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Individual Welfare Gains from Deferred Life-Annuities under Stochastic Lee-Carter Mortality

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Abstract A deferred annuity typically includes an option-like right for the policyholder. At the end of the deferment period, he may either choose to receive annuity payouts, calculated based on a mortality table agreed to at contract inception, or receive the accumulated capital as a lump sum. Considering stochastic mortality improvements, such an option could be of substantial value. Whenever mortality improves less than originally expected, the policyholder will choose the lump sum and buy an annuity on the market granting him a better price. If, however, mortality improves more than expected, the policyholder will choose to retain the deferred annuity. We use a realistically calibrated life-cycle consumption/saving/asset allocation model and calculate the welfare gains of deferred annuities under stochastic Lee-Carter mortality. Our results are relevant both for individual retirement planning and for policymakers, especially if legislation makes annuitization, at least in part, mandatory. Our results also indicate the maximal willingness to pay for the mortality option inherent in deferred annuities, which is of relevance to insurance pricing.

Keywords Stochastic Mortality, Deferred Annuitization, Retirement Decisions, Annuity Puzzle, Intertemporal Utility Maximization

JEL-Classification D14, D81, D91, G11, G22, J11, J26

1. Introduction

A life-annuity guarantees the policyholder (annuitant) an income stream as long he is alive in exchange for paying a certain amount of money (premium) to the insurer. Buying an annuity avoids the risk of outliving one's money. Conversely, if the individual chooses to self-annuitize (i.e., manage his own pay-out plan), the individual may end up with zero wealth and no income while still alive (see Horneff et al., 2008a and the references cited therein). Consequently, in the literature, annuitization is regarded as quite valuable for risk-averse individuals (see Section 2).

A deferred annuity, i.e., an annuity that does not pay out immediately but at some predefined future date, typically comes with an option-like right for the insured. At the end of the deferral period, he may either choose to receive annuity payouts or receive the accumulated capital as a lump sum.

The mortality table used to calculate these payouts is typically agreed upon at contract inception. Considering stochastic mortality improvements, such an option can be of substantial value. Whenever mortality improves less than originally expected, the annuitant will choose the lump sum and buy an annuity on the market at a better price. However, if mortality improves more than expected, he will choose to retain the deferred annuity. A deferred annuity thus protects the individual from the risk of high annuity prices in the future while providing the opportunity to make a better deal in case of low prices.

From the perspective of an insurance company the value of this type of option is estimated by Toplek (2007) using financial pricing methods in a perfect market environment, based on previous work by Milevsky and Promislow

(2001), Boyle and Hardy (2003), Pelsser (2003), Biffis and Millossovich (2006), and Ballotta and Haberman (2006).

In this contribution, we take the perspective of a risk-averse individual facing incomplete markets who wants to maximize expected utility. We use a realistically calibrated life-cycle consumption/saving/asset allocation model and calculate the welfare gains of deferred annuities under stochastic Lee-Carter mortality taking borrowing and short-selling constraints into consideration. Our results are of considerable interest for individual retirement planning and for policymakers, especially if legislation makes annuitization, at least in part, mandatory as has occurred in the United Kingdom (see Cannon and Tonks, 2008). Our results also reveal the maximal price above the expected value a risk-averse individual would pay for deferred annuities and the willingness to pay for the mortality-related option.

Our results confirm the findings of the optimal annuitization literature: annuitization is found to be welfare enhancing considering deferred annuities in a stochastic mortality environment. The option related to fluctuations in mortality, however, appears to be of little value to individuals, with higher values found for middle-aged, patient, and risk-averse individuals.

The remainder of this article is structured as follows. Section 2 contains a literature survey. The stochastic process for mortality is introduced in Section 3. The formal model is developed and calibrated in Section 4. Results are presented in Section 5 and a summary and discussion are found in Section 6.

2. Related Literature

Initiated by the seminal work of Yaari (1965), a broad literature has developed that investigates optimal strategies involving *immediate annuities* under *deterministic mortality*, for example, with respect to the amount of wealth to be annuitized, optimal timing of annuity purchases, or the type of annuity to be purchased.[1] According to this literature, being able to insure longevity risk via annuitization generally increases the utility of a risk-averse individual. In reality however, annuitization rates are much lower than one would expect from the results of these models (see, e.g., Moore and Mitchell, 1997), a contradiction that is called the “annuity puzzle.” The literature suggests several explanations for this puzzle, including: annuities may be too expensive due to adverse selection; annuities may induce a suboptimal consumption profile or asset allocation, bequest motives, the crowding-out effect of government pensions, intra-family risk sharing, the insolvency risk of the insurer, and the background risk of government pensions under stochastic mortality. Furthermore, behavioral biases, like framing, are found to induce a low demand (Agnew et al., 2008; Brown et al., 2008).

In the context of immediate annuitization, *stochastic mortality* is analyzed by Menoncin (2008) and Schulze and Post (2009). Menoncin (2008) studies consumption and asset allocation decisions under stochastic mortality of an agent having access to longevity bonds. The model does not impose borrowing or short-selling constraints and allows for continuous trading in the longevity

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1 See Richard (1975), Kotlikoff and Spivak (1981), Merton (1983), Friedman and Warshawsky (1988), Brugiavini (1993), Mitchell et al. (1999), Brown and Poterba (2000), Brown (2001), Milevsky and Young (2003, 2007), Dushi and Webb (2004), Vidal and Lejárraga (2004, 2006), Davidoff et al. (2005), Babbel and Merrill (2006), Hainaut and Devolder (2006), Post et al. (2006), Gupta and Li (2007), Lopes and Michaelides (2007), Inkmann et al. (2007), Horneff et al. (2008b, 2008c, 2008d, 2008e), Huang et al. (2008), Koijen et al. (2008), Purcal and Piggott (2008), Yogo (2008), Davidoff (2009).

bond. The individual can adjust his mortality risk hedging portfolio continuously. Menoncin (2008) does not account for the irreversibility of annuitization decisions or for the fact that hedging opportunities for most individuals are far less than perfect. It is shown that a longevity bond is always welfare enhancing and that the share invested should decrease over the lifetime because the uncertainty surrounding future mortality developments decreases with the length of the planning horizon. Schulze and Post (2009) analyze the annuity demand of an individual who is able to buy annuities only at a certain age with no opportunity to cancel or sell the contract later, thus taking into consideration the irreversibility of annuitization decisions. The authors show, given shocks to mortality rates are mean preserving, that annuity demand is not influenced by introducing stochastic mortality if the argument of the utility function (consumption) is stochastically independent of mortality risks. However, in their analysis of situations involving mortality-driven insolvency risk of the annuity provider or background risk induced by a mortality dependent government pension income stream, they find that annuity demand may, dependent on the severity of the insolvency risk, increase or decrease compared to a situation without stochastic mortality or without such dependencies.

The optimal demand for *deferred annuities* under *deterministic mortality* is studied in Gong and Webb (2007) and Horneff and Maurer (2008). Horneff and Maurer (2008) find that optimal strategies involving deferred annuities are very similar to strategies involving immediate annuities. Because they do not consider stochastic mortality in their model, they do not take into consideration the option features included in deferred annuities. Gong and Webb (2007) show that under reasonable assumptions about actuarial unfairness, deferred annuities might be preferred over immediate annuities due to a better mortality credit versus loading tradeoff.

Milevsky and Kyrychenko (2008) study the welfare and asset allocation implications of an option, similar to the one considered here, that is included in some variable annuity contracts. The so-called guaranteed minimum income benefit (GMIB) option allows the annuitant to convert a fixed amount of money at a specified date via guaranteed annuity rates or to take the money and buy annuities on the market. However, Milevsky and Kyrychenko (2008) do not consider stochastic mortality. In their analysis, the option's value is solely driven by the stochastic investment return of the money invested in the annuity, which determines whether or not the annuitant should exercise the GMIB option.

In summary, in the literature, immediate annuitization has been analyzed under deterministic and stochastic mortality. Deferred annuitization, however, has only been analyzed under deterministic mortality to date. Thus, our study of deferred annuitization under stochastic mortality is an important contribution to the field.

3. A Lee-Carter Type Stochastic Process for Mortality

Several models for stochastic mortality are discussed in the literature (see, e.g., Cairns et al., 2008), but the model we use is one of the earliest proposed and now one of the most widely used—the Lee-Carter model (Lee and Carter, 1992). According to this model, the log of the central death rate $m_{x,t}$ for a given age x at time t is given by

$$\ln(m_{x,t}) = a_x + b_x k_t \,, \tag{1}$$

where a_x and b_x are age-specific constants and k_t, the mortality index, is a random variable, whose realization defines a complete mortality table for given values of a_x and b_x.[2]

Following Lee and Carter (1992), k_t is assumed to follow a random walk with drift. Thus k_t is given by

$$k_t = k_{t-1} + \theta + \varepsilon_t , \tag{2}$$

where ε_t is normally distributed with $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{Std}[\varepsilon_t] = \sigma_\varepsilon$. The final variable of interest for the expected-utility maximization and annuity pricing framework, the one-period survival probability for age x at time t, $p_{x,t}$, is then given by[3]

$$p_{x,t} = 1 - m_{x,t} / (1 + 0.5 m_{x,t}), \tag{3}$$

which means that the individual and the insurer hold symmetric beliefs as to the distribution of future mortality[4] and that there is no difference between individual mortality and aggregate mortality.

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[2] As in Bauer and Weber (2008), we ignore age-specific mortality shocks.

[3] The conversion of central death rates into survival rates is based on the approximation given in Cairns et al. (2008).

[4] For annuity demand under asymmetric mortality beliefs (i.e., information uncertainty) and heterogeneity of mortality rates in the population of annuitants, see Brugavani (1993) and Sheshinski (2007).

4. Preferences, Decisions Alternatives, and Optimization Problem

4.1 Preferences

The individual derives utility from consumption C (all monetary variables are in nominal terms) over his stochastic lifespan. The intertemporally separable utility function $U(C)$, following the standard discounted utility model, is defined as:

$$U(C) = \sum_{t=0}^{T-x_0} \delta^t \left(\prod_{i=0}^{t} p_{i+x_0,0} \right) U_t(C_t), \tag{4}$$

where T denotes the maximum lifespan, x_0 the individual's current age, δ the subjective discount factor, and $p_{x,0}$ the individual's probability of surviving from age x to $x+1$ given the mortality table information at $t=0$. The individual has no bequest motives; thus, the one-period CRRA-utility function $U_t(C_t)$, with γ as the coefficient of relative risk aversion, is given by:

$$U_t(C_t) = \begin{cases} \log\left(\dfrac{C_t}{(1+\pi)^t} \right), \text{for } \gamma = 1 \\ \dfrac{\left(\dfrac{C_t}{(1+\pi)^t} \right)^{1-\gamma} - 1}{1-\gamma}, \text{otherwise} \end{cases} \tag{5}$$

as long as the individual lives; 0 otherwise. Nominal consumption at time t, C_t, is adjusted for inflation at rate π.

4.2 Decision Alternatives

4.2.1 General Decisions in Each Period

At each point in time, t, the individual must decide on the amount of wealth to be consumed, C_t, which implicitly determines savings, S_t. Wealth at time t is denoted by W_t. Savings $S_t = W_t - C_t$ are invested at the risk-free return R_f. The individual cannot borrow money. Initial wealth is given by W_0.

4.2.2 Only Immediate Annuities are Available

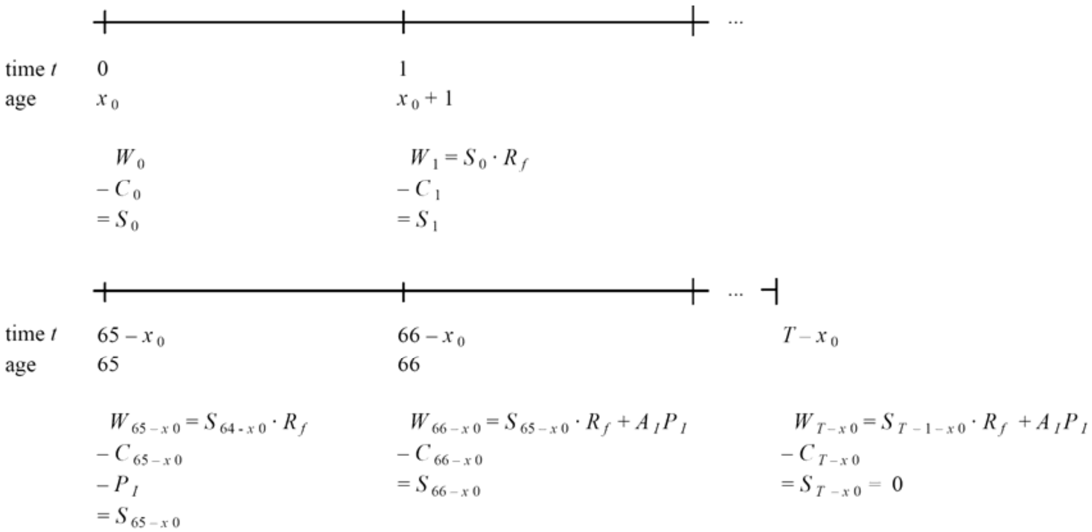
We compare two annuitization decision alternatives. Under the first, the individual can buy only immediate annuities with nominally fixed and constant payouts at age 65 ($t = 65 - x_0$). He pays a premium P_I, with $0 \le P_I \le W_{65-x_0}$. For every \$1 of premium paid, the annuity pays A_I. The insurance company prices the annuity according to the principle of equivalence, given information about the mortality index k_{65-x_0}, but may include a loading factor L, with $L \ge 0$. Given that the individual and the insurer hold symmetric beliefs regarding the distribution of k_t, the annuity payout per \$1 premium paid is derived at age 65 according to:

$$1 = (1+L) \cdot A_I \cdot \mathrm{E}_{65-x_0}\left[\sum_{j=1}^{T-65} \frac{\left(\prod_{i=0}^{j-1} p_{i+65,65-x_0+i+1}\right)}{\left(R_f\right)^j}\right], \tag{6}$$

where E_t denotes the expected value operator with respect to the information available at time t.[5] Immediate annuities, as well as deferred annuities (in the pay-out phase), are irreversible decisions, i.e., the policies cannot be sold or canceled.

Figure 1 illustrates how the individual's consumption and wealth evolve over time (conditional on survival) given that only immediate annuities are available.

Figure 1: Evolution of the Individual's Consumption and Wealth Over Time When Only Immediate Annuities are Available



[5] Pricing the annuity as in Equation (6) does not explicitly account for the possibility that mortality risks may be systematic, i.e., be stochastically dependent of capital market returns. This independence assumption follows the model used in Gründl et al. (2006). Note, however, that the loading factor L, can be interpreted as an implicit risk premium charged for systematic mortality risk. See also Dahl and Møller (2008), Ludkovski and Young (2008), and Delong (2009) for pricing approaches under stochastic mortality, and Van de Ven and Weale (2008) for a general equilibrium analysis of annuity pricing under stochastic mortality. For the case of stochastic interest rates, see Nielsen and Zenios (1996).

The utility the individual receives in this scenario (the first annuitization decision alternative) serves as the benchmark utility, i.e., it is the utility in a world without deferred annuities.

4.2.3 Immediate and Deferred Annuities are Available

Under the second alternative, both deferred annuities and immediate annuities are available. The deferred annuity considered here is a variant of a variable annuity that allows the policyholder a maximum amount of flexibility during the accumulation phase with respect both to consumption purposes and the amount to be annuitized at retirement age. The only parameter that is fixed at $t = 0$ is the future conversion factor, i.e., the payout per \$1 premium paid at age 65 should the individual annuitize instead of taking the lump sum. This flexibility is achieved by a variable annuity having the following contract characteristics:[6]

- Single premium payment, paid at $t = 0$;
- Money that is invested at $t = 0$ is accumulated in a fund earning the risk-free return R_f (as savings outside an annuity would earn),
- Guaranteed minimum death benefit (during the deferral period) (GMDB) equal to the fund value
- Guaranteed minimum withdrawal benefit (GMWB) smaller or equal to the fund value (during the deferral period);
- Guaranteed minimum income benefit (GMIB) granting the annuity payout per \$ of the fund value at retirement age or the right to take the fund value at the end of the deferral period as a lump sum.

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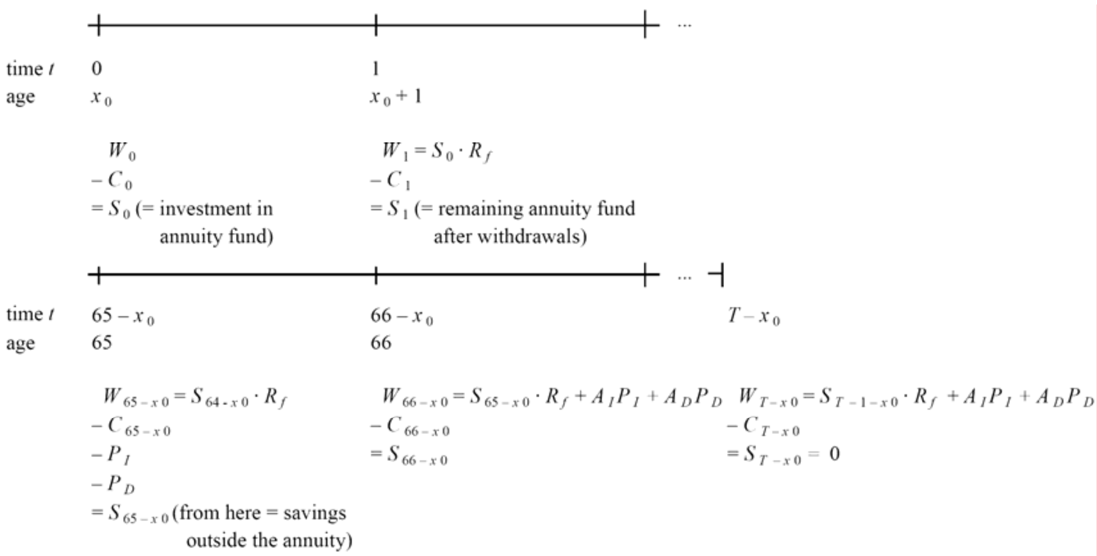
[6] For an overview of contract characteristics and options of variable annuities, see, e.g., Bauer et al. (2008).

In summary, then, because the amount invested at $t = 0$ earns the same return as private savings, withdrawals are possible, and in the case of death during the deferral period all remaining money would be paid out to heirs; the resulting contract structure is identical to the situation before age 65 where no annuities (or only immediate annuities) are available, i.e., private savings are perfectly replicated in the product. Formally, the fund value of the deferred annuity at the beginning of each period can identically be denoted by W_t before taking out money, and by S_t. afterward. Consequently, during the deferral period, we will abstract from the existence of the contract. The only difference between this situation and the one where only immediate annuities are available is that, at retirement age, the individual can choose between the conversion factors of the deferred annuity and, by taking out the fund value as a lump sum, the conversion factors given by the market in annuitizing his money.

Thus, at age 65, the individual can flexibly invest his wealth in this annuity by paying a premium P_D or refuse to invest and buy an immediate annuity with a price based on mortality information available at $t = 65 - x_0$ for paying the premium P_I, when exercising the lump-sum option.

Figure 2 illustrates the evolution of the individual's consumption and wealth over time (conditional on survival) given that both immediate and deferred annuities are available.

Figure 2: Evolution of the Individual's Consumption and Wealth Over Time When Both Immediate and Deferred Annuities are Available



The deferred annuity's payout per \$1 premium is given by:

$$1 = \left(1 + L\right) \cdot A_D \cdot \mathrm{E}_0 \left[\sum_{j=1}^{T-65} \frac{\left(\prod_{i=0}^{j-1} p_{i+65,65-x_0+i+1} \right)}{\left(R_f\right)^j} \right]. \tag{7}$$

This pricing mechanism is very similarly to that of Equation (6); the only difference being in the expected value operator, which is now conditional on the information available at $t = 0$. Note that in order to derive the *maximal* increase in utility an annuitant could derive from this product,[7] we do not include any price adjustment that accounts for the options inherent in this annuity. In other words, we are concerned with how much the individual would be willing to pay to have those options.

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[7] See Toplek (2007) and the references cited therein for the pricing of such options.

In general, the budget restriction at age 65 under the second alternative is $0 \le P_I + P_D \le W_{65-x_0}$, with neither P_I and P_D allowed to be negative. However, at age 65, depending on realization of the mortality index k_{65-x_0}, the individual will buy only one of the two annuity products. Whenever $k_{65-x_0} < \mathrm{E}_0(k_{65-x_0})$, i.e., mortality rates are smaller than expected at $t = 0$, the individual will buy the deferred annuity since doing so will result in obtaining a better price for the annuity than that available on the current market. In case $k_{65-x_0} > \mathrm{E}_0(k_{65-x_0})$, he will buy annuities priced at current market rates. If $k_{65-x_0} = \mathrm{E}_0(k_{65-x_0})$, both types of annuity have the same payout per $ of premium and the individual is indifferent between them, as choosing an immediate annuity will yield the same utility as retaining the deferred one.

4.3 Calibration of Model Parameters

To empirically calibrate the Lee-Carter stochastic process for mortality we use data from 1950 to 2005 from the Human Mortality Database for U.S. males and females and estimate the parameters for the ages 30 to 100 using the demography package provided by Hyndman et al. (2008). The estimated parameters are given in Appendix A.

For the risk-free return, we use the sample mean of U.S. T-Bill returns as a proxy. Using the same sample period as for the Lee-Carter estimation (1950 to 2005), R_f is set to 1.0493 (see Morningstar, 2007). For inflation, we use the same sample period, resulting in a value of 0.0390 (see Morningstar, 2007).

The coefficient of relative risk aversion γ is set to 1, 2, or 3, and the subjective discount factor δ is set to 0.93 or 0.99, both of which are typical values in the literature (see, e.g., Laibson et al., 1998).

The loading factor L is either set to 0 (no loading) or to 0.1, which is in the range of pricing markups for the U.S. annuity market reported in Mitchell et al. (1999).

4.4 Objective Function and Solving Technique

The individual's objective is to maximize the expected utility of consumption:

$$\max_{C_t, P_I, P_D} \mathrm{E}_0\left(U\left(C\right)\right), \tag{8}$$

subject to consumption constraints:

$$\begin{aligned}
&C_0 = W_0 - S_0 \\
&C_t = \underbrace{S_{t-1} R_f}_{W_t} - S_t \quad \forall \quad t \in \{1, 2, ..., 64 - x_0\} \\
&C_{65-x_0} = \underbrace{S_{64-x_0} R_f}_{W_{65-x_0}} - S_{65-x_0} - P_I - P_D \\
&C_t = \underbrace{S_{t-1} R_f + A_I P_I + A_D P_D}_{W_t} - S_t \quad \forall \quad t \in \{66 - x_0, 67 - x_0, ..., T - x_0\},
\end{aligned} \tag{9}$$

subject to borrowing constraints:

$$\begin{aligned}
&C_0 = W_0 - S_0 \\
&0 \le S_t \le W_t \quad \forall \quad t \in \{1, 2, ..., 64 - x_0, 66 - x_0, 67 - x_0, ..., T - x_0\} \\
&0 \le S_{65-x_0} + P_I + P_D \le W_{65-x_0}
\end{aligned} \tag{10}$$

and subject to no-short-sale constraints:

$0 \leq P_I, 0 \leq P_D.$ (11)

The optimization problem (Equations (8)–(11)) is solved backward via stochastic dynamic programming. The Bellman equation for this problem depends on three state variables: time t, wealth W_t, and the mortality index k_t. The Bellman equation (with V denoting the value function) is given for $t = 0, 1, \ldots, T - x_0 - 1$ by

$$V_t\left(W_t, k_t\right) = \max_{C_t, P_I, P_D} \left\{U_t\left(C_t\right) + \delta \,\mathrm{E}_t\left[p_{x,t}\left(V_{t+1}\left(W_{t+1}, k_{t+1}\right)\right)\right]\right\}, \tag{12}$$

subject to the constraints of Equations (9)–(11).[8] In the last period, remaining wealth is consumed, and the value function is given by $U_{T-x_0}\left(W_{T-x_0}\right)$. The Bellman equation (Equation (12)) cannot be solved analytically; hence a numerical technique is used. First, at each point in time t, the W_t-state and the k_t-state spaces are discretized into a grid of $N \times M$ points, W_t^n, with $n = 1, 2, \ldots, N$, and k_t^m, with $m = 1, 2, \ldots, M$. To calculate the distribution of the one-period survival probabilities $p_{x,t}$, the distribution of the mortality index k_t is discretized using Gaussian quadrature methods. Since in the last period (i.e., at $t = T - x_0$), the value function $V_{T-x_0}\left(W_{T-x_0}\right)$ is given by $U_{T-x_0}\left(W_{T-x_0}\right)$, the numerical solution algorithm starts at the penultimate period (i.e., at $t = T - x_0 - 1$). For each (W_t^n, k_t^m) combination, Equation (12) is solved with the MATHEMATICA® 7.0 implemented nonlinear optimizer `NMaximize`, yielding the optimal decisions C_t^{nm} (W_t^n, k_t^m), $P_I^{nm}\left(W_{65-x_0}^n, k_{65-x_0}^m\right), P_D^{nm}\left(W_{65-x_0}^n, k_{65-x_0}^m\right)$, and the function value of $V_t(W_t^n, k_t^m)$. Next, a continuous function is fitted to the points $V_t(W_t^n, k_t^m)$, which delivers a continuous approximation of the value function $V_t(W_t, k_t)$. Finally, the problem is rolled back to the preceding period.

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8 Note that the decision on the optimal values for P_I and P_D is made only at $t = 65 - x_0$.

5. Results

5.1 The Welfare Gain Measure

To calculate the welfare gain of deferred annuitization, we use an equivalent wealth variation measure (see Brown, 2001). The general idea is to compare the expected utility of an individual having access only to immediate annuities with an individual who has access to both immediate and deferred annuities and express it in monetary terms.

The reference point for our analyses is the welfare gain the individual achieves through the availability of only immediate annuities WG_I. This welfare gain is calculated by comparing the expected utility of an individual having no access to any type of annuity versus an individual having access to immediate annuities. WG_I, is derived in Equation (14), i.e., by solving Equation (13) for $\Delta W_{0,I}$ and dividing it by W_0 to obtain a relative measure:

$$V_0\left(W_0, k_0 \mid P_I \geq 0, P_D = 0\right) = V_0\left(W_0 + \Delta W_{0,I}, k_0 \mid P_I = 0, P_D = 0\right), \tag{13}$$

$$WG_I = \Delta W_{0,I} \,/\, W_0. \tag{14}$$

WG_I measures how much expected utility increases translated into monetary terms when the individual can access the immediate annuity market (vs. having no access). Note that due to the CRRA-feature of the one-period utility function (Equation (5)), WG_I, for each combination of model parameters, is a constant, i.e., independent of W_0.

The welfare gain in the case that both immediate and deferred annuities are available (vs. no annuities at all), WG_{ID}, is derived according to Equations (15) and (16):

$$V_0\left(W_0,k_0 \mid P_I \geq 0,P_D \geq 0\right) = V_0\left(W_0 + \Delta W_{0,ID},k_0 \mid P_I = 0,P_D = 0\right), \tag{15}$$

$$WG_{ID} = \Delta W_{0,\,ID} \,/\, W_0. \tag{16}$$

To measure the sole impact of introducing deferred annuities into the market, the incremental welfare gain WG_D is given by:

$$WG_D = WG_{ID} - WG_I\,. \tag{17}$$

5.2 Numerical Results

To illustrate the impact of randomness in future mortality rates on future annuity payouts, we first show, in Figure 3, the distribution of payouts from an immediate annuity A_I and the fixed payout of the deferred annuity A_D at age 65 for an individual aged 30 at $t = 0$.

Figure 3: Distribution of Payouts per $1 Premium for Immediate Annuity A_I and Deferred Annuity A_D at Age 65 for a at t = 0 30-Year-Old Individual; Loading L = 0

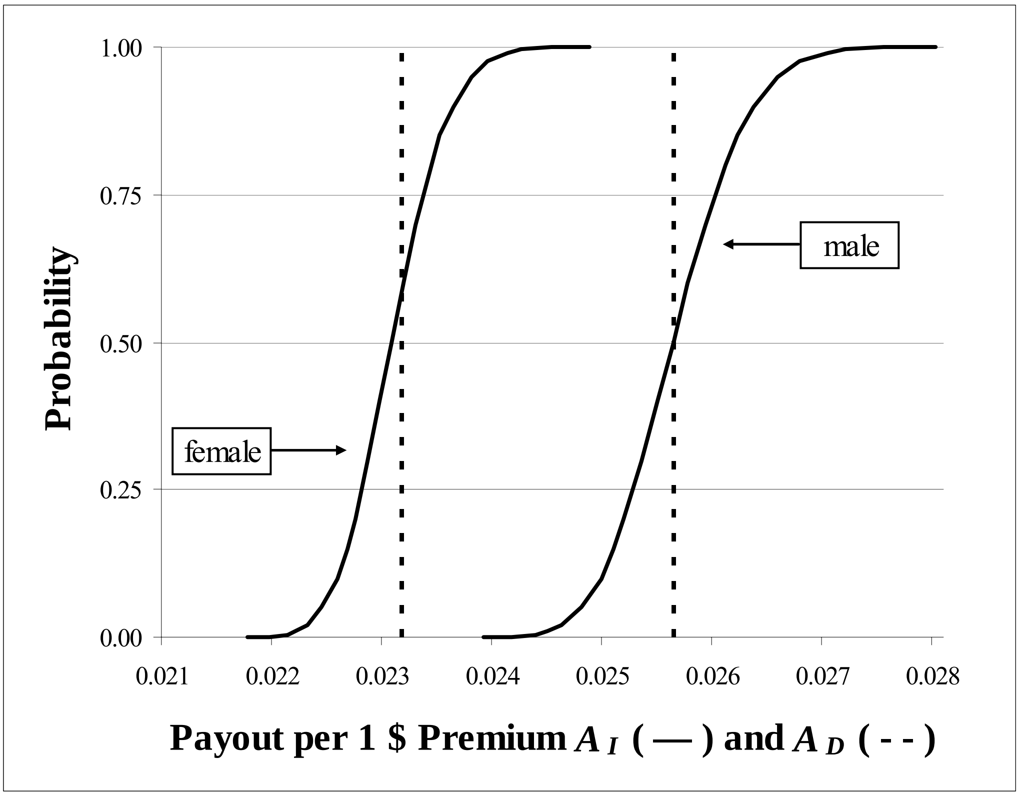

Figure 3 illustrates the option inherent in the deferred annuity. If the payout falls to the left of the dashed horizontal lines depicting the fixed payout from the deferred annuity A_D, the individual would stay with the deferred annuity. If, however, the payout falls to the right of the dashed lines, the individual would exercise the lump-sum option and buy immediate annuities.

The randomness of payouts influences both the welfare gain achieved from immediate annuitization WG_I or deferred annuitization WG_{ID} at age 65, as well as the optimal amount of money, P_I or P_D, to be annuitized. An example of both impacts, again for an individual initially aged 30, is shown in Figures 4 and 5. Here, the welfare gains and optimal amounts of money to be annuitized are shown as a function in the realized value of the mortality index at age 65 k_{65-x_0}.

Figure 4: Welfare Gain of Immediate Annuitization WG_I vs. Deferred Annuitization WG_{ID} at Age 65; Initial Age x_0 = 30, Gender = Male, Loading L = 0

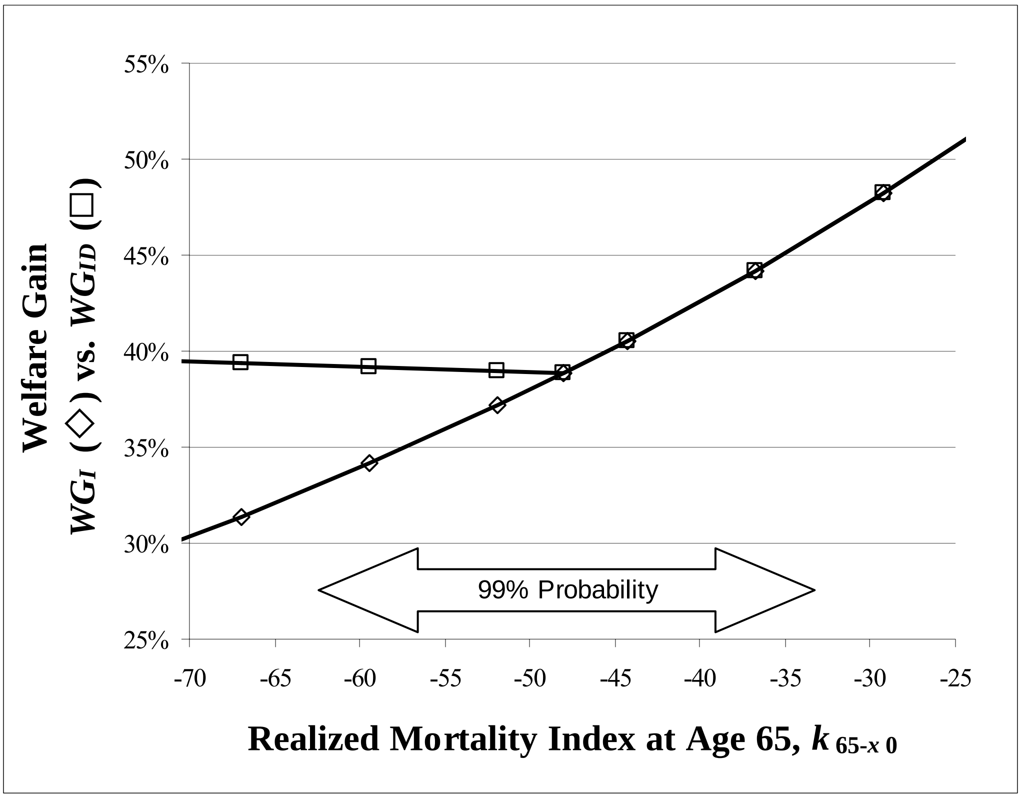

Figure 5: Optimal Amount of Money Annuitized at Age 65 as a Fraction of Wealth at Age 65 P_I / W_{65-x_0} vs. P_D / W_{65-x_0}; Initial Age x_0 = 30, Gender = Male, Loading L = 0

Figure 4 shows that the welfare gain of annuitization is increased by the availability of deferred annuities when the mortality index realizes at relatively low values. This is the case when mortality has decreased more than expected and the conversion factor from the deferred annuity grants better rates than the market. Furthermore, Figure 4 illustrates that stochastic mortality has an impact not only on the price of annuities, but on the utility evaluation as well, because the survival probabilities work as weights for future utility (compare Equations (4) and (12)). Due to this, the welfare gain for an individual who stays with the deferred annuity, even though the conversion factor is a constant, is not independent from the realized mortality index. If the individual buys the deferred annuity, the realized survival probabilities, i.e., the weights for future utility, are comparably high, and thus the welfare gain of deferred annuitization increases the smaller the realized mortality index becomes. This effect also explains why even in case of buying fixed-price deferred annuities, optimal annuitization as shown in Figure 5 is a function in the realized mortality index.

We next analyze the welfare gain at the point in time when the decision about investing savings in the deferred annuity fund must be made ($t = 0$). In particular, we look at the impact of model parameters on the welfare gain. As a measure of welfare gain we concentrate on the incremental welfare gain WG_D the individual experiences through the availability of deferred annuities (compare Equation (17)). Figure 6 plots the incremental welfare gain as a function in the initial age of the individual x_0, the relative risk aversion parameter γ, and the subjective discount factor δ.

Figure 6: Incremental Welfare Gain of Deferred Annuitization WG_{ID} at Time t = 0 as a Function in Age at t = 0 x_0; Gender = Male, Loading L = 0

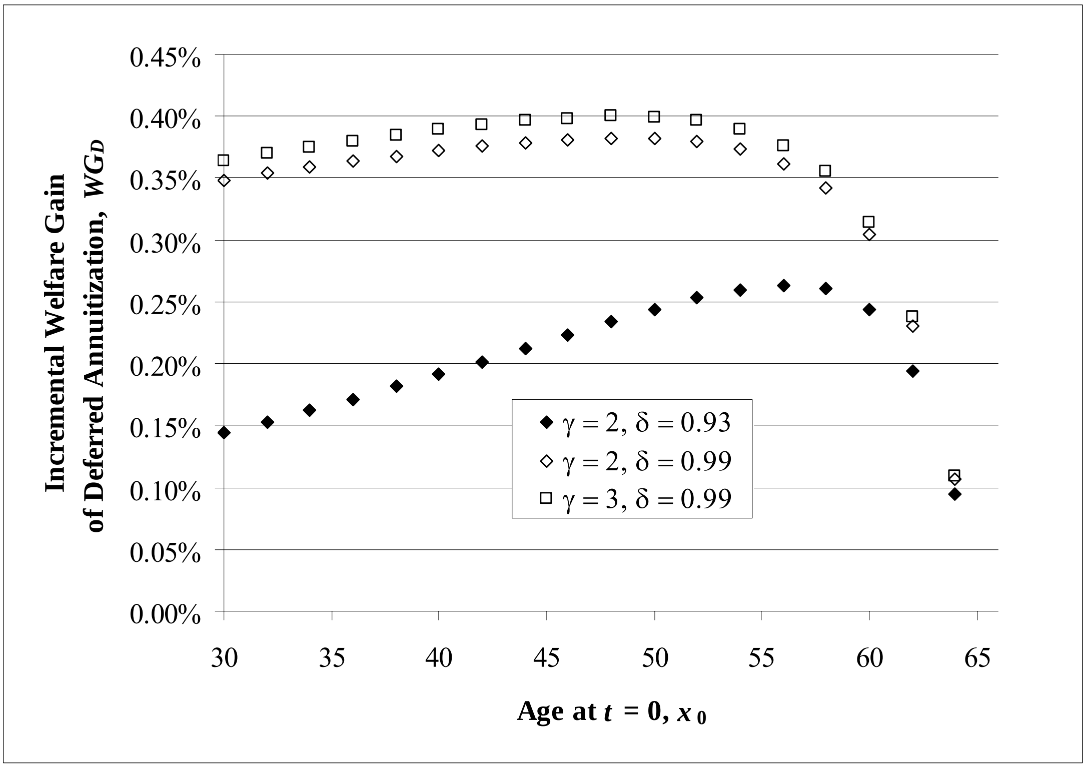

Figure 6 reveals the striking result that the incremental welfare gain at $t = 0$ is small, ranging between 0.09% and 0.4% of the individual's initial resources. Deferred annuitization can improve welfare at age 65 considerably, compared to immediate annuitization (compare Figure 2), but, from the perspective of the present, i.e., the age when the decision on investing savings into a deferred annuity has to be made, the incremental welfare gain is small. Two factors are responsible for this effect. First, the probability of realizing very large welfare gains from deferred annuities is rather small, as can be seen from the 99% confidence band for the realization of the mortality index at age 65, shown in Figure 4 for an individual aged 30. Second, the incremental welfare gain possibly realized at age 65 is evaluated at present time, i.e., after being discounted for many periods with the subjective discount factor δ and the survival probabilities (compare Equation (4)). The discounting effect is confirmed by comparing the curve for the subjective discount factor $\delta = 0.93$ with the higher welfare gains curve showing $\delta = 0.99$.

Both effects result in a hump-shaped age profile of the incremental welfare gain. For younger individuals, future welfare gains are heavily discounted, yielding an increasing function in age first. The older the individual is at $t = 0$, the fewer periods there are for mortality to fluctuate (the 99% confidence band for k_{65-x_0} becomes smaller). Due to this, the option value of deferred annuities decreases in initial age, which explains the decreasing part of the function,[9] where the effect of less heavily discounting is overcompensated by the shrinking option value.

Increasing risk aversion leads to larger incremental welfare gains because optimal annuitization increases, and the welfare gain of annuitization increases.

As a final variation in the model input parameters, we look at the impact of gender and the loading factor L on incremental welfare gains WG_D. The results can be found in Table 1, together with the welfare gains WG_{ID} for initial ages 30 and 50.

[9] This confirms the results of Menoncin (2008), who shows that the demand for mortality hedging instruments is decreasing over the life-cycle.

Table 1: Welfare Gain of Deferred Annuitization WG_{ID}, Incremental Welfare Gain of Deferred Annuitization WG_D, and Optimal Savings S_0 / W_0 at Time $t = 0$ and Impact of Gender, Loading L, Relative Risk Aversion γ, and Subjective Discount Factor δ

					Loading Factor L											
					0						0.1					
					γ						γ					
					1		2		3		1		2		3	
					δ		δ		δ		δ		δ		δ	
					0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99
Age x_0	30	Gender	Male	WG_{ID}	1.00%	9.41%	4.46%	11.65%	7.01%	12.60%	0.60%	6.64%	3.17%	8.60%	5.22%	9.42%
				WG_D	0.05%	0.32%	0.14%	0.35%	0.20%	0.36%	0.04%	0.31%	0.15%	0.35%	0.21%	0.37%
				S_0 / W_0	92.63%	97.49%	95.37%	97.60%	96.21%	97.65%	92.63%	97.49%	95.40%	97.63%	96.25%	97.69%
			Female	WG_{ID}	0.85%	8.26%	3.69%	9.32%	5.56%	9.69%	0.43%	5.15%	2.27%	5.97%	3.58%	6.24%
				WG_D	0.05%	0.33%	0.15%	0.35%	0.21%	0.36%	0.04%	0.32%	0.15%	0.35%	0.21%	0.37%
				S_0 / W_0	92.76%	97.66%	95.48%	97.74%	96.32%	97.77%	92.76%	97.66%	95.51%	97.77%	96.37%	97.82%
	50	Gender	Male	WG_{ID}	5.38%	19.02%	13.70%	23.17%	18.32%	24.84%	3.43%	13.84%	10.04%	17.53%	13.95%	18.99%
				WG_D	0.14%	0.35%	0.24%	0.38%	0.29%	0.40%	0.12%	0.34%	0.24%	0.37%	0.29%	0.39%
				S_0 / W_0	91.74%	96.24%	94.34%	96.45%	95.15%	96.55%	91.74%	96.24%	94.43%	96.53%	95.27%	96.66%
			Female	WG_{ID}	4.42%	16.12%	11.11%	18.12%	14.27%	18.73%	2.43%	10.59%	7.22%	12.25%	9.68%	12.72%
				WG_D	0.08%	0.20%	0.14%	0.21%	0.17%	0.22%	0.06%	0.19%	0.14%	0.21%	0.16%	0.22%
				S_0 / W_0	92.05%	96.61%	94.64%	96.77%	95.45%	96.85%	92.05%	96.61%	94.73%	96.85%	95.57%	96.96%

Gender has an impact on the welfare gains of deferred annuitization WG_{ID}, resulting in higher gains for males. Males have lower survival probabilities and thus the mortality credit of the annuity is larger for them.[10] For males the incremental gains WG_D are also higher at age 50, confirming that annuitization is more utility enhancing for them. The incremental gains are nearly identical for both genders at age 30 because the originally higher gains for males at annuitization age 65 are (due to their lower survival probabilities) more heavily discounted to $t = 0$, which is more pronounced for younger individuals.

The impact of the loading factor on the welfare gains of deferred annuitization WG_{ID}, is straightforward. Making annuities more expensive decreases their attractiveness. The incremental welfare gain WG_D, however, is only barely affected by introducing a loading. Deferred annuitization becomes less attractive but, at the same time, the benchmark for measuring the incremental gain, the welfare gain in a world with only immediate annuities WG_I, also decreases with a positive loading factor.

With respect to the pricing of deferred annuities, Table 1 indicates that the price markups above the expected value of payouts an insurer could charge would be fairly small. The maximal price markup can be calculated by setting the incremental welfare gain WG_D (i.e., the amount of money the individual is willing to give up in order to have access to deferred annuities) in relation to the amount of money invested in the deferred annuity fund at $t = 0$, i.e., savings S_0. With fairly priced (expected value of payouts = price) annuities,

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[10] This result also shows the general direction the results would change if a specific mortality table for annuitants, reflecting their above-average life expectancy, is considered. Welfare gains would decrease for both the typical annuitant and typical nonannuitant. For typical annuitants, the lower mortality credit drives this result; for

the range for maximal price markups is 0.05% to 0.4% of the money paid into the deferred annuity at $t = 0$. If both immediate and deferred annuities already have a 10% loading factor, the additional price markup ranges between 0.04% and 0.4%.

6. Summary and Discussion

Deferred annuities improve the welfare of a risk-averse individual in the presence of stochastic mortality. Our analysis confirms the results in the optimal annuitization literature for the case of both deterministic and stochastic mortality for immediate annuities and in the case of deterministic mortality for deferred annuities.

The incremental gains, i.e., the option value connected to stochastic mortality, of deferred annuities appear to be small. In pricing these products, an insurer can expect that CRRA-individuals are willing to pay only around 0.04% to 0.4% of the money invested in the deferred annuity fund at the beginning of the deferral period in exchange for an option right related to stochastic mortality improvements (given that the benchmark investment, the immediate annuity, comes with the same initial loading factor L). In general, the incremental gains and possible price markups are higher for individuals who are 45 to 60 years of age, are more patient and have greater risk aversion.

In contrast to actual price markups for options related to deferred annuitization in variable annuities (GMIB's), the price markups calculated here seem to leave no room for a market because the actual markups are in the range of

typical nonannuitants, i.e., individuals with average mortality, the increased unfairness of annuities makes annuitization less valuable.

0.5% to 0.75% *per annum* of the fund value during the deferment period (see, e.g., Bauer et al., 2008). It should be noted, however, that the products usually also allow investment in risk assets, such as mutual funds. Thus, the price charged needs to cover more than the stochastic mortality driven part of the option, including, for example, minimum interest rate guarantees, which are, of course, also valuable from the individual's perspective (see Milevsky and Kyrychenko, 2008).

A possible policy implication of our results is that mandatory annuitization schemes should not necessarily require the purchase of deferred annuities because the option value from the individual's perspective is very small and may be easily overcompensated by price markups by insurance companies.

Our work could be extended by considering shocks to individual mortality, e.g., due to health risks as in Horneff et al. (2008b) and Davidoff (2009). In this case, the option value inherent in deferred annuities will increase because the variation of mortality from the individual's perspective will increase. Another idea for future research is to consider deferred annuities where the amount of money is already fixed at t = 0. In this case, the welfare gains of deferred annuitization could either increase or decrease. Increases could occur due to the higher mortality credits of such products (see Gong and Webb, 2007) because, usually, if death occurs during the deferment period no money is returned (while payouts in case of survival are higher). Decreases could occur due to the higher utility costs of inflexibility with respect to consumption needs during the deferment period and the amount of money to be annuitized at retirement age.

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Appendix A. Estimated Parameters for the Lee-Carter Model

	Males		Females			Males		Females	
Drift of k_t: θ	-0.6818		-0.8209						
Standard deviation: σ_ε	0.9766		1.1526						
Age x	a_x	b_x	a_x	b_x	Age x	a_x	b_x	a_x	b_x
30	-6.3127	0.0085	-7.0801	0.0170	66	-3.4843	0.0195	-4.0889	0.0130
31	-6.2663	0.0078	-6.9910	0.0167	67	-3.4027	0.0194	-3.9985	0.0135
32	-6.2160	0.0086	-6.9201	0.0176	68	-3.3223	0.0182	-3.9098	0.0132
33	-6.1676	0.0086	-6.8464	0.0171	69	-3.2524	0.0174	-3.8318	0.0134
34	-6.1361	0.0090	-6.7844	0.0175	70	-3.1577	0.0175	-3.7182	0.0141
35	-6.0676	0.0098	-6.7042	0.0179	71	-3.0931	0.0161	-3.6469	0.0135
36	-6.0042	0.0102	-6.6238	0.0172	72	-2.9928	0.0168	-3.5229	0.0147
37	-5.9372	0.0104	-6.5456	0.0172	73	-2.9212	0.0158	-3.4373	0.0147
38	-5.8428	0.0111	-6.4402	0.0169	74	-2.8412	0.0162	-3.3465	0.0158
39	-5.7948	0.0134	-6.3820	0.0185	75	-2.7567	0.0161	-3.2405	0.0165
40	-5.7111	0.0140	-6.2908	0.0184	76	-2.6804	0.0153	-3.1517	0.0162
41	-5.6254	0.0136	-6.2080	0.0176	77	-2.6034	0.0145	-3.0632	0.0161
42	-5.5307	0.0155	-6.1041	0.0183	78	-2.5252	0.0143	-2.9610	0.0169
43	-5.4539	0.0157	-6.0209	0.0178	79	-2.4405	0.0140	-2.8589	0.0167
44	-5.3787	0.0169	-5.9520	0.0179	80	-2.3416	0.0141	-2.7539	0.0159
45	-5.2841	0.0179	-5.8633	0.0177	81	-2.2672	0.0125	-2.6699	0.0147
46	-5.1960	0.0177	-5.7745	0.0174	82	-2.1742	0.0129	-2.5537	0.0150
47	-5.1048	0.0187	-5.6862	0.0177	83	-2.0837	0.0126	-2.4449	0.0151
48	-5.0079	0.0186	-5.5898	0.0164	84	-1.9961	0.0126	-2.3385	0.0151
49	-4.9389	0.0207	-5.5222	0.0177	85	-1.9138	0.0122	-2.2368	0.0148
50	-4.8350	0.0217	-5.4256	0.0179	86	-1.8272	0.0118	-2.1354	0.0140
51	-4.7484	0.0211	-5.3452	0.0164	87	-1.7463	0.0114	-2.0373	0.0134
52	-4.6526	0.0218	-5.2513	0.0165	88	-1.6758	0.0102	-1.9505	0.0124
53	-4.5734	0.0220	-5.1751	0.0161	89	-1.5901	0.0100	-1.8516	0.0116
54	-4.4915	0.0229	-5.1022	0.0163	90	-1.5085	0.0097	-1.7406	0.0121
55	-4.3999	0.0221	-5.0221	0.0150	91	-1.4471	0.0078	-1.6747	0.0095
56	-4.3183	0.0220	-4.9356	0.0143	92	-1.3567	0.0074	-1.5694	0.0093
57	-4.2369	0.0221	-4.8589	0.0139	93	-1.2755	0.0066	-1.4768	0.0085
58	-4.1351	0.0209	-4.7559	0.0127	94	-1.2085	0.0051	-1.3886	0.0079
59	-4.0633	0.0218	-4.6867	0.0133	95	-1.1437	0.0042	-1.3110	0.0068
60	-3.9609	0.0216	-4.5848	0.0136	96	-1.0715	0.0030	-1.2370	0.0055
61	-3.8862	0.0207	-4.5060	0.0124	97	-1.0183	0.0018	-1.1663	0.0044
62	-3.7818	0.0206	-4.3951	0.0130	98	-0.9831	-0.0003	-1.1101	0.0031
63	-3.7137	0.0205	-4.3290	0.0129	99	-0.9600	-0.0040	-1.0858	-0.0008
64	-3.6384	0.0204	-4.2545	0.0132	100	-0.8900	-0.0023	-0.9931	0.0014
65	-3.5464	0.0207	-4.1519	0.0139					

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