Data Uncertainty in General Equilibrium*

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Abstract

In this paper, using recent empirical results regarding the statistical properties of macroeconomic data revisions, we study the effects of data revisions in a general equilibrium framework. We find that the presence of data revisions, or data uncertainty, creates a precautionary motive and causes significant changes in the decisions of agents. We also find that the model with revisions captures some aspects of the business cycle dynamics of the US data better than the benchmark model with no revisions. Using our model we measure the cost of having data revisions to be about $33 billion, $5 billion of which can be recovered by eliminating the predictability of revisions. Comparing these numbers with the budgets of the major statistical agencies in the US, we conclude that any money spent on the improvement of data collection would be well worth it.

Key Words : Neoclassical growth model, productivity, forecasting, data uncertainty

JEL Codes : C22, C53, C82, E13

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1 Introduction

Macroeconomic data are subject to continuous revisions. In an effort to generate timely data, statistical agencies produce estimates using limited information before they have access to a larger information set. The arrival of new data necessitates often substantial revisions to these initial estimates, sometimes changing the outlook of the economy. These revisions can be considered the symptoms of the underlying *data uncertainty*, the uncertainty that stems from the fact that the current state of the world cannot be perfectly observed. A rational decision-maker recognizes the existence of data uncertainty and does not take announcements at their face value. As such, his decisions will be different from those made under no data uncertainty, and there will be welfare consequences.

In this paper we analyze how decisions change in a general equilibrium model in the presence of revisions, and more importantly, we evaluate the welfare consequences of revisions. Our results indicate that revisions to macroeconomic data in the US are not only large in a statistical sense but are also economically important, significantly altering the decisions of agents and thereby leading to sizeable welfare consequences.

Revisions to macroeconomic data are well understood by economists and have been studied for decades. One of the early examples is Stekler's (1962) analysis of the usefulness of initial data releases for economic analysis. Howrey (1978) considers improving the performance of forecasting models by explicitly using the fact that preliminary data is revised. More recently Croushore and Stark (1999, 2001) show that data vintages are important and that results from empirical analysis can change, often drastically, when different vintages are used.

All the literature cited so far focus on the size of data revisions or data uncertainty. A natural next step is to analyze the statistical properties of data revisions. One of the interesting dimensions to consider is the predictability of data revisions. Mankiw et al. (1984) and Mankiw and Shapiro (1986) find that the revisions to GNP can be considered to be rational forecast errors, while revisions to money stock data are predictable. In a recent paper, Faust et al. (2003) examine the revisions to the GDP growth rates for the G-7 countries and find that while for the US revisions are slightly predictable, for Italy, Japan, and the UK, about half the variability of subsequent revisions can be accounted for by information available at the time of the preliminary announcements.

Despite all the interest in the statistical properties of data revisions, there is hardly any work analyzing their effects on decisions and their welfare implications. In this paper we aim to fill this gap by using a dynamic general equilibrium model that includes data uncertainty.

In the first part of the paper, we summarize some of the statistical properties of revisions which show the extent of data uncertainty. Our results show that over the last 30 years, data revisions have been large and, more importantly, predictable. There is also evidence that the initial announcements by the statistical agencies have been biased. We also provide some evidence from a survey of professional forecasts that shows they seem to ignore this bias and predictability. These empirical observations will be the basis of our model.

There are a number of previous studies that allow for noisy indicators in their models. In one of the earliest examples, Kydland and Prescott (1982) introduce a white noise measurement error for productivity in their model that is used to explain the business cycle dynamics of the US. However, the measurement error does not play a central role in their analysis. A recent paper by Bomfim (2001) considers the effects of white noise measurement errors on business cycle dynamics using a similar model. Both of these papers assume that revisions are unpredictable, zero-mean measurement errors. Our empirical evidence suggests that this may not be the right way to characterize data revisions.

In this paper we use a variant of the neoclassical stochastic growth model in which the agents do not observe true productivity before making their decisions. Instead, they observe an announcement about true productivity by a statistical agency. Unlike earlier papers we calibrate the revision process in the model using a measure we derive from the data, which enables us to use the model to measure the effects of data revisions.

The results of the model show important changes in the behavior of agents when they face data uncertainty. In particular, we find that the agents' optimal response to observed productivity shocks are less extreme for consumption, compared to the case with no data uncertainty. Since the agent knows that the signals he observes might be wrong, he chooses to respond to only a fraction of them. On the other hand, when faced with a positive productivity shock, the agent wants to save more than he would if he knew the true productivity shock. All these changes in decisions can be considered the results of a *precautionary motive* due to data uncertainty. The business cycle dynamics of the model also change with the introduction of data revisions. We find that in the dimensions that the model without revisions fails to match the observations from the US data, the model with revisions performs no worse. However, the model with revisions better captures some aspects of the data, such as volatility of output and labor, and contemporaneous correlation of variables with output.

The last part of the paper attempts to provide a cost-benefit analysis for data collection. Sims (1985) emphasizes the difficulties in applying traditional cost-benefit analysis to data collection. Identifying who obtains and benefits from the data is almost impossible. Moreover, measuring the value of the data to those who benefit from it is difficult, since it requires identifying what their decisions would have been in the absence of the data in question. We are able to address these issues in the framework of our model, since it allows us to measure precisely how much agents benefit from the data.

We find that the existence of revisions, or equivalently not being able to observe the current state of the world, is costly for the agents, around 0.47% of consumption every period. This cost is computed by comparing the welfare in a model with revisions, calibrated to the US data, to a model with no revisions. Due to the timeliness-accuracy trade-off, there will always be some revisions, and thus it is not realistic to expect this cost to be reduced to zero. But to the extent that the predictability of revisions reduce welfare, it may be possible to regain some of this loss by eliminating this predictability. We compute the welfare gain of such a policy to be about 0.08% of consumption. Although it is not possible to compute the cost of implementing such a policy, the monetary value of this gain is about four times the budgets of major statistical agencies in the US. This implies that any money spent on reducing the predictability of data revisions documented in this paper would be worth it. We also compute the benefit of having data announcements to be 0.03% of consumption every period, which is about twice the cost of producing the data.

The paper is organized as follows. In Section 2, we outline our empirical findings regarding the bias and predictability of revisions. In Section 3, we develop a static model of labor supply with revisions. This section shows the effect of revisions in a simple model. In Section 4, we develop a dynamic general equilibrium model with revisions. We use this model to measure the quantitative effects of revisions. Section 5 concludes. Further details about the data used, numerical solution methods, and proofs of claims and propositions are in the Appendix.

2 Statistical Properties of Data Revisions

In Aruoba (2004), we conducted an empirical analysis of data revisions an showed that for a variety of macroeconomic indicators, data revisions do not satisfy the following three desirable properties:

$$E\left(r_t^f\right) = 0 \qquad (P1)$$

$$var\left(r_t^f\right) \text{ is small} \qquad (P2)$$

$$E\left(r_t^f | I_{t+1}\right) = 0 \qquad (P3)$$

where r_t^f is the final revision of a variable and I_{t+1} is the information set at the time of the initial announcement of the variable. In the Introduction, we defined data uncertainty as the uncertainty that arises when we cannot observe the correct state of the world perfectly. Since r_t^f is the difference between the true state of the world and the announcement by the statistical agency, we can characterize data uncertainty with the variance of r_t^f. Similarly when $(P3)$ does not hold, the R^2 of the projection of r_t^f on to I_{t+1} can be used to characterize the level of predictability of revisions.

Before we turn to the models, we summarize our findings from Aruoba (2004). Our analysis covers 22 variables which include key variables such as real output growth, labor productivity growth, capacity utilization and inflation over the period 1966-2000. Our findings are:

- The means of final revisions for all variables we consider are positive and all but four of them are statistically different from zero. In other words, we have significant evidence against $(P1)$. This means that the initial announcements of statistical agencies regarding these variables are biased downwards.
- Final revisions are statistically large, measured by the noise-to-signal ratios, and therefore $(P2)$ is not supported by the data.
- In an ex post forecasting exercise, there are significant gains over a zero forecast which would be correct if $(P3)$ was true. These gains can also be exploited in real time for a number of variables using a very simple forecasting rule.
- Results from the Survey of Professional Forecasters (SPF) suggest that a vast majority of the forecasters choose not to update the initial announcement of the statistical

agency, which implies that they believe that $(P1)$, $(P2)$ and $(P3)$ are true.

In this paper, our main focus will be $(P2)$ and $(P3)$ and we will use the observation from SPF for modeling agent's perceptions of revisions in the model. We do not consider the effects of the bias of the initial announcements of the statistical agencies since doing so requires departure from rationality, which we avoid in this paper. In other words, the agents in our model will know about the non-zero mean revision and adjust the announcements they see from the statistical agency.

In the next section, we construct a simple model with revisions to highlight how revisions may effect the decision making process of agents. We then build a dynamic general equilibrium model to quantify these effects. In particular, we measure the welfare consequences of data uncertainty and the predictability of revisions we summarized in this section.

3 A Static Model of Labor Supply with Revisions

In this section we consider a static model with revisions. This model will set the stage for the notation and, more importantly, provide the intuition for the results we get in the next section, where we develop the full dynamic model. In particular, we show that when faced with uncertainty about the state of the world, the agents respond less to the signals they observe. Moreover, we show why a welfare measure that condition on the observed states may be misleading and we develop the measure of welfare that is used throughout the paper.

We consider the labor supply decision problem of an agent who faces uncertainty about the state of the world (level of productivity) solved by a social planner. In this model, true productivity, A^f, takes one of two possible values, d or $-d$, which we refer to as high and low productivity, respectively.[1] Without loss of generality, we assume both states of the world are equally likely. Instead of observing A^f directly, the agent observes A^o, which is a correct signal with probability p, where $p > 1/2$ by assumption.[2] Formally, we have $p \equiv \Pr\left(A^o = A | A^f = A\right) = \Pr\left(A^f = A | A^o = A\right)$ where the second equality follows from Bayes' Theorem.

[1] We assume that $d \in (0, 1)$, which makes the problem well-defined, as will become clear below.

[2] When $p < 1/2$, we can re-write the problem when $A^o = d$, for example, as the agent observing $A^o = -d$ with the probability of observing A^f correctly given by $1 - p$. In other words, the problem will be symmetric around $p = 1/2$ with both $p = 1$ and $p = 0$ corresponding to the case of certainty.

The social planner maximizes the agent's utility

$$\max_{c,h} E \log (c) + \log (1-h) \tag{1}$$

subject to a trivial resource constraint $c = y$ where output follows

$$y = f(h) = A^f + h \tag{2}$$

and the constraints $c \geq 0$ and $h \in [0,1]$.

Clearly, $f(h)$ is not a constant-returns-to-scale (CRS) production function. We use this production function to make sure A^f affects the decisions of the agent in a problem with a tractable solution. If we use a production function of the form $A^f h^\alpha$, for example, the objective function of the agent would be $\log\left(A^f\right) + \log(h^\alpha) + \log(1-h)$, in which case the solution to this problem would be independent of A^f. In the dynamic model we develop in the next section, the existence of capital eliminates this issue, and we use a CRS production function.

We can write the value function of the agent as follows, being explicit about the expectation operator in (1) and substituting in (2)

$$\begin{aligned} V(A^o) &= \max_h \{ \Pr\left[A^f = \ \ d | A^o\right] \log\left[\ \ d + h\right] \\ &\quad + \Pr\left[A^f = -d | A^o\right] \log\left[-d + h\right] + \log(1-h) \} \end{aligned} \tag{3}$$

where the first term is the utility of consumption when the true shock is d, the second term is the utility of consumption when the true shock is $-d$, and the last term is the disutility of labor. All probabilities are conditional on the observation A^o.

3.1 Solution with No Uncertainty

First we solve the problem in (3) when $p = 1$, which corresponds to the case of no uncertainty, as $A^f = A^o$. The problem is simplified to

$$V^B(A^o) = \max_h \{\log(A^o + h) + \log(1-h)\}$$

and the solution is given by

$$h^B(A^o) = \frac{1 - A^o}{2} \tag{4}$$

which satisfies $h \in [0,1]$ as long as $d \in (-1,1)$. We use the superscript B to reflect the "benchmark" economy, an economy with no revisions. Using (4), consumption is given by

$$c^B(A^o) = \frac{1 + A^o}{2} \tag{5}$$

which always satisfies $c \geq 0$.

In this simple model, the income effect due to the increase in A^f dominates the substitution effect.[3] The solution for h^B shows that the social planner wants to smooth consumption for the agent across the states of the world.[4] We can see this by recognizing that as A^o (and therefore A^f, in this case) goes down, the optimal labor supply goes up, stabilizing consumption in bad states of the world.

3.2 Solution with Uncertainty

When $p < 1$, the social planner considers the possibility that the observation of productivity may be wrong. The objective function becomes

$$V(d) = \max_h \{p \log(d + h) + (1-p)\log(-d+h) + \log(1-h)\} \tag{6}$$

$$V(-d) = \max_h \{p \log(-d + h) + (1-p)\log(d+h) + \log(1-h)\} \tag{7}$$

where we have taken conditional expectations based on the observed value, subject to

$$h \in [0,1] \text{ and } c \geq 0$$

which reduces to

$$h \in (d, 1]$$

since nonnegativity of consumption implies $d + h \geq 0$ and $-d + h \geq 0$.

[3] In fact, since the real wage is constant, there is no substitution effect.

[4] The term "consumption smoothing" is used in a different way than its more familiar usage in a dynamic model where it means consumption changing smoothly over time. A smooth consumption over time means a lower variance. Here, since we have a static model, consumption smoothing refers to a lower variance across the states of the world.

Ignoring the constraints for now, when $A^o = d$, the optimal labor supply, given by $h_H \equiv h^R(d)$, solves the first order condition from (6) given by

$$p\left(\frac{1}{d+h_H}\right)+(1-p)\left(\frac{1}{-d+h_H}\right)=\frac{1}{1-h_H}$$

where the superscript R is used the reflect the economy with "revisions". When $A^o = -d$, the optimal labor supply, given by $h_L \equiv h^R(-d)$, solves the first order condition from (7) given by

$$p\left(\frac{1}{-d+h_L}\right)+(1-p)\left(\frac{1}{d+h_L}\right)=\frac{1}{1-h_L}$$

Note that the social planner chooses labor supply, considering a distribution of possible consumption values, and the realized consumption depends on the true productivity shock, following (2).

These two equations, along with the appropriate second order conditions, characterize the unique solutions to each problem, as long as constraints are not violated. Below we summarize some of the important observations from this model. We provide the solution of the model and the proofs of the claims in Appendix B.

Claim 1 *When $p < 1$, we have the following relationship among optimal decisions for h*

$$h^B(d) < h^R(d) < h^R(-d) < h^B(-d)$$

This result has two implications. First, even when $p < 1$, the social planner wants to smooth consumption for the agent by choosing $h^R(d) < h^R(-d)$. Second, since there is a chance that the observed signal may be wrong, he chooses less extreme actions compared to the $p = 1$ case, i.e. $h^B(d) < h^R(d)$ and $h^R(-d) < h^B(-d)$. This can be labeled as a *precautionary motive* since recognizing the risk of incorrectly observing productivity, the social planner chooses an action not as extreme as it would have been, had he observed the true productivity. In other words, the social planner responds to the signals he observes less than he would if he knew that the signals were correct. Clearly, the precautionary motive and the consumption smoothing motive work in opposite directions, the former reducing the variance of decisions across states and the latter increasing the variance across states.

Claim 2 *When $p < 1$, the variance of the labor supply decisions over states decreases as the level of uncertainty rises, while the variance of realized consumption over states increases.*

We explained the first part of the claim above. The second part, i.e. the variance of consumption increasing, results directly from the first part. The only tool that the social planner has to smooth consumption over states is the labor supply decision. If, in the presence of other motives, he chooses a smoother labor supply, it will lead to more volatile consumption.

3.3 Welfare Comparisons

The ultimate goal of this paper is to compute the welfare consequences of data revisions. In the next section, we use a calibrated, dynamic general equilibrium model for this purpose. In this section we define the welfare measure we use in the context of the simple model and explain why using the value function $V(A^o)$ for this purpose might not be a good idea.

Claim 3 $V^R(d) < V^B(d)$ *and* $V^R(-d) > V^B(-d)$

The interpretation of this result is that when the *observed* state of the world is high, then the agent in the benchmark economy *feels* better off than his counterpart in the revision economy, but when the observed state is low, this is reversed. This result sounds counterintuitive at first since it implies that an agent with more information can be worse off. However, the critical point is that $V(.)$ has the observed state of the world as its argument, and the expectations condition on that. This is why the welfare calculations based on $V(.)$ can be misleading. Thus, we need a welfare measure that conditions on the *true* states of the world instead of the observed states.

We define a new value function, $\hat{V}^R(.)$ as

$$\hat{V}^R\left(A^f\right) = E\left\{\log\left[c^R\left(A^f, A^o\right)\right] + \log\left[1 - h^R\left(A^o\right)\right]\right\} \tag{8}$$

where the expectation is now taken with respect to A^o, conditioning on A^f. Note that when $p = 1$, $\hat{V}^R(A) = V^B(A)$, since the expectation becomes trivial as $A^f = A^o$. This suggests that when $p < 1$, we can interpret $\hat{V}^R(.)$ as the value of the objective function of the agent in the benchmark model, evaluated at the decision rules of the agent who faces revisions.

This interpretation makes welfare comparisons well-defined since we evaluate the decisions of the agents in the two models in the same *true* state of the world.

Proposition 1 *For all states of the world* A^f, $V^B\left(A^f\right) \geq \hat{V}^R\left(A^f\right)$, *and when* $p < 1$, $V^B\left(A^f\right) > \hat{V}^R\left(A^f\right)$.

The formal proof of the theorem is in Appendix B. The result is a straightforward application of Blackwell's Theorem (Blackwell, 1951, 1953) which states that an expected utility maximizer would prefer a larger information set. In our framework, Blackwell's Theorem implies that the agent in the benchmark economy, who has access to a larger information set containing the true level productivity and the signal, must do better than the agent in the revision economy who observes only the signal.

3.4 Summary of Results

We conclude this section by summarizing our findings from the simple model.

- Optimal labor supply increases as the true productivity shock declines. This can be viewed as an effort to stabilize consumption over states.
- In the presence of uncertainty about the true state of the world (data uncertainty), the social planner does not respond to the signals he observes as much as he would, had he observed the true productivity, an action which can be labeled as a precautionary behavior.
- Consumption is not chosen by the social planner but it follows from the resource constraint. Since the labor supply decision is less extreme in the presence of data uncertainty, consumption becomes more extreme.
- Welfare should be compared conditioning on the true states of the world.

In the next section, we construct a dynamic general equilibrium model with data revisions. We use this model as a measurement tool to assess the welfare implications of data revisions.

4 A Dynamic General Equilibrium Model with Revisions

In this section we develop a dynamic general equilibrium model where agents face data uncertainty. We use this model to measure the welfare implications of data revisions and to analyze how the decision making behavior of the agents in the economy changes when faced with revisions.

Our model is a variant of the neoclassical growth model, which is used extensively in the literature and can be considered the workhorse of modern macroeconomics. The model we present in this section, unlike the model in the previous section, is a dynamic model. We use a dynamic model in order to have a role for capital, an element crucial for matching some of the business cycle facts we observe in the data.[5]

4.1 Environment

4.1.1 Endowments, Preferences, and Technology

Time is discrete and continues forever. There is a continuum of identical agents whose measure is normalized to one. Since all agents are identical, we can consider the problem of a representative agent who maximizes his lifetime expected utility given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)$$

where c_t is consumption, l_t is leisure, β is the discount factor, and E_0 is the expectation operator that conditions on the information at time 0. The instantaneous utility function $u\left(.\right)$ is increasing, continuously differentiable, and strictly concave in both arguments.

The agent is endowed with one unit of time every period and an initial capital stock, given by k_0. He supplies labor, h_t, and rents his capital, k_t, to competitive firms, and in return he receives wage and rental payments, given by W_t and R_t per unit, which he takes as given. These factor payments are allocated between investment to augment his capital

[5] Specifically, the existence of capital (and thus investment) makes labor supply procyclical in contrast to a model without capital where the labor supply is, counterfactually, acyclical.

stock, i_t, and consumption. The budget constraint of the agent is thus given by

$$c_t + i_t = W_t h_t + R_t k_t$$

Capital depreciates at rate δ, and the law of motion for the agent's capital holdings is given by

$$k_{t+1} = i_t + (1 - \delta)\, k_t$$

The output in the economy is produced by many identical competitive firms who have access to a constant returns to scale (CRS) production function given by

$$Y_t = \exp\left(A_t^f\right) F\left(H_t, K_t\right)$$

where H_t and K_t are aggregate labor and capital, respectively, and Y_t is aggregate output. The function $F(.)$ is increasing and concave in both arguments. A_t^f is the natural logarithm of the technological shock, and we use the f superscript to denote the true or the *fundamental* value of the shock.[6]

4.1.2 Stochastic Environment

The only source of uncertainty in the model is technological progress which affects production. The true technology shock is assumed to be persistent and its law of motion can be approximated by the first order autoregressive process

$$A_{t+1}^f = \rho A_t^f + \varepsilon_{t+1} \tag{9}$$

where ρ is the persistence parameter. The distribution of the technological innovations, ε_t, is given by

$$\varepsilon_t \sim iid\, N\left(\frac{-\sigma_\varepsilon^2}{2\,(1+\rho)}, \sigma_\varepsilon^2\right) \tag{10}$$

where σ_ε^2 is the variance of ε_t.

We specify the process for ε_t in a slightly non-standard way (with a non-zero mean)

[6] In the rest of the paper, we refer to A_t^f as the technology shock, keeping in mind that what enters the production process is $\exp\left(A_t^f\right)$.

to make sure that changes in σ_ε^2 are mean-preserving spreads, i.e. they do not affect the mean of $\exp\left(A_t^f\right)$.[7] This is important for our analysis since we change some of the variance parameters in our experiments, and we want to make sure we are not distorting the results by changing means.[8]

So far, the model is exactly like the standard growth model. The crucial difference comes from what the agents get to observe when they make their decisions. As in the simple model in the previous section, the agents observe an announcement by a statistical agency about today's productivity, A_t^o, but they do not observe the level of true productivity before they make their decisions. Therefore we add a new layer of uncertainty, the uncertainty about today's productivity, in addition to the uncertainty about future productivity, which is standard.

The announcement and the true productivity shock are related by

$$A_t^f = A_t^o + r_t \tag{11}$$

where r_t is the implied revision. We assume a simple process for r_t given by

$$r_t \sim iid\, N\left(\mu - \frac{\sigma_r^2}{2}, \sigma_r^2\right)$$

where σ_r^2 is the variance of r_t and μ governs the mean of the revision. Similar to the mean of ε_t discussed above, we assume that even when μ is zero, r_t has a non-zero mean. This will ensure that when we change σ_r^2 in our experiments, the mean of $\exp(r_t)$ does not change.[9] Even though we allow for an unconditional mean of revisions, this will be inconsequential as far as agents' decisions are concerned. As long as agents are fully rational, which is the case in this model, they will adjust all the announcements they see by the unconditional mean of revisions and their decisions will be unaffected.

We make two modeling choices regarding the revision process. The first is the normality

[7] Rothschild and Stiglitz (1970, 1971)

[8] See Cho and Cooley (2003) for the proof that when the distribution of ε_t is specified as in (10), the mean of $\exp\left(A_t^f\right)$ is equal to unity and therefore changing σ_ε^2 is a mean-preserving spread.

[9] We are concerned about the mean of $\exp(r_t)$ since we can decompose $\exp\left(A_t^f\right)$ as $\exp(A_t^o)\exp(r_t)$. When we compute the mean of $\exp(r_t)$ with this specification, it is equal to $\exp(\mu)$ which is very close to $1+\mu$, when μ is small.

assumption which will be justified when we calibrate the revision process in Section 4.3.2 using real data. The second choice is that we model revisions as independent processes across time. This assumption seems to contradict the empirical evidence regarding the persistence and predictability of revisions we presented in Aruoba (2004). However, we can justify this by the following argument. Suppose $r_t = \hat{r}_t + \pi_t$, where $\hat{r}_t$ and π_t are the predictable and unpredictable parts of the revision, respectively, and $\hat{r}_t$ comes from a forecasting equation. The representative agent is ultimately interested in forecasting A_t^f, and this decomposition of r_t is irrelevant for him. We can think of a new announcement $\tilde{A}_t^o = A_t^o + \hat{r}_t$ and the new implied revision is π_t, which is by definition an independent process. Therefore, we assume that the announcement A_t^o contains all the information that can be extracted from the information in the economy (including the information that might help to forecast revisions), and the difference between A_t^f and A_t^o is orthogonal to everything in the economy. In other words, the agent does not try to forecast the revisions since he thinks that he is facing *iid* revisions. This setup is in line with the evidence from SPF that we summarized in Section 2, which suggests that agents believe they cannot forecast revisions.[10]

When $\mu = 0$ and $\sigma_r^2 = 0$, we have $r_t = 0$ for all periods. This means, trivially, that $A_t^o = A_t^f$. We label this model as the *benchmark model* since in this case today's productivity is no longer uncertain, and we are back in the standard neoclassical growth model.

4.1.3 The Filtering Problem of the Agent

Every period the agent faces a non-trivial filtering problem. Given A_t^o and the whole history of past realizations of the true shock $\left\{..., A_{t-2}^f, A_{t-1}^f\right\}$, he has to forecast the true technology shock for the current and next period. Formally, he needs to compute $E\left(A_t^f | I_t\right)$ and $E\left(A_{t+1}^f | I_t\right)$ where $I_t = \left\{..., A_{t-2}^f, A_{t-1}^f, A_t^o\right\}$. We assume that the agent uses the Kalman Filter to find the best linear forecast of these objects. Specifically, at period t he considers

[10]Below, when we want to analyze the case where revisions are predictable, we use a similar argument that relates predictability to change in variance of r_t.

the state space system

$$\text{Measurement Equation} \quad : \quad \begin{pmatrix} A^f_{t-1} \\ A^o_t \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} A^f_t \\ A^f_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ -r_t \end{pmatrix}$$
$$\text{Transition Equation} \quad : \quad \begin{pmatrix} A^f_{t+1} \\ A^f_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} A^f_t \\ A^f_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1} \\ 0 \end{pmatrix}$$

where we only include A^f_{t-1} and A^o_t in the state vector due to the Markovian structure of the true technology shock. The optimal forecast of A^f_t and A^f_{t+1} can be obtained using the Kalman Filter and they are given by[11]

$$\begin{aligned} E\left(A^f_t | I_t\right) &= w\rho A^f_{t-1} + (1-w) A^o_t \\ E\left(A^f_{t+1} | I_t\right) &= \rho E\left(A^f_t | I_t\right) \end{aligned}$$

where

$$w = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_\varepsilon^2} \tag{12}$$

To ease notation, we define the filtered signal as

$$\hat{A}_t = w\rho A^f_{t-1} + (1-w) A^o_t$$

and use it as the state variable of the agent since it gives the agent all the information he needs to solve his problem. The errors from the optimal forecasts are given by

$$\begin{aligned} \eta^1_t &= A^f_t - \hat{A}_t = w\varepsilon_t + (1-w) r_t \\ \eta^2_t &= A^f_{t+1} - \rho\hat{A}_t = \rho\left[w\varepsilon_t + (1-w) r_t\right] + \varepsilon_{t+1} = \rho\eta^1_t + \varepsilon_{t+1} \end{aligned}$$

[11] Since the Kalman Filter gives the best linear forecast, we can also solve the following simpler forecast combination problem to find the solution:

$$\begin{aligned} & \min_w var\left[w\left(A^f_t - \rho A^f_{t-1}\right) + (1-w)\left(A^f_t - A^o_t\right)\right] \\ = & \min_w var\left[w\varepsilon_t + (1-w) r_t\right] \end{aligned}$$

since the forecast of the Kalman Filter will essentially be a linear combination of the two forecasts using A^f_{t-1} and A^o_t.

and the law of motion of $\hat{A}_t$ is given by

$$\hat{A}_{t+1} = \rho \hat{A}_t + \eta_t^3$$

where

$$\eta_t^3 = w\rho\varepsilon_t + (1-w)\left[\rho r_t - r_{t+1} + \varepsilon_{t+1}\right]$$

At first glance η_t^3 seems to be autocorrelated due to the existence of multiple lags of ε_t and r_t. However, η_t^3 is an *iid* process since

$$cov\left(\eta_t^3, \eta_{t-k}^3\right) = \begin{cases} w(1-w)\rho\sigma_\varepsilon^2 - (1-w)^2 \rho\sigma_r^2 = 0 & \text{for } k = 1 \\ 0 & \text{for } k > 1 \end{cases}$$

where the results for $k = 1$ follows by substituting the definition of w from (12).

4.1.4 Timing

In the standard growth model, the agents begin the period by observing the productivity shock for the period and use this information along with other state variables to choose labor, investment, and consumption. Since all decisions are made using the same information set, the ordering of the decisions is irrelevant.

The timing of events for the present paper, shown in Figure 4.1, can be summarized as follows:

- The agent starts period t knowing k_t, which he chose last period.
- The statistical agency collects all the information that helps it to forecast A_t^f and makes an announcement, given by A_t^o. The information set it uses, I_t, contains last period's true productivity shock, A_{t-1}^f, among other things.[12]
- Having observed k_t, A_t^o and A_{t-1}^f, the agent chooses his labor supply and his consumption.

[12] We do not model explicitly the way the statistical agency makes its forecasts. We assume that since its information set contains A_{t-1}^f, the forecast error cannot have a variance greater than σ_ε^2, as explained in the previous section.

- Production takes place and the agent receives factor payments. This reveals the true level of productivity.

- Investment follows from the agent's budget constraint.

Note that when we let the statistical agency have A_t^f in its information set, then $A_t^o = A_t^f$, and we are in the benchmark model.

Clearly, once we let a subset of the decisions the agents make depend only on the signal and not on the true level of productivity, there are several variations in timing that one could adopt. The only constraint is that we must let either consumption or investment be the residual to maintain budget balance. Also, having the labor supply decision at the beginning of the period is more sensible. Subject to these two considerations, there are two other timings one can adopt. We can switch the places of consumption and investment or we can have both decisions made at the end of the period, after A_t^f is revealed. The first of these alternatives, the timing when consumption is the residual decision, is not appealing since the optimal decision rules yield a less variable capital path which makes the volatility of investment relative to volatility of output fall to counterfactual levels. On the other hand, when we postpone the consumption and investment decisions to the end of the period, the intertemporal margin is not affected by revisions. This, however, is an artifact of the representative agent framework since when the agent receives his factor payments, he can perfectly infer what the true productivity is. In the real world, due to idiosyncratic uncertainty, this is not the case and thus adopting this timing would be unrealistic. The timing we use seems more natural, since it closely resembles the process an individual agent uses where he chooses consumption and how much to work, and adjusts his savings according to the realization of his earnings.

One restriction of the current framework is that the uncertainty about today's productivity is resolved at the end of today when output is realized. However, since the decisions of the agents for the current period have already been made, this uncertainty will still have welfare consequences. Modifying the model in such a way that extends the length of uncertainty about today's productivity would undoubtedly magnify any welfare consequences we find in this paper.

4.1.5 Optimization Problems

Firms' Problem

Because of the CRS assumption, all the firms will have zero profits, and the number of firms becomes indeterminate. Therefore, without loss of generality we consider the problem of a representative firm. The firm solves a static problem where it maximizes expected profits by choosing factor demands every period. This model differs from the benchmark model in that the firm does not observe the level of true productivity before making its decisions. Instead, like the agents, it observes the announcement of the statistical agency and obtains $\hat{A}$ by running the Kalman Filter. The actual factor payments, on the other hand, depend on the true level of productivity. Hence, the firm takes the factor price functions, $W\left(K,\hat{A},\eta^1\right)$ and $R\left(K,\hat{A},\eta^1\right)$, as given. The solution to the static profit maximization problem implies that the actual wage and rental payments by the firm satisfy

$$W = \exp\left(\hat{A}+\eta^1\right)F_1(H,K) \qquad (13)$$

$$R = \exp\left(\hat{A}+\eta^1\right)F_2(H,K) \qquad (14)$$

which simply set the marginal productivity of the factors equal to their prices.

By Euler's Theorem, total output is exhausted by the factor payments made to the agents, i.e.

$$Y = WH + RK$$

Agents' Problem

The problem that the representative agent faces is a dynamic one in which he forecasts future prices to make his decisions. Moreover, since he does not observe the current level of productivity, he does not know the true wage and rental payments he will receive in return for his factor supply. Instead, he considers a distribution of wage and rental payments, $W\left(K,A^f\right)$ and $R\left(K,A^f\right)$, that depend on the unobserved A^f. Following the timing convention described in Section 4.1.4, the agent's consumption and labor supply decisions are made observing only the announcement by the statistical agency, and the decision rules are denoted by $c\left(k,K,\hat{A}\right)$ and $h\left(k,K,\hat{A}\right)$, respectively. Since the investment decision takes place after true productivity, or equivalently η^1, is realized, the decision rule for capital tomorrow is given by $k\left(k, K, \hat{A}, \eta^1\right)$. Combining all these ingredients and omitting the arguments of the functions, the representative agent's recursive problem can be written as follows

$$V\left(k, K, \hat{A}\right) = \max_{c,h,k'} \quad E\left\{u\left[c, 1-h\right]\right\} + \beta E V\left[k', K', \hat{A}'\right] \tag{15}$$

$$\begin{aligned} \text{subject to} \quad & k' = Wh + Rk + (1-\delta)\,k - c \\ \text{given} \quad & \hat{A}' = \rho \hat{A} + \eta^3 \\ & K' = K\left(K, \hat{A}, \eta^1\right) \end{aligned}$$

where he takes the prices $W(.)$ and $R(.)$, the law of motion for aggregate capital, and initial capital stocks k_0 and K_0 as given. The expectation operator E integrates out r, r', and ε', given the filtered signal $\hat{A}$ since η^1, η^2 and η^3 all depend on those three stochastic objects.[13]

4.2 Equilibrium

We solve for the equilibrium of this economy in a standard way. First, we define a recursive competitive equilibrium (RCE) in this economy. Next, we invoke the equivalence of the allocations from a RCE and those from the solution to the social planner's problem (SPP). Finally, we solve the SPP to obtain the decision rules.

We can define a RCE in this economy as follows.

Definition 1 ***(Recursive Competitive Equilibrium)*** *A Recursive Competitive Equilibrium for this economy is a list of functions,* $V\left(k, K, \hat{A}\right)$, $c\left(k, K, \hat{A}\right)$, $h\left(k, K, \hat{A}\right)$, $k\left(k, K, \hat{A}, \eta^1\right)$, $W\left(K, \hat{A}, \eta^1\right)$, $R\left(K, \hat{A}, \eta^1\right)$, $K\left(K, \hat{A}, \eta^1\right)$, *and* $H\left(K, \hat{A}\right)$ *such that*

- *(Firms Optimize)* $W(.)$, $R(.)$, *and* $H(.)$ *satisfy the first order conditions of the firm, (13) and (14).*

- *(Agents Optimize) Given* $W(.)$, $R(.)$, *and* $K(.)$; $V\left(k, K, \hat{A}\right)$ *solves the Bellman's equation in (15) with associated decision rules* $h(.)$, $c(.)$, *and* $k(.)$.

- *(Consistency)* $h\left(K, K, \hat{A}\right) = H\left(K, \hat{A}\right)$ *and* $k\left(K, K, \hat{A}, \eta^1\right) = K\left(K, \hat{A}, \eta^1\right)$.

[13] Standard Inada conditions on $u(.)$ guarantee that the solution to the agent's problem will be interior, and we do not explicitly put constraints on the decision rules.

In the absence of externalities (such as consumption externalities) and frictions (such as taxes), the allocations from the problem of the social planner, who places equal weights on each agent, are identical to those from the RCE. As shown in Prescott and Mehra (1980), both welfare theorems hold in the current framework, that is, all competitive equilibria are Pareto Optimal, and all Pareto Optimal allocations can be supported as competitive equilibria. Therefore, we can solve for the allocations of the social planner's problem to characterize the allocations in the competitive equilibrium. Note that the results in Prescott and Mehra (1980) require that agents have rational expectations and the existence of a well-defined law of motion for the stochastic variables in the model. These conditions are satisfied in our model.

We can write the SPP recursively as follows

$$\begin{aligned} V\left(k, \hat{A}\right) = \max_{c,h,k'} \quad & E\left\{u\left[c, 1-h\right]\right\} + \beta E V\left[k', \hat{A}'\right] \\ \text{subject to} \quad & k' = y + (1-\delta)\,k - c \\ \text{given} \quad & \hat{A}' = \rho \hat{A} + \eta^3 \end{aligned} \tag{16}$$

where the social planner maximizes the representative agent's utility subject to the resource constraint of the economy, taking as given the initial capital stock, k_0.[14] Combining the first order conditions and the envelope conditions from (16), we obtain the following Euler equations which, along with the budget constraint, characterize the solution of the social planner's problem

$$E\left[u_1(c, 1-h)\exp(\hat{A}+\eta^1)F_1(h,k)\right] = u_2(c, 1-h) \tag{17}$$

$$u_1(c, 1-h) = \beta E\left[u_1\left(c', 1-h'\right)\left\{\exp(\rho\hat{A}+\eta^2)F_2\left(h', k'\right) + (1-\delta)\right\}\right] \tag{18}$$

$$c = \exp\left(\hat{A}+\eta^1\right) F\left(h, k\right) + (1-\delta)\,k - k' \tag{19}$$

Equations (17) and (18) differ from the Euler Equations from the standard growth model by the expectations operator in front of current period terms, on the left hand side of the intra-temporal Euler Equation.[15] This is the result of the new layer of uncertainty generated

[14] Strictly speaking, the functions V, c, h, and k' in (16) are different from those in (15). Due to the equivalence of the allocations in the two problems, using the same function names is innocuous.

[15] See Appendix C.1 where we write these equations using all the arguments of the functions, which clarifies how the expectations are computed.

by revisions.

We use (17), (18), and (19) for finding $c(.)$, $h(.)$, and $k(.)$, the decision rules of the agent. After solving for the allocations, we use the (13) and (14) to decentralize the allocations, i.e. to find the prices that support these allocations as a RCE.

Before turning to solving the dynamic model, we characterize the deterministic steady state of the economy. Since the addition of revisions only changes the stochastic structure of the model, the deterministic steady state of the model is independent of revisions and is identical to the deterministic steady state of the benchmark model. Setting all stochastic variables equal to their means, we obtain the following three equations which characterize the deterministic steady state, $\left(\bar{k}, \bar{c}, \bar{h}\right)$.

$$u_1(\bar{c}, 1-\bar{h})F_1(\bar{h}, \bar{k}) = u_2(\bar{c}, 1-\bar{h}) \tag{20}$$

$$1 = \beta\left[F_2\left(\bar{h}, \bar{k}\right) + (1-\delta)\right] \tag{21}$$

$$\bar{c} = F\left(\bar{h}, \bar{k}\right) - \delta\bar{k} \tag{22}$$

4.3 Functional Forms and Calibration

4.3.1 Preference and Technology Parameters

In order to solve the model, we choose the following functional forms. For the instantaneous utility function, we use a Cobb-Douglas aggregator in a constant relative risk aversion (CRRA) utility function, given by

$$u(c, 1-h) = \frac{\left[c^{\tau}(1-h)^{1-\tau}\right]^{1-\sigma}}{1-\sigma} \tag{23}$$

This utility function is commonly used in the literature since it belongs to a class of utility functions consistent with balanced growth. With this specification the elasticity of substitution of consumption and leisure is unity, and risk aversion is characterized by the parameters τ and σ.[16] For the production function we use a Cobb-Douglas specification

$$F(h, k) = sh^{\alpha}k^{1-\alpha} \tag{24}$$

[16]The coefficient of risk aversion with respect to consumption using the Arrow-Pratt measure of risk aversion is $1-\tau(1-\sigma)$.

where the parameter s is a scale parameter used to normalize the steady state of capital to unity.

We choose a quarterly frequency for our model since most of the data we used in the first part of the paper to document the properties of revisions were quarterly. Moreover, as will be explained in detail below, the measure of the revisions to productivity we use for calibrating the parameters for the process of r_t is only available in quarterly frequency.

In the spirit of Cooley and Prescott (1995), the parameters of the model are calibrated such that the model matches some of the long run characteristics of the US economy. We use the following annualized calibration targets obtained from the US data for the period 1965-2000:[17]

- Steady state investment-capital ratio of 0.04.
- Steady state capital-output ratio of 3.55.
- People work for about a third of their discretionary time.
- Steady state real interest rate of 4%.

These targets, imposed on the steady state conditions given by (20), (21), and (22) along with the functional forms in (23) and (24), fix the values of β, τ, α, and δ.[18]

The value of σ is set equal to 2, which gives a coefficient of relative risk aversion with respect to consumption of 1.37.

4.3.2 Stochastic Processes

To complete the calibration process, we have to fix the values of the parameters that govern the stochastic processes, $\rho, \sigma_\varepsilon, \mu$, and σ_r. For the autoregressive process governing the true technology process, we follow King and Rebello (1999). First, using (24) we obtain a measure of the Solow residual. This gives

$$A_t^f = \log\left(y_t^f\right) - \alpha \log\left(h_t^f\right) - (1-\alpha)\log\left(k_t^f\right) \tag{25}$$

[17]We follow the discussion in Cooley and Prescott (1995) to have a "consistent set of measurements to align the model economy with the data." In particular, our measure of capital stock includes private capital stock (non-residential land, fixed private capital, and inventories), private durables consumption stock, and government capital stock. We use average weekly hours worked from the Establishment Survey as a measure of labor supply.

[18]Details are provided in Appendix C.2.

where the superscript of f denotes the final value of a variable. As noted by many authors, this measure is non-stationary as it is growing over time. The standard growth model implies that all variables (except for hours) grow at the same constant rate on average, implying a linear trend in logarithms. Therefore, we run an AR(1) regression on A_t^f with an additional linear trend in the regression.[19] The detrended Solow residual is shown in Figure 4.3.1, and it is consistent with the productivity slowdown of late 1970s, the recession of early 1980s, and the higher-than-average growth productivity in the second half of 1990s. The AR(1) regression results are given below

$$A_t^f = \underset{(0.0463)}{0.0725} + \underset{(0.0001)}{0.0002}t + \underset{(0.023)}{0.966}A_{t-1}^f + \varepsilon_t, \quad R^2 = 0.999, \quad \sigma_\varepsilon = 0.0081$$

Next, we turn to the revisions to the Solow residual. We can write an *observed* version of (25) as

$$A_t^o = \log(y_t^o) - \alpha \log(h_t^o) - (1-\alpha)\log(k_t^o)$$

which we can use to obtain the revisions to A_t, r_t^A as

$$r_t^A = \log\left(\frac{y_t^f}{y_t^o}\right) - \alpha \log\left(\frac{h_t^f}{h_t^o}\right) - (1-\alpha)\log\left(\frac{k_t^f}{k_t^o}\right)$$

The actual US data for labor supply are based on surveys and do not get revised by definition.[20] Using this fact ($h_t^f = h_t^o$), we get

$$r_t^A = \log\left(1 + \frac{r_t^y}{y_t^o}\right) - (1-\alpha)\log\left(\frac{r_t^k}{k_t^o}\right)$$

where we express the revision to the Solow residual in terms of revisions of output and capital relative to their initial announcements. We already have the revisions of output from our empirical analysis. In Appendix C.2 we create the revision to capital using the perpetual inventory method.[21] The resulting revisions to the Solow residual are given in Figure 4.3.1.

[19] This is, of course, equivalent to detrending A_t^f using a linear trend and using the residuals from this regression in an AR(1) regression. The detrending regression implies a annualized growth rate of 2.1%.

[20] By this we mean that the same survey is not repeated over the same sample in the same period. There are, of course, different measures of labor supply based on different surveys. The only source of labor-based real-time data we know is the Philadelphia Fed Real-Time Dataset's unemployment rate variable. The revisions to unemployment are very small and follow changes in seasonal weights very closely.

[21] While there are some significant revisions to investment, since the capital stock is much larger, the

Table 4.3 - Calibrated Parameters

β	σ	τ	α	s	δ	ρ	σ_ε	μ	σ_r
0.99	2	0.371	0.700	0.153	0.011	0.966	0.0081	0.0049	0.0074

The mean and standard deviation for the revision to the Solow residual are 0.0049 and 0.0074, respectively. Also a standard statistical test fails to reject normality for the revision to the Solow residual, justifying our choice of the normal distribution for r_t in our model.

The values of the calibrated parameters are given in the Table 4.3.

As we argued in Section 4.1.2, we can use the *iid* revisions in our model to analyze the effects of predictability of revisions by changing their variance. If the revisions are predictable, then eliminating this predictability amounts to reducing the variance of the revisions, since the predictable part will be incorporated into the announcement of the statistical agency. To parameterize the case where revisions are not predictable, we conduct an ex-post forecasting exercise for the Solow residual similar to the one discussed in Aruoba(2004). The forecasting equation has an R^2 of 0.14, and this yields a standard deviation for the residuals (which are the revisions we will consider in the no predictability case) of 0.0064. The Solow residual and its forecast from this exercise are shown in Figure 4.3.2. Since the unpredictable part of the revisions come from the residuals of a regression, they will by definition have a zero mean. So, when we analyze the case of no predictability, we use the parameters $\mu = 0$ and $\sigma_r = 0.0064$.

4.4 Solution Method

There is no known analytical solution to the problem given in (16). In order to solve for the equilibrium, we need to approximate the solution. There are many ways of approaching this problem, ranging from linear or higher order approximations to global solution methods, which approximate the decision rules over the whole domain. While the solution method used in similar studies generally depends on the tastes of the authors, sometimes, as in our case, a certain class of solution methods is necessitated by the nature of the problem. The goal of this paper is computing welfare consequences due to the changes in the variances of the stochastic variables. It is well known that linear solution methods display *certainty*

relative importance of the revisions to capital is very small.

equivalence, that is, the solution is independent of the variances of the stochastic variables. This makes using linear methods for this paper unfeasible.

We choose to use the Weighted Residuals Method (or Projection Method) using Chebyshev Polynomials and Orthogonal Collocation to solve the functional equations (17) and (18) in the unknown functions $h(.)$ and $c(.)$. Aruoba et al. (2003) find that for the benchmark problem in this paper, this method performs very well in terms of the size of the approximation error. We provide the details of the process in Appendix C.4.

4.5 Effects of Data Revisions on the Equilibrium

Our first step is to solve for the equilibrium in the model with revisions and compare it with the equilibrium of the benchmark economy. In this section, we highlight the differences in terms of the decision rules and business cycle implications. In the next section, we turn to the question of welfare.

4.5.1 Decision Rules

In Section 3 we described the precautionary motive due to data uncertainty. In the presence of data uncertainty, we observed that the responses of the social planner to the signals he observes were less extreme compared to his responses under no uncertainty. This is a sign of a precautionary motive, since the social planner considers the possibility of observing a wrong signal when making his decisions.

We turn to exploring the effect of data uncertainty on the decision rules in this model. Figure 4.5.1 shows the two decision rules chosen by the agent, $c(.)$ and $h(.)$, along with the implied decisions for investment for the benchmark model and the model with revisions.[22] Since these objects are multi-dimensional objects, we provide a cross section at $k = 1$, its deterministic steady state. We plot the decisions versus $\hat{A}_t - E\left(\hat{A}_t\right)$ so that decision rules for the two models are comparable.

Let us first focus on the decision rules of the agent in the benchmark economy. Unlike the simple model, labor supply is positively related to the technology shock, which means the substitution effect of higher productivity (and therefore wages) dominates the income

[22] Investment follows from the budget constraint (19) as $i = \exp(A^o + r) F(h, k) - c$.

effect.[23] By the same token, the consumption decision rule is also positively related to the technology shock. The difference is due to the dynamic structure of the model and to the existence of investment, which is the link between two periods. When the agent observes a positive technology shock, there are two substitution effects at work. First, the *intratemporal* substitution effect makes the agent work more and consume more compared to a zero shock since a positive in technology shock makes leisure more expensive and consumption cheaper. Second, the *intertemporal* substitution makes the agent work more and invest more. The agent realizes the mean-reverting structure of the stochastic process and he knows that tomorrow's technology shock will be less than today's value (in expectation). Therefore, in order to exploit this better production opportunity he wants to work more and save for tomorrow. Working against these substitution effects is the income effect which makes the agent work less and consume more since he is richer with increased productivity. Clearly, for the current parametrization of the model the two substitution effects dominate the income effect and all three decisions are positively related to productivity.

Comparing the decision rules for the benchmark model and the model with revisions, we can identify the effect of revisions. In the presence of revisions, the agent chooses a labor supply schedule which is more extreme and a consumption schedule which is less extreme than his choices in the benchmark model. These two decisions yield an investment schedule which is more extreme. Two of these findings are in line with the intuition we obtained in the simple model, which suggested that as a result of a precautionary motive the agents choose less extreme decision rules when they are faced with data uncertainty and this leads to more extreme decisions in their residual decisions. In this model the changes in consumption and investment decisions are in line with this intuition. However, we see that labor supply decisions become more extreme. This can be explained by the increased effect of intertemporal substitution for low enough level of uncertainty. In other words, the intertemporal substitution explained above becomes even stronger for low enough σ_r and this counteracts the effects of the precautionary motive.

To understand this further, consider Figure 4.5.2 where we plot the consumption and labor supply decisions of the agent at $k = 1$ and $\hat{A}_t - E\left(\hat{A}_t\right) = 0.05$ as functions of the data uncertainty in the economy given by σ_r. First, we see that consumption decisions for any σ_r are always lower than the benchmark case and similarly the labor decisions are always

[23] The substitution effect was zero in the simple model.

higher than the benchmark case. The interpretation of these results with respect to the slopes of the decision rules in Figure 4.5.1 is that labor supply is always more extreme and consumption is always less extreme as we increase the level of data uncertainty. Moreover, for low levels of σ_r the difference between the benchmark and the revision models are more extreme.

In order to explain this effect, we turn to the conditional distributions of technology shock today and tomorrow, given the observation of the filtered signal today. As we argued above, in the benchmark model (when $\sigma_r = 0$) a positive productivity shock meant that today is more productive than tomorrow and the agent chooses to increase his labor supply to capture this increased production opportunity. Figure 4.5.3 shows the conditional distributions of A_t^f and A_{t+1}^f given $\hat{A}_t = 0.05$ for various values of σ_r. These distributions are characterized by the first two moments since they are both normal distributions and these moments are given by

$$\begin{aligned}
E\left(A_t^f|\hat{A}_t\right) &= \hat{A}_t \\
var\left(A_t^f|\hat{A}_t\right) &= \frac{\sigma_r^2\sigma_\varepsilon^2}{\sigma_r^2+\sigma_\varepsilon^2} \\
E\left(A_{t+1}^f|\hat{A}_t\right) &= \rho\hat{A}_t \\
var\left(A_{t+1}^f|\hat{A}_t\right) &= \rho\left(\frac{\sigma_r^2\sigma_\varepsilon^2}{\sigma_r^2+\sigma_\varepsilon^2}\right)+\sigma_\varepsilon^2
\end{aligned}$$

We see that the conditional means of the two distributions are unchanged and the conditional variances increase as σ_r changes. For $\sigma_r = 0$, the conditional distribution of A_t^f is degenerate at 0.05 and we see that a larger mass of the conditional distribution of A_{t+1}^f lies to the left of 0.05. As we increase σ_r, since the conditional variance of A_t^f increases much faster for small values of σ_r, we observe a bigger mass in the conditional distribution of A_t^f compared to the conditional distribution of A_{t+1}^f. This causes today look even more productive than tomorrow and makes the labor supply decision rule steeper and consumption decision rule less steep since the agent finds it more beneficial to produce and save more today. As we further increase σ_r, the two conditional distributions increase at the same rate and the difference between today and tomorrow reduces. To sum up, due to the dynamic structure of the model the precautionary motive is dominated by the agent's concern for tomorrow and since due to the mean-reverting structure of the technology shock today is always better than tomorrow in expectations he chooses a more extreme labor supply schedule and a less extreme consumption schedule.[24]

4.5.2 Business Cycle Statistics

We compute some statistics to see how the business cycle dynamics are affected by the existence of revisions. We simulate the model 100,000 times for 144 periods and obtain the time series for some key variables. Next, we take their natural logarithm and apply the Hodrick-Prescott filter. We report the averages of the statistics we obtain from the simulations along with the corresponding statistics from the US data[25] for the period 1965-2000 in Table 4.5.[26]

The first two columns report the volatility of the variables in level and relative to the volatility of output. The results indicate that the volatility of output in the benchmark model falls short of the volatility in the data by about 9%. On the other hand, output in the model with revisions is 7% more volatile than in the benchmark model. This result suggests that the extra uncertainty introduced by the revisions increases output volatility to a level more consistent with the data. Both models fail to capture the consumption volatility in the data, as the relative volatility of consumption implied by the models are about half of what the data indicates. On the other hand, both models imply a higher relative volatility of investment than the data, with the model with revisions generating more volatility benchmark model. Similar to consumption, both models produce a labor supply significantly less volatile than what the data shows. Nevertheless, labor in the model with revisions is about 40% more volatile than in the benchmark model, which comes close to the data. Wages in both models fluctuate more than what the data suggests, and there is only a small reduction in the model with revisions. When we look at the volatility of rental rate, both models fail to capture the volatility in the data. We can conclude that the model with revisions is no worse than the benchmark model in matching the volatility of the data for all the variables we consider,

[24] All the conclusions in this argument will change directions when the observed productivity shock is negative. This ensures the suggested changes in thes slope of the decision rules.

[25] See Appendix A for details about the data used in this section.

[26] We report the standard errors of the statistics that we compute using US data. The statistics computed using the model are based on the means over a large number of simulations, and as such, the estimation error of means is negligible.

and it performs better for output and labor.

The next column reports the persistence of the variables, measured by the first order autocorrelation coefficients. All variables are persistent which follows from the persistent structure of the stochastic technology process in the model. We see that, compared to the data, both models produce significantly less persistent variables. Moreover, the existence of data uncertainty lowers the persistence of consumption and labor in the model with revisions compared to the benchmark model, since these decisions depend on the announcement rather than the true productivity shock. The persistence of other variables remains roughly the same.

The rest of the table reports the correlations of the variables with output in different leads and lags. The data shows high contemporaneous correlation and slightly less but still significant correlation in all leads and lags except for wages and the returns on capital. The latter variables have a weak contemporaneous correlation with output, which both models fail to capture. The benchmark model produces variables which are almost perfectly correlated with output contemporaneously, but this correlation dies out within a year in either direction. The same conclusion about the leads and lags is true for the model with revisions, but the contemporaneous correlations are less than perfect. This result is due to the existence of a second source variation in the economy, the revisions, which affect all variables directly except output, which is indirectly affected. This makes variables less correlated with output, and the model with revisions comes closer to matching the data in this dimension.

Overall, the results from this section indicate that in the dimensions that the benchmark model fails to match the data, the model with revisions does no worse and comes closer to matching the data in some dimensions, such as volatility of output and labor and contemporaneous correlation of variables with output.

Before concluding this section, we want to contrast our results with those of Bomfim (2001), who computes the business cycle implications of a noisy indicator. His model is very similar to Kydland and Prescott (1982) except for the strategic complementarity he introduces in the production function. He finds that the volatility of output increases when the noise in the announcements is removed. This result is at odds with our finding that volatility of output decreases when we go from the model with revisions to the benchmark model. While the existence of strategic complementarity in his model might be the reason for the difference, we think the difference is mostly due to the different calibration strategies

Table 4.5 - Business Cycle Statistics

	Volatility (STD %)	Relative Volatility	Persistence	Correlation with y_t $x(t-4)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$
Model with Revisions								
y	1.56	1.00	0.66	0.07	0.66	1.00	0.66	0.07
c	0.54	0.34	0.48	-0.05	0.49	0.91	0.59	0.18
i	7.54	4.77	0.69	0.11	0.68	0.98	0.64	0.03
h	0.87	0.55	0.44	0.10	0.56	0.92	0.52	0.00
W	0.83	0.52	0.68	0.03	0.65	0.91	0.69	0.13
R	0.03	0.02	0.66	0.16	0.68	0.98	0.59	-0.04
Benchmark Model								
y	1.46	1.00	0.70	0.08	0.70	1.00	0.70	0.08
c	0.55	0.38	0.71	-0.01	0.65	0.99	0.73	0.18
i	6.42	4.41	0.69	0.12	0.71	0.99	0.67	0.03
h	0.62	0.42	0.69	0.13	0.71	1.00	0.66	0.01
W	0.84	0.58	0.70	0.04	0.68	1.00	0.71	0.12
R	0.03	0.02	0.69	0.16	0.72	0.99	0.64	-0.03
US Data (1965-2000)								
y	1.61 (0.13)	1.00	0.86 (0.08)	0.23 (0.08)	0.87 (0.08)	1.00 –	0.87 (0.08)	0.23 (0.08)
c	0.99 (0.08)	0.63	0.88 (0.08)	0.44 (0.08)	0.88 (0.08)	0.87 (0.08)	0.72 (0.08)	0.08 (0.08)
i	6.04 (0.50)	3.82	0.85 (0.08)	0.32 (0.08)	0.82 (0.08)	0.92 (0.08)	0.77 (0.08)	0.04 (0.08)
h	1.30 (0.11)	0.82	0.94 (0.08)	-0.08 (0.08)	0.62 (0.08)	0.82 (0.08)	0.90 (0.08)	0.63 (0.08)
W	0.69 (0.06)	0.44	0.91 (0.08)	0.36 (0.08)	0.66 (0.08)	0.66 (0.08)	0.56 (0.08)	0.09 (0.08)
R	1.05 (0.09)	0.66	0.92 (0.08)	0.57 (0.08)	0.32 (0.08)	0.14 (0.08)	-0.09 (0.08)	-0.63 (0.08)

Note : Standard errors are in parentheses.

and solution methods employed in the two papers.

4.6 Welfare Consequences of Data Revisions

In the previous section, we showed that the decision rules of the agents and business cycle dynamics implied by the model change, in some cases significantly, with the introduction of revisions. However, we still do not have a measure of how important these changes are. In this section we turn to this question and compute the welfare consequences of revisions.

4.6.1 Measuring Welfare

The value function defined in (15) measures the lifetime expected utility of the agent. It is, therefore, reasonable to use it to compare the welfare of two agents in two different economies, which differ by the properties of revisions. However, this value function conditions on the filtered signal, $\hat{A}_t$, instead of true shock, A_t^f. As we show in Section 3.3 for the static model, using the value function of the agent may be problematic since it might give a higher expected utility than the benchmark (no uncertainty) model in some *observed* states of the world. This may happen in some bad states of the world, where the agent considers the small possibility that the true state of the world is actually good. Due to the concavity of the utility function, this optimism may increase the expected utility of the agent over and above the utility that an agent, who knows with certainty that the state of the world is bad, enjoys.

In this paper we use a welfare measure that depends only on the true states of the world. We find this measure more appealing since only what the agents *can* do, as opposed to what they *dream they could* do, affect welfare. We define a new value function where we use the true productivity shock as the conditioning information, similar to (8), as

$$\hat{V}\left(k_0, A_0^f\right) = E_t \sum_{t=0}^{\infty} \beta^t (1-\beta) u\left[c\left(k_t, \hat{A}_t\right), 1 - h\left(k_t, \hat{A}_t\right)\right]$$

which solves the Bellman's equation

$$\begin{aligned} \hat{V}\left(k_t, A_t^f\right) &= (1-\beta) E\left(u\left[c\left(k_t, \hat{A}_t\right), 1 - h\left(k_t, \hat{A}_t\right)\right] |A_t^f\right) \\ &+ \beta E\left\{\hat{V}\left[k\left(k_t, \hat{A}_t, A_t^f\right), A_{t+1}^f\right] |A_t^f\right\} \end{aligned} \tag{26}$$

where the expectations are taken with respect to $\hat{A}_t$, conditioning on A_t^f.[27] The first term in (26) evaluates the instantaneous utility at all possible realizations of the announcement, $\hat{A}_t$, given A_t^f, and the second term gives the expected continuation value where all possible values of both $\hat{A}_t$ (for getting k_{t+1}) and A_{t+1}^f are considered. We solve for the function $\hat{V}(.)$ by applying the Chebyshev approximation method to the Bellman's equation (26).[28]

Note that when $\sigma_r = 0$ and $\mu = 0$, we have $\hat{V}\left(k_t, A_t^f\right) = V\left(k_t, \hat{A}_t\right)$ since the expectation with respect to $\hat{A}_t$ becomes trivial. In other words, the new value function is identical to the objective function of the agent in the benchmark economy. We can, therefore, interpret $\hat{V}(.)$ in general as evaluating the decisions of the agent in the model with revisions, in the objective function of an agent in the benchmark economy. This insight facilitates the comparison of the welfare in the benchmark economy and the model with revisions. In what follows, we denote the value of the benchmark economy by $V^B(.)$ and the value of the model with revisions by $\hat{V}^R(.)$.

We can state the following proposition, which is similar to Proposition 1 in Section 3.3.

Proposition 2 $\hat{V}^R\left(k_t, A_t^f\right) \leq \hat{V}^B\left(k_t, A_t^f\right)$ *for all* k_t *and* A_t^f, *and when* $\sigma_r > 0$, $\hat{V}^R\left(k_t, A_t^f\right) < \hat{V}^B\left(k_t, A_t^f\right)$.

A sketch of the proof is in Appendix C.3. This proposition is an implication of Blackwell's (1951, 1953) well-known theorem, which states that an information set is more informative than another if and only if all expected utility maximizers prefer to observe the former. In our case, the benchmark economy contains more information than the economy with revisions. Except for the information structure, the two economies are identical. In particular, the agent in the benchmark economy can choose to mimic the agent in the economy with revisions by using his decision rules. This will be trivially feasible for him. Since he doesn't choose to do so, his welfare cannot be worse than the welfare of the agent in the model with revisions. Note that the definition of $\hat{V}(.)$ and its interpretation as the objective function of an agent in the benchmark economy is crucial for this theorem.

Since comparing value functions, whose units are utils, is not very informative, we use the consumption equivalent variation measure. This measure asks the question, "How much

[27] We multiply instantaneous utility with $(1-\beta)$ so that the value function has the same units as the utility function $u(.)$.

[28] As we show in Appendix C.4.2, the problem is reduced to solving a system of linear equations.

more consumption at every period and in all states of the world would one need to give the agent in one economy to make him as well off as the agent in the other economy?" The answer to the question is a fraction of per-period consumption and can be easily interpreted.

Using the utility function in (23) and the convention that the model with revisions is the "bad" economy, the welfare loss, expressed in terms of consumption, of going from the benchmark economy with initial states S_0^B to the model with revisions with initial states S_0^R is given by

$$w\left(S_0^R, S_0^B\right) = \left[\frac{V^B\left(S_0^B\right)}{\hat{V}^R\left(S_o^R\right)}\right]^{\frac{1}{\tau(1-\sigma)}} - 1 \tag{27}$$

In addition to this state-contingent measure, we define an *ex-ante* measure of welfare. Instead of choosing which state the agents will start their life in, we average over all possible states using the ergodic distribution of the states of each economy. This can be interpreted as the ex-ante value of being in a certain economy at time $t = -1$, that is, before the initial state is realized. This allows us to evaluate welfare behind Rawls' veil of ignorance. We formally define ex-ante value of economy i as

$$V^i = \int_{\mathbb{S}^i} V^i\left(S_0^i\right) F\left(S_0^i\right) dS_0^i \tag{28}$$

where $F\left(S_0^i\right)$ is the ergodic distribution of initial states and $\mathbb{S}^i$ is the set of all possible states. Using (28), the ex-ante welfare measure is defined by

$$w^E = \left(\frac{V^B}{\hat{V}^R}\right)^{\frac{1}{\tau(1-\sigma)}} - 1$$

Finally, we also use a maximin or Rawlsian measure of welfare, which considers the worst possible states in each economy when computing the welfare loss. We define this measure as

$$w^R = \left[\frac{\min\limits_{S_o^B \in \mathbb{S}^B} V^B\left(S_0^B\right)}{\min\limits_{S_o^R \in \mathbb{S}^R} \hat{V}^R\left(S_0^R\right)}\right]^{\frac{1}{\tau(1-\sigma)}} - 1$$

Table 4.6.1 - Welfare Loss Due to Data Uncertainty

	Welfare Loss (% of consumption)
$w\left(\bar{k}, \bar{A}^f\right)$	0.47
Ex-Ante	0.47
Rawlsian	0.43

4.6.2 Welfare Results

Welfare Cost of Data Uncertainty

Before turning to the results, we want to explain the source of the welfare losses we document in this section. The agents are affected by the data uncertainty on two levels. First, their intra-temporal margin is affected since their consumption and labor supply decisions are not optimal responses to the *true* productivity shock. Second, their inter-temporal margin is affected. Since the investment decision is not an independent decision and follows from the budget constraint, the agent cannot change his decision even though he observes the true productivity shock before investment takes place. This makes the capital stock that the agent starts the next period with suboptimal compared to the capital stock of an agent who does not face data uncertainty.

Our first task it to assess the welfare consequences of data uncertainty, that is, the welfare loss due to the existence of revisions, using different measures of welfare. This amounts to comparing the model with revisions, with the calibrated values for σ_r and μ, to the benchmark economy. We report the results in Table 4.6.1. The first row reports the welfare loss when we consider both economies starting at the deterministic steady state of capital and the unconditional mean for the technology shock. In this case, the welfare loss is computed to be 0.62% of consumption every period. The other two measures, the ex-ante measure and the Rawlsian measure, give similar answers.

The next obvious step is to ask the question, "What can be done to reduce this welfare loss?". We cannot talk about eliminating this loss completely since, due to the timeliness-accuracy trade-off that they face, the statistical agencies will always make data announcements which are later revised due to the arrival of new information. In other words, we must accept living with some level of data uncertainty.

Table 4.6.2 - Welfare Results

	μ	σ_r	Ex-Ante Welfare Loss (% of consumption)
Benchmark	0.0000	0.0000	0.00
Calibrated Revisions	0.0049	0.0074	0.47
No Predictability	0.0000	0.0064	0.39
No Announcements	0.0000	0.0081	0.50

One possible way of reducing data uncertainty is to take into account the predictability of revisions. In the version of the model described above, the agents treat the revisions in the economy as *iid* and do not try to forecast revisions. This setup was justified by the evidence from the SPF. As argued in Section 4.1.2, eliminating the predictability of data revisions is equivalent to reducing the variance of the *iid* revisions in our model. We compute the properties of the revisions in the no-predictability economy in Section 4.3.2 as $\mu = 0$ and $\sigma_r = 0.0064$. We repeat the welfare loss computations for this parameterization. The results are reported on the third line of Table 4.6.2 using the ex-ante welfare measure, which also contains the parameter values and welfare measures for the benchmark and the calibrated model. The results show that the welfare loss with this parametrization is 0.39% of consumption every period, which is about 0.08% less than the case when revisions are predictable. This means the agents would enjoy a welfare increase equivalent to a 0.08% increase in consumption every period if the predictability of revisions are reduced or, alternatively, if they use a forecasting equation such as the one in Aruoba (2004) to forecast the revisions.

The final exercise we conduct determines the benefit of having announcements by asking the question "What happens when we eliminate the statistical agency?". As we argued in Section 4.1.2, rationality requires $\sigma_r \leq \sigma_\varepsilon$ since the agents can always use A_{t-1}^f to forecast A_t^f. Therefore, shutting down the statistical agency is equivalent to having the statistical agency announce $A_t^o = \rho A_{t-1}^f$, i.e., $\sigma_r = \sigma_\varepsilon$. The welfare loss in this parameterization is 0.50% of consumption every period, which is about 0.03% more than the loss using the calibrated parameters. We interpret this difference as the welfare gain due to having announcements. Obviously this gain is larger if the data produced by the statistical agency is not predictable.

In the next section, we put the results in Table 4.6.2 into perspective by comparing our results with some of the other results in the literature and conducting a cost-benefit analysis

Table 4.6.3 - Welfare Loss Due to Data Uncertainty - Sensitivity Analysis

	Calibrated Value	Distribution
σ_ε	0.0081	Uniform$[0.0070, 0.0095]$
σ_A	0.0301	Uniform$[0.0261, 0.0357]$
σ_r	0.0074	Uniform$[0.0064, 0.0087]$
ρ	0.966	$[0.936, 0.980]$

for data collection. We close this section by investigating the robustness of our results with respect to parameter uncertainty.

Sensitivity Analysis

We now turn to analyzing the sensitivity of our welfare results to changes in parameters. To that end, we use a Monte Carlo framework similar to the one in Canova (1994). It is a common practice in the literature to consider changing the parameter values to analyze the robustness of the results. This is important for two reasons. First, one can be interested in the contribution of a particular variable to the results. Second, and more importantly, since the calibration targets are often based on estimates, they carry some estimation error and this must be reflected in the results. Canova (1994) suggests a framework which makes the latter point more formal. In particular, in his framework, the researcher chooses distributions for the parameters in the model and solves the model many times for different parametrizations. This yields a distribution for the object of interest instead of a single number. By looking at the width of the distribution one can judge the sensitivity of the results.

Canova (1994) argues that the uncertainty in the parameters that drive the exogenous processes in the models are more important in terms of changing the results compared to the deep parameters, the parameters of the utility function and production function.[29] Following this insight, we conduct a Monte Carlo experiment where we fix the values of the deep parameters to those in Table 4.3 but vary the parameters ρ, σ_ε, and σ_r. As ρ is close to one, the results are very sensitive to small changes. Therefore, instead of choosing a distribution for ρ, we vary $\sigma_A = \frac{\sigma_\varepsilon}{\sqrt{1-\rho^2}}$ and σ_ε and let ρ adjust accordingly. For simplicity we choose a uniform distribution for the parameters σ_A, σ_ε, and σ_r based on the 99% confidence intervals

[29] This can also be justified on the grounds that the deep parameters are calibrated using long-run averages which have small standard errors due to the length of the sample.

of the estimated parameters.[30] The ranges for the three standard deviation parameters and the implied range for ρ is reported in Table 4.6.3 along with the parameters used for obtaining the results reported in this paper.

The kernel density of the distribution of the welfare loss measures obtained in this exercise is given in Figure 4.6. The thick vertical lines show the minimum and maximum values of welfare we found in the simulation and the dashed vertical lines show the 10- and 90-percentile of the empirical distribution that underlies this kernel density. We see that the distribution is roughly centered around the number we report in Table 4.6.1, 0.47. Even though the variance of the distribution is quite high, the distribution is bounded away from zero, which means that regardless of the parameter values we choose, there is a non-negligible welfare loss due to data uncertainty.

4.6.3 Discussion of the Results

One of the obvious benchmarks to which we can compare our results is the literature on welfare costs of business cycles. The seminal work of Lucas (1987) considers a simple model with no labor and capital and computes the welfare gains of replacing a variable consumption stream with its mean. This exercise yields a welfare improvement of about 0.05% of consumption, using parameters that roughly correspond to the post-war US economy. There have been many different approaches to computing the welfare cost of business cycles, from using heterogeneous agent models to using the implications from financial markets.[31] The results from these approaches put the welfare cost of business cycles at roughly 0.1% of consumption.

A recent paper by Cho and Cooley (2003) argues that most of these estimates are biased, potentially very seriously so. They use a representative agent model very similar to the benchmark model in this paper and show that for a wide range of parametrizations (which

[30] The confidence interval of a standard deviation is given by

$$\sqrt{\frac{n-1}{\chi_{\frac{\alpha}{2},n-1}}}s < \sigma < \sqrt{\frac{n-1}{\chi_{1-\frac{\alpha}{2},n-1}}}s$$

where s is the estimate of the standard deviation and n is the sample size. Since the distributions of standard errors are Chi-squared, the end-points of the confidence intervals are not symmetric around the estimate.

[31] Lucas (2003) provides an overview of some of these results.

Table 4.6.4 - Summary of Welfare Results

	Welfare	
	Fraction of Consumption	Monetary Value (in billion $)
Cost of Data Uncertainty	0.47%	32.8
Cost of Predictability	0.08%	5.1
Value of Announcements	0.03%	2.3

includes the parameterization in this paper) business cycles can be welfare-improving.[32] This result is very puzzling if not counterintuitive. Economic theory suggests that risk-averse agents prefer lower risk and they prefer a less variable consumption stream, which would be the result of less severe business cycles. This intuition holds true in the Lucas (1987) economy since the agent simply consumes all output without making any choices. The critical difference in Cho and Cooley (2003) and all models with endogenous labor is that the agents have extra margins to react when faced with variable technological progress, in this case labor and investment. The agents may take advantage of high productivity by working and investing more (and vice versa for lower productivity), and this may increase the mean of output and, therefore, the mean of consumption. While a higher variance (more severe business cycles) will lead to a welfare reduction in the Lucas sense, a higher expected consumption will lead to a welfare improvement. We consider the results from Cho and Cooley (2003) to be evidence that comparisons of our welfare results to the welfare cost of business cycles is not appropriate.

Finally, we turn to the idea of Sims (1985) and conduct a cost-benefit analysis of data collection. Table 4.6.4 shows several welfare measures along with their approximate monetary value.[33] The first row in Table 4.6.4 shows the welfare loss due to data uncertainty which is 0.47% of consumption or $32.8 billion in annual terms. Of course, there is nothing we can do about most of this loss since, due to the tension between timeliness and accuracy, we should be willing to accept some revisions. The next row shows the loss due to the predictability of revisions which is 0.08% of consumption, about $5.1 billion. This means if the statistical agency starts to produce announcements which result in unpredictable revisions, or if the

[32] The parameter restriction they obtain for business cycles to be welfare improving is $\sigma < 3.9$. All the other parameters in their calibration are close to the ones in this paper.

[33] We use annual consumption of $7 trillion to compute the monetary values.

agents in the economy start exploiting the predictability of revisions, the gain will be about $5.1 billion more consumption per year. This means that about a quarter of the loss due to data uncertainty can be recovered by making changes in how data is collected by the statistical agency or processed by private agents. The last row of the table suggests that when we compare the present situation to a case where there are no announcements, the value of the statistical agency is 0.03% of consumption or about $2.3 billion.

We can summarize our statements above as follows:

- The loss due to data uncertainty is $33 billion, and it can be reduced by $5 billion if the statistical agency produces unbiased announcements and unpredictable revisions.

- The current state of data collection is worth $2.3 billion.

These numbers correspond to the benefit part of a cost-benefit analysis. In the framework of our model, we are able to quantify the gains from data collection ($2.3 billion) and able to make a policy proposal which will produce an additional gain of $5.1 billion. The cost side of the analysis, however, is not straightforward. The cost counterpart of our measure of the benefit of data collection is obviously the cost of the statistical agencies that produce the data, measured by their budget allocations. However, the first number is the benefit from a hypothetical policy whose cost is virtually impossible to measure. Moreover, we do not have a clear understanding as to why the revisions are predictable. They may be the results of an optimization problem that the statistical agency solves using a particular loss function. Nevertheless, we believe that the current budget numbers can be of some guidance for this comparison as well.

The FY2004 budget allocates $78 million to the Bureau of Economic Analysis, $662 to the Census Bureau, and $512 million to the Bureau of Labor Statistics.[34] Of the 11 major statistical agencies in the US, these three are the biggest ones with a total budget of $1.3 billion. Also, we use the aggregate data from two of these agencies, BEA and BLS, to compute the revisions to the Solow residual, with the Census Bureau providing some of the micro data underlying the macro aggregates. Clearly this amount is an over-estimate of the cost of producing the relevant aggregates that we use for the Solow residual as these agencies produce hundreds of other data products.

[34]Sources: Various government internet sites.

The benefit of data announcements, computed using our model, is about twice the overall cost of the statistical agencies. On the other hand, the benefit of having unpredictable revisions is about four times the budget of the statistical agencies, which means any money spent towards correcting the predictability of revisions we discussed in this paper would be well worth it.

5 Conclusions

Using recent empirical results regarding the statistical properties of macroeconomic data revisions as the motivation, we analyze the effects of revisions in a general equilibrium framework. We build a variant of the neoclassical growth model where the agents have to make their decisions observing only a signal about the true level of current productivity instead of the true level itself. This information structure induces the agents to change their behavior, in important ways along some dimensions. We find that the presence of data uncertainty creates a precautionary motive where the agents respond to signals of the productivity shock in less extreme fashion when making consumption decisions. This motive also induces the agents to save more when faced with a positive shock. We also find that, due to a more volatile capital stock, all variables in the economy are more volatile. This increase in volatility is as high as 25% in the case of labor supply. We also document important changes in the basic business cycle dynamics. In the presence of data uncertainty, the volatility of output increases by about 8%, and investment and hours become more volatile compared to output. Moreover, we find that the model with revisions performs no worse than the benchmark model in dimensions that this class of models are weak in, and comes closer to matching the data in some dimensions, such as volatility of output and labor, and contemporaneous correlation of variables with output.

We also conduct a cost-benefit analysis for data collection, overcoming the difficulties described by Sims (1985) by using our model to measure the benefits of data collection. We find that around 0.47% of consumption is lost because of data uncertainty, which is equivalent to about $33 billion every year. Moreover, a change in the data collection technology, which would lead to unpredictable revisions, would result in a welfare improvement of 0.08% of consumption, around $5 billion. Although we cannot compute the cost of such a policy, since the benefit is four times the cost of most data production in the US, we conclude that any money spent towards reducing the predictability of revisions would be well worth it. We also find that the benefit of having data announcements is about 0.03% of consumption, or roughly $2.3 billion every year. This is about twice the total annual budgets of the major statistical agencies in the United States.

Our results indicate that data revisions are not only large in a statistical sense, they are also important in an economic sense since they have sizable welfare consequences.

Appendix

A Data Sources

In Section 4.3, we use average weekly hours from the Establishment Survey as a measure of labor supply and real output from the Real-Time Data Set as a measure of output. The measure of capital stock includes private capital stock (non-residential land, fixed private capital and inventories), private durables consumption stock and government capital stock. The revisions to investment that is used to compute the revisions to capital stock and revisions to output are derived from the Real-Time Data Set.

In Section 4.5.2, variables are defined as follows:

- Output: Real GDP.
- Consumption: Non-durable consumption plus services consumption.
- Investment: Private fixed investment expenditures plus durable goods consumption.
- Hours: Total nonfarm employment.
- Wages: Average hourly earnings, deflated by GDP deflator.
- Rental Rate: Real return to capital, computed along the lines of Cooley and Prescott (1995).

B Details about the Static Model

B.1 The Optimization Problem

In this section, we provide the details about the optimization problem when $p < 1$ given by (6) and (7). All the results extend to the case when $p = 1$, by imposing this condition on the equations.

To solve this problem we consider two cases. If the agent observes $A^o = d$. Then the first order condition from (6) is

$$p\left(\frac{1}{d+h_H}\right)+(1-p)\left(\frac{1}{-d+h_H}\right)-\frac{1}{1-h_H}=0 \tag{29}$$

where h_H is the labor supply decision when the signal is high. (29) is a quadratic equation in h_H and can be simplified to

$$-2h_H^2-(d-1-2pd)\,h_H+d^2-2pd+d=0$$

and the solutions are given by

$$h_H=\frac{1}{2}dp-\frac{1}{4}d\pm\frac{1}{4}\sqrt{6d-12dp+9d^2-4d^2p+4d^2p^2+1}+\frac{1}{4} \tag{30}$$

The second order condition from (6) is

$$-4h_H-(d-1-2pd)<0$$

and it reduces to

$$\pm\sqrt{6d-12dp+9d^2-4d^2p+4d^2p^2+1}>0$$

using (30). Clearly, the second order condition is satisfied when $\pm$ is replaced by $+$, i.e.

$$h_H=\frac{1}{2}dp-\frac{1}{4}d+\frac{1}{4}\sqrt{6d-12dp+9d^2-4d^2p+4d^2p^2+1}+\frac{1}{4} \tag{31}$$

Similarly, if the agent observes $A^o = -d$, the first order condition of (7) is

$$p\left(\frac{1}{-d+h_L}\right)+(1-p)\left(\frac{1}{d+h_L}\right)-\frac{1}{1-h_L}=0$$

which can be simplified as

$$-2h_L^2+(d+1-2pd)\,h_L+d^2+2pd-d=0$$

and the solution is given by

$$h_L = -\frac{1}{2}dp + \frac{1}{4}d \pm \frac{1}{4}\sqrt{-6d + 12dp + 9d^2 - 4d^2p + 4d^2p^2 + 1} + \frac{1}{4}$$

The second order condition of this problem is

$$-4h_L + (d + 1 - 2pd) < 0$$

which imply

$$\pm\sqrt{-6d + 12dp + 9d^2 - 4d^2p + 4d^2p^2 + 1} > 0$$

and is satisfied when $\pm$ is replaced by $+$, i.e.

$$h_L = -\frac{1}{2}dp + \frac{1}{4}d + \frac{1}{4}\sqrt{-6d + 12dp + 9d^2 - 4d^2p + 4d^2p^2 + 1} + \frac{1}{4} \tag{32}$$

Note that the solutions for h_H and h_L satisfy the condition $h \in (d, 1]$ which guarantees a positive consumption.

B.2 Proofs of Claims and Propositions[35]

B.2.1 Claim 1

We first show analytically that $h^R(d) < h^R(-d)$. Comparing h_L and h_H from (32) and (31), we see that

$$\begin{aligned} h_L - h_H = {} & \frac{1}{4}\sqrt{12dp - 6d + 9d^2 - 4d^2p + 4d^2p^2 + 1} \\ & + \frac{1}{4}\sqrt{6d - 12dp + 9d^2 - 4d^2p + 4d^2p^2 + 1} + \frac{1}{2} - d > 0 \end{aligned} \tag{33}$$

which imply $h_L > h_H$. The inequality in (33) holds unambiguously for all parameter values d and p but this may not be clear due to the last term. In particular, $1/2 - d$ is negative when $d \in (0.5, 1)$. Using the following steps, we can verify analytically that the inequality holds:

[35] All claims and propositions in this section are also verified numerically, using a very fine grid of $d \in (0, 1)$ and $p \in (0.5, 1)$.

- At $d = 0.5$, $h_L - h_H > 0$ holds.
- The terms in square roots are monotonically increasing in d for $d \in (0.5, 1)$.
- As $d \to 1$, $h_L - h_H > 0$ holds.

We verify the second part of the claim numerically.

B.2.2 Claim 2

The first part of the claim follows from the ordering of the decision rules given by

$$h^B(d) < h^R(d) < h^R(-d) < h^B(-d)$$

which shows that the two decisions under uncertainty are less extreme than the decisions under certainty.

When we compute realized consumptions for the two economies we get

$$c^R(d,-d) < c^R(-d,-d) < c^B(-d) < c^B(d) < c^R(d,d) < c^R(-d,d) \tag{34}$$

where $c^R\left(A^f, A^o\right) \equiv A^f + h^R(A^o)$ and $c^B(A^o) \equiv A^o + h^B(A^o)$. We prove this ordering by pairwise:

- $c^R(d,-d) = -d + h^R(d) < -d + h^R(-d) = c^R(-d,-d)$ since $h^R(d) < h^R(-d)$ from Claim 1.
- $c^R(-d,-d) = -d + h^R(-d) < -d + h^B(-d) = c^B(-d)$ since $h^R(-d) < h^B(-d)$ from Claim 1.
- $c^B(-d) = \frac{1-d}{2} < \frac{1+d}{2} = c^B(d)$ which follows directly from (5).
- $c^B(d) = d + h^B(d) < d + h^R(d) = c^R(d,d)$ since $h^B(d) < h^R(d)$ from Claim 1.
- $c^R(d,d) = d + h^R(d) < d + h^R(-d) = c^R(-d,d)$ since $h^R(d) < h^R(-d)$ from Claim 1.

B.2.3 Claim 3

$V(.)$ has two parts, one coming from the expected utility of consumption and the other coming from the disutility of labor. As we have argued in Claim 1 above, the labor supply decisions are ranked such that

$$\log\left[1 - h^B(d)\right] > \log\left[1 - h^R(d)\right] \quad \text{and} \quad \log\left[1 - h^B(-d)\right] < \log\left[1 - h^R(-d)\right]$$

which are in line with the ordering in Claim 3. Given the ordering of consumption levels in (34), when the observed productivity is high, i.e. $A^o = d$, the expected utility is given by $p\log\left[c^R(d,d)\right] + (1-p)\log\left[c^R(d,-d)\right]$ which, for sufficiently low p, is less than $\log\left[c^B(d)\right]$ due to the concavity of the utility function.[36] Similarly, when $A^o = -d$, the expected utility is given by $p\log\left[c^R(-d,-d)\right] + (1-p)\log\left[c^R(-d,d)\right]$, which is greater than $\log\left[c^B(-d)\right]$ for sufficiently low p.

These arguments show that when p is sufficiently low, the ordering in the statement of the claim holds unambiguously. Our numerical results show that the ordering holds for all p and d values.

B.2.4 Proposition 1

When $p = 1$, $\hat{V}^R\left(A^f\right)$ is by definition equal to $V^B\left(A^f\right)$. Suppose $p < 1$. We want to show that $V^B\left(A^f\right) > \hat{V}^R\left(A^f\right)$ for all A^f. In words, this implies that the agent who observes the announcements and conditions his decisions on them must be strictly worse off than the agent who observes the true productivity. We can rewrite the objective function of the agent in the benchmark economy, $V^B\left(A^f\right)$, as

$$\begin{aligned} V^B\left(A^f\right) &= \max_h \Pr\left(A^o = A^f\right)\left\{\log\left[A^f + h\right] + \log\left[1-h\right]\right\} \\ &\quad + \Pr\left(A^o \neq A^f\right)\left\{\log\left[A^f + h\right] + \log\left[1-h\right]\right\} \end{aligned} \tag{35}$$

[36] That is, when the observed shock is low, the remote chance that the true productivity is high makes expected utility higher, compared to knowing the (low) state of the world with certainty.

When we plug in the optimal decision rules we derived above, we have

$$V^B\left(A^f\right) = \Pr\left(A^o = A^f\right)\left\{\log\left[c^B\left(A^f\right)\right] + \log\left[1 - h^B\left(A^f\right)\right]\right\}$$
$$+\Pr\left(A^o \neq A^f\right)\left\{\log\left[c^B\left(A^f\right)\right] + \log\left[1 - h^B\left(A^f\right)\right]\right\}$$

where, as we showed above, $c^B(.)$ and $h^B(.)$ are the unique functions that maximize $V^B\left(A^f\right)$. The agent in the benchmark economy can also observe the announcements and may choose to condition his decisions on them. In other words, he may choose to use the decision rules of the agent in the revision economy, $h^R(.)$ and $c^R(.)$. Since there is a trivial resource constraint, these decisions are feasible for the agent in the benchmark economy. If we plug in $h^R(.)$ and $c^R(.)$ in to the quantity that is maximized in (35), we get

$$\Pr\left(A^o = A^f\right)\left\{\log\left[c^R\left(A^o, A^f\right)\right] + \log\left[1 - h^R\left(A^o\right)\right]\right\}$$
$$+\Pr\left(A^o \neq A^f\right)\left\{\log\left[c^R\left(A^o, A^f\right)\right] + \log\left[1 - h^R\left(A^o\right)\right]\right\}$$

which is by definition equal to $\hat{V}^R\left(A^f\right)$. Since $V^B\left(A^f\right)$ is the unique maximum, it must be the case that $V^B\left(A^f\right) > \hat{V}^R\left(A^f\right)$.

C Details about the Dynamic Model

C.1 Euler Equations

Here, we write (17), (18) and (19) from Section 4.2, being explicit about the arguments of functions and expectations.

$$\begin{aligned} &\int_{\boldsymbol{R}} \left\{u_1\left[\boldsymbol{c}\left(k,\hat{A}\right), 1-\boldsymbol{h}\left(k,\hat{A}\right)\right]\exp(\hat{A}+\eta^1)F_1\left[\boldsymbol{h}\left(k,\hat{A}\right), k\right]\right\} dF\left(r|\hat{A}\right) \\ = \;\; & u_2\left[\boldsymbol{c}\left(k,\hat{A}\right), 1-\boldsymbol{h}\left(k,\hat{A}\right)\right] \end{aligned}$$

$$\int_{\boldsymbol{R}} \left\{ u_1 \left[\boldsymbol{c}\left(k, \hat{A}\right), 1 - \boldsymbol{h}\left(k, \hat{A}\right) \right] \right\} dF\left(r|\hat{A}\right)$$

$$\begin{aligned} = \; & \beta \int_{\boldsymbol{R}} \int_{\boldsymbol{E}} \int_{\boldsymbol{R}} u_1 \left(\boldsymbol{c}\left[\boldsymbol{k}'\left(k, \hat{A}, r\right), \underbrace{\rho\hat{A} + \eta^3}_{A^{o\prime}} \right], 1 - \boldsymbol{h}\left[\boldsymbol{k}'\left(k, \hat{A}, r\right), \rho\hat{A} + \eta^3 \right] \right) \\ & \times \left\{ \exp(\rho\hat{A} + \eta^2) F_2 \left(\boldsymbol{h}\left[\boldsymbol{k}'\left(k, \hat{A}, r\right), \rho\hat{A} + \eta^3 \right], \boldsymbol{k}'\left(k, \hat{A}, r\right) \right) + (1 - \delta) \right\} \\ & \times dF\left(r|\hat{A}\right) dF\left(\varepsilon'\right) dF\left(r'\right) \end{aligned}$$

$$\boldsymbol{c}\left(k, \hat{A}\right) = \exp\left(\hat{A} + \eta^1\right) F\left[\boldsymbol{h}\left(k, \hat{A}\right), k \right] + (1 - \delta) k - \boldsymbol{k}'\left(k, \hat{A}, r\right)$$

The function names are shown in boldface and $\boldsymbol{R}$ and $\boldsymbol{E}$ are the spaces r and ε lie in. $F\left(r|\hat{A}\right)$ is the conditional distribution of r given $\hat{A}$ which is given by

$$r|\hat{A} \sim N\left(\tilde{\mu}, \tilde{\sigma}^2\right)$$

where $\tilde{\mu}$ and $\tilde{\sigma}^2$ follow from the conditional expectation rules for bivariate normal variables. Note that $\hat{A}$ and r follow a bivariate normal distribution given by

$$\begin{pmatrix} r \\ \hat{A} \end{pmatrix} \sim N\left[\begin{pmatrix} \mu \\ \mu_{\hat{A}} \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & -(1 - w)\sigma_r^2 \\ -(1 - w)\sigma_r^2 & \sigma_{\hat{A}}^2 \end{pmatrix} \right]$$

C.2 Calibration

The steady state conditions (20), (21) and (22) can be written as follows using our choices of functional forms

$$\left(\frac{\tau}{1 - \tau}\right)\left(\frac{1 - \bar{h}}{\bar{c}}\right) s\alpha \left(\frac{\bar{h}}{\bar{k}}\right)^{\alpha - 1} = 1 \tag{36}$$

$$\frac{1}{\beta} = s(1 - \alpha)\left(\frac{\bar{h}}{\bar{k}}\right)^{\alpha} + (1 - \delta) \tag{37}$$

$$\bar{c} = s\bar{h}^{\alpha}\bar{k}^{1 - \alpha} - \delta\bar{k} \tag{38}$$

We have five calibration targets to match and five parameters to choose, α, β, δ, τ and s.

Two of these targets can be imposed without reference to the steady state conditions above:

- Steady state real interest rate of 4% imply that $\beta = (1 + 0.4)^{-\frac{1}{4}} = 0.99$
- Steady state annual investment-capital ratio of 0.04 imply that $\delta = 1 - (1 - 0.04)^{\frac{1}{4}} = 0.011$, since the steady state investment capital ratio is δ.

We can write (37) as

$$\frac{1}{\beta} = (1 - \alpha) \left(\frac{y}{k} \right) + (1 - \delta)$$

which can be solved for $\left(\frac{y}{k} \right)$. Our third calibration target sets the annual capital-output ratio to 3.55, which gives $\alpha = 0.700$.

We impose $\bar{k} = 1$ as a normalization and solve (37) for s and find $s = 0.153$. Finally we impose $\bar{h} = 0.33$ following the observation that people work for about a third of their discretionary time. Substituting (38) in to (36) we can solve for $\tau = 0.371$.

In the main text, we did not discuss how we derive the revisions to capital using the perpetual investment method. Here, we give the details of this process. We can write the initial announcement (on date $t + 1$) and the announcement on date $t + 1 + k$ as follows

$$\begin{aligned} k_t^{t+1} &= k_0^{t+1} + \sum_{s=0}^{t} (1 - \delta)^s \, i_{t-s}^{t+1} \\ k_t^{t+1+k} &= k_0^{t+1+k} + \sum_{s=0}^{t} (1 - \delta)^s \, i_{t-s}^{t+1+k} \end{aligned}$$

where i_{t-s}^{t+1} and i_{t-s}^{t+1+k} denote different observations of investment at date $t - s$, δ is the depreciation rate and k_0 is the capital stock at date zero. We can define the k^{th} revision to capital at date t as

$$r_t^k = r_0^k + \sum_{s=0}^{t} (1 - \delta)^s \left[i_{t-s}^{t+1+k} - i_{t-s}^{t+1} \right]$$

and we can calculate $\left[i_{t-s}^{t+1+k} - i_{t-s}^{t+1} \right]$ by using the real-time data set we described in Appendix A. In the empirical application we assumed that $r_0^k = 0$, which capital at date zero is perfectly observed. By using this assumption, we can measure the revisions in capital, without measuring capital.

C.3 Proof of Proposition 2

The proof is very similar to the proof of Proposition 1 in Appendix B.2.4. We will provide only a sketch of the proof here since the full proof will require working with sequences, which in turn introduce some additional notation. The steps of the proof are as follows:

- The value function of the social planner in the benchmark economy can be defined in terms of sequences as

$$V^B\left(k_0, A_0^f\right) = \max_{\{c_t, h_t, k_t\}} E \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-h_t\right) \tag{39}$$

  subject to the resource constraint

$$c_t = y_t + (1-\delta) k_t - k_{t+1} \tag{40}$$

  The expectation operator is with respect to $\left\{A_t^f\right\}$. $V^B\left(k_0, A_0^f\right)$ denotes the maximized value of the value function, which is achieved by the unique optimal sequences $\left\{c_t^B, h_t^B, k_t^B\right\}$ which can be obtained by using the decision rules $c^B(.)$, $h^B(.)$ and $k^B(.)$. These sequences, by definition, obey the resource constraint (40).

- We can rewrite this value function by adding trivial integrals with respect to $\{r_t\}$ since nothing depends on revisions. When we rewrite the problem with revisions, we allow the agent to observe revisions (and therefore A_t^o) and react to them.

- The sequences $\left\{c_t^R, h_t^R, k_t^R\right\}$ are the optimal sequences for the model with revisions which can be obtained by using the decision rules $c^R(.)$, $h^R(.)$ and $k^R(.)$. These sequences obey the resource constraint (40) and therefore they are feasible for the social planner. The value of using these decision rules will be

$$\begin{aligned} & E \sum_{t=0}^{\infty} \beta^t u\left(c_t^R, 1-h_t^R\right) \qquad (41) \\ k_{t+1}^R &= y_t + (1-\delta) k_t^R - c_t^R \end{aligned}$$

  which is by definition equal to $\hat{V}^R\left(k_0, A_0^f\right)$. Note that integration is with respect to A^o, given A^f. As such, the quantity in (41) ia not the expected lifetime utility of the agent in the revision economy since that expectation would condition on the announcement.

- Since the sequences $\{c_t^B, h_t^B, k_t^B\}$ are the unique maximizers of the problem in (39), the sequences $\{c_t^R, h_t^R, k_t^R\}$ cannot give a higher value and we have $V^B\left(k_0, A_0^f\right) > \hat{V}^R\left(k_0, A_0^f\right)$.
- When $\sigma_r = 0$, then we have $\{c_t^B, h_t^B, k_t^B\} = \{c_t^R, h_t^R, k_t^R\}$ and therefore $V^B\left(k_0, A_0^f\right) = \hat{V}^R\left(k_0, A_0^f\right)$.

C.4 Computational Details

C.4.1 Solution Method

The decision rules $c(.)$ and $h(.)$ are solutions to the two functional equations (17) and 18). The value function $\hat{V}(.)$ is the solution to the functional equation given by 26. In other words all unknown objects in this paper will be the solutions to a functional equation, or a system of functional equations. In this section, we summarize how our solution method works for a generic problem.

Let $F[g(x)] = 0$ denote a functional equation where the unknown function is $g(x)$. We approximate the unknown function by

$$\tilde{g}(x;\theta) = \sum_{i=1}^{n} \psi_i(x)\,\theta_i$$

where $\{\psi_i(.)\}$ are basis functions, which are flexible functions whose linear combination can take many different shapes and θ is a set of n weights of these basis functions. We define the residual from this approximation by

$$R(x;\theta) = F[\tilde{g}(x;\theta)]$$

The Weighted Residual Method[37] solve for the weights by setting a weighted average of the residual function equal to zero, in other words, theta is given by

$$\int R(x;\theta)\,\phi_i(x)\,dx = 0, \ \ i = 1, ..., n$$

[37] For more details about this and other methods for solving functional equations, see Judd (1998).

where $\{\phi_i\}$ are a set of weighting functions. Clearly, the infinite dimensional problem of solving for $g(x)$ is reduced to solving n equations in n unknowns.

In this paper, we choose Chebyshev Polynomials as the basis functions $\{\psi_i\}$, and choose $\{\phi_i(x)\}$ to be Dirac delta functions $\phi_i(x) = \delta(x - x_i)$ where the points $\{x_i\}$ are the roots of the n^{th} order Chebyshev Polynomial. These choices are collectively called Orthogonal Collocation. The system of equations that needs to be solved reduce to

$$R(x_i; \theta) = 0, \;\; i, j = 1, ..., n \tag{42}$$

which amounts to picking weights θ such that at the collocation points the approximation is exact. Chebyshev Interpolation Theorem (see, for example, Judd, 1992) states that as n increases, the approximation given by $\tilde{g}(x; \theta)$ becomes arbitrarily close to the true $g(x)$.

For evaluating the integrals in the Euler Equations (17) and (18) and the Bellman's equation (26), we use Gauss-Hermite integration. We choose the collocation points for A^o such that the first two moments of A^f and the mean of r match the implied moments in the approximated economy.

C.4.2 Finding $\hat{V}(.)$ by Chebyshev Approximation

As shown in (42), approximating an unknown function $g(.)$ using Chebyshev approximation amounts to solving n equation in n unknowns. These equations are potentially non-linear equations. While there are some methods to reduce the non-linearity of these equations, for approximating value functions, the equations turn out to be linear.

To see this, consider a generic equation $R^j(k^j, A^j; \theta)$ from the Bellman's equation (26)

$$R^j\left(k^j, A^j; \theta\right) = \hat{V}\left(k^j, A^j\right) - (1 - \beta) Eu\left[c(.), 1 - h(.)\right] - \beta EV\left[k'(.), A'\right] = 0 \tag{43}$$

where $c(.)$, $h(.)$ and $k'(.)$ are appropriate decision rules and E operator integrates out all stochastic variables conditional on A^f. We approximate the value function as

$$\hat{V}\left(k^j, A^j\right) = \sum_{i=1}^{mn} \theta_i \psi_i\left(k^j, A^j\right)$$

where $\{\theta_i\}$ are the unknown set of weights and $\psi_i(.)$ are the basis functions. We can rewrite

(43) as

$$R^j\left(k^j, A^j; \theta\right) = \sum_{i=1}^{n} \theta_i \psi_i \left(k^j, A^j\right) - (1-\beta) Eu\left[c(.), 1-h(.)\right] - \beta E \sum_{i=1}^{mn} \theta_i \psi_i \left[k'(.), A'\right] = 0$$

$$\Longrightarrow \sum_{i=1}^{n} \theta_i \left\{\psi_i \left(k^j, A^j\right) - \beta E \psi_i \left[k'(.), A'\right]\right\} = (1-\beta) Eu\left[c(.), 1-h(.)\right] \quad (44)$$

where (44) shows that the equation is linear in $\{\theta_i\}$. Combining all equations we get a linear system of equations

$$A\theta = b$$

where

$$A = \begin{bmatrix} \vdots \\ \psi_i\left(k^j, A^j\right) - \beta E \psi_i \left[k'(.), A'^j\right] \\ \vdots \end{bmatrix} \text{ and } b = \begin{bmatrix} \vdots \\ (1-\beta) Eu\left[c(.), 1-h(.)\right] \\ \vdots \end{bmatrix}$$

Note that this result is valid for any Bellman's equation and is not specific for $\hat{V}(.)$.

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Figure 2.4.1 Timing of Events

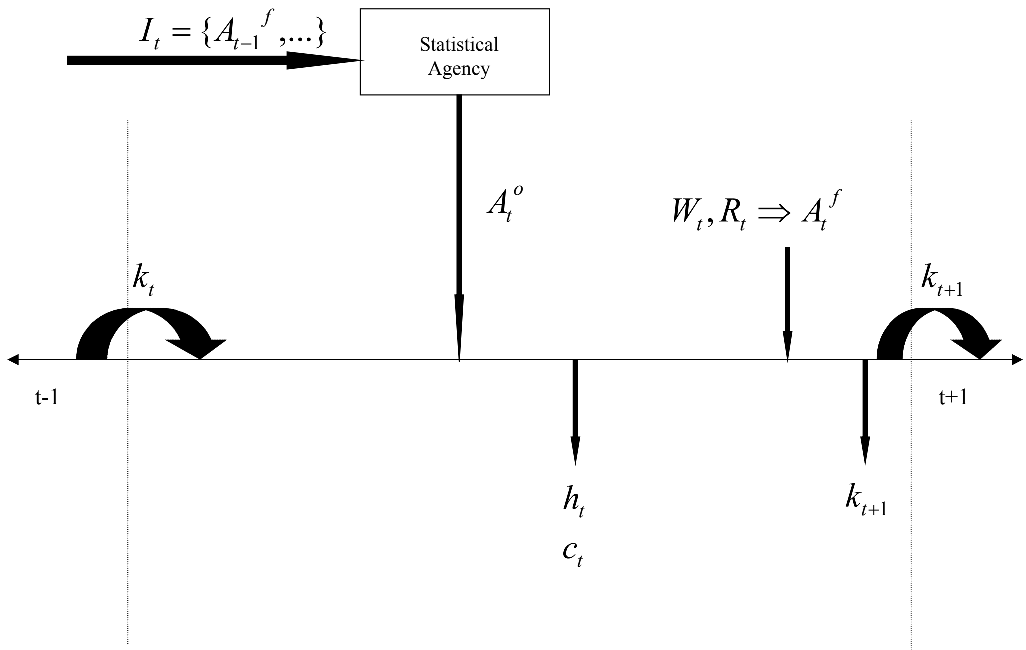


Figure 2.4.3.1 – The Solow Residual and Its Revision

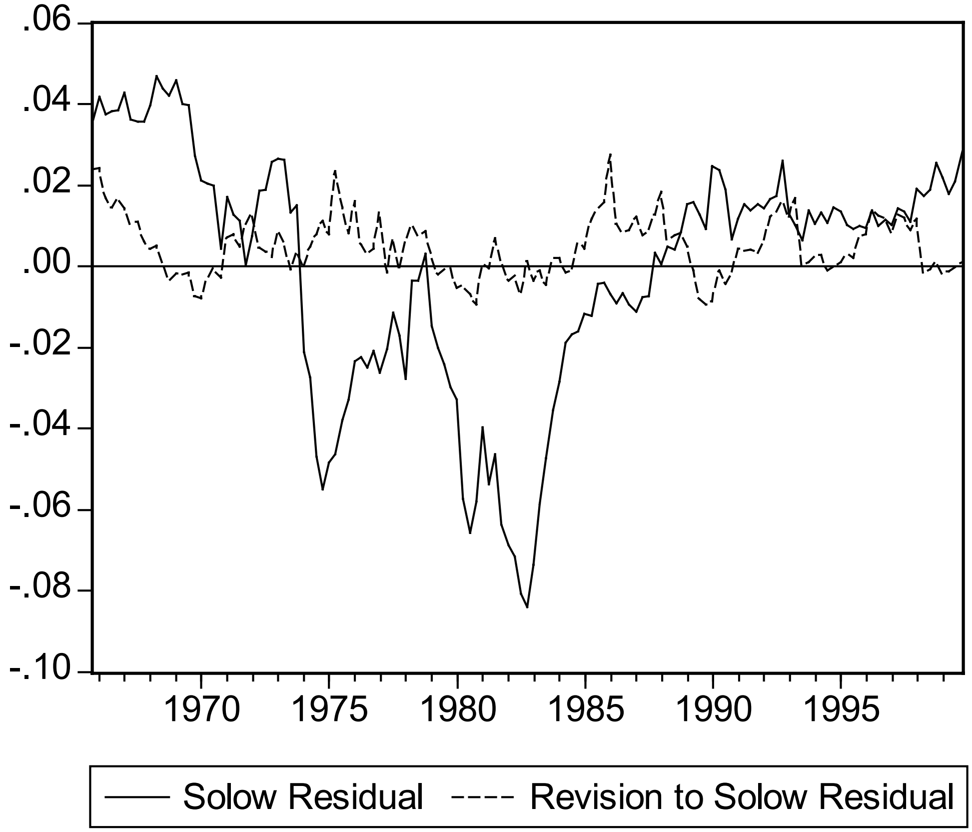


Figure 2.4.3.2 – Revision to Solow Residual and Its Unpredictable Component

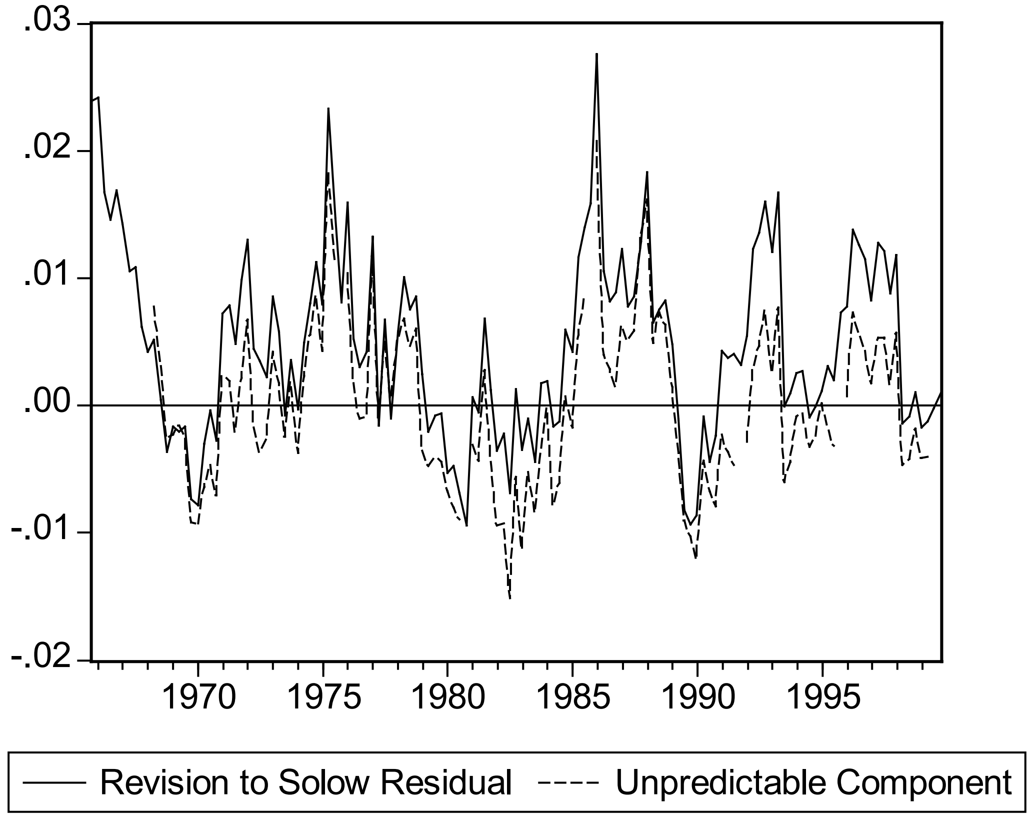
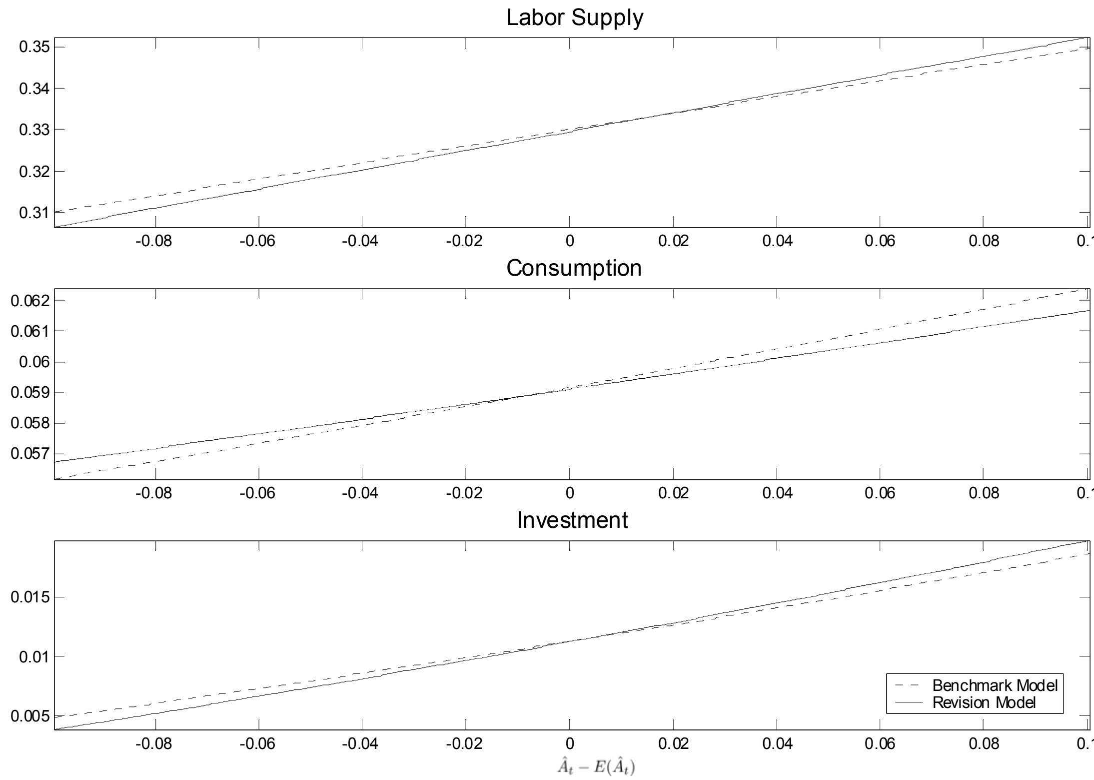


Figure 2.4.5.1 – Decision Rules for the Benchmark Model and the Model with Revisions



Note: This figure plots a cross-section of the decision rules at $k = 1$.

Figure 2.4.5.2 – Consumption and Labor Supply at $k = 1$ and $\hat{A}_t - E(\hat{A}_t) = 0.05$

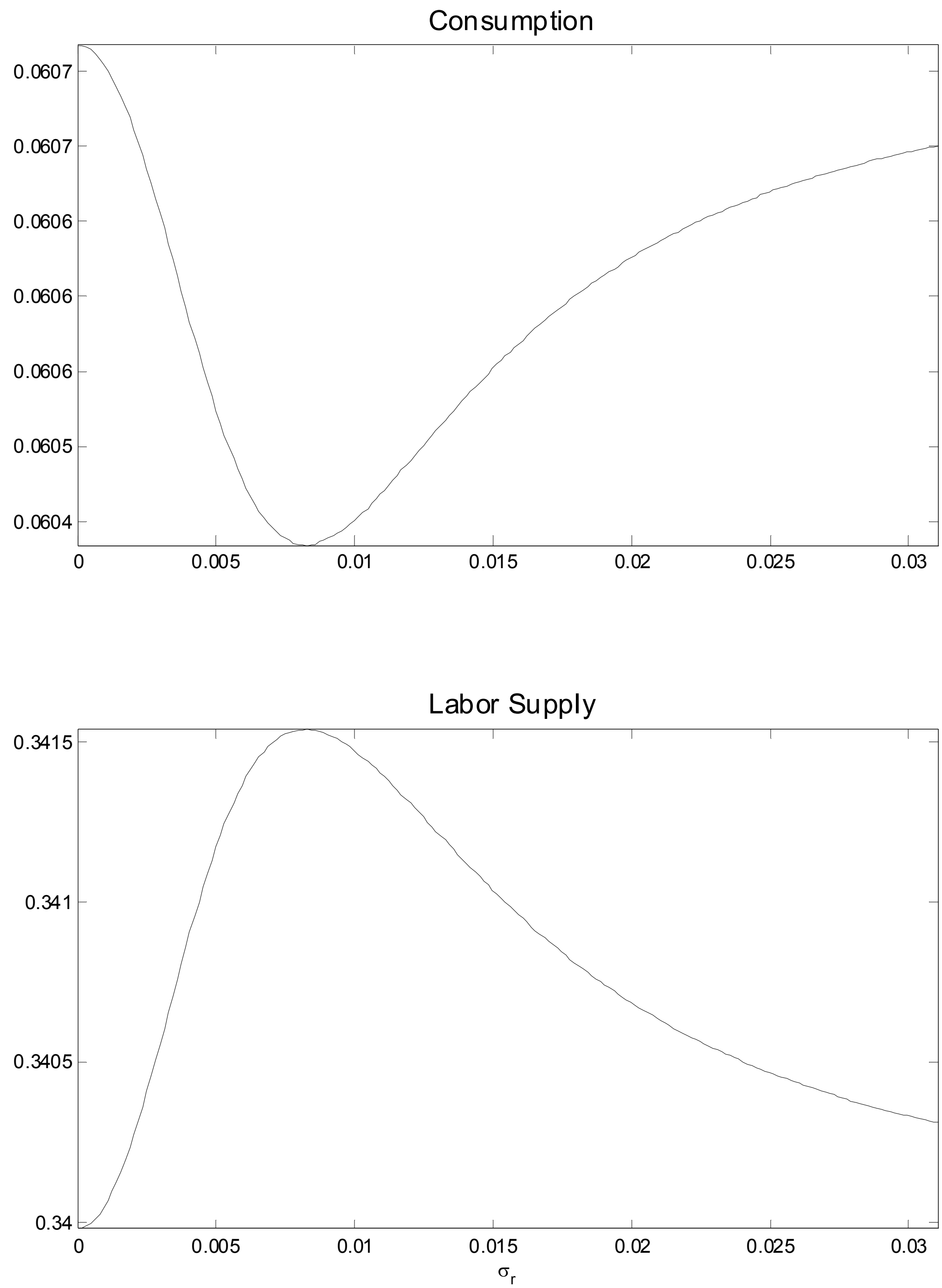


Figure 2.4.5.3 Conditional Distribution of A_t^f and A^f_{t+1} given $\hat{A}_t - E(\hat{A}_t) = 0.05$

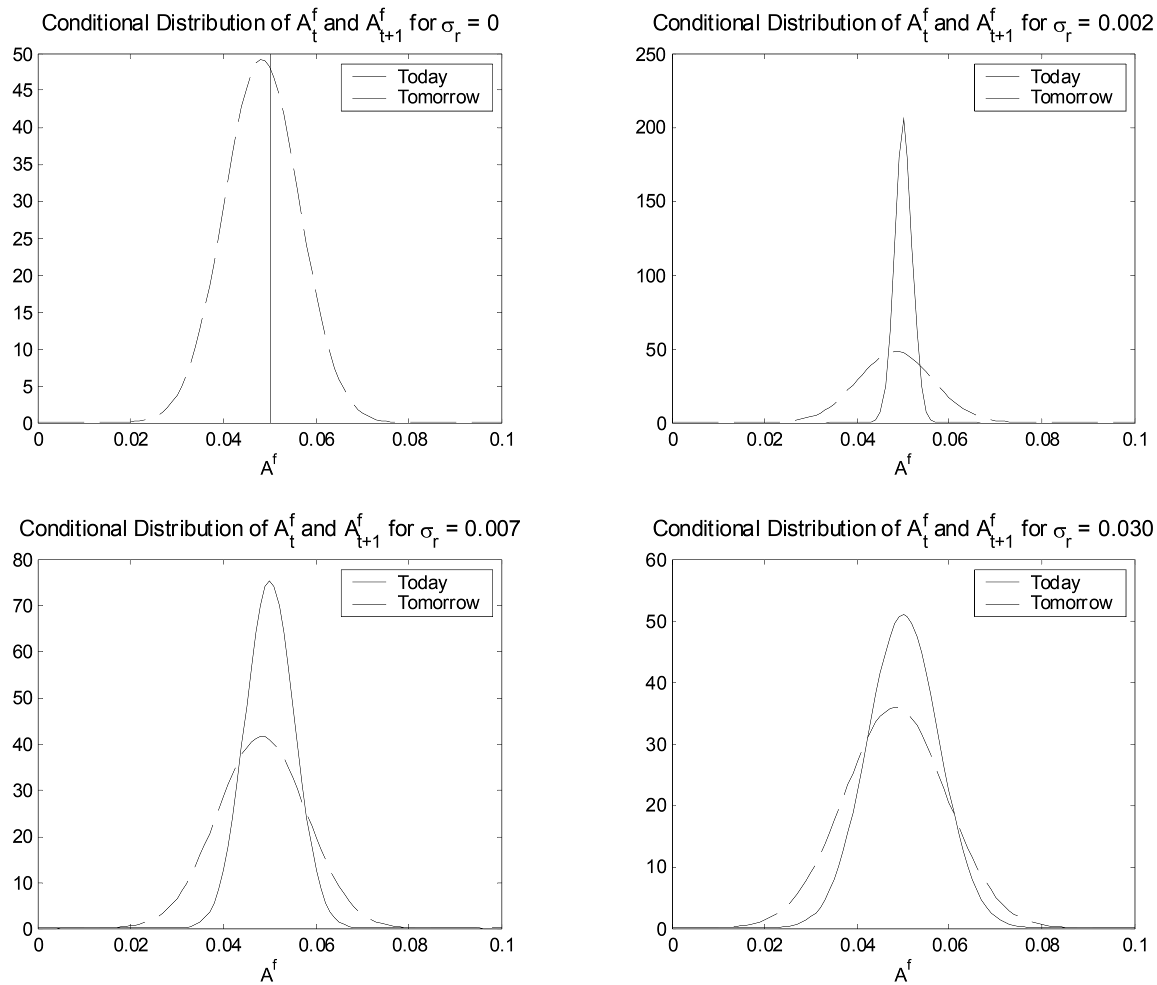
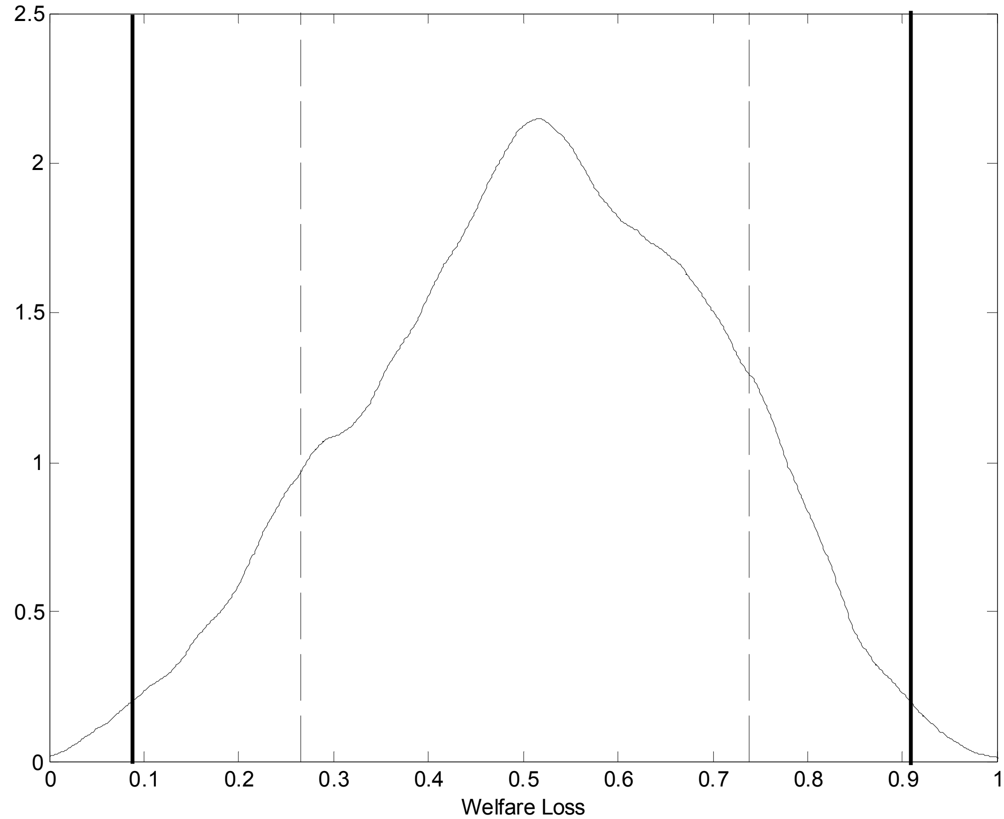


Figure 2.4.6 – Distribution of Welfare Loss Measures



Notes: The two thick vertical lines show the minimum and maximum point of the empirical distribution from the simulation. The dashed lines show the 10- and 90-percentile of the empirical distribution.