforward to calculate the implied volatility (IV). The surface (on day t) given by the mapping from moneyness κ (a measure of strikes) and from time to maturity τ $(\kappa, \tau) \rightarrow \widehat{\sigma}_t(\kappa, \tau)$ is called implied volatility surface (IVS). The observed IVS, Figure 1, reveals a non-flat profile across moneyness (called "smile" or "smirk") and across time to maturity.

Despite the deficiencies of the BS model it is popular among practitioners to quote option prices in terms of IV due to its intuitive simplicity. However there were many efforts to model a non constant IVS. One possible approach is to change the dynamics of the process of the underlying asset by increasing its degree of freedom. This leads to parametric models with jumps like in [14], stochastic volatility like in [13] and [12] or models based on Lévy processes like the generalized hyperbolic in [4] among many others. The models reproduce the smile phenomenon and their parameters are calibrated from the option prices by minimizing cost functional. One may also consider local volatility (LV) models like in [3] where the volatility is assumed to be a function of time and the price of the underlying. There exist an analytical formula which allows to calibrate this surface (LVS) straightforward from the IVS.

A drawback even of the most sophisticated models is the failure to correctly describe the dynamics of the IVS. This can be inferred from frequent recalibration of the model and has been best understood in the context of LV-models [10]. Consequently, studying the IVS as an additional market factor has become a vital stand of research. The main focus is on a low dimensional approximation of the IVS based on principal components analysis (PCA). The PCA is applied both to the term structure of the IVS ([16] or [7]) and strike dimension (eg. [15]). The common PCA for several maturity groups is studied in [8] and the functional PCA was discussed in [1] and [2].

Our approach is to represent the IVS as the sum of factors treated as two dimensional functions depending on moneyness and maturity. In [2] the factors are obtained using the functional PCA for the IVS fitted on a grid for each particular day. However this fit may be biased due to the degenerated data design. In [9] the IVS is obtained as a projection on parametric factors which has to be initially specified. In DSFM the factors are estimated from the data, which allows flexible modelling. Contrary to [2] the IVS is obtained as a fit to the factors smoothed in time, which reflects the dynamics of the whole system.

The paper is organized as follows: in the next section we describe the DSFM and present the estimation procedure. In Section 3 we discuss the estimation issues and proposed improvements of the algorithm. Section 4 presents the empirical results on DAX options and discusses briefly possible applications.

2. DSFM

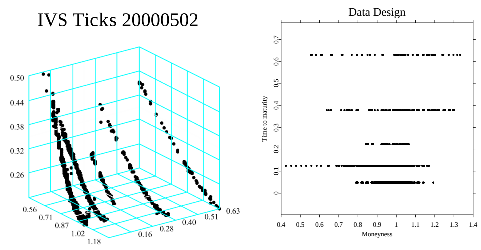

Figure 1. Left panel: implied volatilities observed on 2nd May, 2000. Right panel: data design on 2nd May, 2000.

Institutional conventions of the option market entail a specific degenerated IV data design. Each day one observes only a small number of maturities which form typical 'strings'. The usual data pattern is visible in the right panel of Figure 1. Options belonging to the same string have a common time to maturity but different moneyness. In the left panel of the Figure 1 IV smiles are presented. One can easily see different curvatures for a each time to maturity. As time passes not only do the strings move through the space towards expiry but they also change their shape and level randomly.

In order to capture this complex dynamic structure of the IVS a DSFM was proposed in [6]. It offers a low-dimensional representation of the IVS, which is approximated by basis functions in a finite dimensional function space. The basis functions are unknown and have to be estimated from the data. The IVS dynamics are explained by loading coefficients, which form a multidimensional time series.

Let $Y_{i,j}$ be the log-implied volatility observed on a particular day. The index i is the number of the day, while the total number of days is denoted by I ($i = 1, ..., I$). The index j represents an intra-day trade on day i and the number of trades on that day is J_i ($j = 1, ...J_i$). Let $X_{i,j}$ be a two-dimensional variable containing moneyness $\kappa_{i,j}$ and maturity $\tau_{i,j}$. Among many moneyness settings we define it as $\kappa_{i,j} = \frac{K_{i,j}}{F_{t_i}}$, where $K_{i,j}$ is a strike and F_{t_i} the underlying futures price at time t_i. The DSFM regresses $Y_{i,j}$ on $X_{i,j}$ by:

$$Y_{i,j} = m_0(X_{i,j}) + \sum_{l=1}^{L} \beta_{i,l} m_l(X_{i,j}), \qquad (1)$$

where m_0 is an invariant basis function, m_l ($l = 1, ...L$) are the 'dynamic' basis functions and $\beta_{i,l}$ are the factor weights depending on time i.

2.1. Estimation

The estimates $\widehat{\beta}_{i,l}$ and $\widehat{m}_l$ are obtained by minimizing the following least squares criterion ($\beta_{i,0} = 1$):

$$\sum_{i=1}^{I}\sum_{j=1}^{J_i} \int \left\{ Y_{i,j} - \sum_{l=0}^{L} \widehat{\beta}_{i,l}\widehat{m}_l(u) \right\}^2 K_h(u - X_{i,j})\, du, \qquad (2)$$

where K_h denotes a two-dimension kernel function. The possible choice for two-dimensional kernels is a product of one dimensional kernels $K_h(u) = k_{h_1}(u_1) \times k_{h_2}(u_2)$, where $h = (h_1, h_2)^\top$ are bandwidths and $k_h(v) = k(h^{-1}v)/h$ is a one dimensional kernel function.

The minimization procedure searches through all functions $\widehat{m}_l : \mathbb{R}^2 \longrightarrow \mathbb{R}$ ($l = 0, ..., L$) and time series $\widehat{\beta}_{i,l} \in \mathbb{R}$ ($i = 1, ..., I; l = 1, ..., L$).

To calculate the estimates an iterative procedure is applied. First we introduce the following notation for $1 \le i \le I$:

$$\widehat{p}_i(u) = \frac{1}{J_i}\sum_{j=1}^{J_i} K_h(u - X_{i,j}), \qquad (3)$$

$$\widehat{q}_i(u) = \frac{1}{J_i}\sum_{j=1}^{J_i} K_h(u - X_{i,j}) Y_{i,j}. \qquad (4)$$

We denote by $\widehat{m}^{(r)} = (\widehat{m}_0^{(r)}, ..., \widehat{m}_L^{(r)})^\top$ the estimate of the basis functions and $\widehat{\beta}_i^{(r)} = (\widehat{\beta}_{i,l}^{(r)}, ..., \widehat{\beta}_{i,L}^{(r)})^\top$ the factor loadings on the day i after r iterations. By replacing each function $\widehat{m}_l$ in (2) by $\widehat{m}_l + \delta g$ with arbitrary function g and taking derivatives with respect to δ one obtains:

$$\sum_{i=1}^{I}\sum_{j=1}^{J_i} \left\{ Y_{i,j} - \sum_{l=0}^{L} \widehat{\beta}_{i,l}\widehat{m}_l(X_{i,j}) \right\} \widehat{\beta}_{i,l'} K_h(u - X_{i,j}) = 0. \qquad (5)$$

Rearranging terms in (5) and plugging in (3)-(4) yields:

$$\sum_{i=1}^{I} J_i \widehat{\beta}_{i,l'} \widehat{q}_i(u) = \sum_{i=1}^{I} J_i \sum_{l=0}^{L} \widehat{\beta}_{i,l'}\widehat{\beta}_{i,l}\widehat{p}_i(u)\widehat{m}_l(u), \qquad (6)$$

for $0 \le l' \le L$. In fact (6) is a set of $L+1$ equations. Define the matrix $B^{(r)}(u)$ and vector $Q^{(r)}(u)$ by their elements:

$$\left(B^{(r)}(u)\right)_{l,l'} = \sum_{i=1}^{I} J_i \widehat{\beta}_{i,l'}^{(r-1)} \widehat{\beta}_{i,l}^{(r-1)} \widehat{p}_i(u), \qquad (7)$$

$$\left(Q^{(r)}(u)\right)_l = \sum_{i=1}^{I} J_i \widehat{\beta}_{i,l}^{(r-1)} \widehat{q}_i(u). \tag{8}$$

Thus (6) is equivalent to:

$$B^{(r)}(u)\widehat{m}^{(r)}(u) = Q^{(r)}(u) \tag{9}$$

which yields the estimate of $\widehat{m}^{(r)}(u)$ in the r-th iteration. A similar idea has to be applied to update $\widehat{\beta}_i^{(r)}$. Replacing $\widehat{\beta}_{i,l}$ by $\widehat{\beta}_{i,l} + \delta$ in (2) and taking once more the derivative with respect to δ yields:

$$\sum_{j=1}^{J_i} \int \left\{ Y_{i,j} - \sum_{l=0}^{L} \widehat{\beta}_{i,l} \widehat{m}_l(X_{i,j}) \right\} \widehat{m}_{l'}(u) K_h(u - X_{i,j}) du = 0, \tag{10}$$

which leads to:

$$\int \widehat{q}_i(u)\widehat{m}_{l'}(u)\, du = \sum_{l=0}^{L} \widehat{\beta}_{i,l} \int \widehat{p}_i(u)\widehat{m}_{l'}(u)\widehat{m}_l(u)\, du, \tag{11}$$

for $1 \leq l' \leq L$. The formula (11) is now a system of L equations. Define the matrix $M^{(r)}(i)$ and the vector $S^{(r)}(i)$ by their elements:

$$\left(M^{(r)}(i)\right)_{l,l'} = \int \widehat{p}_i(u)\widehat{m}_{l'}(u)\widehat{m}_l(u)\, du, \tag{12}$$

$$\left(S^{(r)}(i)\right)_l = \int \widehat{q}_i(u)\widehat{m}_l(u)\, du - \int \widehat{p}_i(u)\widehat{m}_0(u)\widehat{m}_l(u)\, du. \tag{13}$$

An estimate of $\widehat{\beta}_i^{(r)}$ is thus given by solving:

$$M^{(r)}(i)\widehat{\beta}_i^{(r)} = S^{(r)}(i). \tag{14}$$

The algorithm stops when only minor changes occur:

$$\sum_{i=1}^{I} \int \left(\sum_{l=0}^{L} \widehat{\beta}_{i,l}^{(r)} \widehat{m}_l^{(r)}(u) - \widehat{\beta}_{i,l}^{(r-1)} \widehat{m}_l^{(r-1)}(u) \right)^2 du \leq \epsilon \tag{15}$$

for some small ϵ. Obviously one needs to set initial values of $\widehat{\beta}_i^{(0)}$ in order to start the algorithm.

2.2. Orthogonalization and normalization

The estimates $\widehat{m} = (\widehat{m}_1, ..., \widehat{m}_L)^\top$ of the basis functions are not uniquely defined. They can be replaced by functions that span the same affine space. Define $\widehat{p}(u) = \frac{1}{I}\sum_{i=1}^{I} \widehat{p}_i(u)$ and the $L \times L$ matrix Γ by its elements

$$\Gamma_{l,l'} = \int \widehat{m}_l(u)\widehat{m}_{l'}(u)\widehat{p}(u)du.$$

The estimates $\widehat{m}$ are replaced by new functions $\widehat{m}^{new} = (\widehat{m}_1^{new}, ..., \widehat{m}_L^{new})^\top$:

$$\begin{aligned} \widehat{m}_0^{new} &= \widehat{m}_0 - \gamma^\top \Gamma^{-1} \widehat{m} \\ \widehat{m}^{new} &= \Gamma^{-1/2} \widehat{m} \end{aligned}$$

such that they are now orthogonal in the $L^2(\widehat{p})$ space. The loading time series estimates $\widehat{\beta}_i = (\widehat{\beta}_{i,1}, ..., \widehat{\beta}_{i,L})^\top$ need to be substituted by:

$$\widehat{\beta}_i^{new} = \Gamma^{-1/2}(\widehat{\beta}_i + \Gamma^{-1}\gamma) \tag{16}$$

where γ is $(L \times 1)$ vector with $\gamma_l = \int \widehat{m}_0(u)\widehat{m}_l(u)\widehat{p}(u)du$. The next step is to choose an orthogonal basis such that for each $w = 1, ..., L$ the explanation achieved by the partial sum:

$$m_0(u) + \sum_{l=1}^{w} \beta_{i,l} m_l(u)$$

is maximal. One proceeds as in PCA. First define a matrix B with $B_{l,l'} = \sum_{i=1}^{I} \widehat{\beta}_{i,l}\widehat{\beta}_{i,l'}$ and $Z = (z_1, ..., z_L)$ where z_1,...,z_L are the eigenvectors of B. Then replace $\widehat{m}$ by $\widehat{m}^{new} = Z^\top \widehat{m}$ and $\widehat{\beta}_i$ by $\widehat{\beta}_i^{new} = Z^\top \widehat{\beta}_i$.
The orthonormal basis $\widehat{m}_1, ..., \widehat{m}_L$ is chosen such that $\sum_{i=1}^{I} \widehat{\beta}_{i,1}^2$ is maximal and given $\widehat{\beta}_{i,1}, \widehat{m}_0, \widehat{m}_1$ the quantity $\sum_{i=1}^{I} \widehat{\beta}_{i,2}^2$ is maximal and so forth.

3. Estimation issues

The estimation procedure encounter several computational challenges. The basis factor functions can be represented on the finite grid only, which obviously may not cover the whole desired estimation space. One also needs to choose some kernel function, the bandwidths and the initial loading time series $\widehat{\beta}_i^{(0)}$. Due to the degenerated design of the IV data proper decision of these points is a key issue in successful model estimation.

3.1. Implementation

As a numerical result of the estimation L time series $(\widehat{\beta}_{i,l})$ and $L+1$ functions $(\widehat{m}_l)$ given on the finite grid are obtained. The choice of the grid needs to be arbitrary and depends on the density of the data points. The data ($X_{i,j}$ and $Y_{i,j}$) comes into the computation only in $\widehat{p}_i(u)$ and $\widehat{q}_i(u)$, which has to be calculated before the main iteration procedure. The calculation of $\widehat{p}_i(u)$ and $\widehat{q}_i(u)$, however, is the main computation effort in the estimation procedure. Therefore we believe that the DSFM can be used efficiently in a 'sliding window' type of analysis. Updating of $\widehat{p}_i(u)$ and $\widehat{q}_i(u)$ requires calculations only for one additional day, which is not a big computational issue.

3.2. Bandwidths dependence

In derivative market one can observe fairly many different types of option contracts. Each day one may trade options with several different time to maturities and many different strikes. However the number of possible strikes is much higher than the number of maturities, which results in the string structure. Moreover the contracts with smaller maturities are traded more intensively and there tend to exist more contracts for the smaller time to maturities for which the difference between two successive expiry days is one month (1M, 2M, 3M), but for the next maturity range it increases to three months (6M, 9M, 12M).

Since the strings are moving in the maturity vs. moneyness plane towards expiry one needs to pool many days in order to fill the plane with observations. However due to an unequal distribution of data points one needs even more days to fill the range with bigger maturities than with smaller ones. Otherwise one faces gaps for some particular maturity range.

These gaps may obstruct the estimation procedure. If in any point u' the function $\widehat{p}(u') = 0$ in (3) then obviously matrix $B^{(r)}(u')$ in (7) contains only 0 and is singular. This means that one may not estimate successfully any value of the IVS in this point.

This problem may be solved by increasing the bandwidths but it may lead also to a larger bias. One may also use a kernel function with infinite support like the Gaussian kernel but instead of analytical zeros numerical zeros creep in. Another possibility is to use the k-nearest neighbor estimator. In the range with many data, however, one takes into consideration only very few observations closest to the grid points. On the other hand in the range with few points the estimator is based on the observations far from the grid points. In order to cope with the degenerated data design local bandwidths can be applied. In (3) and (4) the fixed bandwidths are replaced by bandwidths dependent on time to maturity and moneyness:

$$\widehat{p}_i(u) = \frac{1}{J_i}\sum_{j=1}^{J_i} K_{h(u)}(u - X_{i,j}), \qquad (17)$$

$$\widehat{q}_i(u) = \frac{1}{J_i}\sum_{j=1}^{J_i} K_{h(u)}(u - X_{i,j})Y_{i,j}. \qquad (18)$$

Due to the described data design we propose to keep the bandwidths in the moneyness direction constant and linearly increasing in the maturity dimension. For the optimal choice of the bandwidths we refer to [11].

3.3. Initial parameter dependence

The problem of gaps in the data cannot only be handled with the size of the bandwidths. Of course it is obligatory that $\widehat{p}_i(u)$ needs to be non-zero for at least one i. However this is not a sufficient condition to ensure non singularity of the matrix $B^{(r)}(u')$. The initial estimates of $\widehat{\beta}_i^{(0)}$ play also an important role.

In [6] a piecewise constant initial time series was proposed. The subintervals $I_1, ..., I_L$ are pairwise disjoint subsets of $\{1, ..., I\}$ and $\bigcup_{l=1}^{L} I_l$ is a strict subset of $\{1, ..., I\}$. The initial estimates are now defined by $\widehat{\beta}_{i,l}^{(0)} = 1$ if $i \in I_l$ and $\widehat{\beta}_{i,l}^{(0)} = 0$ if $i \notin I_l$. To complete the setting $\widehat{\beta}_{i,0}^{(0)} = 1$ for each i.

However this kind of setting requires even more data to obtain the final estimates. For each subset I_l there needs to exist at least one day i such that $\widehat{p}_i(u') \neq 0$, otherwise the row of zeros in (7) appears. The smaller is the length of I_l intervals the bigger bandwidths need to be taken. This deficiency can be removed by taking a random initial time series.

4. Results

For our analysis we employ tick statistics on DAX index options from January 1999 to February 2003. By inverting the BS formula one easily obtains IV. We regard as outliers observations with IV bigger that 0.8 and smaller than 0.04. We also remove observations with maturity less than 10 day since their behavior in this range is irregular due to expiry effect.

We apply the algorithm on an equidistant grid covering moneyness $\kappa \in [0.8, 1.2]$ and time to maturity measured in years $\tau \in [0.05, 1.00]$. In each direction our grid consists of 25 points. We set the number of dynamic basis functions to $L = 3$ like in [6]. In the moneyness direction we apply constant bandwidths $h_1 = 0.03$ and in order to get smoother estimates of the basis functions in the maturity direction we use linearly increasing bandwidths. On the smallest maturity grid points we set bandwidths on 0.02 and increase them linearly to 0.2 for the greatest maturity points. As the starting values of $\widehat{\beta}^{(0)}$ we take a piecewise constant series on disjoint time intervals. The initial weights selection is discussed below.

Figure 2 presents the estimated factors loading $\widehat{\beta}_1$,$\widehat{\beta}_2$ and $\widehat{\beta}_3$ respectively. The magnitude and variance of the $\widehat{\beta}_1$ are much higher than for the other two time series, which suggests that the first basis function has the biggest explanatory power of the IVS variation. This is actually not surprising since the basis functions were ordered with respect to the biggest variance of loading factors.

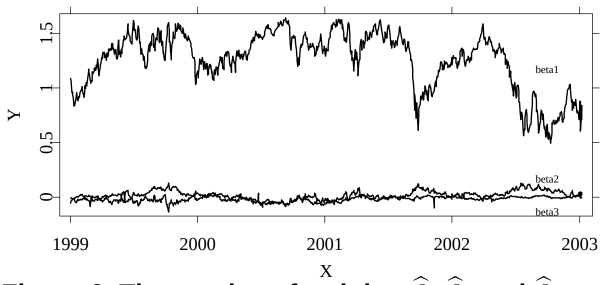

Figure 2. Time series of weights $\widehat{\beta}_1$,$\widehat{\beta}_2$ and $\widehat{\beta}_3$.

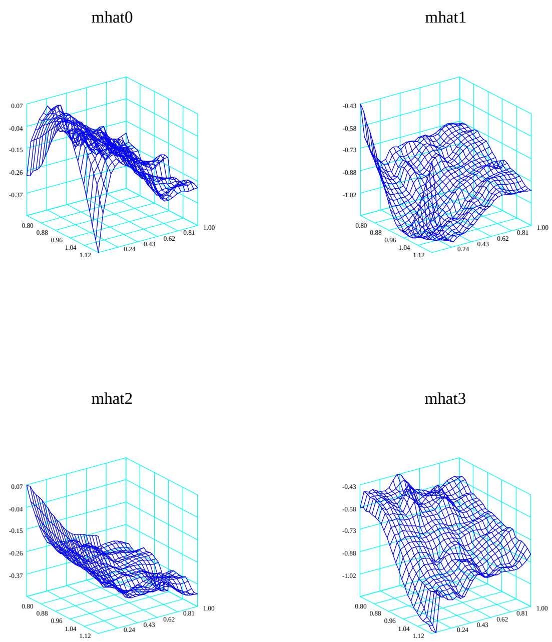

Figure 3. Invariant basis function $\widehat{m}_0$ and dynamic basis functions $\widehat{m}_1$, $\widehat{m}_2$ and $\widehat{m}_3$.

Figure 3 displays the estimated basis functions $\widehat{m}_0$ - $\widehat{m}_3$. We find similar interpretations of the factors as in [6] or [2]. The first dynamic factor $\widehat{m}_1$ is relatively flat on almost the whole range and negative on all grid points. It reflects the up and down shifts of the entire log-IVS. For the small maturities a strong curvature can be seen. It corresponds to the empirical fact that near the expiry the 'smile' effect becomes stronger. The second function is positive for the small maturities and negative for the bigger maturities. The positive $\widehat{\beta}_2$ increases short term maturities IVs and simultaneously decreases the long term ones. The negative $\widehat{\beta}_2$ causes the opposite effect. This function provides term structure changes of the IVS. The last function $\widehat{m}_3$ reveals a strong slope in the moneyness direction changing from positive to negative near at-the-money. It reflects changes of the moneyness slope and smile curvature.

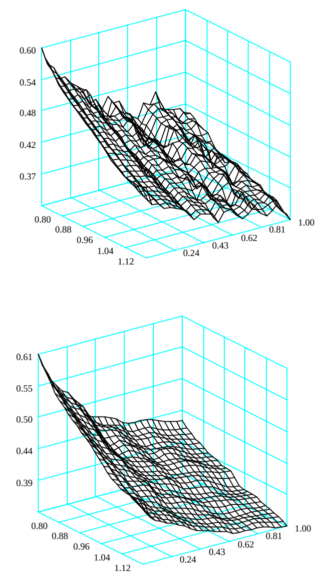

Figure 4. IVS estimates on February 25, 2003, fixed bandwidths $h_1 = 0.03, h_2 = 0.02$ (top), linearly increasing bandwidths (bottom).

In our estimation we used the local bandwidths linearly increasing in maturity. Figure 4 presents the comparison of the two different IVS estimates on February 25, 2003 obtained with fixed bandwidths $h_1 = 0.03, h_2 = 0.02$ and with local bandwidths. While in the fixed bandwidths approach in bigger maturities the estimated IVS is rough, in the local bandwidths approach it becomes smoother.

Another estimation issue is the choice of the initial times series $\widehat{\beta}^{(0)}$. We have recalculated the estimates for different starting values. Denote by PC_1, PC_2, PC_3 as the different settings of piecewise constant starting values as described in Section 3.3. Denote also by WN_1, WN_2, WN_3 settings where the algorithm starts from a white noise and BM_1, BM_2, BM_3 from a Brownian Motion. For each of the 9 settings we have obtained different estimates of weights. The correlation between different estimates of $\widehat{\beta}_1$, $\widehat{\beta}_2$ and $\widehat{\beta}_3$ are respectively:

$$\begin{pmatrix} \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{1.0} & -0.9 & 0.9 & -0.9 \\ & \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & 0.9 & -0.9 & 0.9 \\ & & \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & 0.9 & -0.9 & 0.9 \\ & & & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & 0.9 & -0.9 & 0.9 \\ & & & & \mathbf{1.0} & \mathbf{-1.0} & 0.9 & -0.9 & 0.9 \\ & & & & & \mathbf{1.0} & -0.9 & 0.9 & 0.9 \\ & & & & & & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{1.0} \\ & & & & & & & \mathbf{1.0} & \mathbf{-1.0} \\ & & & & & & & & \mathbf{1.0} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & 0.3 & -0.3 & -0.3 \\ & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & 0.3 & -0.3 & -0.3 \\ & & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & 0.3 & -0.3 & -0.3 \\ & & & \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & -0.3 & 0.3 & 0.3 \\ & & & & \mathbf{1.0} & \mathbf{1.0} & 0.3 & 0.3 & 0.3 \\ & & & & & \mathbf{1.0} & -0.3 & 0.3 & 0.3 \\ & & & & & & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} \\ & & & & & & & \mathbf{1.0} & \mathbf{1.0} \\ & & & & & & & & \mathbf{1.0} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{-1.0} & \mathbf{1.0} & -0.8 & 0.8 & -0.8 \\ & \mathbf{1.0} & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{1.0} & 0.8 & -0.8 & 0.8 \\ & & \mathbf{1.0} & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{1.0} & 0.8 & -0.8 & 0.8 \\ & & & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{1.0} & 0.8 & -0.8 & 0.8 \\ & & & & \mathbf{1.0} & \mathbf{-1.0} & -0.8 & 0.8 & -0.8 \\ & & & & & \mathbf{1.0} & 0.8 & -0.8 & 0.8 \\ & & & & & & \mathbf{1.0} & \mathbf{-1.0} & \mathbf{1.0} \\ & & & & & & & \mathbf{1.0} & \mathbf{-1.0} \\ & & & & & & & & \mathbf{1.0} \end{pmatrix}$$

where the sequence of the settings is following: $PC_1, PC_2, PC_3, WN_1, BM_1, BM_2, WN_2, WN_3, BM_3$. The algorithm converges to two different solutions depending on the starting values since the settings form clearly two clusters: $(PC_1, PC_2, PC_3, WN_1, BM_1, BM_2)$ and (WN_2, WN_3, BM_3). Inside the clusters the weights are almost perfectly correlated - top left and bottom right corners of the matrices contain 1 or -1. Of course if the correlation of the time series estimates is -1 the same factors are considered because they are identifiable only up to sign. Between the clusters the correlation is not so strong. In order to choose one solution other criteria like explained variance or smoothness of IVS need to be taken into account.

The DSFM can easily be applied in hedging or risk management. Computing sensitivity with respect to factor loadings changes simplify the vega hedge since the whole dynamics of the IVS is reduced to L factors. After estimating stochastic model for $\widehat{\beta}$, like in [5] where VAR(2) was detected, it can be used for scenario generation in Monte Carlo framework. Therefore it allows to compute the VaR for portfolios containing options.

5. Conclusion

We discuss estimation issues of the DSFM, which gives a flexible way of handling IV data and is a convenient modelling tool. We study the dependence on the starting $\widehat{\beta}$ and the bandwidths settings. These are the key issues in efficient application of the model, which is left for future research.

6. Acknowledgement

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References

[1] M. Benko and W. Härdle. *Common Functional Implied Volatility Analysis* in P. Čížek, W. Härdle and R. Weron (eds) *Statistical Tools for Fianance and Insurance*, chapter 5, pages 115–134. Springer Verlag, 2005.

[2] R. Cont and J. da Fosenca. Dynamics of implied volatility surfaces. *Quantitative Finance*, 2:45–60, February 2002.

[3] B. Dupire. Pricing with a smile. *RISK*, 1(7):18–20, 1994.

[4] E. Eberlein and K. Prause. *The generalized hyperbolic model: Financial derivatives and risk measures* in H. German, D. Madan, S. Pliska, T. Vorst (eds) *Mathematical Finance - Bachelier Congress 2000* pages 245–267. Springer Verlag, 2002.

[5] M. Fengler. *Semiparametric Modelling of Implied Volatility*. PhD thesis, Humboldt-Universität zu Berlin, 2004.

[6] M. Fengler, W. Härdle, and E. Mammen. A Dynamic Semiparametric Factor Model for Implied Volatility String Dynamics. CASE Discussion Paper, Humboldt-Universität zu Berlin, 2004.

[7] M. Fengler, W. Härdle, and P. Schmidt. Common factors govering VDAX movements and the maximum loss. *Journal of Financial Markets and Portfolio Managment*, 1(16):16–19, 2002.

[8] M. Fengler, W. Härdle, and C. Villa. The Dynamics of Implied Volatilities: A Common Principal Components Approach. *Review of Derivatives Research*, 6:179–202, 2003.

[9] R. Hafner. *Stochastic Implied Volatility*. Springer-Verlag, Berlin Heidelberg, 2004.

[10] P. Hagan, D. Kumar, D. Lesniewski and D. Woodward. Manging smile risk. *Wilmott magazine*, 1:84–108, 2002.

[11] W. Härdle. *Applied Nonparametric Regression*. Cambridge University Press,1990.

[12] S. Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6:327–343, 1993.

[13] J. Hull, A. White The pricing on option on assets with stochastic volatilities. *Journal of Finance*, 42:281–300, 1987.

[14] R. Merton. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3:125–144, 1976.

[15] G. Skiadopoulos, S. Hodges, and L. Clewlow. The dynamics of the S&P 500 implied volatility surface. *Review of Derivatives Research*, 3:263–282, 1999.

[16] Y. Zhu and M. Avellaneda. An E-ARCH model for the term-structure of implied volatility of FX options. *Applied Mathematical Finance*, 4:81–100, 1997.

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