Joint Tests for Long Memory and Non-linearity: The Case of Purchasing Power Parity

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Abstract

A pervasive finding of unit roots in macroeconomic data often runs counter to intuition regarding the stochastic nature of the process under consideration. Two econometric techniques have been utilized in an attempt to resolve the finding of unit roots, namely long memory and models that depart from linearity. While the use of long memory and stochastic regime switching models have developed almost independently of each other, it is now clear that the two modeling techniques can be intimately linked. In particular, both modeling techniques have been used in isolation to study the dynamics of the real exchange rate. To determine the importance of each technique in this context, I employ a testing and estimation procedure that allows one to jointly test for long memory and non-linearity (regime switching behavior) of the STAR variety. I find that there is substantial evidence of non-linear behavior for the real exchange rate for many developing and European countries, with little evidence for ESTAR non-linearity for countries outside the European continent including Japan and Canada. In cases where non-linearity is found, I also find significant evidence of long memory for the majority of the countries in my sample. Thus, long memory and non-linearity can also be viewed as compliments rather than substitutes. The linear model in isolation appears to be inadequate for breaking down the paradox known as the PPP puzzle. On the other hand, a combination of long memory and non-linearity may be a promising research avenue for pursuing an answer to the paradox.

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1 Introduction

A controversy is brewing in time series econometrics. The spark of this controversy emerged with the seminal work of Nelson and Plosser (1982) who found that most macroeconomic series were well characterized as an infinite variance unit root process. The earlier work of Granger and Newbold (1974) and subsequently the work of Phillips (1986) turned empirical research in macroeconomics on its head. The seemingly robust finding that macroeconometric data were characterized as unit root processes contradicted the practical insight that a strict unit root process is impossible for bounded series such as interest rates and unemployment rates.

Almost simultaneously two schools of thought emerged to bridge the gap between the finding of unit roots and the theoretical implausibility of processes that can increase or decrease without bound. The concept of fractional integration was introduced by Granger and Joyeux (1980) as a means of bridging the gap between covariance stationary linear processes and unit root processes. The concept of fractional integration is surely one of the foremost advances in time series econometrics and has been used extensively over the last twenty years to model virtually every macroeconomic series including interest rates (Backus and Zin, 1993 and Dueker and Startz, 1998), inflation (Hassler and Wolters, 1995 and Baillie, Chung, and Tieslau, 1996), unemployment (Diebold and Rudebusch, 1989), and particularly exchange rate dynamics (Diebold, Husted, and Rush, 1991, Baillie and Bollerslev, 1994, and Cheung and Lai, 2001). Shortly after the introduction of fractional dynamics, Perron (1989) established the fact that the unit root finding itself may be spurious if the actual data is characterized as a process having a broken trend. A proliferation of empirical research has emerged since Perron's study employing models that accommodate structural change or regime switching. In particular, Stock and Watson (1996) found evidence of parameter instability among 76 different monthly time series including employment, monetary aggregates, and prices. Garcia and Perron (1996) find evidence that the real interest rate is subject to regime switching behavior, while Bianchi and Zoega (1998) find evidence for structural breaks in the unemployment rate. Recently, evidence has emerged detailing the finding that exchange rate dynamics can be characterized by models allowing for stochastic regime switching behavior (Obstfeld and Taylor, 1997, Michael, Nobay, and Peel, 1997, and Taylor Peel and Sarno, 2001).

The seemingly divergent paths of regime switching or non-linearity and long memory have frequently enabled researchers to avoid the strict assertion that data are characterized as having a unit root. Recently, however, it has become clear that the divergent paths actually run parallel to each other and that it is frequently difficult to know which road we are on. Diebold and Inoue (2001) have each shown that regime switching and long memory are intimately linked. In particular, it would appear that a regime switching process can produce dynamics that are arbitrarily close to the stochastic properties of long memory processes. Herein lies the controversy. Both long memory and non-linear models have been successfully employed to analyze the behavior of the same processes.

The divergence of the two approaches can be highlighted through the purchasing power parity (PPP) paradox. The initial finding of a unit root in the real exchange rate (see for example, Meese and Rogoff, 1988) is at odds with the theory that prices in two countries, expressed in the same currency, should be equal. Deviations from PPP, as measured by the persistence in the real exchange rate, are too large relative to the predictions of economic models (Rogoff, 1996). Attempts to resolve these issues have utilized both long memory and non-linear models. In this paper, I analyze the interrelationship between non-linear and long memory models for this extremely important example. Clearly there are an infinite number of potential non-linear models that one could consider. Recently, both theoretical and empirical research suggests that a non-linear model that allows for a smooth transition between regimes may be most appropriate. Thus, attention is confined to the STAR family of non-linear models. I extend an approach developed by van Dijk, Franses, and Paap (2002) that allows one to jointly test for the existence of long memory and STAR non-linearity. My results are based on a large set of developed and developing countries relative to the United States. These are several major findings. First, while I do not find overwhelming evidence that all real exchange rates are characterized by non-linear behavior, I find strong evidence of complimentary and substitutability between long memory and non-linear models. From a statistical standpoint, each modeling strategy appears to adequately capture the behavior for many of the real exchange rates considered in this paper. It is important to note that the linear models in isolation are incapable of explaining the PPP paradox. On the other hand, when non-linearity is present, there is typically strong evidence of the dual existence of long memory. The results suggest that in modeling exchange rate dynamics, it is important to recognize the potential for both long memory and non-linearity. Clearly, these results likely extend to other economic and financial variables.

The remainder of the paper will be organized as follows. In the next section, I discuss the purchasing power parity paradox and the role of non-linear and long memory models in the debate. In section 3, I introduce the FI-STAR model and derive an LM-type statistic to jointly test for long memory and ESTAR non-linearity that heavily borrows from van Dijk, Franses, and Paap. In section 4, I discuss the data and present the results relative to my tests and estimation. The final section presents concluding remarks and suggestions for future research.

2 The Purchasing Power Parity Puzzle

The absolute form of purchasing power parity contends that the price of a country's currency relative to another country should equal the ratio of the two countries price indices. If the strict form of the PPP hypothesis holds then the log of the real exchange rate is equal to zero. In other words, the following most hold:

$$\log(S_t) - \log(P_t^*) + \log(P_t) = 0, \quad (1)$$

where S_t is the foreign currency price of the domestic currency, and P_t and P_t^* are the domestic and foreign price levels respectively. As alluded to by Rogoff (1996), the empirical findings suggest that the real exchange rate is highly volatile and that shocks are persistent. That is, deviations from PPP persist for some time, typically having a half life in the neighborhood of 3-5 years. Presumably, the source of these shocks is pecuniary in nature. While nominal rigidities including wage and price stickiness are plausible modeling alternatives for temporary deviations from purchasing power parity, the persistence of shocks are implausibly long for macroeconomic models of exchange rate behavior given the volatility of the real exchange rate. Therein lies the puzzle.

There appear to be few examples in empirical macroeconomics that better illuminate the heated rivalry between long memory and non-linearity than the purchasing power parity debate. Researchers who have used long memory models in an attempt to resolve the finding of a unit root in the relationship between the nominal exchange rate and goods prices include Diebold, Husted, and Rush (1991), and Cheung and Lai (1993, 2001). On the other hand, Obstfeld and Taylor (1997), Michael, Nobay, and Peel (1997), Baum, Barkoulas, and Caglayan (2001), and Taylor, Peel, and Sarno (2001) attempt to use non-linear regime switching models to describe the dynamics associated with purchasing power parity. There have also been recent attempts to address the question using elements of both approaches. Baum, Barkoulas, and Caglayan (1999) consider both a long memory model and a model that allows for a double mean shift for the real exchange rate and are unable to revive purchasing power parity. Kapetanios (2002) finds evidence of both long memory and a general form of non-linearity for Yen based real exchange rates.

There have also been advances in the theoretical formulation of exchange rates that accommodate both types of modeling approaches, particularly non-linear models. Cheung and Lai (2001) argue that under imperfect information and bounded rationality long swings in financial data are plausible. Cheung and Lai argue that the now familiar appreciation of the US real exchange rate during the beginning of the 1980's and the subsequent depreciation is consistent with the view that speculative markets can be characterized by sharp swings. The fractional process is a natural modeling alternative for variables characterized by long swing dynamics given the underlying behavior of the fractional process. More rigorous approaches have suggested that non-linear models may be an appropriate approach. The theory of PPP is based on the law of one price, where a differential between prices in two countries expressed in the same currency presents an arbitrage opportunity that should quickly eliminate a disparity between exchange rates and goods prices. Empirically, deviations from PPP, of course, do not imply the availability of any trading rule that allows profitable arbitrage. Price indices do not represent the price of any single tradeable commodity, and the different problems with variable indices render the empirical version of PPP a distant cousin to the theoretical version. On a more basic level, impediments to trade such as transportation costs, tariffs, and transactions costs complicate the underlying notion of the law of one price. Sercu, Uppal, and Hulle (1995) develop a model where traded goods are subject to transactions costs through a familiar iceberg cost mechanism. The intuitively appealing model suggests that there is no a trade region when transactions costs are large enough. In this way, the arbitrage necessary to insure the law of one price is absent within a transactions band. Once the band is broached, arbitrage takes place and the deviations are quickly eliminated. In this way, there are two regimes, a no-trade regime and an arbitrage regime. Thus, under the modeling scenario where transactions costs mitigate trade, non-linear models are an attractive alternative. Furthermore, given the time aggregation of price indices and potentially exchange rates, the movement from one regime to another is unlikely to be sudden. Rather it would appear plausible that adjustment is smooth and larger deviations result in a stronger attraction process to equilibrium for the real exchange rate. For these reasons, the STAR family of models has emerged as an attractive candidate for exchange rate dynamics.

Empirically, it would be difficult to suggest that either non-linear models or long memory models have thoroughly resolved the purchasing power parity puzzle, especially for the US. Taylor, Peel, and Sarno (2001) are able to show that for large shocks, the half lives of deviations from PPP are considerably less than 3-5 years as originally implied by Rogoff (1996). However, for smaller shocks, the impulse response function dies out at a rate roughly consistent with the previous literature. Cheung and Lai (2001) also use impulse response analysis for yen based real exchange rates. While the half lives are typically smaller than 3 years for the majority of the yen based real exchange rates, the estimated half life for the dollar/yen real exchange rate based on the estimated fractional model is 2.9 years. Furthermore, it is somewhat difficult to directly interpret Cheung and Lai's results, as they fail to include estimated ARMA parameters for their results. Nevertheless, the use of both long memory and non-linear models in characterizing the dynamics associated with PPP is a recent breakthrough in the study of international finance that is not without merit particularly given the theoretical implications discussed above. Given the potential relationship between the two modeling approaches, it would be interesting to determine the significance of each approach jointly as it relates to the real exchange rate. In the next section, I introduce a methodology that allows one to directly test for both non-linearity and long memory simultaneously.

3 A Long Memory Non-linear Model

As discussed above, there is an infinite set of potential non-linear models that could be employed for a time series variable. The theoretical and empirical literature suggests that the STAR family of models is an attractive alternative to explain exchange rate dynamics. I thus concentrate on a testing procedure that incorporates long memory and STAR non-linearity, although the approach can be extended to other types of nonlinearity.

The concept of long memory was popularized with the seminal work on fractional integration by Granger and Joyeux (1980). For many time series processes, Granger and Joyeux concluded that the spectral density function of the differenced process appeared to be over-differenced, while the level of the series exhibited long run dependence that was inconsistent with stationary ARMA dynamics. Their approach was to employ a fractional differencing operator that produced a stationary ARMA series. Mathematically, the ARFIMA model for a time series process y_t is written as,

$$\phi(L)(1-L)^d(y_t-\mu)=\theta(L)\varepsilon_t, \tag{2}$$

where $\phi(z)=0$ and $\theta(z)=0$ have all roots lying outside the unit circle, d is any real number, and $\{\varepsilon_t\}$ is a martingale difference sequence. While unnecessary for semi-parametric estimation such as the Geweke-Porter-Hudak (1983) estimator, maximum likelihood estimation, and its variants, typically employ a Gaussian assumption for the sequence of innovations, ε_t. The differencing operator, $(1-L)^d$, is defined as,

$$(1-L)^d=\sum_{k=0}^{\infty}\frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)}L^k, \tag{3}$$

where $\Gamma()$ is the gamma or generalized factorial function.

Several important features of the ARFIMA model merit brief discussion.[1] The ARFIMA model is stationary provided $d<1/2$ and is invertible if $d>-1/2$. The autocorrelations of the fractional process decay at a hyperbolic rate for $0<d<1/2$ and for positive values of the differencing parameter, the spectral density function if unbounded at the origin. The ARFIMA model also has the property that for $d<1$, the impulse response weights converge to zero, such that the process is said to mean reverting.

The STAR model has a rich tradition in time series econometrics, although Terasvirta (1994) is generally credited with establishing the necessary conditions to make the model empirically applicable.[2] Generally speaking, the two regime STAR (p) model can be written as:

$$y_t=(\phi_{1,0}+\phi_{1,1}y_{t-1}+...+\phi_{1,p}y_{t-p})+(\phi_{2,0}+\phi_{2,1}y_{t-1}+...+\phi_{2,1}y_{t-p})G(s_t;\gamma,c)+\varepsilon_t. \tag{4}$$

[1]For an excellent survey on long memory models and their applications in economics and finance, the interested reader is referred to Baillie (1996).

[2]For a tremendous survey of STAR models, the interested reader is referred to van Dijk, Terasvirta, and Franses (2002).

The first or inner regime corresponds to the first set of p autoregressive terms, while the outer regime corresponds to the sum of the two sets of autoregressive parameters (when $G(s_t; \gamma, c) = 1$). For the development of tests for non-linearity it is typically assumed that the sequence $\{\varepsilon_t\}$ is Gaussian white noise. The function $G(s_t; \gamma, c)$ is the transition function governing the movement from the inner regime to the outer regime and is a function of the transition variable s_t, the argument γ, which determines the degree of curvature of the transition function, and the argument c, which is the threshold parameter. In most applications, including the one considered in this paper, the transition function is confined to be an exponential function (resulting in an ESTAR model) or a logistic function (resulting in an LSTAR specification). Specifically, for the ESTAR family of models, the transition function takes on the following form:

$$G(s_t; \gamma, c) = 1 - \exp[-\gamma(s_t - c)^2]. \tag{5}$$

Clearly, the transition function is symmetric in that its value does not depend on whether the transition variable lies above or below the threshold c. Clearly, the parameter γ controls the degree of non-linearity. As $\gamma \to 0$, the transition function function depicted in (5) goes to zero, such that the model in 4 becomes a simple autoregressive model. As $\gamma \to \infty$, the transition function in (5) converges to unity, and the model in (4) becomes a different autoregressive model, whose autoregressive coefficients are equal to the sum of the autoregressive coefficients in the two regimes. For the LSTAR family of non-linear models, the transition function is given by:

$$G(s_t; \gamma, c) = (1 + \exp[-\gamma(s_t - c)])^{-1}. \tag{6}$$

It is clear that the LSTAR model is preferred when asymmetric behavior is expected in the transition variable. As $\gamma \to 0$, the LSTAR transition function converges to 0.50, such that the model in (4) becomes an autoregressive specification whose coefficients correspond to a geometric average of the autoregressive coefficients, whereas, as $\gamma \to \infty$ the transition function converges to 1. In this case, the model in (4) again becomes an autoregressive specification whose coefficients are the sums of the autoregressive coefficients in the two regimes.

While not necessary, it is typically assumed that the transition variable is simply a lag of the dependent variable. In other words, $s_t \equiv y_{t-d^*}$ where d^* denotes the delay parameter. In the present exercise, Taylor, Peel, and Sarno (2001) argue that the most sensible transition function is a single lag of the real exchange rate. While attention is not confined to a single lag of the real exchange rate, the analysis below strongly supports the conjecture that the transition variable is most appropriately defined as a lag of the real exchange rate, rather than, for example, a lagged difference of the real exchange rate or some other macroeconomic variable. Conceptually, it is intuitively appealing that, to the extent non-linearities are present, the degree of mean reversion is related strictly to the degree of misalignment in the real exchange rate. I therefore proceed with the discussion assuming $s_t \equiv y_{t-d^*}$.

3.1 Testing for STAR Non-linearity

The testing procedure employed in this paper borrows heavily on the approach of Terasvirta (1994), and thus a discussion of the general testing procedure for STAR non-linearity is merited. Tests for STAR non-linearity are complicated by the fact that the parameters of the STAR model are not identified under the null hypothesis of linearity. Intuitively, the hypothesis of linearity can be expressed in at least two ways, $\gamma = 0$ or $\phi_{1,0} = \phi_{2,0}$, $\phi_{1,1} = \phi_{2,1}$, $\cdots$, $\phi_{1,p} = \phi_{2,p}$. The complication of identification can be resolved easily, however, by taking an appropriate Taylor series expansion of the transition function about $\gamma = 0$. My results below show that the ESTAR model is the more appropriate choice for modeling exchange rate dynamics. Thus, I discuss only testing for ESTAR non-linearity. The reader interested in the similar testing procedure for LSTAR non-linearity is referred to Terasvirta (1994). For the ESTAR model, under the assumption that $d^* \leq p$ (which is overwhelmingly the case in my analysis), a first order Taylor series expansion of the transition function about $\gamma = 0$ results in the following auxiliary regression:

$$y_t = \sum_{j=1}^{p}(\beta_{1,0} + \beta_{1,j}y_{t-j}) + \sum_{j=1}^{p}(\beta_{2,j}y_{t-j}y_{t-d^*}) + \sum_{j=1}^{p}(\beta_{3,j}y_{t-j}y_{t-d^*}^2) + e_t. \quad (7)$$

where e_t is related to the disturbance ε_t in (4) and the remainder from the first order Taylor series expansion. The terms $\beta_{2,0}y_{t-d^*}$ and $\beta_{3,0}y_{t-d^*}$ are excluded to avoid multicollinearity since $d^* \leq p$. Under the null hypothesis of linearity, the remainder from the Taylor series expansion of the transition function around $\gamma = 0$ is equal to zero and the disturbance in (7) is equivalent to the disturbance in (4). The null and alternative hypotheses are given by:

$$\begin{aligned} H_0 &: \beta_{2,j} = \beta_{3,j} = 0, \text{ j=1,...,p} \\ H_A &: \beta_{2,j} \neq 0 \text{ or } \beta_{3,j} \neq 0, \text{ for at least one j.} \end{aligned} \quad (8)$$

The χ^2 version of the LM type test statistic is calculated as $\text{LM}_{\chi^2}=\frac{T(SSR_R-SSR_{UR})}{SSR_R}$, where T is the number of usable observations, SSR_R is the sum of squared errors calculated under the null hypothesis, and SSR_{UR} is the sum of squared errors for the regression in equation number 7. The hypothesis is distributed as a $\chi^2(2p)$ statistic. The F-version of the test is given by $\text{LM}_F=\frac{(SSR_R-SSR_{UR})/p}{SSR_{UR}/(T-2p)}$ and is distributed as an $F(2p, T-3p-1)$ statistic.

3.2 The FI-STAR Model

The model depicted in (4) is a simple non-linear model that combines the aspects of non-linearity with autoregressive models. Recently, van Dijk, Franses, and Paap (2002) extended this concept to allow for fractional integration. In particular, the FI-STAR model for a time series process y_t is defined as

$$(1-L)^d y_t = \{\phi_{1,0}+\sum_{j=1}^{p}\phi_{1,j}(1-L)^d y_{t-j}\}+\{\phi_{2,0}+\sum_{j=1}^{p}\phi_{2,j}(1-L)^d y_{t-j}\}G(y_{t-d^*};\gamma,c)+\varepsilon_t, \tag{9}$$

where I assume that ε_t is a martingale difference sequence. In this case, the fractional difference of the time series process is a STAR model. The details for estimation and testing for non-linearity have been developed by van Dijk, Franses, and Paap for the case where the transition function is given by the logistic function. I extend these results to consider the ESTAR specification as well. As above, when $d^* \leq p$ a first order Taylor series expansion of the exponential transition function results in the following auxiliary regression for (9):

$$(1-L)^d y_t = \{\phi_{1,0}+\sum_{j=1}^{p}\phi_{1,j}(1-L)^d y_{t-j}\}+\{\sum_{j=1}^{p}\phi_{2,j}(1-L)^d y_{t-j}y_{t-d^*}\}+\{\sum_{j=1}^{p}\phi_{3,j}(1-L)^d y_{t-j}y_{t-d^*}^2\}+e_t. \tag{10}$$

For the construction of the test, the Gaussian assumption will be applied. The null hypothesis for linearity is given by:

$$H_0 : \phi_{2,j} = \phi_{3,j} = 0, \text{ j=1,...,p.} \tag{11}$$

Under the null hypothesis, the time series process is distributed as a long memory ARFIMA$(p,d,0)$ process and $e_t = \varepsilon_t$. The lack of a moving average component does not appear to be a major short-coming for the FI-ESTAR model in terms of modeling the real exchange rate given the apparent dominance of the 1st lag in the partial autocorrelation function and subsequent insignificance of remaining lags in the PACF for the real exchange rate (c.f. Taylor, Peel, and Sarno, 2001). The existence of the fractional differencing parameter, however, complicates the construction of the LM type test statistic. In particular, I follow van Dijk, Franses, and Paap, and use the conditional likelihood function under the assumption of normality and a constant variance.[3] The likelihood function for the tth observation is given by:

$$\pounds_t = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\sigma^2 - \frac{e_t^2}{2\sigma^2}, \tag{12}$$

[3] An important consideration, especially in the context of a developing country's real exchange rate process, is the possibility that the variance of the disturbance may be time varying. We do not consider this complication here, but note that the use of the conditional likelihood function eases the construction of GARCH errors, by allowing one to replace σ^2 with σ_t^2.

where e_t is calculated by recursion using (3). The construction of the LM test statistic for non-linearity differs from the discussion above due to the gradient of the likelihood function with respect to d. In particular,

$$\frac{\partial \pounds_t}{\partial d} = -\frac{e_t}{\sigma^2}\frac{\partial e_t}{\partial d} \tag{13}$$

The calculation of the derivative in (13) results in the following:

$$\begin{aligned} \frac{\partial e_t}{\partial d} &= (1-L)^d \log(1-L) y_t - \sum_{j=1}^{p} \{\phi_{1,j}(1-L)^d \log(1-L) y_{t-j}\} - \\ &\quad \{\sum_{j=1}^{p} \phi_{2,j}(1-L)^d \log(1-L) y_{t-j} y_{t-d^*}\} - \{\sum_{j=1}^{p} \phi_{3,j}(1-L)^d \log(1-L) y_{t-j} y_{t-d^*}^2\} \end{aligned} \tag{14}$$

The calculation of the above under the null hypothesis of linearity yields

$$\frac{\partial e_t}{\partial d}|_{H_0} = \log(1-L)\hat{\varepsilon}_t, \tag{15}$$

where $\hat{\varepsilon}_t$ denotes the residual obtained from the ARFIMA (p,d,0) model. Finally, under the hypothesis of linearity, we have

$$\frac{\partial \pounds_t}{\partial d}|_{H_0} = -\frac{\hat{\varepsilon}_t}{\sigma^2}\sum_{j=1}^{t-1}\frac{\hat{\varepsilon}_{t-j}}{j} \tag{16}$$

The last step follows from Gradshteyn and Ryzhik (1980) equation 1.511. The remainder of the construction of the LM test for ESTAR non-linearity follows directly from Terasvirta (1994) and more particularly, van Dijk, Franses, and Paap (2002). In particular, one first estimates an ARFIMA(p,d,0) model, obtains the set of residuals $\hat{\varepsilon}_t$ and the estimate for d denoted $\hat{d}$. The sum of squared errors, denoted SSR_R, is then constructed from the residuals, $\hat{\varepsilon}_t$. A regression of $\hat{\varepsilon}_t$ is run on $-\sum_{j=1}^{t-1}\frac{\hat{\varepsilon}_{t-j}}{j}$, 1, $(1-L)^{\hat{d}}y_{t-1}$, ..., $(1-L)^{\hat{d}}y_{t-p}$, $(1-L)^{\hat{d}}y_{t-1}y_{t-d^*}$, ..., $(1-L)^{\hat{d}}y_{t-p}y_{t-d^*}$, and $(1-L)^{\hat{d}}y_{t-1}y_{t-d^*}^2$, ..., $(1-L)^{\hat{d}}y_{t-p}y_{t-d^*}^2$. From this regression, form the sum of squared errors denoted SSR_{UR}. The χ^2 version of the LM test statistic is calculated as $\text{LM}_{\chi^2}=\frac{T(SSR_R-SSR_{UR})}{SSR_R}$ and is distributed as a $\chi^2(2p)$ statistic. The F version of the LM test statistic is calculated as $\text{LM}_F=\frac{(SSR_R-SSR_{UR})/2p}{SSR_{UR}/(T-3p-1)}$ and is distributed as an $F(2p, T-3p-1)$ statistic.

3.3 Estimation of the FI-STAR Model

The test statistic for FI-STAR non-linearity depends on the estimated value of the differencing parameter d. Thus, it is important to obtain a consistent estimate of d. In the section, I describe estimation of the ARFIMA(p,d,0) model and more importantly the FI-STAR model. A plethora of advances have been made in the area of long memory estimation. In particular, Fox and Taqqu (1986) develop an estimator for the ARFIMA model that is a frequency based approximation to maximum likelihood estimation (MLE), while Sowell (1992) develops the theory necessary for maximum likelihood estimation (MLE) of the ARFIMA model. Beran (1995) develops an estimator for potentially non-stationary models that is based on the conditional likelihood function of the time series process. Geweke and Porter-Hudak (1983) develop a log periodogram regression based estimator for the differencing parameter that allows one to estimate this parameter free of the ARMA components. Andrews and Guggenberger (2003) have recently provided log periodogram regression based estimators that adapt the Geweke-Porter-Hudak technique. In this paper, I wish to estimate the autoregressive parameters with the differencing parameters, which makes the time based estimators a more appropriate choice. Furthermore, the advances of Sowell have established the conditions necessary to calculate the autocovariances of the ARFIMA(p,d,0) model. However, conditions have not been established for the FI-STAR model. Thus, it would appear clear that the estimator of Beran is the most appropriate choice for the present exercise.[4]

The estimator of Beran is based on the approximate maximum likelihood function and results in minimizing the sum of squared residuals from either an ARFIMA(p,d,0) model or the FI-STAR(p) model. In particular, Beran's estimator chooses the model parameters to minimize the following formula:

$$S(\theta) = \sum_{t=2}^{T} \varepsilon_t^2(\theta), \tag{17}$$

where θ denotes the model parameters for either model. The residuals are calculated using the formula in (9). In particular, for the FI-ESTAR model estimated below, the residuals are fit as follows,[5]

[4]Beran uses the sample mean as an estimate of μ. Chung and Baillie (1993) show that the use of the sample mean in estimating μ can result in sizeable small sample biases when d is positive. The biases tend to be smaller when μ is estimated jointly using the conditional sum of squared errors estimator reported below. Thus, I choose to estimate μ jointly with the other parameters rather than use the sample mean.

[5]Following Terasvirta (1994), we standardize γ by the variance of the transition variable. Among other things, this makes the selection of starting values relatively straightforward. In my algorithm, I select starting values by first estimating an ARFIMA (p,d,0) model and then fit an ESTAR model to the fractional difference of the real exchange rate process. Starting values for γ are selected using a grid search before joint estimation of all of the model

$$\varepsilon_t = (1-L)^d y_t - \{\phi_{1,0} + \sum_{j=1}^{p}(1-L)^d y_{t-j}\} \quad (18)$$
$$-\{\phi_{2,0} + \sum_{j=1}^{p}(1-L)^d y_{t-j}\}[1-\exp\{-\frac{\gamma}{\sigma^2_{y_{t-d}}}(y_{t-d}-c)^2\}].$$

Estimation of the model requires non-linear least squares and necessarily implies that the residuals must be truncated. The estimation procedure used in this paper generates a numerical gradient that is used in construction of the model's standard errors. As alluded to by Terasvirta (1994) for ESTAR models and van Dijk, Franses, and Paap (2002) for FI-STAR models, it can be difficult to estimate the model parameters jointly. In particular, accurate estimation of the smoothness parameter γ is quite difficult when this parameter is large, since small changes in the transition function result from even large changes in γ. Among other things, this implies that γ will not have a standard t-distribution under the null hypothesis of linearity. Furthermore, a large resulting standard error for γ, in this case, is necessarily a result of the numerical difficulty in estimating γ and should not be taken as evidence against non-linearity. Because of difficulty in joint estimation of the model parameters as outlined by Terasvirta (1994), van Dijk, Franses, and Paap (2002) propose an algorithm that concentrates the sum of squares function. Conditional on the differencing parameter, the threshold parameter c, and the smoothness parameter γ, the FI-STAR model is linear in the remaining parameters. Thus, one can estimate γ, d, and c using non-linear least squares, and estimates of the autoregressive parameters in the two regimes can be obtained via ordinary least squares. Terasvirta (1994) recommends a similar approach when the algorithm fails to converge. For the examples considered here, I find that normalizing the exponential function by the variance of the transition variable is all that is required to insure convergence of the algorithm. I therefore estimate all of the model parameters jointly using non-linear least squares.

There are several technical details that briefly merit discussion. Here I propose a general modeling strategy for the FI-STAR model that again borrows from Terasvirta (1994) and van Dijk, Franses, and Paap (2002). The first step is to determine the number of autoregressive parameters to be estimated. In choosing a value for the number of autoregressive parameters, I use the SIC and AIC and check the residuals to insure a lack of serial correlation. I generally find that p=1 is sufficient, although there are several detractors as alluded to below. Once an appropriate selection has been established for the number of autoregressive coefficients, one must decide on the appropriate delay parameter and the preferred transition function. Taylor, Peel, and Sarno (2001) argue that it is difficult to envision a scenario under which a delay parameter larger than 1 is necessary. Furthermore, they suggest that an ESTAR model is more

parameters occurs.

appropriate than the LSTAR counterpart. They argue that it is there is no economic rationale to expect exchange rates to adjust differently when above equilibrium than below equilibrium, and thus a symmetric transition function is appropriate. On anecdotal grounds, I do not necessarily agree. It would certainly seem plausible that monetary authorities might react to a rapid real appreciation of their currency through market intervention, while leaving a real depreciation relatively unchecked. In choosing the appropriate transition function and delay parameter, I employ the modeling cycle of Terasvirta (1994). In particular, I choose the delay parameter that minimizes the p-value associated with the linearity tests described above. To the extent that non-linearity is found, I generally conclude that the delay parameter is equal to 1. Furthermore, in spite of my contention that LSTAR non-linearity is plausible, there is absolutely no evidence of FI-LSTAR non-linearity over FI-ESTAR non-linearity in any of the real exchange rates referred to below.

Finally, Kapetanios (2002) recommends a test for a general form of non-linearity given the fact that the estimate of d above is constructed under the null hypothesis of linearity. Of course, under the null, the estimate of d is consistent, but under the alternative of non-linearity the model is misspecified and the estimate of d is likely inconsistent. Here, given the discussion above on the theoretical and empirical improvements that STAR models introduce to exchange rate dynamics, I prefer to concentrate on a specific rather than general form of non-linearity. Therefore, as a robustness check, I also estimate the full FI-ESTAR model, obtain the estimate of d, and perform the same set of tests as described above. The results are largely consistent with the original findings, although there are several instances in which non-linearity is found.

4 Data and Estimation

In this paper I consider the real exchange rate process for 20 countries against the United States. My sample includes Argentina, Belgium, Brazil, Canada, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Japan, Korea, Mexico, the Netherlands, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The data employed in this study were obtained from the IFS CD-ROM for June 2002. All of the data are monthly and begin in January of 1973. The data set ends in December of 1998 for all of the EU member states. For Israel, the available data extends through November 2001, while for Japan I have data through February 2002. For Switzerland and Korea, the available data extend through April 2002, while for the remaining countries the available data extends through March of 2002. Whenever possible I have used the consumer price index in the construction of the real exchange rate. For Ireland I used the wholesale price index and for Israel it was necessary to use the producer price index. Following Taylor, Peel, and Sarno (2001), the real exchange rate is calculated as in equation 1, where the nominal exchange rate is defined as the foreign currency price of the dollar. The real exchange rate is then normalized to have the value 0 in January 1973.

To gain a sense of the real exchange rate process under consideration refer to figure 1, which depicts the real exchange rate process for Germany. The most striking aspect of the plot is the unprecedented real appreciation of the dollar during the beginning of the 1980's and subsequent depreciation after the Plaza Accord in 1985. Interestingly, ocular inspection reveals about four periods after 1987 in which the dollar steadily appreciates and then unexpectedly depreciates after reaching approximately the same threshold. This is entirely consistent with the theoretical foundation of Sercu, Uppal, and Hulle (1995) and provides some support for the use of non-linear models.

[FIGURE 1 ABOUT HERE]

4.1 ARFIMA (p,d,0) estimation

I now turn to estimation of the ARFIMA (p,d,0) model in table 1. As alluded to above, the modeling strategy generally suggested that the appropriate number of autoregressive parameters is equal to 1. In particular, in the vast majority of cases both the SIC and AIC were smallest for the ARFIMA(1,d,0). The strategy employed was to minimize the SIC under the constraint that the residuals are white noise. In instances where the residuals are not WN, I either use the AIC (if it differs from the SIC), or increase the number of autoregressive terms until serial correlation is removed. There were three detractors. For Canada, using the Ljung Box Q-statistic there was some evidence of higher order serial correlation for every model estimated. The inclusion of additional autoregressive terms did not eliminate the higher order serial correlation, nor did it impact tests for non-linearity. In both cases the AIC and SIC selected one autoregressive term, and I therefore set p=1 for Canada throughout. For Korea and Mexico, every linear model considered was inadequate for removing serial correlation, although the FI-ESTAR(1) model was adequate for Korea, while the FI-ESTAR (4) model produced an adequate fit for Mexico.

From table 1, there is little evidence that supports PPP. For 14 of the countries in the sample, the estimated value of d is not significantly different from unity using the constructed numerical standard errors at the 5% value. For Canada, the estimated value of d is not statistically different from 0, while the autoregressive specification points to an extremely persistent real exchange rates in line with the analysis reported in Rogoff (1996). The results for the United Kingdom and Spain are somewhat difficult to interpret. While the estimated value of d for these two countries is not significantly different from 0, there is a strong first order autoregressive coefficient exceeding one, while the sum of the autoregressive coefficients is only marginally less than unity. There is some evidence of mean reversion for Argentina, Israel, and Mexico, where the estimated differencing parameter is not significantly different from 0 and the sum of the autoregressive coefficients is substantially less than unity. This is somewhat in line with previous analysis that has suggested purchasing power parity is more likely to hold for countries with high inflation rates. Table 2 reports diagnostic statistics associated with the estimated ARFIMA (p,d,0) models. It is not at all surprising that the residuals from the developing countries exhibit a high degree of kurtosis and appeared to be skewed. In particular, the economies of Latin America have endured a history of exchange rate turmoil. By and large, the residuals from the ARFIMA estimation appear to be serially uncorrelated.

To shed more light on the use of long memory, I also estimated an ARFIMA model for the first difference of the real exchange rate process. The results, which are available upon request, very strongly support the estimation results reported in table 1. In fact, there are only two cases where there is any discernible disparity between the results. For both Spain and the United Kingdom, the estimated value for d for the differenced series is insignificantly different from 0, suggesting the potential presence of a unit root. The estimated differencing parameters for Argentina, Israel, and Mexico are all significantly different (and less) than 0. In addition, the autoregressive parameters sum to less than unity. For Canada, the estimated differencing parameter is not statistically different from -1 (suggesting d=0 for the level of the series), while the estimated autoregressive parameter remains in line with the estimates found in table 1. For the remaining countries, there is no direct evidence for purchasing power parity. Each of the remaining differencing parameters are not statistically different from zero using the numerical standard errors, and thus I am unable to reject the hypothesis that the real exchange rate processes have unit roots. It is well known, however, that the asymptotic distribution of the approximate MLE estimator of the differencing parameter for a stationary and invertible ARFIMA model with known mean is given by the following (c.f. Li and McLeod, 1986):

$$\sqrt{T}(\hat{d} - d) \sim N(0, \frac{6}{\pi^2}) \tag{19}$$

Recall from above, that sample size varies from 352 to 312, and thus the asymptotic standard errors range from 0.0416 to 0.0442. With the asymptotic standard errors, there is mild support for PPP in that the differencing parameters for the real exchange rates of Argentina, Brazil, Canada, France, Ireland, Israel, Italy, Japan, Korea, Mexico, Spain, and the UK are all statistically different from 0 at the 5% level. Nonetheless, even using this relatively informal metric, it would be difficult to validate PPP on these grounds. The estimated differencing parameters typically indicate a strong degree of non-stationarity in the level of the real exchange rate, and while the estimated ARFIMA models for these 12 countries may point to some degree of mean reversion, the amount of persistence implied by the parameter estimates is too large to resolve the PPP puzzle of Rogoff (1996).

4.2 Tests for FI-STAR Non-linearity

Evidence of STAR(p) non-linearity has been reported by several authors, most recently Taylor, Peel, and Sarno (2001) and Baum, Barkoulas, and Caglayan (2001). In table 3, I report the results of the tests for ESTAR non-linearity based on the modeling cycle described above. Again, I vary the delay parameter and report the findings associated with the smallest probability for linearity.[6] There is very strong evidence of non-linearity for Argentina, Brazil, Korea, and Mexico. Among these countries, the probability that the fractionally differenced series is linear is at most 3×10^{-7} (Argentina). There is also strong evidence of non-linearity in the German real exchange rate given a p-value of 0.0154. Of the remaining countries, the p-values for linearity are less than 10% for Israel, Spain, and Sweden. I also estimated the FI-ESTAR model for each of the countries where I failed to reject linearity. The estimated value of d for these countries indicates additional evidence of non-linearity for the Netherlands, Portugal, and Spain. In each case, the estimated p-value associated with the hypothesis of linearity is less than 10%. Furthermore, while the tests fail to reject linearity both with the estimated value of d under the null and the value under the alternative hypothesis, the estimated FI-ESTAR models for the United Kingdom and France produce a smaller AIC than the simple fractional model. I report the estimation results for these two countries as well. Nonetheless while I concede the possibility of non-linear effects for these two countries, given the results of the linearity tests, I conclude that the UK and French real exchange rate are best defined as long memory linear processes. Therefore, of the 20 countries in my sample, the results indicate that the real exchange rates for 9 countries are best defined as linear long memory processes, while 11 countries have real exchange rates that appear to be defined well by ESTAR non-linear dynamics. Of these 11 countries, 7 have estimated values of d that are significantly greater than 0 but less than 1 indicating the significant presence of long memory coupled with non-linearity.[7]

In the next section, I turn to estimation of the FI-ESTAR model for the

[6] I allow the delay parameter to vary from 1 to 4 and select the value that yields the most support for non-linearity as in Terasvirta (1994). I experimented with higher values, but never encountered a situation where a delay greater than 4 was necessary.

[7] Clearly, while non-linearity can be quite important in explaining the dynamics of the real exchange rate in this paper, I find that not all real exchange rates are characterized by non-linearity when long memory is present. Rapach and Wohar (2003) also present results that suggest non-linearity may be somewhat limited in capturing the dynamics of the real exchange rate process, while Koop and Potter (2001) caution that the finding of non-linearity could result from an underlying structural change. This seems like a possibility especially for the developing countries in my sample.

eleven countries that showed evidence of non-linearity. I also include the estimation results for the UK and France, and I comment on the lack of finding of non-linearity in the other countries in my sample, especially Canada.

4.3 Estimation Results for the FI-ESTAR Model

In this section, I consider estimation of the FI-ESTAR model. Table 4 contains the estimation results for the 11 countries where non-linearity was found plus the UK and France. As alluded to above, I estimate the model parameters jointly and report the standard errors associated with the numerical procedure. The underlying estimates of the smoothness parameter γ are quite large, and as such the estimated Hessian matrix can be volatile. As a consequence, the numerical standard errors are relatively large when compared to table 1. Nonetheless, there is ample evidence to support the hypothesis that the real exchange rate processes considered in table 4 can be described as having both long memory and non-linear components. In particular, for 7 of the countries considered, Argentina, Brazil, Germany, Korea, the Netherlands, Portugal, and Switzerland, the estimated value for d is significantly different from both 0 and 1 at the 5% for a two tailed test even using the large numerical standard errors. It is also interesting to note, that when one allows for non-linearity the degree of persistence as measured through the differencing parameter decreases substantially. In particular, the estimated value of d has fallen for all countries relative to the results in table 1, except Argentina, Israel, Mexico, and Spain. The estimated values of d for the latter countries are of the same magnitude as the quantity estimated in table 1.

Notice that if the differencing parameter is equal to zero, the result is a simple ESTAR (p) model. The ESTAR model appears to be the preferred model using the numerical standard errors for Israel, Mexico, Spain, and Sweden. Again, the FI-ESTAR model appears most appropriate for the remaining 7 countries. There is particular interest in Germany. Below, is an estimated ESTAR(1) model for the German real exchange rate process where R_t denotes the real exchange rate,

$$R_t = 0.067 + 0.793R_{t-1} + (-0.078 + 0.148R_{t-1})\{1 - \exp[-38.3442 * 0.363(R_{t-1} - 0.42)^2]\}. \tag{20}$$

Analysis indicates that there are two regimes associated with the ESTAR(1) model, the inner regime corresponding to an AR(1) process with an estimated autoregressive coefficient equal to 0.7930, and an outer regime, corresponding to an AR(1) process with an autoregressive parameter equal to 0.9410. The German real exchange rate lies within the outer regime for much of the sample, implying that the real exchange rate can be persistent. Nonetheless, the results are interesting, as the two regimes produce autoregressive models that are less persistent that a linear autoregressive model estimated in isolation, which would produce an AR coefficient equal to 0.9851.

Turning to the estimated FI-ESTAR model, I first point out that the model fits the data markedly better than the ESTAR counterpart. First, the estimated value of d is significantly different from 0. Second, the sum of squared errors of the ESTAR model is 0.2467 compared to the much lower value (0.2175) that results from the FI-ESTAR model. The introduction of the additional differencing parameter allows the SIC to drop from -7.0287 to -7.1546. The inner regime is associated with a relatively transient regime given by

$$(1-L)^{0.4011}R_t = 0.3179(1-L)^{0.4011}R_{t-1} \tag{21}$$

The outer regime is the more persistent of the two regimes resulting in an ARFIMA process with an estimated autoregressive coefficient equal to 0.9123.

Insight can be gained into the estimated model for Germany by observing the transition function against time for the estimated FI-ESTAR model, which is depicted in figure 2. The rapid appreciation of the dollar during the early 1980s is accompanied with a move to the relatively persistent regime. Referring to figure 1, there are 4 smaller episodes following the original run-up and associated decline of the dollar in which the pattern repeats itself. Interestingly, during the periods of rapid appreciation of the dollar, the transition function depicted in figure 2 suggests that the real exchange rate lies in the persistent regime, before the deviation from the equilibrium value becomes too large, and the process begins to transition to the less persistent regime.

[Figure 2 About Here]

Table 4 also contains the estimated results for the United Kingdom and France. It is interesting to note, that in spite of the failure to reject linearity for these two countries, that the estimated models do provide some possible evidence favoring the FI-ESTAR model. The estimated values of d are of the same magnitude as the estimated differencing parameter for Germany, especially France's, although using the numerical standard values, we fail to reject that these parameters are 0. The inner regime for France is associated with an ARFIMA model with three autoregressive parameters which sum to 0.3309, while the outer regime has autoregressive parameters equal to 0.8639, -0.1434, and 0.1860. The estimates for France suggest that the process tends to be quite near the outer regime, implying dynamics that are comparable to those of Germany.

Overall, the FI-ESTAR model appears to be an attractive alternative for modeling real exchange rate dynamics, especially for the developing countries and European countries in our sample vis-a-vis the US dollar. Nonetheless, note that there is not rampant evidence of non-linearity in the real exchange rate process once long memory has been introduced. In particular, there is little evidence supporting the contention that non-linearity is present in the real exchange rate in Canada. The estimated FI-STAR model seems to collaborate the findings in table 1. The transition function associated with the estimated FI-STAR model for Canada is depicted in figure 3. Note, the transition function is always quite near zero, deviating slightly away from zero near the end of the sample. This implies that the process is frequently near the inner regime, which has an ARFIMA specification with d=0.2057 and an autoregressive coefficient equal to 0.9540. While the outer regime results in a process with an autoregressive parameter equal to -0.3472, the actual process never appears to approach this regime, and we are left with a model that is virtually indistinguishable form the one depicted in the first table.

[Figure 3 About Here]

5 Conclusion

A debate has emerged in economics as to who has the better answer concerning the paradox that many bounded series appear to be well described as unit root processes. Both long memory and structural change are elegant tools that have been introduced in the past quarter century as a means of bridging the gap between stationary ARMA models and non-stationary infinite variance unit root processes. While both techniques have proven to be important from a modeling perspective, it is now clear that the picture has been clouded again by the apparent similarity between the two approaches. Indeed, Diebold and Inoue (2001) establish conditions under which structural change results in a stochastic structure that can be characterized as long memory. Therefore, it is important to consider the relationship between these models.

Nowhere in economics is the intersection of structural change and long memory more apparent than it is for the purchasing power parity puzzle. In particular, I argue that the strongest empirical and theoretical modeling approaches for purchasing power parity with regard to structural change point to models that allow structural change to occur endogenously through non-linear stochastic regime switching models. The appealing argument suggests that a model that incorporates a smooth transition between regimes is most appropriate given various impediments to the arbitrage conditions that underlie the theory of PPP and the time aggregation of price indices and exchange rates. Thus, numerous authors including Michael, Nobay, and Peel (1997), Taylor, Peel, and Sarno (2001), and Baum, Barkoulas, and Caglayan (2001) have recently employed the STAR family of models to analyze real exchange rate movements. At the same time, others such as Cheung and Lai (2001) have put forth arguments suggesting that the real exchange rate process is best defined as a long memory process.

Given the disparate modeling approaches with regard to purchasing power parity and it importance in international economics, the theory provides a natural environment for analyzing the interaction between non-linear and long memory models. To this end, I employ a modeling approach that allows for joint estimation of a non-linear long memory STAR model that extends the approach of van Dijk, Paap, and Franses (2002) to allow for FI-ESTAR models. Again the STAR model is selected given the obvious breakthroughs with this modeling procedure in the last ten years, although the procedure readily extends to other forms of non-linearity. I find that it can be important to view the two modeling techniques not only as potential substitutes but also as potential complements. Of the 11 countries where evidence of non-linearity was found, 7 also produced significant evidence of long memory. Thus, future analysis of the real exchange rate process should consider the potential of both long memory and non-linearity. The results also accord with those of Diebold and Inoue (2001) in that frequently the dynamics of the real exchange rate process are captured well by both models. While the findings of non-linearity are not rampant, it is crucial to note that in terms of the PPP puzzle, linear long memory models offer little empirical relief for the paradox. On the other hand, from both an empirical and theoretical perspective, the non-linear techniques used in conjunction with long memory may help explain the findings of strong persistence in the real exchange rate. In particular, when non-linearity is found, my results typically indicate a strong reduction in the differencing parameter. In particular, the German real exchange rate has an inner and outer regime that appear to be largely stable.

Finally, it is important to note what I am able to do and what I am unable to do. I only test for one form of non-linearity, and it would be interesting to extend my results to include different types of non-linearity such as band TAR models and Markov switching models. Second, more research may be needed for several of the countries in my sample to determine the appropriate model when non-linearity is found. In particular, the strongest support for non-linearity obtained in this paper results for economies that have a history of institutional instability with regard to their exchange rate policy. Especially for the developing countries in my sample, it would be sensible to associate non-linearity with underlying parameter instability associated with structural change. Finally, especially for the developing countries, there is some evidence of time varying conditional volatility and non-normality in the residuals. The primary assumption employed by van Dijk, Franses, and Paap in the construction of their estimation procedure only relies on the assumption that the disturbance is a martingale difference sequence. Nonetheless, it is clear, as pointed out by Eitrheim and Terasvirta (1996), that conditional heteroskedasticity and non-normality can influence the testing procedure for ESTAR non-linearity. In the present exercise, one could easily accommodate the assumption of a time varying conditional volatility by explicitly modeling it in the conditional likelihood function. I leave these issues for future research.

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Table I
Results for Estimation of ARFIMA (p,d,q) Model

Country	ϕ_1	ϕ_2	ϕ_3	ϕ_4	d	μ
Argentina	0.7703	0.1676	N/A	N/A	0.1274	0.3527
	[0.1727]	[0.1254]			[0.1680]	[0.2827]
Belgium	0.3476	N/A	N/A	N/A	0.9689	-0.0837
	[0.1710]				[0.1444]	[0.0604]
Brazil	0.4302	N/A	N/A	N/A	0.8142	-0.0428
	[0.1873]				[0.1645]	[0.0817]
Canada	0.9960	N/A	N/A	N/A	0.1042	0.4488
	[0.0108]				[0.0662]	[0.8844]
Denmark	0.3859	N/A	N/A	N/A	0.9370	-0.0671
	[0.1802]				[0.1546]	[0.0582]
Finland	0.3853	N/A	N/A	N/A	0.9374	-0.0675
	[0.1803]				[0.1547]	[0.0583]
France	0.5815	-0.0962	0.1946	N/A	0.7057	-0.1822
	[0.2346]	[0.0928]	[0.0810]		[0.2312]	[0.1371]
Germany	0.4467	N/A	N/A	N/A	0.8510	-0.1206
	[0.1811]				[0.1599]	[0.0755]
Ireland	0.5249	N/A	N/A	N/A	0.7976	-0.0322
	[0.2242]				[0.2108]	[0.0655]
Israel	0.9572	N/A	N/A	N/A	0.0794	-0.1369
	[0.0313]				[0.0865]	[0.0692]
Italy	0.4476	N/A	N/A	N/A	0.8932	-0.0313
	[0.2024]				[0.1797]	[0.0615]

Notes: The numerical standard errors are displayed in brackets under the respective coefficients. We used the SIC and AIC, coupled with a criteria that the residuals from the estimated models must be serially uncorrelated to generate the number of AR coefficients.

Table I (cont).
Results for Estimation of ARFIMA (p,d,q) Model

Country	ϕ_1	ϕ_2	ϕ_3	ϕ_4	d	μ
Japan	0.5138	-0.099	0.1271	N/A	0.8352	-0.1828
	[0.2016]	[0.0834]	[0.0831]		[0.1917]	[0.1181]
Korea	0.5149	N/A	N/A	N/A	0.8627	0.0101
	[0.2205]				[0.2105]	[0.0697]
Mexico	0.8701	-0.0421	-0.0905	0.1946	0.1852	0.0298
	[0.2267]	[0.1300]	[0.0998]	[0.0831]	[0.2258]	[0.1477]
Netherlands	0.4053	N/A	N/A	N/A	0.8995	-0.1052
	[0.1795]				[0.1564]	[0.0689]
Portugal	0.3101	N/A	N/A	N/A	0.9299	-0.0639
	[0.1656]				[0.1369]	[0.0574]
Spain	1.1637	-0.1912	N/A	N/A	0.1291	-0.3004
	[0.2358]	[0.2075]			[0.2132]	[0.1459]
Sweden	0.3747	-0.1064	N/A	N/A	1.0000	-0.0487
	[0.1412]	[0.0804]			[0.1218]	[0.0470]
Switzerland	0.4145	N/A	N/A	N/A	0.8773	-0.1482
	[0.1648]				[0.1441]	[0.0792]
UK	1.1576	-0.3623	0.1494	N/A	0.1935	-0.1656
	[0.2553]	[0.1883]	[0.0799]		[0.2543]	[0.0946]

Notes: The numerical standard errors are displayed in brackets under the respective coefficients.
We used the SIC and AIC, coupled with a criteria that the residuals from the estimated models must be serially uncorrelated to generate the number of AR coefficients.

Table II
Statistics Associated with FI Model

Country	Excess Kurtosis	Skewness	Jacque Bera	Q Stat Pvalue[5]	Q Stat Pvalue[10]	Q Stat Pvalue[20]	AIC	SSE
Argentina	36.5209	4.7096	20567.00	0.8096	0.129	0.2957	-4.3025	4.6301
Belgium	0.4538	0.1359	3.6024	0.2923	0.2656	0.6176	-7.271	0.2121
Brazil	9.6344	1.5574	1486.60	0.5763	0.2334	0.308	-6.7507	0.4026
Canada	-0.1158	0.2029	2.5827	0.749	0.0418	0.0009	-9.019	0.0417
Denmark	2.4120	0.5339	89.5831	0.5892	0.4571	0.8553	-7.4397	0.1792
Finland	2.4119	0.5337	89.562	0.5907	0.4579	0.8557	-7.4397	0.1792
France	1.1238	0.3197	21.3851	0.9083	0.9121	0.9454	-7.3142	0.2006
Germany	0.4565	0.0137	2.6931	0.8339	0.6392	0.9043	-7.2081	0.2259
Ireland	0.2132	0.0433	0.6819	0.8289	0.6883	0.5974	-7.5574	0.1593
Israel	24.1936	3.319	9021.3000	0.9254	0.9538	0.8967	-7.3419	0.2203
Italy	0.9432	0.2442	14.5246	0.8749	0.8576	0.8189	-7.4323	0.1805
Japan	0.8404	-0.5294	26.269	0.9867	0.8306	0.3739	-7.1299	0.2716
Korea	73.6714	6.3334	81258.00	0.0006	0.0000	0.0003	-7.4160	0.2076
Mexico	35.0047	4.3974	18726.00	0.0141	0.0174	0.2600	-5.9989	0.8392
Netherlands	0.2096	0.1351	1.5064	0.8974	0.8543	0.9607	-7.2439	0.2180
Portugal	1.6523	0.4599	46.0417	0.9474	0.7204	0.759	-7.1910	0.2298
Spain	2.7178	0.5282	109.1154	0.4776	0.5136	0.482	-7.3948	0.1862
Sweden	2.6122	0.6211	120.9666	0.9715	0.4001	0.7683	-7.4701	0.1950
Switzerland	0.296	0.0373	1.3551	0.6957	0.5407	0.8542	-7.0453	0.3007
UK	1.2948	-0.1275	25.1086	0.7237	0.8844	0.2431	-7.4036	0.2072

Table III
Tests Results for Linearity

	LM χ^2	LM F	p value χ^2	p value F	Selected Model	Selected delay	Transition Function
Argentina	34.521821	9.4156975	5.82E-07	3.11E-07	FI-ESTAR	2	ESTAR
Belgium	3.2579	1.6303	0.1961	0.1976	FI	1	ESTAR
Brazil	36.563133	20.24544	1.1493E-08	4.814E-09	FI-ESTAR	1	ESTAR
Canada	0.6498	0.3217	0.7226	0.7251	FI	1	ESTAR
Denmark	3.9293	1.9706	0.1402	0.1411	FI	1	ESTAR
Finland	3.9229	1.9674	0.1407	0.1416	FI	1	ESTAR
France	5.1914	0.8544	0.5195	0.5289	FI	1	ESTAR
Germany	8.3148	4.2306	0.0156	0.0154	FI-ESTAR	1	ESTAR
Ireland	2.9251	1.4622	0.2316	0.2333	FI	1	ESTAR
Israel	4.9508	2.4896	0.0841	0.0844	FI-ESTAR	1	ESTAR
Italy	3.9601	1.9798	0.1381	0.1398	FI	1	ESTAR
Japan	6.9412	1.1498	0.3263	0.333	FI	1	ESTAR
Korea	33.6594	18.4608	4.9086E-08	2.393E-08	FI-ESTAR	1	ESTAR
Mexico	66.4741	9.9286	2.465E-11	1.98E-12	FI-ESTAR	1	ESTAR
Netherlands	2.9759	1.4878	0.2258	0.2275	FI	1	ESTAR
Portugal	0.4726	0.2336	0.7896	0.7918	FI	4	ESTAR
Spain*	7.9936	2.0116	0.0918	0.0927	FI-ESTAR	2	ESTAR
Sweden	8.005	2.013	0.0914	0.0922	FI-ESTAR	2	ESTAR
Switzerland	2.8493	1.4199	0.2406	0.2431	FI	3	ESTAR
UK*	9.8635	1.6433	0.1305	0.1344	FI	4	ESTAR

* For Span and the United Kingdom, the p-values reported for 2 and 4 lags of the delay parameter are marginally smaller than the p-values for 1 lag. However, the estimated models reported below are based on the delay parameter set equal to 1, as this model produced a better fitting non-linear model.

Table IV
FI-ESTAR Estimation Results

Coefficient	Country: Argentina	Brazil	Germany	Israel	Korea	Mexico	Nether.	Portugal
$\phi_0^{(1)}$	0.0129	0.0017	-0.0299	0.0172	-0.0143	0.0004	-0.0072	0.0303
	[0.0142]	[0.0034]	[0.0314]	[0.0372]	[0.0197]	[0.0090]	[0.0088]	[0.0318]
$\phi_1^{(1)}$	0.8264	1.1015	0.3179	1.1772	3.4397	0.6445	1.1988	1.2853
	[0.1373]	[0.1098]	[0.5166]	[0.2724]	[0.3077]	[0.2110]	[0.2000]	[0.3193]
$\phi_2^{(1)}$	0.3506	N/A	N/A	N/A	N/A	0.4373	N/A	N/A
	[0.1513]					[0.1643]		
$\phi_3^{(1)}$	N/A	N/A	N/A	N/A	N/A	-0.3926	N/A	N/A
						[0.1464]		
$\phi_4^{(1)}$	N/A	N/A	N/A	N/A	N/A	0.2778	N/A	N/A
						[0.0844]		
$\phi_0^{(2)}$	0.0308	0.1094	0.0278	-0.0209	0.0139	-0.0071	0.0067	-0.034
	[0.0717]	[0.9989]	[0.0318]	[0.0378]	[0.0198]	[0.0246]	[0.0096]	[0.0319]
$\phi_1^{(2)}$	-0.8777	-4.0668	0.6034	-0.2643	-2.9919	1.7517	-0.2797	-0.3365
	[0.4080]	[35.7836]	[0.5029]	[0.2487]	[0.3221]	[1.6481]	[0.1911]	[0.3180]
$\phi_2^{(2)}$	0.1752	N/A	N/A	N/A	N/A	-3.5	N/A	N/A
	[0.3303]					[3.3280]		
$\phi_3^{(2)}$	N/A	N/A	N/A	N/A	N/A	2.0105	N/A	N/A
						[2.2312]		
$\phi_4^{(2)}$	N/A	N/A	N/A	N/A	N/A	-0.4193	N/A	N/A
						[0.8038]		
γ	0.2855	0.0258	3.4207	10.5394	690.8707	0.2373	18.9115	1410.70
$1/\sigma^2_s$	5.7847	37.7403	38.3442	90.8149	48.8415	27.6396	40.3676	27.9313
	[01.980]	[0.2451]	[0.5131]	[16.2099]	[394.5694]	[0.3153]	[19.0115]	[4230.00]
c	0.2156	-0.0825	-0.359	-0.208	0.0059	0.2711	-0.1373	0.0787
	[0.1023]	[0.0285]	[0.0833]	[0.0023]	[0.0017]	[0.0185]	[0.0137]	[0.0053]
d	0.2186	0.3759	0.4011	0.1166	0.7977	0.1983	0.2932	0.2042
	[0.0969]	[0.0947]	[0.1613]	[0.0997]	[0.1015]	[0.1804]	[0.0996]	[0.0959]
Ex. Kurt.	42.8037	9.2229	0.2359	23.654	21.4036	38.6593	0.0874	1.7026
Skewness	5.2834	1.2689	-0.0664	3.2293	2.5618	4.8483	0.1106	0.4465
J. Bera	27699.00	1311.5453	0.931	8517.40	6962.70	22372.00	0.7183	46.9714
Pval- Q (5)	0.8625	0.1520	0.5554	0.8995	0.9976	0.4732	0.2633	0.6457
Pval- Q (10)	0.0547	0.1206	0.4669	0.8660	0.1646	0.5778	0.5335	0.3943
Pval- Q (20)	0.1664	0.0749	0.7494	0.8385	0.3853	0.8950	0.8738	0.6124
AIC	-4.3970	-6.8318	-7.2171	-7.339	-8.035	-6.1158	-7.2394	-7.2074
SSE	4.0692	0.3617	0.2175	0.2152	0.1089	0.7086	0.2127	0.2174

Table IV (continued)
FI-ESTAR Estimation Results

	Country: Spain	Sweden	Switz.	Country: France*	UK*
Coefficient					
$\phi_0^{(1)}$	-0.0385	-0.0317	-0.0334	-0.0250	-0.1442
	[0.0351]	[0.2828]	[0.0551]	[0.0387]	[0.1514]
$\phi_1^{(1)}$	1.6946	1.4158	-1.4921	-3.3783	1.7905
	[0.9586]	[0.1510]	[7.7717]	[8.3796]	[1.3875]
$\phi_2^{(1)}$	0.1122	-0.6173	N/A	2.9733	0.5736
	[0.9324]	[1.2810]		[7.6902]	[0.4796]
$\phi_3^{(1)}$	N/A	N/A	N/A	0.7359	0.1857
				[1.5792]	[0.4983]
$\phi_4^{(1)}$	N/A	N/A	N/A	N/A	N/A
$\phi_0^{(2)}$	0.0323	0.2359	0.0294	0.0248	0.1405
	[0.0357]	[2.0749]	[0.0551]	[0.0388]	[0.1508]
$\phi_1^{(2)}$	-0.5532	-0.4531	2.4282	4.2422	-0.5939
	[0.8864]	[1.6024]	[7.7652]	[8.3188]	[1.2320]
$\phi_2^{(2)}$	-0.2931	0.3597	N/A	-3.1167	-1.0015
	[0.8999]	[0.5754]		[7.6837]	[0.5265]
$\phi_3^{(2)}$	N/A	N/A	N/A	-0.5499	-0.0169
				[1.5625]	[0.4982]
$\phi_4^{(2)}$	N/A	N/A	N/A	N/A	N/A
γ	9.9316	0.0874	242.3588	164.8797	8.569
$1/\sigma^2_s$	29.1519	26.4049	34.5878	43.9756	57.6385
	[7.5488]	[0.6370]	[404.5597]	[182.9398]	[6.4255]
c	0.0223	-0.2392	0.0502	0.0100	0.1095
	[0.0247]	[0.9856]	[0.0032]	[0.0040]	[0.0138]
d	0.1291	-0.0134	0.2910	0.4340	0.1916
	[0.1056]	[0.0954]	[0.0996]	[0.2638]	[0.1852]
Ex. Kurt.	2.9427	2.2488	0.3209	1.2812	1.3121
Skewness	0.5845	0.485	0.0455	0.3778	-0.1815
J. Bera	126.5782	85.4718	1.5992	27.7469	26.2567
Pval- Q (5)	0.469	0.3196	0.0943	0.7194	0.8322
Pval- Q (10)	0.5621	0.1987	0.1121	0.3761	0.8207
Pval- Q (20)	0.4212	0.6583	0.4655	0.7435	0.1571
AIC	-7.397	-7.4726	-7.0470	-7.3281	-7.422
SSE	0.1787	0.1878	0.2908	0.1884	0.1948

* Tests indicate that the linear models are more appropriate for these two countries.

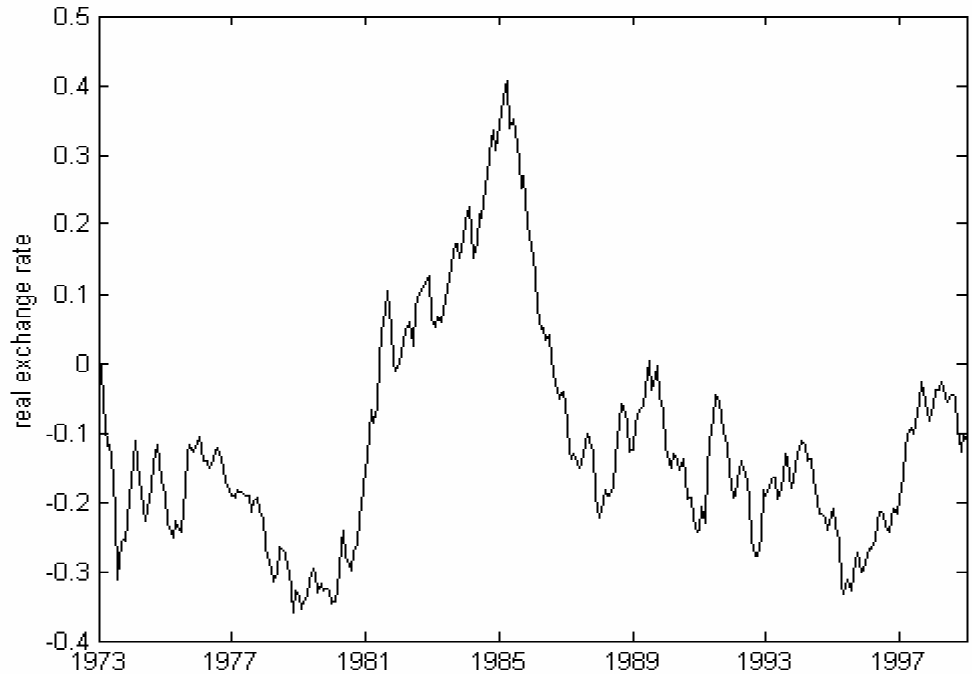

Figure 1
Real Exchange Rate Germany

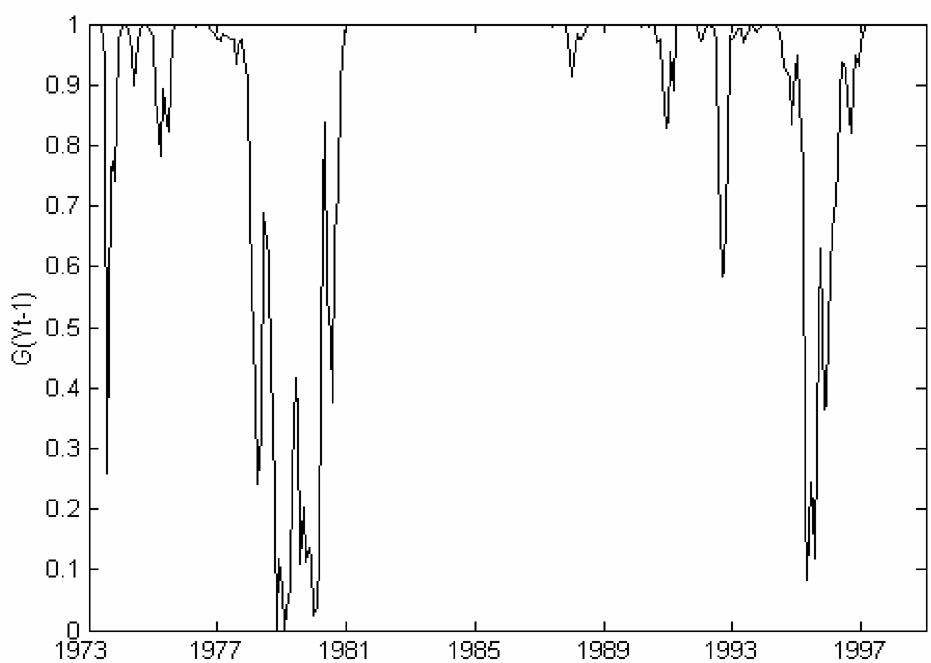

Figure 2
Estimated Transition Function Germany

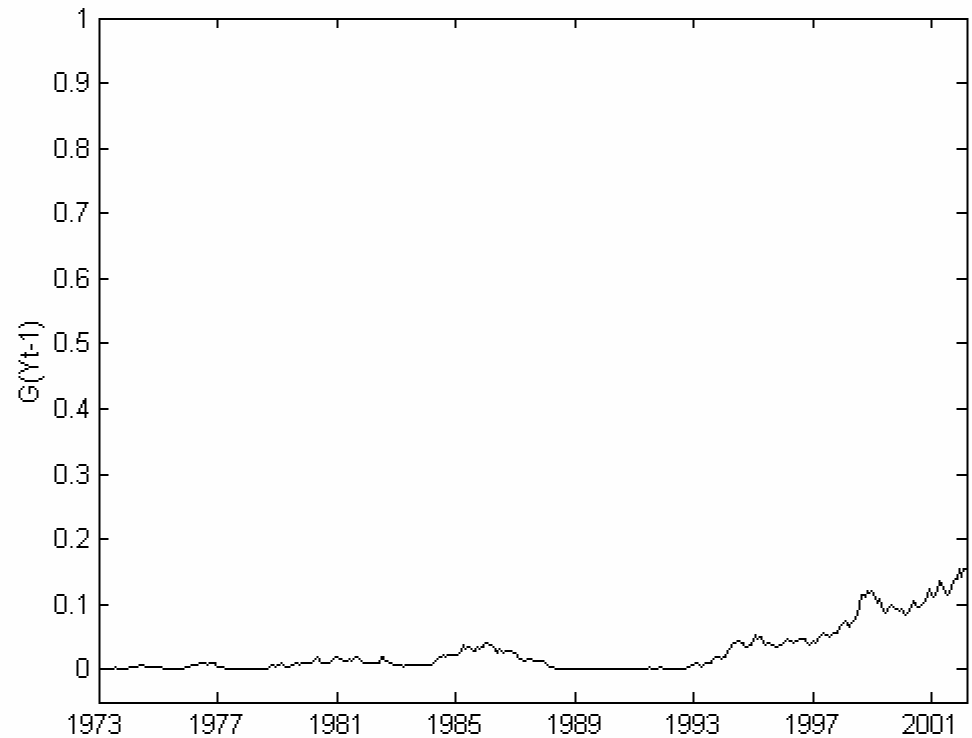

Figure 3
Estimated Transition Function Canada