

SFB 649 Discussion Paper 2007-064

# Correlation vs. Causality in Stock Market Comovement

Enzo Weber\*



\* Freie Universität Berlin, Germany

This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>  
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin  
Spandauer Straße 1, D-10178 Berlin



SFB 649 ECONOMIC RISK BERLIN

# Correlation vs. Causality in Stock Market Comovement<sup>1</sup>

Enzo Weber

Universität Mannheim and Freie Universität Berlin

Boltzmannstr. 20, 14195 Berlin, Germany

eweber@wiwiss.fu-berlin.de

phone: +49 30 838-55792      fax: +49 30 838-54142

First version: 09/2007

This version: 10/2007

## Abstract

This paper seeks to disentangle the sources of correlations between high-, mid- and low-cap stock indexes from the German prime standard. In principle, such comovement can arise from direct spillover between the variables or due to common factors. By standard means, these different components are obviously not identifiable. As a solution, the underlying study proposes specifying ARCH-type models for both the idiosyncratic innovations and a common factor, so that the model structure can be identified through heteroscedasticity. The seemingly surprising result that smaller caps have higher influence than larger ones is explained by asymmetric information processing in financial markets. Broad macroeconomic information is shown to enter the common factor rather than the segment-specific shocks.

*Keywords:* Identification, Spillover, Common Factor, Structural EGARCH, DAX

*JEL classification:* C32, G10

---

<sup>1</sup>This research was supported by the Deutsche Forschungsgemeinschaft through the CRC 649 "Economic Risk". I am grateful to Jürgen Wolters and Cordelia Thielitz for their help. Of course, all remaining errors are my own.

# 1 Introduction

Different stocks or portfolios often reveal a high degree of coherence in their fluctuations. For example, from 2001 till 2007 daily open-to-close returns of the German blue chip index DAX exhibit a 70% correlation with the mid-cap index MDAX and a 43% correlation with the small-cap index SDAX. In every-day business, this type of comovement is normally taken for granted, and the financial press often finds seemingly plausible explanations for any observed market outcome. Econometrically however, determining reasons and sources of contemporaneous correlations turns out to be an intricate task.

On a very general basis, comovement could result from instantaneous spillover between the relevant variables or from common exogenous factors driving all of them alike. Concerning the first alternative, the direction of the transmission effect represents a further refinement. In statistics, the negative statement, "correlation does not imply causation" is widespread; in short, this paper intends to develop a positive answer to the question, to which extent a specific correlation has a real causal nature, and to which it is based on third-party intervention.

In the language of the financial markets example, equal development of different stock indexes can have two reasons: First, observing the realised index movements in one market might influence the decisions of participants in another. This logically represents the case of direct causal spillover. Second, certain information obtained by participants in several markets during parallel trading time might be judged equally relevant, logically generating immediate and similar stock price reactions in the segments concerned. It follows that the current study has to unite two types of econometric analyses, one occupied with financial transmission in terms of direct causality, and the other one seeking for the effect of "news" as fundamental factors triggering market responses.

A straightforward way of examining links between financial variables is given by choosing markets with non-overlapping trading time. For example, when the New York Stock Exchange opens, closing prices in Tokyo are already established. Therefore, the direction of propagation can be defined running from the respective daytime to overnight returns. Nonetheless, even here a real direct causal impact is not yet separated from pure incorporation of news arrived overnight, yet already manifested in the daytime trade of a different time zone. What's more, in case of parallel trading, naturally given for equity indexes of the same nationality, the issue is additionally complicated by the possibility of bi-directional spillover. Inevitably, the discussion results in a classical econometric identification problem. In this context, consider for illustrational purposes the standard solu-

tion to identifying a structural vector autoregressive (SVAR) model: Usually, a Choleski decomposition is applied to the reduced-form residual covariance-matrix, leading to a triangular matrix of recursive instantaneous effects. Furthermore, the correlation of the structural innovations has to be assumed zero. Consequently, two possible sources of contemporaneous correlation, that is half of the causal impacts (the *interdependence*) and the non-causal connections, are assumed inexistent in this setting.

A small strand of recent literature introduced a method that exploits non-constant variances mostly of financial variables to address the simultaneity problem: Following the idea that every shift in the covariance-matrix yields more determining equations than unknown coefficients, the model structure can be identified "through heteroscedasticity" (see Rigobon 2003). Building upon this logic, further research proposed estimating ARCH-type processes either for the reduced-form (Rigobon 2002) or structural residuals (Weber 2007a), thereby providing a continuum of variance regimes. Further contributions in this area comprise King et al. (1994) and Sentana and Fiorentini (2001).

As a drawback, the existent methodology either implicitly assumes that the contemporaneous correlation results exclusively from common factors, or that it is to the full extent a product of causal interaction only between the included variables.<sup>2</sup> The first variant obviously fails to detect causality between the observed endogenous variables, which might play an important role for indexes as closely connected as the DAX segments. Concerning the second variant, in presence of neglected exogenous shocks, the estimation is bound to overstate the bilateral linkage. While this problem might in principle be treated by augmenting the model with essential missing variables, much relevant information will be unobservable or can hardly be completely covered by necessarily low-dimensional time series systems. As a consequence, the importance of allowing for contemporaneous interaction in the structural innovations is stressed. However, as will be shown in the following section, unrestricted time-varying covariances would simply undo the identifiability created by heteroscedasticity.

Contributing to the progress in this research field, this paper tackles the discussed issue by including an unobservable common factor into a structural heteroscedastic VAR of DAX, MDAX and SDAX returns. Since this establishes only a single additional unknown magnitude (the factor variance) per volatility regime, full identification can still be achieved.<sup>3</sup> Furthermore, latent factor modelling allows covering general exogenous

---

<sup>2</sup>A notable exception is the SCCC model in Weber (2007b), which assumes constant conditional correlation of the structural innovations.

<sup>3</sup>For a general discussion of identification in heteroscedastic factor models see as well Sentana and

influences, which do not have to be observed or even predisposed. Estimates of model parameters, factors states and conditional variances are obtained by a Kalman filtering Quasi-Maximum-Likelihood (QML) procedure.

In the equity market application, one might intuitively ascribe the strongest causal influences to the leading blue chip index DAX. Nonetheless, empirical results reveal just the opposite: Besides a strong common factor, the contemporaneous correlation of stock index returns mainly arises from spillovers running from "smaller" to "larger" indexes. Before I present details and interpretation of the obtained results and further assess the identified factor in section 3, the methodological concept is discussed at length in the following. In the end, a summary provides a short overview of outcome, merits and further potentials of the present examination.

## 2 Methodology

In the following paragraphs, a model is constructed that shall finally feature and identify both mutual and common influences among a set of variables. To begin with, think of the data generating process of the  $n$  stock returns  $y_{it}$  being approximated by the SVAR model with lag length  $q$

$$Ay_t = \mu_0 + \mu_1 d_t + \sum_{j=1}^q B_j y_{t-j} + \varepsilon_t, \quad (1)$$

where the  $B_j$  represent  $n \times n$  coefficient matrices of lagged effects, and  $\varepsilon_t$  is an  $n$ -dimensional vector of uncorrelated structural residuals. The contemporaneous impacts are included in the matrix  $A$  with diagonal elements normalised to one. The deterministic terms are a constant and centred daily seasonal dummies ( $d_t$ ), which control for possible day-of-the-week effects.

Representing a classical simultaneous equation system, (1) as it stands is not identified and therefore cannot be consistently estimated. A first step thus derives the reduced-form VAR

$$y_t = \mu_0^r + \mu_1^r d_t + \sum_{j=1}^q B_j^r y_{t-j} + u_t. \quad (2)$$

All coefficients are obtained by premultiplying  $A^{-1}$  in (1), therefore being marked by the superscript  $r$  for "reduced". Accordingly, the new residuals are given by  $u_t = A^{-1}\varepsilon_t$ .

---

Fiorentini (2001), amongst others.

While the reduced form (2) can be estimated simply applying OLS, it proves impossible to recover the structural parameters without further constraints: In the matrix  $A$  with normalised diagonal,  $n(n - 1)$  simultaneous impacts have to be estimated. Since those are not included in the systematic part of the reduced form, all contemporaneous interaction is reflected in residual cross-correlation. However, the information contained in the covariance-matrix of the reduced-form residuals is not sufficient for identification; due to its symmetry, it delivers only  $n(n - 1)/2$  equations for simultaneous covariances, for instance leading to a lack of  $3(3 - 1)/2 = 3$  equations in the above-mentioned three-dimensional DAX example. The standard solution now would be to impose a recursive structure on the contemporaneous impacts, thereby restricting  $A$  to a triangular matrix. However, this would effectively imply that the research question of uncovering direction and strength of mutual spillovers would have to be answered *a priori* for some theoretical, but not for empirical reasons. Albeit, one might want to exclude contemporaneous impacts running from MDAX and SDAX on the DAX and from SDAX on MDAX, based on the intuition that major segments should not be affected by minor ones. Section 3 will show that such a strategy would be totally at odds with underlying economic reality.

Heading towards a more appropriate solution, assume for example that it is possible to identify two separate time regimes with differing variances of the uncorrelated structural residuals  $\varepsilon_t$ : The variance shift between the regimes delivers two distinct reduced-form covariance-matrices, so that  $n(n - 1)/2$  additional covariance equations and  $n$  additional variance equations are obtained from the second matrix. Since the number of free parameters rises only by  $n$ , the number of structural variances, identification can be achieved "through heteroscedasticity" (e.g. Rigobon 2003). While time-varying volatility has become a common feature throughout the empirical financial literature, determining a valid date for imposing a single shift in variance is naturally problematic. Therefore, in this point I will follow the econometric procedure in Weber (2007a), who specifies multivariate EGARCH processes for the structural residuals. This basically keeps up the intuition of identification through volatility regimes. An ARCH-type model however practically defines a distinct variance state for every single observation. This can be thought of as modelling a quasi continuum of regimes, which is reflected in the estimated conditional variances.

Before tackling model setup and estimation in more detail, let us turn our attention to a last problematic point: The effectiveness of the strategy of defining volatility regimes critically hinges on conditional uncorrelatedness of the structural residuals. Allowing for unrestricted time-varying covariances would lead to the unfavourable situation that each

shift in variance introduces as many structural parameters (variances and covariances) as additional equations from the reduced-form covariance-matrices. Thus, the volatility regimes could not deliver the additional information needed to identify the model structure. However, maintaining the uncorrelatedness assumption implies that the contemporaneous correlation of the variables in  $y_t$  is to be fully taken into account by instantaneous causal spillovers between the included variables. Returning to the DAX example, it seems extremely unlikely that the three German indexes are not subject to any exogenous common factors, which might at least partly trigger the observed substantial correlations.

As the main contribution to the literature, the present study formally includes a common factor into a simultaneous heteroscedastic VAR. This allows for time-varying interaction in the structural innovations as opposed to the generally unrealistic uncorrelatedness assumption. Keeping with the DAX example, this makes it necessary to estimate one factor variance per volatility regime, on top of the three variances belonging to the idiosyncratic innovations. Since a  $3 \times 3$  reduced-form covariance-matrix delivers six additional equations per shift, it proves still possible to make up the gap between available information and unknown coefficients. Identifiability is thus preserved by representing the dynamic covariance structure in a parsimonious factor setup.

Formalising the preceding argumentation, add the scalar<sup>4</sup> factor  $z_t$  multiplied by the  $n \times 1$  vector of loadings  $\beta$  to equation (1). Since  $z_t$  is not assumed observable, the reduced-form residuals  $u_t$  then result as

$$u_t = A^{-1}(\beta z_t + \varepsilon_t). \quad (3)$$

All  $\varepsilon_{jt}$  and  $z_t$  are conditionally uncorrelated, including leads and lags.

Furthermore, define  $e_t = \left( \varepsilon_{1t} \quad \dots \quad \varepsilon_{nt} \quad z_t \right)'$ , and denote the conditional variances of the elements in  $e_t$  by

$$\text{Var}(e_{jt}|\Omega_{t-1}) = h_{jt} \quad j = 1, \dots, n, z, \quad (4)$$

where  $\Omega_{t-1}$  stands for the whole set of available information at time  $t - 1$ .

Then, stack the conditional variances in the vector  $H_t = \left( h_{1t} \quad \dots \quad h_{nt} \quad h_{zt} \right)'$ .

At last, denote the standardised white noise innovations by

$$\tilde{e}_{jt} = e_{jt}/\sqrt{H_{jt}} \quad j = 1, \dots, n + 1. \quad (5)$$

The multivariate EGARCH(1,1)-process following Weber (2007a) is then given by

$$\log H_t = C + G \log H_{t-1} + D(|\tilde{e}_{t-1}| - \iota\sqrt{2/\pi}) + F\tilde{e}_{t-1}, \quad (6)$$

---

<sup>4</sup>The extension to multiple factors is straightforward.

where  $C$  is a  $(n + 1)$ -dimensional vector of constants<sup>5</sup>,  $G$ ,  $D$  and  $F$  are  $(n + 1) \times (n + 1)$  coefficient matrices, and  $\iota$  denotes a column vector of  $(n + 1)$  ones. The absolute value operation is to be applied element by element.  $\sqrt{2/\pi}$  is subtracted to demean the absolute value terms, see Nelson (1991).

Due to the conditional uncorrelatedness of the idiosyncratic and common factors, the multivariate extension (6) of Nelson (1991) can avoid explicitly modelling any conditional covariances. Asymmetric effects are incorporated by including  $\tilde{\varepsilon}_t$  without taking absolute values: Any parameters in  $F$  differing from zero indicate that besides the magnitude of a shock its sign contains valuable information for forecasting the conditional variances. The process orders 1,1 seem to be appropriate for most series, what will be shown by ARCH-LM tests and is quite usual in financial econometrics (see Nelson 1992). Apart from that, higher-order lags would considerably complicate the likelihood optimisation.

By definition (3), the conditional covariance-matrix of the reduced-form residuals  $u_t$  is obtained as

$$\Sigma_t = A^{-1}(\beta h_{zt}\beta' + \begin{pmatrix} h_{1t} & 0 \\ & \ddots \\ 0 & h_{nt} \end{pmatrix})(A^{-1})'. \quad (7)$$

Since the log-linearised EGARCH system (6) necessarily delivers positive conditional variances, the quadratic form (7) conveniently solves the common problem of assuring the covariance matrix to be positive definite. Furthermore, two sources of cross-correlation, as represented by non-zero off-diagonal elements, become evident: First, the common factor  $z_t$  naturally produces a certain degree of comovement, and second, changes in a variable can instantaneously spill over according to the coefficients in  $A^{-1}$ . The task is to determine the contributions of both effects to the overall correlation as well as the specific directions of spillover.

At this stage, identifiability can be discussed concretely for the given model. In the trivariate DAX example with one factor, the variance process (6) contains three free parameters in  $C$  and 16 each in  $G$ ,  $D$  and  $F$ . Together with the six parameters from the structural matrix  $A$  and three from  $\beta$ , this sums up to 60 coefficients. This has to be compared to the number arising from the reduced-form process for  $vech(\Sigma_t)$ , where the  $vech$  operator stacks the lower triangular portion of a matrix into a column vector. For the given example, this vector includes three variances and three covariances. Thus, in a completely general trivariate MGARCH, the equivalent of  $C$  would have dimension  $6 \times 1$  and the equivalents of  $G$ ,  $D$  and  $F$  would be  $6 \times 6$ . However, from (7) it can be seen that

---

<sup>5</sup>The last element is set to zero, in order to normalise the unconditional factor variance to unity.



all six reduced-form second moments are linear combinations of the three idiosyncratic variances and the single factor variance. This implies that only four of the moments are distinct, whereas the remaining two linearly depend on the first four. As a consequence, two of the six reduced-form MGARCH equations are redundant, reducing the relevant row dimension to four. The number of actually free parameters then comes to a total of  $4 + 3(4 \cdot 6) = 76$ , which still exceeds 60 and therefore satisfies the necessary summing-up constraint. As a sufficient condition, linear independence of the structural variances is required. Otherwise, the reduced-form moments could be written in terms of even less than four structural variances. Since this would render further reduced-form MGARCH equations redundant, the number of assessable parameters could fall below the number of unknowns. Under normal conditions, ARCH-type processes should meet the latter criterion.

The estimation is done by Maximum Likelihood. Since the common factor is non-observable, approximate Kalman filtering is used to construct the log-likelihood under the assumption of conditional normality as

$$L(A, \beta, C, G, D, F) = -\frac{1}{2} \sum_{t=1}^T (n \log 2\pi + \log |\Sigma_t| + u_t' \Sigma_t^{-1} u_t) . \quad (8)$$

Maximisation of (8) yields estimates both of the EGARCH parameters as well as the structural coefficients in  $A$  and  $\beta$ . From the latter, the respective influences underlying the stock correlation can be retained. Furthermore, the procedure delivers estimates of the states of the common and idiosyncratic factors from the Kalman filter as well as their conditional variances from the EGARCH process. In the preceding theoretical model equations, the unobservable magnitudes are straightforwardly replaced by these estimates.

Since assuming conditional normality is often problematic using financial markets data, the estimation relies on Quasi-Maximum-Likelihood (see Bollerslev and Wooldridge 1992). While excess kurtosis may be taken as an argument for adopting for example a Student-t-distribution, QML has the advantage of consistency even if the distributional assumption is violated. The Kalman filter likelihood optimisation is implemented applying the BHHH algorithm (Berndt et al. 1974).

## 3 The German Stock Market

### 3.1 Data

The data consists of daily returns of three indexes from the German prime standard, namely the DAX, the Mid-Cap-DAX "MDAX" and the Small-Cap-DAX "SDAX". While the DAX contains the 30 strongest German companies according to market capitalisation and transaction volume, the MDAX represents the 50 stocks following the DAX and the SDAX the 50 following the MDAX. All stocks are traded in the Xetra-system of the Frankfurt stock exchange during the same trading hours. The sample begins in January 2001, from whereon SDAX opening prices were available, and ends in August 2007; the data source is Reuters.

Open-to-close returns are used since this paper is concerned with spillover effects, which cannot appear during overnight periods when stock prices are not updated. Especially for the smaller segments, recorded opening prices might be plagued by the well-known stale-quote problems. However, since a robustness check with close-to-close data did not lead to substantial deviations, the empirical outcome should not be sensitive in this respect. A possible extension is given by using higher-frequency data to circumvent the difficulties connected to opening prices (e.g. Baur and Jung 2006). By the same token, the term "contemporaneous" has to be interpreted in relation to daily data; naturally, at least in continuous time, any "causality" necessarily results from a certain time structure, which could be further assessed for instance on the basis of tick data. Nonetheless, the present application avoids the whole range of market microstructure "contamination" like bid-ask-spreads, asynchronous trading or price discreteness. Above all, it should be emphasized that the developed methodology represents a general solution to identification problems, which carries over to a large set of variables irrespective of available data frequencies.

Figure 1 presents the return series and an overview of the index development. Falling stock prices appear after 2000 due to the "new economy" bubble burst and a general recession. Naturally, this is exactly the period with the highest stock market volatility, in addition to a shorter turbulence after the 9/11 terrorist attacks. Note that the rise in variance is most pronounced for the DAX, followed by MDAX and SDAX. The unconditional standard deviations in the same order are given by 1.50, 0.88 and 0.69.

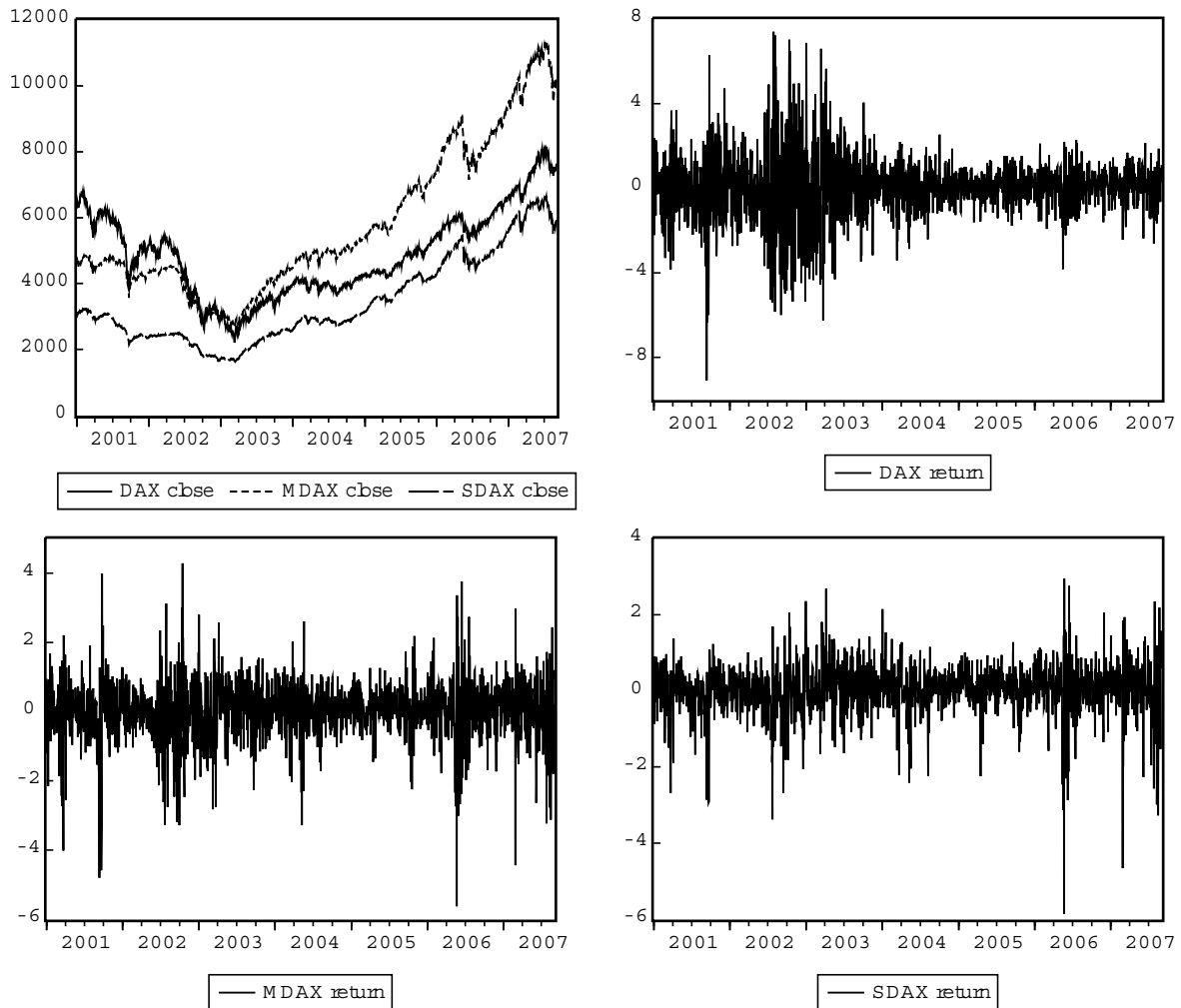


Figure 1: DAX, MDAX and SDAX indexes and open-to-close returns

### 3.2 Specification and Estimation

As a first step, I specify a trivariate reduced-form model as in (2), including the DAX, MDAX and SDAX returns as endogenous variables. Five lags have been chosen following the Akaike information criterion. Ljung-Box tests, mostly with p-values around 0.9, confirmed that this is sufficient to capture any possible serial correlation. While one might naturally exclude any lagged regressors for theoretical reasons, the chosen specification avoids spurious results and does practically not reduce the number of degrees of freedom relative to the sample size of 1740 observations. The next steps are based on the residuals obtained from the estimated VAR.

Due to the well-known problems with numerical ML optimisation of highly interdependent systems, for convenience I deleted all cross-coefficients in  $G$  and  $D$  as well as the

asymmetry matrix  $F$  from the model.<sup>6</sup> While this has the disadvantage of neglecting causality-in-variance and leverage effects, identification of the mean equations as the key issue can still be achieved. Starting values were obtained as follows: The initial factor was extracted in a static ML estimation and standardised to unit variance. Then, using the respective loadings in  $\beta$ , the factor scores were subtracted from the reduced-form residuals.  $A$  was thus initialised as the identity matrix. The EGARCH parameters were then obtained from univariate models for the initial series of the factor and the idiosyncratic residuals. The variance processes were started at the according sample moments. Even though the ML optimisation came along with the natural difficulties, the outcome from various attempts with different starting values, algorithms etc. should justify a certain confidence in having found the global maximum. The estimations were carried out in a Gauss program using the CML module.

### 3.3 Results and Discussion

This section presents numerical estimates for the identified equation system and provides further analyses and economic interpretation. At first, equations (9) display the contemporaneous interactions in the German stock markets, based on the parameters from the structural matrix  $A$  and the vector of loadings  $\beta$ . The variable names denote open-to-close returns at time  $t$ , QML standard errors are in parentheses and significant coefficients in bold.

$$\begin{aligned}
 \text{DAX}_t &= \mathbf{0.099} \text{ MDAX}_t + \mathbf{0.153} \text{ SDAX}_t + \mathbf{1.280} \hat{z}_t + \hat{\varepsilon}_{1t} \\
 &\quad (0.026) \qquad (0.054) \qquad (0.060) \\
 \text{MDAX}_t &= 0.026 \text{ DAX}_t + \mathbf{0.508} \text{ SDAX}_t + \mathbf{0.431} \hat{z}_t + \hat{\varepsilon}_{2t} \\
 &\quad (0.030) \qquad (0.054) \qquad (0.047) \\
 \text{SDAX}_t &= 0.050 \text{ DAX}_t + \mathbf{0.148} \text{ MDAX}_t + \mathbf{0.101} \hat{z}_t + \hat{\varepsilon}_{3t} \\
 &\quad (0.030) \qquad (0.048) \qquad (0.042)
 \end{aligned} \tag{9}$$

Surprisingly, it is not the blue chip index DAX, which dominates the direct transmission effects. Rather the MDAX and, even more astonishingly, above all the SDAX development spill over into the other indexes. Concerning the common factor, the loadings shrink with the "size" of the segments.

Before entering the discussion, first let us shortly check whether results change when the entire system dynamics from the SVAR(5) are considered. Of course, looked at from the finance standpoint, the contemporaneous interactions, which take place within the same trading day, should cover all relevant market processes. Nevertheless, since for example

---

<sup>6</sup>Note however that these restrictions are *not* necessary for identification.

delayed information processing or frictions from market microstructure may trigger lagged reactions, long-run impacts may be of additional interest. For instance, one might want to assure that the immediate processes do not simply reflect some type of overshooting later on corrected as time passes by. Alternatively, the hypothesis of large capitalisation stocks tending to lead smaller caps (e.g. Lo and MacKinley 1990) could be checked. Therefore, in the following the long-run effects of the structural shocks  $\varepsilon_j$  on the index levels, obtained from the total impact matrix  $(A - \sum_{j=1}^5 B_j)^{-1}$ , are given.

$$\begin{aligned} \varepsilon_2 &\rightarrow \text{DAX} : 0.49 & \varepsilon_3 &\rightarrow \text{DAX} : 0.47 \\ \varepsilon_1 &\rightarrow \text{MDAX} : 0.09 & \varepsilon_3 &\rightarrow \text{MDAX} : 0.78 \\ \varepsilon_1 &\rightarrow \text{SDAX} : 0.14 & \varepsilon_2 &\rightarrow \text{SDAX} : 0.49 \end{aligned}$$

Obviously, the long-run reactions simply confirm the impression from the instantaneous analysis that it is the smaller caps triggering the main cross-index spillovers. So, does this mean that the DAX is less important an index than the SDAX? - Surely not. Although the empirical results seem to contradict intuitive *a priori* beliefs, the following economic reasoning should provide the revealed pattern with a firm plausibility: Direct spillover between stocks or stock indexes implies that market participants observe the development of one index in order to gain information for their trading decisions in other segments. Naturally, the observed index is driven by a flow of information arriving during trading time. Logically, one should ask whether traders prefer (or are able) to collect this relevant information directly from its sources, or rather observe the *outcome* of the market.

It is well known that large stocks have generally lower transaction and information costs than smaller ones. Much of the relevant information is normally released publicly and available from easily accessible sources, including mass media. Furthermore, numerous analysts, funds and financial institutions intensively monitor the blue chip segment. Such a constellation implies that market participants can obtain firsthand information without major difficulties and delays. In contrast, smaller caps are far less in the public eye; firm-specific news might be characterised to a more sizeable degree as "private" information, and the alleged effect of widely perceived macroeconomic, financial and international events might be less pronounced than for large global companies (see as well section 3.4). Accordingly, these obstacles might render well-informed market participation unattractive for "outsiders". In conclusion, the more or less private - and therefore asymmetric - information is revealed to the general market through the development of stock prices, which can for instance be observed in the MDAX or SDAX. At this stage, as far as the underlying news are ascribed a broader economic importance, movements in these indexes might spill over for example into the DAX blue chip segment. Against this explanation

one could raise the objection that price making may be dominated by "big players", who should easily be able to intensively engage as well in smaller segments. However, even in light of this argument the proposed explanation does not lose its appeal, because such global players normally follow trading and diversification strategies based on broad international first-hand information. Thus, in the sense of rational inattention, even if they could, it is unlikely that they actually do bear the costs for thoroughly covering a wide range of small capitalisation enterprises.

Besides the fundamental channel, purely "psychological" reasons might play an additional role in stock market transmission. However, spillover unrelated to fundamental connections is not necessarily based on irrational behaviour: In presence of asymmetric information, the type of shock underlying a price movement might not be learned even after the latter has been observed. Consequently, market participants might rationally react to certain developments simply because of the mere possibility that these are relevant. To complete the picture, identical or at least financially connected investors are likely to be present in the different German stock segments. Transmission via liquidity or wealth channels as well as cross-market hedging is thus a further potential source of equity comovement.

As has been stated above and as a consequence of the preceding discussion, the results do of course not entail that the DAX is economically not important. Far more, the bulk of DAX-relevant information is included in the factor, which does well matter for the smaller segments. Intuitively, one should thus expect broad macroeconomic and financial news governing the factor development, at least to a certain extent. Section 3.4 empirically verifies this conclusion and consequently provides support for the identification scheme and the discussed economic rationale. Furthermore, a few words of caution may be appropriate after all: While the unconditional variances of all idiosyncratic terms  $\varepsilon_{jt}$  uniformly lie around 0.3, the factor variance equals unity by normalisation. This implies that in terms of variance contributions, the spillovers in (9) do not play an important role as compared to the variance proportions explained by the factor and the respective index-specific shocks. In short, the DAX is of course not *governed* by MDAX and SDAX, just because significant spillovers appear in the simultaneous equation system. Nonetheless, non-trivial transmission does exist, giving ground for the preceding discussion.

The initial research question asked for sources of stock return comovement. In this context, one can compare the contributions of the common factor and the mutual transmission based on the outcome from equations (9). At first, the total unconditional correlations calculated from (7) amount to

$Cor(DAX_t, MDAX_t) = 0.70$ ,  $Cor(DAX_t, SDAX_t) = 0.43$  and  $Cor(MDAX_t, SDAX_t) = 0.68$ .

If there was no spillover, that is  $A = I$ , then these reduce to

$Cor(DAX_t, MDAX_t) = 0.58$ ,  $Cor(DAX_t, SDAX_t) = 0.16$  and  $Cor(MDAX_t, SDAX_t) = 0.11$ .

As could already be seen from the parameter estimates, the considerable comovement of DAX and MDAX mainly arises from the presence of a common factor. Opposingly, the factor is of minor importance for the MDAX-SDAX correlation, which is dominated by a large spillover from the latter to the former. The DAX-SDAX relation represents an intermediary case. In this context, one should be aware of the fact that only a single common factor had been specified. Even though this might be appropriate for indexes of the same nationality and prime standard and naturally keeps the practical difficulties under control, extending the system still may valuably amend the analysis. Going beyond the unconditional perspective, Figure 2 presents the time-varying reduced-form correlations.

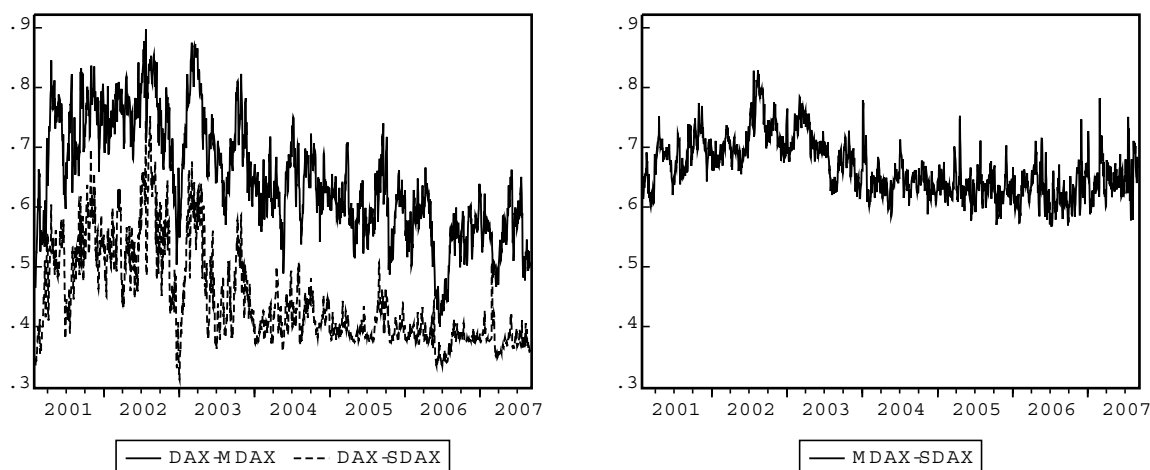


Figure 2: Conditional correlations

Here one can see that the correlations including the DAX were highest during the crisis time in 2002/2003 and fell afterwards. In contrast, the MDAX-SDAX correlation is relatively unaffected by these events and stays on a high level during the latest period. Naturally, such pure correlations may as well be inferred from reduced-form MGARCH models. In the present context however, the comovement is shown to result from distinct structural market processes. That is, the differences in the correlation patterns from Figure 2 are evidently driven by the factor variance, presented together with the idiosyncratic variances in Figure 3.

The turbulences in the first sample half are to a large extent picked up by the common factor. This makes sense, since the 9/11 attacks and the recessive crisis after the "new

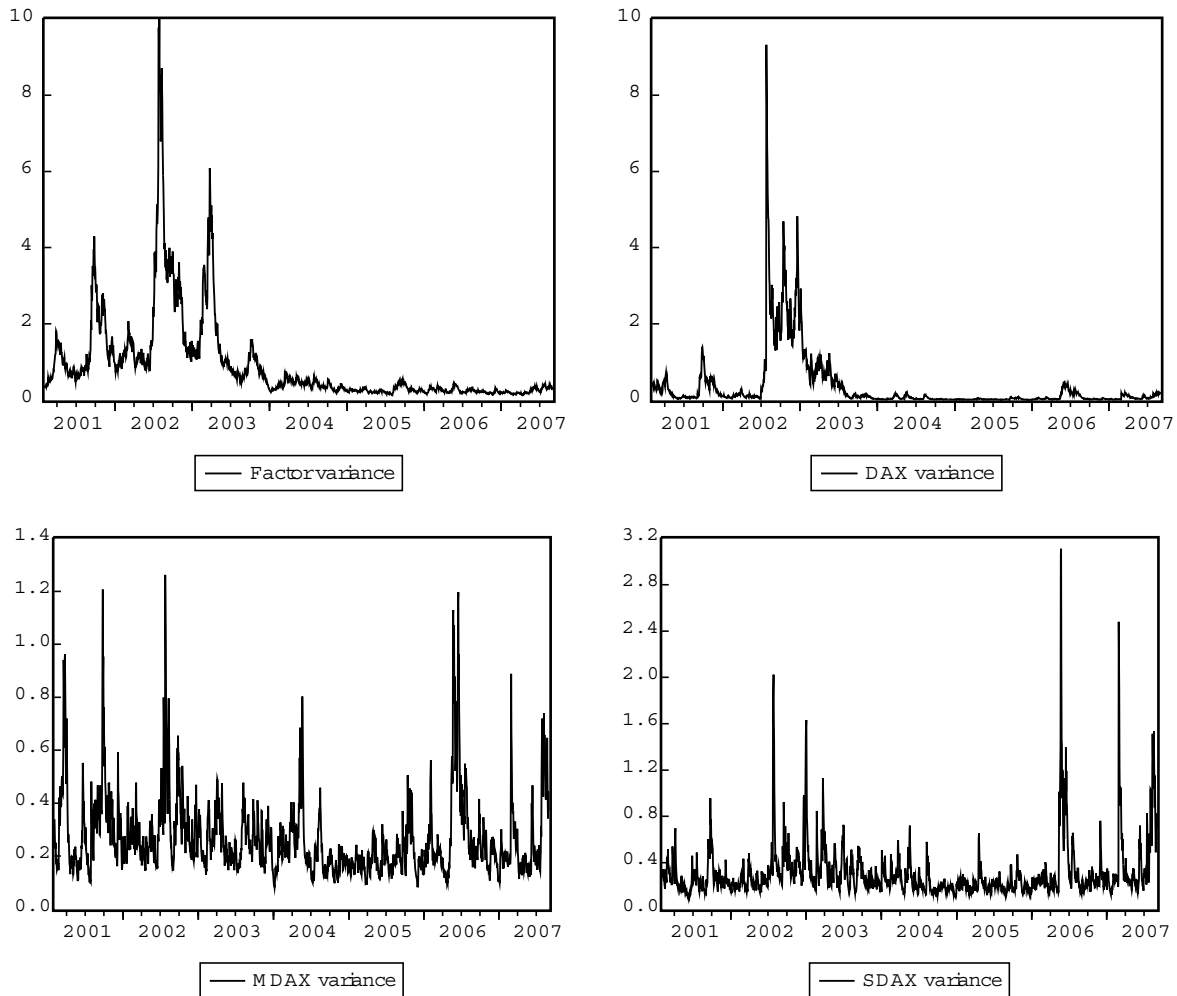


Figure 3: Conditional factor and idiosyncratic variances

economy” bubble do certainly not represent news that are specific to one of the indexes. Necessarily, the high conditional factor variances at the time increase the cross-correlations in that period. Note that the smaller volatile phases in 2006 and 2007 are apparently more specific to the MDAX and SDAX and are not tracked by the factor variance; see as well the according pronounced return movements in Figure 1.

As a test for appropriate model specification, the autocorrelations of the squares of the standardised disturbances  $\tilde{\epsilon}_{jt}$  were checked to not exceed their two standard error bands. In general, this confirms the common literature result that GARCH-type models of orders 1, 1 are fairly suitable for financial markets data. Only for the common factor, significant autocorrelations were found from the third until the sixth order, which were however not persistent. ARCH-LM tests could additionally corroborate the model adequacy. Furthermore, the stability criterion is fulfilled, since all autoregressive parameters in the matrix  $G$  lie below unity. Thus, with the structural conditional variances being stationary, the same



applies to the reduced-form second moments that are calculated as linear combinations according to (7).

### 3.4 Looking behind the Shocks

The preceding analysis has delivered insights into causalities and correlations in the German stock market without relying on specific sets of further explanatory variables. Now, for the sake of economic interpretation and plausibility check, this section provides a short examination of model-exogenous linkages. More precisely, common financial variables are employed in order to uncover possible connections of the identified factors from the structural model. Thereby, without claiming to be exhaustive, I focus on the euro / US dollar exchange rate, the Dow Jones Industrial index, the 10-year Euro government benchmark bonds and the Brent crude oil price. While the first two variables are defined as open-to-close returns, the interest rate is transformed to close-to-close differences and the oil variable is measured in full day spot returns. Due to different trading hours and the transatlantic time shift, these data are not totally synchronous to the DAX; however, a more refined analysis based on intraday data is left for future research.

Table 1 reports correlations of the common factor and the idiosyncratic innovations with the selected variables. The asymptotic standard error of correlations with 1735 observations is 0.024. Hence, the segment-specific shocks hardly display any relevant correlations; the DAX-Dow correlation as one of two exceptions worth mentioning might be well explained by wealth and liquidity as well as portfolio rebalancing effects due to the likely presence of common (international) investors in the two leading indexes. The lack of correlation supports the appropriateness of the model identification and the considered economic rationale, since the selected observable variables are likely to affect stock prices on a broader base. Nevertheless, such interpretations should be aware of the fact that no actual causation is identified; for instance, in case the DAX on his part affects the chosen "explaining" variables, correlation with  $\varepsilon_{1t}$  would arise as a natural consequence. Possibly, the DAX-Dow correlation is amenable to such a kind of explanation, the more so as one takes into account the time difference. As a task for ongoing research, firm-specific news instead of macro data could be employed to verify systematic idiosyncratic linkages.

Here, let us investigate the relations of the common factor in more detail. Thereby, the positive link of the factor to euro depreciations, strengthening German competitiveness, and to "good" news in the Dow Jones as well as the negative one to oil price increases were to be expected; thus, the factor seems to be conveniently identified. The positive

correlation with the bond yields reflects the fact that real activity innovations are likely to result both in rising interest rates and stock prices. Supposing that large DAX firms might be more directly connected to economy-wide growth shocks might then as well explain the leftover idiosyncratic DAX-bond correlation.

	common factor	DAX residual	MDAX residual	SDAX residual
euro/dollar	0.23	0.05	0.02	-0.05
Dow Jones	0.54	0.20	0.08	0.03
Govt Bond	0.32	0.18	0.02	0.03
Brent oil	-0.08	0.00	0.01	0.03

Table 1: Correlation of factors with exogenous variables

A simple OLS regression yields the following result:

$$\hat{z}_t = 0.003 + 0.172 \text{euro}_t + 0.478 \text{dow}_t + 4.679 \text{bond}_t - 0.014 \text{oil}_t + \hat{v}_t, \quad R^2 = 0.349. \quad (10)$$

(0.019)      (0.035)                      (0.020)                      (0.528)                      (0.007)

Hence, all partial effects are significant, and a non-trivial portion of the factor variation can be explained by the four observable variables. At the same time, this outcome demonstrates the merits of a latent factor approach: Whereas including a large set of conditioning variables might be considered as a strategy, it is unlikely to cover all common influences by observable variables (see as well King et al. 1994). Furthermore, as discussed above, the question of causality between the examined series and the exogenous variables would be answered simply by assumption.

## 4 Concluding Summary

Stock market returns, like those of the German indexes DAX, MDAX and SDAX, are often correlated to a substantial degree. This paper aimed at distinguishing the part of a contemporaneous correlation arising from causal spillover between the relevant variables from the one that is due to any third-party influences affecting all of them alike. Logically, an appropriate model has to feature a structural character and must additionally include common factors as sources of model-exogenous impulses. However, such a specification obviously runs into classical identification problems.

This study developed a customised adequate solution based on the idea of identification through heteroscedasticity: Both the idiosyncratic innovations of the stock returns as well as their common factor are allowed to display ARCH-type effects, so that the additional

information needed for identifying the model structure can be achieved from the continually modelled shifts in conditional variance. Parameter estimates as well as factor states and conditional variances are obtained by means of QML Kalman filtering techniques.

The stunning results showed that instantaneous spillovers run from the smaller to the larger caps, but hardly into the opposite direction. An economic interpretation suggested that most of the DAX-relevant information might be included in the common factor and is directly observed by market participants in all segments; supporting this argument, about one third of the factor variation could be explained by the euro / US dollar exchange rate, the Dow Jones index, the 10-year Euro government benchmark bonds and the Brent oil price. In contrast, DAX traders evidently do not collect all news specific to the MDAX or SDAX directly, but rather react to the outcome of these segments by observing the index development. In view of higher information and transaction costs connected to smaller stocks, such behaviour can represent a rational strategy.

This paper contributed to the literature by allowing the researcher to determine common driving forces of different variables while retaining the possibility of mutual contemporaneous interaction between them. Through this methodological innovation, it was possible to uncover structural market processes that are normally hidden behind reduced-form correlations. Future research might exploit this advance in methodology for finding sources of correlation in further significant applications, respectively for re-examining econometric approaches, which traditionally had to rely on non-testable assumptions. Moreover, interest could focus on econometric refinements in terms of theoretical model elaboration, for instance concerning the specification of the factor structure, as well as simplified estimation procedures.

## References

- [1] Baur, D., R.C. Jung (2006): Return and volatility linkages between the US and the German stock market. *Journal of International Money and Finance*, 25, 598-613.
- [2] Berndt, E., B. Hall, R. Hall, J. Hausman (1974): Estimation and Inference in Non-linear Structural Models. *Annals of Social Measurement*, 3, 653-665.
- [3] Bollerslev, T., J.M. Wooldridge (1992): Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances. *Econometric Reviews*, 11, 143-172.

- [4] King, M., E. Sentana, S. Wadhvani (1994): Volatitliy and Links between National Stock Markets. *Econometrica*, 62, 901-933.
- [5] Lo, A., C. MacKinlay (1990): When are contrarian profits due to stock market overreaction? *Review of Financial Studies*, 3, 175-205.
- [6] Nelson, D.B. (1991): Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59, 347-370.
- [7] Nelson, D.B. (1992): Filtering and Forecasting with Misspecified ARCH Models I: Getting the Right Variance with the Wrong Model. *Journal of Econometrics*, 52, 61-90.
- [8] Rigobon, R. (2002): The Curse of Non-Investment Grade Countries. *Journal of Development Economics*, 69, 423-449.
- [9] Rigobon, R. (2003): Identification through heteroscedasticity. *Review of Economics and Statistics*, 85, 777-792.
- [10] Ross, S.A. (1989): Information and volatility: the no-arbitrage martingale approach to timing and resolution irrelevancy. *Journal of Finance*, 44, 1-17.
- [11] Sentana, E., G. Fiorentini (2001): Identification, estimation and testing of conditionally heteroskedastic factor models. *Journal of Econometrics*, 102, 143-164.
- [12] Weber, E. (2007a): Volatility and Causality in Asia Pacific Financial Markets. CRC 649 Discussion Paper 2007-004, Humboldt-Universität zu Berlin.
- [13] Weber, E. (2007b): Structural Constant Conditional Correlation. CRC 649, Humboldt-Universität zu Berlin.

## SFB 649 Discussion Paper Series 2007

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 001 "Trade Liberalisation, Process and Product Innovation, and Relative Skill Demand" by Sebastian Braun, January 2007.
- 002 "Robust Risk Management. Accounting for Nonstationarity and Heavy Tails" by Ying Chen and Vladimir Spokoiny, January 2007.
- 003 "Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model." by Harald Uhlig, January 2007.
- 004 "Volatility and Causality in Asia Pacific Financial Markets" by Enzo Weber, January 2007.
- 005 "Quantile Sieve Estimates For Time Series" by Jürgen Franke, Jean-Pierre Stockis and Joseph Tadjuidje, February 2007.
- 006 "Real Origins of the Great Depression: Monopolistic Competition, Union Power, and the American Business Cycle in the 1920s" by Monique Ebell and Albrecht Ritschl, February 2007.
- 007 "Rules, Discretion or Reputation? Monetary Policies and the Efficiency of Financial Markets in Germany, 14th to 16th Centuries" by Oliver Volckart, February 2007.
- 008 "Sectoral Transformation, Turbulence, and Labour Market Dynamics in Germany" by Ronald Bachmann and Michael C. Burda, February 2007.
- 009 "Union Wage Compression in a Right-to-Manage Model" by Thorsten Vogel, February 2007.
- 010 "On  $\sigma$ -additive robust representation of convex risk measures for unbounded financial positions in the presence of uncertainty about the market model" by Volker Krätschmer, March 2007.
- 011 "Media Coverage and Macroeconomic Information Processing" by Alexandra Niessen, March 2007.
- 012 "Are Correlations Constant Over Time? Application of the CC-TRIG<sub>t</sub>-test to Return Series from Different Asset Classes." by Matthias Fischer, March 2007.
- 013 "Uncertain Paternity, Mating Market Failure, and the Institution of Marriage" by Dirk Bethmann and Michael Kvasnicka, March 2007.
- 014 "What Happened to the Transatlantic Capital Market Relations?" by Enzo Weber, March 2007.
- 015 "Who Leads Financial Markets?" by Enzo Weber, April 2007.
- 016 "Fiscal Policy Rules in Practice" by Andreas Thams, April 2007.
- 017 "Empirical Pricing Kernels and Investor Preferences" by Kai Detlefsen, Wolfgang Härdle and Rouslan Moro, April 2007.
- 018 "Simultaneous Causality in International Trade" by Enzo Weber, April 2007.
- 019 "Regional and Outward Economic Integration in South-East Asia" by Enzo Weber, April 2007.
- 020 "Computational Statistics and Data Visualization" by Antony Unwin, Chun-houh Chen and Wolfgang Härdle, April 2007.
- 021 "Ideology Without Ideologists" by Lydia Mechtenberg, April 2007.
- 022 "A Generalized ARFIMA Process with Markov-Switching Fractional Differencing Parameter" by Wen-Jen Tsay and Wolfgang Härdle, April 2007.

**SFB 649, Spandauer Straße 1, D-10178 Berlin**  
**<http://sfb649.wiwi.hu-berlin.de>**

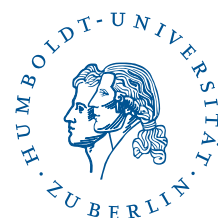
This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



- 023 "Time Series Modelling with Semiparametric Factor Dynamics" by Szymon Borak, Wolfgang Härdle, Enno Mammen and Byeong U. Park, April 2007.
- 024 "From Animal Baits to Investors' Preference: Estimating and Demixing of the Weight Function in Semiparametric Models for Biased Samples" by Ya'acov Ritov and Wolfgang Härdle, May 2007.
- 025 "Statistics of Risk Aversion" by Enzo Giacomini and Wolfgang Härdle, May 2007.
- 026 "Robust Optimal Control for a Consumption-Investment Problem" by Alexander Schied, May 2007.
- 027 "Long Memory Persistence in the Factor of Implied Volatility Dynamics" by Wolfgang Härdle and Julius Mungo, May 2007.
- 028 "Macroeconomic Policy in a Heterogeneous Monetary Union" by Oliver Grimm and Stefan Ried, May 2007.
- 029 "Comparison of Panel Cointegration Tests" by Deniz Dilan Karaman Örsal, May 2007.
- 030 "Robust Maximization of Consumption with Logarithmic Utility" by Daniel Hernández-Hernández and Alexander Schied, May 2007.
- 031 "Using Wiki to Build an E-learning System in Statistics in Arabic Language" by Taleb Ahmad, Wolfgang Härdle and Sigbert Klinke, May 2007.
- 032 "Visualization of Competitive Market Structure by Means of Choice Data" by Werner Kunz, May 2007.
- 033 "Does International Outsourcing Depress Union Wages? by Sebastian Braun and Juliane Scheffel, May 2007.
- 034 "A Note on the Effect of Outsourcing on Union Wages" by Sebastian Braun and Juliane Scheffel, May 2007.
- 035 "Estimating Probabilities of Default With Support Vector Machines" by Wolfgang Härdle, Rouslan Moro and Dorothea Schäfer, June 2007.
- 036 "Yxilon – A Client/Server Based Statistical Environment" by Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen, June 2007.
- 037 "Calibrating CAT Bonds for Mexican Earthquakes" by Wolfgang Härdle and Brenda López Cabrera, June 2007.
- 038 "Economic Integration and the Foreign Exchange" by Enzo Weber, June 2007.
- 039 "Tracking Down the Business Cycle: A Dynamic Factor Model For Germany 1820-1913" by Samad Sarferaz and Martin Uebele, June 2007.
- 040 "Optimal Policy Under Model Uncertainty: A Structural-Bayesian Estimation Approach" by Alexander Kriwoluzky and Christian Stoltenberg, July 2007.
- 041 "QuantNet – A Database-Driven Online Repository of Scientific Information" by Anton Andriyashin and Wolfgang Härdle, July 2007.
- 042 "Exchange Rate Uncertainty and Trade Growth - A Comparison of Linear and Nonlinear (Forecasting) Models" by Helmut Herwartz and Henning Weber, July 2007.
- 043 "How do Rating Agencies Score in Predicting Firm Performance" by Gunter Löffler and Peter N. Posch, August 2007.

**SFB 649, Spandauer Straße 1, D-10178 Berlin**  
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche  
 Forschungsgemeinschaft through the SFB 649 "Economic Risk".



- 044 "Ein Vergleich des binären Logit-Modells mit künstlichen neuronalen Netzen zur Insolvenzprognose anhand relativer Bilanzkennzahlen" by Ronald Franken, August 2007.
- 045 "Promotion Tournaments and Individual Performance Pay" by Anja Schöttner and Veikko Thiele, August 2007.
- 046 "Estimation with the Nested Logit Model: Specifications and Software Particularities" by Nadja Silberhorn, Yasemin Boztuğ and Lutz Hildebrandt, August 2007.
- 047 "Risiken infolge von Technologie-Outsourcing?" by Michael Stephan, August 2007.
- 048 "Sensitivities for Bermudan Options by Regression Methods" by Denis Belomestny, Grigori Milstein and John Schoenmakers, August 2007.
- 049 "Occupational Choice and the Spirit of Capitalism" by Matthias Doepke and Fabrizio Zilibotti, August 2007.
- 050 "On the Utility of E-Learning in Statistics" by Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen, August 2007.
- 051 "Mergers & Acquisitions and Innovation Performance in the Telecommunications Equipment Industry" by Tseveen Gantumur and Andreas Stephan, August 2007.
- 052 "Capturing Common Components in High-Frequency Financial Time Series: A Multivariate Stochastic Multiplicative Error Model" by Nikolaus Hautsch, September 2007.
- 053 "World War II, Missing Men, and Out-of-wedlock Childbearing" by Michael Kvasnicka and Dirk Bethmann, September 2007.
- 054 "The Drivers and Implications of Business Divestiture – An Application and Extension of Prior Findings" by Carolin Decker, September 2007.
- 055 "Why Managers Hold Shares of Their Firms: An Empirical Analysis" by Ulf von Lilienfeld-Toal and Stefan Ruenzi, September 2007.
- 056 "Auswirkungen der IFRS-Umstellung auf die Risikoprämie von Unternehmensanleihen - Eine empirische Studie für Deutschland, Österreich und die Schweiz" by Kerstin Kiefer and Philipp Schorn, September 2007.
- 057 "Conditional Complexity of Compression for Authorship Attribution" by Mikhail B. Malyutov, Chammi I. Wickramasinghe and Sufeng Li, September 2007.
- 058 "Total Work, Gender and Social Norms" by Michael Burda, Daniel S. Hamermesh and Philippe Weil, September 2007.
- 059 "Long-Term Orientation in Family and Non-Family Firms: a Bayesian Analysis" by Jörn Hendrich Block and Andreas Thams, October 2007
- 060 "Kombinierte Liquiditäts- und Solvenz-kennzahlen und ein darauf basierendes Insolvenzprognosemodell für deutsche GmbHs" by Volodymyr Perederiy, October 2007
- 061 "Embedding R in the Mediawiki" by Sigbert Klinke and Olga Zlatkin-Troitschanskaia, October 2007
- 062 "Das Hybride Wahlmodell und seine Anwendung im Marketing" by Till Dannewald, Henning Kreis and Nadja Silberhorn, November 2007
- 063 "Determinants of the Acquisition of Smaller Firms by Larger Incumbents in High-Tech Industries: Are they related to Innovation and Technology Sourcing? " by Marcus Wagner, November 2007

**SFB 649, Spandauer Straße 1, D-10178 Berlin**  
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche  
 Forschungsgemeinschaft through the SFB 649 "Economic Risk".



064 "Correlation vs. Causality in Stock Market Comovement" by Enzo Weber, October 2007

**SFB 649, Spandauer Straße 1, D-10178 Berlin**  
**<http://sfb649.wiwi.hu-berlin.de>**

This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

