Keynesian Dynamics and the Wage-Price Spiral. Estimating a Baseline Disequilibrium Approach*

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Abstract:

We reformulate the baseline disequilibrium AS-AD model of Asada, Chen, Chiarella and Flaschel (2004) to make it applicable for empirical estimation. The model now exhibits a Taylor interest rate rule in the place of an LM curve, a dynamic IS curve and dynamic employment adjustment. It is based on sticky wages and prices, perfect foresight of current inflation rates and adaptive expectations concerning the inflation climate in which the economy is operating. The implied nonlinear 5D model of real markets disequilibrium dynamics avoids striking anomalies of the Neoclassical synthesis (Stage I). It exhibits Keynesian feedback structures with asymptotic stability of its steady state for low adjustment speeds and with cyclical loss of stability – by way of Hopf bifurcations – when certain adjustment speeds are made sufficiently large.

In the second part we estimated the equations of the model to study its stability features from the empirical point of view with respect to the feedback chains it exhibits. Based on these estimates we also study to which extent a stronger Blanchard / Katz error correction mechanism, more pronounced interest rate feedback rules or downward wage rigidity can stabilize the dynamics in the large when the steady state is found to be locally repelling. The achievements of this baseline disequilibrium AS-AD model, its Keynesian feedback channels and our empirical findings can be usefully contrasted with those of the microfounded, but in scope more limited now fashionable New Keynesian alternative (the Neoclassical Synthesis, Stage II).

Keywords: DAS-DAD growth, wage and price Phillips curves, estimation, adverse real wage adjustment, (in)stability, persistent business cycles, monetary policy rules.

JEL CLASSIFICATION SYSTEM: **E24**, **E31**, **E32**.

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1 Introduction

In this paper we reformulate and extend the standard AS-AD growth dynamics of the Neoclassical Synthesis (Stage I) with its traditional microfoundations, as it is for example treated in detail in Sargent (1987, Ch.5). Our reformulation, based on our earlier presentation and analysis of a DAS-AD alternative to the neoclassical AS-AD framework, see Asada, Chen, Chiarella, Flaschel (2004), now replaces their LM curve with a Taylor interest rate policy rule, as in the New Keynesian approaches. The model, as well as its predecessor, exhibits sticky wages as well as sticky prices, underutilized labor as well as capital stock, myopic perfect foresight of current wage and price inflation rates and adaptively formed medium run expectations concerning the inflation climate in which the economy is operating. Moreover we now employ a dynamic IS-equation in the place of the static one of Asada, Chen, Chiarella, Flaschel (2004) and will also make use of a dynamic form of Okun's law to a certain extent. The resulting nonlinear 5D model of labor and goods market disequilibrium dynamics (with a Taylor type treatment of the financial part of the economy) avoids the striking anomalies of the conventional AS-AD model of the Neoclassical synthesis under myopic perfect foresight, stage I.[1] Instead it exhibits Keynesian feedback dynamics proper with in particular asymptotic stability of its unique interior steady state solution for low adjustment speeds of wages, prices, and expectations among others. The loss of stability occurs cyclically, by way of Hopf bifurcations, when some of these adjustment speeds are made sufficiently large, even leading eventually to purely explosive dynamics sooner or later. This latter fact – if it occurs – implies the need to look for appropriate extrinsic nonlinearities that can bound the dynamics in an economically meaningful domain, such as downward rigidity of wages and prices and the like, if the economy departs too much from its steady state position.

Locally we thus obtain and can prove the existence of in general damped, persistent or explosive fluctuations in the real and the nominal part of the dynamics, in the rates of capacity utilization of both labor and capital, and of wage and price inflation rates which here induce interest rate adjustments by the monetary authority that attempt to stabilize the observed output and price level fluctuations. Our modification and extension of traditional AS-AD growth dynamics, as investigated from the orthodox point of view in Sargent (1987), thus provides us with a Keynesian theory of the business cycle, including a modern approach to monetary policy. This is even true in the case of myopic perfect foresight, where the structure of the traditional approach radically dichotomizes into independent classical supply-side and real dynamics – that cannot be influenced by monetary policy – and a subsequently determined inflation dynamics, that are purely explosive if the price level is taken as a predetermined variable, a situation that forced convergence by an inconsistent application of the jump-variable technique[2] in Sargent (1987,ch.5), see again Asada, Chen, Chiarella and Flaschel (2004) for details. In our new type of Keynesian labor and goods market dynamics we however can treat myopic perfect foresight of both firms and wage earners without the need for the methodology of the rational expectations approach to unstable saddlepoint dynamics.

[1]These anomalies include in particular saddle point dynamics that imply instability unless some poorly motivated jumps – and indeed flawed – are imposed on certain variables, here on both the price and the wage level, see Asada, Chen, Chiarella and Flaschel (2004) for details.

[2]since the nominal wage is transformed into a non-predetermined variable there.

From the global perspective, if our model loses asymptotic stability for higher adjustment speeds, in the present framework specifically of prices and the inflationary climate, purely explosive behavior is the generally observed outcome, as is easily checked by means of numerical simulations. The considered so far only intrinsically nonlinear model type therefore cannot be considered as being complete in such circumstances, since some mechanism is then required to bound the fluctuations to economically viable regions. Downward money wage rigidity is the mechanism we have often used for this purpose and which we will use here again, see the numerical investigations in Asada, Chiarella, Flaschel and Hung (2004). Extended in this way, by simply excluding deflation (to some extent) for m occurring, we obtain and study a baseline model of the DAS-DAD variety with a rich set of stability implications and with various types of business cycle fluctuations that it can generate endogenously or by adding stochastic shocks to the considered dynamics.

The dynamic outcomes of this baseline disequilibrium AS-AD or DAS-DAD model can be usefully contrasted with those of the currently fashionable New Keynesian alternative (the Neoclassical Synthesis, stage II) that in our view is more limited in scope, at least as far as the treatment of interacting Keynesian feedback mechanisms and the thereby implied dynamic possibilities are concerned. This comparison reveals in particular that one does not always end up with the typical (in our view strange) dynamics of rational expectation models, due to certain types of forward looking behavior, if such behavior is coupled with plausible backward looking behavior for the medium-run evolution of the economy. This basic insight is now also stressed in New Keynesian approaches due to the complete empirical failure of their New Phillips curve.

Our dual Phillips curves approach to the wage-price spiral indeed performs quite well, when estimated empirically, as we shall show in this paper,[3] in particular does not give rise to the situation observed for the New (Keynesian) Phillips curve, found to be completely at odds with the facts in the literature [4]. In our approach standard Keynesian feedback mechanisms are coupled with a wage-price spiral having a considerable degree of inertia, with the result that these feedback mechanisms work – as is known from partial analysis – in their interaction with the added wage and price level dynamics.

The present paper therefore intends to provide a proper baseline model of the Keynesian DAS-DAD variety, not plagued by the theoretical anomalies of the traditional AS-AD model and the empirical anomalies of the New Keynesian approach. It does so on the basis of the fully specified DAS-AD growth dynamics of Asada, Chen, Chiarella and Flaschel (2004), by transforming this dynamics into a reduced DAS-DAD form that can be estimated empirically. It discusses the feedback structure of this reduced form and its stability implications, first on a general level and then on the level of the sign restrictions obtained from empirical estimates of the five laws of motion of the dynamics. These estimates also allow us to show asymptotic stability for the estimated parameter sizes and to determine stability boundaries (with respect to price flexibility and the speed of

[3]See also Flaschel and Krolzig (2004), Flaschel, Kauermann and Semmler (2004) and Chen and Flaschel (2004).

[4]In this connection, see for example Mankiw (2001) and with much more emphasis Eller and Gordon (2003), whereas Gali, Gertler and Lopez-Salido (2003) argue in favor of a hybrid form of the New Phillips Curve.

adjustment of the inflationary climate) where the need for further (behavioral) nonlinearities therefore becomes established, to be investigated and discussed in a companion paper to the present one (Asada, Chiarella, Flaschel and Hung, 2004).

Section 2 presents our reformulation of the baseline Keynesian DAS-AD growth dynamics of Asada, Chen, Chiarella and Flaschel (2004) as a DAS-DAD growth dynamics in order to make this model applicable to empirical estimation. Section 3 considers the feedback chains of the reformulated model and derives cases of local asymptotic stability and of loss of stability by way of Hopf-bifurcations. In section 4 we estimate the model to find out sign restrictions and which type of feedback mechanisms may apply to the US-economy after World War II. Section 5 then investigates again the stability of the dynamics on the basis of these sign restrictions and determines stability boundaries with respect to the adjustment speed of prices and the inflationary climate variable. Section 6 concludes and considers briefly behavioral assumptions that may provide global stability to the economy in the cases where the steady state is surrounded by centrifugal forces. We there also discuss in which way the resulting persistent fluctuations will be influenced through a stronger conduct of interest rate policy rules.

2 Keynesian disequilibrium dynamics: Empirically oriented reformulation of a baseline model

In this section we reformulate the theoretical disequilibrium model of AS-AD growth of Asada, Chen, Chiarella and Flaschel (2004) in order to make it applicable for empirical estimation and for the study of the role of contemporary interest rate policy rules. We thus dismiss the LM curve of the original approach and replace it now by a Taylor interest rate policy rule and now also use dynamic IS as well as employment equations in the place of static ones of the original approach, where with respect to the former the dependence of consumption and investment on income distribution now only appears in aggregated or reduced type format. We furthermore now use Blanchard and Katz (2000) type error correction terms both in the wage and the price Phillips curve that give income shares a role to play in wage as well as in price dynamics. Finally, we will add inflationary inertia to a world of myopic perfect foresight through the inclusion of a medium-run variable, the inflationary climate in which the economy is operating, and its role in the wage and price dynamics of the considered economy.

We start from the observation that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found for this in Keynes' General Theory) allow for under- (or over-)utilized labor *as well as* capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes, IS-LM is (or should be) based on imperfectly flexible wages <u>and</u> prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that medium-run aspects count both in wage and price adjustments, here still formulated in simple terms by the introduction of the concept of an inflation climate. This economic climate term is based on past observation, while we have model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS-AD model that we shall introduce thus treats expectations in a hybrid way, myopic perfect foresight of the current rates of wage and price inflation on the one hand and an adaptive updating of an economic climate expression with exponential or other weighting scheme on the other hand.

We assume two Phillips Curves or PC's in the place of only one. In this way we can discuss wage and price dynamics separately from each other, in their structural forms, both based on their own measure of demand pressure, namely $V^l - \bar{V}^l, V^c - \bar{V}^c$, in the market for labor and for goods, respectively. We here denote by V^l the rate of employment on the labor market and by $\bar{V}^l$ the NAIRU-level of this rate, and similarly by V^c the rate of capacity utilization of the capital stock and $\bar{V}^c$ the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation, $\hat{w}, \hat{p}$, are here both augmented by a weighted average of cost-pressure terms based on forward looking myopic perfect foresight and a backward looking measure of the prevailing inflationary climate, symbolized by π^m. Cost pressure perceived by workers is a weighted average of the currently evolving rate of price inflation $\hat{p}$ and a medium-run concept of price inflation, π^m, the inflationary climate in which the economy is operating, which is based on past observations. Similarly, cost pressure perceived by firms is given by a weighted average of the currently evolving (perfectly foreseen) rate of wage inflation $\hat{w}$ and again the measure of the inflationary climate in which the current state of the economy is embedded.

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS-AD dynamics are – qualitatively seen – still formulated in a fairly symmetric way.[5] We stress that we have included forward-looking behavior here, without the (later) need for the jump variable technique of the rational expectations school.

Structural form of the wage-price dynamics:

$$\begin{aligned} \hat{w} &= \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^m, \\ \hat{p} &= \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^m. \end{aligned}$$

Inflationary expectations over the medium run, π^m, i.e., the inflationary climate in which current inflation is operating, may be adaptively following the actual rate of inflation (by use of some linear or exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other possibilities for updating expectations. For simplicity of the exposition we shall make use of the conventional theoretical expression of an adaptive expectations mechanism:

[5] With respect to empirical estimation we could also consider the role of labor productivity growth, which we here assume to be zero to ease the presentation of the model. This role is found to be of second order only in the empirical estimates of the considered wage and price Phillips curves, as far as trend terms in wage inflation are concerned. And with respect to the distinction between real wages, unit wage costs and the wage share we shall detrend the corresponding time series such that the following types of PC's can still be applied. empirically, we therefore concentrate on the cyclical features of the model by the formulation of the structural equations of the model and the choice of detrended time series.

$$\dot{\pi}^m = \beta_{\pi^m}(\hat{p} - \pi^m)$$

in the presentation of the full model below. Besides demand pressure we thus use (as cost pressure expressions) in the two PC's weighted averages of this economic climate and the (foreseen) relevant temporary cost pressure term for future wage and price setting. In this way we get two PC's with very analogous building blocks, which despite their traditional outlook will have a variety of interesting and novel implications. Somewhat simplified versions of these two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2004), Flaschel, Kauermann and Semmler (2004) and Chen and Flaschel and Chen (2004) and found to represent a significant improvement over single reduced-form Phillips curves, with in fact wage flexibility being greater than price flexibility with respect to their demand pressure item in the market for goods and for labor, respectively. Note that such a finding is not possible in the conventional framework of a single reduced-form Phillips curve.

We have added here to these earlier studies Blanchard and Katz type error correction mechanisms (the second β terms in each equation) not only in the wage inflation equation, but also in the price inflation equation. The minus sign in front of β_{w_2} is motivated as in the article of Blanchard and Katz (2000), see also Flaschel and Krolzig (2004) in this regard, while the plus sign in front of β_{p_2} simply represents a second measure of cost pressure, in addition to the weighted average of inflation rates shown thereafter. To simplify steady state calculations we measure these error correction terms in deviation from their steady state values.

Note that for our current version, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by ($\kappa = 1/(1 - \kappa_w \kappa_p)$):

$$\hat{\omega} \;=\; \kappa[(1-\kappa_p)(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o)) - (1-\kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o))]$$

This follows easily from the following obviously equivalent representation of the above two PC's:

$$\begin{aligned} \hat{w} - \pi^m &= \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o) + \kappa_w(\hat{p} - \pi^m), \\ \hat{p} - \pi^m &= \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o)) + \kappa_p(\hat{w} - \pi^m), \end{aligned}$$

by solving for the variables $\hat{w} - \pi^m$ and $\hat{p} - \pi^m$. It also implies the two across-markets or reduced form PC's given by:

$$\begin{aligned} \hat{p} = \quad & \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o) + \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o))] + \pi^m, \\ \hat{w} = \quad & \kappa[\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o)) + \kappa_w(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o))] + \pi^m, \end{aligned}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market.

The remaining laws of motion of the private sector of the model are as follows:

$$\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\omega - \omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) \\ \hat{V}^l &= \beta_{V^l_1}(V^c - \bar{V}^c) - \beta_{V^l_2}(\omega - \omega_o) + \beta_{V^l_3}\hat{V}^c \end{aligned}$$

The first law of motion is of the type of a dynamic IS-equation, see also Rudebusch and Svensson (1999) in this regard, here expressed in terms of the growth rate of the rate of capacity utilization of firms and linearized around the steady state of the model. It reflects the dependence of excess goods demand on aggregate (income) supply and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect, it shows the joint dependence of consumption and investment on the real wage (which in the aggregate allows for positive or negative signs before α_ω depending on whether consumption or investment is more responsive to real wage changes) and shows finally the negative influence of the real rate of interest on the evolution of economic activity. Note here that we have generalized this law of motion in comparison to the original baseline model of Asada, Chen, Chiarella and Flaschel (2004), since we now allow for the possibility that also consumption, not only investment, depends on income distribution as measured by the real wage.

In the second law of motion, for the rate of employment, we assume that the employment policy of firms follows their rate of capacity utilization (and the thereby implied rate of over- or underemployment of the employed workforce) with a lag (measured by $1/\beta_{V_1^l}$). Employment is thus assumed to adjust to the level of current activity in delayed form which is a reasonable assumption from the empirical point of view. We also include (via the parameter $\beta_{V_2^l}$) an influence of income distribution on the rate of change of the employment rate. The last term finally, $\beta_{V_2^l}\hat{V}^c$, is added to take account of the possibility that Okun's is to be formulated in level form rather than by a law of motion, since this term is equivalent to the use of *const* $(V^c)^{\beta_{V_2^l}}$, the form of Okun's law in which this law was originally specified by Okun himself.

The above two laws of motion therefore summarize the static IS-curve and the employment this curve implies of the paper of Asada, Chen, Chiarella and Flaschel (2004) in a dynamic form. They also reflect the there assumed influence of smooth factor substitution in production and the measurement of the potential output this implied in Asada, Chen, Chiarella and Flaschel (2004) in an indirect form, as another positive influence of the real wage on the rate of capacity utilization and its rate of change. This helps to avoid the estimation of separate equations for consumption and investment C, I and for potential output Y^p as they were discussed and used in detail in Asada, Chen, Chiarella and Flaschel (2004).

Finally, we have no longer to employ a law of motion for real balances as still was the case in Asada, Chen, Chiarella and Flaschel (2004). Money supply is now accommodating to the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we here assume the following type of Taylor rule:

$$\hat{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) + \gamma_\omega(\omega - \omega_o)$$

Note that we allow for interest rate smoothing in this rule. Furthermore, the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target $\bar{\pi}$ of the central bank. There is next a positive influence of the output gap in this law of motion for the rate of interest, here measured by the rate of capacity utilization of firms. Note finally that we have included a new kind of gap into the above Taylor rule, the real wage gap, since we have in our model a dependence of aggregate demand on income distribution and the real wage. The state of income distribution matters for the dynamics of our model and thus should also play a role in the decisions of the central bank. All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with balanced growth.

We note that the steady state of the considered Keynesian dynamics is basically the same as the one considered in Asada, Chen, Chiarella and Flaschel (2004), with $\epsilon_o = 0, V_o^c = \bar{V}^c, V_o^l = \bar{V}^l, \pi_o^m = \bar{\pi}$. The values of ω_o, r_o are in principle determined as in Asada, Chen, Chiarella and Flaschel (2004), but are here just assumed as given, underlying the linear approximation of the present model around the steady state of the original framework (when adjusted to the considered modifications of the baseline model).

The steady state of the dynamics is locally asymptotically stable under certain sluggishness conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability by way of cycles (by way of so-called Hopf-bifurcations) if the system becomes too flexible, and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high, as we shall show below. If the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior – like downward wage rigidity – to come into being at least far off the steady state in order to bound the dynamics to an economically meaningful domain in the considered 5D state space. Asada, Chiarella, Flaschel and Hung (2004) provide details for such an approach with extrinsically motivated nonlinearities and undertake its detailed numerical investigation. In sum, therefore, our dynamic AS-AD growth model here and there will exhibit a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modelling of AS-AD growth dynamics or its reformulation by the New Keynesians.

Taken together the model of this section consists of the five laws of motion:

$$\hat{V}^c = -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\omega - \omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) \tag{1}$$

$$\hat{V}^l = \beta_{V_1^l}(V^c - \bar{V}^c) - \beta_{V_2^l}(\omega - \omega_o) + \beta_{V_3^l}\hat{V}^c \tag{2}$$

$$\hat{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) + \gamma_\omega(\omega - \omega_o) \tag{3}$$

$$\hat{\omega} = \kappa[(1 - \kappa_p)(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o)) - (1 - \kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o))] \tag{4}$$

$$\dot{\pi}^m = \beta_{\pi^m}(\hat{p} - \pi^m) \quad \text{or} \quad \pi^m(t) = \pi^m(t_o)e^{-\beta_{\pi^m}(t-t_o)} + \beta_{\pi^m}\int_{t_o}^t e^{\beta_{\pi^m}(t-s)}\hat{p}(s)ds \tag{5}$$

where the following reduced form expression for the price inflation rate

$$\hat{p} = \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\omega - \omega_o) + \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\omega - \omega_o))] + \pi^m$$

has to be inserted into the third and fifth equation in order to get an autonomous system of differential equations in the state variables: capacity utilization V^c, the rate of employment V^l, the rate of interest r, the real wage rate ω and the inflationary climate π^m.

This modification and extension of the baseline disequilibrium AS-AD model of Asada, Chen, Chiarella and Flaschel (2004) in particular goes beyond this earlier approach as it now also allows for positive effects of real wage changes on aggregate demand, not yet present in the AD component of our original modification of the conventional AS-AD dynamics.

The above model – though not microfounded by making the representative household assumption – is microfounded in the way Keynesian theory was microfounded after Patinkin and it also makes use of recent approaches, to labor market dynamics as in Blanchard and Katz (2000). With respect to empirically relevant restructuring of the theoretical framework it is as pragmatic as for example the approach employed by Rudebusch and Svensson (1999). By and large we therefore believe that it represents a working alternative to the New Keynesian approach, in particular when the critique of the latter approach is taken into account. It overcomes the weaknesses and the logical inconsistencies of the Neoclassical synthesis, stage I, and it does so in a minimal way from a mature traditional Keynesian perspective (that is not really 'New'). It preserves the problematic nature of the real rate of interest channel, where the stabilizing Keynes effect (or the interest rate policy of the central bank) is interacting with the destabilizing, expectations driven Mundell effect. And it preserves the real wage effect of the Neoclassical synthesis, stage I, where – due to a negative dependence of aggregate demand on the real wage – we have that price flexibility is destabilizing, while wage flexibility is not. This real wage channel is not a topic in the New Keynesian approach, due to the specific form of wage-price dynamics there considered, see for example Woodford (2003, p.225), and it is summarized in the figure 1 for the situation where investment dominates consumption with respect to real wage changes. In the opposite case, the situations considered will be reversed with respect to their stability implications.

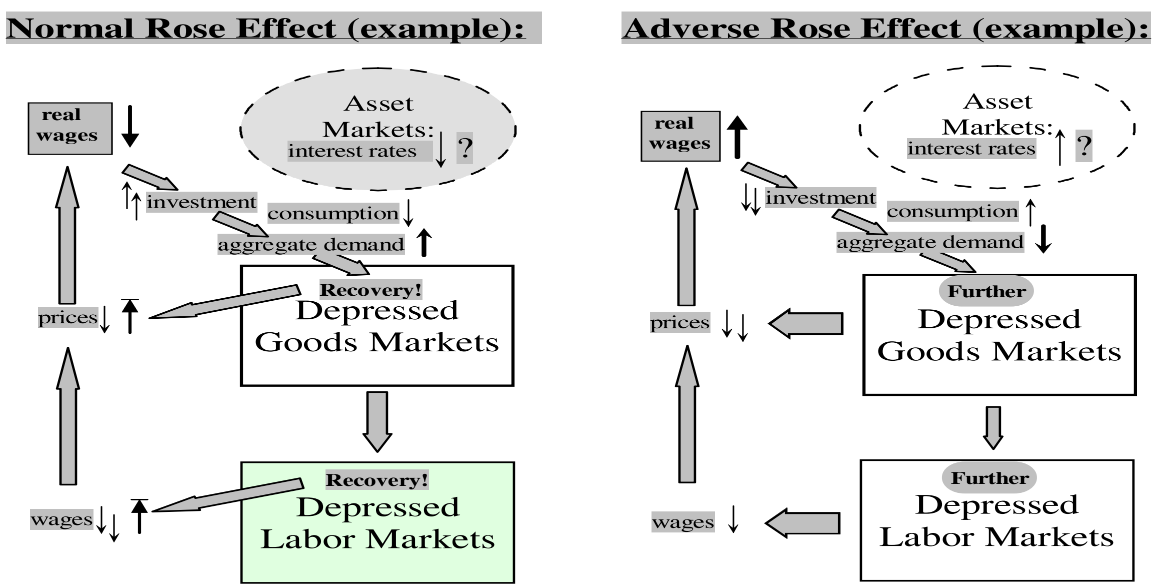

Figure 1: Rose effects: The real wage channel of Keynesian macrodynamics .

The feedback channels just discussed will be the focus of interest in the now following stability analysis of the DAS-DAD dynamics.

We note that the rate of employment, if above the NAIRU level, may also act negatively on the growth rate of capacity utilization, i.e., on the first law of motion, when the Kaleckian view of the political business cycle (bosses do not like full employment) is taken into account in addition. There may of course also be derivative influences (time derivatives of the considered variables) added to the considered equations, which indeed should have the same sign as the level influence, since they add to the impact of high levels if the considered change is positive. Such extensions of the present dynamics must here however be left for their future investigation.

We have employed reduced-form expressions in the above system of differential equations whenever possible. We have thereby obtained a dynamical system in five state variables that is in a natural or intrinsic way nonlinear. We note however that there are many items that reappear in various equations or are similar to each other implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms with respect to the stability assertions it gives rise to. A rigorous proof of local asymptotic stability and its loss by way of Hopf bifurcations can be found in Asada, Chen, Chiarella and Flaschel (2004) for the original baseline dynamic AS-AD form of the considered disequilibrium AS-AD growth dynamics. For the present model variant we shall supply more detailed stability proofs in Asada, Chiarella, Flaschel and Hung (2004) and also detailed numerical simulations of the model.

3 Feedback-guided stability analysis

In this section we illustrate an important method to prove local asymptotic stability of the interior steady state of the dynamical system (1) – (5) through partial motivations from the feedback chains that characterize this baseline model of Keynesian dynamics. Since the model is an extension of the standard AS-AD growth model we know from the literature that there is a real rate of interest effect typically involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). Instead of the stabilizing Keynes-effect, based on activity-reducing nominal interest rate increases following price level increases, we have here a direct steering of economic activity by the interest rate policy of the central bank. Secondly, if the correctly expected short-run real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is the activity stimulating (partial) effect of increases in the rate of inflation that may lead to accelerating inflation under appropriate conditions. This is the so-called Mundell-effect that normally works opposite to the Keynes-effect, and through the same real rate of interest channel as this latter effect.

Due to our use of a Taylor rule in the place of the conventional LM curve, the Keynes-effect is here exploited in a more direct way towards a stabilization of the economy and it works the stronger the larger the parameters γ_p, γ_{V^c} are chosen. The Mundell-effect by contrast is the stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have a positive influence of this climate variable both on price as well as on wage inflation and from there on rates of employment of both capital and labor. Excess profitability depends positively on the inflation rate and thus on the inflationary climate as the reduced-form price Phillips curve in particular shows.

There is a further important potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital ρ and thus negatively on the real wage ω. We thus get – since consumption may also depend (positively) on the real wage – that real wage increases can depress or stimulate economic activity depending on whether investment or consumption is dominating the outcome of real wage increases (we here neglect the stabilizing role of the Blanchard / Katz type error correction mechanisms). In the first case, we get from the reduced-form real wage dynamics:

$$\hat{\omega} \quad = \quad \kappa[(1-\kappa_p)\beta_w(V^l - \bar{V}^l) - (1-\kappa_w)\beta_p(V^c - \bar{V}^c)].$$

that price flexibility should be bad for economic stability due to the minus sign in front of the parameter β_p while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation as it was already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal and price flexibility to adverse Rose effects as far as real wage adjustments are concerned (if it is assumed – as in our baseline model – that only investment depends on the real wage). Besides real rate of interest effect, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh and Semmler (2000) for further discussion of these as well as other feedback mechanisms in Keynesian growth dynamics. We stress again that our DAS-AD growth dynamics – due to their origin in the baseline model of the Neoclassical Synthesis, stage I – allows for negative influence of real wage changes on aggregate demand solely, and thus only for cases of destabilizing wage level flexibility, but not price level flexibility. In the empirical estimation of the model we will indeed find that this case seems to be the typical one in dynamic models of the AS-AD variety.

This adds to the description of the dynamical system (1) – (5) whose stability properties are now to be investigated by means of varying adjustment speed parameters. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, here $\pi_o^m = \bar{\pi}$, if $\beta_{\pi^m} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\alpha_\omega = 0, \gamma_\omega = 0, \beta_{w_2} = 0, \beta_{p_2} = 0$ is assumed, since this decouples the ω-dynamics from the remaining system V^c, V^l, r. We will consider the stability of these 3D subdynamics – and its subsequent extensions – in informal terms here only, reserving rigorous calculations to the alternative scenarios provided in Asada, Chiarella, Flaschel and Hung (2004). We nevertheless hope to show to the reader how one can proceed from low to high dimensional analysis in such stability investigations. This method has been already applied to various other, often much more complicated, dynamical systems, see Asada, Chiarella, Flaschel and Franke (2003) for a variety of typical examples.

Before we start with these stability investigations we establish that loss of stability can in general only occur in the considered dynamics by way of Hopf-bifurcations, since the following proposition can be shown to hold true under mild – empirically plausible – parameter restrictions. Note that we assume for the dynamics of the employment rate the simple rule $\hat{V}^l = \beta_{V^l}(V^c - \bar{V}^c)$ throughout this section.

Proposition 1:

> *Assume that the parameters γ_ω, γ_r are chosen sufficiently small and that the parameters $\beta_{w_2}, \beta_{p_2}, \kappa_p$ fulfill $\beta_{p_2} > \beta_{w_2}\kappa_p$. Then: The 5D determinant of the Jacobian of the dynamics at the interior steady state is always negative in sign.*

Sketch of proof: We have for the sign structure in this Jacobian under the given assumptions the following initial situation (we here assume as limiting situation $\gamma_r = \gamma_\omega = 0$):

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ + & + & 0 & 0 & + \\ - & + & 0 & - & 0 \\ + & + & 0 & + & - \end{pmatrix}.$$

We note that the ambiguous sigh in the entry J_{11} in the above matrix is due to the fact that the real rate of interest is a decreasing function of the inflation rate which in turn depends positively on current rates of capacity utilization.

Using second row and the last row in its dependence on the partial derivatives of $\hat{p}$ we can reduces this Jacobian to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & - \end{pmatrix}$$

without change in the sign of its determinant. In the same way we can now use the third row to get another matrix without any change in the sign of the corresponding determinants

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

The last two columns can under the considered circumstance be further reduced to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}$$

which finally gives

$$J = \begin{pmatrix} 0 & 0 & - & 0 & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}.$$

This matrix is easily shown to exhibit a negative determinant which proves the proposition, also for all values of $\gamma_r = \gamma_\omega$ which are chosen sufficiently small. ■

Proposition 2:

Assume in addition that the parameters $\beta_{w_2}, \beta_{p_2}, \alpha_\omega, \gamma_\omega$ and β_{π^m} are all set equal to zero which decouples the dynamics of V^c, V^l, r from the rest of the system. Assume furthermore that the partial derivative of the first law of motion depends negatively on V^c, i.e., the dynamic multiplier process, characterized by α_{V^c}, dominates this law of motion with respect to the impact of V^c.[6] *Then: The interior steady state of the implied 3D dynamical system*

$$\hat{V}^c = -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) \tag{6}$$

$$\hat{V}^l = \beta_{V_1^l}(V^c - \bar{V}^c) \tag{7}$$

$$\hat{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) \tag{8}$$

is locally asymptotically stable if the interest rate smoothing parameter γ_r and the employment adjustment parameter β_{V^l} are chosen sufficiently small.

Sketch of proof: In the considered situation we have for the Jacobian of these reduced dynamics at the steady state:

$$J = \begin{pmatrix} - & + & - \\ + & 0 & 0 \\ + & + & - \end{pmatrix}$$

The determinant of this Jacobian is obviously negative if the parameter γ_r is chosen sufficiently small. Similarly, the sum of the minors of order 2: a_2, will be positive if β_{V^l} is chosen sufficiently small. The validity of the full set of Routh-Hurwitz conditions then easily follows, since trace J $= -a_1$ is obviously negative and since det J is part of the expressions that characterize the product $a_1 a_2$. ■

Proposition 3:

Assume now that the parameter α_ω is negative, but chosen sufficiently small, while the error correction parameters β_{w_2}, β_{p_2} are still kept at zero (as is the

[6] i.e., $\alpha_{V^c} > \alpha_p \kappa \kappa_p \beta_w$.

policy parameter γ_ω). Then: The interior steady state of the resulting 4D dynamical system (where the state variable ω is now included)

$$\begin{aligned}
\hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_\omega(\omega - \omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) && (9)\\
\hat{V}^l &= \beta_{V_1^l}(V^c - \bar{V}^c) && (10)\\
\hat{r} &= -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) && (11)\\
\hat{\omega} &= \kappa[(1-\kappa_p)\beta_{w_1}(V^l - \bar{V}^l) - (1-\kappa_w)\beta_{p_1}(V^c - \bar{V}^c) && (12)
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof: It suffices to show in the considered situation that the determinant of the resulting Jacobian at the steady state is positive, since small variations of the parameter α_ω must then move the zero eigenvalue of the case $\alpha_\omega = 0$ into the negative domain, while leaving the real parts of the other eigenvalues – shown to be negative in the preceding proposition – negative. The determinant of the Jacobian to be considered here – already slightly simplified – is characterized by

$$J = \begin{pmatrix} 0 & + & - & - \\ + & 0 & 0 & 0 \\ 0 & + & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

This can be simplified to

$$J = \begin{pmatrix} 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

without change in the sign of the corresponding determinant which proves the proposition. ■

We note that this proposition also holds where $\beta_{p_2} > \beta_{w_2}\kappa_p$ holds true as long as the thereby resulting real wage effect is weaker than the one originating from α_ω. Finally – and in sum – we can also state that the full 5D dynamics must also exhibit a locally stable steady state if β_{π^m} is made positive, but chosen sufficiently small, since we have already shown that the full 5D dynamics exhibits a negative determinant of its Jacobian at the steady state under the stated conditions.

A weak Mundell effect, (here still) the neglect of Blanchard-Katz error correction terms, a negative dependence of aggregate demand on real wages, coupled with nominal wage and also to some extent price level inertia (in order to allow for dynamic multiplier stability), a sluggish adjustment of employment towards actual capacity utilization and a Taylor rule that stresses inflation targeting therefore are (for example) the basic ingredients that allow for the proof of local asymptotic stability of the interior steady state of the dynamics (1) – (5). We expect however that indeed more general situation of convergent dynamics can be found, but have to leave this here for future research and numerical simulations of the model. Instead we now attempt to estimate the signs and sizes of the parameters of the model in order to gain insight into the question to what extent for example the US economy supports the types of real wage effects considered in figure 1 and also the possibility for overall asymptotic stability for such an economy, despite a destabilizing Mundell effect in the real interest rate channel.

4 Estimating the model

In this section we first of all provide single equation estimates for the laws of motion (1) – (5) of our disequilibrium AS-AD model. These estimates, on the one hand, serve the purpose of confirming the parameter signs we have specified in the initial formulation of the model and to determine the size of these parameters in addition. On the other hand, we have three situations where we cannot specify the parameter signs on purely theoretical grounds and where we therefore aim at obtaining these signs from the empirical estimates of the equations where this happens. There is first of all, see eq. (1), the ambiguous influence of real wages on (the dynamics of) the rate of capacity utilization, which should be a negative one if investment is more responsive than consumption to real wage changes and a positive one in the opposite case. There is secondly, with an immediate impact effect if the rates of capacity utilization for capital and labor are perfectly synchronized, the fact that real wages rise with economic activity through money wage changes and the labor market, while they fall with it through price level changes and the goods market, see eq. (4). Finally, we have in the theory of price level inflation a further ambiguous effect of real wage increases, which there lower wage inflation while speeding up price inflation, effects which work into opposite directions in the reduced form price PC shown below eq.s (1) – (5).

In all of these three cases we expect that empirical analysis will provide us with information which of these opposing forces will be the dominant one. Furthermore, we shall also see that the Blanchard and Katz (2000) error correction terms do play a role in the US-economy, in contrast to what has been found out by these authors for the money wage PC. Finally, we will also attempt to estimate the parameter β_{π^m} that characterizes the evolution of the inflationary climate. This however will be done only after we have applied a moving average representation with linearly declining weights in a first approach to the treatment of our climate expression.

We take a general to specific approach to specify the empirical model for the Keynesian dynamics described in eq.s (1) – (5), i.e. we catch up first all dynamic properties of the relevant variables in a general statistical model, in this linear case a VAR model. We then test whether the theoretically motivated hypotheses on the parameters of the dynamic model are supported by the data. If the theoretically hypotheses cannot be rejected, then we will estimated a specific model where all these theoretical restrictions are present.

The relevant variables are the wage inflation rate, the price inflation rate, the rates of utilization of labor and of capital, and unit wage cost, to be denoted in the following by: $dw_t, dp_t, V_t^l, V_t^c, r_t, ukbp_t$, where $ukbp_t$ is the cycle component of the time series for the unit wage cost, filtered by the bandpass filter.

4.1 Data Description

The empirical data of the corresponding time series are taken from the Federal Reserve Bank of St. Louis data set (see http:/www.stls.frb.org/fred). The data are quarterly , seasonally adjusted and are all available from 1948:1 to 2001:2. Except for the unemployment rates of the factors labor, U^l, and capital, U^c, the log of the series are used (see table 1).

Variable	Transformation	Mnemonic	Description of the untransformed series
$U^l = 1 - V^l$	UNRATE/100	UNRATE	Unemployment Rate
$U^c = 1 - V^c$	1-CUMFG/100	CUMFG	Capacity Utilization: Manufacturing, Percent of Capacity
w	log(COMPNFB)	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
p	log(GNPDEF)	GNPDEF	Gross National Product: Implicit Price Deflator, 1992=100
$yn = y - l^d$	log(OPHNFB)	OPHNFB	Nonfarm Business Sector; Output Per Hour of All Persons, 1992=100
$u = w - p - yn$	$\log\left(\frac{COMPRNFB}{OPHNFB}\right)$	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Output Unit, 1992=100
r			Federal Funds Rate

Table 1: Data used for empirical investigation

Note that w_t and p_t now represent logarithms, i.e., their first differences dw_t, dp_t give the current rate of wage and price inflation. We use $dp12_t$ and $dfp_t$to denote now specifically the moving average with equal weight and linearly decreasing weight of price inflation over the past 12 quarters (as an especially simple measure of the employed inflationary climate expression), and denote by V^l, V^c the rates of utilization of the stock of labor and the capital stock. The graphs of the time series of these variables are shown in the figure 2.

There is a pronounced downward trend in part of the employment rate series (over the 1970's and part of the 1980's) and in the wage share (normalized to 0 in 1996). The latter is not the topic of this paper, but will be briefly considered in the concluding section. Wage inflation shows three to four trend reversals, while the inflation climate representation clearly show two periods of low inflation regimes and in between a high inflation regime.

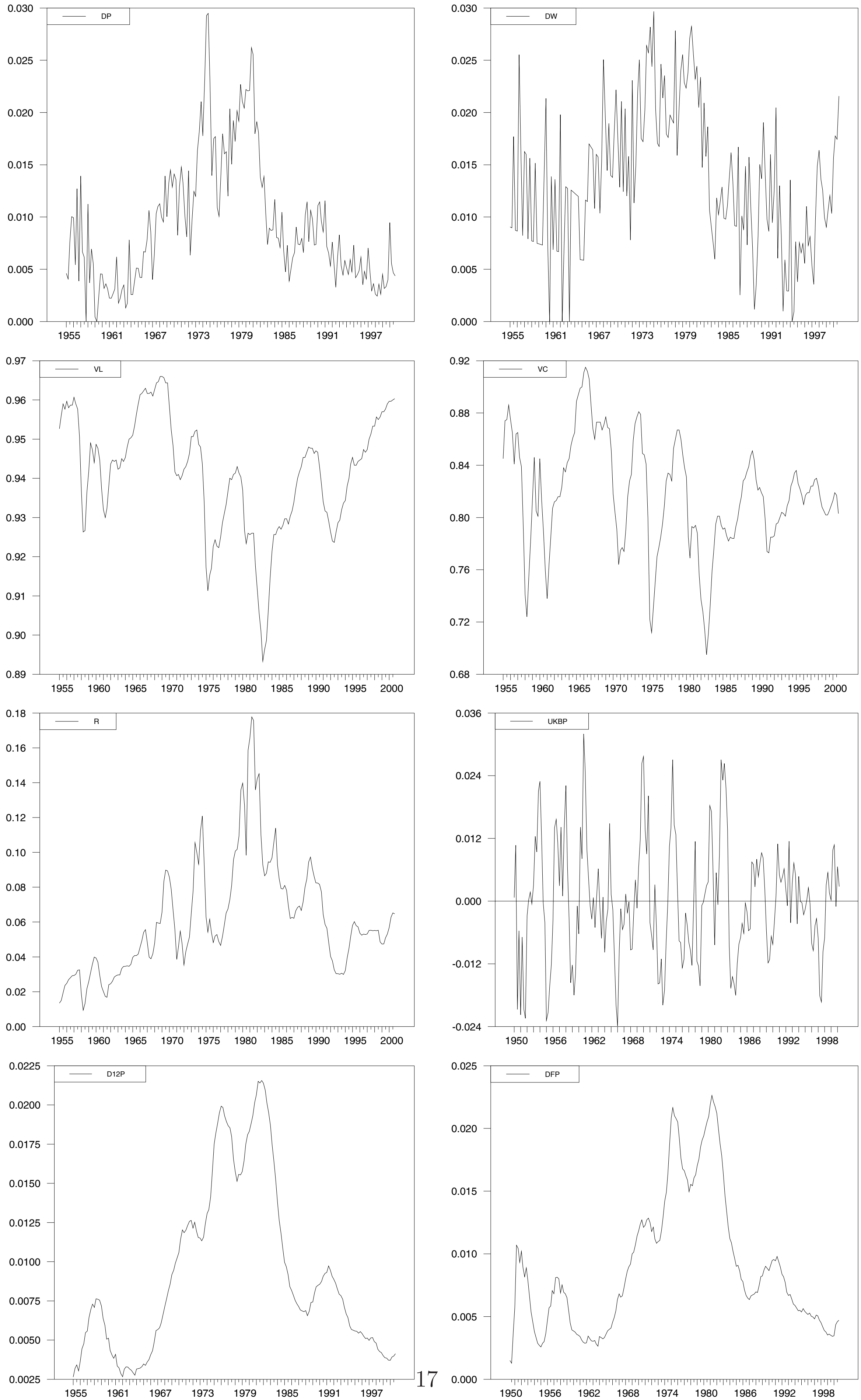

Figure 2: The fundamental data of the model.

We expect that these five time series are stationary. The graphs of the series wage and price inflation, capacity utilization rates and labor productivity growth, $dw_t, dp_t, V_t^l, V_t^c, dyn_t$, confirm our expectation. In additional we carry out DF unit root test for each series. The test results are shown in table 2.

Variable	Sample	Critical value	Test Statistic
dw	1947:02 TO 2000:04	-3.41000	-3.74323
dp	1947:02 TO 2000:04	-3.41000	-3.52360
V^l	1948:02 TO 2000:04	-1.95000	-0.73842
V^c	1948:02 TO 2000:04	-3.41000	-4.13323
ukbp	1950:01 TO 2000:04	-3.41000	-7.09932
R	1955:01 TO 2000:04	-1.95000	-0.94144

Table 2: Summary of DF-Test Results

4.2 Estimation of unrestricted VAR

The unit root test confirms our expectation with the exception of V_t^l and r_t. Although the test cannot reject the null of unit root, there is no reason to expect the rate of unemployment and the federal funds rate as being a unit root process. Indeed we expect that they are constrained in certain limited ranges, say from zero to 0.3. Due to the lower power of DF test, the test result should only provide hints that the rate of unemployment and the federal funds rate have a strong autocorrelation, respectively.

Given this stationarity we can construct a VAR model for these 6 variables to mimic the DGP of these 6 variables.

$$\begin{pmatrix} dw_t \\ dp_t \\ V_t^l \\ V_t^c \\ r_t \\ ukbp_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix} d74 + \sum_{k=1}^{P} \begin{pmatrix} a_{11k} & a_{12k} & a_{13k} & a_{14k} & a_{15k} & a_{16k} \\ a_{21k} & a_{22k} & a_{23k} & a_{24k} & a_{25k} & a_{26k} \\ a_{31k} & a_{32k} & a_{33k} & a_{34k} & a_{35k} & a_{36k} \\ a_{41k} & a_{42k} & a_{43k} & a_{44k} & a_{45k} & a_{46k} \\ a_{51k} & a_{52k} & a_{53k} & a_{54k} & a_{55k} & a_{56k} \\ a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} dw_{t-k} \\ dp_{t-k} \\ V_{t-k}^l \\ V_{t-k}^c \\ r_{t-k} \\ ukbp_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{pmatrix} \tag{13}$$

To determine the lag length of the VAR we apply sequential likelihood tests. We start with a lag length of 24, at which the residuals can be taken as WN process. The sequence likelihood ratio test procedure gives a lag length of 9. The test results are listed as follows.

- $H_0 : P = 20$ v.s. $H_1 : P = 24$
  Chi-Squared(144)= 150.01095 with Significance Level 0.34880
- $H_0 : P = 16$ v.s. $H_1 : P = 20$
  Chi-Squared(144)= 148.82953 with Significance Level 0.37424
- $H_0 : P = 12$ v.s. $H_1 : P = 16$
  Chi-Squared(25)= 120.97749 with Significance Level 0.91874

- $H_0 : P = 11$ v.s. $H_1 : P = 12$
  Chi-Squared(25)= 42.86003 with Significance Level 0.20055

- $H_0 : P = 10$ v.s. $H_1 : P = 11$
  Chi-Squared(25)= 52.30518 with Significance Level 0.03868

According to these test results we use VAR(12) to represent a general model that should be an approximation of the DGP. Because in the dynamic system of (1) – (5) the variable $ukbp_t$ is treated as exogenous, we factorize the VAR(12) process into a conditional process of $dw_t, dp_t, V_t^l, V_t^c, r_t$ given $ukbp_t$ and the lagged variables, and the marginal process of $ukbp_t$ given the lagged variables:

$$\begin{pmatrix} dw_t \\ dp_t \\ V_t^l \\ V_t^c \\ r_t \end{pmatrix} = \begin{pmatrix} c_1^* \\ c_2^* \\ c_3^* \\ c_4^* \\ c_5^* \end{pmatrix} + \begin{pmatrix} b_1^* \\ b_2^* \\ b_3^* \\ b_4^* \\ b_5^* \end{pmatrix} d74 + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} ukpb_t \tag{14}$$

$$+ \sum_{k=1}^{P} \begin{pmatrix} a_{11k}^* & a_{12k}^* & a_{13k}^* & a_{14k}^* & a_{15k}^* & a_{16k}^* \\ a_{21k}^* & a_{22k}^* & a_{23k}^* & a_{24k}^* & a_{25k}^* & a_{26k}^* \\ a_{31k}^* & a_{32k}^* & a_{33k}^* & a_{34k}^* & a_{35k}^* & a_{36k}^* \\ a_{41k}^* & a_{42k}^* & a_{43k}^* & a_{44k}^* & a_{45k}^* & a_{46k}^* \\ a_{51k}^* & a_{52k}^* & a_{53k}^* & a_{54k}^* & a_{55k}^* & a_{56k}^* \end{pmatrix} \begin{pmatrix} dw_{t-k} \\ dp_{t-k} \\ V_{t-k}^l \\ V_{t-k}^c \\ r_{t-k} \\ ukbp_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t}^* \\ e_{2t}^* \\ e_{3t}^* \\ e_{4t}^* \\ e_{5t}^* \end{pmatrix}$$

$$ukbp_t = c_6 + \sum_{k=1}^{P} \begin{pmatrix} a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} dw_{t-k} \\ dp_{t-k} \\ V_{t-k}^l \\ V_{t-k}^c \\ r_{t-k} \\ ukbp_{t-k} \end{pmatrix} + e_{6t} \tag{15}$$

Now we examine if $ukbp_t$ can be taken as "exogenous" variable. The partial system (14) is exactly identified. Hence the variables u_t are weakly exogenous for the parameters in the partial system,[7] For the strong exogeneity of $ukbp_t$, we test whether $dw_t, dp_t, V_t^l, V_t^c, r_t$ Granger cause $ukbp_t$.

The test is carried out by testing the hypothesis: $H_0 : a_{ijk} = 0, (i = 6; j = 1, 2, 3, 4, 5; k = 1, 2, ..., 12)$ in (15) based on likelihood ratio

- Chi-Squared(60)= 69.157150 with Significance Level 0.19572294

[7]For detailed discussion see Chen (2003)

4.3 Estimation of the Structural Model

As discussed in section 2, the law of motion for real wage rate, eq. (4), can be considered as the reduced form of two structural equations for dw_t and dp_t. Assuming that the inflation climate variable is a function of past price inflation rates, the dynamics of the system (1) – (5) is equivalently presented by the following equations:

$$dw_t = \beta_{w_1}(V^l - \bar{V}^l)_{t-1} + \kappa_w dp_{t-1} + (1-\kappa_w)dp12_{t-1} - \beta_{uw}ukbp_t + e_{1t} \tag{16}$$

$$dp_t = \beta_{p_1}(V^c - \bar{V}^c)_{t-1} + \kappa_p dw_t + (1-\kappa_p)dp12_{t-1} + \beta_{up}ukbp_t + e_{2t} \tag{17}$$

$$\hat{V}^l_t = \beta_{V^l_1}(V^c - \bar{V}^c)_{t-1} - \beta_{V^l_2}(\omega - \omega_o)_{t-1} + \beta_{V^l_3}(V^l - V^l)_{t-1} + e_{3t} \tag{18}$$

$$\hat{V}^c_t = -\alpha_{V^c}(V^c - \bar{V}^c)_{t-1} \pm \alpha_\omega(\omega - \omega_o) - \alpha_r((r-\hat{p}) - (r_o - \bar{\pi})) + \alpha_l \hat{V}^l + \alpha_u ukbp_t + e_{4t} \tag{19}$$

$$\hat{r}_t = -\gamma_r(r - r_o)_{t-1} + \gamma_p(\hat{p} - \bar{\pi})_{t-1} + \gamma_{V^c}(V^c - \bar{V}^c)_{t-1} + \gamma_\omega(\omega - \omega_o) + e_{5t} \tag{20}$$

Obviously the model (16) and (20) is nested in the VAR(12) of (14). Therefore we can use (14) to evaluate the empirical relevance of the model (16) to (20). First we test whether the parameter restrictions on (14) implied by (16) to (20) are valid. If the null of these restrictions cannot be rejected, we will estimate (16) and (20) and then calculate the empirical estimates for the model (1) to (5).

The structural model (16) to (20) puts 408 restrictions on the unconstrained VAR(12) of system (14). Applying likelihood ratio method we can test the validity of these restrictions. We get the result:

- Chi-Squared(408)= 536.377763 with Significance Level 0.00001921

Obviously, for the period from 1955:1 to 2000:4, the structural model is too restrictive. However, for the period from 1965:1 to 2000:4 we can not reject the null of these restrictions. The test result is the following:

- Chi-Squared(408)= 414.104639 with Significance Level 0.39322678

Obviously, the specification from (16) to (20) is a valid one for the data set from 1965:1 to 2000:4. The estimation results are listed in the appendix. This result shows strong empirical relevance of the law of motions described in (1) – (5) as a model for the economy from 1965:1 to 2000:4. It is worthwhile to note that altogether 408 restrictions are implied through (1) – (5) on the VAR(12) model. A p-value of 0.39 means that (1) –

(5) is a much more parsimonious presentation of the DGP than VAR(12), and henceforth a much more efficient model to describe the economic dynamics for this period.

Omitting the insignificant parameters in the structural models and putting the NAIRU state variables and the like into the constant terms we then get following estimation result:

$$\begin{aligned}
dw_t &= 0.16V^l_{t-1} + 0.29dp_{t-1} + 0.71dp12_{t-1} - 0.08ukbp_t - 0.15 + e_{1t} && (21)\\
dp_t &= 0.04V^c_{t-1} + 0.08dw_t + 0.92dp12_{t-1} + 0.01d74_t - 0.03 + e_{2t} && (22)\\
\hat{V}^l_t &= 0.42\hat{V}^c_{t-1} - 0.10ukbp_t - 0.003d74_t + 0.02 + e_{3t} && (23)\\
\hat{V}^c_t &= -0.08V^c_{t-1} - 0.08(r_t - dp_t) + 0.97\hat{V}^l - 0.38ukbp_t + 0.08 + e_{4t} && (24)\\
\hat{r}_t &= -0.06r_{t-1} + 0.44dp_{t-1} + 0.08V^c_{t-1} - 0.06 + e_{5t} && (25)
\end{aligned}$$

Alternatively we also estimate a slightly modified version of (26) – (30) where we look at the time rate of change of labor utilization and capacity utilization instead of their growth rate.

$$dw_t = \beta_{w_1}(V^l - \bar{V}^l)_{t-1} + \kappa_w dp_{t-1} + (1-\kappa_w)dp12_{t-1} + e_{1t} \tag{26}$$

$$dp_t = \beta_{p_1}(V^c - \bar{V}^c)_{t-1} + \kappa_p dw_t + (1-\kappa_p)dp12_{t-1} - s\cdot\kappa_p dyn_{t-1} \tag{27}$$

$$\dot{V}^c = -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\omega - \omega_o) - \alpha_r((r-\hat{p}) - (r_o - \bar{\pi})) \tag{28}$$

$$\dot{V}^l = \beta_{V^l_1}(V^c - \bar{V}^c) - \beta_{V^l_2}(\omega - \omega_o) + \beta_{V^l_3}\dot{V}^c \tag{29}$$

$$\dot{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) + \gamma_\omega(\omega - \omega_o) \tag{30}$$

Omitting the insignificant parameters in the structural models (26) – (30) and putting all constants of the theoretical model into a single constant term, we get following estimation result:

$$\begin{aligned}
dw_t &= 0.16V^l_{t-1} + 0.26dp_{t-1} + 0.74dp12_{t-1} - 0.07ukbp_t - 0.15 + e_{1t} && (31)\\
dp_t &= 0.04V^c_{t-1} + 0.08dw_t + 0.92dp12_{t-1} + 0.01d74_t - 0.03 + e_{2t} && (32)\\
\dot{V}^l_t &= 0.04\dot{V}^c_{t-1} - 0.10ukbp_t - 0.004d74_t + e_{3t} && (33)\\
\dot{V}^c_t &= -0.12V^c_{t-1} - 0.12(r_t - dp_t) - 0.57ukbp_t + 0.1 + e_{4t} && (34)\\
\hat{r}_t &= -0.08r_{t-1} + 0.55dp_{t-1} + 0.06V^c_{t-1} - 0.05 + e_{5t} && (35)
\end{aligned}$$

Obviously these alternative specifications give similar result as in the formulation of restricted VAR(12) we considered beforehand.

5 Analyzing the estimated model

In the preceding section we have provided definite answers with respect to the type of real wage effect present in the data of the US economy after World War II, concerning the dependence of aggregate demand on the real wage and the degrees of wage and price flexibilities. The resulting combination of effects suggest that it is favorable for stability. We stress however that the inflation climate is so far only measured by a moving average of past inflation rates with linearly declining weights. So role of the parameter β_{π^m} – which when increased will destabilize the economy – is thus not yet present in the considered situation.

We start the stability analysis of the model with estimated parameters in this section from a basic 3D core situation which we obtain by totally ignoring adjustments in the inflationary climate term by setting $\pi^m = \bar{\pi}$ and by interpreting the law of motion for V^l in level terms, i.e., by concentrating on the influence of $\hat{V}^c$ on $\hat{V}^l$. Integrating this relationship gives approximately $V^l = +const\ (V^c)^{0.42}$ as can also be confirmed by estimating this level form relationship directly. Under these assumptions, the laws of motion (1) – (5) can be reduced to:

$$\hat{V}^c = const - const\ V^c - const\ r - const\ \omega \quad (36)$$

$$\hat{r} = -const + const\ V^c - const\ r + const\ \omega \quad (37)$$

$$\hat{\omega} = const + const\ V^c - const\ \omega \quad (38)$$

We note here that we have inserted here the reduced form expression for the price inflation rate into the first law of motion for the activity dynamics and rearranged terms such that the influence of V^c and ω appears only once, though both terms appear via two channels in this law of motion, one direct channel and one via the price inflation rate. The result of our estimate of this equation is that the latter channel is not changing the signs of the direct effects of capacity utilization (via the dynamic multiplier) and the real wage (via consumption and investment behavior).[8]

A similar treatment applies to the law of motion for the nominal rate of interest, where price inflation is again dissolved into its constituent part (in its reduced form expression) and where again the influence of V^l in this expression is replaced by V^c through Okun's Law. Again the direct effect of ω in the Taylor rule is assumed to dominate the indirect on (via the inflation rate), as this was confirmed by our empirical estimate of this law of motion.[9]

Finally, the law of motion for real wages themselves is obtained from the two estimated structural laws of motion for wage and price inflation in the way shown in section 2. We have a positive influence of capacity utilization on the growth rate of real wages, since the wage Phillips curve dominates the outcome here and a negative influence of real wages on their rate of growth due to the signs of the Blanchard / Katz error correction terms in the wage and the price dynamics.[10]

[8] to be estimated in this form: sign of $\beta_{p_2} - \kappa_p\beta_{w_2}$ does not matter?

[9] to be estimated in this form: sign of $\beta_{p_2} - \kappa_p\beta_{w_2}$ does not matter?

[10] to be estimated in this form: DFP to be suppressed?

On this basis we arrive at the following sign structure for the Jacobian of the 3D dynamics at the interior steady state of the above reduced model:

$$J = \begin{pmatrix} - & - & - \\ + & - & + \\ + & 0 & - \end{pmatrix}.$$

We therefrom immediately get that the trace of this matrix is negative, the sum a_2 of principal minors of order two is positive and a determinant of the whole matrix that is negative. The coefficients $a_i, i = 1, 2, 3$ of the Routh Hurwitz polynomial of this matrix are therefore all positive as demanded by the Routh Hurwitz stability conditions. The remaining stability condition is

$$a_1 a_2 - a_3 = (-traceJ)a_2 - detJ > 0.$$

With respect to this condition we first of all see that the determinant of J is given by:

$$J_{33}(J_{11}J_{22} - J_{12}J_{21}) + J_{31}(J_{12}J_{23} - J_{13}J_{22}).$$

With respect to this expression we see that the first term is dominated by $(-traceJ)a_2$ and can thus be canceled from the calculation of $a_1a_2 - a_3$. The same holds true for the term $-J_{31}J_{13}J_{22})$ in the determinant of J, while the remaining, non-neutralized term $J_{31}J_{12}J_{23}$ in this determinant can be made arbitrary small if the dependence of the interest rate policy rule on the unconventional influence of the real wage on this interest rate setting is made sufficiently small. the may however exist a variety of other situations where the above sign structure of the Jacobian of the considered 3D dynamics will lead to asymptotic stability, in particular if the actual size of the estimated parameters is taken into account in addition. The real wage effect that is now included into the dynamics of the private sector therefore seems to create not much harm for the stability of the steady state of the considered dynamics, in particular due to its negative influence on the rate of change of economic activity.

Increasing price flexibility may however change this situation, since growth rate of economic activity can thereby be made to depend positively on the level of economic activity, leading to an unstable dynamic multiplier process in the trace of J under such circumstances. Furthermore, such increasing price flexibility will also give rise to a negative dependence of the real wage on economic activity and thus lead to further sign changes in the Jacobian J. A further destabilizing mechanism is introduced if we add again the law of motion for the inflationary climate surrounding the current evolution of price inflation.

Under this latter extension to a 4D dynamical system the Jacobian J is augmented in its sign structure in the following way:

$$J = \begin{pmatrix} - & - & - & + \\ + & - & + & + \\ + & 0 & - & 0 \\ + & 0 & \pm & 0 \end{pmatrix}.$$

As the positive entries J_{14}, J_{41} show there is now a destabilizing feedback chain, leading from increases in economic activity to increases in inflation and expected inflation and from there back to increases in economic activity, through the real rate of interest channel. This destabilizing so-called Mundell effect must become dominant as the adjustment speed of the climate expression β_{π^m} is increased. The Blanchard / Katz error correction terms in the fourth row of J, obtained from the reduced form price Phillips curve, that are (as only further terms) associated with the speed parameter β_{π^m}, are of no help here, since they do not appear in combination with the parameter β_{π^m} in the sum of principal minors of order 2. In this sum the parameter β_{π^m} thus only enters once and with a negative sign implying that this sum can be made negative if this parameter is chosen sufficiently large.

Furthermore, making use of the reduced form expression for the term $\hat{p}-\pi^m$ one can easily show – under one mild assumption – that the sign structure in the above 4D Jacobian can be reduced to the following form without change in the sign of the corresponding Jacobians:

$$J = \begin{pmatrix} 0 & - & 0 & 0 \\ 0 & 0 & 0 & + \\ 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix}.$$

This follows, since we can reduce the first two laws of motion to the use of π^m in the place of $\hat{p}$ and since the last two laws can be reduced to β_w and β_p terms solely, respectively, which in turns implies a further reduction to a negative influence of only ω on its rate of growth and a positive sole influence of V^c on π^m, everything without change of sign in the considered determinants. Assuming then that interest rate smoothing is sufficiently weak allows for the conclusion that the 4D determinant exhibits a positive sign throughout.

We therefrom in sum get that the 4D dynamics will be convergent for small speeds of adjustments β_{π^m}, while it will be divergent for parameters β_{π^m} chosen sufficiently large. The Mundell effect thus works as expected from a partial perspective. There will be a unique Hopf bifurcation point $\beta^H_{\pi^m}$ in between where the system loses asymptotic stability in a cyclical fashion by the death of an unstable or the birth of a stable limit cycle. Yet sooner or later purely explosive behavior will be established, where there is no room any more for persistent economic fluctuations in the real and the nominal magnitudes.

Remark: The Livingston index for consumer price index inflation may be used to measure the size of the parameter β_{π^m} on the basis of this measure for an inflationary climate. Comparison with DFP?

Modifying the above model finally in order to incorporate into it a simple dynamic version of Okun's law, see (2), gives rise to its following respecification:

$$\begin{aligned} \hat{V}^c &= const - \alpha_{V^c} V^c - \alpha_\omega \omega - \alpha_r (r - \hat{p}) && (39) \\ \hat{V}^l &= const + \beta_{V^l_1} V^c + \beta_{V^l_2} \hat{V}^c && (40) \\ \hat{r} &= const - \gamma_r r + \gamma_p \hat{p} + \gamma_{V^c} V^c + \gamma_\omega \omega && (41) \\ \hat{\omega} &= const + \kappa[(1-\kappa_p)(\beta_{w_1} V^l - \beta_{w_2}\omega) - (1-\kappa_w)(\beta_{p_1} V^c + \beta_{p_2}\omega)] && (42) \\ \dot{\pi}^m &= \beta_{\pi^m}(\hat{p} - \pi^m) && (43) \end{aligned}$$

with

$$\hat{p} = const + \kappa[\beta_{p_1} V^c + \beta_{p_2}\omega + \kappa_p(\beta_{w_1} V^l - \beta_{w_2}\omega)] + \pi^m$$

with the variables: capacity utilization V^c, the rate of employment V^l, the rate of interest r, and the inflationary climate π^m, and the real wage ω.

Inserting finally the estimated values into these reformulated equations gives rise to the following numerical specification of this model type

$$\begin{aligned}
\hat{V}^c &= const - 0.08V^c - 0.38\omega - 0.089(r - \hat{p}) \\
\hat{V}^l &= const + 0.01V^c + 0.15\hat{V}^c \\
\hat{r} &= const - 0.08r + 0.44\hat{p} + 0.08V^c \\
\hat{\omega} &= const + 0.10V^l - 0.025V^c - 0.067\omega \\
\dot{\pi}^m &= \beta_{\pi^m}(\hat{p} - \pi^m), \quad \beta_{\pi^m} \text{ to be determined still}
\end{aligned}$$

with

$$\hat{p} = const + 0.04V^c + 0.13V^l + 0.01\omega + \pi^m,$$

based on the estimates $\beta_{w_1} = 0.16$, $\beta_{w_2} = -0.08$, $\beta_{p_1} = 0.04$, $\beta_{p_2} = 0$, $\kappa_w = 0.29$, and $\kappa_p = 0.08$ ($\kappa = 1.08$).

We clearly see again in these equations the stabilizing role of the dynamic multiplier, the dominance of investment demand in the determination of real wage influences on aggregate demand and the multiplier, as well as the negative real rate of interest effect on changes in goods markets' activity levels.

In the law of motion describing the evolution of the real wage, we have the expected positive influence of the rate of employment and the negative influence of the rate of capacity utilization (that drives the price rate of inflation), as well as the joint working of the Blanchard and Katz (2000) error correction mechanisms, but only in the wage dynamics. We know from the estimates of the dw, dp equations that their difference must contain $0.326dyn$ as resulting influence of labor productivity growth, but do neglect this here, since unit wage costs have been detrended by the bandpass filter in the estimation of the wage and price Phillips curves.

6 Conclusions and outlook

We have considered in this paper an significant extension and modification of the traditional approach to AS-AD growth dynamics that allows us to avoid dynamical inconsistencies of the traditional Neoclassical synthesis, stage I, and also to overcome empirical weaknesses of the New Keynesian approach, the Neoclassical synthesis, stage II, that arise from the assumption of purely forward looking behavior. Conventional wisdom avoids the stability problems then generated in these model types by just assuming global asymptotic stability through the adoption of non-predetermined variables and the application of the so-called jump-variable technique.

This approach of the Rational Expectations School is however much more than just the consideration of rational expectations, but in fact the assumption of hyperperfect foresight coupled with a solution method that avoids all potential instabilities of macrodynamic economic systems by assumption. In the present context, this approach would impose the condition that prices – and also nominal wages – must be allowed to jump in a particular way in order to establish by assumption the stability of the investigated dynamics.

By contrast, our alternative approach – which allows for sluggish wage as well as price adjustment and also for certain economic climate variables, representing the medium-run evolution of inflation (and in Asada, Chen, Chiarella and Flaschel (2004) also excess profitability) – completely bypasses such stability assumptions. Instead it allows to demonstrate in a detailed way, guided by the intuition behind important macroeconomic feedback channels, local asymptotic stability under certain plausible assumptions (indeed very plausible from the perspective of a Keynesian feedback channel theory), cyclical loss of stability when these assumptions are violated (if speeds of adjustment become sufficiently high), and even explosive fluctuations in the case of further increases of the crucial speeds of adjustment of the model. In the latter case extrinsic nonlinearities have to be introduced in order to tame the explosive dynamics as in some of the examples in Chiarella and Flaschel (2000, Ch.6,7) where a kinked Phillips curve is employed to achieve global boundedness.

The stability features of this – in our view properly reformulated – Keynesian dynamics are based on specific interactions of traditional Keynes- and Mundell-effects or real rate of interest effects (here present only in the employed investment function) with so-called Rose or real-wage effects, see Chiarella and Flaschel (2000) for their introduction, which in the present framework simply means that increasing wage flexibility is stabilizing and increasing price flexibility destabilizing, based on the fact that aggregate demand here depends negatively on the real wage (due to the assumed investment function) and due to the extended types of Phillips curves we have employed in our new approach to traditional Keynesian growth dynamics. The interaction of these three effects is what explains the obtained stability results under the in this case not very important assumption of myopic perfect foresight, on wage as well as price inflation, and thus gives rise to a traditional type of Keynesian business cycle theory, not at all plagued by the anomalies of the textbook AS-AD dynamics, see Chiarella, Flaschel and Franke (2004) for a detailed treatment and critique of this textbook approach.

The model of this paper will be numerically explored in a companion paper, Asada, Chiarella, Flaschel and Hung (2004), in order to analyze in greater depth, and also with the empirical background here generated, the interaction of the various feedback channels present in the considered dynamics. At that point we will make use of LM curves as well as Taylor interest rate policy rules, kinked Phillips curves and Blanchard / Katz error correction mechanisms in order to investigate in detail the various ways by which a locally unstable dynamics can be made bounded and thus viable. The question then is which behavioral assumption on private behavior and fiscal and monetary policy – once viability is achieved – can reduce the volatility of the resulting persistent fluctuations.

Our work on related models suggests that the interest rate policy rule may not be sufficient to tame the explosive dynamics in all conceivable cases, or even make it convergent. But when viability is achieved – for example by downward wage rigidity –

we can investigate the parameter corridor where monetary policy for example can reduce the endogenously generated fluctuations of this approach to Keynesian business fluctuations.

Taking all this together our general conclusion will be that this framework not only overcomes the anomalies of the Neoclassical Synthesis, Stage I, but also provides a coherent alternative to the New Keynesian theory of the business cycle, as sketched in Gali (2000). This alternative is based on disequilibrium in the market for goods and labor, on sluggish adjustment of prices as well as wages and on myopic perfect foresight interacting with certain economic climate expression with a rich array of dynamic outcomes that provide great potential for further generalizations. Some of these generalizations are considered in Chiarella, Flaschel, Groh and Semmler (2000) and Chiarella, Flaschel and Franke (2004). Our overall approach, which may be called a disequilibrium approach to business cycle modelling, thus provides a theoretical framework within which to consider the contributions of authors such as Zarnowitz (1999) who also stresses the dynamic interaction of many traditional macroeconomic building blocks.

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7 Appendix: Estimation Results

```
Single Equation Estimation of (21) to (25)

Linear Regression - Estimation by Least Squares
Dependent Variable DW
Quarterly Data From 1965:01 To 2000:04
Usable Observations    143     Degrees of Freedom   138
 Total Observations    144      Skipped/Missing        1
Centered R**2     0.624227      R Bar **2   0.613335
Uncentered R**2   0.932527      T x R**2     133.351
Mean of Dependent Variable      0.0144268217
Std Error of Dependent Variable 0.0067728907
Standard Error of Estimate      0.0042115453
Sum of Squared Residuals        0.0024477217
Regression F(4,138)                  57.3108
Significance Level of F           0.00000000
Durbin-Watson Statistic             1.623078

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DP{1}                     0.197140530  0.148555607      1.32705  0.18668405
2.  DFP                       0.929962056  0.189391042      4.91027  0.00000253
3.  VL{1}                     0.195780837  0.032693348      5.98840  0.00000002
4.  UKBP{1}                  -0.068700307  0.034212646     -2.00804  0.04659112
5.  Constant                 -0.181534056  0.031220303     -5.81462  0.00000004


Linear Regression - Estimation by Least Squares
Dependent Variable DP
Quarterly Data From 1965:01 To 2000:04
Usable Observations    144     Degrees of Freedom   139
Centered R**2     0.835171      R Bar **2   0.830428
Uncentered R**2   0.959113      T x R**2     138.112
Mean of Dependent Variable      0.0105799813
Std Error of Dependent Variable 0.0060979256
Standard Error of Estimate      0.0025110731
Sum of Squared Residuals        0.0008764628
Regression F(4,139)                 176.0747
Significance Level of F           0.00000000
Durbin-Watson Statistic             1.599777

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DW{1}                     0.087894111  0.046473210      1.89129  0.06066693
2.  DFP                       0.935253173  0.061707858     15.15614  0.00000000
3.  VC{1}                     0.046153826  0.005654010      8.16303  0.00000000
4.  D74                       0.009987686  0.001840223      5.42743  0.00000025
```

```
5.  Constant                 -0.038381571  0.004726692     -8.12018  0.00000000
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable DVL
Quarterly Data From 1965:01 To 2000:04
Usable Observations    142     Degrees of Freedom   136
 Total Observations    144      Skipped/Missing        2
Centered R**2     0.529833      R Bar **2   0.512547
Uncentered R**2   0.530008      T x R**2      75.261
Mean of Dependent Variable      0.0000688254
Std Error of Dependent Variable 0.0035759981
Standard Error of Estimate      0.0024966844
Sum of Squared Residuals        0.0008477469
Regression F(5,136)                  30.6517
Significance Level of F            0.00000000
Durbin-Watson Statistic              2.056523

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  VL{1}                      0.505469389  0.085304889      5.92544  0.00000002
2.  VL{2}                     -0.520280734  0.078411280     -6.63528  0.00000000
3.  VC{1}                     -0.007799417  0.009128566     -0.85440  0.39438776
4.  UKBP                      -0.109893359  0.024329306     -4.51691  0.00001348
5.  D74                       -0.003587478  0.001851895     -1.93719  0.05479480
6.  Constant                   0.020331826  0.017135470      1.18653  0.23747990
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable DVC
Quarterly Data From 1965:01 To 2000:04
Usable Observations    142     Degrees of Freedom   137
 Total Observations    144      Skipped/Missing        2
Centered R**2     0.481545      R Bar **2   0.466407
Uncentered R**2   0.481817      T x R**2      68.418
Mean of Dependent Variable      -0.000323944
Std Error of Dependent Variable  0.014185973
Standard Error of Estimate       0.010362488
Sum of Squared Residuals        0.0147112173
Regression F(4,137)                  31.8116
Significance Level of F            0.00000000
Durbin-Watson Statistic              1.638300

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  VC{1}                     -0.124199856  0.021085861     -5.89020  0.00000003
2.  DVL{1}                     1.335384864  0.298778877      4.46948  0.00001629
3.  RRATE{1}                  -0.157024873  0.034250470     -4.58460  0.00001015
4.  UKBP                      -0.389488266  0.101354961     -3.84281  0.00018539
5.  Constant                   0.110333381  0.017685736      6.23855  0.00000001
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable DRATE
Quarterly Data From 1965:01 To 2000:04
Usable Observations    144     Degrees of Freedom   140
Centered R**2     0.107048      R Bar **2   0.087914
Uncentered R**2   0.107363      T x R**2      15.460
Mean of Dependent Variable      0.0002011574
Std Error of Dependent Variable 0.0107603680
Standard Error of Estimate      0.0102764963
Sum of Squared Residuals        0.0147848928
Regression F(3,140)                   5.5945
Significance Level of F           0.00118480
Durbin-Watson Statistic             1.689986

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DFP                      0.425074027  0.218103715      1.94895  0.05330045
2.  VC{1}                    0.067787859  0.021178390      3.20080  0.00169594
3.  RATE{1}                 -0.097427370  0.036990822     -2.63383  0.00939262
4.  Constant                -0.052760785  0.018009545     -2.92960  0.00396402


3SLS System Estimation of () to ()


Linear Systems - Estimation by Seemingly Unrelated Regressions

Iterations Taken 2
Quarterly Data From 1965:01 To 2000:04
Usable Observations    142
 Total Observations    144      Skipped/Missing        2

Dependent Variable DW
Centered R**2     0.628606      R Bar **2   0.465648
Uncentered R**2   0.932844      T x R**2     132.464
Mean of Dependent Variable      0.0144055704
Std Error of Dependent Variable 0.0067920795
Standard Error of Estimate      0.0049649699
Sum of Squared Residuals        0.0024157908
Durbin-Watson Statistic             1.652995

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DP{1}                    0.210329356  0.143144866      1.46935  0.14173893
2.  DFP                      0.934167371  0.182447249      5.12021  0.00000031
3.  VL{1}                    0.191462660  0.031671401      6.04529  0.00000000
4.  UKBP{1}                 -0.086606587  0.033212145     -2.60768  0.00911586
5.  Constant                -0.177708375  0.030236448     -5.87729  0.00000000
```

```
Dependent Variable DP
Centered R**2     0.832898      R Bar **2   0.759578
Uncentered R**2   0.959052      T x R**2     136.185
Mean of Dependent Variable      0.0106653293
Std Error of Dependent Variable 0.0060978384
Standard Error of Estimate      0.0029899457
Sum of Squared Residuals        0.0008760980
Durbin-Watson Statistic             1.605294

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
6.  DW{1}                     0.082390164  0.045812523      1.79842  0.07211045
7.  DFP                       0.942049199  0.061753718     15.25494  0.00000000
8.  VC{1}                     0.046450989  0.005597851      8.29800  0.00000000
9.  D74                       0.010522748  0.001763341      5.96751  0.00000000
10. Constant                 -0.038614621  0.004684016     -8.24391  0.00000000


Dependent Variable DVL
Centered R**2     0.522405      R Bar **2   0.305765
Uncentered R**2   0.522583      T x R**2      74.207
Mean of Dependent Variable      0.0000688254
Std Error of Dependent Variable 0.0035759981
Standard Error of Estimate      0.0029795500
Sum of Squared Residuals        0.0008611387
Durbin-Watson Statistic             1.899020

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
11. VL{1}                     0.442497469  0.075962496      5.82521  0.00000001
12. VL{2}                    -0.453181272  0.071466282     -6.34119  0.00000000
13. VC{1}                    -0.007408317  0.007570160     -0.97862  0.32776736
14. UKBP                     -0.102245939  0.022041647     -4.63876  0.00000351
15. D74                      -0.002718577  0.001376292     -1.97529  0.04823509
16. Constant                  0.016128849  0.013268927      1.21554  0.22416201


Dependent Variable DVC
Centered R**2     0.453939      R Bar **2   0.214341
Uncentered R**2   0.454225      T x R**2      64.500
Mean of Dependent Variable      -0.000323944
Std Error of Dependent Variable  0.014185973
Standard Error of Estimate       0.012574082
Sum of Squared Residuals        0.0154945396
Durbin-Watson Statistic             1.538509

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
```

17. VC{1}	-0.107396969	0.020385633	-5.26827	0.00000014
18. DVL{1}	0.990268556	0.260254619	3.80500	0.00014180
19. RRATE{1}	-0.087342645	0.025881467	-3.37472	0.00073891
20. UKBP	-0.389421984	0.087005531	-4.47583	0.00000761
21. Constant	0.092439937	0.016944697	5.45539	0.00000005

Dependent Variable DRATE

Centered R**2	0.098748	R Bar **2	-0.283601
Uncentered R**2	0.099027	T x R**2	14.062
Mean of Dependent Variable		0.0001899061	
Std Error of Dependent Variable		0.0108345824	
Standard Error of Estimate		0.0122751627	
Sum of Squared Residuals		0.0149172824	
Durbin-Watson Statistic		1.735485	

Variable	Coeff	Std Error	T-Stat	Signif
22. DFP	0.425510274	0.194647582	2.18605	0.02881160
23. VC{1}	0.071522021	0.020739654	3.44856	0.00056358
24. RATE{1}	-0.063107081	0.033596640	-1.87837	0.06032991
25. Constant	-0.058282751	0.017563513	-3.31840	0.00090535

Likelihood Ratio Test of the Structural Restrictions

Chi-Squared(408)= 414.104639 with Significance Level 0.39322678

Chi-Squared(3)= 3.929183 with Significance Level 0.26921333

3SLS - System Estimation under Restrictions

Linear Model - Estimation by Restricted Regression
Dependent Variable UKBP

Variable	Coeff
1. DP{1}	0.285296856
2. DFP	0.714703144
3. VL{1}	0.159703912
4. UKBP{1}	-0.078237242
5. Constant	-0.146329289
6. DW{1}	0.081392827
7. DFP	0.918607173
8. VC{1}	0.045327673
9. D74	0.010708802
10. Constant	-0.037436331

11. VL{1}	0.461799467
12. VL{2}	-0.461799467
13. VC{1}	-0.010854974
14. UKBP	-0.103630314
15. D74	-0.002665601
16. Constant	0.008902589
17. VC{1}	-0.108166568
18. DVL{1}	1.002987661
19. RRATE{1}	-0.091396432
20. UKBP	-0.386270535
21. Constant	0.093312776
22. DFP	0.438595499
23. VC{1}	0.071940548
24. RATE{1}	-0.064939038

3SLS Estimation of the System (31) to (35)

Linear Systems - Estimation by Seemingly Unrelated Regressions

```
Iterations Taken 2
Quarterly Data From 1965:01 To 2000:04
Usable Observations    140
 Total Observations    144      Skipped/Missing        4

Dependent Variable DW
Centered R**2     0.622469      R Bar **2   0.469931
Uncentered R**2   0.933152      T x R**2     130.641
Mean of Dependent Variable      0.0145273297
Std Error of Dependent Variable 0.0067628354
Standard Error of Estimate      0.0049237398
Sum of Squared Residuals        0.0024000782
Durbin-Watson Statistic             1.647187
```

Variable	Coeff	Std Error	T-Stat	Signif
1. DP{1}	0.190786620	0.144850244	1.31713	0.18779501
2. DFP	0.944474099	0.185285895	5.09739	0.00000034
3. VL{1}	0.194794094	0.031937339	6.09926	0.00000000
4. UKBP{1}	-0.078907652	0.033873529	-2.32948	0.01983374
5. Constant	-0.180706804	0.030494684	-5.92585	0.00000000

```
Dependent Variable DP
Centered R**2     0.831840      R Bar **2   0.763897
Uncentered R**2   0.959287      T x R**2     134.300
Mean of Dependent Variable      0.0107511047
Std Error of Dependent Variable 0.0060983439
```

```
Standard Error of Estimate      0.0029632119
Sum of Squared Residuals        0.0008692818
Durbin-Watson Statistic             1.611661

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
6.  DW{1}                     0.084652477  0.046020765      1.83944  0.06585037
7.  DFP                       0.945391907  0.061584060     15.35124  0.00000000
8.  VC{1}                     0.048304943  0.005649662      8.55006  0.00000000
9.  D74                       0.010211416  0.001762877      5.79247  0.00000001
10. Constant                 -0.040181448  0.004711322     -8.52870  0.00000000


Dependent Variable DLVL
Centered R**2     0.373032      R Bar **2   0.137144
Uncentered R**2   0.373142      T x R**2      52.240
Mean of Dependent Variable      0.0000472956
Std Error of Dependent Variable 0.0035954978
Standard Error of Estimate      0.0033398603
Sum of Squared Residuals        0.0011266214
Durbin-Watson Statistic             1.432012

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
11. DLVC{1}                   0.046349029  0.011887032      3.89913  0.00009654
12. UKBP{1}                  -0.103750948  0.023763966     -4.36589  0.00001266
13. D74                      -0.003998848  0.001592853     -2.51049  0.01205625


Dependent Variable DLVC
Centered R**2     0.291811      R Bar **2   0.015617
Uncentered R**2   0.292685      T x R**2      40.976
Mean of Dependent Variable      -0.000625869
Std Error of Dependent Variable  0.017872618
Standard Error of Estimate       0.017732507
Sum of Squared Residuals        0.0314441821
Durbin-Watson Statistic             1.377719

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
14. VC{1}                    -0.118574486  0.023993282     -4.94199  0.00000077
15. RRATE{1}                 -0.125010402  0.036489460     -3.42593  0.00061269
16. UKBP{1}                  -0.578025142  0.113297116     -5.10185  0.00000034
17. Constant                  0.103163866  0.019990019      5.16077  0.00000025


Dependent Variable DRATE
Centered R**2     0.214185      R Bar **2  -0.103316
Uncentered R**2   0.214349      T x R**2      30.009
Mean of Dependent Variable      0.0001569048
```

```
Std Error of Dependent Variable 0.0109072628
Standard Error of Estimate      0.0114568646
Sum of Squared Residuals        0.0129947148
Durbin-Watson Statistic             1.885672

   Variable                    Coeff       Std Error      T-Stat     Signif
*******************************************************************************
18. DP{1}                    0.560533027  0.163625222      3.42571  0.00061319
19. VC{1}                    0.062208851  0.017968086      3.46219  0.00053581
20. RATE{1}                 -0.081392279  0.031553949     -2.57946  0.00989537
21. Constant                -0.050592198  0.015051926     -3.36118  0.00077611
22. D74{1}                  -0.022586355  0.006685026     -3.37865  0.00072843


Chi-Squared(404)=    450.332794 with Significance Level 0.05538152
Chi-Squared(2)=      3.229649 with Significance Level 0.19892557


Linear Model - Estimation by Restricted Regression
Dependent Variable UKBP

   Variable                    Coeff
*****************************************
1.  DP{1}                    0.262618684
2.  DFP                      0.737381316
3.  VL{1}                    0.164985342
4.  UKBP{1}                 -0.070252058
5.  Constant                -0.151247052
6.  DW{1}                    0.083696433
7.  DFP                      0.916303567
8.  VC{1}                    0.046957797
9.  D74                      0.010443958
10. Constant                -0.038759866
11. DLVC{1}                  0.046621832
12. UKBP{1}                 -0.103603694
13. D74                     -0.004016235
14. VC{1}                   -0.118759579
15. RRATE{1}                -0.126515345
16. UKBP{1}                 -0.578622963
17. Constant                 0.103404222
18. DP{1}                    0.559054506
19. VC{1}                    0.062268453
20. RATE{1}                 -0.081677196
21. Constant                -0.050604874
22. D74{1}                  -0.022604143
```