Public opinion formation in policy issues. An evolutionary approach

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Abstract

In this paper we seek to shed new light on the social process of political opinion formation from an evolutionary perspective. Drawing on previous contributions in evolutionary economics, we propose and analyze a model in which heterogenous agents (citizens) collectively learn and modify their opinions about the size of public expenditure that is most convenient for the economy. The assumption of bounded rationality on the part of agents gives social interaction, institutional transparency and the citizens' political memory a key role in the dynamics of public opinion formation.

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1 Introduction

Public choice theory studies collective action in political processes by applying economic analysis. Thus, most of public choice theory centers around the equilibrium-oriented, comparative static methodology typical of neo-classical economics, where individuals are modelled as perfectly rational utility maximizers with no information problems (Mueller, 1976). Although the theoretical advances within the realm of public choice theory over the past half century are, without doubt, among the most important achievements of the economic science, some authors have suggested the possibility of extending the comparative static interpretations to incorporate a more dynamical approach (see e.g. Witt 2003a). The processes of formation of political ideals and of public opinion, as well as the way in which actions are carried out in areas such as economic policy, have an essentially dynamic character (Metcalfe, 1995; Witt, 2003b) that is not well represented in traditional public choice models. Moreover, some of the authors that have been most influential in the development of these theories and models (see e.g. Mueller, 1993, pp. 489-514) consider that it is desirable to revise some of the postulates of public choice related to the behavior of the agents (voters, politicians, citizens); concretely, they point out the convenience of revising the assumption of perfect rationality, allowing more bounded types of rational behavior (Simon, 1983) and learning processes. The incorporation of such aspects would allow the models to capture far more realistically certain aspects of policy making and public opinion formation, such as the ongoing creation and acquisition of new factual and normative knowledge that takes place during the political process.

This situation offers evolutionary economics a unique opportunity to enrich its particular research program[1] , by accepting the challenge of incorporating certain public choice

[1] Excellent surveys of this research program, its frontiers and present challenges are Nelson (1995),

issues into dynamical evolutionary frameworks. There are at least two reasons why this should be considered a worthy venture. Firstly, political phenomena share important features with other social and economic processes such as technical or institutional change (Nelson and Winter, 1982; Nelson, 1995). Diversity in political options and heterogenous views of voters and citizens, social interactions through local interactive networks, competitive processes on the part of political parties to capture voters' attention and the imperfect understanding of the present and future environmental conditions of policy makers and policy recipients, are some of the reasons that signal political processes as being good candidates for analysis from an evolutionary perspective. Secondly, although the research program of evolutionary economics claims to offer original insights in many fields of economics, it is time to address economic policy making from a formal theoretical perspective since the absence of solid theoretical foundations for policy making is an important shortcoming of the evolutionary approach (Witt, 2003b).

In this paper we take modest steps towards evolutionary theorizing on economic policy making by constructing a model in which heterogenous citizens interact and learn at different levels to come up with a precise opinion on a specific policy issue: the question of the most suitable size of the public sector in the economy. Although we will not propose any voting procedure in the model, nor will we be able to determine the internal structure of policy making, the analysis of public opinion formation is a first step in approaching political processes from an evolutionary perspective. In fact, at least in democratic societies, the dynamics of public opinion are often assumed to be the legitimate source of political power and governmental change.

The analysis of the model proposed here clearly shows that, the recollections of recent

Foster and Metcalfe (2001), and Fagerberg (2003).

socio-political events in the minds of citizens, the extent to which they trust public institutions, and their permeability to the political opinions that they get from daily social interaction, are key factors in explaining the dynamics of the political preferences in a society. The degree to which political opinions differ over a society's members is another element that determines the trajectory followed by public opinion. In order to arrive at these and other results the paper is organized as follows: in section 2 we consider the characteristics that an evolutionary approach to political dynamics should have. In section 3 we propose a model with these characteristics, that allows us to study the concrete problem of how public opinion on issues of economic policy is formed. Sections 4 and 5 contain the dynamic analysis of the model and the interpretation of the results. Finally we end with the conclusions.

2 Towards an evolutionary approach to policy issues

An evolutionary approach to policy making should provide new aspects of realism to the analysis of political process by considering, at least, the following aspects (Witt, 2003b):

1. *Bounded rationality* on the part of agents (citizens that are affected by policy actions, policy makers, etc.). The cognitive limitations of individuals, and the way in which they concentrate their attention temporarily on specific matters while neglecting others, in general serves to impede their ability to establish complete, stable and perfectly coherent political preferences (Simon, 1983). Besides, bounded rationality introduces the need of the agents to attempt to improve their knowledge through learning, but learning takes time. Thus, this assumption on the behavior of agents necessarily places us within a *dynamical approach* to policy issues (Metcalfe, 1995; Dosi, 2000).

2. *Heterogeneity*: The agents have heterogeneous and incomplete views of the values, ends and interests that they find desirable.

3. Limited attention and information processing capacity implies selective learning (*agenda setting effects*; Witt, 2003b). Thus, in the process of formation of political opinions not all issues are treated with the same importance; certain aspects and measures gain special relevance for individuals at particular moments of time.

4. *Selective learning is socially contingent*, in the sense that face-to-face communication, conversation circles and local networks play a key role in setting the agents' political agenda and in generating new normative and factual knowledge.

5. Given the socially contingent nature of political learning, we can argue that the process of political opinion formation is a collective learning process involving heterogenous agents with ever-changing opinions. Therefore, we can interpret the trajectory of formation and transformation of public opinion within a society as an *emerging property* of this collective learning process.

6. Not only social learning but also *trust, habits, fears and disappointments from past experiences* play important roles in the process of public opinion formation (Witt, 2003b).

In the model that follows, we consider these aspects with the objective of analyzing the process of transformation of public opinion as to the appropriate size of the public sector in the economy. There exist as many possible different political opinions as we want, and the society is initially distributed over them. We call this distribution the initial state of public opinion. We assume that citizens can revise their opinions according to the frequency with which they perceive other opinions that are moderately different from their own. This process of interaction and interchange influences the way in which the different opinions are valued within the society. As time goes on, those political opinions that are most valued socially gain weight. Because we assume bounded rationality, we do not consider individual processes of rational choice on behalf of the citizens. Instead, we assume that the population situates itself along the different political opinions, and that the development of this process over time depends on how each possible opinion goes gaining or losing social favor.

3 Modelling public opinion formation

3.1 The range of political opinions

Assume that there exist n $(i = 1, ..., n)$ different political opinions in a society, all related to a particular issue of public interest. Concretely, we assume that in a society made up of free and heterogeneous citizens, there are n different opinions as to the appropriate size of the public sector in the economy. We shall assume that the participation of the public sector in the economy can be approximated by a particular variable: the proportion of total GDP that is represented by the public administrations' demand for goods and services[2] . If we denote the public sector's participation in the economy as defined by opinion i by g_i , $(0 < g_i < 1)$, then we have a vector of possible opinions $(g_1, ..., g_n)$ along which the citizens may position themselves[3] . We can order the components of this vector from greatest to smallest, so that political opinion "1" favors the least possible presence of the public sector in the economy, while opinion "n" defends the greatest level of public spending as a fraction of GDP. For simplicity, and without affecting any results in any significant way, we

[2] Note that other options are possible, like for example the relative weight of the public deficit in GDP, fiscal pressure, etc.

[3] We should not associate directly an opinion g_i of the appropriate weight of the public sector in the economy with other ideological positions related to other issues. Historically we have examples of political opinions that are profoundly favorable to public intervention and yet that maintain very different ideas as to other issues.

assume that the difference between any two neighboring political opinions is a distance a, $(0 < a < 1)$. The value of this parameter determines the degree of differentiation between the political opinions. Thus, given g_1 we have $g_2 = g_1 + a$, $g_3 = g_2 + a = g_1 + 2a$, etc. Generalizing to the case of n different opinions, we get:

$$g_i = g_1 + (i-1)a, \text{ with } \ i = 1, ..., n \ \text{ and } g_n < 1 \tag{1}$$

Since we can, in principle, assume that n is as large as we want, the variety of political opinions that exist in the society can also be as wide as we want.

3.2 The evolving structure of public opinion

Let s_i denote the proportion of all citizens whose political opinion as to the appropriate weight of the public sector is g_i. Thus, $0 \leq s_i \leq 1$ and $\sum_i s_i = 1$. The distribution $\{s_i(t)\}_{i=1}^n$ synthesizes the positioning of the public opinion concerning our specific political issue at instant t. If we are able to establish the social dynamics that determines the evolution of the distribution $\{s_i(t)\}_{i=1}^n$, then we will be able to analyze different evolutions of public opinion, as well as the sensitivity of these evolutions to changes in particular social and economic conditions.

In order to model the evolution of public opinion, we assume (as has been justified in Section 2) that the society is made up of boundedly rational citizens. In our model, this means that given the ongoing social change in public opinion and the cognitive limitations of individuals, the citizens are not able to define rational preferences over the range of different political opinions that are present in the society. Consequently, they must undergo continuous *processes of adaptive political learning.* In their daily lives, the citizens interact with other individuals holding different political opinions in such a way that, in certain circumstances, the valuation of a citizen of his own opinion can be influenced by the opinions of others. These processes of social interaction give rise to certain social tendencies according to which some opinions become more highly valued than others. Likewise, the way in which the social valuation of the different political positions evolves also affects the citizens, perhaps resulting in some of them changing their own opinion. In order to formalize these elements, we shall begin with valuation functions of a citizen's own political opinion like those suggested in what follows.

3.3 The valuation function

Given the state of public opinion at a particular instant $\{s_i(t)\}_{i=1}^n$, we assume that the citizens that hold opinion i at instant t consider if it is convenient or not to continue to hold this opinion. In doing so, they begin by valuing their position according to a function $u_i(g_i)$ like the following:

$$u_i\left(g_i\right) = \left[\alpha + \beta(s_{i+1} - s_{i-1})\right] g_i \quad , \quad \alpha, \beta \in (0,1) \quad , \quad i = 1, ..., n \tag{2}$$

The parameter $\alpha \in (0,1)$ captures the basic social valuation of public activities. The value of α depends on the political memory of the society, on its confidence in the public institutions, on the mechanisms under which the citizens maintain control over public organizations (transparency), and on the efficiency with which the public sector habitually carries out its activity. We shall assume that this basic social valuation is a general characteristic, in the sense that α is common to all citizens. The basic social valuation allows all citizens to evaluate the expected flow and quality of the public services that are derived from having dedicated a proportion g_i $(i = 1, ..., n)$ of GDP to public activities. The valuation αg_i will be greater the greater is the proportion g_i of resources used in public activities, and the greater is the confidence that this spending is done appropriately (α).

When citizens interact amongst themselves, doubts may arise as to the convenience of lending support to a particular level of spending. Under the assumption of bounded rationality, the individuals do not form a definitive opinion as to the best level of g_i, but rather they listen to each other, and in certain conditions, they incorporate into their own valuation the opinions expressed by those others holding political positions that are moderately different from their own. We shall assume that the permeability of the individuals in a society when revising their political valuation, depending on the opinions of others, is given by β. Concretely, the way in which we propose to capture the processes of social interaction within u_i $(i = 1, ..., n)$ is: $\beta(s_{i+1} - s_{i-1})g_i$. This assumption deserves three comments:

- *Local influence:* We assume that the individuals holding opinion i, think about their position under the influence, to a greater or lesser extent, of the opinions of citizens that are politically near-by. Formally, we assume that a citizen who favors a proportion of spending of g_i determines his own political valuation under the influence of the political opinions of those favoring g_{i+1} and g_{i-1}.

- The predominate proportion of individuals whose opinion is moderately different to that of opinion i will have the effect of forcing the preferences of the holder of position i in that direction. Thus if $s_{i+1} - s_{i-1} > 0$ the individuals who favor g_i will revise their basic valuation αg_i upward, with an intensity that increases with β. On the other hand, if $s_{i+1} - s_{i-1} < 0$ then the individuals who favor g_i will be influenced with a greater frequency by those favoring a lower level of public spending, and this can end up being reflected in their own opinion.

- The component $\beta(s_{i+1} - s_{i-1})g_i$ introduces a revaluation or devaluation effect (derived from social interaction) on the basic valuation (αg_i). The possibility that one's own opinion is erroneous implies that citizens can evaluate the consequences of committing errors. Thus, the consequences of errors (that can occur in the hypothetical scenario of a vote and the consequent application of the policies that are voted[4]) are greater the greater is the level of spending that is defended. If serious doubt ($s_{i+1} - s_{i-1} < 0$) is cast upon the feasibility of a particular spending program g_i, the costs derived from the infeasibility of such a political proposition are increasing with g_i, and therefore we would see a significant downward revision of the political valuation of this opinion. If, on the other hand, the opinion of individuals who value g_i is reinforced (from local interaction under conditions of $s_{i+1} - s_{i-1} > 0$) in the sense that the necessities that would be covered using spending level g_i are unavoidable, the valuation of political opinion i would be reinforced by more the greater is the set of public necessities thus identified, that is, the greater is g_i.

Note that in the extreme cases ($i = 1$; $i = n$) the valuation function takes the form $u_1(g_1) = \alpha g_1 + \beta s_2 g_1$ in the first case and $u_n(g_n) = \alpha g_n - \beta s_{n-1} g_n$ in the second. Both political opinions are so extreme that the individuals who hold them can only be influenced by citizens who hold more moderate positions.

3.4 Public opinion formation as a social learning process

As a result of the distribution of public opinion at any given moment and of the permeability of the citizens to social interaction, and depending on the value of the basic social valuation parameter α, the individuals who favor opinion g_i ($i = 1, ..., n$) calculate their political valuation u_i for this option. In what follows, we propose that the process of transformation of public opinion can be modelled as a flow of social interactions between citizens with different opinions and valuations who, because of their bounded rationality, are open

[4] We do not consider such a scenario in our model.

to the possibility of eventually revising their opinion. In as much as social contacts and interchanges of opinion between citizens with different political opinions, it is reasonable to assume that the more highly valued opinions (those with a greater value of $u_i(\cdot)$) will be able to attract citizens who are less content with their own opinion. These people can decide to alter their political position, after having noted that there exist other citizens with opinions that are different from their own, and that appear to be more satisfied than them.

Formally, if we denote by f_{ij} the rate at which citizens with opinion j switch to opinion i, we assume that this switching rate is given by

$$f_{ij} = \gamma \left[u_i - u_j\right]_+ = \gamma \max\left(u_i - u_j; 0\right), \quad \gamma > 0$$

So we are assuming that citizens only switch to more highly valued political opinions; if $u_i > u_j$ we assume that the only changes of opinion that occur are in the direction $j \to i$.

Assuming that the product $\delta s_i s_j$ gives the probability for a random and independent interaction between a citizen with opinion i and another with opinion j, in a small time interval Δt, the flow of citizens from opinion j to opinion i is given by

$$\delta s_i s_j f_{ij} \Delta t$$

and the change in the proportion of citizens who favor opinion g_i $(i = 1, ..., n)$ is

$$\Delta s_i = \sum_{j=1}^{n} \delta s_i s_j (f_{ij} - f_{ji}) \Delta t$$

where

$$f_{ij} - f_{ji} = \gamma \left[u_i - u_j\right]_+ - \gamma \left[u_j - u_i\right]_+ = \gamma \left(u_i - u_j\right)$$

In this way, the continuous evolution of the proportion of citizens favoring opinion i is described by the differential equation

$$\dot{s}_i = \sum_{j=1}^{n} \delta s_i s_j \left(f_{ij} - f_{ji}\right) = \delta s_i \sum_{j=1}^{n} s_j \gamma \left(u_i - u_j\right) = \gamma \delta s_i \left(u_i - \sum_{j=1}^{n} s_j u_j \right)$$

or equivalently (taking $s_i(t) = s_i(\delta\gamma\tau)$, which only represents a change in velocity)

$$\dot{s}_i = s_i \left(u_i - \sum_{j=1}^{n} s_j u_j \right)$$

Therefore, the dynamics of social transformation of public opinion can be represented by the following replicator dynamics system:

$$\dot{s}_i = s_i \left(u_i - \bar{u}\right) \quad \text{with} \quad \bar{u} = \sum_{i=1}^{n} s_i u_i \quad , \quad i = 1, ..., n \tag{3}$$

The difference $u_i - \bar{u}$ indicates, at each instant t, whether opinion i $(i = 1, .., n)$ is above or below the average. The distances $u_i - u_j$, $\forall i, j$ are perceived progressively by the citizens, leading to changes of opinion of some citizens in favor of opinions that are more highly valued socially. The flow of citizens changing opinion appears in the endogenous change in the distribution $\{s_i(t)\}_{i=1}^{n}$. Note that the valuation $u_i = u_i(\cdot)$ made by those citizens favoring option i, when they must decide whether to change their opinion or not, does not by itself determine the final decision of the citizen.

The social process proposed in (3) simply assumes that there exists a continuous process of relocation of the population between the different political opinions. This process is such that the opinions that are more highly valued tend to capture new supporters while the less valued opinions lose support. However the gaining of support is neither immediate nor total. Due to problems of information and of bounded rationality, and due to inherent fondness to certain thought habits, process (3) admits the possibility that even those opinions that are least valued continue to have supporters. Besides, it can be seen from the assumptions in (2) that the transformation of public opinion changes the valuation indexes of each different option, which can make this a perpetual process. The society is thus modelled as an enormous communication system in which local and global connectivity, frequency and the strength of interactions play key roles (Birner, 1999). In the next section, we shall analyze the type of dynamic trajectory that can be generated in this model, and we study their ability to explain certain political phenomena.

4 Analysis of dynamics

The evolution of the model with n political opinions is determined by the system of differential equations:

$$\dot{s}_i = g_i(\mathbf{s}) = s_i f_i(\mathbf{s}) = s_i[u_i - \overline{u}] = s_i[\alpha g_i + \beta(A\mathbf{s})_i - \mathbf{s}(\alpha\mathbf{g} + \beta A\mathbf{s})],\ i = 1, \ldots, n \tag{4}$$

where $\mathbf{s} = (s_1, \ldots, s_n)$, $\mathbf{g} = (g_1, \ldots, g_n)$, A is the $n \times n$ matrix

$$A = \begin{pmatrix} 0 & g_1 & 0 & 0 & \cdots & 0 \\ -g_2 & 0 & g_2 & 0 & \cdots & 0 \\ 0 & -g_3 & 0 & g_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -g_{n-1} & 0 & g_{n-1} \\ 0 & 0 & \cdots & 0 & -g_n & 0 \end{pmatrix} \tag{5}$$

and $(A\mathbf{s})_i$ denotes the i-th element of $A\mathbf{s}$.

The simplex

$$S_n = \left\{ (s_1, \ldots, s_n) \in \mathbb{R}^n : \sum_{i=1}^{n} s_i = 1;\ s_i \geq 0,\ i = 1, \ldots, n \right\}$$

is invariant under (4), and so the phase space of the evolutionary model with n political opinions is given by an $(n-1)$-dimensional simplex. The surface of the simplex consists of n hyperplanes $s_i = 0$, each of which gives an invariant set. Therefore, any intersection of these hyperplanes is also an invariant set. As a direct consequence we conclude that every vertex $\mathbf{e}^k = (0,\ldots,0,\overset{k}{1},0,\ldots,0)$ of the simplex S_n is an equilibrium point of system (4). If (4) has interior equilibrium points they are given by the solutions of

$$\begin{cases} \alpha g_1 + \beta(A\mathbf{s})_1 = \alpha g_2 + \beta(A\mathbf{s})_2 = \ldots = \alpha g_n + \beta(A\mathbf{s})_n \\ s_1 + \ldots + s_n = 1 \end{cases} \tag{6}$$

that satisfy $s_i > 0$.

It is not difficult to prove that $\mathbf{e}^k$, $k = 1,\ldots,n$, are the only equilibrium points of (4) when $\frac{\alpha}{\beta} > \frac{g_n}{a}$. Indeed, let $\mathbf{p}$ be an equilibrium point of (4), i the first integer such that $p_i \neq 0$, and j the last integer such that $p_j \neq 0$ $(1 \leq i \leq j \leq n)$. Let us suppose $i < j$, then

$$\alpha g_i + \beta(A\mathbf{p})_i = \alpha g_j + \beta(A\mathbf{p})_j \Longrightarrow (\alpha + \beta p_{i+1}) g_i = (\alpha - \beta p_{j-1}) g_j \Longrightarrow$$

$$\Longrightarrow \alpha(g_j - g_i) = \beta(p_{i+1} g_i + p_{j-1} g_j) \leq \beta g_j \Longrightarrow \frac{\alpha}{\beta} \leq \frac{g_j}{g_j - g_i} \leq \frac{g_n}{a}$$

Hence, if $\frac{\alpha}{\beta} > \frac{g_n}{a}$, every $\mathbf{p}$ equilibrium point of (4) must satisfy $i = j$ and therefore $\mathbf{p} = \mathbf{e}^k$ for some $k = 1,\ldots,n$.

When $\frac{\alpha}{\beta} < \frac{g_n}{a}$ equation (4) can have more equilibrium points, both interior to S_n and on its boundary. However, the same reasoning leads us to prove that, on the one hand, no equilibrium points exist on the edges $\mathbf{e}^i\mathbf{e}^j$ for $j > i+1$ (if they exist: $\alpha g_i + \beta(A\mathbf{p})_i = \alpha g_j + \beta(A\mathbf{p})_j$, which implies $\alpha g_i = \alpha g_j$), and, on the other hand, neither can they exist on the edges $\mathbf{e}^i\mathbf{e}^{i+1}$ if $\frac{\alpha}{\beta} < \frac{g_i}{a}$ ($\alpha g_i + \beta(A\mathbf{p})_i = \alpha g_{i+1} + \beta(A\mathbf{p})_{i+1}$ implies $\alpha a = \beta(g_i + a p_i) > \beta g_i$).

Although the number of equilibrium points of (4) (and, as we will see, their stability) depends on the values of the parameters g_1, a, α and β, its orbits always converge to $\operatorname{bd} S_n$. This can be proved by transforming (4) into the $(n-1)$-dimensional system

$$\dot{s}_i = \widehat{g}_i(\widehat{\mathbf{s}}) = g_i(\widehat{\mathbf{s}}, s_n),\ i = 1,\ldots,n-1 \tag{7}$$

with $\widehat{\mathbf{s}} = (s_1, \ldots, s_{n-1})$ and $s_n = 1 - \sum_{i=1}^{n-1} s_i$, on the simplex

$$\widehat{S}_{n-1} = \left\{ (s_1, \ldots, s_{n-1}) \in \mathbb{R}^{n-1} : \sum_{i=1}^{n-1} s_i \leq 1;\ s_i \geq 0,\ i = 1, \ldots, n-1 \right\}$$

and determining the sign of the divergence of the vector field $\widehat{\mathbf{g}} = (\widehat{g}_1, \ldots, \widehat{g}_{n-1})$ in $\operatorname{int} \widehat{S}_{n-1}$, which is the same as that corresponding to $\widehat{D}\widehat{\mathbf{g}}$ for any real-valued positive function $\widehat{D}(\widehat{\mathbf{x}})$ defined in $\operatorname{int} \widehat{S}_{n-1}$. Taking

$$D(\mathbf{s}) = \frac{1}{\prod_{i=1}^{n} s_i} \Longrightarrow \widehat{D}(\widehat{\mathbf{s}}) = \frac{1}{\prod_{i=1}^{n-1} s_i \left(1 - \sum_{i=1}^{n-1} s_i\right)}$$

after some computations we obtain

$$\begin{aligned} \operatorname{div} \widehat{D}\widehat{\mathbf{g}} &= \sum_{i=1}^{n-1} s_i \frac{\partial(\widehat{D}\widehat{g}_i)}{\partial s_i} = D \cdot \left[\sum_{i=1}^{n-1} s_i \left(\frac{\partial f_i}{\partial s_i} - \frac{\partial f_i}{\partial s_n} \right) + \sum_{i=1}^{n-1} s_i \frac{f_i}{s_n} \right] = \\ &= -D(\mathbf{s}) \cdot \beta \mathbf{s} A \mathbf{s} = \beta D(\mathbf{s}) \sum_{i=1}^{n-1} (g_{i+1} - g_i) s_i s_{i+1} > 0 \end{aligned}$$

So the divergence of $\mathbf{f}$ is positive at any point of S_n, that is, the replicator equation (4) is volume-expanding in $\operatorname{int} S_n$. Therefore, every orbit of (4) starting in $\operatorname{int} S_n$ converges to $\operatorname{bd} S_n$.

In order to analyze the asymptotic behavior[5] of the orbits of (4) in a more precise way we study the (local) stability of $\mathbf{e}^k$ as follows. The Jacobian of (4) at $\mathbf{e}^k$ is of the form

$$\frac{\partial g_i}{\partial x_j}(\mathbf{e}^k) = \delta_{ij} f_i(\mathbf{e}^k) + \mathbf{e}_i^k \frac{\partial f_i}{\partial x_j}(\mathbf{e}^k)$$

(where δ_{ij} is the Kronecker delta: $\delta_{ij} = 1$ if $i = j$; $\delta_{ij} = 0$ otherwise). If $\mathbf{e}_i^k = 0$ $(i \neq k)$, this reduces to $f_i(\mathbf{e}^k)$ for $i = j$ and to 0 for $i \neq j$, so $f_i(\mathbf{e}^k)$ is an eigenvalue for the (left) eigenvector $\mathbf{e}^i$. $\lambda_{k,i} = f_i(\mathbf{e}^k)$ is called a transversal eigenvalue at the corner $\mathbf{e}^k$ belonging to the eigenvector pointing towards the corner $\mathbf{e}^i$. For our system, we obtain

$$\lambda_{k,i} = \alpha g_i + \beta (A\mathbf{e}^k)_i - \alpha \mathbf{e}^k \mathbf{g} - \beta \mathbf{e}^k A \mathbf{e}^k = \alpha g_i + \beta (A\mathbf{e}^k)_i - \alpha g_k$$

[5] See Hofbauer and Sigmund (1998) for a comprehensible revision of the mathematical concepts and methods used in this paper.

(note that $\mathbf{e}^k A \mathbf{e}^k = 0$ since the main diagonal of A consists of zeros) which leads to

$$\begin{aligned} \lambda_{k,i} &= \beta g_{k-1} - a\alpha, \;\; i = k-1 \\ \lambda_{k,i} &= -\beta g_{k+1} + a\alpha, \;\; i = k+1 \\ \lambda_{k,i} &= \alpha(g_i - g_k), \;\; i \neq k-1, k, k+1 \end{aligned} \tag{8}$$

When $\frac{\alpha}{\beta} > \frac{g_n}{a}$, we obtain that $\lambda_{k,i} < 0$ for $i < k$ and $\lambda_{k,i} > 0$ for $i > k$. Then $\mathbf{e}^1$ is a source, $\mathbf{e}^n$ is a sink, and every $\mathbf{e}^k$ with $k = 2, \ldots, n-1$ is a saddle point, and the edges of the simplex S_n are isoclines of the system that determine the stable and unstable manifolds of the saddle points. In consequence, every orbit of (4) starting in $\operatorname{int} S_n$ tends to $\mathbf{e}^n$; that is the evolution of public opinion shows a long term concentration towards g_n, and the diversity of political opinions gradually disappears.

However, for $\frac{\alpha}{\beta} < \frac{g_1}{a}$, the orbits of system (4) show a qualitatively very different evolution. In this case, again the unique equilibrium points on the edges of S_n are their own vertices $\mathbf{e}^k$, $k = 1, \ldots, n$, and so each edge (not including the corners) is exactly one orbit of system (4). The transversal eigenvalues at every $\mathbf{e}^k$ can be represented as an $n \times n$ matrix, where the entry in row k and column i is $\lambda_{k,i} = f_i(\mathbf{e}^k)$ (and it is 0 if $s_i > 0$ at $\mathbf{e}^k$); so if element $\lambda_{k,i}$ of this matrix is positive then the orbit $\mathbf{e}^k\mathbf{e}^i$ evolves from $\mathbf{e}^k$ to $\mathbf{e}^i$, and from $\mathbf{e}^i$ to $\mathbf{e}^k$ when $\lambda_{k,i} < 0$. For $\frac{\alpha}{\beta} < \frac{g_1}{a}$ the sign structure of the (characteristic) matrix takes the form

$$\begin{pmatrix} 0 & - & + & \cdots & + \\ + & 0 & - & \ddots & \vdots \\ - & + & 0 & \ddots & + \\ \vdots & \ddots & \ddots & \ddots & - \\ - & \cdots & - & + & 0 \end{pmatrix} \tag{9}$$

and so we deduce that a heteroclinic cycle appears on bd S_n.

Indeed, the edges connecting $\mathbf{e}^k$ and $\mathbf{e}^{k-1}$ $(k = 2, \ldots, n)$ are orbits of (4) which evolve from $\mathbf{e}^k$ towards $\mathbf{e}^{k-1}$ since the transversal eigenvalues $\lambda_{k,k-1} = \beta g_{k-1} - a\alpha$ are all positive; and the edge $\mathbf{e}^1\mathbf{e}^n$ is an orbit evolving from $\mathbf{e}^1$ to $\mathbf{e}^n$ since $\lambda_{1,n} = \alpha(g_n - g_1)$ is also positive. So $\Gamma_{n,1} = \{\mathbf{e}^n \longrightarrow \mathbf{e}^{n-1} \longrightarrow \ldots \longrightarrow \mathbf{e}^2 \longrightarrow \mathbf{e}^1 \longrightarrow \mathbf{e}^n\}$ is a heteroclinic cycle of system (4). But it is not the only one: any other sequence $\Gamma_{i,j} = \{\mathbf{e}^i \longrightarrow \mathbf{e}^{i-1} \longrightarrow \ldots \longrightarrow \mathbf{e}^j \longrightarrow \mathbf{e}^i\}$, with $i \geq j + 2$, is also a heteroclinic cycle (note that however there is no 'planar' heteroclinic cycle, connecting only two corners).

Thus, for $\frac{\alpha}{\beta} < \frac{g_1}{a}$, system (4) has a heteroclinic network on bd S_n consisting of one n-cycle, two $(n-1)$-cycles, three $(n-2)$-cycles, ..., and $(n-2)$ 3-cycles; so the number of heteroclinic cycles in the network is $\frac{1}{2}(n-1)(n-2)$. The concrete attracting cycle[6] depends on the values of the parameters: so, the simulations that were carried out show that for $n = 4$ political options and $g_1 = 0.1$, $a = 0.05$ (which generates the sequence $g_1 = 0.1$, $g_2 = 0.15$, $g_3 = 0.2$, $g_4 = 0.25$ of political options), $\beta = 1$, the evolution of the orbits tends to the $\mathbf{e}^3\mathbf{e}^2\mathbf{e}^1$ cycle if $\alpha = 0.5$, the $\mathbf{e}^4\mathbf{e}^3\mathbf{e}^2\mathbf{e}^1$ cycle is attracting when $\alpha = 0.63$, and the attracting cycle is $\mathbf{e}^4\mathbf{e}^3\mathbf{e}^2$ for $\alpha = 1$ (see Figure 1).

(Figure 1 about here)

Finally we analyze the last case: $\frac{g_1}{a} < \frac{\alpha}{\beta} < \frac{g_n}{a}$. As we will see, the dynamical behavior of the orbits of system (4) depends on the relative position of $\frac{\alpha}{\beta}$ in relation to the values $\frac{g_i}{a}$. First of all, note that if $\frac{\alpha}{\beta} > \frac{g_2}{a}$ then all the transversal eigenvalues $\lambda_{1,i}$ are positive, and so

[6] Stability conditions for cycles in heteroclinic networks are only known in very specific situations (see Brannath, 1994; Hofbauer and Sigmund, 1998).

$\mathbf{e}^1$ is a source, which implies that s_1 tends to 0 when $t \longrightarrow \infty$. The asymptotic behavior of the orbits of (4) is then described by the same system (4) but with (the last) $n-1$ variables $(s_2, \ldots, s_n)$. Applying reiterately this procedure we obtain that, if $\frac{g_m}{a} < \frac{\alpha}{\beta} < \frac{g_{m+1}}{a}$ for some $1 \leq m \leq n-1$, the evolution of every s_i with $1 \leq i \leq m-1$ vanishes to 0, and these political options are gradually disappearing from the society. The model is then (asymptotically) reduced to the $(n-m+1)$-dimensional system (4) concerning to the variables $s_m, \ldots, s_n$.

The dynamical behavior of the reduced model depends on the number of values $\frac{g_i}{a}$ greater than $\frac{\alpha}{\beta}$. In particular, if $\frac{g_{n-1}}{a} < \frac{\alpha}{\beta} < \frac{g_n}{a}$ (one value greater), the sign structure of the 2×2 characteristic matrix takes the form

$$\begin{pmatrix} 0 & - \\ - & 0 \end{pmatrix}$$

and, in consequence, we deduce that $\mathbf{e}^n$ and $\mathbf{e}^{n-1}$ are sinks (note that this is possible because a new equilibrium point exists on the edge $\mathbf{e}^{n-1}\mathbf{e}^n$). Hence, in this case, we get a long term concentration on one political option, which depends on the initial conditions. This long term concentration on one political option also takes place when $\frac{g_{n-2}}{a} < \frac{\alpha}{\beta} < \frac{g_{n-1}}{a} < \frac{g_n}{a}$ (two values greater than $\frac{\alpha}{\beta}$): indeed, the (now 3×3) characteristic matrix takes the form

$$\begin{pmatrix} 0 & - & + \\ - & 0 & - \\ - & + & 0 \end{pmatrix}$$

and so $\mathbf{e}^{n-2}$ and $\mathbf{e}^n$ are saddle points, and $\mathbf{e}^{n-1}$ is a sink (note that a new equilibrium point exists on the edge $\mathbf{e}^{n-2}\mathbf{e}^{n-1}$) which represents the asymptotic behavior of the orbits of system (4).

On the other hand, the case $\frac{\alpha}{\beta} < \frac{g_{n-2}}{a}$ (three or more values of $\frac{g_i}{a}$ greater than $\frac{\alpha}{\beta}$) generates time evolutions qualitatively quite different from the previous one ($\frac{\alpha}{\beta} > \frac{g_{n-2}}{a}$), where the diversity of political options is gradually disappearing from the society and a long term concentration on $\mathbf{e}^{n-1}$ or $\mathbf{e}^n$ is produced. This is a consequence of the sign structure of the characteristic matrix, which for $\frac{\alpha}{\beta} < \frac{g_{n-2}}{a}$ is of dimension $n - m + 1 \geq 4$ and takes the form

$$\left(\begin{array}{c|cccc} 0 & - & + & \cdots & + \\ \hline - & & & & \\ - & & & & \\ \vdots & & C & & \\ - & & & & \end{array}\right)$$

where C is the matrix given by (9). So we deduce that every $\mathbf{e}^k$ ($m \leq k \leq n$) is a saddle point (again note that a new equilibrium point appears on the edge $\mathbf{e}^m\mathbf{e}^{m+1}$) and there exists a heteroclinic cycles network connecting the vertices $\mathbf{e}^{m+1}, \ldots, \mathbf{e}^n$. Then the orbits of system (4) evolve towards a heteroclinic cycle and now ($\frac{\alpha}{\beta} < \frac{g_{n-2}}{a}$) the diversity of political options is permanently present over time.

5 Different regimes in the dynamics of public opinion

The dynamic analysis of the model clearly indicates the factors that condition the evolution of public opinion, given an initial range of different options $(g_1, ..., g_n)$, where the difference between each option is determined by the political proximity parameter a. The dynamic model is determined by the value of the ratio $\frac{\alpha}{\beta}$ as compared to the components of the vector $\left(\frac{g_1}{a}, \ldots, \frac{g_i}{a}, \ldots, \frac{g_n}{a}\right)$, and so the fundamental aspect of the interpretation of the results lies in the interpretation of the parameters α and β.

Recall that the value of $\frac{\alpha}{\beta}$ depends on the relationship between the basic social valuation of the public sector's activity, and the ease with which citizens adapt to the different political opinions that they note in their immediate neighborhood. Thus, given a value for β, the greater is social confidence in the public sector, the transparency of public institutions and the efficiency with which the state carries out its activities in the economy, the greater will be the social valuation of the public sector, and the greater will be the ratio $\frac{\alpha}{\beta}$. As a consequence, elements such as corruption and its effects on the political memory of a society, the accumulation of public inefficiencies that tend to erode the confidence of citizens, or the deterioration of the mechanisms of civil control over public institutions will have a negative effect on the parameter α, thereby reducing the ratio $\frac{\alpha}{\beta}$. On the other hand, however, the actions of powerful social agents like, for example, the mass-media, can also affect (in either direction) the value of this ratio. In that way, propaganda in favor of the government, or to the contrary, criticisms of the public administration in the media, can also influence the value of $\frac{\alpha}{\beta}$ (reducing or increasing it, respectively), thereby conditioning the evolution of public opinion.

Following on from the analysis in Section 4 of the exact way in which $\frac{\alpha}{\beta}$ conditions the dynamics of the model, we can establish certain conclusions on the influence of the social aspects concerning the evolution of public opinion that have just been mentioned. Firstly, it was shown in Section 4 that for values of $\frac{\alpha}{\beta} > \frac{g_{n-2}}{a}$ and consequentially, for changes in the aforementioned social conditions that can increase the value of the ratio to such levels, we observe a long run concentration of public opinion on one of the options that assigns maximum weight to the state within the economy, g_n or g_{n-1}. This particular evolution of public opinion is characterized by a progressive erosion in the diversity of opinions, and has its origin in a very high social valuation of the public sector (or in a very low intensity of local interaction between citizens).

If, on the other hand, the society is characterized by a ratio that satisfies $\frac{\alpha}{\beta} < \frac{g_{n-2}}{a}$, that is, if the basic social valuation is low, or if it deteriorates significantly compared to the intensity of the social interactions of the citizens, the evolution of public opinion becomes more complex. Note that it is precisely this type of situation that we can expect to see after certain periods of corruption or when public inefficiency introduces doubts into the minds of citizens. Such an evolution regime for public opinion is characterized by the appearance of a heteroclinic network with the following features:

1. In this evolution regime we find that several political opinions persist indefinitely; that is, we do not get a progressive concentration of the population on a single opinion, as would be the case if $\frac{\alpha}{\beta} > \frac{g_{n-2}}{a}$.

2. As can be seen in Figure 1, the way in which the system tends to $\mathrm{bd}\, S_n$, reveals the fact that, in spite of a general persistence of many opinions, during most of the time two particular opinions predominate over the others (most of the time the orbits are close to the edges of S_n). This property can perhaps explain, within a framework in which political parties incorporate dominant opinions into their electorial campaigns, why we often get two dominant parties. Given the frequency with which such a situation can be found in real-world democracies, we note that our model makes realistic predictions, at least as far as this result is concerned, under certain conditions.

3. The change in the attracting heteroclinic cycle resulting from changes in the values of the parameters (bifucation phenomenon) indicates that small shocks that affect the parameter values slightly can have important effects. We can think of the consequences for public opinion of the denouncement in the mass-media of a case of

corruption. If this news affects the parameters and gives rise to a change in the attracting cycle, we could observe the reappearance of certain political opinions that are not highly valued socially, or even the disappearance of traditionally important options.

4. The analysis in Section 4 also shows that if $\frac{g_m}{a} < \frac{\alpha}{\beta} < \frac{g_{m+1}}{a}$ for some $1 \leq m \leq n-1$, the evolution of every s_i with $1 \leq i \leq m-1$ vanishes to 0, and these political options gradually disappear from the society. This result allows us to understand how, in some situations, we can get processes of successive and ordered disappearance of certain political opinions, precisely those that society considers to be too extreme.

Finally, note that the parameter of political proximity between the different opinions, a, also plays a central role in the evolutions that have just been interpreted: when opinions are closer together (and so we have a greater number of opinions) then we get a greater region of persistence of diversity.

Now that we have analyzed the relationships between the basic social valuation of the public sector (and its determinants), the process of social interaction between citizens, and the concrete evolution of public opinion in a society, we now turn to suggestions of how our model may be extended to cater for the evolutionary analysis of political processes. Without going into too much detail, and since in our model we have limited the opinions of the citizens to the concrete case of the appropriate weight of the public sector in the economy, one particular extension would consist in the study of the way in which different trajectories of public opinion can materialize themselves in the actual trajectories of public spending. Thus, assuming adaptive behavior (vote searching) by certain political parties competing in a democratic election, we could consider the question of to whether the average public opinion trajectory $g(t) = \sum_i s_i(t)\, g_i$, is a relevant indicator of the actual evolution of the weight of the public sector.

A second extension to the model could consist of proposing voting rules that allow the general population to democratically elect from among a set of political parties. These parties would include in their electorial campaigns proposals that reflect some of the political opinions present in the society, and they can adapt these proposals over time according to the evolution of public opinion. In any case, such an extension would imply the need to define a menu of issues (not only the relative size of the public sector) about which the citizens may form opinions.

6 Summary

In this paper we have considered the possibility that public opinion undergoes a transformation as a result of evolutionary processes of social change. Local interactions between boundedly rational citizens, who perceive and adapt to certain social tendencies when they formulate their own political opinions, generate emergent dynamic properties of public opinion that can be quite different. We have seen how, within this type of process, the basic social valuation of public institutions can be a determinant factor for social change.

If our strategy is a valid manner in which to study the dynamics of certain political processes, the next step would be to introduce voting rules under which different political options can be incorporated into democratic election campaigns. To do this, we would need to provide an adequate foundation for the behavior of political parties and policy-makers within a bounded rationality framework. The comparison of the results obtained in such a framework with those already available from Public Choice theory would be of great interest. Such a comparative study would allow us to check if the dynamics of political processes, with boundedly rational agents, implies a step forward in realism or not, in comparison with the rational-static frameworks of Public Choice. However, the complexity of such extensions and the current state of the art in this area of research force us to leave such thoughts on the future research agenda.

References

Birner, J., 1999. Making Markets. In: Dow, S. and Earl, P. (eds.), *Economic Organization and Economic Knowledge.* Edward Elgar.

Brannath, W., 1994. Heteroclinic networks on the tetrahedron. *Nonlinearity* 7, 1367-1384.

Chawanya, T., 1995. A new type of irregular motion in a class of game dynamics systems. *Progress of theoretical physics* 94, 163-179.

Dosi, G., 2000. *Innovation, Organization and Economic Dynamics.* Edward Elgar.

Fagerberg, J., 2003. Schumpeter and the revival of evolutionary economics. *Journal of Evolutionary Economics* 13, 125-159.

Foster, J. and Metcalfe, S. (eds.), 2001. *Frontiers of Evolutionary Economics.* Edward Elgar.

Hofbauer, J. and Sigmund, K., 1998. *Evolutionary Games and Population Dynamics.* Cambridge University Press.

Metcalfe, S., 1995. The Economic Foundations of Technology Policy: Equilibrium and Evolutionary Perspectives. In: Stoneman, P. (ed.), *Handbook of the Economics of Innovation and Technological Change.* Blackwell.

Mueller, D.C., 1976. Public Choice: A Survey. *Journal of Economic Literature* 14, June, 395-433.

Mueller, D.C., 1993. *The public choice approach to politics.* Edward Elgar.

Nelson, R.R., 1995. Recent evolutionary theorizing about economic change. *Journal of Economic Literature* 33, 48-90.

Nelson R.R. and Winter, S., 1982. *An Evolutionary Theory of Economic Change.* Harvard

University Press.

Simon, H.A., 1983. *Reason in Human Affairs.* Stanford University Press.

Witt, U., 2003a. *The Evolving Economy.* Edward Elgar.

Witt, U., 2003b. Economic policy making in evolutionary perspective. *Journal of Evolutionary Economics* 13, 77-94.

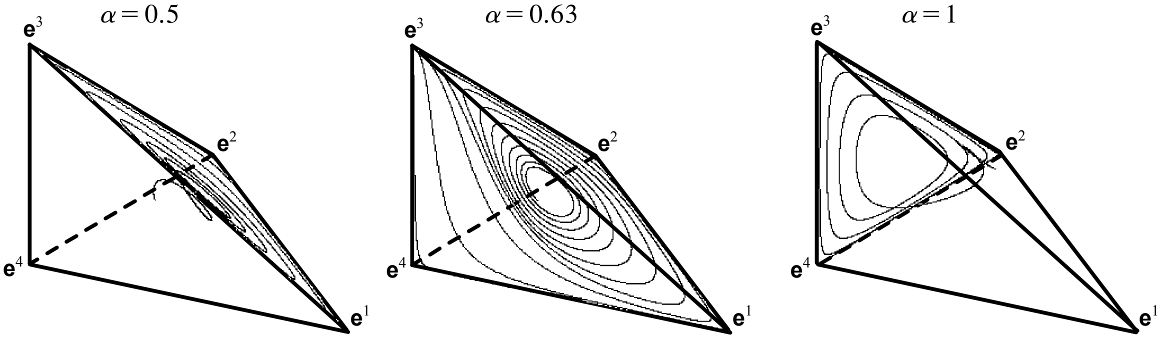

Figure 1: Attracting heteroclinic cycles for $n = 4$