Monetary Rules, Indeterminacy, and the Business-Cycle Stylised Facts*

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Abstract

Several papers—see, e.g., Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004)—have documented how the reaction function of the U.S. monetary authority has been passive, and destabilising, before the appointment of Paul Volcker, and active and stabilising since then. In this paper we first compare and contrast the two sub-periods in terms of several key business-cycle 'stylised facts'. The latter period appears to be characterised by a lower inflation persistence; a smaller volatility of reduced-form innovations to both inflation and real GDP growth; and a systematically smaller amplitude of business-cycle frequency fluctuations.

Working with the Smets-Wouters (2003) sticky-price, sticky-wage DSGE model of the U.S. economy, we then investigate how such stylised facts change systematically with changes in the parameters of a simple forward-looking monetary rule. We solve the model under indeterminacy via the procedure introduced by Lubik and Schorfheide (2003). The determinacy and indeteminacy regions appear to be characterised by two markedly different sets of macroeconomic stylised facts. Further, in several cases the relationship between the parameters of the monetary rule and key stylised facts under indeterminacy is a sort of mirror image of what it is under determinacy: both inflation persistence and the volatility of its reduced-form innovations, for example, are *increasing* in the coefficient on inflation under indeterminacy, *decreasing* under determinacy.

Finally, we compare the stylised facts identified in the data with those generated by the Smets-Wouters model conditional on estimated monetary rules.

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Although variation in the monetary rule across sub-periods *can*, in principle, explain the broad features of the variation in the macroeconomic stylised facts we consider, results are in general not consistent across different inflation measures and output gap proxies. In particular, some of our estimates imply that the pre-Volcker era, too, was characterised by a determinate equilibrium, in spite of the lower activism of the monetary rule.

Keywords: monetary policy rules; indeterminacy; business cycles; frequency domain; median-unbiased estimation.

1 Introduction

In recent years, several papers—see in particular Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004)—have documented marked changes in the conduct of U.S. monetary policy over the post-WWII era. Specifically, the reaction function of the U.S. monetary authority is estimated to have been passive, and destabilising, before Volcker, and active and stabilising since then.[1] A second group of studies—see, e.g., Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Kim, Nelson, and Piger (2003), and Stock and Watson (2002)—has documented a marked increase in U.S. economic stability over (roughly) the last two decades, with the volatility of reduced-form innovations to both inflation and output growth being estimated to have drastically fallen compared to previous years.[2]

These two strands of literature prompt two obvious questions:

(1) 'What is the relationship between historical changes in the conduct of U.S. monetary policy and the increase in U.S. economic stability?'

(2) 'At a more general level, what is the impact of changes in the conduct of monetary policy on key macroeconomic 'stylised facts', like inflation persistence, or the amplitude of business-cycle frequency fluctuations?'

In this paper we first compare and constrast the two sub-periods preceding and, respectively, following the appointment of Paul Volcker as Chairman of the Board of Governors of the Federal Reserve System in terms of a number of key business-cycle 'stylised facts'. The latter period appears to be characterised by a lower inflation persistence; a smaller volatility of reduced-form innovations to inflation and output growth; and a systematically lower amplitude of business-cycle frequency fluctuations for all the macroeconomic indicators we consider, with the only exception of base

[1] For a contrarian view, see Sims (1999), Sims and Zha (2002), and Hanson (2002).

[2] See also Blanchard and Simon (2001), Chauvet and Potter (2001), Kahn, McConnell, and Perez-Quiros (2002), and Ahmed, Levin, and Wilson (2002) on the increased stability of the U.S. economy; Cogley and Sargent (2002) and Cogley and Sargent (2003), on changes in the stochastic properties of U.S. inflation (in particular, in inflation persistence) since the beginning of the 1960s; and Brainard and Perry (2000) on changes in the slope of the U.S. Phillips curve.

money growth. Finally, consistent with Brainard and Perry (2000)'s finding of a decrease in the slope of the U.S. Phillips curve over the last two decades, the gain and the coherence between cyclical indicators (the rate of unemployment, and an 'activity factor' constructed along the lines of Stock and Watson (1999b)) and inflation at the business-cycle frequencies appear to have decreased, after 1979, compared with the pre-Volcker era, although the change is not statistically significant at conventional levels.

Working with the sticky-price, sticky-wage DSGE model of the U.S. economy recently estimated via Bayesian methods by Smets and Wouters (2003), and preliminarly, to build intuition, with the standard workhorse New Keynesian model of Clarida, Gali, and Gertler (1999), we then investigate how such stylised facts change systematically with changes in the coefficients on inflation and the output gap in a simple Taylor rule. Given that, as documented in the previously mentioned papers, the pre-Volcker period appears to have been characterised by a passive monetary rule, we do not restrict our investigation uniquely to the determinacy region, solving the model under indeterminacy via the procedure recently introduced by Lubik and Schorfheide (2003). The determinacy and indeterminacy regions appear to be characterised by a markedly different set of macroeconomic stylised facts. Further, in several cases the relationship between the parameters of the monetary rule and key stylised facts under indeterminacy is a sort of mirror image of what it is under determinacy: both inflation persistence and the volatility of its reduced-form innovations, for example, are *increasing* in the coefficient on inflation under indeterminacy, *decreasing* under determinacy. Finally, we compare the stylised facts identified in the data with those generated by the Smets-Wouters model conditional on estimated monetary rules. Although variation in the monetary rule across sub-periods *can*, in principle, explain the broad features of the variation in the macroeconomic stylised facts we consider, results are in general not consistent across different inflation measures and output gap proxies. In particular, some of our estimates imply that the pre-Volcker era, too, was characterised by a determinate equilibrium, in spite of the lower activism of the monetary rule.

The paper is organised as follows. The next section describes the dataset. In section 3 we identify key business-cycle stylised facts for the sub-periods of interest. Section 4 investigates the relationship between changes in the conduct of monetary policy, and changes in the very same stylised facts we previously investigated in the data. In section 5 we compare the stylised facts identified in section 4 with those generated by the Smets-Wouters model conditional on estimated forward-looking monetary rules. Section 6 concludes, and outlines several possible directions for future research.

2 The Data

The quarterly real GDP[3] and GDP deflator series[4] are from *U.S. Department of Commerce: Bureau of Economic Analysis*. Quarterly series for real private consumption and investment in billions of 2000 chained dollars are from Table 1.1.6. of the *National Income and Product Accounts*, downloadable from the *Bureau of Economic Analysis* site on the web. Both series are seasonally adjusted and quoted at an annual rate. The *Congressional Budget Office* (henceforth, CBO) output gap measure has been constructed as the difference between the logarithms of quarterly real GDP and the CBO potential real GDP series.[5] For all series the sample period is 1954:3-2003:3.

Turning to monthly series, the consumer price index[6] and employment[7] series are from the *U.S. Department of Labor: Bureau of Labor Statistics*. The federal funds rate[8] and base money[9] series are from FRED II, the Federal Reserve Bank of St. Louis web data search engine. The activity factor we use in section 3.4 has been constructed, following Stock and Watson (1999b), as the first principal component extracted from a matrix of (logged) HP-filtered indicators. Following Stock and Watson (1999b), filtering is implemented by exploiting the state-space representation of the Hodrick-Prescott filtering problem.[10] A detailed list of the series used in the construction of the activity factor is contained in appendix A. The producer price index for fuels and related products[11] used in the next section is from the *U.S. Department of Labor: Bureau of Labor Statistics*. Both the interest rate on 10-year constant maturity Treasury bills,[12] and the rate on 3-month Treasury bills quoted on the secondary market,[13] are from the *Federal Reserve Board*. For all monthly series the sample period is 1954:7-2003:9. Monthly series have been converted to the quarterly frequency either by taking averages within the quarter (this is the case, for example, of the Federal funds rate, and of employment and unemployment series), or by keeping the last observation from each quarter (this is the case, for example, of the CPI and of

[3] 'GDPC1: Real Gross Domestic Product, 1 Decimal, Seasonally Adjusted Annual Rate, Quarterly, Billions of Chained 1996 Dollars'.

[4] 'GDPDEF: Gross Domestic Product: Implicit Price Deflator, Seasonally Adjusted, Quarterly, Index 1996=100'.

[5] 'GDPPOT: Real Potential Gross Domestic Product, U.S. Congress: Congressional Budget Office, Quarterly, Billions of Chained 1996 Dollars'.

[6] 'CPIAUCSL: Consumer Price Index For All Urban Consumers: All Items, Consumer Price Index, Seasonally Adjusted, Monthly, Index 1982-84=100'

[7] 'CE16OV: Civilian Employment: Sixteen Years & Over, Seasonally Adjusted, Monthly, Thousands'.

[8] 'FEDFUNDS: Effective Federal Funds Rate, Averages of Daily Figures, Monthly, Percent'.

[9] 'AMBNS: St. Louis Adjusted Monetary Base; Billions of Dollars; NSA'.

[10] For technical details, see Stock and Watson (1999b).

[11] 'PPIENG: Producer Price Index: Fuels & Related Products & Power, Producer Price Index, Not Seasonally Adjusted, Monthly'.

[12] 'GS10: 10-Year Treasury Constant Maturity Rate, Averages of Business Days, Monthly, Percent'.

[13] 'TB3MS: 3-Month Treasury Bill: Secondary Market Rate, Averages of Business Days, Discount Basis, Monthly, Percent'.

the producer price index for fuels and related products). A detailed list of all the series can be found in appendix A.

3 Macroeconomic Stylised Facts

In this section we present some key macroeconomic stylised facts for the three sub-periods of interest, the ones preceding and, respectively, following the appointment of Paul Volcker as Chairman of the Board of Governors of the Federal Reserve System, and the post-1982 era.[14] We focus on inflation persistence; the volatility of reduced-form innovations to inflation and real GDP growth; the amplitude of business-cycle frequency fluctuations for key macroeconomic series; and the correlation between inflation and two alternative cyclical indicators, the rate of unemployment and a 'real activity' dynamic factor. Although in recent years several papers have documented a widespread increase in U.S. economic stability over the last two decades, to the best of our knowledge this is the first study to systematically compare the pre-Volcker and the post-1979 eras in terms of a broad list of key macroeconomic facts.[15]

From a methodological point of view, the techniques we use—univariate autoregressions; band-pass filtering techniques; and cross-spectral methods—characterise themselves for being based on a minimal set of identifying assumptions. Our hope is that, by eschewing methods based on complex identification schemes, like structural VAR analysis, we will be capable of 'nailing down' a set of reasonably robust stylised facts. For this reason, we ignore conditional stylised facts like the shape of impulse-responses to a monetary shock.[16]

3.1 Inflation persistence, and the volatility of reduced-form inflation innovations

Table 1 reports results from estimating AR(p) models for both quarterly CPI and quarterly GDP deflator inflation, for the three sub-periods of interest. Specifically, we estimate

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t \tag{1}$$

via OLS, choosing the lag order, p, based on the Bayes information criterion, with an upper bound P=6 on the possible number of lags. For each sub-sample we report the

[14] There are at least two reasons for excluding the period between Volcker's appointment and the end of 1982 from the Volcker-Greenspan era. First, as shown for example by Bernanke and Mihov (1998), during a significant portion of this period the Fed pursued a policy of targeting non-borrowed reserves. Second, the Volcker disinflation can arguably be considered as a highly idiosyncratic, one-shot episode marking the transition between two different monetary policy regimes, and as such should probably not be ascribed to any of the two.

[15] Clarida, Gali, and Gertler (2000, table __) contain a brief and informal comparison based on the standard deviation of inflation.

[16] On this, see Hanson (2002) and Sims and Zha (2002).

estimated mean, the innovation variance, and the median-unbiased estimate of our preferred measure of persistence—which, following Andrews and Chen (1994), we take it to be the sum of the autoregressive coefficients[17]—computed via the Hansen (1999) 'grid bootstrap' procedure. Specifically, following Hansen (2000, section III.A) we recast (1) into the augmented Dickey-Fuller form $y_t = \mu + \rho y_{t-1} + \gamma_1 y_{t-1} + \ldots + \gamma_{p-1} y_{t-(p-1)} + u_t$, where ρ is the sum of the AR coefficients in (1), and we simulate the sampling distribution of the t-statistic $t = (\hat{\rho}\text{-}\rho)/\hat{S}(\hat{\rho})$, where $\hat{\rho}$ is the OLS estimate of ρ, and $\hat{S}(\hat{\rho})$ is its estimated standard error, over a grid of possible values $[\hat{\rho}\text{-}4\hat{S}(\hat{\rho}); \hat{\rho}+4\hat{S}(\hat{\rho})]$, with step increments equal to 0.01. For each of the possible values in the grid, we consider 999 replications. For each sub-sample we report both the median-unbiased estimate of ρ and the 90%-coverage confidence interval computed based on the bootstrapped distribution of the t-statistic. Estimates for both the mean and the innovation variance have been computed conditional on the median-unbiased estimate of ρ. The estimate of the standard error for the mean, a non-linear function of the estimated parameters, has been computed via the delta method.

Table 1 Estimated AR(p) models for U.S. inflation, by sub-period

Sub-periods:	Lag order	Mean* (st.err.)	$\hat{\rho}$, and 90% conf. interval	Innovation variance* (st.err.)
	Based on quarterly CPI inflation			
Before Volcker	2	—	1.00 [0.86; 1.04]	3.84 (0.55)
Volcker-Greenspan	3	3.03 (0.31)	0.64 [0.51; 0.78]	3.42 (0.51)
Post-1982	3	3.04 (0.25)	0.64 [0.37; 1.00]	2.51 (0.41)
	Based on quarterly GDP deflator inflation			
Before Volcker	2	6.91 (30.89)	0.96 [0.84; 1.03]	2.20 (0.32)
Volcker-Greenspan	6	2.21 (0.15)	0.76 [0.68; 0.84]	0.60 (0.09)
Post-1982	3	2.24 (0.39)	0.85 [0.66; 1.03]	0.64 (0.10)

* In percentage points. ρ = sum of the AR coefficients. St. err. = standard error.

Several findings stand out. First—consistent with the results reported in, e.g. Kim, Nelson, and Piger (2003), Stock and Watson (2002), and Benati (2003)—a fall in the volatility of reduced-form innovations. The decrease is particularly marked, and statistically significant, for GDP deflator inflation, while it is less marked, and not statistically significant at conventional levels, for CPI inflation. Estimating (1) based on the month-on-month rate of growth of the CPI (quoted at annual rate), however,

[17]As shown by Andrews and Chen (1994), the sum of the autoregressive coefficients maps one-to-one into two alternative measures of persistence, the cumulative impulse-response function to a one-time innovation and the spectrum at the frequency zero. Andrews and Chen (1994) also contain an extensive discussion of why an alternative measure favored, e.g., by Stock (1991) and DeJong and Whiteman (1991), the largest autoregressive root, may provide a misleading indication of the true extent of persistence of the series depending on the specific values taken by the other autoregressive roots.

produces sharper results, with the volatility of reduced-form shocks estimated, for the three sub-periods, at 8.32 (0.69), 5.34 (0.46), and 4.44 (0.41).[18] Although our focus is on the quarterly frequency, the overall impression is therefore of a clear fall in the innovation variance for CPI inflation, too. Second, based on both indices, inflation is estimated to have been very highly persistent during the pre-Volcker era. In particular, CPI inflation is estimated as an exact unit root process, while for GDP deflator inflation the null of a unit root cannot be rejected at the 10% level, and the median-unbiased estimate of ρ is still extremely high, at 0.96. As for the post-1979 years evidence is not clear-cut. While results based on the CPI suggest a marked fall in persistence[19] (although, for the post-1982 period, the 90% confidence interval is very wide, to the point that it is not possible to reject the null of a unit root), results based on GDP deflator inflation are mixed. In particular, while a comparison between the pre- and post-1979 periods clearly suggests a fall in persistence, results for the post-1982 years point towards a smaller and not statistically significant decrease, to the point that the null of a unit root cannot be rejected.

3.2 The volatility of reduced-form innovations to real GDP growth

Table 2 reports Hansen (1999) median-unbiased estimates of ρ; and estimates of the mean and the innovation variance in (1), computed conditional on the median-unbiased estimate of the sum of the autoregressive coefficients, for real GDP growth. Consistent with the evidence reported in, e.g., Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), the Volcker-Greenspan and post-1982 sub-periods appear to be characterised by a markedly lower volatility of reduced-form innovations than the pre-Volcker era. In particular, the innovation variance is estimated to have decreased by 58.7 and respectively 75.4% compared with the former period. Persistence, on the other hand, appears to have increased, although the changes are not statistically significant at conventional levels.

Table 2 Estimated AR(p) models for U.S. real GDP growth, by sub-period

Sub-periods:	Lag order	Mean* (st.err.)	$\hat{\rho}$, and 90% conf. interval	Innovation variance* (st.err.)
Before Volcker	1	3.84 (0.36)	0.29 [0.13; 0.46]	18.35 (2.62)
Volcker-Greenspan	3	3.26 (0.37)	0.44 [0.22; 0.67]	7.57 (1.13)
Post-1982	2	3.28 (0.26)	0.61 [0.41; 0.83]	4.51 (0.72)

* In percentage points. ρ = sum of the AR coefficients. St. err. = standard error.

[18] The full set of results is available upon request.

[19] Estimates based on the month-on-month rate of growth of the CPI (quoted at annual rate) confirm the decrease in persistence, with the median-unbiased estimate of ρ for the three sub-periods being equal to 0.94 [0.84; 1.02], 0.80 [0.69; 0.91], and respectively 0.46 [0.31; 0.62].

3.3 The amplitude of business-cycle fluctuations

Table 3 reports standard deviations of business-cycle components for (the logarithms of) several macroeconomic indicators for the three sub-periods of interest. The variables we consider are the same as those analysed in Smets and Wouters (2003). Following established conventions in business-cycle analysis[20], we define the business-cycle frequency band as the one containing all the components of a series with a frequency of oscillation between 6 and 32 quarters. Business-cycle components are extracted via the optimal approximated band-pass filter recently proposed by Christiano and Fitzgerald (2003).[21]

Table 3 The amplitude of business-cycle fluctuations: standard deviations of band-pass filtered (logarithms of) macroeconomic indicators, by sub-period*

	Real GDP	Consumption	Investment	Employment
Before Volcker	0.018	0.015	0.079	0.011
Volcker-Greenspan	0.012	8.0E-3	0.060	7.4E-3
Post-1982	9.9E-3	6.5E-3	0.055	6.7E-3
	CPI inflation	Price level, CPI	GDP deflator inflation	Price level, GDP deflator
Before Volcker	1.795	0.014	1.189	9.1E-3
Volcker-Greenspan	1.248	8.7E-3	0.715	5.7E-3
Post-1982	0.952	6.6E-3	0.553	4.4E-3
	Federal funds rate	Base money growth		
Before Volcker	1.540	1.116		
Volcker-Greenspan	1.298	2.614		
Post-1982	1.121	2.740		

* All series have been logged except the Federal funds rate, inflation measures, and base money growth.

All the series, with the single exception of base money growth, exhibit the same

[20] See for example King and Watson (1996), Baxter and King (1999), Stock and Watson (1999a), and Christiano and Fitzgerald (2003).

[21] The Christiano-Fitzgerald band-pass filtered series is computed as the linear projection of the ideal band-pass filtered series onto the available sample. (For a definition of the ideal band-pass filter, see for example Sargent (1987).) The filter weights are chosen to minimise a weighted mean squared distance criterion between the ideal band-pass filtered series and its optimal approximation. The criterion is computed directly in the frequency domain, weighting the squared distance between the two objects frequency by frequency, based on the series' spectral density. This clearly requires, in principle, knowledge of the series' stochastic properties. In practice, Christiano and Fitzgerald show that for 'typical' time series representations—i.e., for representations that fit macroeconomic data well—the filter computed under the assumption the series is a random walk is always nearly optimal. In what follows we use such a recommended filter for all series.

ranking among the three sub-periods in terms of amplitude of business-cycle fluctuations, with the pre-Volcker and the post-1982 eras being characterised by the highest and, respectively, the lowest volatilities, and the Volcker-Greenspan period in between. For several series, the contrast between the pre-Volcker and the post-1982 periods is striking. For the log of real GDP, for example, volatility in the former period is 82.6% higher than in the latter, while for consumption the corresponding figure is beyond 125%. The volatility of inflation measures has decreased by 47% for the CPI and, respectively by 53.5% for the GDP deflator (the logaritms of both price indices display analogous marked decreases in volatility). Finally, the Federal funds rate exhibits a still respectable 27.2% decrease in volatility. As for a comparison between the pre- and post-1979 periods, due to the inclusion of the turbulent Volcker disinflation episode into the latter sub-period, the fall in volatility is less striking. Still, however, the decrease in the amplitude of business-cycle fluctuations is generalised (again, with the exception of base money growth), and for some series—consumption, GDP deflator inflation, and the logarithms of both the CPI and the GDP deflator—it is quite marked, between 40 and 45%, while for the logarithms of real GDP and employment it is around 34-35%.

3.4 The correlation between inflation and cyclical indicators at the business-cycle frequencies

Table 4 reports average cross-spectral statistics, and 90% confidence intervals, between monthly CPI inflation and two cyclical indicators—the unemployment rate, and the previously discussed 'real activity' dynamic factor—at the business-cycle frequencies (again, we define the business-cycle frequency band as the one containing all the components of a series with a frequency of oscillation between 6 and 32 quarters). Specifically, let π_t and x_t be inflation and the relevant cyclical indicator; let $F_\pi(\omega_j)$ and $F_x(\omega_j)$ be the smoothed spectra of the two series at the Fourier frequency ω_j; let $C_{x,\pi}(\omega_j)$ and $Q_{x,\pi}(\omega_j)$ be the smoothed co-spectrum and, respectively, quadrature spectrum between x_t and π_t corresponding to the Fourier frequency ω_j; and let Ω_{BC} be the set of all the Fourier frequencies belonging to the business-cycle frequency band. The estimated *average* smoothed gain, phase angle and coherence between x_t

and π_t at the business-cycle frequencies can then be computed according to[22]

$$\Gamma_{BC} = \frac{\left\{\left[\sum_{\omega_j \in \Omega_{BC}} C_{x,\pi}(\omega_j)\right]^2 + \left[\sum_{\omega_j \in \Omega_{BC}} Q_{x,\pi}(\omega_j)\right]^2\right\}^{\frac{1}{2}}}{\sum_{\omega_j \in \Omega_{BC}} F_x(\omega_j)} \tag{2}$$

$$\Psi_{BC} = \arctan\left[-\frac{\sum_{\omega_j \in \Omega_{BC}} Q_{x,\pi}(\omega_j)}{\sum_{\omega_j \in \Omega_{BC}} C_{x,\pi}(\omega_j)}\right] \tag{3}$$

$$K_{BC} = \left\{\frac{\left[\sum_{\omega_j \in \Omega_{BC}} C_{x,\pi}(\omega_j)\right]^2 + \left[\sum_{\omega_j \in \Omega_{BC}} Q_{x,\pi}(\omega_j)\right]^2}{\left[\sum_{\omega_j \in \Omega_{BC}} F_x(\omega_j)\right]\left[\sum_{\omega_j \in \Omega_{BC}} F_\pi(\omega_j)\right]}\right\}^{\frac{1}{2}} \tag{4}$$

We estimate both the spectral densities of x_t and π_t, the co-spectrum, and the quadrature spectrum, by smoothing the periodograms and, respectively, the cross-periodogram in the frequency domain by means of a Bartlett spectral window. Following Berkowitz and Diebold (1998), we select the bandwidth automatically via the procedure introduced by Beltrao and Bloomfield (1987).

We compute confidence intervals via the multivariate spectral bootstrap procedure introduced by Berkowitz and Diebold (1998) (for technical details, see appendix B): given that we are here dealing with the average values taken by the cross-spectral statistics at the business-cycle frequencies, traditional formulas for computing confidence intervals for the gain, the phase angle, and the coherence at the frequency ω—as found for example in Koopmans (1974), ch. 8—cannot be applied, and the spectral bootstrap is therefore the only possibility. We do not report confidence intervals for the phase angle: given the periodicity of the tangent function, stochastic realisations of the (average) phase angle obtained by bootstrapping the spectral density matrix cannot be properly interpreted. Intuitively, a sufficiently large positive (negative) stochastic realisation is converted by the inverse tangent function into a negative (positive) one, with the result that confidence percentiles for the phase angle cannot literally be constructed.

[22] Given that the Fourier frequencies are uncorrelated, an average value for the two spectra, for the co-spectrum, and for the quadrature spectrum can be computed as a simple average within Ω_{BC}. Given the non-linearities involved in computing gains, phase angles, and coherences, the resulting values are different from the ones we would get by simply taking the averages of estimated gains, phase angles, and coherences within the band. I wish to thank Fabio Canova for extremely helpful discussions on these issues.

Table 4 The correlation between inflation and cyclical indicators at the business-cycle frequencies: average cross-spectral statistics and 90% confidence intervals

	Based on unemployment:			Based on the activity factor:		
	Before Volcker	Volcker-Greenspan	Post-1982	Before Volcker	Volcker-Greenspan	Post-1982
Gain	0.77	0.60	0.42	0.22	0.15	0.10
	[0.32; 1.63]	[0.17; 1.61]	[0.11; 1.34]	[0.07; 0.44]	[0.03; 0.39]	[0.02; 0.33]
Coherence	0.24	0.20	0.16	0.27	0.21	0.16
	[0.11; 0.43]	[0.06; 0.46]	[0.04; 0.42]	[0.09; 0.50]	[0.05; 0.48]	[0.04; 0.44]
Phase angle	-0.75	-0.46	-0.71	-0.40	-0.31	-0.61

Confidence intervals (in parentheses) have been computed via the Berkowitz-Diebold (1998) multivariate spectral bootstrap procedure.

Three findings clearly emerge from the table. First, the imprecision of the estimates, with wide confidence intervals for both the gain and the coherence for all the three sub-periods, and based on either cyclical indicator. Second, in spite of the width of the confidence intervals for the three sub-periods, which systematically overlap with one another, the overall impression is of some decrease in both the gain and the coherence over the post-1979 period, compared with the pre-Volcker era.[23] This is consistent with Brainard and Perry (2000)'s finding of a decrease in the slope of the U.S. Phillips curve over the last two decades, based on a time-varying parameters Phillips curve for U.S. inflation. Third, consistent with a vast body of evidence, for all the three sub-periods, and based on either cyclical indicator, inflation clearly appears to lag the cyclical component of economic activity, as shown by the negative values taken by the phase angle.

4 Monetary Rules and Macroeconomic Stylised Facts

4.1 The Smets-Wouters (2003) model of the U.S. economy

The model we use is a slightly modified version of the sticky-price, sticky-wage DSGE model of the U.S. economy recently proposed by Smets and Wouters (2003), and estimated via Bayesian techniques. Since the structure of the model is extensively discussed in Smets and Wouters (2003, section 2), in what follows we proceed rapidly, and we refer the reader to the original paper for further details.

[23] For the United Kingdom, on the other hand, Benati (2004) documents marked changes in the correlation between inflation and unemployment at the business-cycle frequencies over the last several decades, with the correlation being comparatively steeper during the high inflation of the 1970s, and strikingly flat over the last decade, associated with the introduction of an inflation targeting regime.

Household j maximises the following intertemporal utility function[24]

$$E_0 \sum_{t=0}^{\infty} \beta^t \epsilon_t^b \left\{ \left[\frac{(C_t^j\text{-}H_t)^{1\text{-}\sigma_c}}{(1\text{-}\sigma_c)} + \epsilon_t^L \right] \exp\left[\frac{\sigma_c\text{-}1}{1+\sigma_L} \left(L_t^j\right)^{1+\sigma_L} \right] + \epsilon_t^M \frac{\left(M_t^j/P_t\right)^{1\text{-}\sigma_M}}{(1\text{-}\sigma_M)} \right\} \tag{5}$$

subject to the budget constraint

$$b_t \frac{B_t^j}{P_t} + \frac{M_t^j}{P_t} = \frac{B_{t-1}^j}{P_t} + \frac{M_{t-1}^j}{P_t} + w_t^j L_t^j + \left[r_t^k z_t^j\text{-}\Psi(z_t^j) \right] K_{t-1}^j + D_t^j\text{-}C_t^j\text{-}I_t^j \tag{6}$$

where β is the discount factor; C_t^j, L_t^j, and (M_t^j/P_t) are consumption, labor supply, and real money balances of household j; ϵ_t^b, ϵ_t^L and ϵ_t^M are preference disturbances; σ_c is the relative risk aversion coefficient (within this setup, the same as the inverse of the elasticity of intertemporal substitution); σ_L is the inverse of the elasticity of the work effort with respect to the real wage; σ_M is the inverse of the elasticity of money holdings with respect to the interest rate; H_t, the external habit stock, is a function of aggregate past consumption, $H_t = hC_{t-1}$, with $0 \leq h < 1$; (B_t^j/P_t) is real bond holdings, with price b_t; w_t^j is the real wage; K_t^j is the stock of physical capital owned by household j; $[r_t^k z_t^j\text{-}\Psi(z_t^j)]$ is the net return on capital, where z_t^j is effective capital utilisation, r_t^k is the real return on capital, and $\Psi(z_t^j)$ represents costs associated with the level of capital utilisation; I_t^j is gross investment; and D_t^j are dividends distributed to the households.

Maximisation of (5) subject to (6) implies the following first order conditions for household j:

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t P_t}{P_{t+1}} \right] \tag{7}$$

$$\lambda_t = \epsilon_t^b (C_t^j\text{-}H_t)^{\text{-}\sigma_c} \exp\left[\frac{\sigma_c\text{-}1}{1+\sigma_L} \left(L_t^j\right)^{1+\sigma_L} \right] \tag{8}$$

here put FOC for money (9)

where λ_t is the marginal utility of consumption, and $R_t = 1/b_t = 1 + i_t$ is the gross nominal bond rate.

Households set nominal wages Calvo-style. Following Christiano, Eichenbaum, and Evans (2004) and Smets and Wouters (2002), those households that, in a given period, do not receive the random Calvo signal, index their own wage to a weighted average of last period's inflation and the inflation target. This results in the following law of motion for the aggregate wage level, W_t:

$$W_t^{-\frac{1}{\lambda_w}} = \xi_w \left[W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \bar{\pi}^{1-\gamma_w} \right]^{-\frac{1}{\lambda_w}} + (1 - \xi_w) \tilde{W}_t^{-\frac{1}{\lambda_w}} \tag{10}$$

[24]Different from Smets and Wouters (2003), and in order to study how key business cycle stylised facts involving money growth change as a function of the parameters of the monetary rule, we here introduce real balances in the utility function along the lines of Smets and Wouters (2002).

where $\tilde{W}_t$ is the optimal reset wage at time t; $\bar{\pi}$ is the inflation target of the monetary authority; ξ_w is the fraction of households that do not receive the Calvo signal; and γ_w and λ_w are the degree of indexation to past inflation, and respectively, the wage markup. At the set wage, households supply any amount of labour that is demanded. Intermediate goods producers set the nominal price for their individual product in an analogous way, resulting in the following law of motion for the aggregate price level for final goods, P_t,

$$P_t^{-\frac{1}{\lambda_p}} = \xi_p \left[P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \bar{\pi}^{1-\gamma_p} \right]^{-\frac{1}{\lambda_p}} + \left(1 - \xi_p\right) \tilde{P}_t^{-\frac{1}{\lambda_p}} \tag{11}$$

where the notation is obvious.

Households own the capital stock, which rent to intermediate goods producers at the rate r_t^k. They choose investment, and the utilisation rate of the existing capital stock, in order to maximise their intertemporal objective function, subject to (6) and to the capital accumulation equation

$$K_{t+1} = K_t \left(1 - \tau\right) + I_t \left[1 + \epsilon_t^I - S\left(\frac{I_t}{I_{t-1}} \right) \right] \tag{12}$$

where I_t is gross investment; τ is the depreciation rate; $S(\cdot)$ is the adjustment cost function; and ϵ_t^I is a shock to the relative efficiency of investment goods. Optimisation results in the following first-order conditions for the real value of capital, Q_t, investment, and tehe rate of capital utilisation:

$$Q_t = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1} \left(1 - \tau\right) + z_{t+1} r_{t+1}^k - \Psi(z_{t+1}) \right] \right\} \tag{13}$$

$$Q_t \left[S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \left(1 + \epsilon_t^I\right) \right] = \beta E_t \left[Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} S'\left(\frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}}{I_t} \right] - 1 \tag{14}$$

$$r_t^k = \Psi(z_t) \tag{15}$$

Finally, goods market equilibrium implies

$$Y_t = C_t + I_t + \Psi(z_t) K_{t-1} \tag{16}$$

Different from Smets and Wouters (2003), in what follows we assume that all disturbances are serially uncorrelated. A key reason for doing so is to better highlight the difference between the properties of the model economy within the determinacy and indeterminacy regions.

Log-linearising (7)-(16) around a non-stochastic steady-state, we obtain the following key equations of motion for the endogenous variables:

$$\hat{C}_t = \frac{h}{1+h} \hat{C}_{t-1} + \frac{1}{1+h} \hat{C}_{t+1|t} + \frac{\sigma_c\text{-}1}{\sigma_c(1+\lambda_w)(1+h)} (\hat{L}_t\text{-}\hat{L}_{t+1|t})\text{-}$$

$$-\frac{(1\text{-}h)}{\sigma_c(1+h)}(\hat{R}_t\text{-}\hat{\pi}_{t+1|t})+\frac{(1\text{-}h)}{\sigma_c(1+h)}(\hat{\epsilon}_t^b\text{-}\hat{\epsilon}_{t+1|t}^b) \tag{17}$$

$$\hat{I}_t=\frac{1}{1+\beta}\hat{I}_{t-1}+\frac{\beta}{1+\beta}\hat{I}_{t+1|t}+\frac{1}{\varphi(1+\beta)}\left(\hat{Q}_t+\hat{\epsilon}_t^I\right) \tag{18}$$

$$\hat{Q}_t=\text{-}(\hat{R}_t\text{-}\hat{\pi}_{t+1|t})+\beta(1\text{-}\tau)\hat{Q}_{t+1|t}+[1\text{-}\beta(1\text{-}\tau)]\hat{r}_{t+1|t}^k+\hat{\epsilon}_t^Q \tag{19}$$

$$\hat{K}_t=(1\text{-}\tau)\hat{K}_{t-1}+\tau\hat{I}_{t-1}+\tau\hat{\epsilon}_{t-1}^I \tag{20}$$

$$\hat{\pi}_t=\frac{\beta}{1+\gamma_p\beta}\hat{\pi}_{t+1|t}+\frac{\gamma_p}{1+\gamma_p\beta}\hat{\pi}_{t-1}+\frac{1}{1+\gamma_p\beta}\frac{(1\text{-}\beta\xi_p)(1\text{-}\xi_p)}{\xi_p}\left[\alpha\hat{r}_t^k+(1\text{-}\alpha)\hat{w}_t\text{-}\hat{\epsilon}_t^a\right] \tag{21}$$

$$\hat{w}_t=\frac{\beta}{1+\beta}\hat{w}_{t+1|t}+\frac{1}{1+\beta}\hat{w}_{t-1}+\frac{\beta}{1+\beta}\hat{\pi}_{t+1|t}\text{-}\frac{1+\gamma_w\beta}{1+\beta}\hat{\pi}_t+\frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1}\text{-}$$

$$\text{-}\frac{1}{1+\beta}\frac{(1\text{-}\beta\xi_p)(1\text{-}\xi_p)}{\xi_w[1+\lambda_w^{-1}(1+\lambda_w)\sigma_L]}\left[\hat{w}_t\text{-}\sigma_L\hat{L}_t\text{-}\frac{\sigma_c}{1\text{-}h}(\hat{C}_t\text{-}h\hat{C}_{t-1})+\hat{\epsilon}_t^L\right] \tag{22}$$

$$\hat{L}_t=\text{-}\hat{w}_t+(1+\psi)\hat{r}_t^k+\hat{K}_{t-1} \tag{23}$$

$$\hat{Y}_t=\phi\hat{\epsilon}_t^a+\phi\alpha\hat{K}_{t-1}+\phi\alpha\psi\hat{r}_t^k+\phi(1\text{-}\alpha)\hat{L}_t=\hat{C}_t(1\text{-}\tau k_Y)+\tau k_Y\hat{I}_t \tag{24}$$

$$\hat{M}_t\text{-}\hat{P}_t=-\frac{1}{\sigma_M}\hat{r}_t+\left(\frac{\sigma_C}{\sigma_M}\right)\frac{\hat{C}_t\text{-}h\hat{C}_{t-1}}{1\text{-}h}-\left(\frac{\sigma_C\text{-}1}{\sigma_M}\right)\hat{L}_t+\frac{1}{\sigma_M}\epsilon_t^M \tag{25}$$

where a ˆ above a variable indicates the percentage deviation from the non-stochastic steady-state, α is the capital income share, and $\hat{\epsilon}_t^a$ is a productivity shock. Equations (17) to (19) are the Euler equations for consumption, investment, and, respectively, the real value of capital; equation (20) is the law of motion for capital; equations (21) and (22) are Phillips curves for nominal prices and, respectively, nominal wages; (23) describes labor demand; (24) is a good market equilibrium condition; and equation (25) is the law of motion for real balances. We close the model with the following monetary rule

$$\hat{r}_t=\rho\hat{r}_{t-1}+(1-\rho)\left[\phi_\pi\hat{\pi}_{t+1|t}+\phi_Y\hat{Y}_{t+1|t}\right]+\epsilon_{r,t} \tag{26}$$

where the notation is obvious.

Finally, two important points concerning the way the model has actually been estimated in Smets and Wouters (2003). First, the monetary rule they use to close the model (see their equation 36) is very different from (26)—specifically, (i) it is purely backward-looking; (ii) it features a time-varying inflation target evolving according to a random walk; and (iii) it contains, as additional terms, the first differences of the output gap, and of the deviation of inflation from target. An obvious rationale for using such a rule is that it leads to a better fit, but unfortunately this has the drawback that the way parameters' estimates have been obtained is not fully consistent with the structure of the model we will be using. A simple justification for our approach of using Smets and Wouters's parameters' estimates is to treat it as a sort of 'informed calibration', where parameters' values are calibrated to estimates based on a closely related structure. A more serious problem is that Smets and Wouters (2003), in line with existing literature—with the single exception of Lubik and Schorfheide (2004)—perform estimation by restricting the parameter space to the determinacy region. Given the strong evidence that, for a significant portion of the post-WWII era, the U.S. monetary rule has been such as to give rise to an indeterminate equilibrium,[25] this, as stressed by Lubik and Schorfheide (2004), has the potential to introduce a bias in parameter's estimates. Unfortunately, to this problem there seems to be, at the moment, no solution, as Lubik and Schorfheide (2004)—the only paper estimating a DSGE New Keynesian model without imposing the restriction that the parameters lay within the determinacy region—is based on a markedly simpler model (we will be briefly using their estimated model in section 4.3). Given that it is not possible even to gauge an idea about the size or direction of such a potential bias, in what follows we have decided to simply use the Smets-Wouters estimates to calibrate the model. In evaluating the results we will obtain, however, it is important to keep such a *caveat* in mind.

4.2 Solution method under determinacy and indeterminacy

We define $\xi_t \equiv [\hat{\pi}_{t+1|t},\hat{w}_{t+1|t},\hat{K}_t,\hat{Q}_{t+1|t},\hat{I}_{t+1|t},\hat{C}_{t+1|t},\hat{R}_t,\hat{r}^k_t,\hat{L}_t,X^C_t,X^I_t,X^\pi_t,X^w_t,X^Q_t]'$, where the auxiliary variables X^C_t, ..., X^Q_t are defined as X^C_t=$\hat{C}_t$, ..., X^Q_t=$\hat{Q}_t$. We also define the vector of structural shocks $\epsilon_t \equiv [\hat{\epsilon}^a_t,\ \hat{\epsilon}^b_t,\ \hat{\epsilon}^I_t,\ \hat{\epsilon}^I_{t-1},\ \hat{\epsilon}^Q_t,\ \hat{\epsilon}^L_t, \epsilon_{r,t}]'$, and the vector of forecast errors $\eta_t \equiv [\eta^C_t,\eta^L_t,\eta^I_t,\eta^Q_t,\eta^\pi_t,\eta^w_t,\eta^{r^k}_t]'$, where $\eta^C_t \equiv \hat{C}_t$-$\hat{C}_{t|t-1}$, ..., $\eta^{r^k}_t \equiv \hat{r}^k_t$-$\hat{r}^k_{t|t-1}$. Model (17)-(24) can then be put into the 'Sims canonical form'[26]

$$\Gamma_0\xi_t = \Gamma_1\xi_{t-1} + \Psi\epsilon_t + \Pi\eta_t \tag{27}$$

where Γ_0, Γ_1, Ψ and Π are matrices conformable to ξ_t, ξ_{t-1}, ϵ_t and η_t.

In order to solve the model under both determinacy and indeterminacy, following Lubik and Schorfheide (2003) we exploit the QZ decomposition of the matrix pencil (Γ_0-$\lambda\Gamma_1$). Specifically, given a pencil (Γ_0-$\lambda\Gamma_1$), Moler and Stewart (1973) prove the existence of matrices Q, Z, Λ, and Ω such that QQ'=ZZ'=I_n, Λ and Ω are upper triangular, Γ_0=$Q'\Lambda Z$, and Γ_1=$Q'\Omega Z$. By defining w_t=$Q'\xi_t$, and by premultiplying (27) by Q, we have:

$$\left[\begin{array}{c|c}\Lambda_{11} & \Lambda_{12}\\ \hline 0 & \Lambda_{22}\end{array}\right]\left[\begin{array}{c} w_{1,t}\\ w_{2,t}\end{array}\right] = \left[\begin{array}{c|c}\Omega_{11} & \Omega_{12}\\ \hline 0 & \Omega_{22}\end{array}\right]\left[\begin{array}{c} w_{1,t-1}\\ w_{2,t-1}\end{array}\right] + \left[\begin{array}{c} Q_{1\cdot}\\ \hline Q_{2\cdot}\end{array}\right](\Psi\epsilon_t + \Pi\eta_t) \tag{28}$$

where the vector of generalised eigenvalues, λ (equal to the ratio between the diagonal elements of Ω and Λ) has been partitioned as λ=[λ_1', λ_2']', with λ_2 collecting all the explosive eigenvalues, and Ω, Λ, and Q have been partitioned accordingly. In

[25] Besides Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004), see our estimates in section 5.1 below.

[26] See Sims (2002).

particular, $Q_{j\cdot}$ collects the blocks of rows corresponding to the stable (j=1) and, respectively, unstable (j=2) eigenvalues. The explosive block of (28) can then be rewritten as

$$w_{2,t} = \Lambda_{22}^{-1}\Omega_{22}w_{2,t-1} + \Lambda_{22}^{-1}\left(\Psi_x^*\epsilon_t + \Pi_x^*\eta_t\right) \tag{29}$$

where Ψ_x^*=$Q_{2\cdot}\Psi$, and Π_x^*=$Q_{2\cdot}\Pi$. Given that λ_2 is purely explosive, obtaining a stable solution to (27) requires $w_{2,t}$ to be equal to 0 for any $t \geq$0. This can be accomplished by setting $w_{2,0}$=0, and by selecting, for each $t >$0, the forecast error vector η_t in such a way that $\Psi_x^*\epsilon_t + \Pi_x^*\eta_t$=0.

Under determinacy, the dimension of η_t is exactly equal to the number of unstable eigenvalues, and η_t is therefore uniquely determined. Under indeterminacy, on the other hand, the number of unstable eigenvalues falls short of the number of forecast errors, and the forecast error vector η_t is therefore not uniquely determined, which is at the root of the possibility of sunspot fluctuations. Lubik and Schorfheide (2003), however, prove the following. By defining UDV'=Π_x^* as the singular value decomposition of Π_x^*, and by assuming that for each ϵ_t there always exists an η_t such that $\Psi_x^*\epsilon_t + \Pi_x^*\eta_t$=0 is satisfied, the general solution for η_t is given by

$$\eta_t = \left[-V_{\cdot 1}D_{11}^{-1}U_{\cdot 1}'\Psi_x^* + V_{\cdot 2}M_1\right]\epsilon_t + V_{\cdot 2}M_2 s_t^* \tag{30}$$

where D_{11} is the upper-left diagonal block of D, containing the square roots of the non-zero singular values of Π_x^* in decreasing order; s_t^* is a vector of sunspot shocks; and M_1 and M_2 are matrices whose entries are not determined by the solution procedure, and which basically 'index' (or parameterise) the model's solution under indeterminacy.

As (30) shows, there are two consequences of indeterminacy. First, assuming $M_1 \neq 0$, the impact of structural shocks is no longer uniquely identified. Second, assuming $M_2 \neq 0$, sunspot shocks may influence aggregate fluctuations. Concerning M_1 and M_2 we follow Lubik and Schorfheide (2004), first, by setting $M_2 s_t^*$=s_t, where s_t can therefore be interpreted as a vector of 'reduced-form' sunspot shocks. Second, we choose the matrix M_1 in such a way as to preserve continuity of the impact matrices of the impulse-responses of the model at the boundary between the determinacy and the indeterminacy region.[27] Ideally, this should be done by setting M_2=0 in (30), and by coupling the resulting expression with the solution under determinacy,

$$\eta_t = -(\Pi_x^*)^{-1}\Psi_x^*\epsilon_t \tag{31}$$

Unfortunately, this requires knowledge of *which*, among the generalised eigenvalues that are currently stable under indeterminacy, *would become* explosive under determinacy. As pointed out by Lubik and Schorfheide (2004), the generalised Schur

[27] As discussed in Lubik and Schorfheide (2004), a key reason for doing so is that allowing the model's response to structural shocks to jump discontinuously between the two regions appears as highly unattractive. Results based on the alternative 'orthogonality normalisation', in which M_1 is set to 0, so that fundamental and sunspot shocks perturb the system in completely unrelated 'directions', are available upon request.

decomposition, as implemented by the Moler-Stewart (1973) QZ algorithm, suffers from the drawback that, following a perturbation of the matrix pencil, it does not necessarily preserve the ordering of the generalised eigenvalues.[28] As a result, (31) cannot literally be computed, and the whole method for getting M_1 breaks down. Once again we have therefore followed Lubik and Schorfheide (2004), by adopting the following approach. Let θ be the parameters' vector, and let Θ_I and Θ_D be the sets of all the θ's corresponding to the indeterminacy and, respectively, to the determinacy regions. For every $\theta \in \Theta_I$ we identify a corresponding vector $\tilde{\theta} \in \Theta_D$ laying just on the boundary between the two regions.[29] By definition, the two impact matrices for the impulse-responses of the model conditional on θ and $\tilde{\theta}$ are given by

$$\frac{\partial \xi_t(\theta, M_1)}{\partial \epsilon_t} = \Psi^*(\theta)\text{-}\Pi^*(\theta)V_{\cdot 1}(\theta)D_{11}^{-1}(\theta)U_{\cdot 1}'(\theta)\Psi_x^*(\theta)+\Pi^*(\theta)V_{\cdot 2}(\theta)M_1 \equiv \quad (32)$$

$$\equiv B_1(\theta) + B_2(\theta)M_1 \quad (33)$$

[28] The problem is not unique to the QZ decomposition. In our quest for a solution, we explored another algorithm for the numerical implementation of the generalized Schur decomposition, Kaufman's LZ algorithm, as exposed in Kaufman (1974). (Kaufman's original FORTRAN program, as found in Kaufman (1975), is available from the NAG library. A MATLAB code based on Kaufman (1974) is available from us upon request.) Exactly as the QZ algorithm, the LZ one does not preserve the ordering of the eigenvalues.

We then tried the following approach based on matrix perturbation theory—as expounded in, e.g., Stewart and Sun (1990). The key intuition is that, given a certain perturbation in the matrix pencil (A-λB), e.g. (A-λB)$\Longrightarrow$($\tilde{A}$-$\tilde{\lambda}\tilde{B}$), where $\tilde{A}$=A+dA and $\tilde{B}$=B+dB, with dA and dB being the perturbations in the two matrices A and B, theory puts an upper limit to the perturbations in the corresponding vectors of generalised eigenvalues. Specifically, let λ_h and $\tilde{\lambda}_k$ be the h-th and, respectively k-th generalised eigenvalues corresponding to the two pencils (A-λB) and ($\tilde{A}$-$\tilde{\lambda}\tilde{B}$). The distance between λ_h and $\tilde{\lambda}_k$ is typically measured by means of the 'chordal metric'

$$\chi(\lambda_h, \tilde{\lambda}_k) = (|\lambda_h - \tilde{\lambda}_k|)\Big/[(1+|\lambda_h|^2)^{0.5}(1+|\tilde{\lambda}_k|^2)^{0.5}]$$

As found, e.g., in Stewart and Sun (1990, p. 294, equation 2.3), the upper limit for $\chi(\lambda_h, \tilde{\lambda}_h)$ is given by $\bar{\chi}(\lambda_h, \tilde{\lambda}_h) = (|[dA\ dB]|)/(|a|^2 + |b|^2)^{0.5}$, where a=$y'Ax$ and b=$y'Bx$, with y and x being the normed generalised eigenvectors corresponding to λ_h. In principle, by comparing $\chi(\lambda_h, \tilde{\lambda}_k)$ with $\bar{\chi}(\lambda_h, \tilde{\lambda}_h)$ for each h, k=1, 2, ..., M, with M being the size of λ_h, it should be possible to recover the correct ordering of the generalised eigenvalues. Based on our experience, this unfortunately only works for small perturbations, so that, in our case, if the parameter vector is relatively far away from the boundary between the determinacy and indeterminacy regions, it is not possible to recover the correct ordering of the eigenvalues.

[29] Specifically, for any $[\phi_\pi,\ \phi_y]'$ such that $\theta \in \Theta_I$, we choose the vector $[\tilde{\phi}_\pi,\ \tilde{\phi}_y]'$, such that the resulting $\tilde{\theta} \in \Theta_D$ lies just on the boundary between the two regions, by minimising the criterion $\tilde{C}$=$[(\phi_\pi\text{-}\tilde{\phi}_\pi)^2+(\phi_y\text{-}\tilde{\phi}_y)^2]^{1/2}$. It is important to stress that, in general, there is no clear-cut criterion for choosing a specific vector on the boundary. Minimisation of $\tilde{C}$ is based on the intuitive notion of taking, as the 'benchmark' $\tilde{\theta}$, the one that is closest in vector 2-norm to θ.

and, respectively,

$$\frac{\partial \xi_t(\tilde{\theta})}{\partial \epsilon_t} = \Psi^*(\tilde{\theta}) - \Pi^*(\tilde{\theta}) V_{\cdot 1}(\tilde{\theta}) D_{11}^{-1}(\tilde{\theta}) U_{\cdot 1}'(\tilde{\theta}) \Psi_x^*(\tilde{\theta}) \equiv B_1(\tilde{\theta}) \tag{34}$$

where $\Psi^*(\cdot) \equiv Q\Psi(\cdot)$, and $\Pi^*(\cdot) \equiv Q\Pi(\cdot)$. We minimise the difference between the two impact matrices, $B_1(\tilde{\theta})$-$[B_1(\theta)+B_2(\theta)M_1]$=$[B_1(\tilde{\theta})$-$B_1(\theta)]$-$B_2(\theta)M_1$ by means of a least-squares regression of $[B_1(\tilde{\theta})$-$B_1(\theta)]$ on $B_2(\theta)$, thus setting M_1=$[B_2(\theta)' B_2(\theta)]^{-1} \times B_2(\theta)'[B_1(\tilde{\theta})$-$B_1(\theta)]$.

The solution to (28) is now completely characterised. The forecast error η_t can be substituted into the law of motion for $w_{1,t}$,

$$w_{1,t} = \Lambda_{11}^{-1} \Omega_{11} w_{1,t-1} + \Lambda_{11}^{-1} Q_{1\cdot} (\Psi \epsilon_t + \Pi \eta_t) \tag{35}$$

thus obtaining the final solution for ξ_t as ξ_t=Qw_t=$Q[w_{1,t}', w_{2,t}']'$.

4.3 A digression: the simple case of the standard workhorse New Keynesian model

Before analysing the results based on the Smets-Wouters model, in this section we briefly discuss results based on the standard workhorse New Keynesian model of Clarida, Gali, and Gertler (1999). There are several reasons for doing so. First, as we already stressed, such a model is currently the only one for which we have a set of estimates obtained without imposing the (very likely, implausible) restriction that the parameters only lay within the determinacy region.[30] As such, it should provide us with a useful robustness check: if some of the stylised facts produced by this model as we vary the parameters of the monetary rule turn out to be broadly similar to those produced by the Smets-Wouters model based on a set of estimates possibly biased by the fact of neglecting the possibility of indeterminacy, this should provide some reassurance on the robustness/meaningfulness of the second set of facts. Second, the simple structure of the model makes it possible to obtain a purely analytical solution under both determinacy and indeterminacy,[31] thus eliminating the need to resort to the previously described approximated numerical solution.

The model is described by the following equations:

$$\hat{Y}_t = \hat{Y}_{t+1|t} - \vartheta^{-1} \left[\hat{r}_t - \hat{\pi}_{t+1|t} \right] + g_t \tag{36}$$

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1|t} + \kappa \hat{Y}_t + u_t \tag{37}$$

[30] Here, too, there is however a small fly in the ointment, as the Taylor rule estimated by Lubik and Schorfheide (2004) is the contemporanous version of (26)—i.e., $\hat{\pi}_{t+1|t}$ and $\hat{Y}_{t+1|t}$ have been replaced by $\hat{\pi}_t$ and $\hat{Y}_t$. We regard this, however, as a minor problem.

[31] See Lubik and Schorfheide (2003) and the technical appendix to Lubik and Schorfheide (2004), available at Frank Schorfheide's web page.

where the notation is obvious, with g_t and u_t being white noise demand and supply shocks. Finally, given that the monetary rule estimated by Lubik and Schorfheide (2004) is the contemporanous version of (26)—i.e., $\hat{\pi}_{t+1|t}$ and $\hat{Y}_{t+1|t}$ have been replaced by $\hat{\pi}_t$ and $\hat{Y}_t$, in this section (and only in this section) we close the model with such a rule. Based on Lubik-Schorfheide's (2004) Bayesian estimates, we calibrate the key parameters as follows: β=0.99; κ=0.65; ϑ=1.8. Finally, we set σ_s, the standard deviation of sunspot shocks, to 0.2, and σ_g and σ_u, the standard deviations of demand and supply shocks, to 0.25 and respectively 1.1.

Figures 1 and 2 show how key business-cycle stylised facts—inflation and output gap persistence; the volatility of reduced-form innovations to inflation and the output gap; and the correlation between inflation and the output gap—change with changes in the parameters of the monetary rule. We consider a grid of 100 values of ϕ_π over the interval [0.5; 2], and of 75 values of ϕ_Y over the interval [0; 1]. We set ρ to 0.9.[32] For each combination of ϕ_π and ϕ_Y in the grid we solve the model under either determinacy or indeterminacy—the full analytical solution, computed along the lines of Lubik and Schorfheide (2003) is available upon request—and we simulate it for 10,000 periods.[33] We estimate AR(p) models for inflation and the output gap, selecting the lag order based on the Bayes information criterion, for an upper bound on p equal to 8. The sum of the AR coefficients is, again, our measure of persistence. We do not report Hansen (2000) grid-bootstrap corrected estimates of the sum of the AR coefficients for two reasons. First, with 7.500 points in the ϕ_π-ϕ_Y grid, it would be simply infeasible. Second, the length of the simulation guarantees that the distortion the grid bootstrap should correct for is not an issue here. Consistent with our investigation of the correlation between inflation and cyclical indicators in section 3.4, in figure 2 we report estimates of the average gains, phase angles, and coherences between inflation and the output gap at the business-cycle frequencies, based on the same methodology we described in that section. Finally, following established practice in econometrics—see, e.g., Hendry (1984), Ericsson (1991), and Diebold and Chen (1996)—we do not report the raw results from the simulations, but rather estimated response-surfaces.[34]

Several facts clearly emerge from the figures. First, figure 1 points to significant differences between the determinacy and the indeterminacy regions as far as the persistence and the volatility of reduced-form innovations to either inflation or the output gap are concerned. Given that the model is purely forward-looking, within the determinacy region neither inflation nor the output gap exhibit any persistence. Within the indeterminacy region, on the other hand, persistence is positive for both

[32] Experimentation with two alternative values, 0.5 and 0.7, showed that the *qualitative* features of the results are invariant to the specific value chosen for ρ.

[33] More precisely, for 10,100, then discarding the first 100 observations to make the impact of initial conditions irrelevant.

[34] The methodology we use exactly follows Diebold and Chen (1996, p. 225), to which the reader is referred, with the only difference that we consider expansions up to the third power (instead of the second).

variables—although not especially high, given that the model does not possess any intrinsic inertia—and in the case of inflation it exhibits an interesting relation with the parameters of the monetary rule, being, somehow counterintuitively, *increasing in* ϕ_π *and decreasing in* ϕ_Y. As we will see in the next section, within the Smets-Wouters model, too, inflation persistence is increasing in ϕ_π within the indeterminacy region, and such a result should therefore probably be regarded as reasonably robust. As for the output gap, persistence is, as intuition would suggest, decreasing in ϕ_Y, but, again counterintuitively, it appears to be mostly decreasing in ϕ_π too.

Turning to the volatility of reduced-form innovations, within the determinacy region we get the expected results, with the standard deviation of innovations to inflation (the output gap) being decreasing (increasing) in ϕ_π, and increasing (decreasing) in ϕ_Y. As for the indeterminacy regions, however, several results are, again, counterintuitive, with the volatility of reduced-form shocks to inflation, in particular, being markedly *increasing in* ϕ_π *and decreasing in* ϕ_Y. As for the output gap, the volatility is clearly increasing in both parameters.

Turning to figure 2, the two regions exhibit quite markedly different properties in terms of the correlation between inflation and the output gap, too. First, while within the determinacy region inflation and the output gap move exactly in synch, as shown by the zero value taken by the phase angle, within the indeterminacy region the output gap leads inflation, with the lead being increasing in ϕ_Y and decreasing in ϕ_π. Second, concerning the gain, the two regions appear to exactly mirror each other, with the average gain at the business-cycle frequencies being decreasing in ϕ_π and increasing in ϕ_Y within the determinacy region, and the opposite within the indeterminacy region. Finally, the coherence is increasing in ϕ_π within both regions, while the impact of an increase in ϕ_Y is positive within the determinacy region, and negative within the indeterminacy region.

Summing up, two findings emerge from such an (admittedly limited) exercise. First, in most cases, key macroeconomic stylised facts appear to be quite markedly different within the two regions. Second, for several facts of particular interest—inflation persistence, and the strength of the correlation between inflation and the output gap—the two regions appear to be a sort of mirror image of each other.

4.4 Monetary rules and the business-cycle stylised facts in the Smets-Wouters model

Let's now turn to the Smets-Wouters model. Figures 3-5 shows results from simulations analogous to those we just discussed. Table 5 illustrates details of the calibration: all the parameters' values have been calibrated based on the posterior modes of Smets and Wouters (2003)'s Bayesian estimates, with the exception of the standard deviation of sunspot shock, which we calibrated based on the posterior mode of Lubik and Schorfheide (2004)'s estimates; and of σ_M, which we calibrated based on Smets and Wouters (2002).

Table 5 The calibration for the Smets-Wouters model							
β	τ	α	φ	σ_M	λ_w	λ_p	ϕ
0.99	0.025	0.24	6.14	5	0.5	0.5	1.584
ξ_w	ξ_p	γ_w	γ_p	ψ	σ_c	h	σ_L
0.809	0.902	0.324	0.47	0.27	1.815	0.636	1.942

Standard deviations of structural shocks:			
Money demand	0.5	*Labor supply*	2.111
Wage markup	0.259	*Productivity*	0.48
Price markup	0.186	*Consumption*	1.271
Equity premium	0.615	*Sunspot*	0.5
Investment	0.357		

Figure 3 shows how inflation and output gap persistence, and the volatility of reduced-form innovations to inflation and the output gap, change with changes in the parameters of the monetary rule. Again, all simulations are conditional on ρ=0.9. Several findings emerge from the four panels. First, inflation persistence is monotonically increasing in ϕ_Y within both the determinacy and the indeterminacy regions, and—as intuition would suggest—monotonically decreasing in ϕ_π within the determinacy region. Counterintuitively, however, and in line with the results based on the standard New Keynesian model we discussed in section 4.3, it is monotonically *increasing* in ϕ_π under indeterminacy, for low values of ϕ_Y quite markedly so. For ϕ_Y=0, for example, it increases from 0.65, corresponding to ϕ_π=0.5, to 0.98 corresponding to ϕ_π=0.99. Second, the *responsiveness* of inflation persistence to changes in the parameters of the monetary rule is markedly greater under indeterminacy than under determinacy. For ϕ_Y=0, for example, an increase in ϕ_π from 1 to 2 is associated with a comparatively modest fall in persistence from 0.41 to 0.30. For higher values of ϕ_Y the responsiveness to changes in ϕ_π is even smaller, and for ϕ_Y=1 it is barely discernible, going from 0.465 to 0.462. The same holds for ϕ_Y. While for ϕ_π=0.5 an increase in ϕ_Y from 0 to 0.74 (close to the boundary between the two regions) is associated with an increase in persistence from 0.65 to 0.99, for ϕ_π=2 the corresponding increase is from 0.30 to 0.45. Third—and not surprisingly—the actual *extent* of persistence within the two regions is significantly different, with the inderminacy region being characterised by a markedly higher persistence. In particular, the 'jump' from indeterminacy to determinacy associated with small increases in ϕ_π and/or ϕ_Y for parameters' configurations initially within Θ_I, and close to the boundary, is associated with drastic falls in inflation persistence. For ϕ_Y=0, for example, the 'jump into determinacy' is associated with a fall in inflation persistence from 0.98 to 0.41.

Turning to output gap persistence, two findings stand out. First, again not surprisingly, a markedly greater persistence under indeterminacy than under determinacy. Second, an increase in ϕ_Y causes, as expected, a decrease in persistence under determinacy, but, again counterintuitively, it causes an *increase* under indeterminacy.

Such a result is independent of the specific value taken by ϕ_π and shows, once again, that for key macroeconomic stylised facts the two regions seem to behave as mirror images of each other.

Let's now turn to the volatility of reduced-form innovations. Once again, the two regions appear to be characterised by a markedly different set of stylised facts. First, for both variables, the volatility of shocks appears to be markedly greater under indeterminacy than under determinacy. Second, once again, the responsiveness of volatility to changes in the parameters of the monetary rule under inderminacy appears, in general, quite markeldy greater than under determinacy. This is expecialy clear for the volatility of reduced-form inflation innovations, but also holds for the output gap. Finally, once again, the two regions appear to behave, along a number of dimensions, as mirror images of each other. Exactly as in the case of the standard worhorse New Keynesian model we discussed in section 4.3, the volatility of reduced-form inflation innovations is increasing in ϕ_π under indeterminacy, decreasing under determinacy. The impact of an increase in ϕ_Y on the volatility of reduced-form output gap innovations, on the other hand, appears to be negative within either region, with the exception of a small portion of the indeterminacy region.

Figure 4 shows the average gain, phase angle and coherence between the output gap and inflation at the business-cycle frequencies. Consistent with the results we discussed in section 4.3 based on the standard New Keynesian model, the two regions exhibit, once again, quite markedly different properties. First, the strength of the correlation—as measured by the average gain—is markedly lower under determinacy, and is virtually unresponsive to changes in the parameters of the monetary rule. Under indeterminacy, on the other hand, an increase in either ϕ_π or ϕ_Y causes, somehow counterintuitively, an *increase* in the average gain. Second, the coherence, too, is markedly higher under indeterminacy—further, the impact of an increase in ϕ_Y on the coherence is positive under indeterminacy, negative under determinacy. Finally, the phase angle is decreasing in ϕ_π within both regions, while it is increasing in ϕ_Y under determinacy, and decreasing under indeterminacy. An interesting finding is that, while the output gap leads inflation for nearly all the parameters' configurations considered herein, for a small region of the parameters' space characterised by extremely low values of both ϕ_π or ϕ_Y the opposite holds true, with inflation leading the output gap.[35]

Finally, figure 5 shows the logarithms of the standard deviations of the business-cycle components of several macroeconomic time series. Consistent with the empirical investigation in section 3.3., business-cycle components have been extracted by means of the Christiano-Fitzgerald (2003) band-pass filter. [Here finish to comment. Key

[35] In his investigation of changes in U.K. macroeconomic performance over the post-WWII era, Benati (2004) estimates a *positive* phase angle between the unemployment rate and inflation (namely, a *lead* of inflation over the output gap) during the period of the high inflation of the 1970s. Admittedly, such a result is based on a relatively short sample period. Still, however, it is an intriguing one.

points are:

(a) Nominal rate (i) the volatility of the business-cycle component of the nominal rate is monotonically increasing in both phipi and phiy within both regions; (ii) there are parameters's configurations for which the volatility is greater under determinacy than under indeterminacy (stress the intuition here).

(b) The volatilities of output, consumption, investment, and employment exhibit a very similar pattern: (i) a marked difference between the two regions, with indeterminacy being characterised by a markedly larger volatility (so a move from indeterminacy before Volcker to determinacy after 1979 can indeed explain, in principle, what we see in the data); (ii) all four volatilities are clearly decreasing in phiy under indeterminacy, though not for phiy tending to 1; under determinacy they seem to be slightly decreasing in phiy; (iii) the impact of phipi is not clear under indeterminacy, while under determinacy it is clearly positive.

(c) Money growth: (i) again, huge difference between determinacy and indeterminacy, with latter regions characterised by a markedly greater volatility; (ii) under determinacy, a negative impact of phipi and a positive one of phiy; (iii) a move from indeterminacy (before Volcker) to determinacy (after 1979) cannot possibly explain the increase in the volatility of the business-cycle componeent of money growth we see in the data.

(d) Inflation and the price level show an analogous pattern (stress that this is not necessarily to be expected, given that the stylised facts concerning the two are not the same: for example, inflation is typically pro-cyclical while the price level is counter-cyclical): (i) their volatilities are increasing in both phipi and phiy within the indeterminacy regions; (ii) under determinacy they are (as expected) increasing in phiy and decresing in phipi, but the effect is almost negligible; (iii) the difference between the two regions in terms of volatilities is quite huge.]

5 Can Shifts in Monetary Policy Explain Changes in the Macroeconomic Stylised Facts?

Given that the indeterminacy region, compared with the determinacy region, is characterised by a markedly different set of 'macroeconomic stylised facts'—inflation is more persistent; the volatility of reduced-form innovations to both inflation and output growth is greater; and the business-cycle components of key macro series are more volatile—a possible explanation for the changes in key stylised facts we documented in section 3 is that, as first suggested by Clarida, Gali, and Gertler (2000), during the highly volatile period preceding the appointment of Paul Volcker the economy was operating under indeterminacy, and that the more aggressive monetary policy of the post-1979 era moved it well inside the determinacy region. On the other hand, given the overall modest responsiveness of macroeconomic stylised facts to changes in the parameters of the monetary rule under determinacy, the alternative hypothesis that (a) the economy has been operating within the determinacy region during both sub-periods, and (b) after 1979 monetary policy just got more activist, appears, *by itself*—namely, without being accompanied by *other* changes in the economic environment—incapable of explaining the marked changes we documented in section 3. Further, it is difficult to make sense of the highly persistent behavior of inflation before Volcker's appointment based on the results contained in figure (3a): with inflation persistence under determinacy being always smaller than 0.5, estimates of the sum of the AR coefficients close to 1 are difficult to rationalise.[36] On the other hand, as we will stress in what follows, the increase, post-1979, in the volatility of the business-cycle component of money growth documented in section 3.3 appears as impossible to rationalise in terms of a move from indeterminacy to determinacy. In this section we therefore try to assess the plausibility of the hypothesis that the differences between the two periods, in terms of macroeconomic stylised facts, may originate from the fact that, before and after Volcker's appointment, the economy has been operating within the indeterminacy and, respectively, determinacy regions. We therefore start by estimating forward-looking monetary rules for the three sub-periods of interest.

5.1 Estimating forward-looking monetary rules

Table 6 reports results from estimating the following standard forward-looking monetary rule:

$$r_t = \rho_1 r_{t-1} + \rho_2 r_{t-2} + (1 - \rho_1 - \rho_2)\,\tilde{r}_t + \epsilon_{R,t} \tag{38}$$

$$\tilde{r}_t = \tilde{r} + \phi_\pi \left(\pi_{t+h|t} - \tilde{\pi}\right) + \phi_y y_{t+k|t} \tag{39}$$

where r_t is the Federal funds rate; $\tilde{r}_t$ is the target rate at time t; ρ_1 and ρ_2 are partial adjustment coefficients; $\tilde{\pi}$ is the inflation target; $\tilde{r}=\varphi+\tilde{\pi}$ is the long-run target for the Federal funds rate, with φ being the long-run equilibrium real rate; $\epsilon_{R,t}$ is a zero-mean, serially uncorrelated monetary policy shock; and $\pi_{t+h|t}$ and $y_{t+k|t}$ are expected inflation and the expected output gap at time $t+h$ and, respectively, $t+k$, based on information at time t. Following Clarida, Gali, and Gertler (2000) we set φ equal to the full-sample mean of the *ex-post* real Federal funds rate, which allows us to separately identify $\tilde{\pi}$.

Estimation is performed via two-stage least squares based on quarterly CPI inflation, quoted at an annual rate,[37] and using three alternative proxies for the output

[36] Adding a vector of autocorrelated structural disturbances to the model as in, e.g., **?** does not solve the problem unless (a) the disturbances were very strongly correlated before Volcker, and (b) they have become much less correlated after 1979. This can be easily illustrated by means of the simple process $y_t=\rho y_{t-1}+u_t$, where the disturbance u_t evolves according to $u_t=\phi u_{t-1}+\epsilon_t$. Assuming ρ=0.5, for the sum of the AR coefficients of the reduced-form expression for y_t to be equal to 0.95, ϕ should be equal to 0.9. Further, a near-unit root for y_t can only be obtained based on a near-unit root u_t.

[37] Qualitatively similar results based on the 3-month CPI inflation (quoted at an annual rate)

gap: the CBO output gap measure, constructed as described in section 2, and either one-sided or two-sided HP-filtered log real GDP. The rationale for considering also the one-sided estimate is that, compared with the two-sided estimate, the CBO output gap measure, or (linearly or quadratically) detrended log real GDP, it presents the advantage of not being based on future information, and therefore of partially addressing Orphanides' criticism.[38] The set of instruments includes a constant and four lags of the Federal Funds rate, the inflation rate, the output gap measure, the quarter-on-quarter rate of change of the producer price index for fuels and related products (quoted at an annual rate); and the spread between the rate on 10-year constant-maturity Treasury bills and the rate on the 3-month Treasury bill quoted in the secondary market. Since, as shown in Clarida, Gali, and Gertler (2000), results for alternative forecasting horizons are broadly similar, in what follows we uniquely focus on the one-quarter ahead horizon. Very similar results based on IV estimation (in which we instrument π_{t+1} with π_{t-1}) for either quarterly CPI or quarterly GDP deflator inflation, based on either of the three output gap proxies, are available upon request.

We also estimated (38)-(39) via GMM, based on the same set of instruments, and using a Newey and West (1987) estimate of the covariance matrix to compute the weighting matrix for the GMM criterion. (Numerical minimisation of the criterion was performed via the Nelder-Mead simplex algorithm, as implemented by the MATLAB subroutine `fminsearch.m`.) Quite surprisingly, in the light of the results reported in Clarida, Gali, and Gertler (2000), we were not able to obtain a consistent set of meaningful results. First, in some cases the algorithm converged to 'non-sensical' solutions. Second, in the other cases results were not consistent across inflation and output gap measures. Given the idiosyncracies which are unfortunately typical of numerical optimisation methods, we have therefore reluctantly decided to resort to 2SLS, less high-tech but, we believe, more robust.

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sampled at the monthly frequency, and based on two alternative output gap proxies—either the one-sided, or the two-sided activity factor used in section 3.4—are available upon request.

[38] See in particular Orphanides (2001). We say *partially* as we are dealing with revised data, instead of real-time data.

Table 6 Estimated forward-looking Taylor rules based on quarterly CPI inflation, quoted at an annual rate

	$\tilde{\pi}^{(a)}$	ϕ_π	ϕ_y	ρ_1	ρ_2	ρ
	Based on one-sided HP-filtered log real GDP					
Before Volcker	-6.2E-4	0.72	0.09	1.02	-0.32	0.70
	(0.02)	(0.09)	(0.25)	(0.11)	(0.09)	(0.08)
Volcker-Greenspan	0.03	2.50	1.12	0.78	0.12	0.90
	(0.04)	(1.01)	(1.21)	(0.09)	(0.09)	(0.05)
Post-1982	0.03	1.17	2.13	1.33	-0.40	0.93
	(0.37)	(0.84)	(1.04)	(0.11)	(0.10)	(0.03)
	Based on two-sided HP-filtered log real GDP					
Before Volcker	-0.03	0.74	0.86	0.95	-0.17	0.78
	(0.06)	(0.11)	(0.54)	(0.10)	(0.10)	(0.07)
Volcker-Greenspan	0.05	2.42	3.27	0.77	0.16	0.93
	(0.07)	(1.24)	(3.21)	(0.08)	(0.09)	(0.05)
Post-1982	-0.02	0.68	2.48	1.17	-0.30	0.87
	(0.04)	(0.47)	(0.66)	(0.11)	(0.11)	(0.03)
	Based on the CBO output gap measure					
Before Volcker	-9.6E-3	0.74	0.37	0.95	-0.23	0.72
	(0.02)	(0.09)	(0.16)	(0.10)	(0.09)	(0.06)
Volcker-Greenspan	0.03	2.96	0.89	0.78	0.14	0.91
	(0.03)	(1.50)	(1.26)	(0.09)	(0.09)	(0.05)
Post-1982	0.02	1.74	1.10	1.33	-0.40	0.92
	(0.06)	(0.80)	(0.72)	(0.11)	(0.11)	(0.03)

[a] In percentage points.

Results based on either output gap proxy are broadly in line with those reported in Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004), with the period preceding Volcker's appointment being characterised, first, by a significantly lower inertia in interest rate setting; and second, by a markedly less aggressive monetary policy than during the post-1979 era. Analogous results based on (a) quarterly GDP deflator inflation (quoted at an annual rate), and the same three output gap proxies; and (b) a backward-looking version of (38)-(39) estimated based on either CPI or GDP deflator inflation, and the same three output gap proxies, are available upon request. Overall, empirical evidence seems therefore to lend strong support to the conventional wisdom notion of a drastic change in the conduct of monetary policy following Volcker's appointment. For our purposes, however, the key issue is not whether U.S. monetary policy has become *more aggressive* after 1979, but rather whether it was such to give rise to an *indeterminate equilibrium* before Volcker's appointment. As we previously stressed, indeed, given (a) the weak reasponsiveness of key macroeconomic stylised facts to changes in the extent of activism of monetary policy which appears to be typical of the determinacy region; and (b) the fact that some stylised facts typical of the pre-Volcker era—like high inflation persistence—cannot possibly arise (at least, conditional on the calibration adopted herein) under determinacy, the only way to generate differences in the macroeconomic stylised facts between the two sub-periods comparable to those we observed in the data is to have the U.S. economy operating within the indeterminacy region before Volcker's appointment.

5.2 Stylised facts generated by estimated monetary rules

Table 7 reports a series of stylised facts generated by the Smets-Wouters model conditional on the estimates of forward-looking rules reported in table 6. Estimates based on two output gap proxies out of three—one-sided HP-filtered log real GDP, and the CBO output gap measure—imply that during the pre-Volcker era the U.S. economy was operating under indeterminacy. Results based on the third output gap proxy, as well as results based on GDP deflator inflation and either of the three output gap proxies, suggest instead that, in spite of a marked increase in the extent of activism post-1979, the U.S. economy was operating under determinacy in the pre-Volcker era, too. As the table clearly shows, the two sets of estimates generate markedly different sets of stylised facts. Focusing on the estimates based on two-sided HP-filtered log GDP and on the CBO output gap measure—which are broadly representative of the two sets—the former ones imply virtually no change in the stylised facts before and after Volcker's appointment, with the only exception of the phase angle between the output gap and inflation. Crucially, inflation persistence and the volatility of reduced-form innovations to inflation and output growth exhibit either no variation, or a negligible amount of variation, between the two periods. This is in spite of the quite marked increase in the extent of activism post-1979 documented in table 6, and reflects the previously stressed overall lack of responsiveness of most macroeconomic facts to the conduct of monetary policy within the determinacy region. Finally, inflation persistence is never greater than 0.48 for either of the three sub-periods.

Estimates based on the CBO output gap measure, on the other hand, imply marked changes in the model-generated stylised facts corresponding to the pre- and post-1979 eras, broadly capable of replicating the main features we have seen in the data for most series. Inflation persistence, for example, falls from 0.99 to less than 0.5, while the volatility of reduced-form innovations to inflation and output growth falls by 83% and respectively 67%. In the data, the fall in volatility for the two post-1979 sub-periods is equal to 11% (Volcker-Greenspan) and 35% (post-1982) based on quarterly CPI; to 73% and 71% based on the GDP deflator; and to 36% and 47% based on monthly CPI.

Turning to the correlation between the output gap and inflation, estimated monetary rules imply an increase in the lead of the output gap over inflation, while the data do not point towards any clear-cut indication. On the other hand, exactly as we have seen in the data both the gain and the coherence experience a decrease, post-1979, compared with the pre-Volcker era.

Table 7 Stylised facts generated by the Smets-Wouters model conditional on estimated monetary rules

	Output gap measure:								
	HP-filtered log real GDP						*CBO gap measure*		
	one-sided			*two-sided*					
	BV	VG	PO	BV	VG	PO	BV	VG	PO
Inflation:									
persistence	0.94	0.45	0.45	0.48	0.47	0.47	0.99	0.45	0.47
volatility of shocks	0.31	0.07	0.07	0.07	0.07	0.07	0.42	0.07	0.07
Output growth:									
volatility of shocks	0.78	0.16	0.15	0.13	0.13	0.12	0.53	0.18	0.17
St. dev. of filtered:									
output	1.37	0.48	0.44	0.36	0.38	0.32	0.73	0.55	0.55
consumption	1.30	0.48	0.45	0.37	0.39	0.34	0.73	0.56	0.55
investment	3.29	0.55	0.55	0.42	0.49	0.44	1.37	0.70	0.70
employment	1.14	0.54	0.53	0.50	0.50	0.47	0.71	0.57	0.58
inflation	0.56	0.20	0.21	0.20	0.21	0.20	0.72	0.21	0.21
prices	1.80	0.49	0.50	0.52	0.52	0.51	2.40	0.51	0.50
nominal rate	0.49	0.20	0.20	0.19	0.20	0.19	0.68	0.21	0.21
money growth	0.61	0.23	0.24	0.23	0.23	0.22	0.69	0.23	0.24
Output-inflation:									
phase angle	0.27	-0.88	-0.27	-0.51	-1.16	-0.60	-0.03	-1.21	-0.56
coherence	0.40	0.09	0.06	0.04	0.06	0.07	0.36	0.09	0.076
gain	0.16	0.04	0.03	0.02	0.04	0.05	0.36	0.03	0.03

BV=before Volcker; VG=Volcker-Greenspan; PO=post-1982

Finally, turning to the volatility of the business-cycle components of macroeconomic indicators, for all of them estimated monetary rules for the three sub-periods imply a fall in volatility post-1979 compared with the pre-Volcker era. This is in line with what we have seen in the data with the only exception of money growth, whose component's volatility has actually increased

[To be finished. Stress the following:

- The increase in the volatility of the business-cycle component of money growth clearly runs against a determinacy/indeterminacy based explanation of differences between the sets of stylised facts for the two periods.

- On the other hand, it appears as very difficult to explain high inflation persistence within this setup without appealing to indeterminacy. Stress once again why the story of the autocorrelated shocks is not convincing.

- If we do not believe in the estimates that imply that before Volcker the U.S. economy was operating under indeterminacy, and we believe on the contrary that it has been operating under determinacy all along, then, what can explain the pattern we've seen in the data? Some candidates that have been advanced in the literature—like better inventory management, by McConnell and Peres-Quiros—*may* explain one of these changes, an increase in output stability. But they can't possibly explain the broad set of stylised facts we've documented. How can you explain, based on such a notion, the fall in inflation persistence, in the volatility of its reduced-form innovations, etc.?

- Conclude with an agnostic tone, but stress that if before Volcker the U.S. economy was not operating under indeterminacy, then we must be able to find *some* difference between the two periods that has a broad-based impact on many different variables at the same time, and that may account for the overall set of changes we have documented.]

6 Conclusions

Several papers—see, e.g., Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004)—have documented how the reaction function of the U.S. monetary authority has been passive, and destabilising, before the appointment of Paul Volcker, and active and stabilising since then. In this paper we first compare and contrast the two sub-periods in terms of several key business-cycle 'stylised facts'. The latter period appears to be characterised by a lower inflation persistence; a smaller volatility of reduced-form innovations to both inflation and real GDP growth; and a systematically smaller amplitude of business-cycle frequency fluctuations.

Working with the Smets-Wouters (2003) sticky-price, sticky-wage DSGE model of the U.S. economy, we then investigate how such stylised facts change systematically with changes in the parameters of a simple forward-looking monetary rule. We solve the model under indeterminacy via the procedure introduced by Lubik and Schorfheide (2003). The two regions appear to be characterised by a markedly different set of macroeconomic stylised facts. Further, in several cases the relationship between the parameters of the monetary rule and key stylised facts under indeterminacy is a sort of mirror image of what it is under determinacy: both inflation persistence and the volatility of its reduced-form innovations, for example, are increasing in the coefficient on inflation under indeterminacy, decreasing under determinacy.

Finally, we compare the stylised facts identified in the data with those generated by the Smets-Wouters model conditional on estimated monetary rules. Although variation in the monetary rule across sub-periods can, in principle, explain a significant fraction of the variation in the stylised facts we consider, results are in general not consistent across different inflation measures and output gap proxies. In particular, some of our estimates imply that the pre-Volcker era, too, was characterised by a determinate equilibrium, in spite of the lower activism of the monetary rule.

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A The Monthly Indicators Used in the Construction of the Activity Factor

Here follows a list of the monthly indicators used in the construction of the activity factor we used in section 3.4. For reasons of space, we only report acronyms and data sources.

Interest rates: GS10, GS1, GS3, GS5, TB3MS, MPRIME, AAA, BAA (source: *Board of Governors of the Federal Reserve System*).

Employment and unemployment indicators: UNRATE, USCONS, USFIRE, USGOOD, USSERV, USPBS, USTRADE, SRVPRD, USTPU, USWTRADE, MANEMP, UEMP15OV, UEMPLT5, UNEMPLOY, PAYEMS, EMRATIO, CE16OV (source: *U.S. Department of Labor: Bureau of Labor Statistics*).

Banking indicators: BUSLOANS, CONSUMER, OTHSEC, REALLN, TOTALSL, INVEST, LOANINV, LOANS, NONREVSL, USGSEC (source: *Board of Governors of the Federal Reserve System*).

Cyclical indicators: INDPRO (source: *Board of Governors of the Federal Reserve System*), NAPM (source: *Institute for Supply Management*), HELPWANT (source: *Conference Board*),

Producer prices: PPIFCG, PPIFCF, PPIENG, PPICRM, PPICPE, PPIACO, PFCGEF, PPIITM, PPIIDC, PPIFGS (source: *U.S. Department of Labor: Bureau of Labor Statistics*).

B The Berkowitz and Diebold (1998) Multivariate Spectral Bootstrap Procedure

The Berkowitz-Diebold spectral bootstrap—a multivariate generalisation of the Franke and Hardle (1992) univariate bootstrap—can be briefly described as follows. Let Z_t=$[x_t,\ \pi_t]'$, and let $\Phi_Z(\omega_j)$, $I_Z(\omega_j)$, and $F_Z(\omega_j)$ be the population spectral density matrix; the unsmoothed sample spectral density matrix; and the smoothed sample spectral density matrix (i.e., the consistent estimator of $\Phi_Z(\omega_j)$), for the random vector Z_t, all corresponding to the Fourier frequency ω_j. As it is well known[39], $I_Z(\omega_j)$ converges in distribution to a N-dimensional—in the present case, a 2-dimensional—complex Wishart distribution with one degree of freedom and scale matrix equal to $F_Z(\omega_j)$, namely

$$I_Z(\omega_j) \xrightarrow{d} W_{2,C}\left(1, F_Z\left(\omega_j\right)\right) \tag{40}$$

where $W_{s,C}\left(h, H\right)$ is a s-dimensional complex Wishart distribution with h degrees of freedom and scale matrix H. Berkowitz and Diebold (1998) propose to draw from

$$I_Z^k(\omega_j) = F_Z\left(\omega_j\right)^{\frac{1}{2}} W_{2,C}^k\left(1, I_2\right) F_Z\left(\omega_j\right)^{\frac{1}{2}} \tag{41}$$

[39] See for example Brillinger (1981).

for all the Fourier frequencies $\omega_j=2\pi j/T$, j=1,2, ..., $[T/2]$, with T being the sample length, and $[\cdot]$ meaning 'the largest integer of'. Confidence intervals are computed by first getting a smoothed estimate of the spectral density matrix, $F_Z(\omega_j)$. Then, for each $\omega_j=2\pi j/T$, j=1,2, ..., $[T/2]$, we generate 1000 random draws from (41), thus getting bootstrapped, artificial (unsmoothed) spectral density matrices, we smooth them exactly as we previously did with $I_Z(\omega_j)$, and we compute the average cross-spectral statistics at the business-cycle frequencies based on (2)-(4). For each sub-sample, the 95% confidence interval is then given by the 95% upper and lower percentiles of the bootstrapped distribution.